

## LIKELIHOOD AND STATISTICAL EVIDENCE IN SURVEY SAMPLING

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### ABSTRACT

A comparison is made of the two concepts, generalized likelihood (from Bjørnstad, 1996) and Royall's measure of empirical statistical evidence in prediction problems, here called evidential likelihood, applied to survey sampling when a population model is assumed. As shown by Bjørnstad (1996), the likelihood principle based on the generalized likelihood is implied by conditionality and sufficiency principles, generalizing the fundamental result from Birnbaum (1962). The main difference between the two likelihood concepts is that the generalized likelihood is a basis for statistical inference containing all available statistical information, while the evidential likelihood concentrates on the empirical evidence per se which does not contain all available statistical information about the variable to be predicted. One can regard the evidential likelihood as the sample evidence of the generalized likelihood, changing the prior distribution of the population total to the generalized likelihood after the data have been obtained.

*Key words:* likelihood, prediction, survey sampling, population model.

### 1. Introduction

The purpose of this paper is to present some ideas on foundations of survey sampling. It deals with likelihood concepts for general statistical inference in prediction problems, applied to survey sampling. The aim here is at comparing and clarifying properties of two likelihood concepts that have been suggested.

We consider the problem of estimating population totals based on a sample from the population, assuming a population model. This problem can be regarded as a prediction problem under a population model; the unobserved part of a population total is a realized value of a random variable. We therefore start by considering the two likelihood concepts for prediction problems in sections 2 and

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3. Section 4 deals with these concepts for survey sampling under a simple population model.

## 2. Generalized likelihood in prediction problems

Let  $Y = y$  denote the data. The aim is to predict the value  $z$  of some unobservable (like a population total in model based sampling) or future random variable  $Z$ . The joint model of  $(Y, Z)$  is given as  $P = \{f_\theta(y, z), \theta \in \Theta\}$ . For simplicity let us only consider the discrete case, such that  $f_\theta(y, z) = P_\theta(Y = y, Z = z)$ . The continuous case is completely similar. In general  $f_\theta(\cdot)(f_\theta(\cdot|\cdot))$  denotes the probability distribution (conditional probability distribution) of the enclosed variables. The usual parametric likelihood is defined as the probability of observing what we have observed in a random experiment, considered as a function of the unknown parameters in the probability model. Generalizing this concept to prediction problems, we might loosely say that the likelihood should be the probability of observing what we have observed in a random experiment jointly or conditional on a given value  $z$  of the predictand, considered as a function of the unknown quantities in the experiment. Several such suggestions have been made in the literature. A discussion is given in Section 4.1 in Bjørnstad (1996). To define the generalized likelihood function for prediction, the guide is the sufficiency principle and conditionality principle in the sense that we require the likelihood to be such that the corresponding likelihood principle is implied by (and implies) the sufficiency principle and conditionality principle, generalizing Birnbaum's fundamental result. In Bjørnstad (1996), it is shown that the generalized likelihood must be a function of the model parameter  $\theta$  and predictand  $z$  and is given by  $L(z, \theta) = f_\theta(y, z)$ . We observe that we can express  $L$  in the following way:

$$L(z, \theta) = f_\theta(z) f_\theta(y|z).$$

Here,  $f_\theta(z)$  is the prior information about  $z$ , i.e., what we know before observing  $Y$ , while  $f_\theta(y|z)$  measures what we learn from the data about  $z$ .

The likelihood principle states that  $L(z, \theta)$  contains all information, given the data and the model, about the problem at hand and hence should serve as the basis for the statistical analysis. The term *likelihood* is used for  $L(z, \theta)$  because this property and the equivalence with the sufficiency principle and the conditionality principle are regarded as the most important features of the *parametric* likelihood in parametric inference. Berger and Wolpert (1988) use the same terminology. Bedrick & Hill (1999) prefer to use the term *summary function* for  $L(z, \theta)$ . They show the utility of  $L$  as a summarization procedure in prediction problems, giving many examples of  $L$  as a data analytic tool. As noted

by Bedrick & Hill (1999),  $L$  is neither a likelihood in the usual sense nor a probability distribution being a product of an indexed predictive distribution ( $f_\theta(z|y)$ ) and a likelihood function for the index ( $l(\theta) = f_\theta(y)$ ).

Since the inference problem of interest is predicting the value of  $Z$ , the model parameter  $\theta$  plays the role of a nuisance parameter. Likelihood based prediction analysis can be done by eliminating  $\theta$  from the joint likelihood  $L$  resulting in a partial likelihood for  $z$  usually called a *predictive likelihood* (for a review see Bjørnstad (1990) or Bjørnstad (1998)). Two examples are:

- I) The profile predictive likelihood:  $L_p(z) = \max_{\theta} L(z, \theta)$ .
- II) The conditional predictive likelihood:  $L_c(z) = f_\theta(y, z) / f_\theta(r(y, z))$ , where  $R = r(Y, Z)$  is a minimal sufficient statistic of  $\theta$  based on  $(Y, Z)$ .

It is important to note that the generalized likelihood principle says that  $L(z, \theta)$  contains all information about  $z$  and  $\theta$ . It does not say *how* to use  $L$  to do inference about  $z$ . A chosen predictive likelihood can then be regarded as *one* likelihood based method.

### 3. Evidential likelihood in prediction problems

According to Royall (2003), generalizing the concept for parametric inference in Royall (1997), the strength of empirical evidence *per se* in the observation  $y$  supporting one value  $z_1$  of  $Z$  versus another value  $z_2$  is measured by the factor by which their probability ratio is changed by the observation  $y$ . Before  $Y$  is observed, the ratio of the probability that  $Z$  equals the value  $z_1$  to the probability that it equals  $z_2$  is

$$\frac{P_\theta(Z = z_1)}{P_\theta(Z = z_2)}$$

After the observation  $y$ , the ratio of the probability that  $Z$  equals the value  $z_1$  to the probability that it equals  $z_2$  is changed to

$$\frac{P_\theta(Z = z_1 | y)}{P_\theta(Z = z_2 | y)} = \frac{P_\theta(Y = y | z_1)}{P_\theta(Y = y | z_2)} \cdot \frac{P_\theta(Z = z_1)}{P_\theta(Z = z_2)}$$

Hence, the factor by which this probability ratio is changed is

$$\frac{P_\theta(Y = y | z_1)}{P_\theta(Y = y | z_2)} = \frac{f_\theta(y | z_1)}{f_\theta(y | z_2)}$$

We shall prefer to call  $f_\theta(y|z)$  the *evidential likelihood*  $L_{ev}$  of  $z$ . It may, however, depend on  $\theta$ . Therefore we use the notation  $L_{ev}(z|\theta) = f_\theta(y|z)$  in

general and the notation  $L_{ev}(z)$  when it is independent of  $\theta$ . The strength of evidence in the data is measured by the evidential likelihood ratio

$$L_{ev}(z_1|\theta) / L_{ev}(z_2|\theta).$$

Note the fundamental difference between generalized likelihood and evidential likelihood: As noted in Section 2 for the generalized likelihood, the main feature is to use the likelihood as a basis for statistical analysis, in contrast to evidential likelihood which is a measure of evidence in the sample per se. We shall later illustrate that these two concepts are not the same.

We have the following two results regarding the evidential likelihood.

**Result 1.** *If  $Y$  and  $Z$  are stochastically independent given  $\theta$ , then for any observation  $y$  the evidence in the sample is identically the same for all possible values of  $z$ ,  $L_{ev}(z_1|\theta) = L_{ev}(z_2|\theta)$  for all  $z_1, z_2$ .*

Result 1 says that there is no direct evidence about  $z$  in the data according to  $L_{ev}$ .

**Result 2.** *Assume  $Z$  is (minimal) sufficient for  $(Y, Z)$ , i.e., the conditional distribution of  $Y$  given  $Z$  is independent of  $\theta$ . Then the evidential likelihood for  $z$  equals the conditional predictive likelihood  $L_c$ .*

Here, the evidential likelihood coincides with one particular tool for prediction. Another choice is the profile predictive likelihood or some modified version of it.

Comparing the generalized likelihood and the evidential likelihood, we note that the effect of the statistical evidence measure  $L_{ev}$  is to change  $f_\theta(z)$  to  $L(z, \theta) = L_{ev}(z|\theta)f_\theta(z)$ . Hence,  $L_{ev}$  can be regarded as the part of the generalized likelihood that changes the prior predictive distribution of  $z$  to the generalized likelihood, after the data are obtained.  $L$  uses  $L_{ev}$  to obtain a proper basis for prediction analysis. In prediction, the measure of statistical evidence is not enough to form a proper likelihood basis for statistical analysis. This is contrary to parametric inference, where the evidential likelihood is  $L_{ev}(\theta) = f_\theta(y)$ , the usual likelihood function. So here the evidential likelihood is also the (likelihood) basis for making statistical inference. Since the likelihood principle states that the generalized likelihood  $L$  should serve as a basis for statistical analysis, while  $L_{ev}$  is a measure of empirical evidence, in prediction problems we need to make a distinction between measuring empirical evidence and forming a basis for predictive inference. That is, the measure of empirical evidence *in itself* will not be a proper basis for prediction. The cases where it seems to work well *as a method for predictive inference* is when it coincides with the conditional predictive likelihood. When the predictand and data are independent it does not work. Also, when the evidential likelihood depends on the model parameter it is inappropriate since the corresponding LP is not implied

by the conditionality and sufficiency principles with the evidential likelihood as likelihood.

It is illustrative to compare  $L$  and  $L_{ev}$  when there has been no experiment (or the experiment was empty). In that case the generalised likelihood becomes proportional to the distribution of the future random variable while the evidential likelihood is constant corresponding to no information gained.

We may conclude:  $L_{ev}$  is the sample evidence of the generalized likelihood for prediction, but, as the example below will show, does not contain all available information about  $z$ .  $L$  and  $L_{ev}$  constitute two different concepts aiming at two different types of information.

#### 4. Survey sampling under a simple population model

Consider a finite population of size  $N$  with  $y$  as variable of interest. The  $N$   $y$ -values in the population are assumed to be values of independent identically distributed random variables, each with distribution  $p_\theta(y) = P_\theta(Y_i = y)$ . Let  $t$  be the total of  $y$  in the population,  $t = \sum_{i=1}^N y_i$ . The problem of interest is to estimate  $t$  based on a sample  $s$  with  $y$ -values  $y_s = (y_i : i \in s)$ . We assume that the sampling design is uninformative (selection probabilities are independent of the population vector of  $y$ -values). This means we can, from the point of view of the likelihood principle, consider the statistical analysis conditional on the sample  $s$  of selected units.

Let  $t_s$  be the sample total and  $t_{\bar{s}}$  the total outside the sample  $s$ , such that  $t = t_s + t_{\bar{s}}$ . Under a population model, estimating  $t$  can be regarded as a prediction problem since  $t$  is an unobservable value of a random variable. For simplicity, we restrict attention to models where  $T_s$  is sufficient for  $\theta$ , such that  $f_\theta(y_s) = f_\theta(t_s)f(y_s|t_s)$ . In addition, we assume that  $T$  is minimal sufficient for  $(Y_s, T)$ . Then  $f(t_s|t) = P(T_s = t_s|T = t)$  is independent of  $\theta$ .

Given the data, the problems of predicting  $t$  and  $t_{\bar{s}}$  are equivalent since  $t_s$  is observed. A prediction tool should give the same result whether we consider the inference problem in terms of  $z = t$  or  $z = t_{\bar{s}}$ . For the two equivalent versions we get the following generalized likelihood:

$$1) z = t : L_1(t, \theta) = f_\theta(y_s, t) \propto f_\theta(t_s)f_\theta(t_{\bar{s}} = t - t_s).$$

$$\text{Hence we can let the likelihood be } L_1(t, \theta) = P_\theta(T_s = t_s)P_\theta(T_{\bar{s}} = t - t_s).$$

$$2) z = t_{\bar{s}} : L_2(t_{\bar{s}}, \theta) = f_\theta(y_s, t_{\bar{s}}) \propto f_\theta(t_s)f_\theta(t_{\bar{s}}).$$

$$\text{Hence we can let } L_2(t_{\bar{s}}, \theta) = P_\theta(T_s = t_s)P_\theta(T_{\bar{s}} = t_{\bar{s}}).$$

We see that the generalized likelihood is invariant under these alternative versions in the sense that  $L_1(t, \theta) = L_2(t - t_s, \theta)$ . Hence, whether we consider prediction of  $t$  or  $t_{\bar{s}}$ , the generalized likelihood is the same in either case. This is not the case for the evidential likelihood:

$$(i) z = t: L_{ev}(t|\theta) = f_{\theta}(y_s|t) \propto \frac{P_{\theta}(T_s = t_s)P_{\theta}(T_{\bar{s}} = t - t_s)}{P_{\theta}(T = t)}.$$

That is,  $L_{ev}(t) = f(t_s|t) = L_c(t)$ .

$$(ii) z = t_{\bar{s}}: L_{ev}(t_{\bar{s}}|\theta) = f_{\theta}(y_s|t_{\bar{s}}) = P_{\theta}(Y_s = y_s), \text{ independent of } t_{\bar{s}}.$$

One way to interpret case (i) is that in the absence of prior information one should assume exchangeability and infer that the remaining units have the same mean as in the sample. The second one seems worthless for prediction, saying that no sample total carries evidence about  $t_{\bar{s}}$ . The sample data carries only information about  $\theta$ , not about  $t_{\bar{s}}$  in the sample per se. Hence, the evidential likelihood gives seemingly inconsistent evidence for  $t$  and  $t_{\bar{s}}$ .

We note that in the second case, since the data does not contain any information on the remaining units, one should intuitively use the prior information  $f_{\theta}(t_{\bar{s}})$  without change for prediction. This is exactly what is expressed by the generalised likelihood, on the form  $L(t_{\bar{s}}, \theta) = L_{ev}(t_{\bar{s}}|\theta)f_{\theta}(t_{\bar{s}})$ .

We shall now illustrate why the evidential likelihood does not contain all available information about  $t$  and therefore cannot be used as a basis for statistical analysis. Assume that  $y$  is binary taking values 0 or 1 with  $\theta = P(Y = 1) = 1/2$ . We can regard the population of  $y$ -values as being generated by tossing fair coin  $N$  times, and  $t$  is the number of heads. Then a simple random sample of  $n$  tosses is chosen with  $t_s$  being the observed number of heads. Since the model parameter is known the generalized likelihood becomes a likelihood for  $t$  alone:

$$L(t) = P_{1/2}(T_s = t_s)P_{1/2}(T_{\bar{s}} = t - t_s) \propto P_{1/2}(T_{\bar{s}} = t - t_s).$$

Hence, we use the version

$$L(t) = P_{1/2}(T_{\bar{s}} = t - t_s) = \binom{N - n}{t - t_s} \left(\frac{1}{2}\right)^{N - n}.$$

The data contains no information on  $t_{\bar{s}}$ , and the inference should not depend on the distribution of  $Y$ . This is exactly what is achieved by the generalized likelihood. It is proportional to the known binomial distribution ( $N - n$ ,  $\theta = 1/2$ ) of  $T_{\bar{s}}$  and does not use the information in the data when predicting  $t_{\bar{s}}$ . However, when predicting  $t$ ,  $L$  clearly uses the information in the data.

Next, consider the evidential likelihood in (i):

$$L_{ev}(t) = f(t_s|t) = \binom{n}{t_s} \binom{N-n}{t-t_s} / \binom{N}{t} = \binom{t}{t_s} \binom{N-t}{n-t_s} / \binom{N}{n},$$

a hypergeometric probability of  $t_s$  successes in a sample of  $n$  units from a population of  $N$  units with  $t$  successes. We note that the evidential likelihood is the same when  $\theta$  is unknown.

We see that  $L(t) = f(t|t_s)$ , the predictive distribution, while  $L_{ev}(t) = f(t_s|t)$ .

Consider a population of  $N = 100$  and  $n = 50$ , and suppose we get 20% successes in the sample,  $t_s = 10$ . (A very unlikely outcome, having probability .000009). The evidential likelihood is then

$$L_{ev}(t) = \binom{t}{10} \binom{100-t}{40} / \binom{100}{50} = P(X = 10|t)$$

where  $X$  has a hypergeometric distribution with  $t$  successes in the population, giving the following evidential distribution:

$t$	10	13	18	19	20	21	22	25	30	35
$L_{ev}(t)$	.0006	.028	.180	.194	.197	.188	.170	.095	.017	.0012

We see that the evidential likelihood disregards the knowledge of  $\theta$  and measures only the evidence in the sample per se, not taking into account that we know how  $t_s$  is generated. As a basis for predicting  $t$  or equivalently  $t_s$  it will fail badly. For example, consider the 1/8 and 1/32 likelihood intervals suggested by Royall (1997):

$$LI_{1/8} = \{t: L_{ev}(t) \geq \frac{1}{8} \max L_{ev}\} = \{t: L_{ev}(t) \geq .0246\} = [13,29]$$

$$LI_{1/32} = \{t: L_{ev}(t) \geq \frac{1}{32} \max L_{ev}\} = \{t: L_{ev}(t) \geq .00615\} = [12,32]$$

These intervals are clearly very unreliable. The conditional coverages are given by

$$P(13 \leq T \leq 29 | t_s = 10) = P(3 \leq T_s \leq 19) = .06$$

$$P(12 \leq T \leq 32 | t_s = 10) = P(2 \leq T_s \leq 22) = .24.$$

The generalized likelihood is  $L(t) = P_{1/2}(T_{\bar{s}} = t - 10) = \binom{50}{t-10} \left(\frac{1}{2}\right)^{50}$ . This gives the following likelihood distribution:

$t$	20	26	30	34	35	36	40	44	50
$L(t)$	.000009	.0044	.042	.108	.112	.108	.042	.0044	.000009

We see that the most likely value according to  $L$  is equal to 35, the sum of the observed 10 in the sample and predicted 25 for the remaining units.

The 1/8 and 1/32 likelihood intervals based on  $L(t)$  :

$$LI_{1/8} = \{t: L(t) \geq \frac{1}{8} \max L\} = \{t: L(t) \geq .014\} = [28, 42]$$

$$LI_{1/32} = \{t: L(t) \geq \frac{1}{32} L(35)\} = \{t: L_v(t) \geq .0035\} = [26, 44].$$

The conditional coverages are given by

$$P(28 \leq T \leq 42 | t_s = 10) = P(18 \leq T_{\bar{s}} = 32) = .967$$

$$P(26 \leq T \leq 44 | t_s = 10) = P(16 \leq T_{\bar{s}} \leq 34) = .993.$$

## 5. Conclusions

$L_{ev}$  is not suitable as a tool for prediction in this case. It does not contain *all available* information about  $t$  as required by the likelihood principle. There are two sources of information about  $t = t_s + t_{\bar{s}}$  :

$$\text{a) } P(T_{\bar{s}} = t_{\bar{s}}) = \binom{50}{t_{\bar{s}}} \left(\frac{1}{2}\right)^{50}$$

$$\text{b) } t_s = 10.$$

The evidential likelihood uses only the second source and ignores part a). Consider the two possible values  $t = 20$  and  $t = 50$  that the generalized likelihood considers equally likely. It is true that the *sample itself* presents evidence supporting  $t = 20$  over  $t = 50$ , but the generalized likelihood uses *all available* information, given the data, as a basis for prediction and then clearly  $t = 20$  and  $t = 50$  are equally likely, since  $t = 20, 50$  means that  $t_{\bar{s}} = 10, 40$  and these two values have the same probabilities, also given  $t_s = 10$ .

The sample total of 10 is a case of misleading evidence. Note that the generalized likelihood corrects for this in this admittedly simplistic example.



As measured by  $L_{ev}$ , the strength of empirical evidence is the same whether  $\theta$  is known or not. However, the available statistical information about  $t$  depends on whether  $\theta$  is known or unknown, as measured by the generalized likelihood.

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