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# SIMULTANEOUS ESTIMATION UNDER NESTED ERROR REGRESSION MODEL

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## ABSTRACT

Zhang (2003) proposed a frequentist method of simultaneous small area estimation under hierarchical models. This can be useful when various ensemble characteristics of the small area parameters are of interest in addition to area-specific prediction. In this paper we extend the approach under the nested error regression model (Battese, Harter and Fuller, 1988), which allows for use of auxiliary information at the unit level. Simulations based on monthly wage data suggest that the simultaneous estimator has much better ensemble properties than the empirical best linear unbiased predictor, without losing much of the precision of the latter in area-specific prediction.

***Key words:*** area-specific prediction; ensemble statistics; bootstrap.

## 1. Introduction

In small area estimation problems, the ensemble characteristics of the small area estimators (Judkins and Liu, 2000), i.e. when these are viewed as a collection of statistics, is often of as much interest as each area-specific estimator. Such ensemble characteristics include the variance, the rank ordering, the mini- and maximum, the range, the percentiles, *etc.* of the small area parameters. Estimators that are optimal for prediction of each specific area may have unsatisfactory ensemble properties. For instance, the between-area variation of the estimates can be much smaller than the true variation in the population, which is known as over-shrinkage. Various constrained Bayes approaches have been developed (Louis, 1984; Spjøtvoll and Thomsen, 1987; Lahiri, 1990; Ghosh, 1992). For a two-stage hierarchical model without auxiliary covariates, Shen and Louis (1998) proposed "triple-goal" estimators that produce good ranks, a good distribution and good area-specific estimators. The authors also noted that

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the approach can be generalized under models with a regression intercept and slope.

Zhang (2003) proposed a frequentist method of simultaneous estimation under basically the same two-stage hierarchical model. Suppose we are interested in, say, the mean of a variable from a large number of small areas, denoted by $\theta_i$, for $i = 1,...,m$. At the lower level of the model, we assume a parametric distribution of $\theta_i$; at the upper level, we assume a conditional distribution of the data given $\theta_i$. The simultaneous estimates are derived in two steps. Firstly, since the number of small areas is finite, one of them must have the smallest value among all the $\theta_i$'s, another must have the second smallest value, and so on. Let $\theta_{(i)}$ be the $i$th order statistic of $\{\theta_i\}$, i.e. the set of all the $\theta_i$'s, where $\theta_{(1)} \le \theta_{(2)} \le \cdots \le \theta_{(m)}$. Denote the expectation of $\theta_{(i)}$ by

$$\eta_i = E[\theta_{(i)}; \xi]$$

where $\xi$ is the parameter of the distribution of $\theta_i$. It follows that $\eta_1$ is the best predictor for $\theta_{(1)}$, and $\eta_2$ is the best predictor for $\theta_{(2)}$, and so on, and $\{\eta_i\}$ is the best ensemble predictor of $\{\theta_i\}$. Let $\hat{\xi}$ be an estimator of $\xi$ and $\hat{\eta} = E[\theta_{(i)}; \hat{\xi}]$, then $\{\hat{\eta}_i\}$ is the estimated best ensemble predictor.

Secondly, we match $\{\hat{\eta}_i\}$ with the small areas. Let $\hat{\theta}_i$ be the estimated best area-specific predictor, for $i = 1,...,m$. Instead of using $\hat{\theta}_i$ directly, we obtain the rank of $\hat{\theta}_i$ among all the $\hat{\theta}_i$'s, denoted by $r_i = rank(\hat{\theta}_i)$. These are now used to match the estimated best ensemble predictors, and the simultaneous estimator of area $i$ is given by

$$\dot{\theta}_i = \hat{\eta}_{r_i}.$$

In this way, the simultaneous estimates have the same rank ordering as the area-specific estimates. In the special case of ties among the $\hat{\theta}_i$'s, we assign the corresponding $\hat{\eta}_i$'s randomly.

The simultaneous estimator is not optimal for area-specific prediction. However, they have better ensemble properties due to the use of the best ensemble estimators $\hat{\eta}_i$'s. Empirical results (Zhang, 2003), validated by the true population values, suggest that the simultaneous estimator performs similarly as the Bayesian alternatives, i.e. producing estimates with good ensemble as well as area-specific properties. Exactly how big is the trade-off between the gain in ensemble statistics and the loss in area-specific precision, however, depends on the particular situation and must be evaluated on a case-to-case basis.

In this paper, we extend the approach of Zhang (2003) to regression models. In our derivation we concentrate on the one-fold nested error regression model (Battese, Harter and Fuller, 1988), which allows us to incorporate auxiliary information at the individual level. We also allow for nonparametric specification of the distribution of the random errors, which is another difference from the case of two-stage hierarchical model above. In Section 2, we show that the best linear unbiased predictor (BLUP) entails loss of between-area variation under the nested error regression model. In Section 3 we present the simultaneous estimator. Section 4 contains a simulation study based on the monthly wage data, where the simultaneous estimator is compared to the empirical best linear unbiased predictor (EBLUP) and the direct estimator, both in situations with and without auxiliary covariates. A short summary is given in Section 5.

## 2. Between-area variation under nested error regression model

The nested error regression model to be considered is given as

$$y_{ij} = x_{ij}^T \beta + u_{ij} \quad \text{and} \quad u_{ij} = v_i + e_{ij} \tag{2.1}$$

where $j$ is the subscript for unit $j$ within area $i$, $y_{ij}$ is the variable of interest and $x_{ij}$ is the vector of unit-level covariates, and $\beta$ contains the regression coefficients. The random error $u_{ij}$ is the sum of an area-level effect $v_i$ and a unit-level effect $e_{ij}$. The random errors $v_i$'s and $e_{ij}$'s are assumed to be independent, with zero mean and variance $\sigma_v^2$ and $\sigma_e^2$, respectively. Apart form the first two moments, we do not require full specification of the distribution of the random errors in general.

Suppose we are to estimate the small area means of $y_{ij}$, denoted by $\overline{Y}_i = N_i^{-1} \sum_{j=1}^{N_i} y_{ij}$, where $N_i$ is the size of area $i$. Let $\theta_i = \overline{X}_i^T \beta + v_i$, where $\overline{X}_i$ is the mean of $x_{ij}$ within area $i$, which is the expected area mean conditional on $v_i$. The difference between $\theta_i$ and $\overline{Y}_i$ is the within-area population average of the unit-level random effects. Given the covariates of the population, we have

$$E[\frac{1}{m-1} \sum_{i=1}^{m} (\theta_i - \overline{\theta})^2 \mid \overline{X}_1, \ldots, \overline{X}_m] = \Delta + \sigma_v^2 \tag{2.2}$$

where $\overline{\theta}$ is the average of all the $\theta_i$'s, and $\overline{X}$ is the average of all the $\overline{X}_i$'s, and

$$\Delta = \frac{1}{m-1} \beta^T \sum_i (\overline{X}_i - \overline{X})(\overline{X}_i - \overline{X})^T \beta .$$

From (2.2), it is seen that the variation in $\theta_i$ decomposes into two parts, where the first part is what can be accounted for by the covariates through the regression model, and the second part is what needs to be attributed to random effects. By comparing $\Delta$ with $\sigma_v^2$, we may get an idea of how good the covariates of the model are in a particular situation.

Let $\bar{y}_i = n_i^{-1}\sum_{j=1}^{n_i} y_{ij}$ be the sample mean of $y_{ij}$ within area $i$, where $n_i$ is the within-area sample size, which is an unbiased direct estimator of $\theta_i$ conditional on $v_i$. The BLUP of $\theta_i$ is given by

$$\hat{\theta}_i = \bar{X}_i^T \beta + \gamma_i(\bar{y}_i - \bar{x}_i^T \beta) = \bar{X}_i^T \beta + \gamma_i(v_i + \bar{e}_i)$$

where $\gamma_i = \sigma_v^2/(\sigma_v^2 + \sigma_e^2/n_i)$ when $\beta$, $\sigma_v^2$ and $\sigma_e^2$ are all known. In the special case of $n_1 = \cdots = n_m$, let $\varphi = \sigma_e^2/n_i$ be a constant for all the areas. We have

$$E[\frac{1}{m-1}\sum_i(\hat{\theta}_i - \bar{\hat{\theta}})^2 \mid \bar{X}_1,\ldots,\bar{X}_m] = \Delta + \gamma^2(\sigma_v^2 + \varphi) = \Delta + \gamma\sigma_v^2 < \Delta + \sigma_v^2 .$$

In other words, the BLUPs are under-dispersed compared to the $\theta_i$'s. Notice that, in the absence of auxiliary variables, i.e. $\Delta = 0$, the result reduces to that in Zhang (2003).

## 3. Simultaneous estimator

The empirical best linear unbiased predictor (EBLUP) $\hat{\theta}_i$ can be written as

$$\hat{\theta}_i = \bar{X}_i^T \hat{\beta} + \hat{v}_i \qquad \text{where} \qquad \hat{v}_i = \hat{\gamma}_i(\bar{y}_i - \bar{x}_i^T \hat{\beta}) .$$

Loss of between-area variation is essentially due to over-shrinkage in estimation of $v_i$. This amounts to using too small shrinkage-factor $\hat{\gamma}_i$. We now consider two adjustments, depending on whether the distribution of $v_i$ is fully specified or not.

In the first place, we might assume a fully parametric distribution of $v_i$, denoted by $G(v;\xi)$ with parameters $\xi$. Let $\{v_{(i)}\}$ be the order statistics of $\{v_i\}$, for $i = 1,\ldots,m$. Let

$$\eta_i = E[v_{(i)};\xi]$$

be the expectation of $v_{(i)}$. It follows that $\{\eta_i\}$ is the best ensemble predictor of $\{v_i\}$. Let $\hat{\xi}$ be an estimator of $\xi$ and $\hat{\eta} = E[v_{(i)};\hat{\xi}]$, then $\{\hat{\eta}_i\}$ is the estimated best ensemble predictor. Instead of using $\hat{v}_i$ directly, we obtain the rank $\hat{v}_i$ of among all the $\hat{v}_i$'s, denoted by $r_i = rank(\hat{v}_i)$. The simultaneous estimator of the random effect of area $i$ is then given by

$$\dot{v}_i = \hat{\eta}_{r_i}$$

and the simultaneous estimator of $\theta_i$ is

$$\dot{\theta}_i = \overline{X}_i^T \beta + \dot{v}_i .$$

In the case when all the parameters are known, we have, as $m \to \infty$,

$$E[\frac{1}{m-1}\sum_i (\dot{\theta}_i - \bar{\dot{\theta}})^2 \mid \overline{X}_1,\dots,\overline{X}_m] \to \Delta + \sigma_v^2 = E[\frac{1}{m-1}\sum_i (\theta_i - \bar{\theta})^2 \mid \overline{X}_1,\dots,\overline{X}_m]$$

provided $\frac{1}{m-1}\sum_i (\overline{X}_i - \overline{X})^T \beta \eta_{r_i} = 0$, because $\frac{1}{m-1}\sum_i (\eta_i - \bar{\eta})^2 \to \sigma_v^2$ as $m \to \infty$.

In many situations, however, it may be too difficult or restrictive to fully specify the distribution of $v_i$. Let $\tau_{\hat{v}}^2 = (m-1)^{-1}\sum_i (\hat{v}_i - \bar{\hat{v}})^2$ be the empirical variance of the EBLUP $\hat{v}_i$'s. We observe over-shrinkage of the EBLUPs if

$$\tau_{\hat{v}}^2 < \hat{\sigma}_v^2 .$$

A simple nonparametric simultaneous estimator can be given as

$$\dot{v}_i = \bar{\hat{v}} + (\hat{v}_i - \bar{\hat{v}})\hat{\sigma}_v / \tau_{\hat{v}} \qquad \text{and} \qquad \dot{\theta}_i = \overline{X}_i^T \beta + \dot{v}_i . \tag{3.1}$$

Notice that, in this way, the empirical variance of the $\dot{\theta}_i$'s always equals to $\hat{\sigma}_v^2$.

To evaluate the MSE (or variance) of $\dot{\theta}_i$, we use a bootstrap procedure. Firstly, we fix the parameters of the model at the estimated values. Secondly, we generate a bootstrap sample in area $i$:

1) let $\theta_i^* = \overline{X}_i^T \hat{\beta} + v_i^*$, where $v_i^*$ is drawn randomly and with replacement from $\{\dot{v}_i\}$;
2) let $y_{ij}^* = x_{ij}^T \hat{\beta} + v_i^* + e_{ij}^*$, where $e_{ij}^*$ is drawn randomly and with replacement from $\{\dot{e}_{ij}\}$, and $\dot{e}_{ij} = \hat{e}_{ij}\hat{\sigma}_e / \tau_{\hat{e}}$, and $\hat{e}_{ij} = y_{ij} - x_{ij}^T \hat{\beta} - \hat{v}_i$, and $\tau_{\hat{e}}^2$ is the empirical variance of the $\hat{e}_{ij}$'s.

Finally, based on a bootstrap sample $\{y_{ij}^*\}$, we re-estimate the model (2.1) and derive the simultaneous estimates in the same way as based on the original sample, denoted by $\dot{\theta}_i^*$. A bootstrap replicate of the error in the original simultaneous estimator is given by $\dot{\theta}_i^* - \theta_i^*$. Independent bootstrap replicates can then be used to produce Monte Carlo approximation to the bootstrap MSE (or variance).

## 4. Simulations

### 4.1. Data

The Norwegian Wage Survey (NWS) is based on a yearly sample of clusters of wage earners. The clusters correspond to establishments enlisted in the Establishment Register, stratified according to the size of the establishment. The NWS includes all the employees from each selected establishment. The primary variable of interest is the monthly wage, classified by sex, age, education, type of position, and so on. For our simulations, we use the sample from industry group 52 (retailing) and occupation group 5 (sales, service) in 2000, 2001 and 2002. We use the municipalities as small areas, and estimate the average monthly wage in each municipality.

### 4.2. Case without auxiliary information

In this case model (2.1) contains only an intercept, denoted by $\mu$. We fit the model separately for men and women in all the 3 years. Estimates of the model parameters are given in Table 1. As expected, the estimated overall average monthly wage, i.e. $\hat{\mu}$, increases from 2000 to 2002, and is higher for men than for women. The estimated variance components vary from one year to another, with $\hat{\sigma}_v^2$ having the largest variation. The estimated residuals are far from normal. Student-t distribution, on the other hand, appears to fit the estimated residuals quite well, albeit after deletion of a few largest and/or smallest values. In the simulations below, we shall consider only the nonparametric version of the simultaneous estimates.

We now set up the model parameters for simulation based on the sample in 2001. We use the mean and variance of the observed area sample means as the true $\mu$ and $\sigma_v^2$. Whereas we use the variance of the observed within-area deviations, i.e. $e_{ij} = y_{ij} - \bar{y}_i$, calculated across all the areas as the true $\sigma_e^2$. We draw a simulated sample in two steps:

a) draw $\theta_i^*$ randomly and with replacement from all the observed area sample means;
b) draw $e_{ij}^*$ randomly and with replacement from $\{e_{ij}\}$ and set $y_{ij}^* = \theta_i^* + e_{ij}^*$, for all $(i, j)$.

**Table 1.** Estimated model parameters in 2000, 2001 and 2002. (Data: NWS)

| Men | | | | | |
|---|---|---|---|---|---|
| Year | Sample size | $m$ | $\hat{\mu}$ | $\hat{\sigma}_v$ | $\hat{\sigma}_e$ |
| 2002 | 6062 | 316 | 20954 | 573.4 | 4601.5 |
| 2001 | 6353 | 305 | 19768 | 780.1 | 4589.3 |
| 2000 | 5444 | 303 | 19318 | 623.9 | 3916.6 |

| Women | | | | | |
|---|---|---|---|---|---|
| Year | Sample size | $m$ | $\hat{\mu}$ | $\hat{\sigma}_v$ | $\hat{\sigma}_e$ |
| 2002 | 10424 | 365 | 19318 | 359.6 | 3494.1 |
| 2001 | 10659 | 269 | 18475 | 1025.0 | 4556.9 |
| 2000 | 9025 | 366 | 17552 | 503.9 | 2949.2 |

The within-area sample sizes are the same as the observed ones in 2001. Given each simulated sample, we estimate the model parameters, and derive the direct estimate, the EBLUP, and the simultaneous estimate of all the $\theta_i^*$'s. The results for the parameter estimators based on 1000 simulated samples are given in Table 2.1. Both the estimator for $\mu$ and $\sigma_e^2$ seem to be unbiased. Whereas the estimator of $\sigma_v^2$ ("fitting-of-constants" method, Rao, 2003) appears to be slightly downward biased. In addition, $\hat{\sigma}_v$ has large (just below 30%) relative standard error. The estimation of the between-area variation is more demanding than the estimation of the within-area variation.

**Table 2.1.** Simulation results for parameter estimators, 1000 simulations.

| Parameter | Men | | | Women | | |
|---|---|---|---|---|---|---|
| | $\hat{\mu}$ | $\hat{\sigma}_e$ | $\hat{\sigma}_v$ | $\hat{\mu}$ | $\hat{\sigma}_e$ | $\hat{\sigma}_v$ |
| True value | 19852 | 4500 | 2679 | 18287 | 4493 | 1515 |
| Expectation | 19856 | 4475 | 2592 | 18285 | 4474 | 1425 |
| Relative standard error (%) | 1.0 | 2.6 | 27.1 | 0.7 | 8.2 | 28.4 |

The three small area estimators are compared to each other with respect to (i) the average, and maximum, *absolute relative error (ARE)* given, respectively, by

$$m^{-1}\sum_{i=1}^{m} |\hat{\theta}_i^* / \theta_i^* - 1| \qquad \text{and} \qquad \max_{i=1,\ldots,m} |\hat{\theta}_i^* / \theta_i^* - 1|;$$

(ii) the average *absolute relative distributional error (ARDE)* given by

$$m^{-1}\sum_{i=1}^{m} |\hat{\theta}_{(i)}^* / \theta_{(i)}^* - 1|$$

where $\theta^*_{(i)}$ is the $i$th order statistic of $\{\theta^*_i\}$, and $\hat{\theta}^*_{(i)}$ is the $i$th order statistic of $\{\hat{\theta}^*_{(i)}\}$; and (iii) the *relative error (RE)* of the range estimator given by

$$(\max_i \hat{\theta}^*_i - \min_i \hat{\theta}^*_i)/(\max_i \theta^*_i - \min_i \theta^*_i) - 1.$$

**Table 2.2.** Simulation results for small area estimators, 1000 simulations

| | Men | | | Women | | |
|---|---|---|---|---|---|---|
| Estimator | Direct | EBLUP | Simultaneous | Direct | EBLUP | Simultaneous |
| Average ARE | 8.6 | 5.7 | 6.4 | 6.8 | 3.9 | 4.4 |
| Maximum ARE | 89.7 | 42.0 | 46.5 | 116.4 | 39.2 | 45.8 |
| Average ARDE | 4.5 | 2.1 | 2.0 | 4.1 | 2.0 | 1.2 |
| RE in range | 47.9 | -17.1 | 6.3 | 121.7 | -27.6 | 6.2 |

Based on the results given in Table 2.2, we observe that, (I) on average, the model-based estimators improve the area-specific estimation compared to the direct estimator. They are also more robust since the maximum AREs are much smaller than that of the direct estimator. The simultaneous estimator is slightly worse than the EBLUP, without losing the essential gains of the modeling approach. (II) The model-based estimators are also much better than the direct estimator for estimation of the distribution of the small area means, both with respect to the average ARDE and the range. (III) Not unexpectedly, the simultaneous estimators have better ensemble properties than the EBLUPs. In the present simulation, the gains are substantial with respect to the range (and variance) of the small area means.

### 4.3. Case with auxiliary information

To create the case with auxiliary information, we take the joint sample of 2001 and 2002, which contains 8459 persons of both sex. We now treat the monthly wage in NWS 2001 as the known covariates, denoted by $x_{ij}$, and the monthly wage in NWS 2002 as the variable of interest. The parameters of the model (2.1) are fixed as follows. Firstly, we obtain the sample means $\bar{x}_i$ and $\bar{y}_i$. The ordinary least square fit of regressing $\bar{y}_i$ on $\bar{x}_i$ (including an intercept term) yields the regression coefficients for the simulations below, denoted by $\beta$. Whereas the residuals will be used as the area-level random effects, denoted by $v_i = \bar{y}_i - \bar{x}_i^T \beta$. Finally, we obtain the within-area deviations as $\varepsilon_{ij} = x_{ij} - \bar{x}_i$ and $e_{ij} = y_{ij} - \bar{y}_i$. For the particular data we used, we find

$$\Delta/(\Delta + \sigma_v^2) = 0.430,$$

such that the covariate accounts for about half of the variation in the variable of interest.
For each simulation, we generate the population and the sample as follows:

1) draw $\overline{X}_i^*$ randomly and with replacement from $\overline{x}_i$, and draw $v_i^*$ randomly and with replacement from $\{v_i\}$, and set $\theta_i^* = \overline{X}_i^{*T}\beta + v_i^*$;
2) draw $\varepsilon_{ij}^*$ randomly and with replacement from $\{\varepsilon_{ij}\}$, and set $x_{ij}^* = \overline{X}_i^* + \varepsilon_{ij}^*$;
3) draw $e_{ij}^*$ randomly and with replacement from $\{e_{ij}\}$, and set $y_{ij}^* = x_{ij}^{*T}\beta + v_i^* + e_{ij}^*$.

The within-area sample sizes are the same as in the observed panel. Based on each simulated sample, we derive the parameter estimates, the direct estimates, the EBLUP and the simultaneous estimates of all the $\theta_i^*$'s. The results based on 1000 simulations are given in Table 3.1 and 3.2.

The parameter estimators perform similarly as in the case without auxiliary information. The estimators of $\beta$ and $\hat{\sigma}_e$ are apparently unbiased. Whereas $\hat{\sigma}_v$ appears to be slightly downward biased, and is associated with the largest uncertainty.

**Table 3.1.** Simulation results for parameter estimators with auxiliary information, 1000 simulations.

| | $\hat{\beta}$ | $\hat{\sigma}_e$ | $\hat{\sigma}_v$ |
|---|---|---|---|
| True value | (8613.9, 0.603) | 3713.9 | 1195.2 |
| Expectation | (8623.4, 0.602) | 3705.0 | 1157.6 |
| Relative standard error (%) | (2.6, 1.8) | 2.1 | 29.5 |

**Table 3.2.** Simulation results for small area estimators with auxiliary information, 1000 simulations.

| Estimator | Direct | EBLUP | Simultaneous |
|---|---|---|---|
| Average ARE | 7.2 | 3.1 | 3.6 |
| Maximum ARE | 73.7 | 62.8 | 55.8 |
| Average ARDE | 4.3 | 1.1 | 0.9 |
| RE in range | 53.0 | -23.6 | -8.8 |

Next, we compare the three estimators with respect to the average and maximum ARE, the average ARDE and the RE in range (Table 3.2). The conclusions are similar to those in the case without auxiliary information: (i) the model-based estimators improve the direct estimator both in terms of area-specific and ensemble properties, and (ii) the simultaneous estimator improves the ensemble properties of the EBLUP, without losing much of the precision oi the EBLUP in area-specific prediction. Notice that the nonparametric simultaneous estimator can be expected to give good estimation of the variance oi the small area means, since it is based on an adjustment of the empirical variance of the EBLUPs. The simulation results above suggest that this typically also leads to better estimation of the other ensemble statistics such as the range.

## 5. Summary

We considered a nested error regression model, which is a very basic mode for small area estimation. We showed, in theory as well as by empirical example that the (empirical) best linear unbiased prediction entails loss of the between area variation of the small area means. In general, estimators that are optimal fo area-specific prediction may have unsatisfactory ensemble properties. We exten the simultaneous estimation approach of Zhang (2003) for the nested erro regression model. This allows us to make use of auxiliary information at the unit level when it is available. Simulations suggest that the simultaneous estimato may substantially improve the estimation of the ensemble characteristics of th small area parameters, without losing much of the precision in area-specifi prediction. Our approach provides a frequentist alternative to the existin Bayesian methods.

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