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Author: Dagsvik, John K.
Brunborg, Helge
Flaatten, Ane Seip

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A BEHAVIORAL TWO-SEX MARRIAGE MODEL[1]

by

John K. Dagsvik, Ane S. Flaatten and Helge Brunborg

ABSTRACT:

In this paper we propose a particular marriage model, i.e., a model for the number of marriages for each age combination as a function of the vectors of the number of single men and women in each age group. The model is based on Dagsvik (2000) where it is demonstrated that a general type of matching behavior imply, under specific assumptions about the distribution of the preferences of the women and men, a convenient expression for the corresponding marriage model.

Data from the Norwegian Population Register for nine years are applied to estimate the model. We subsequently test the hypothesis that, apart from a random “noise” component, the age-specific parameters change over time according to a common trend. We find that the hypothesis is not rejected by our data.

KEYWORDS: Two-sex demographic models, Marriage models, Two-sided matching.

JEL CLASSIFICATION: C78, J11, J12

Address: John K. Dagsvik, Research Department, Statistics Norway, P.O.Box 8131 Dep.

N-0033 Oslo, Norway

e-mail: john.dagsvik@ssb.no

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1. INTRODUCTION

In this paper we discuss a particular approach to the modeling of marriage behavior, and we estimate an empirical version of this model from annual files of data on marriages in Norway.

The classical stable population models rests on a one-sex theory represented by age-specific fertility and death rates for the female population. It is, however, recognized that when there are substantial differences between the female and the male population, the one-sex models may lead to quite unrealistic predictions, see for example Pollak (1990), and Kuczynski (1932, pp. 36-38). Kuczynski pointed out that since more than 50 per cent of the newborns are boys, predictions based on the male population may imply an increasing population while the opposite may be the case for one-sex models based on the female population.

The two-sex problem was already discussed by Lotka (1922). Several researchers have proposed different types of theories based on two-sex marriage models, that is, models that yield the number of marriages of each possible age combination as a function of the number of unmarried females and males, in each age group. These contributions include Fredrickson (1971), Keyfitz (1971), Feeney (1972), McFarland (1972), Das Gupta (1973), Pollard (1977), Schoen (1977, 1981), Keilman (1985), Pollard and Höhn (1993).

Although these authors have made seminal contributions to the literature on two-sex marriage models, the proposed models are nonetheless unsatisfactory from a behavioral point of view since they are not derived from a theory of individual behavior. Without such a theory, it is difficult to give a precise interpretation of key concepts and parameters in the marriage model. In other words, the models are ad hoc from a theoretical perspective.

The analysis in this paper is based on a two-sex marriage model that is derived from a theory of two-sided matching. The point of departure is the theoretic analysis of marriage markets summarized in Roth and Sotomayor (1990). The literature on matching behavior does not, however, consider the aggregation problem of predicting the number of matches of each type as a function of the number of agents of each type and parameters that represent the corresponding distribution of preferences. This aggregation problem was analyzed by Dagsvik (2000) who derived a particular aggregate matching model from assumptions about the distribution of preferences of the agents in the market and assumptions about the rules of the matching behavior. The model proposed by Dagsvik (2000) offers therefore the possibility of establishing a behavioral two-sex marriage model.

While the discussion in Dagsvik (2000) was intended to apply to different types of matching markets, the focus in this paper is on empirical modeling and estimation of a two-sex marriage model based on Dagsvik's framework. The empirical analysis is based on population register data from Statistics Norway for the years 1985 to 1994.

The paper is organized as follows: In Section 2 we outline the theoretical point of departure and the structure of the (aggregate) marriage model. In Section 3 qualitative properties of the model are addressed, and in Section 4 a particular extension of the model is discussed. Section 5 describes the data, and in the last section we report the empirical results.

2. A BEHAVIORAL TWO-SEX MODEL

In this section we outline the key elements of a behavioral theory for the marriage market and the implied two-sex model. For a more detailed analysis including proofs we refer to Dagsvik (2000).

As mentioned above, our theory is based on a particular two-sided matching setting which has been extensively analyzed by numerous authors, and discussed in Roth and Sotomayor (1990). We shall now describe a particular matching algorithm called the *"deferred acceptance"* algorithm, which is an explicit example of a particular type of matching behavior.

Consider a population of men and women who are looking for a partner to form a match (marriage). Each man and each woman are supposed to have sufficient information about the population of the opposite sex so as to be able to establish preference lists, i.e., lists of rankings of all potential partners, including the alternative of being single. The matching process towards equilibrium takes place in several stages. There are no search costs and the agents have no information about the preferences of potential partners, which means that they are ignorant about their "chances" in the market. The deferred acceptance algorithm goes as follows: Either the women or the men make offers, that is, if the men make the offers no woman is allowed to make offers.

Let us first introduce some basic terminology. The following concepts are borrowed from Roth and Sotomayor (1990).

A man is acceptable to a woman if the woman prefers to be married (matched) to the man rather than staying single. Consider a matching denoted by μ that matches a pair (m,f) who are *not* mutually acceptable. Then at least one of the agents would prefer to be single rather than being matched to the other. Such a matching μ is said to be blocked by the unhappy agent. Consider next a matching μ such that there exist a man m and a woman f who are matched to one another, but who prefer each other to their assignment at μ (given the rules of the game). The pair (m,f) is said to block the matching μ. We say that a matching μ is stable if it is not blocked by any individual or pair of agents.

Gale and Shapley (see Roth and Sotomayor (1990)) have demonstrated that stable matchings exist for every matching market. Specifically they prove that the "deferred acceptance" procedure produces a stable matching for any set of preferences, provided the preferences are strict, i.e., that indifference is ruled out. The algorithm goes as follows: Suppose the men make the offers. First each man makes an offer to his favorite woman. Each woman rejects the offer from any man who is unacceptable to her, and each woman who receives more than one offer from any man rejects all but her most preferred among these. Any man whose offer is not rejected at this point is kept temporarily "engaged" until better offers arrive. At any step any man who was rejected at the previous step makes an offer to his next choice i.e., to his most preferred woman among those who have not rejected him. Each woman receiving offers rejects any from unacceptable men, and also rejects all but her most preferred among the group of the new offers and any man she may have kept engaged from the previous step. There are no "costs" associated with the temporary "engagements". The algorithm stops after any step in which no man is rejected. (The final stage.) The matches are now consummated with each man being married to the woman he is engaged.

The stability argument goes as follows: Suppose that man m and woman f are not matched to each other, but m prefers f to his partner. Then woman f must be acceptable to man m, and so he must have made an offer to f before making an offer to his current partner. Since m was not engaged to f when the algorithm stopped, m must have been rejected by f in favor of someone she (f) liked more. Therefore, f is matched to a man whom f likes more than man m, and so m and f do not block the matching. Since the matching is not blocked by any individual or any pair, it is stable. Similarly one could apply a rule where the women make offers to the men. However, this would not necessarily produce a matching that is equal to the former one.

As discussed in Dagsvik (2000), the aggregate marriage model which will be outlined below is consistent with *any* matching algorithm—be it this deferred acceptance algorithm or not—provided the matching is stable.[1]

Next we shall introduce some concepts and notations which will enable us to describe formally the marriage model. We assume that the preferences of the individuals are represented by latent utility indexes. Let F_i be the number of single women in age group i and M_j the number of single men in age group j, $i=1,2,\ldots,S$, $j=1,2,\ldots,D$. Let U_{ij}^{fm} be the utility of female f in age group i of being married to man m in age group j, and let U_{i0}^{f} be the utility of female f in age group i of being single. Let V_{ji}^{mf} be the utility of man m in age group j of being married to female f in age group i, and let V_{j0}^{m} be the corresponding utility of being single. We assume that the utilities have the following structure

$$U_{ij}^{fm}=a_{ij}\,\varepsilon_{ij}^{fm},\quad U_{i0}^{f}=\varepsilon_{i0}^{f}$$

and

$$V_{ji}^{mf}=b_{ji}\,\eta_{ji}^{mf},\quad V_{j0}^{m}=\eta_{j0}^{m}$$

where $\{a_{ij}\},\{b_{ji}\}$ are positive deterministic terms, while $\{\varepsilon_{ij}^{fm}\},\{\varepsilon_{i0}^{f}\},\{\eta_{ji}^{mf}\}$ and $\{\eta_{j0}^{m}\}$ are positive i.i.d. random variables (taste-shifters) with cumulative distribution function $\exp(-1/y)$ for $y>0$. Note that since we are only concerned with preference orderings we can take any increasing transformation of the utilities without altering the rank orderings. For example, if we take the logarithm of the utilities we get an (equivalent) additive formulation instead of the multiplicative one above, and the corresponding cumulative distribution function of the error terms will have the form $\exp\left(-e^{-y}\right)$. The justification for this particular distribution function can be found in the theory of random utility models for discrete choice, see for example McFadden (1984).

Before we state the implications of the assumptions above, it may be instructive to outline a somewhat informal argument to provide the intuition behind the basic idea that underlies the model. To this end we ignore the fact that the sets of available partners to a specific individual at each stage in the adjustment process will vary across the population due to the effect of the random taste-shifters. Moreover, we only discuss the simple setting in which there are no age groups, which means that all men, as well as women, are observationally identical.

Let C_f be the set of men that are available to woman f in the final stage of the game, and let n be the (mean) number of men in C_f. Let r be the (mean) number of women that are available to man m in the final stage of the game. Since there are F women, the probability that a woman shall prefer to be married to man m equals r/F. But this probability can also be expressed as

$$P\left(U^{fm} = \max\left(\max_{s\in C_f} U^{fs}, U_0^f\right)\right).$$

The probability statement above means the probability that man m yields the highest utility to woman f, among all feasible men and the utility of being matched to man m is also higher than the utility of being single. From the above distributional assumption it follows from standard results in discrete choice theory, cf. for example McFadden (1984), that

$$P\left(U^{fm} = \max\left(\max_{s\in C_f} U^{fs}, U_0^f\right)\right) = \frac{a}{1+a\cdot n} = \frac{1}{\alpha+n}$$

where $\alpha = 1/a$. But this probability is also equal to r/F. Consequently, the following equation must be true

$$\frac{r}{F} = \frac{1}{\alpha + n}. \tag{2.1}$$

By symmetry we also must have that

$$\frac{n}{M} = \frac{1}{\beta + r} \tag{2.2}$$

where $\beta = 1/b$. It is easily verified that these equations determine r and n uniquely. Consider next the probability that a woman and a man shall marry. Since the probability that a woman makes an offer to a particular man equals r/F, and there are n available men to this woman the probability that the woman shall marry any of the men available to her must be equal to $n \cdot r/F$. Since F is the number of women the number of marriages, X (say), is therefore equal to $r \cdot n$. When equations (2.1) and (2.2) are solved for r and n we find that X satisfies the equation

$$(F - X)(M - X) = \alpha \beta X. \tag{2.3}$$

This equation has only one acceptable solution which is equal to

$$X = \frac{1}{2}\left(\alpha\beta + M + F - \sqrt{(\alpha\beta + M + F)^2 - 4MF}\right). \tag{2.4}$$

From (2.3) we realize that α and β are not separately identified, only the product $\alpha\beta$ can be identified given F, M and X. The intuitive and informal derivation above ignores the fact that the women's and the men's choice sets are stochastic in that they depend on all the random error terms in the utility functions. For a more rigorous treatment, where the stochastic dependencies between the different choice sets are taken into account, we refer to Dagsvik (2000).

Let us next return to the general case. By using analogous arguments to the ones used in the case with observationally identical men and women considered above, it is possible to derive a convenient expression for the number of marriages in the case where the women and men are characterized by age. Let X_{ij} be the number of marriages where the wife has age i and the husband has age j. Let X_{i0}^{f} be the number of women of age i that remain single and X_{j0}^{m} the number of men of age j that remain single. Dagsvik (2000) has demonstrated that X_{ij}, X_{i0}^{f} and X_{j0}^{m} are given by

$$X_{ij} = \frac{F_i M_j c_{ij}}{A_i B_j}, \tag{2.5}$$

$$X_{i0}^{f} = \frac{F_i}{A_i} \tag{2.6}$$

and

$$X_{j0}^{m} = \frac{M_j}{B_j}, \tag{2.7}$$

where $c_{ij} \equiv a_{ij} b_{ji}$, and $\{A_i\}$ and $\{B_j\}$ are uniquely determined by the system of equations

$$A_i = 1 + \sum_{k=1}^{D} \frac{c_{ik} M_k}{B_k} \tag{2.8}$$

and

$$B_j = 1 + \sum_{k=1}^{S} \frac{c_{kj} F_k}{A_k}. \tag{2.9}$$

Unfortunately, the solution of (2.8) and (2.9) cannot be expressed in closed form. However, we realize from (2.5), (2.6) and (2.7) that we can express the preference parameters $\{c_{ij}\}$ as

$$c_{ij} = \frac{X_{ij}}{X_{i0}^{f} X_{j0}^{m}}. \tag{2.10}$$

This expression is very convenient because it allows us to recover the structural parameters $\{c_{ij}\}$ from data on the number of marriages and the number of unmarried men and women in a very simple way. If the population is large (2.10) will provide precise estimates of $\{c_{ij}\}$. Similarly to the simple case considered above we realize that $\{a_{ij}\}$ and $\{b_{ji}\}$ cannot be separately identified unless further structure on the preferences is imposed.

3. QUALITATIVE PROPERTIES OF THE MARRIAGE MODEL

Let us next discuss some additional qualitative properties of the marriage function, i.e., the number of marriages X_{ij} as a function of the population vectors of single men and women.

McFarland (1972) has proposed seven axioms which a marriage model should satisfy. To describe these axioms, let now $X_{ij}(\mathbf{F},\mathbf{M})$ denote the marriage function where **F** and **M** are the vectors of the number of single women and men in the respective age groups. The axioms are as follows:

A1. $X_{ij}(\mathbf{F},\mathbf{M})$ should be defined for all vectors **F** and **M** whose elements are non-negative integers.

A2. $X_{ij}(\mathbf{F},\mathbf{M})$ must be non-negative.

A3. $\sum_j X_{ij}(\mathbf{F},\mathbf{M}) = F_i$ and $\sum_i X_{ij}(\mathbf{F},\mathbf{M}) = M_j$.

A4. The number of marriages should depend heavily on the ages of the males and females.

A5. $X_{ij}(\mathbf{F},\mathbf{M})$ should be a non-decreasing function of F_i and M_j, and be strictly increasing for some values of F_i and M_j.

A6. $X_{ij}(\mathbf{F},\mathbf{M})$ should be a non-increasing (and over some interval a strictly decreasing function) of F_r and M_s for $r \neq i$ and $s \neq j$.

A7. The negative effect on $X_{ij}(\mathbf{F},\mathbf{M})$ of an increase in M_s should be greater than the negative effect on $X_{ij}(\mathbf{F},\mathbf{M})$ of an equivalent increase in M_r if s is closer to j than r is. Likewise with the sexes interchanged.

The most important of these axioms are **A5** to **A7**. Axiom **A7** requires that a metric is introduced. A natural metric is to define s as closer to i than r (for men of age j) if

$$\left| b_{js} - b_{ji} \right| > \left| b_{jr} - b_{ji} \right| ,$$

i.e. the distances are expressed as the difference between the respective structural terms of the preferences.

We shall now demonstrate that our marriage model does not satisfy all axioms above unless further assumptions about the preferences are imposed. Unfortunately, we have not been able to prove whether or not **A5** and **A7** hold. In some cases, **A6** does not hold. Given the sizes of the age-specific population groups of unmarried females and males and the parameter estimates of $\{c_{ij}\}$ reported in Section 6 we have checked whether or not **A5**, **A6** and **A7** are violated. This is done by successively increasing the sizes of the female and male age groups, from the respective observed levels of $\{F_j\}$ and $\{M_i\}$. In the period 1985-1994 we did not find any case where **A5**, **A6** and **A7** was violated.

We shall next discuss a particular case, where $a_{ij} = a_j$ and $b_{ji} = b_i$, i.e., the deterministic components of the agent's utility function do not depend on his age, and demonstrate that in this case **A6** does not hold. From (2.8) and (2.9) we obtain that

$$A_i = 1 + b_i K_1, \quad B_j = 1 + a_j K_2 \tag{3.1}$$

where K_1 and K_2 are determined by

$$K_1 = \sum_k \frac{M_k a_k}{B_k} = \sum_k \frac{M_k}{\alpha_k + K_2}, \tag{3.2}$$

$$K_2 = \sum_k \frac{F_k b_k}{A_k} = \sum_k \frac{F_k}{\beta_k + K_1} \tag{3.3}$$

and $\alpha_j = 1/a_j$ and $\beta_i = 1/b_i$. From (2.5) we get that

$$X_{ij} = \frac{F_i M_j b_i a_j}{A_i B_j}. \tag{3.4}$$

Consequently, for $r \neq j$

$$\frac{\partial \log X_{ij}}{\partial M_r} = -\frac{1}{\beta_i + K_1}\frac{\partial K_1}{\partial M_r} - \frac{1}{\alpha_j + K_2}\frac{\partial K_2}{\partial M_r}. \tag{3.5}$$

By implicit differentiation, (3.2) and (3.3) yield

$$(1-D)\frac{\partial K_1}{\partial M_r} = \frac{1}{\alpha_r + K_2} \tag{3.6}$$

and

$$(1-D)\frac{\partial K_2}{\partial M_r} = -\frac{1}{\alpha_r + K_2} \cdot \sum\nolimits_k \frac{F_k}{(\beta_k + K_1)^2} \tag{3.7}$$

where

$$D \equiv \sum\nolimits_k \frac{M_k}{(\alpha_k + K_2)^2} \sum\nolimits_k \frac{F_k}{(\beta_k + K_1)^2}. \tag{3.8}$$

Note that

$$D < \sum_k \frac{M_k}{K_2(\alpha_k + K_2)} \sum_k \frac{F_k}{K_1(\beta_k + K_1)} = \frac{K_1}{K_2} \cdot \frac{K_2}{K_1} = 1.$$

According to McFarland, $\partial \log X_{ij} / \partial M_r$ should be nonpositive which would be true provided

$$\alpha_j + K_2 - (\beta_i + K_1) \sum_k \frac{F_k}{(\beta_k + K_1)^2} \geq 0.$$

It is straight forward to demonstrate that there exists a $\beta^* \in \left(\min_k \beta_k, \max_k \beta_k\right)$ such that

$$\sum_k \frac{F_k}{(\beta_k + K_1)^2} = \frac{1}{\beta^* + K_1} \sum_k \frac{F_k}{(\beta_k + K_1)} = \frac{K_2}{\beta^* + K_1}.$$

Hence

$$\alpha_j + K_2 - (\beta_i + K_1) \sum_k \frac{F_k}{(\beta_k + K_1)^2} = \alpha_j + K_2 - \frac{(\beta_i + K_1) K_2}{\beta^* + K_1}.$$

Suppose that $\beta_i \geq \beta^*$ and that α_j is close to zero. Then, evidently

$$\alpha_j + K_2 - \frac{(\beta_i + K_1) K_2}{\beta^* + K_1} \approx K_2 \left(1 - \frac{\beta_i + K_1}{\beta^* + K_1}\right) < 0.$$

Thus if $a_j \equiv 1/\alpha_j$ is sufficiently large and $b_i \equiv 1/\beta_i$ is sufficiently small then X_{ij} will increase when M_r increases, which means that axiom **A6** is violated. The intuition here is as follows: If more men become available the demand from women of age i for men of age j is in general likely to decrease. Similarly the demand for women of age i from men of age j is likely to increase since the competition becomes harder when new men enter. However, since demand from men of age r for women of ages *other* than age i is high compared to the demand for women of age i, this implies that new men of age r who enter the market will increase the demand pressure towards women of other ages than i. Similarly, women of other ages than i will have lower preferences for men of age j than for men of age r when a_j is sufficiently high. Consequently, the competition for men of age j the women of age i are facing, will in this case *decrease* because women of other ages tend to prefer new men of age r. Similarly, new men of age r will tend to fancy women of other ages than i, which thus reduces the competition for women of age i facing men of age j. Accordingly, X_{ij} will increase when new men of age r enter the market.

In the appendix we derive analytic expressions for the elasticities of X_{ij}, X_{i0}^{f} and X_{j0}^{m} with respect to F_i and M_j for all i and j.

4. AN EXTENSION OF THE MODEL

In this section we shall describe a particular extension of the model discussed above. Specifically, we shall now allow some of the random error terms to be correlated. As above we only give a brief summary here; for more precise details we refer to Dagsvik (2000). We define $\theta_1 \in [0,1]$ by

$$\operatorname{corr}\left(\log \varepsilon_{ij}^{fm}, \log \varepsilon_{ij}^{fs}\right) = 1 - \theta_1^2 \tag{4.1}$$

for $s \neq m$. Similarly, $\theta_2 \in [0,1]$ is defined by

$$\operatorname{corr}\left(\log \eta_{ji}^{mf}, \log \eta_{ji}^{ms}\right) = 1 - \theta_2^2, \tag{4.2}$$

for $s \neq f$. The motivation for this correlation is that there may be unobservable factors affecting the utility for potential partners, which are correlated across potential partners. These correlations are the only ones that are allowed to be different from zero, i.e.

$$\operatorname{corr}\left(\log \varepsilon_{ij}^{fm}, \log \varepsilon_{rk}^{pq}\right) = \operatorname{corr}\left(\log \eta_{ji}^{mf}, \log \eta_{kr}^{qp}\right) = 0$$

for $k \neq j$, or $r \neq i$ (or both), and finally

$$\operatorname{corr}\left(\log \varepsilon_{ij}^{fm}, \log \varepsilon_{r0}^{p}\right) = \operatorname{corr}\left(\log \eta_{ji}^{mf}, \log \eta_{kr}^{q}\right) = 0,$$

$$\operatorname{corr}\left(\log \varepsilon_{ij}^{fm}, \log \eta_{kr}^{qp}\right) = \operatorname{corr}\left(\log \varepsilon_{ij}^{fm}, \log \eta_{k0}^{q}\right) = \operatorname{corr}\left(\log \eta_{ji}^{mf}, \log \varepsilon_{r0}^{p}\right) = 0$$

for all f, m, p, q, i, j, k and r. Dagsvik (2000) demonstrates that the marriage model in this case turns out to have the structure

$$X_{ij} = \left(\frac{F_i\, a_{ij}}{\tilde{A}_i}\right)^{\theta_2/\theta} \left(\frac{M_j\, b_{ji}}{\tilde{B}_j}\right)^{\theta_1/\theta}, \tag{4.3}$$

$$X_{i0}^{f} = \frac{F_i}{\tilde{A}_i} \tag{4.4}$$

and

$$X^{m}_{j0} = \frac{M_j}{\tilde{B}_j} \tag{4.5}$$

where $1-\theta = (1-\theta_1)(1-\theta_2)$, and $\{\tilde{A}_i\}$ and $\{\tilde{B}_j\}$ are uniquely determined by

$$\tilde{A}_i = 1 + \left(\frac{\tilde{A}_i}{N_i}\right)^{1-\theta_2/\theta} \sum_{k=1}^{D} a_{ik}^{\theta_2/\theta} M_k^{\theta_1/\theta} \left(\frac{b_{ki}}{\tilde{B}_k}\right)^{\theta_1/\theta} \tag{4.6}$$

and

$$\tilde{B}_j = 1 + \left(\frac{\tilde{B}_j}{M_j}\right)^{1-\theta_1/\theta} \sum_{k=1}^{S} b_{kj}^{\theta_1/\theta} F_k^{\theta_2/\theta} \left(\frac{a_{kj}}{\tilde{A}_k}\right)^{\theta_2/\theta}, \tag{4.7}$$

for $i = 1,2,\ldots,S,$ and $j = 1,2,\ldots,D$. For the purpose of estimation it is convenient that we can express the preference parameters as

$$\tilde{c}_{ij} = \frac{X_{ij}}{\left(X^{f}_{i0}\right)^{\theta_2/\theta}\left(X^{m}_{j0}\right)^{\theta_1/\theta}} \tag{4.8}$$

where $\tilde{c}_{ij} = a_{ij}^{\theta_2/\theta}\, b_{ji}^{\theta_1/\theta}$. Similarly to the model considered in Section 2 we cannot identify a_{ij} and b_{ji} separately. However, with data for several periods it is possible to identify θ_1/θ and θ_2/θ.

5. DATA

The data come from the annual files of marriages at Statistics Norway, which are obtained from the Central Population Register for Norway and based on the personal identification numbers introduced in Norway in 1964. A number of variables are available for each new marriage, such as date of birth of the spouses, date of marriage, marriage number (1^{st}, 2^{nd}, etc.), previous marital status (single, divorced, widow(er)ed) and citizenship. In this preliminary/first analysis we have included all non-married persons who were residents of Norway at the time of marriage, to secure consistency between flows (marriages) and stocks (marriageable persons). From these files we have constructed marriage matrices by age at the end of the year, to make stocks and flows refer to the same birth cohorts. For the stock of potential marriage partners we use the number of non-married men and women, respectively, implicitly assuming that never married and previously married have the same preferences, and vice versa, that they are equally attractive in the marriage market, (which is probably not quite true in practice). As our model assumes that the population is closed, i.e. there being no deaths, immigrations and emigrations, we use the mean population of non-married persons at the beginning and end of the year as estimates of the number of non-married men and women in each age group, respectively, to adjust for actual deaths and migrations.[2]

6. EMPIRICAL RESULTS

As regards estimation results for the preference parameters $\{c_{ijt}\}$ based on (2.10) for all the years from 1985 to 1994 we refer to Dagsvik et al. (1998). Here, t indexes year.

On the basis of these results we have tested an implication of a particular hypothesis which we shall explain below. To this end let $\{a_{ijt}\}$ and $\{b_{jit}\}$ denote the preference matrix in year t. Consider the hypothesis

$$a_{ijt} = h_{ij1}\, q_1(t), \tag{6.1}$$

and

$$b_{jit} = h_{ij2}\, q_2(t) \tag{6.2}$$

where h_{ij1} and h_{ij2} are parameters that are constant over time. The equations (6.1) and (6.2) mean that, apart from the noise implied by the random error terms, the preferences for potential partners will not change over time as long as the option to remain single is ruled out. This follows from the fact that

$$U_{ij}^{fm} > U_{ik}^{fq}$$

is equivalent to

$$h_{ij1}\, \varepsilon_{ijt}^{fm} > h_{ik1}\, \varepsilon_{ikt}^{fq}$$

since the factor $q_1(t)$ cancels in utility comparisons. Thus $q_1(t)$ and $q_2(t)$ only affect the propensity to marry.

In the following we shall test a slightly weaker hypothesis. Without loss of generality we may write

$$\log c_{ijt} = \gamma_{ij} + m_t + \eta_{ijt} \tag{6.3}$$

where $\{\gamma_{ij}\}$ are constants that do not depend on t while $\{m_t\}$ are constants that do not depend on i and j. The terms $\{\eta_{ijt}\}$ are random variables with zero mean. Note that when $\eta_{ijt} = 0$, (6.3) is implied by (6.1) and (6.2) with $m_t = \log q_1(t) + \log q_2(t)$ and $\gamma_{ij} = \log h_{ij1} + \log h_{ij2}$.

We wish to test the hypothesis H_0 that the random variables $\{\eta_{ijt}\}$ are i.i.d. against the alternative that $\eta_{ijt}, i = 1,2,\ldots, j = 1,2,\ldots,$ are independent random variables with zero mean and with a distribution which may depend on t. For this purpose the T-sample analogue to the Kolmogorov-Smirnov or, alternatively, Cramér-von Mises test procedure can be used. To this end let

$$Z_{ijt} = \log\left(\frac{X_{ijt}}{X_{i0t}^{f} X_{j0t}^{m}}\right). \tag{6.4}$$

Recall that by (2.10), Z_{ijt} is a “natural” estimator of $\log c_{ijt}$. Without loss of generality we can normalize so that the mean of $\{m_t\}$ (over time) is equal to zero. Hence, under the assumption that $\{\eta_{ijt}\}$ have zero mean across time as well as across all age combinations (i,j), it follows that $\{\eta_{ijt}\}$ can be estimated as

$$\hat{\eta}_{ijt} = Z_{ijt} - \bar{Z}_{ij\cdot} - \bar{Z}_{\cdot\cdot t} + \bar{Z}_{\cdots} \tag{6.5}$$

where $\bar{Z}_{ij\cdot}, \bar{Z}_{\cdot\cdot t}$ and $\bar{Z}_{\cdots}$ are the respective means over time, age combinations, and combinations of age and time. The estimator (6.5) follows from the least squares procedure. To avoid estimation errors due to the limited number of marriages in certain age combinations, particularly for large age differences, we have only used data with $-3 \le j - i \le 7$.

Consider next the test procedures. Let $\hat{F}_t(y)$ be the cumulative empirical distribution of $\hat{\eta}_{ijt}$ in year t, and let $F_t(y)$ be the corresponding theoretical cumulative distribution function. Let n_t be the number of observations in year t, i.e., n_t is the number of combinations (i,j) given the constraints above. Finally, let $\tilde{F}(y)$ be the mean empirical distribution over all years, i.e.,

$$\tilde{F}(y) = \sum_{t=1}^{T} \frac{n_t}{n} \hat{F}_t(y) \tag{6.6}$$

where T is the number of years for which we have observations of marriages, and

$$n = \sum_{t=1}^{T} n_t \,.$$

Define

$$Q_1(T) = \left(\sup_{y \geq 0} \sum_{t=1}^{T} n_t \left| \hat{F}_t(y) - \tilde{F}(y) \right| \right)^{1/2} \tag{6.7}$$

and

$$Q_2(T) = \sum_{t=1}^{T} n_t \int_{0}^{\infty} \left(\hat{F}_t(y) - \tilde{F}(y) \right)^2 d\tilde{F}(y) \,. \tag{6.8}$$

The statistics $Q_1(T)$ and $Q_2(T)$ are known, respective as the T-sample analogue to the Kolmogorov-Smirnov, and the Cramér-von Mises statistics, which provide two alternative test statistics for testing H_0, where H_0 now can be formulated as

$$H_0 : F_1 = F_2 = \cdots = F_T \,.$$

Kiefer (1959) has derived the asymptotic distributions of Q_1 and Q_2 and he has provided tables of critical values for $T \leq 6$. In our data set $n_t = 131$, which we assume is sufficiently large to allow us to apply asymptotic test criteria. In the case with $T = 6$ the five per cent critical value for $Q_1(6)$ is equal to 2.00, and for $Q_2(6)$ it is equal to 1.47. In our case $T = 9$, but since it follows from (6.7) and (6.8) that $Q_1(T)$ and $Q_2(T)$ are increasing in T, the respective five per cent critical values for $Q_1(9)$ and $Q_2(9)$ are larger than the ones for $Q_1(6)$ and $Q_2(6)$. Our data yields $Q_1(9) = 1.81$ and $Q_2(9) = 1.38$. We can therefore conclude that neither the test based on $Q_1(9)$ nor the one based on $Q_2(9)$ imply that H_0 is rejected.

Our next concern is the distribution of $\{\hat{\eta}_{ijt}\}$. In Figure 1 we display the QQ normal plot[3] of $\{\eta^*_{ijt}\}$, where $\eta^*_{ijt} = \exp \hat{\eta}_{ijt}$. This plot suggests that the normal distribution provides a fairly good representation of the distribution of $\{\eta^*_{ijt}\}$. The corresponding Kolmogorov-Smirnov statistics for a test of the hypothesis that F is a normal distribution equals 0.030. With $n = 1179$, the 5 per cent critical level is 0.039, which implies that the hypothesis is not rejected. Figures 2 and 3 display the empirical density and cumulative distribution together with the estimated normal density and cumulative distribution, respectively. In Table 1 we report the estimates of $\{\gamma_{ij}\}$ and $\{m_t\}$. The mean and the standard deviation of $\{\eta^*_{ijt}\}$ are estimated to 1.002 and 0.064, respectively.

Thus, the data suggest that $\{c_{ijt}\}$ are approximately normally distributed. It is interesting that one can in fact provide theoretical arguments that support the hypothesis that $\{\eta^*_{ijt}\}$ are Gaussian random variables. These arguments stem from the property that the behavioral model discussed above is in fact derived from a matching model in which men and women in addition to having preferences over potential partners also have preferences over a set of available "contracts", cf. Dagsvik (2000), pp. 36-37. By a contract we understand terms of an agreement between wife and husband. In the present context it seems reasonable to assume that contracts are associated with the couples' social, demographic, cultural and economic choice opportunities related to residential location, lifestyle, type of housing, number of children, etc. The men and the women are assumed to behave so as to maximize utility with respect to the feasible contracts and partners.

Let $w = 1,2,\dots$, index the contract possibilities, and analogous to the exposition in Section 2 let $a_{ijt}(w)$ and $b_{jit}(w)$ be the respective structural terms of the utility functions of the women and the men at time t. Let $c_{ijt}(w) = a_{ijt}(w)\, b_{jit}(w)$. The corresponding matching model analysed in Dagsvik (2000) is a direct extension of the one presented in Section 2, and it yields a model for $X_{ijt}(w)$, where $X_{ijt}(w)$ is the number of (i,j) marriages at time t with for which the contract is equal to w. In Dagsvik (2000) it is demonstrated that the total number of marriages, $X_{ijt} \equiv \sum_w X_{ijt}(w)$, depends on the preference parameters $\{c_{ijt}(w)\}$ through $\{c_{ijt}\}$ where

$$c_{ijt} = \sum_w c_{ijt}(w).$$

Thus c_{ijt} may be interpreted as the sum of a large set of random variables, $\{c_{ijt}(w)\}$. Under rather general assumptions about the dependence structure between these variables the Central Limit Theorem applies, which implies that c_{ij} is approximately normally distributed. Recall that the classical Central Limit Theorem requires the variances of the original variables be bounded. In the more general case with unbounded variances there also exists a Central Limit Theorem which yields the class of *Stable distributions*, see for example Lamperti (1996). Recall that the class of Stable distributions is characterized by four parameters, namely $\alpha \in (0,2]$, $\sigma > 0$,

$\beta \in [-1,1]$ and μ, where α may be interpreted as a measure of how heavy the tail of the distribution is, σ is a scale parameter, β represents skewness and μ is a location parameter. When $\alpha = 2$, we obtain the normal distribution in which case β vanishes. Now provided one finds the theoretical arguments above convincing and assume that c_{ijt} is a Stable variable, then data suggest that the hypothesis of normality may not be true. We have applied a method suggested by McCulloch (1986) to estimate α[4]. Specifically, we obtained the estimate, $\hat{\alpha} = 1.75$ with asymptotic standard deviation equal to 0.09. This means that α seems to be significantly less than 2. The data indicate that if we test the hypothesis that $\{\eta^*_{ijt}\}$ are normally distributed against the alternative that they are generated from a Stable distribution, then the hypothesis will be rejected. Thus, we conclude that when the class of Stable distributions is postulated apriori the distribution of $\{c_{ijt}\}$ seems to be non-normal, which implies that the right tail is (asymptotically) Pareto distributed.

[Figure 1 here]

[Table 1 here]

[Figures 2 to 5 here]

In Figure 4 we get an impression of how the parameters $\{\gamma_{ij}\}$ are distributed. The difference between the two pictures is due to the fact that the wife is usually younger than the husband. According to these pictures, there seems to be a strong relationship between the γ-parameters for different age combinations.

In Figure 5 we have plotted the parameter m_t as a function of time. We notice that m_t decreases almost linearly from 1986 to 1994. Recall that m_t may, loosely speaking, be interpreted as the overall preference for marrying. The decline in m_t may be due to the substantial growth in consensual unions and an increasing age at (first) marriage.

Let us finally consider the significance of the random terms $\left\{\eta^*_{ijt}\right\}$. Recall that the estimation result yields that $\left\{\eta^*_{ijt}\right\}$ are i.i. Stably distributed random variables. If, however, we are willing to assume the Gaussian approximation then we can write

$$\eta^*_{ijt} \cong 1 + 0.064\, u_{ijt}$$

where $\left\{u_{ijt}\right\}$ are i.i. N(0,1) distributed. Since

$$c_{ijt} = \eta^*_{ijt} \exp\left(\gamma_{ij} + m_t\right) = \left(1 + 0.064\, u_{ijt}\right) \exp\left(\gamma_{ij} + m_t\right)$$

the systematic term $\exp\left(\gamma_{ij} + m_t\right)$ will predict c_{ijt} apart from the multiplicative random term, $1 + 0.064\, u_{ijt}$, which with probability 0.95 will vary within (0.872, 1.128).

7. CONCLUSION

In this paper we have discussed a particular model for two-sex marriage behavior. In contrast to earlier work in this field this model is derived from assumptions about the behavior of women and men in the marriage market. We have estimated the parameters of the model on annual marriage data for the years 1985-1994. We have also demonstrated that for this time period, the overall preference for marriage versus staying single decreases (m_t declines over time). However, conditional on marriage, the preferences over age of the potential partners seem to remain unchanged throughout this period, apart from random “noise”, which is represented by a

Stably- or alternatively a normally distributed random variable. The empirical results seem somewhat surprising, given the general belief of systematic changes in marriage behavior during this period.

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APPENDIX

Elasticities

In this appendix we derive expressions for the elasticities of X_{ij}, X^f_{i0} and X^m_{j0} with respect to F_i and M_j for all i and j. Let $\partial_M Q^f_0, \partial_F Q^f_0, \partial_M Q^m_0$ and $\partial_F Q^f_0$ denote the matrices with elements

$$\partial_M Q^f_{0ij} = \frac{\partial \log\left(X^f_{i0}/F_i\right)}{\partial \log M_j}, \quad \partial_M Q^m_{0ij} = \frac{\partial \log\left(X^m_{i0}/M_i\right)}{\partial \log M_j},$$

$$\partial_F Q^f_{0ij} = \frac{\partial \log\left(X^f_{i0}/F_i\right)}{\partial \log F_j} \quad \text{and} \quad \partial_F Q^m_{0ij} = \frac{\partial \log\left(X^m_{i0}/M_i\right)}{\partial \log F_j}$$

and let Q^f and Q^m be the matrices with elements

$$Q^f_{ij} = \frac{X_{ij}}{F_i} \quad \text{and} \quad Q^m_{ij} = \frac{X_{ji}}{M_i}.$$

Then it follows readily from (2.6) to (2.9) that

$$\partial_M Q^f_0 = -\left(I - Q^f\, Q^m\right)^{-1} Q^f, \tag{A.1}$$

$$\partial_F Q^f_0 = \left(I - Q^f\, Q^m\right)^{-1} Q^f\, Q^m, \tag{A.2}$$

$$\partial_F Q^m_0 = -\left(I - Q^m\, Q^f\right)^{-1} Q^m \tag{A.3}$$

and

$$\partial_M Q_0^m = \left(I - Q^m Q^f\right)^{-1} Q^m Q^f . \tag{A.4}$$

Note that X_{i0}^f / F_i and X_{j0}^m / M_j may be interpreted as, respectively the fraction of women of age i and fraction of men of age j that remain single. Consequently, the matrices may be interpreted as elasticities of the probability of remaining single with respect to the respective age group sizes of men and women. From (2.10) it follows that the elasticities of X_{ij} can be computed as

$$\frac{\partial \log X_{ij}}{\partial \log M_k} = \frac{\partial \log\left(X_{i0}^f / F_i\right)}{\partial \log M_k} + \frac{\partial \log\left(X_{j0}^m / M_j\right)}{\partial \log M_k} + \delta_{jk} , \tag{A.5}$$

where δ_{jk} is the Kronecker delta. Thus, to compute the elasticities we only need to know Q^f and Q^m.

By using a suitable metric on the space of quadratic matrices, it is easy to show that

$$\left(I - Q^f Q^m\right)^{-1} = \sum_{n \geq 0} \left(Q^f Q^m\right)^n > 0$$

and similarly when f and m are interchanged. Consequently, (3.1) to (3.4) imply that

$$\partial_M Q_0^f < 0, \quad \partial_F Q_0^f > 0,$$

$$\partial_M Q_0^m > 0 \quad \text{and} \quad \partial_F Q_0^m < 0 .$$

This means that when the number of women in some age group increases then the fraction of single women increases while the fraction of single men decreases. By symmetry the same result holds when women and men are interchanged.

Footnotes

[1] Several of the modeling assumptions made above seem rather strong. Athough we are able to relax some of the assumptions, as will be discussed in Section 4, the assumption of for example no search costs can only be relaxed at the cost of analytic intractability.

[2] The potential number of marriage partners is not greatly affected by such changes, however, as the mortality is negligible in the ages with the highest marriage rates, 20-35, and the number of immigrants is approximately the same as the number of emigrants, although there has been an immigration surplus of young men in recent years.

[3] Recall that the QQ normal plot is obtained by plotting $\Phi^{-1}\left(\hat{G}(x)\right)$ where $\Phi^{-1}(y)$ is the inverse of the cumulative standard normal distribution and $\hat{G}(x)$ is the cumulative empirical distribution function of the variable under study. (In this case η^{*}_{ijt}.)

[4] When estimated α we have set $\beta = 1$. This is necessary to ensure that the probability mass on the negative part of the real line is negligible.

Figures

Figure 1. QQ-plot of the empirical distribution of $\left\{\eta^{*}_{ijt}\right\}$

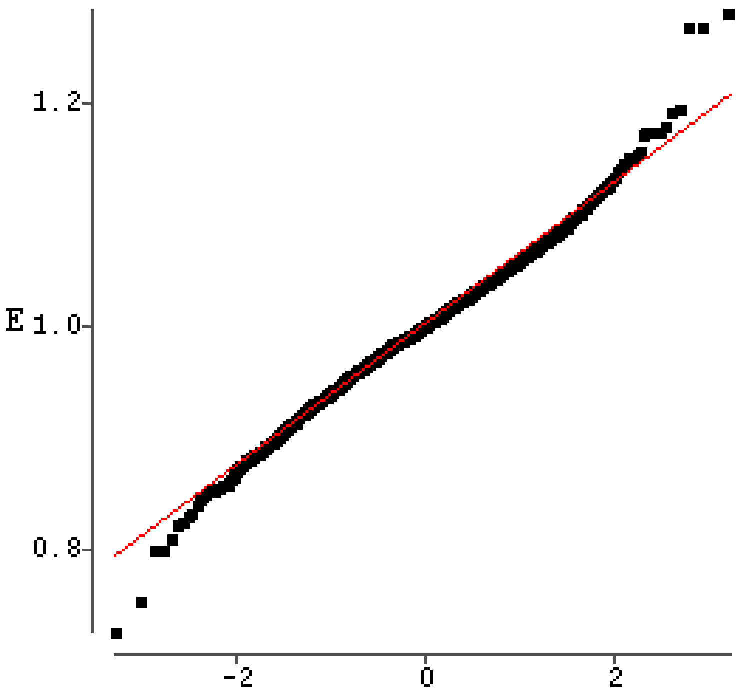


Figure 2. The empirical and the fitted normal density of $\left\{\eta^*_{ijt}\right\}$

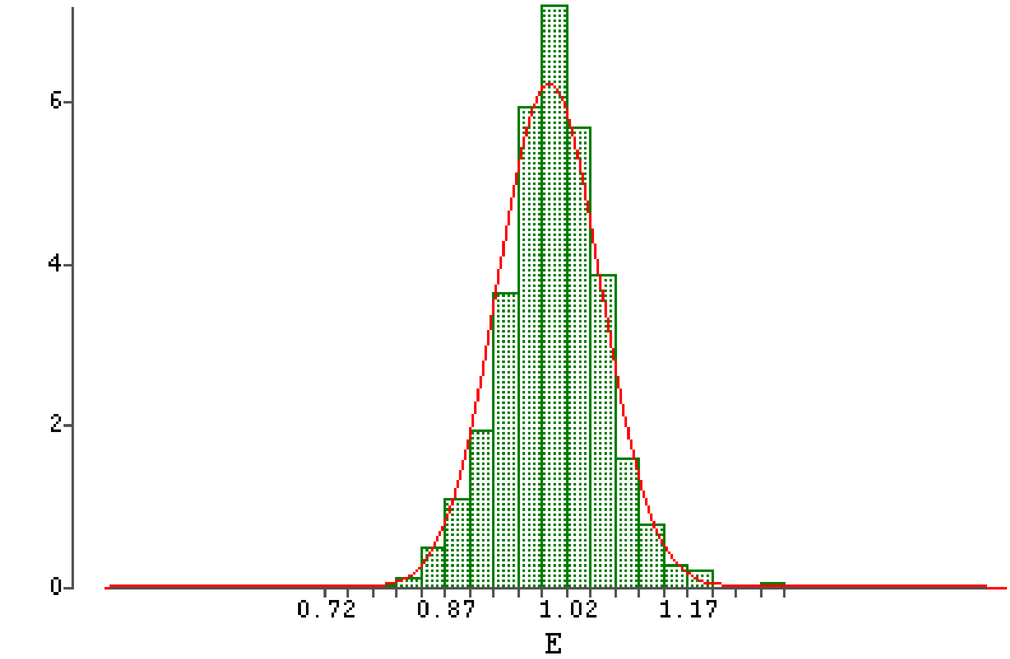


Figure 3. The cumulative empirical and fitted normal distribution of $\left\{\eta^*_{ijt}\right\}$

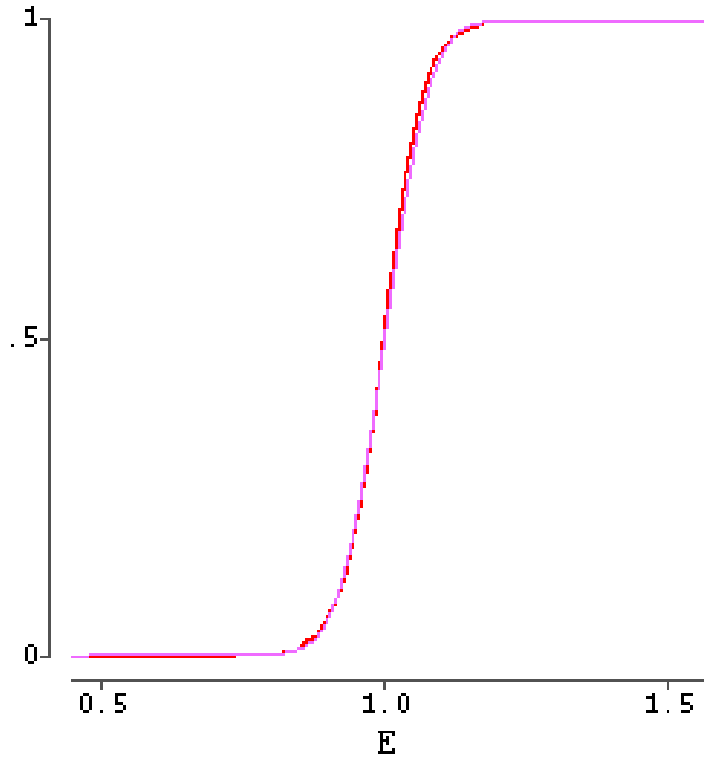


Figure 4. Plots of $\{\gamma_{ij}\}$

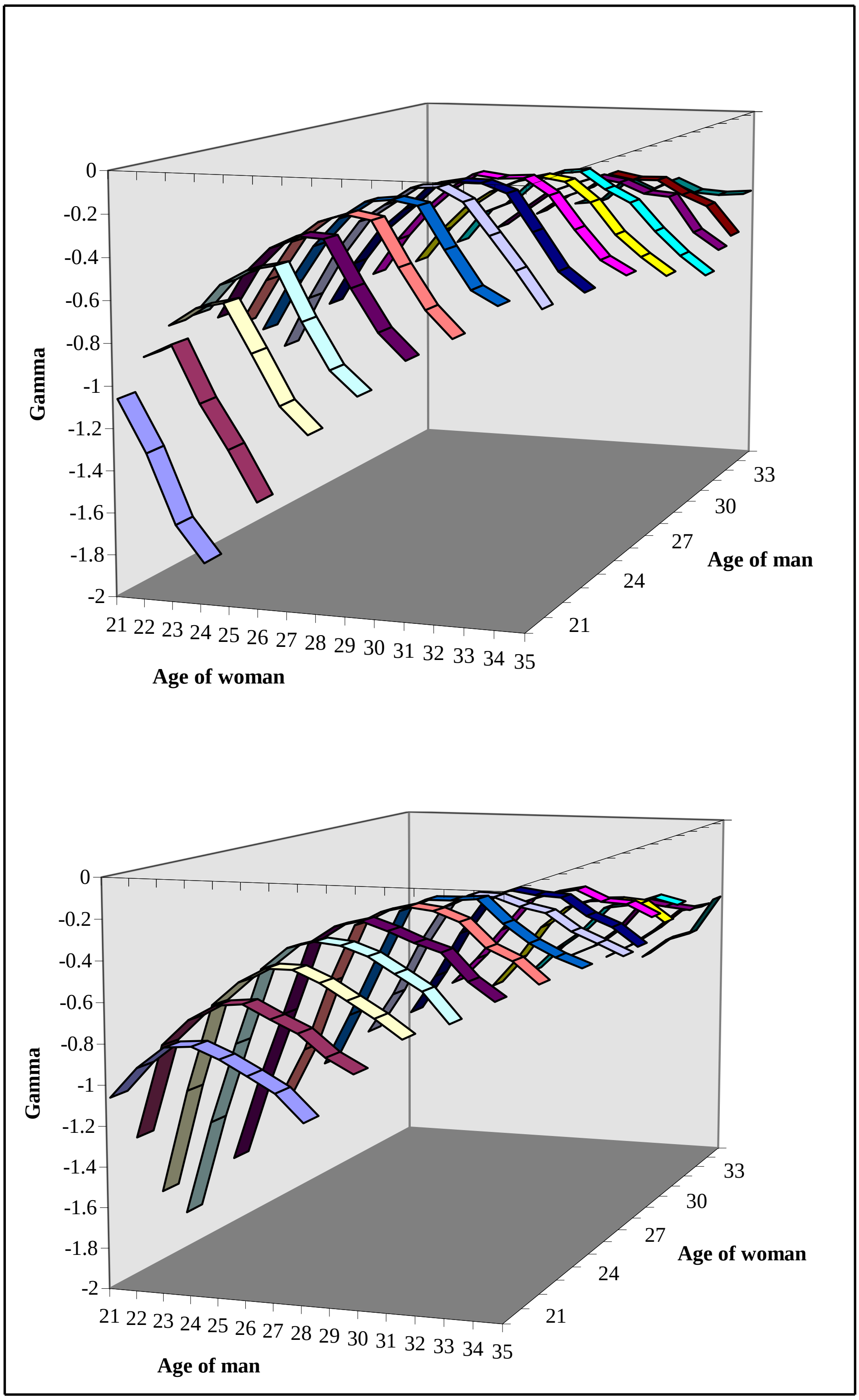
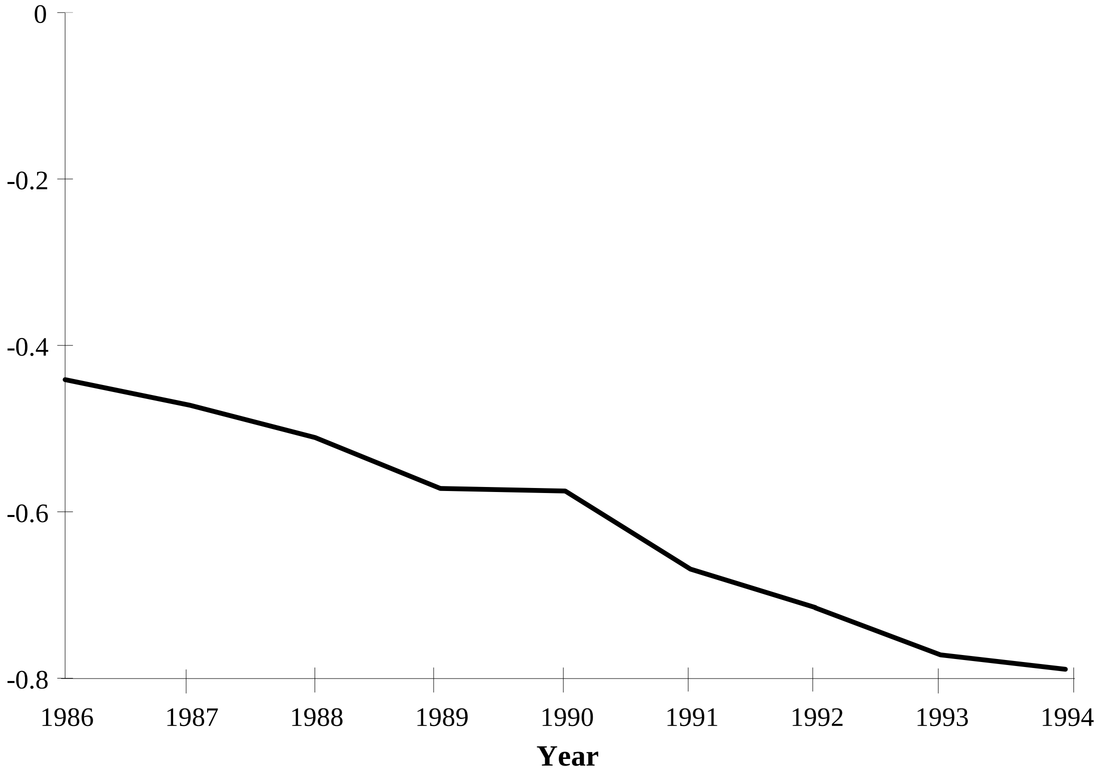


Figure 5. Plot of $\{m_t\}$ from 1986 to 1994



Tables