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Macroeconomic stability or cycles?
The role of the wage-price spiral

Abstract:
We derive aggregate supply (AS) relationships for an intermediate-run macro model.The wage-price spiral provides the conceptual framework for a synthesis of different contesting theoretical and empirical perspectives on the AS curve: the Phillips curve model (PCM) and the wage-price equilibrium correction model (WPECM). The generalized AS curve is grafted into a small macro model. We analyze stability conditions, steady states, and dynamic solutions, using a combination of algebra and simulations. The specification of the AS curve, as a PCM or a WPECM, is shown to be important for all aspects of the model's solution, but within each model also the detailed parameterization is of qualitative importance. For example, endogenous cyclical fuctuations are typical for both nominal and real variables, e.g. inflation and unemployment.

Keywords: AS-AD, cycles, dynamics, equilibrium correction, macroeconomics, nominal rigidity, Phillips curve, unemployment, wage-price spiral.

JEL classification: E24, E30, J50.

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1 Introduction

Models for medium-term macroeconomic forecasting and policy analysis include dynamic formalizations of the demand side behavior, of the policy response, and of the wage-price spiral. It is custom to refer to these parts of the macro models as the aggregated demand (AD) curve, the Taylor rule, and the aggregate supply (AS) curve. It is well known that the specifications of the AD curve and the Taylor rule, in order to represent different monetary policy regimes, give rise to models that display different dynamic responses to supply and demand shocks. In comparison, the supply side of the models has received less attention. However, there are important recent contributions that study the consequences of changing the specification of the supply side. Specifically, Blanchard and Galí (2007) introduce real-wage rigidities to the supply side of the New-Keynesian macro model. More generally, there is case for investigating the 'system implications' of different models of the wage price spiral.[1] The main alternative to the Phillips curve model (PCM hereafter) is the approach that incorporates wage-bargaining and monopolistic price setting aspect in the form of wage and price equations with equilibrium correction (WPECM hereafter), see e.g. Nymoen (1991), Blanchard and Katz (1999), and Montuenga-Gomez and Ramos-Parreno (2005). In this paper, we show that the choice of model for the AS relationship affects the existence of equilibrium, and also the properties of the stable equilibrium if it exists. The paper extends the analysis of the wage-price spiral in Kolsrud and Nymoen (1998) to a macro model of a small open economy.

A separate motivation for focusing on the wage-price spiral AS curve is that slow adjustment processes of nominal prices and wages induce nominal as well as real shocks to have effects on *real* economic variables. Unless all nominal adjustment are perfectly synchronized, relative prices will be dynamically affected by (even nominal) shocks to the economy. Hence, nominal rigidity is an integral part of the propagation mechanism of shocks, and specifically the transmission mechanism through which monetary policy affects the real economy.

From one point of view, sluggish response of macro variables might be problematic. If persistence is ascribed to not fully rational or other inefficient behavior, one could think that the sluggishness implies disequilibrium and instability. But is that necessarily so? We address the equilibrium consequences of nominal rigidity in price and wage setting, and show that the details of the chosen model of nominal rigidity, specifically PCM versus WPECM, are important for overall *dynamic stability.* More generally, our analysis support the view that the wage-price spiral contributes to the total set of macroeconomic frictions that gives rise to different dynamics than the conventional natural rate view, see e.g. Bårdsen and Nymoen (2003) and Karanassou et al. (2009). For example, our results show that if the real economy, as represented by the rate of unemployment, is stabilized at any targeted unemployment level, there is no logically or empirically compelling reason for why the inflation rate should not be dynamically stable. Hence, the natural rate property is not 'natural' at all, but follows from choosing one specification of the wage-price spiral (PCM with additional restrictions) instead of another, equally relevant specification (WPECM).[2]

The rest of this paper is organized as follows. In section 2 we lay out the model of the wage-price spiral. We use a joint framework for two model alternatives; namely a PCM, see Fuhrer (1995), Gordon (1997), and a WPECM consistent with a bargaining model of the long-run wage level, and monopolistic mark-up price setting in steady-state, see Bårdsen et al. (2005). The PCM version of our model is also representative of specifications that contain a hybrid New Keynesian Phillips curve, see Clarida et al. (1999) and Gali et al.

[1] Akram and Nymoen (2009) studies the consequences of different specifications of the supply side for model-based optimal monetary policy.

[2] The asymptotically stable equilibrium rate of unemployment can correspond to a natural rate independently of the foreign steady-state inflation rate, or to a NAIRU which depends on such an inflation rate, see Bårdsen and Nymoen (2009a), but often we will simply use the term natural rate for brevity.

(2001). This is because the rational expectations solution of the hybrid New Keynesian model equation gives the inflation rate as a function of lagged inflation and the current and lagged forcing variable, see Bårdsen et al. (2004). Care is taken to secure logical consistency between the assumptions made about the stochastic properties of the variables and the specification of equations that constitute the dynamic model.

In section 3 we analyze and investigate the dynamic properties of the different versions of the wage-price spiral (AS relationship). We first consider the case of partial equilibrium, where the unemployment rate is exogenous. This step allows us to analyze theoretically the dynamic properties of the different models of the wage-price spiral without intervention from the demand side. We give the conditions under which the WPECM gives an asymptotically stable solution for the rate of inflation, the wage share and the real exchange rate. Since the stable parameter constellations do not cover the special case of the PCM, that specification of the AS curve is generally unstable (as expected). The partial analysis of the wage-price spiral also aids the understanding of the full system, where the unemployment rate is endogenous. As expected, unemployment provides a separate equilibrating mechanism. Asymptotic stability is therefore more typical in this version than with targeted unemployment. That said, we find that the standard Phillips curve (with no equilibrium correction in either wage of price setting) implies a non-stationary wage share. Compared to conventional macro models, this is a surprising result. It is a logical implication of a more structural modelling of the AS than what has become custom elsewhere in macroeconomics. To substantiate our results, in all cases we establish final form expressions for the endogenous variables.

While section 3 establishes the long-run stability properties of the system, section 4 investigates the short- and medium-term dynamic properties by numerical analysis and simulations. The simulated models are furnished with parameter values that are representative of estimation results of PCM and WPECM models for small open economies. First order stability in the form of stationarity is often a logical requirement on a real variable. Second order *in*stability in the form of cyclical fluctuations is a less addressed property of a real variable. We discover that when unemployment is endogenous and interacts with the wage-price spiral, cycles appear in both WPECM and PCM models with realistic parameterizations. It appears that the cycles are inherent properties of the models, created by the delayed feedbacks (nominal rigidity) in the models. They are *not* propagations of imported exogenous cycles. Cycles appear even when all exogenous variables are monotonous and smooth. Hence, in a business cycle perspective, endogenous cycles due to propagation mechanisms in the wage-price spiral appear as a typical feature of the models[3]. This may provide a rationale for stabilization policies even though there is "enough" equilibrium correction in the economy to secure first order asymptotic stability.

In section 5 we summarize our findings and discusses the consequences of certain assumptions. To improve the readability of the paper we have moved all the mathematics and all the numerical and simulation details to the appendices.

2 The model

The basic nominal variables in the model we formulate are: hourly wage w, domestic producer price q, domestic consumer price p, and foreign prices pf in foreign currency, and a nominal exchange rate e. The average labour productivity a and the unemployment rate u are real variables. All variables are in logarithmic scale to facilitate relationships that are linear in the parameters. Appendix A lists all variables and parameters.

[3]This result links all the way back to the 1930s and the business cycle models of Frisch and Kalecki. Both men shared Aftalions idea that a major source of booms and depressions is "time to build" or, in the present context, frictions and nominal rigidities. We return to Frisch and Kalecki in section 5.

2.1 Optimal price and wage levels

Following custom, we refer to the wage and price levels that firms and unions would decide if there were no costs or constraints on adjustment, as the 'optimal' or 'target' values of prices and wages. Another interpretation, following from the essentially static nature of these models, says that optimal prices are those that would prevail in a hypothetical completely deterministic steady-state situation.

We have the following two theoretical propositions of price and wage setting:

$$q^f = m_q + w \qquad - a - \vartheta u, \tag{1}$$
$$w^b = m_w + q + \omega (p - q) + \iota a - \varpi u, \tag{2}$$

with $m_q, m_w > 0$, $0 \leq \omega \leq 1$, $0 < \iota \leq 1$, $\vartheta, \varpi \geq 0$. The variable q^f in (1) refers to the theoretical price determined by monopolistic firms in a situation characterized by known and stable growth in the hourly wage, and in labour productivity. From the profit maximizing conditions it is implied that the mark-up coefficient m_q is positive, because firms choose a point on the elastic part of the demand curve. We follow custom and approximate marginal labour costs with $w - a - \vartheta u$. With reference to Okun's law, we use the rate of unemployment as a proxy for capacity utilization. The case of $\vartheta = 0$ is so often considered as the relevant case that it has earned its own name, namely *normal cost pricing*.

Equation (2) is derived from a theory of wage bargaining, see e.g. Bårdsen et al. (2005, Ch 5). The variable w^b represents the theoretical concept of a *bargained wage*. The right hand side contains variables that might systematically influence the bargained wage. The producer price q and productivity a are central variables in the model of wage formation, see e.g. Nymoen and Rødseth (2003) and Forslund et al. (2008). Based on theory and the empirical evidence, we expect the elasticity ι to be close to one. The impact of unemployment on the bargained wage is given by the elasticity $-\varpi \leq 0$ and is the slope of the wage-curve, see Blanchflower and Oswald (1994).

Equation (2) is seen to include the variable $p - q$, called the wedge (between the producer and the consumer real wage), with elasticity ω. If wage bargaining is first and foremost about sharing of the value-added created by capital and labour then $\omega = 0$ is a logical implication, see Forslund et al. (2008). However, this is a strong assumption to make when we have the total economy in mind. In the service sectors, where unions have less bargaining power, wage setting might be dominated by efficiency wage considerations. Equation (2) is formulated to be consistent with both theories. Since we have in mind a model of the total economy, it is relevant to consider the behavior of the model both with a wedge ($0 < \omega < 1$) and without ($\omega = 0$). The no-wedge model is abbreviated NWM hereafter.[4]

Even though they are static relationships, equation (1) and (2) will play an important role in the dynamic model of the wage-price spiral, as attractors for wages (w_t), and price (q_t), where the subscript t, denote time period.

2.2 Nominal exchange rate and foreign nominal prices

At this point we introduce simple equations for the nominal exchange rate e_t and a foreign price index pf_t. We start by writing pf_t as a random-walk with a positive drift:

$$\Delta pf_t = g_{pf} + \varepsilon_{pf,t}, \quad \text{with} \quad g_{pf} > 0 \quad \text{and} \quad \varepsilon_{pf,t} \sim \mathsf{IN}(0, \sigma^2_{pf}), \tag{3}$$

[4] In empirical studies of wage setting in manufacturing in the Nordic countries, where union-firm bargaining dominates, $\omega = 0$ is typically *not* rejected, see e.g., Nymoen and Rødseth (2003). However, in empirical studies that use aggregate (nation wide) data, $\omega > 0$ is typically reported. Other considerations than profit-sharing might play an important role in the publics sector and in some private sectors, i.e., efficiency wages, or product prices and productivity might be poorly measured in the data.

where $\Delta pf_t \equiv pf_t - pf_{t-1}$ and subscript t denotes the time period. The positive drift term g_{pf} represents 'world price' inflation. It is well known that a null hypothesis of random walk behavior is rarely rejected for nominal price indices in particular, so (3) is intended as a realistic assumption. It is the foreign price index in domestic currency,

$$pi_t = pf_t + e_t, \tag{4}$$

that feeds into the domestic wage-price system. If we let the nominal exchange rate e_t follow a stationary process,

$$\Delta e_t = -\theta_e \, e_{t-1} + \varepsilon_{e,t}, \quad \text{with } \ 0 < \theta_e < 2 \ \text{ and } \ \varepsilon_{e,t} \sim \mathsf{IN}(0, \sigma_e^2), \tag{5}$$

the random walk property of the foreign price index pf_t will nevertheless dominate the stationary exchange rate so that the import price index pi_t is a random-walk variable with drift. For simplicity, we might therefore just as well use a random walk model also for the nominal exchange rate,

$$\Delta e_t = \varepsilon_{e,t}, \tag{6}$$

which is the same as imposing the non-stationary value $\theta_e = 0$ in (5). If the solution of the complete model for domestic inflation is dynamically stable when (6) is used, then stability of the system will also hold for a fixed exchange rate regime ($\theta_e = 1$), or a target-zone regime ($-1 < \theta_e < 1$). Another alternative, a purchasing power mechanism of the form $\Delta e_t = -\theta_e \, (e_{t-1} + pf_{t-1} - p_{t-1}) + \varepsilon_{e,t}$, would also stabilize rather than de-stabilize the wage-price spiral.

2.3 The wage-price spiral

We first use (1) and (2) to define the two optimal *real* wages as stochastic variables rw_t^f and rw_t^b that are driven by q_t, p_t, a_t and u_t:

$$\begin{aligned} rw_t^f &\equiv w_t - q_t^f = -m_q \qquad\qquad\qquad + \; a_t + \vartheta \, u_t & (7) \\ rw_t^b &\equiv w_t^b - q_t = \quad m_w + \omega \, (p_t - q_t) + \iota \, a_t - \varpi \, u_t. & (8) \end{aligned}$$

rw_t^b and rw_t^f are random walk variables by implication, because the random walk variable a_t is a common driving factor in both (7) and (8), and $\iota > 0$ has been assumed above. For the rate of unemployment, u_t, we maintain stationarity throughout the paper (but with the understanding that deterministic regime shifts have been filtered out). The specification of the process for u_t is the topic of the next subsection.

With rw_t^f and rw_t^b being random walks, logical consistency requires that also the actual real wage $rw_t \equiv w_t - q_t$ is a random walk variable. Next, define the firms' and the workers' real wage "gap":

$$\begin{aligned} ecm_t^f &\equiv rw_t - rw_t^f = q_t^f - q_t = w_t - q_t - \; a_t \qquad\qquad\qquad - \vartheta \, u_t + m_q, & (9) \\ ecm_t^b &\equiv rw_t - rw_t^b = w_t - w_t^b = w_t - q_t - \iota \, a_t - \omega \, (p_t - q_t) + \varpi \, u_t - m_w. & (10) \end{aligned}$$

If the economic theory is empirically relevant then both ecm_t^b and ecm_t^f are stationary variables, i.e. they have finite variability around constant levels. This is tantamount to assuming two cointegrating relationships between the three random walk variables rw_t^b, rw_t^f, and rw_t, cf. Engle and Granger (1987).

Cointegration between real wages is the same as cointegration between q_t and q_t^f, and between w_t and w_t^b. Cointegration implies equilibrium correction dynamics, and we get the following equilibrium correction model for wages and prices:[5]

$$\begin{aligned} \Delta q_t &= c_q + \psi_{qw} \, \Delta w_t + \psi_{qpi} \, \Delta pi_t - \varsigma \, u_{t-1} + \theta_q \, ecm_{t-1}^f + \varepsilon_{q,t}, & (11) \\ \Delta w_t &= c_w + \psi_{wq} \, \Delta q_t + \psi_{wp} \, \Delta p_t - \varphi \, u_{t-1} - \theta_w \, ecm_{t-1}^b + \varepsilon_{w,t}, & (12) \end{aligned}$$

[5] We use the simultaneous equation representation since it is convenient for economic interpretation.

where $\psi_{qw}, \psi_{qpi}, \psi_{wq}, \psi_{wp}, \varsigma, \varphi, \theta_q, \theta_w \geq 0$, $\varepsilon_{q,t} \sim \mathsf{IN}(0, \sigma_q^2)$ and $\varepsilon_{w,t} \sim \mathsf{IN}(0, \sigma_w^2)$. Substituting the right hand sides of (9)-(10) for the *ecm*s and using the following definition of the consumer price[6],

$$p = \phi\, q_t + (1-\phi)\, pi_t \quad \text{with } 0 < \phi < 1 \text{ reflecting the openness of the economy,} \tag{13}$$

we obtain a dynamic system that corresponds to the supply-side of standard macroeconomic models for medium-term analysis:

$$\begin{aligned} \Delta q_t &= (c_q + \theta_q\, m_q) + \psi_{qw}\, \Delta w_t + \psi_{qpi}\, \Delta pi_t - \mu_q\, u_{t-1} \\ &\quad + \theta_q\, (w_{t-1} - q_{t-1} - a_{t-1}) + \varepsilon_{q,t}, \end{aligned} \tag{14}$$

$$\begin{aligned} \Delta w_t &= (c_w + \theta_w\, m_w) + \psi_{wq}\, \Delta q_t + \psi_{wp}\, \Delta p_t - \mu_w\, u_{t-1} \\ &\quad - \theta_w\, (w_{t-1} - q_{t-1} - \iota\, a_{t-1}) + \theta_w\, \omega\, (p_{t-1} - q_{t-1}) + \varepsilon_{w,t}, \end{aligned} \tag{15}$$

$$\Delta p_t = \phi\, \Delta q_t + (1-\phi)\, \Delta pi_t, \tag{16}$$

We have introduced $\mu_q = \theta_q\, \vartheta + \varsigma$ and $\mu_w = \theta_w\, \varpi + \varphi$. They will be discussed below. Equation (16) is (13) in differenced form[7].

The coefficient θ_w in (15) determines the degree or speed of equilibrium correction in the wage setting. It is thus a key parameter. In the case of $\theta_w > 0$, the wage increase in the current period is negatively affected by last period's real wage and the rate of unemployment, and positively affected by productivity and the wedge.[8] As noted above, this case captures the main implication of both wage bargaining models and efficiency wage models. A strictly positive θ_w also implies that when we consider (15) as a single equation model for wages, that model is asymptotically stable and the long-run steady-state solution takes the form given in (2), so the dynamic relationship and the long-run wage equation are internally consistent.

2.4 Wage bargaining and Phillips curves

In the case of wage bargaining/efficiency wage model ($\theta_w > 0$), the rate of unemployment u_t is already affecting wage growth via the term $\theta_w\, \varpi\, u_{t-1}$. Then the only logically consistent value of φ is zero. In the following we use the convention:

$$\text{Wage bargaining model: } \theta_w > 0,\ \varpi > 0 \text{ and } \varphi = 0 \ \Rightarrow\ \mu_w = \theta_w\, \varpi. \tag{17}$$

We also consider the case of $\theta_w = 0$, where wage dynamics clearly do no support a long-run wage equation of the bargaining type. With $\varphi > 0$ the specification corresponds to a wage Phillips curve (WPCM hereafter), typically found to represent the relationship between aggregate wage inflation and unemployment in the United States, see Blanchard and Katz (1999). For use in the following, we define:

$$\text{Wage Phillips curve model (WPCM): } \theta_w = 0 \text{ and } \varphi > 0 \ \Rightarrow\ \mu_w = \varphi. \tag{18}$$

We make a similar distinction in firms' price-setting between the case where the rate of unemployment affects the mark-up relationship ($\vartheta > 0$) and the Phillips-curve case of $\theta_q = 0$:

$$\text{Price mark-up model} \ :\ \theta_q > 0 \text{ and } \varsigma = 0 \ \Rightarrow\ \mu_q = \theta_q\, \vartheta, \tag{19}$$

$$\text{Price Phillips curve model} \ :\ \theta_q = 0 \text{ and } \varsigma > 0 \ \Rightarrow\ \mu_q = \varsigma. \tag{20}$$

In the latter case there is an effect of u_{t-1} directly on Δq_t by $\varsigma > 0$.

[6]Note that, due to the log-form, $\phi = im/(1-im)$ where im the import share in private consumption.

[7]For the coefficients ψ_{wq}, ψ_{qw} and ψ_{wp}, ψ_{qpi}, the non-negative signs are standard in economic models. Negative values of θ_w and θ_q imply explosive evolution in wages and prices (hyperinflation), which is different from the low to moderately high inflation scenario that we have in mind for this paper.

[8]Although equilibrium corrections in wage setting ($\theta_w > 0$) and price setting ($\theta_q > 0$) stabilize the dynamics of the system, "too much" equilibrium correction, for example $\theta_w \geq 2$ can endanger stability. However, values of θ_w in the region $1 < \theta_w < 2$ are usually not regarded as economically meaningful, because the implied negative autocorrelation ("volatility") in the nominal wage level is unrealistic.

The productivity a_t is an important conditioning variable of the price and wage system. In order to solve the model, a process for a_t has to be formulated. For simplicity, we assume an unstable process with a positive constant growth rate g_a:

$$\Delta a_t = g_a + \varepsilon_{a,t}, \quad \text{with} \;\; g_a > 0 \;\; \text{and} \;\; \varepsilon_{w,t} \sim \mathsf{IN}(0, \sigma_w^2). \tag{21}$$

The equation reflect a trend-like growth that we typically observe for average labour productivity. The residual $\varepsilon_{a,t}$ represents productivity shocks.

The above specification of the supply side does not exclude that expectations errors can be added in a more enhanced version of the model though, see Nymoen (1991). In its present form the model conforms to perfect expectations about current period wage and price increases.

2.5 Aggregate demand relationship and macroeconomic regimes

In order to close the model we need to take account of how the rate of unemployment is related to aggregate demand, which in turn is influenced by one or more of the variables that appear in the supply-side model above. Because focus is on the role of equilibrium correction and nominal rigidity in the supply side, we keep the model of the demand side down to a minimal version. We notice first that the *real exchange rate* $re \equiv pi - q$ reflects the price competitiveness of the domestic production relative to the imports. According to standard macroeconomic theory, aggregate demand increases if there is a real depreciation (re increases), and, with reference to *Okun's law,* the rate of unemployment is reduced. The only other economic variable that we introduce explicitly is the variable gs_t. It represents a measure of government real expenditure or possibly another measure of fiscal policy stance. Hence, the aggregate demand relationship is simply represented by the log of the unemployment rate in percent:

$$u_t = (c_{u0} + c_{u1} D_t) + \alpha\, u_{t-1} - (\rho\, re_{t-1} + \tau\, gs_t) + \epsilon_{u,t}, \quad \text{with } \varepsilon_{w,t} \sim \mathsf{IN}(0, \sigma_w^2). \tag{22}$$

Except for c_{u0} and c_{u1} the coefficients are logically non-negative: $\alpha, \rho, \tau \geq 0$. We presume that $\alpha < 1$, but we shall see below that this limitation is generally *not* necessary for stationarity. An increase in price competitiveness (re) or government expenditure (gs) reduces unemployment (or increases capacity utilization). We assume, for simplicity, that unemployment reacts to a real depreciation (re) with a lag. Without a lag the result would be qualitatively equal.

In order to simulate the dynamic response a large shock to the economy, we include a step dummy $D_t \in \{0, 1\}$, with $D_t = 1$ implementing an exogenous permanent shock (or shift) of size c_{u1} to the unemployment level. In the analysis below the shift term is not needed, and we simplify the constant term to c_u. The error term $\epsilon_{u,t}$ might represent a temporary shock to the aggregated demand or to labour supply.

The most conspicuous omission from (22) is perhaps the real interest rate, which will have to be included in more realistic versions of the model. A possible interpretation of the present formulation of the model is that the real interest rate is kept constant, by nominal interest rate adjustments, at a long-run equilibrium level, perhaps motivated by a wish to keep an 'even flow' of real investments. Logically, the monetary policy will then have to be accommodative in order to equilibrate the domestic money market (through quantitative easing and tightening).

We investigate the dynamics of the model macro economy where unemployment is endogenous and interacts with the price and wage formation. That requires $\rho > 0$, and that gs_t is an exogenous variable. To emphasize the coordinating role of unemployment for the price and wage growth, and thereby its stabilizing function in the model, we contrast the results with those in a regime where unemployment is an exogenous variable in the model. In such a regime, we imagine that the equilibrium level of unemployment is targeted by economic policy. If u^* denotes the targeted level of unemployment, this regime is characterized by $u_t \to u^*$ from any given initial level u_0. Since the lagged real exchange rate re_{t-1} is pre-determined in (22), it follows that government expenditure $gs_t = c_{gs} - \rho\, re_{t-1}/\tau$ keeps unemployment at a constant level $u^* = (c_u - \tau\, c_{gs})/(1-\alpha)$. Consequently, unemployment has no interactive role to play in the dynamics of the system. For simplicity and without loss of generality we let $\rho = \tau = 0$ in the numerical simulations in the regime with targeted unemployment. The regime with *endogenous* unemployment is implemented by $\rho > 0$ and $\tau = 0$. We can do that since government expenditure only appears in the unemployment equation and nowhere else in the model.

In both regimes, u_t is subject to a shift. Specifically, in the simulations reported below, D_t changes from 0 to 1 early in the simulation period. The coefficient c_{u1} is positive, so that the permanent shock increases unemployment in both regimes. The shock is *not* counteracted by policy in any regime. In the regime with targeted unemployment the whole effect of the shift is therefore on unemployment. In the regime with endogenous unemployment the feedback from the exchange rate moderates the effect of the shock. This makes it possible to compare the responses of the system — with different specifications of the wage-price spiral — to identical shocks in the two regimes.

2.6 First and second order (in)stability

The wage-price spiral (14)-(16) is characterized by both nominal rigidity and friction. The responses of the nominal variables to each other are partial (parameters are less than 1) and delayed (explanatory variables are backdated). Inertia allows the variables to develop differently over time. At the same time, the lagged equilibrium correcting terms serve as 'attractors', and might coordinate the development of the variables.

There is a positive trend in foreign prices (3) and consequently in the import price (4). There is also a trend in productivity (21). The wage-price spiral passes the trending properties of these exogenous variables onto the nominal wage w and the nominal prices q and p. Although the trends in these nominal variables are not equal, there might be linear combinations that have no trend. We shall see that the real wage aspirations (7)-(8) and endogenous unemployment ($\rho > 0$ in (22)) are able to synchronize the nominal growth processes so that certain linear combinations among non-stationary nominal variables become stationary real variables. The nominal instability gets harnessed into proportional or 'real' stability. Specifically, even though there is no stable equilibrium for the nominal exchange rate, the real exchange rate may have a stable equilibrium solution.

To avoid a trend in the composite real variables — the *productivity corrected real wage* $prw \equiv w - q - \iota\, a$ and the real exchange rate $re \equiv pi - q$ — they need to influence wage growth Δw and producer price inflation Δq. The equilibrium correction terms (9)-(10) bring information about these real variables into the wage-price spiral. The wedge $p - q$ represents price competitiveness, and it is proportional to the real exchange rate. Information about the real exchange rate is also brought into the wage-price spiral through endogenous unemployment ($\rho > 0$). The real information in the wage-price spiral is not distributed equally between the wage growth and inflation. There is an information asymmetry which causes instability in certain model versions.

It is common to call a variable stable if it is stationary, and unstable if it is trending. In addition to this 'first order' (in)stability, we note a 'second order' instability: cyclical fluctuations. They are well known features of economic variables, and in the present model an endogenous variable might fluctuate around a stable level or a trend. The cycles might persist or cease over time. They are generated by the interaction of the endogenous variables in the model. The cycles do *not* have exogenous causes. If a model is cyclical, all endogenous variables — nominal and real — move in cycles because they are interconnected.

In the next section we analyze under which conditions certain real variables are stable or not, and whether their instabilities are due to trends and/or cyclical fluctuations.

3 Dynamic analysis

The model consists of 8 equations: (3), (4), (6), (14)-(16), (21) and (22). They determine time series for $pf_t, pi_t, e_t, q_t, w_t, p_t, a_t$ and u_t as functions of start values, gs_t and disturbances $\varepsilon_{i,t} \sim \mathsf{IN}(0, \sigma_i^2)$, $i = pf, e, q, w, a, u$. w_t and q_t are simultaneously determined, pi_t and p_t are identities, while pf_t, e_t and a_t are autonomous. When unemployment is targeted at u^*, it is effectively exogenous. When it is endogenous, u_t is predetermined.

3.1 Reduced form model

The wage-price spiral is given by the three equations (14)-(16). The structural form model for the two interacting nominal variables q and w can be transformed into a reduced form model for two interacting real variables: the real exchange rate and the productivity corrected producer real wage. The reduced form equation for the real exchange rate is

$$re_t = l\, re_{t-1} - k\, prw_{t-1} + e\, \Delta pi_t + b\, a_{t-1} + n\, u_{t-1} - d + \epsilon_{re,t}. \qquad (23)$$

The reduced form equation for the productivity corrected producer real wage is

$$prw_t = \lambda\, re_{t-1} + \kappa\, prw_{t-1} + \xi\, \Delta pi_t - \iota\, \Delta a_t + \beta\, a_{t-1} - \eta\, u_{t-1} + \delta + \epsilon_{ws,t}. \qquad (24)$$

The domains of the structural parameters in equation (14)-(16) imply that all reduced form coefficients in (23) and (24), except d and δ, lie in the interval [0,1]. Appendix B contains the derivation of (23) and (24), explicit expressions for the composite reduced form coefficients as functions of the structural (form) parameters, and also expressions for the reduced form error/shock terms. The unemployment rate (22) is a real variable, and is already on a reduced form.

The dynamic system of three reduced form equations (22)-(24) can be expressed as a single vector equation $\mathbf{y}_t = \mathbf{R}\,\mathbf{y}_{t-1} + \mathbf{P}\,\mathbf{x}_t + \boldsymbol{\epsilon}_t$, where the vector $\mathbf{y} = (re, prw, u)'$ contains the endogenous variables, the vector $\mathbf{x} = (\Delta pi, \Delta a, a_{-1}, gs, 1)'$ contains the exogenous variables and 1 (for the constant term), and the vector $\boldsymbol{\epsilon}$ contains the reduced form shocks. The reduced form coefficients are the elements of the 3×3 matrix $\mathbf{R}$ and the 3×5 matrix $\mathbf{P}$. The vector equation for the reduced form of the model is

$$\underbrace{\begin{pmatrix} re_t \\ prw_t \\ u_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k & n \\ \lambda & \kappa & -\eta \\ -\rho & 0 & \alpha \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ prw_{t-1} \\ u_{t-1} \end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & b & 0 & -d \\ \xi & -\iota & \beta & 0 & \delta \\ 0 & 0 & 0 & -\tau & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ a_{t-1} \\ gs_t \\ 1 \end{pmatrix}}_{\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{prw,t} \\ \epsilon_{u,t} \end{pmatrix}}_{\boldsymbol{\epsilon}_t}. \qquad (25)$$

3.2 Real trends

The reduced form equations (23) and (24) show that each period t the real exchange rate re_t and the productivity corrected producer real wage prw_t both get a positive and *increasing* contribution from the trending productivity a_{t-1} as long as $b, \beta > 0$. That makes the system (25) unstable. Appendix B shows that both $b, \beta > 0$ if

1. $0 < \iota < 1$ (less than full reward for productivity in wage target)
   and $\theta_q > 0$ (no Phillips curve in producer price setting).

In case 1 above, inflation (14) is influenced by the *wage share* $ws \equiv w - q - a$ while nominal wage growth (15) is influenced by the productivity corrected real wage $prw \equiv w - q - \iota\, a$. The different influences cause imbalanced nominal growth rates, which induce trends in the real exchange rate and the productivity corrected real wage (or the wage share).

We have to impose $b = \beta = 0$ to purge the system from trends due to the deterministic growth in productivity. If inflation is determined by a Phillips curve ($\theta_q = 0 \Rightarrow b = \beta = 0$), the differences in equilibrium correcting terms (wage share vs. productivity corrected real wage) for price and wage growth is eliminated, and with it a cause of instability. Note that there is no symmetry. If wage growth is determined by a Phillips curve ($\theta_w = 0$), the difference in the equilibrium correcting term is also eliminated. But with $\theta_w = 0$ an effect from the real exchange rate gets eliminated too. The wedge is an equilibrium correcting term necessary for stability of the real exchange rate when unemployment is targeted at a constant rate. Alternatively, if productivity growth is fully reflected ($\iota = 1 \Rightarrow b = \beta = 0$) in the nominal wage target (2), the price and wage growth get affected by the *same* productivity corrected real wage, and the cause of instability has turned into a stabilizer. This is a more attractive stability condition than the other alternative in condition 1 above, a price Phillips curve ($\theta_q = 0$). Hence, we impose $\iota = 1$ to make $b = \beta = 0$. Then the productivity corrected real wage $prw \equiv w - q - \iota\, a$ becomes the wage share $ws \equiv w - q - a$, which is used in the following. While this is necessary for stability, it is not sufficient.

We see from (22) that government *real* expenditure gs has to be a non-trending variable to avoid a trend in unemployment. Alternatively, gs could cointegrate with a trending real exchange rate re. But a trending real exchange rate would be a cause of instability itself. The only other alternative is $\tau = 0$, which eliminates government expenditure from the model. But that would remove our mechanism for switching between regimes with endogenous and exogenous unemployment. We assume that government expenditure is integrated of the same order as the real exchange rate. That keeps unemployment from trending.

After the elimination of trends due to exogenous causes, the stability properties of the system in (25) depends on the recursion matrix $\mathbf{R}$ and its eigenvalues. The general analytic expressions for the eigenvalues of the 3×3 matrix $\mathbf{R}$ are too large and complex to be of much help. But, in the PCM, the restrictions $\theta_w = \theta_q = 0$ simplify the recursion matrix $\mathbf{R}$ and make $l = \kappa = 1$ and $\lambda = k = 0$, cf. Appendix B and C. The appendices show that the wage share is trending if

2. $\theta_w = \theta_q = 0$ (a Phillips curve in both wage and price setting (PCM)).

The restrictions remove all information about the wage share from the wage-price spiral. That causes a trend in the wage share. The price wedge also gets eliminated. But since the real exchange rate indirectly affects the wage and price growth through the unemployment rate, the real exchange rate is not trending unless unemployment is targeted.

While analysis shows that the PCM is unstable, we have to resort to numerical investigations into the question of stability when conditions 2 do not hold. In Appendix D, we calculate the magnitudes of the eigenvalues for a number of combinations of parameter and coefficient values. Before we discuss the results, we address (in)stability in the regime with a targeted rate of unemployment.

3.3 Real trends in a regime with targeted unemployment

A targeted unemployment rate $u_t = u^*$ does not react to any other variable than exogenous government expenditure gs_t, and therefore cannot serve a stabilizing function. This suggests that the model with targeted unemployment is unstable in more cases than the model with endogenous unemployment. A targeted unemployment rate requires $gs_t = c_{gs} - \rho\, re_{t-1}/\tau$ to cancel any effect of the real exchange rate. Alternatively, $\rho = 0$

and real government expenditure is constant, and unemployment (22) is an autoregressive process. In any case, unemployment is effectively exogenous in (25). The 3-dimensional vector equation can therefore be reduced to a 2-dimensional vector equation, cf. (31) in Appendix B. That makes eigenvalue analysis feasible. The regime with targeted unemployment is interesting because it provides analytical insights into the dynamics of the wage-price spiral.

Conditions 1 and 2 are *necessary* for non-trending behavior of both the real exchange rate and the wage share. But they are *not sufficient* when unemployment is targeted and unable to serve as a stabilizer. Certain other parameters in the structural model (14)-(16) have to be strictly positive not to eliminate stabilizing mechanisms, or less than one to avoid too strong effects. Appendix B shows that the model has one eigenvalue $r = 1$, and thus a trend in the real exchange rate or the wage share, in the following five cases:

3. $\omega = 0$ (no price-wedge in wage formation, denoted NWM),
4. $\theta_w = 0$ (Phillips curve in wage setting, denoted WPCM),
5. $\theta_q = \psi_{qw} = 0$ (Phillips curve in price setting and no regard for wage growth),
6. $\theta_q = 0$, $\psi_{qw} = 1$ (Phillips curve in price setting, full pass-through of wage growth),
7. $\psi_{qw} = \psi_{wq} + \phi\,\psi_{wp} = 1$.

Not all the unstable cases 1-7 are economically interesting. We address case 2, 3, and 4 in a regime with targeted unemployment and in a regime with endogenous unemployment.

The wage-price spiral passes trends in productivity and foreign prices on to the nominal wage and domestic prices. Trends in the nominal variables cause trends in the real exchange rate and the wage share *unless* the nominal growth rates are aligned by equilibrium correcting mechanisms in the wage-price spiral. The real exchange rate is trending in case 2, 3 and 4 because the restriction(s) eliminate the wedge. Without the wedge there is no information about relative prices in the wage-price spiral to align domestic inflation with foreign inflation, and thus keep the real exchange rate from trending. The wage share is trending only in case 2.

Another case also worth assessing is motivated from economics. Dynamic homogeneity is often regarded as a necessary feature of a model that is to be used for policy advise in order to avoid 'monetary illusion' or give a false impression of the existence of a menu between rates of unemployment and inflation. *Dynamic price and wage homogeneity* entails the following restrictions on the structural parameters: $\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$. We exclude the extreme form of homogeneity where $\psi_{qw} = \psi_{wq} = 1$ and $\psi_{qpi} = \psi_{wp} = 0$. It is a special case of the unstable case 7. As a matter of fact the model we have formulated above does not have a solution when conditions 7 hold.

Stability of the model when unemployment is targeted and thus effectively exogenous, is synonymous with *no trend* in the real exchange rate nor the wage share. Only the unrestricted model (WPECM) is free of trends and stable. Dynamic wage and price homogeneity is not able to stabilize the model when there is reduced or no equilibrium correction.

If unemployment is endogenous, the model still does not have a solution when conditions 7 hold. On the other hand, conditions 3-6 no longer make the model generally unstable. The reason is that the wage-price spiral interacts with unemployment, so that unemployment takes on a stabilizing role.

3.4 (In)stability in a regime with endogenous unemployment

The unemployment equation (22) alone suggests that $\alpha = 1$ makes unemployment a random walk, and thus *destabilizes* the system. But, the question of stability is not decided by the properties of a single equation. When the equation is an integral part of a system of equations, *(in)stability is a system property.* There is feedback between unemployment and the real exchange rate. Substituting the reduced form expression (23) for the real exchange rate into the unemployment equation (22) allows us write unemployment as a lag polynomial:

$$(1 - \alpha\, L + \rho\, n\, L^2)\, u_t = const - \rho\, l\, re_{t-2} + \rho\, k\, ws_{t-2} - \rho\, e\, \Delta pi_{t-1} + \tau\, gs_t + shock_t,$$

where $const = c_{u0} + \rho\, d$, $shock_t = \epsilon_{u,t} - \rho\, \epsilon_{re,t-1}$ and the lag operator $L^s : u_t \to u_{t-s}$. We see directly that $\alpha = 1$ and $\rho = 0$ make unemployment a random walk and autonomous, i.e. effectively exogenous. But that model does not belong to the present regime, which presupposes $\rho > 0$. Alternatively, $\alpha = 1$ and $n = 0$ would cause instability, but Appendix B shows that $n > 0$ always. With unemployment depending on the real exchange rate re, and the real exchange rate depending on the wage-price spiral and on unemployment, we cannot infer about (in)stability from the unemployment equation alone.

From section 3.3 we know that the real exchange rate is stationary for any given rate of unemployment as long as none of conditions 1-7 hold. Equation (22) says that unemployment is stationary if the real exchange rate is stationary. It is thus tempting to presume that all three variables are stationary. But, again, general stability of the system cannot be established by such a shortcut, since the reasoning is partial and circular. We shall see that conditions 3-6 do not entail instability.

It is not feasible to investigate stability of the dynamic system (25) purely analytically and in general. We thus resort to a mix of analysis, numerical investigations and simulations of the model. Appendices B, C and D summarize the methods and the findings. The main result is that endogenous unemployment forms a feedback loop with the real exchange rate, and thereby makes the system more stable for parameters and coefficient values within 'realistic' ranges. This is not surprising, and it is indeed in accordance with the aggregate demand and supply model in macroeconomic textbooks, see e.g. Sørensen and Whitta-Jacobsen (2005). But we also discover unexpected dynamics and instability in cases with not 'unrealistic' parameter and coefficient values. In our linear model, going from a regime with two interacting real variables to a regime with three interacting real variables opens for more complicated dynamics. While the real wage targets and the equilibrium corrections are able to neutralize trends, nominal and real rigidity (also called frictions) due to delayed and partial responses might cause cyclical fluctuations.

Trends in the nominal variables do not cause a trend in a real variable as long as there is information about the level of the real variable in the growth processes of its constituent nominal variables. Unemployment carries information about the real exchange rate into the wage-price spiral. That keeps the real exchange rate from trending in all models. The wage share is eliminated entirely from the wage-price spiral by restrictions 2. Consequently the wage share is trending only in the PCM. However, absence of real trends does not imply that the model is stable.

Cyclical fluctuations are possible in all models with endogenous unemployment. Cycles around a constant level makes a 'stable' variable less stable. The more persistent cycles, the more unstable variable. If cycles occur in a model, all endogenous variables are cyclical since they are interdependent. Occurrence, frequency, amplitudes and persistence of cyclical fluctuations depend on the parameter and coefficients values that govern the intra-action of the wage-price spiral and in particular its inter-action with the endogenous unemployment process. Damped, persistent and increasing oscillations are possible in *all* models (WPECM, NWM, WPCM, PCM) for parameterizations that are not 'unrealistic'.

Stability of the model requires that the real variables (re, ws and u) all converge to constant steady-state levels in the absence of shocks. It follows that the nominal variables (q, w and p) must converge to constant steady-state growth rates determined by constant productivity growth and constant foreign inflation. While absence of real trends is a necessary condition for stability, it is not sufficient. If there are cyclical fluctuations around constant real levels, asymptotic stability requires that the cycles are damped. They might nevertheless dominate in the short and medium run, and be revitalized by temporary shocks.

Apart from the PCM which has a trending wage share, the other models are all stable for 'realistic' parameterizations. But, with certain 'not unrealistic' parameterizations do all models display non-damped cycles around stable levels or, in the PCM, around a trend.

3.5 Steady-states in the regime with endogenous unemployment

With help from the analytic results in the regime with a targeted unemployment rate, we derive expressions for steady-state levels in WPECM, NWM and WPCM, and the steady-state growth rates in the PCM. Appendix C contains the derivations.

Stable models

The dynamic system (25) is driven by the exogenous variables $\Delta pi, \Delta a, gs$ and temporary shocks. Stability condition $\iota = 1$ eliminates the exogenous variable a. If we presume that the parameterization is 'realistic' and makes the system stationary, all three variables will be asymptotically constant in the absence of temporary shocks to the system. Ignoring temporary shocks and substituting deterministic growth rates g_{pf} and g_a for Δpi_t and Δa_t, the steady-state solutions for the real variables are:

$$re = e'_{ss}\, g_{pf} + \underset{\pm}{b'_{ss}}\, g_a - \varsigma_{ss}\, gs + \underset{\pm}{d'_{ss}}, \tag{26}$$

$$ws = \underset{\pm}{\xi'_{ss}}\, g_{pf} - \beta'_{ss}\, g_a - \zeta_{ss}\, gs - \underset{\pm}{\delta'_{ss}}, \tag{27}$$

$$u = -\,\mathfrak{e}_{ss}\, g_{pf} - \underset{\pm}{\mathfrak{b}_{ss}}\, g_a - \mathfrak{c}_{ss}\, gs + \mathfrak{d}_{ss}. \tag{28}$$

The equations show that the *stable* endogenous real variables fluctuate around levels that are determined by constant productivity growth (g_a), constant foreign *nominal* price inflation (g_{pf}) and government expenditure (gs). Less stable variables cycle around the same levels. Appendix C contains explicit expressions for the steady-state coefficients. The expressions involve structural parameters from the wage and price formation and coefficients from the unemployment equation. The stable level for each variable thus depends on *all* parameters and coefficients in the model. The explicit sign of each term in a sum on the right hand side follows analytically from the structural form. Five of the twelve coefficients might be positive or negative depending on the parameterization. Below those coefficient we have put a $\pm$ sign. The other coefficients are positive.

In a steady-state, a higher level of government expenditure gs implies lower levels of price competitiveness ($re \equiv pi - q$), i.e. a real appreciation. It also implies lower levels of wage share $ws \equiv w - q - a$ and unemployment u. Government expenditure is not a variable in the wage-price spiral. It affects only unemployment directly. A higher level of spending causes higher employment and higher capacity utilization in general. A lower unemployment level implies a higher wage level, and a higher wage level causes a higher price level through the wage-price spiral. A lower unemployment level is also a proxy for higher capacity utilization, which implies a higher price level. The price level is thus more affected by the unemployment level than is the wage level. Hence, the wage share is lower the higher government expenditure is. Since government expenditure stimulates the domestic price, it moderates the price competitiveness or the real exchange rate.

A higher constant productivity growth (g_a) implies a higher level of price competitiveness, but a lower wage share and, consequently, a lower unemployment rate. A higher constant productivity growth makes the productivity level (a_t) higher, which implies lower price growth (14), a lower producer price level and thus improved price competitiveness.

A higher productivity level stimulates wage growth (15) and inhibits price growth, but despite this the increase in the producer real wage ($rw \equiv w_t - q_t$) is less than the increase in the productivity level. The stable wage share is therefore lower the higher the productivity growth. Higher productivity growth implies higher capacity utilization and lower unemployment. *If* $\theta_w \psi_{qw} > \theta_q$ then productivity stimulates wage growth so much that it also stimulates domestic inflation. Higher producer price reduces the real exchange rate, and through it also unemployment.

A higher exogenous foreign inflation (g_{pf}) implies a higher level of both price competitiveness and wage share, and lower unemployment. Higher foreign inflation increases domestic producer price inflation, but to a lesser degree, so that with unchanged exchange rate the difference between import price and domestic producer price — the real exchange rate — increases. The wedge helps the wage level increase more than the price level, and consequently the wage share increases. Without the wedge it decreases. Increased price competitiveness lowers the unemployment rate directly. Appendix B shows that *dynamic price and wage homogeneity* ($\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$) eliminates the short-run effect of foreign *nominal* inflation on the real exchange rate and the wage share since $e = \xi = 0$ in the reduced form (25). It follows directly that this also holds for unemployment (22) as a function of the real exchange rate. Since it holds for the short term dynamics it must also hold for the long-run steady-states. Appendix C confirms that dynamic wage and price homogeneity makes $e'_{ss} = \mathfrak{e}_{ss} = \xi'_{ss} = 0$.

The steady-state expressions (26)-(28) hold asymptotically also in the NWM and WPCM. But the expressions for the steady-state coefficients change with the restrictions that define the models, cf. Appendix C. The appendix shows that government expenditure gs only has effect on the stationary level of the real exchange rate ($\zeta_{ss} = \mathfrak{c}_{ss} = 0$). Compared to the long-run effects of gs in the WPECM, there is no moderating effect from the reduced price competitiveness through the wedge in the wage-price spiral. An increase in government expenditure thus leads to a larger real appreciation. That fully counters the initial reduction in the unemployment rate, and brings it back to its steady-state level. Hence, government expenditure cannot permanently change the steady-state unemployment rate. Neither can a permanent shift in the constant term in the unemployment equation (22), as seen in Figure 2, 3 and 4. This is consistent with a 'natural rate of unemployment' property.

Lack of concern for lost price competitiveness allows the temporary reduction in the unemployment to temporary increase the wage share, before the wage share equilibrium correction term in domestic inflation aligns the wage and price levels, and restores the wage share level, independent of the new level of government expenditure.

In a stable model the deterministic real growth rates are $\Delta re \equiv \Delta pi - \Delta q \equiv 0$ and $\Delta ws \equiv \Delta w - \Delta q - \Delta a \equiv 0$. It follows from these definitions and equation (13) that in steady-state the deterministic nominal growth rates are determined by foreign inflation and productivity growth: $\Delta q = \Delta p = g_{pf}$ and $\Delta w = \Delta q + \Delta a = g_{pf} + g_a$.

Unstable wage and price Phillips curve model

In the PCM there is no equilibrium correction of price and wage growth by the wage share. That makes it trending, and the long-run solution is

$$ws_t = ws_0 + t \times (\xi'_{ss} \, g_{pf} - g_a + \delta'_{ss}). \tag{29}$$

The steady-state expressions for the real exchange rate (26) and the unemployment rate (28) still hold, but with different expressions for the coefficients, cf. Appendix C.

Government expenditure gs has effect on the stationary level of the real exchange rate only ($\zeta_{ss} = \mathfrak{c}_{ss} = 0$). The explanation in the NWM and WPCM still holds for the real exchange rate and unemployment. But not for the wage share. The trending wage share is uncoupled from the real exchange rate, and consequently its growth rate is independent of government expenditure.

Without any equilibrium correction, productivity affects *no* nominal variable in the model. Hence, productivity affects only the wage share ($b'_{ss} = \mathfrak{b}_{ss} = 0$), and, by definition, productivity growth affects wage share growth in full, as expressed by (29). This is true regardless of whether unemployment is endogenous or targeted.

Since the real exchange rate re is stable, $\Delta q = g_{pf}$ in the PCM too. From the long-run growth rate in the trending wage share (29) it follows that $\Delta ws = \xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}$, and hence $\Delta w = (1 + \xi'_{ss})\, g_{pf} + \delta'_{ss}$. The trend in the wage share is caused by $\Delta w \neq g_{pf} + g_a$. Like in the stable models, dynamic wage and price homogeneity eliminates any effect of foreign nominal inflation on the steady-states *and* the long-run growth rate.

4 Numerical simulations

The analysis in the previous section provides information about (in)stability of the long-run levels of the variables, but it provides no information about the possibility of cyclical fluctuations around a steady-state level or a trend. In general, a linear dynamic model with three interacting variables (re, ws and u) is able to generate cycles, depending on the parameterization of the model. The long-run stability analysis does not address any short term behavior the models. For instance, if a model is in equilibrium and the unemployment rate experiences a permanent exogenous shock, how does the model respond? How fast, how much and for how long do the variables react to the shock? To shed some light on the short to medium term dynamics and cyclical properties of a model, we supplement the theoretical steady-state analysis with numerical simulations.

Each model is explored by numerous simulations with different sets of parameter values selected from a wide domain. In this paper we present only a handful of simulations that illustrate the different dynamics possible within each model. The selected parameterizations are denoted b for basis, h for dynamic wage and price homogeneity in addition to the basis values, and 1-4 for other different values. The parameterizations are guided by estimation results on quarterly data, as reviewed in Bårdsen et al. (2005). Parameterization b is intended as typical, while 1-4 are realistic alternatives since the differences from b are statistically *in*significant. Parameterization h is relevant in its own right, since it is a common assumption in theoretical models of the wage-price spiral. In the simulations, the time period can thus be thought of as a quarter. The start value of each variable is the same in all simulations. All models are simulated once with temporary shocks and once without, and with a targeted (constant) as well as endogenous unemployment rate. After 50 periods the unemployment rate (22) experiences a permanent exogenous shock $c_{u,1} = 0.1$. The dynamic response to that shock varies with the model, its parameterization, and the unemployment regime (endogenous vs. exogenous). Appendix D contains more information and details about all simulations, and about the dynamic properties of the models.

4.1 Wage and price equilibrium correction model (WPECM)

Equilibrium correction of wage and price growth is able to stabilize *both* the real exchange rate and the wage share. Their stable levels depend on the stable rate of unemployment, and change when the rate of unemployment changes. When unemployment is endogenous and interacts with the wage-price spiral, unemployment might contribute to the stabilization of the system — or make the system less stable. It depends on the parameterization. Equilibrium correction in the wage and price formation is able to prevent a trend, but not able to prevent cycles if the feedback between unemployment and the real exchange rate is ‘too’ strong. Then the equilibrium correcting terms (9) and (10) become cyclical too.

Figure 1 illustrates different dynamics of the unrestricted WPECM. Each column of panels displays simulations of the model with a specific parameterization, denoted b, 3 and

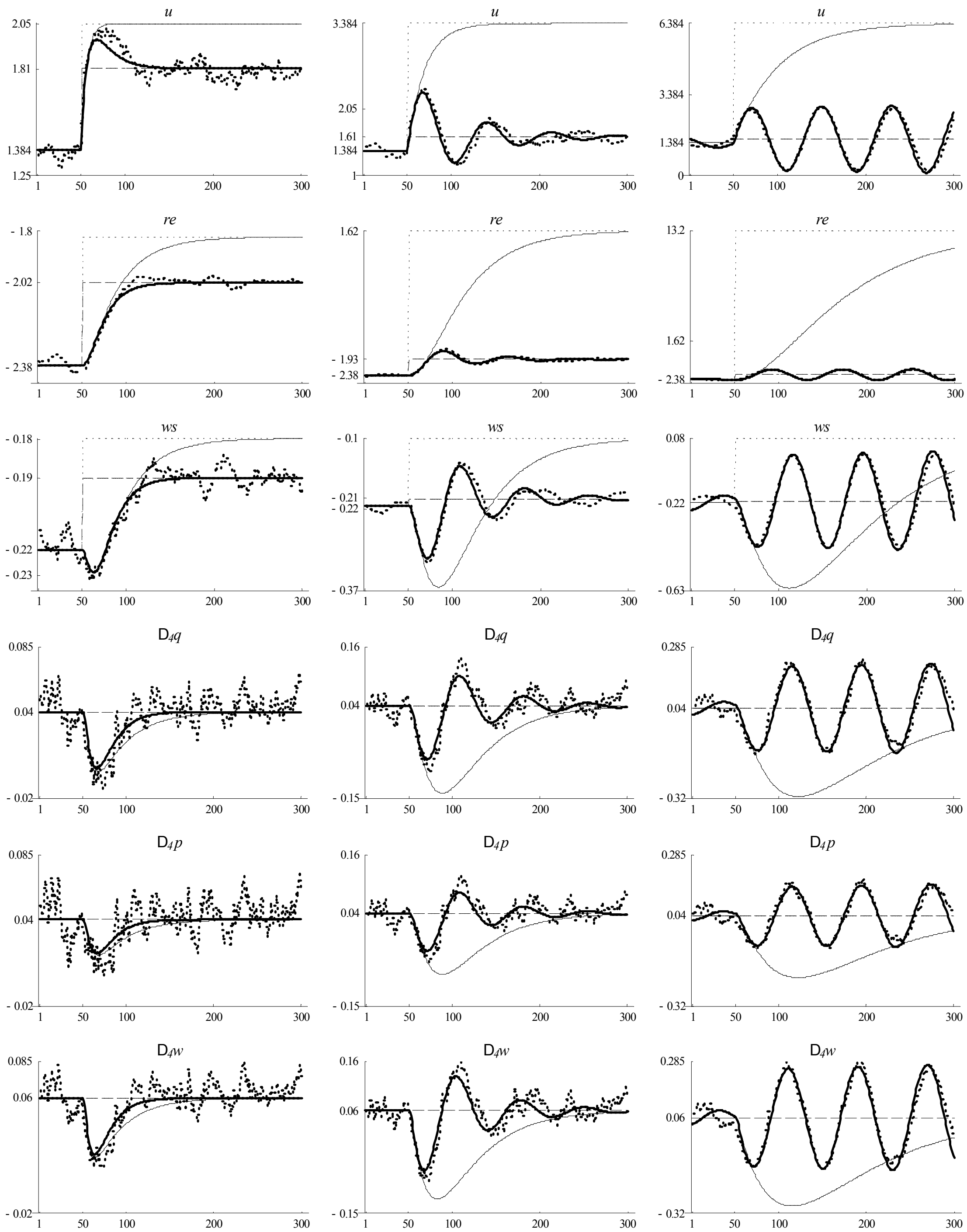

Figure 1: Simulations of the unrestricted model (WPECM) with parameterization b (left panels), 3 (centre panels) and 4 (right panels). The upper 9 panels show levels of real variables, while the lower 9 panels show changes (growth rates) in nominal variables. Bold graphs are simulations with endogenous unemployment. Thin graphs are simulations with targeted (exogenous) unemployment. Solid graphs are steady-state simulations with endogenous unemployment. Dotted graphs are simulations with endogenous unemployment *and* temporary shocks. Dashed and dotted straight lines are steady states. The simulations start in period $t = 1$. In period $t = 51$ there is an upward exogenous shift in unemployment ($c_{u1} = 0.1$). From that point in time the graphs show the dynamic responses of the different variables. The main text explains the parameterizations and the simulations, while details are found in Appendices D and E.

4. The basis parameterization b (left column of panels) has values close to estimated values. The parameter values are shown in row U_b in Table 1 in Appendix D. Figure 1 shows the time paths of the unemployment rate (u), the real exchange rate (re), the wage share (ws), and the annual producer price inflation ($\Delta_4 q$), consumer price inflation ($\Delta_4 p$) and nominal wage growth ($\Delta_4 w$). Bold solid graphs are simulations with endogenous unemployment. Thin solid graphs are simulations with targeted unemployment. Dashed straight lines are the steady-state solutions (26)-(28). Dotted straight lines are the steady-state solutions (34)-(35). Apart from different constants, the parameterizations are identical in the two regimes. The differences in dynamics and levels between the bold and thin graphs in each panel are due to endogenous unemployment only ($\rho > 0$ vs. $\rho = 0$).

Relative to the basis model, parameterizations 3 and 4 (shown in row U_3 in U_4 in Table 1 in Appendix D) imply slightly weaker equilibrium correction, dynamic wage and price homogeneity, and — if endogenous — a more responsive unemployment rate. That fits with the lesser stability seen in the panels in the right two columns. Table 2 in Appendix D relates the dynamic properties to eigenvalues of the recursion matrix $\mathbf{R}$.

In the basis model, the stability of the variables shows clearly in all panels in the left column of Figure 1, irrespective of whether unemployment is targeted or endogenous. The exogenous positive permanent shock to the unemployment rate gets multiplied almost seven times by the autoregressive dynamics of unemployment. The shift is technical and illustrative, but it can be given the interpretation that the government has permanently increased the unemployment rate. As predicted by equation (23) alone, the permanent upward shift in the unemployment rate causes a permanent real depreciation. The reason is that the rise in unemployment makes domestic price inflation drop below the international inflation rate for a period of time, as seen in the fourth panel from the top. When unemployment is endogenous, the depreciation of the real exchange rate partly counters the autoregressive multiplier. The new steady-state unemployment rate is thus below the new targeted level by about a third. The same holds for the real exchange rate.

The wage share is also permanently affected by the increase in unemployment. The immediate response to the increase is a reduction in the wage share, as predicted by bargaining theory and the reduced form equation (24). But then the wage-price spiral kicks in, and price growth is reduced more than wage growth. That increases the wage share, and makes its post-break level higher than the pre-break level. This is a general equilibrium result, and opposite of the partial equilibrium result from the single equation (24). The equation for the long-run (33) in Appendix B expresses the wage share as an increasing function of the targeted unemployment rate ($\eta_{ss} = \mu_q/\theta_q = \vartheta$). This result is due to the departure from normal cost pricing. With $\vartheta = 0$ in equation (1), pre- and post-break levels would be equal. The steady-state equation (27) expresses the wage share as a decreasing function of exogenous government expenditure, or an increasing function of endogenous unemployment. Since steady-state unemployment is lower than in the regime where it is targeted, so is the steady-state wage share.

The center column of panels show simulations of a model with slightly weaker equilibrium correction and dynamic homogeneity in the wage-price spiral, and more responsive unemployment, cf. U_3 in Table 1 in Appendix D. That explains the more lasting and larger responses to the exogenous shift in the targeted unemployment rate (thin graphs). Stronger interaction of endogenous unemployment with the wage-price spiral via the real exchange rate causes damped cycles in all variables (bold graphs). These mechanisms are even stronger and the effects more pronounced in the simulations shown in the right column of panels. Due to a complex root of unit magnitudes the cycles do not cease, cf. U_4 in Table 2 in Appendix D.

The three parameterizations displayed in Figure 1 illustrate the three types of dynamics possible in a model with equilibrium correction in wage and price formation: stability, damped cycles, and constant or increasing cycles. Even though numerical investigations suggest that a trend is not possible for any economically sensible parameterization, pronounced and lasting cycles superimposed on the stable long-run levels constitute a significant instability.

The thin graphs in the upper nine panels of Figure 1 illustrate that the *existence* of a steady-state is independent of the level of the targeted unemployment rate. The model has *non-accelerationist* properties: the wage-price spiral stabilizes wage and price inflation independent of the unemployment rate. There is no need for a unique natural rate of unemployment to stabilize the variables' levels or growth rates. Inflation is stable at any targeted rate of unemployment. There is no trade-off (or "menu") between high inflation with low unemployment and low inflation with high unemployment. The expected stable rate of inflation is given by the trend in foreign price growth ($\Delta_4 p = \Delta_4 q = 4\, g_{pf} = 0.04$) and the average rate of depreciation ($\theta_e = 0$ according to equation (6)). That is *contrary* to conventional macro models that can be described as *accelerationist*: "there is a degree of supply-demand balance of the economy as a whole, measured by the unemployment rate although capacity utilization or output-gap can also be used, with the property that inflation speeds up if the economy is tighter and decelerates if the economy is slacker. That special state of the real economy is usually called the 'natural rate' of unemployment, or the NAIRU, cf. Solow (1999).

The WPECM has non-accelerationist properties also when unemployment is endogenous. The exogenous permanent shock to unemployment triggers an adjustment process which brings about new stable real levels. Wage and price inflation returns to the levels determined by foreign inflation ($4\, g_{pf} = 0.04$) and productivity growth ($4\, g_a = 0.02$).

4.2 No-wedge model (NWM)

As noted earlier, the relevance of the wedge term $p - q$ in the wage target equation (2) is not clear. If we allow for efficiency wage effects, a wedge may be allowed: $\omega > 0$. According to some bargaining models that hold currency, prices have no effect on the bargained real wage, and consequently there should be no wedge: $\omega = 0$. Figure 2 shows simulations of two NWMs. Compared to the same models *with* a wedge in Figure 1, there are both qualitative and quantitative differences.

The wedge is proportional to the real exchange rate, since $p - q = (1 - \phi)(pi - q) \equiv (1 - \phi)\, re$. No wedge ($\omega = 0$) eliminates a direct influence from the real exchange rate on the wage growth, and via the wage-price spiral also on domestic inflation. When unemployment is constant at a target rate there is no price-equilibrium correction, and domestic inflation is no longer tied to foreign inflation. The producer price then grows slower than the import price, and the real exchange rate appreciates continuously. A permanent increase in the unemployment rate speeds up the improvement of price competitiveness.

A permanent increase in unemployment reduces inflation more than wage growth. Without a wedge, they do not return to the rates implied by foreign inflation and productivity growth. That permanently reduces the wage share, see the thin graphs in the left column of plots in Figure 2. Slightly weaker equilibrium correction and dynamic homogeneity in the wage and price formation, and more sensitive unemployment, contribute to increasing the effects of a raised unemployment target, as seen in the right plots.

Endogenous unemployment moves in the opposite direction of the real exchange rate, and lack of a wedge no longer makes the real exchange rate a random walk with drift. Repeated substitution for unemployment in the restricted reduced form equation (23) shows that the real exchange rate is a lag polynomial:

$$re_t = (1 - n\,\rho\,L - n\,\rho\,\alpha\,L^2 - n\,\rho\,\alpha^2\,L^3 - ... - n\,\rho\,\alpha^{t-2}\,L^{t-1})re_{t-1} - k\,ws_{t-1} + ..., \quad (30)$$

The real exchange rate feeds back into unemployment ($\rho > 0$ in (22)), and information about relative price levels remains in the wage and price formation despite no direct effect

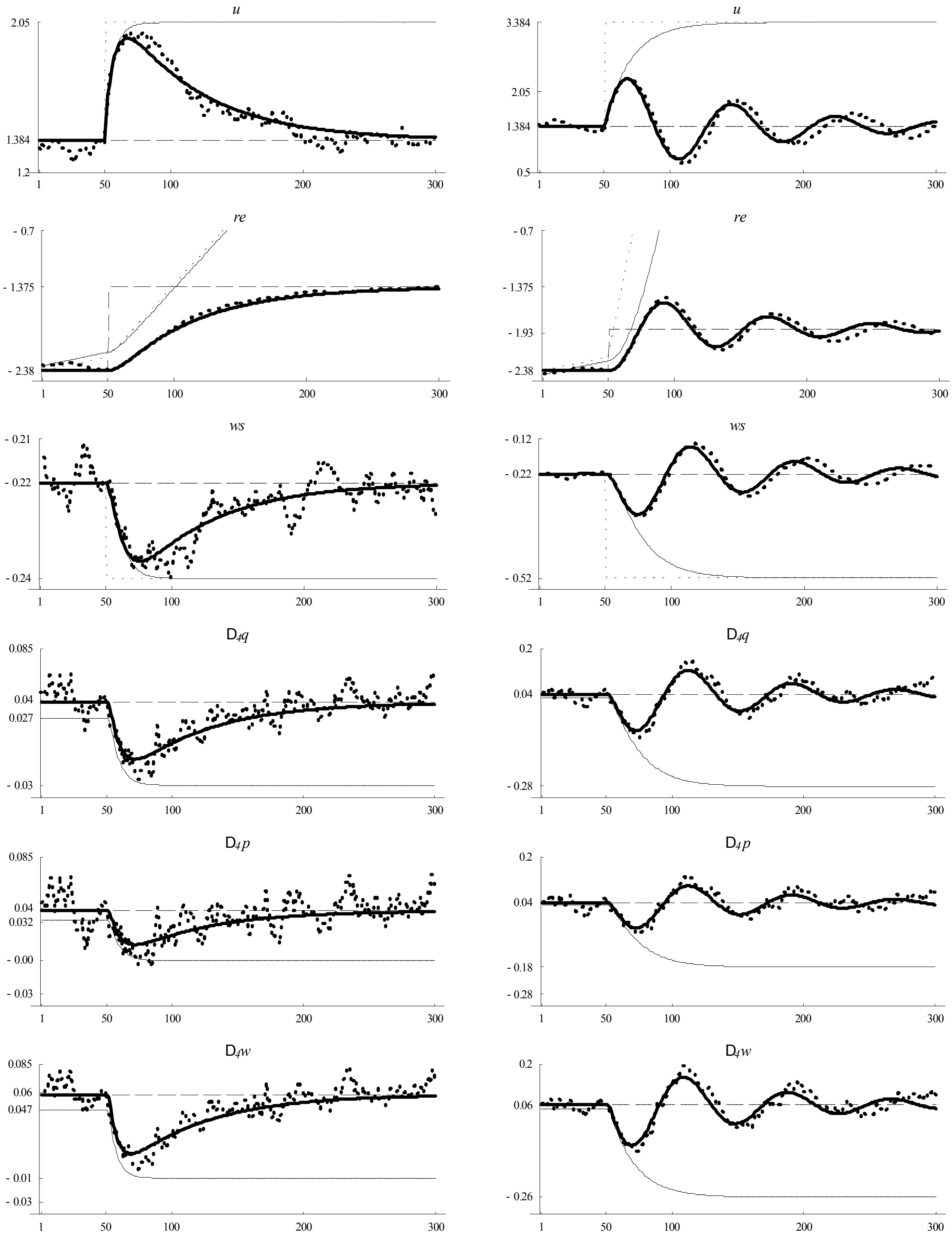

Figure 2: Simulations of the no-wedge model (NWM) with parameterization b (left panels) and 3 (right panels). The upper 6 panels show levels of the real variables, while the lower 6 panels show changes (growth rates) in nominal variables. See Figure 1 for explanations of the graphs. While all variables are asymptotically stable when unemployment is endogenous (bold graphs), the real exchange rate re is trending when unemployment is targeted/exogenous (thin solid graphs in the second row of panels). The trend is caused by domestic inflation $\Delta_4 q$ being less than foreign inflation $4\, g_{pf} = 0.04$. Since wage growth $\Delta_4 w$ equals the sum of domestic inflation and productivity growth $4\, g_a = 0.02$, both before and after the shift in unemployment, the wage share ws is stable. See the main text and Appendix D and E for full explanations of the parameterizations and the simulations.

through the wedge. That keeps the system stable, in the sense of avoiding a trend in the real exchange rate. But, whether the three real variables (re, ws and u) converge to constant steady-state levels or cycle around them depends on the coefficients in all three reduced form equations. Of particular importance is the interaction between the real exchange rate and unemployment, governed by the coefficients n and ρ, and the responsiveness of unemployment, determined by the autoregressive coefficient α. Figure 2 illustrates both asymptotic stability and damped cycles. Non-decreasing cycles is a third possibility, cf. Table 2 in Appendix D.

There are two noticeable qualitative differences between the NWM and the WPECM. In the NWM, the unemployment rate and the wage share return to their 'natural' rates after the permanent shock to unemployment. The steady-state equations (27)-(28) with $\omega = 0 \Rightarrow \zeta_{ss} = \mathfrak{c}_{ss} = 0$ (cf. Appendix C) show no long-run influence from government expenditure (gs) on neither unemployment nor the wage share. Government expenditure is an exogenous variable that appears only in the unemployment equation. A permanent shift to its level affects the system the same way as a permanent shock to unemployment via its constant term. Hence, an exogenous permanent shift to unemployment has no lasting effect on unemployment.

Without a wedge there is less equilibrium correction in the wage and price formation. It takes longer to restore the nominal growth rates after the negative shock from increased unemployment, and the wage and domestic price levels are lower. That makes the real exchange rate appreciate so much that it counters the permanent shock to unemployment and brings it back to its pre-break rate. In the absence of a wedge the only (direct) equilibrium correction going on in the wage and price formation (14)-(15) is caused by the wage share. After the temporary reduction in inflation and wage growth caused by the increase in unemployment, the wage share makes both the price and wage growth, and their levels, align again so that the wage share return to its 'natural' level.

4.3 Wage Phillips curve model (WPCM)

While the no-wedge restriction $\omega = 0$ eliminates a direct correction of nominal wage growth by relative price, the WPCM restriction $\theta_w = 0$ in (15) cancels equilibrium correction by the wage share in addition. But equilibrium correction of the producer price by the wage share remains. That makes the WPCM qualitatively no different from the NWM. In a regime with targeted unemployment, the real exchange rate is trending due to lack of equilibrium correction by relative price. The wage share coordinates domestic inflation with wage and productivity growth so that the wage share is stable. When unemployment is endogenous, it brings information about relative price into in the wage-price spiral. The direct equilibrium correction of domestic inflation by the wage share and the indirect equilibrium correction of wage growth and domestic inflation by the real exchange rate through unemployment keep all three real variables from trending. Depending on the parameterization, they might be stable or display cyclical fluctuations.

Compared to the WPECM and NWM, the WPCM restriction $\theta_w = 0$ makes the negative effect of the wage share on the real exchange rate stronger and decreases the rigidity of the wage share ($l = 1$ and k is larger in (23), and $\lambda = 0$ and κ is larger in (24), cf. Appendix B. Equation (30) still holds). Expressions in Appendix B and C also show that the steady-state levels in the WPCM are more sensitive to the direct effect (ρ) of the real exchange rate on unemployment than they are in the NWM. With reduced *nominal* equilibrium correction compared to the WPECM or NWM, the *real* system appears to be more sensitive and more prone to cyclical dynamics. This is supported by Table 2 in Appendix D, which shows that for the same parameterizations (subscripts b, h, 3 and 4) more eigenvalues are complex and of larger magnitude than in the WPECM or NWM.

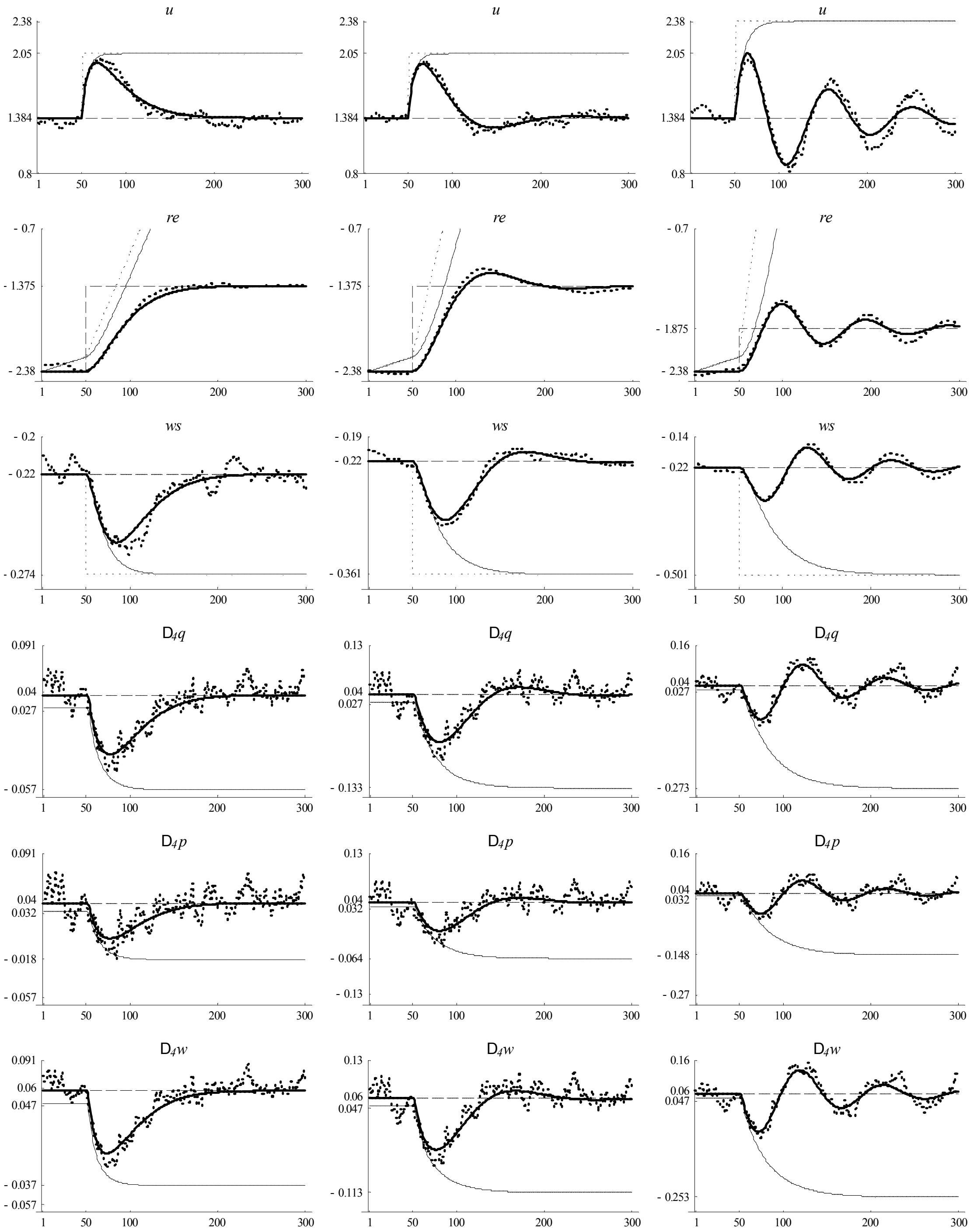

Figure 3: Simulations of the wage Phillips curve model (WPCM) with parameterization b (left panels), h (centre panels) and 2 (right panels). See Figure 1 for explanations of the panels and graphs. All variables are asymptotically stable (bold graphs) except the the trending real exchange rate re in the regime with targeted (exogenous) unemployment (thin solid graph in the second row of panels), see Figure 2 for a brief explanation. Dynamic wage and price homogeneity (centre panels) increases the sensitivity of the variables to each other relative to the basis parameterization (left panels). Each variable responds stronger/faster to movements in the others. That causes oscillations. They are quickly damped compared to parameterization 2 (right panels), which has a more responsive unemployment rate. Other parameterizations display persistent and increasing cycles. The main text and Appendix D and E provide the details of the parameterizations and the simulations.

Figure 3 shows simulations of the WPCM with three different sets of values for the parameters and coefficients. In addition to the basis parameterization (left), we have increased ψ_{qpi} and ψ_{wp} to impose dynamic wage and price homogeneity ($\psi_{qw}+\psi_{qpi} = \psi_{wq}+\psi_{wp} = 1$). Changes to the impact effects in the wage-price spiral makes no real variable trend in any model. But they can make the variables cyclical. While just visible with parameterization h in the center panels, that is clearly visible with parameterization 2 in the right column of panels (ψ_{wp} smaller and ψ_{wq} larger, see Table 1 in Appendix D).

Eliminating equilibrium correction by relative prices (i.e. the wedge which is proportional to the real exchange rate) in the NWM and WPCM makes the real exchange rate trend — unless the real exchange influences wage and price growth through endogenous unemployment. Since relative prices equilibrium corrects only nominal wage setting (15) and not domestic pricing (14), a *price* Phillips curve alone ($\theta_q = 0$ and $\theta_w > 0$) would not damage stability of the model. Equilibrium correction of the nominal wage ($\theta_w > 0$) by both the wage share and the wedge brings information about the real levels into the wage-price spiral. That information is necessary and sufficient to stabilize the dynamic behavior of the system of real variables. The next section shows that in a wage *and* price Phillips curve model is level information so limited that the real system is unstable, also with endogenous unemployment

4.4 Wage and price Phillips curve model (PCM)

The WPECM has been formulated so that $\theta_w = 0$ cancels direct equilibrium correction of the nominal wage, and leaves us with a wage Phillips curve. Symmetrically, $\theta_q = 0$ cancels direct equilibrium correction of the producer price, and leaves us with a price Phillips curve. The PCM is defined by imposing $\theta_w = \theta_q = 0$ and $\varphi, \varsigma > 0$ on the supply side equations (14)-(15). This version of the model has many traits in common with the standard aggregate demand and supply (AD-AS) model found in modern textbooks in macroeconomics, see Sørensen and Whitta-Jacobsen (2005). The main difference is that we have an explicit model of the wage-price spiral, while the textbook model only includes a price Phillips curve, but that is due to simplification in the textbooks. The intended interpretation is always that the underlying process of nominal adjustments is of a wage-price spiral type. Another difference is that in textbooks the Phillips curve, and therefore also the AS schedule, are in terms of an output-gap variable. We use unemployment as a proxy for capacity utilization, but due to Okun's law this difference does not affect the interpretation. Finally, the textbook version has more variables that represent determinants of aggregate demand, while our model only includes the real exchange rate and a variable for fiscal policy (gs). We focus on the stability properties of the model when there is a single endogenous variable in the AD schedule (22), namely the real exchange rate.

The PCM can be regarded as a way to emulate what Blanchard and Katz (1999) have dubbed the standard NAIRU model for the US economy. As shown in Bårdsen and Nymoen (2009b), this model is obtained in our framework by setting $\theta_w = \theta_q = 0$, and allow for a wage Phillips curve ($\varphi > 0$), but no price Phillips-curve ($\varsigma = 0$). The economic rationale might be that prices are adjusted to the developments in normal costs, but it is mainly a simplification since it is clear that $\varsigma > 0$ would only strengthen the equilibrating mechanism already in the WPCM.

The PCM restrictions $\theta_w = \theta_q = 0$ and $\varphi, \varsigma > 0$ make $l = \kappa = 1$ and $k = \lambda = 0$ in equation (23) and (24). That uncouples the real exchange rate from the wage share. Since the real exchange rate moderates unemployment, the two form a feedback loop that is able to stabilize both variables. The wage share is no longer part of a feedback loop with the real exchange rate, and becomes a random walk with drift, cf. Appendix C.

Figure 4 shows simulations of the PCM with basis parameterization (left panels) and parameterization 4 (right panels). The wage share is trending since it has no coordinating

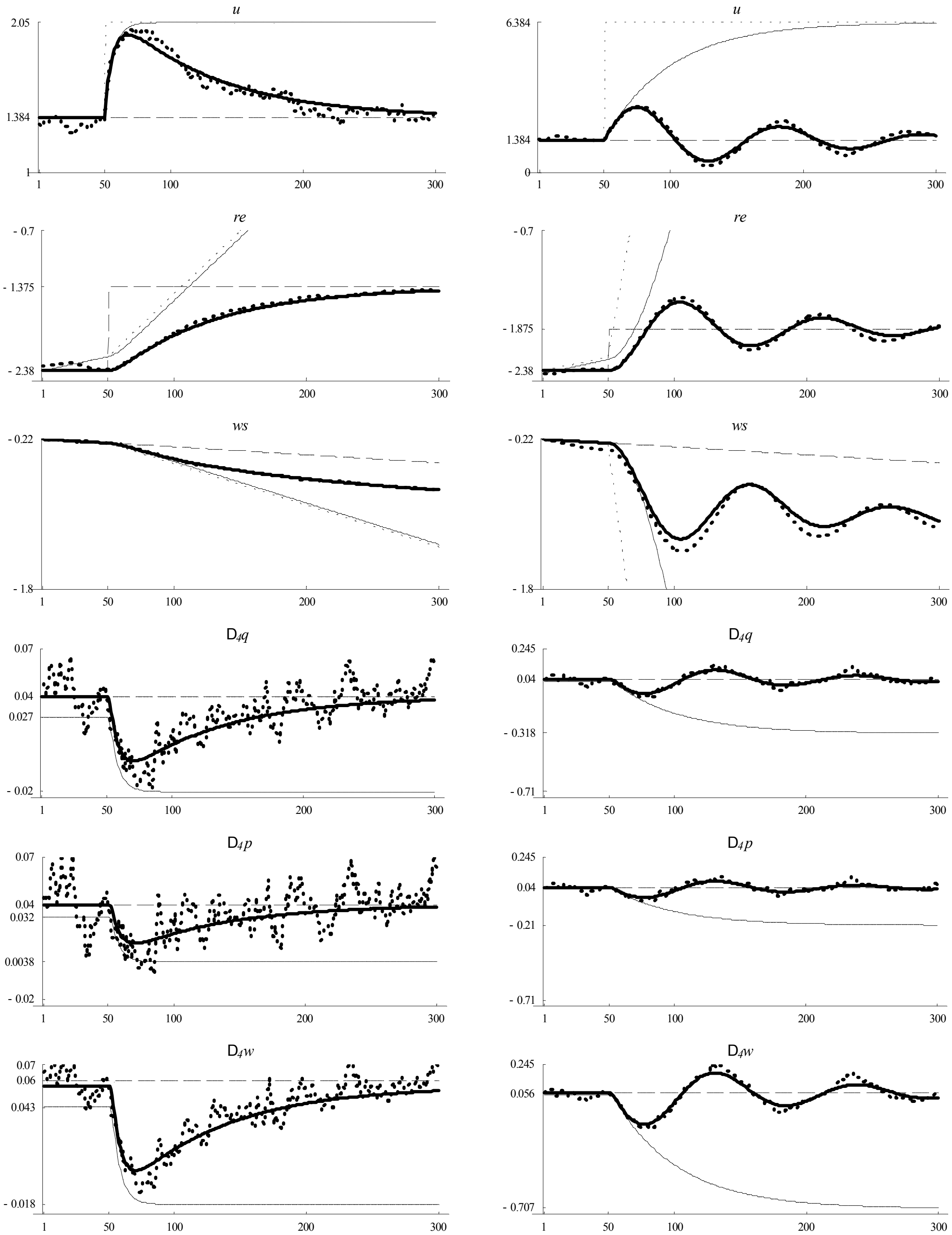

Figure 4: Simulations of the wage and price Phillips curve model (PCM) with parameterization b (left panels), and 4 (right panels). See Figure 1 for explanations of the panels and graphs. The real exchange rate re is trending in the regime with targeted (exogenous) unemployment (thin solid graph in second row of panels). The trend is caused by domestic inflation $\Delta_4 q$ being less than foreign inflation $4\,g_{pf} = 0.04$. The negative trend in the wage share ws — with endogenous as well as targeted (exogenous) unemployment — is caused by the wage growth $\Delta_4 w$ being less than the sum of domestic inflation $\Delta_4 q$ and productivity growth $4\,g_a = 0.02$, both before and after the shock to unemployment. The trends in the real exchange rate and in the wage share can be both positive and negative, as long as they are of opposite sign. For a full explanation of the parameterizations and the simulations, confer the main text and Appendix D and E.

influence on the wage and price growth. The real exchange is trending when it does not influence the wage and price growth directly, nor indirectly through endogenous unemployment. The rest of the variables are non-trending, and return gradually to their pre-break level when unemployment is endogenous.

5 Summary and further work

We have formulated a dynamic, multivariate and simultaneous macroeconomic model, and explored its dynamic properties by a combination of theoretical analyses, numerical investigations and computer simulations. The model versions demonstrate a full range of dynamics, and the results show that the dynamic properties of the endogenous variables are *system properties*. In particular, the choice of model for the AS curve (the wage-price spiral) conditions many important system properties, among them being a dynamically stable rather than "natural" unemployment rate.

The model's supply side is a wage-price spiral, with wage bargaining and mark-up pricing characterized by nominal rigidity and adjustments towards real wage goals. The real wage goals bring real attractors into the wage-price spiral. The wage share and a price wedge are able to coordinate the nominal wage and price growth, and thereby stabilize their own levels *independent of the unemployment rate*. As long as there is information about the wage share and about relative prices — by the price wedge or, equivalently, the real exchange rate — in the wage-price spiral, these real variables have constant steady-state levels. The steady-state levels as well as the nominal wage growth and domestic inflation are determined by exogenous productivity growth, foreign inflation and the level of government expenditure.

In the NWM or WPCM there is no equilibrating level information about relative prices in the wage-price spiral. But there is indirect information about the real exchange rate through unemployment. That keeps the real exchange rate stable and domestic inflation equal to foreign inflation. In a wage and price Phillips curve model there is no information at all about the wage share in the wage-price spiral. The wage share trends because nominal wage growth does not equal inflation plus productivity growth. This result does *not* depend on the assumption that unemployment reacts to a real depreciation with a lag[9]. Even in the Phillips curve versions of the model, (in)stability is independent of the unemployment rate.

The quantitative dynamic properties of all model versions depend on all parameters in the wage-price spiral and the coefficients in the unemployment equation. The qualitative dynamic properties of the models are independent of the actual processes for the exogenous variables[10]. The dynamic properties are fully endogenous, and are determined

[9]Without a lag, (22) would not be the reduced form equation for unemployment. Substituting the reduced form (23) into (22) would replace the 0 in the $\mathbf{R}$ matrix in (25) with ρk. According to Appendix B, in the wage and price Phillips curve model $\theta_q = \theta_w = 0 \Rightarrow k = 0$. Hence, the 0 would reappear in $\mathbf{R}$ and there would be a trend in the wage share.

[10]There is a chain of implications from the exogenous trends to endogenous trends to the equilibrium correction formulation of a wage-price spiral (14)-(15). But there is no implication chain running all the way in the the other direction. Endogenous trends in the wage-price spiral do not require exogenous trends. The wage-price spiral passes a trend in the exogenous import price pi and/or a trend in productivity a onto domestic wage w and prices q and p. But, in the absence of exogenous trends, the wage-price spiral is still able to keep domestic wage and prices growing. The reason is that the wage growth Δw_t (14) and the price inflation Δq_t (15) might tend to be positive even if pi and a should be stationary ($\Delta pi \approx 0$ and $a \approx const$). Hence, should the import price (4) and productivity *temporarily* stop growing, the domestic wage and prices would keep on trending upwards. But, for domestic wage and prices to be trending variables and equations (11)-(12) to be valid formulations in an economy where the exogenous import price and productivity are stationary variables (permanently, not temporarily), we would need to rationalize the constant terms c_q and c_w. Self-fulfilling expectations is a possibility, which might also rationalize a continued wage and price growth during a temporary stop in foreign inflation and productivity growth.

by the existence and strength of transmission channels in the model. Stationarity of the real variables requires that their levels coordinate growth in their constituent nominal variables, through equilibrium correcting terms or unemployment. Depending on their relative strengths, the impact effects and the equilibrating mechanisms might induce cyclical fluctuations in all endogenous variables despite lack of exogenous cycles. In other words, stationarity or a trend depends on the existence of coordinating mechanisms (non-zero parameters/coefficients), while cyclical fluctuations depend on relative strength of impacts and coordination (magnitudes of parameters/coefficients). In the 1930s, Kalecki and Frisch had different views on the causes of cycles. Kalecki saw *persistent* cycles as an intrinsic feature of a capitalist economy, while Frisch believed them to *highly damped*, but revitalized by shocks[11]. Our models accommodate both views (simply by different relative parameter values, cf. Appendix D).

The duality — first order stability and second order instability — make us wonder whether inclusion of more variables and mechanisms is able to reduce or eliminate the cycles. Replacing the random walk specification we have used for the nominal exchange rate with an equilibrium condition for the market for foreign exchange is an obvious extension of the model. Even more importantly, the finding that second order instability (cycles) is a typical feature of the solution, motivates the inclusion of a reaction function for the interest rate, say a Taylor type rule, in order to study the stabilizing role of monetary policy. The interdependence between the exchange rate and the interest rate makes it natural to incorporate both in a model that may represent an inflation targeting regime for example. Then the issues of expectations and foresight have to be addressed.

In the present paper we have focused on the supply side, and deliberately kept the model simple in order to manage a thorough — both theoretical and numerical — analysis. From this basis model we plan to include the extensions mentioned above and build our understanding step by step.

[11] Zambelli (2007) has an interesting discussion of Frisch's work and macro dynamics in the 1930s — and claims that Frisch's famous 'rocking-horse' model does *not* generate cycles for plausible parameter values!

A Definitions of variables, parameters and coefficients

Variables, parameters and coefficients are explained in the main text, but they are collected here for convenience. All variables are logarithmically transformed, and are listed in alphabetical order:

a_t	Labour productivity (autonomous/exogenous), eq. (21)
D_t	Step dummy to facilitate a shift in unemployment level, in eq. (22)
e_t	Exchange rate, eq. (5) and (6)
ecm_t^b	Workers' (bargained) real wage gap $rw_t - rw_t^b$, eq. (10)
ecm_t^f	Firms' real wage gap $rw_t - rw_t^f$, eq. (9)
gs_t	Government real expenditure, eq. (22)
p_t	Consumer price, eq. (13) and (16)
pf_t	Foreign price in foreign currency (autonomous/exogenous), eq. (3)
pi_t	Import price in domestic currency $pf_t + e_t$, eq. (4)
prw_t	Productivity corrected real wage $rw_t - \iota\, a_t$, $\iota \in (0, 1]$, eq. (24) and (25)
$p_t - q_t$	Wedge between consumer and producer real wage, in eq. (2) and (15)
q_t	Producer price, eq. (11) and (14)
q_t^f	Price goal of producers in a steady growth economy, eq. (1)
re_t	Real exchange rate $pi_t - q_t$, eq. (23) and (25)
rw_t^b	Bargained real wage $w_t^b - q_t$, eq. (8)
rw_t^f	Optimal producer real wage $w^f - q$, eq. (7)
u_t	Unemployment (endogenous or exogenous), eq. (22)
w_t	Nominal hourly wage, eq. (12) and (15)
ws_t	Wage share $re_t - a_t$, , eq. (38), equal to prw_t with $\iota = 1$ in eq. (24)
w_t^b	Bargained wage (goal), eq. (2)
$\varepsilon_{\text{variable},t}$	Temporary shock (or residual) to nominal 'variable'
$\epsilon_{\text{variable},t}$	Temporary shock (or residual) to real 'variable'

The parameters and coefficients are grouped according to the equation they belong to:

Equation for the producer price inflation (14)

c_q	Constant in the expression for price growth
m_q	Mark-up on marginal labour cost
ψ_{qw}	Elasticity of nominal wage growth
ψ_{qpi}	Elasticity of import price inflation
θ_q	Degree (speed) of equilibrium correction in price setting
μ_q	$= \theta_q\, \vartheta$ or ς, where...
ϑ	is the effect of unemployment (capacity utilization) on marginal labour cost
ς	is the effect of unemployment in case of no equilibrium correction

Equation for the nominal wage growth (15)

c_w	Constant in the expression for wage growth
m_w	Constant in the expression for bargained wage
ψ_{wq}	Elasticity of producer (domestic) price inflation
ψ_{wp}	Elasticity of consumer price inflation
θ_w	Degree (speed) of equilibrium correction in wage formation
μ_w	$= \theta_w\, \varpi$ or φ, where...
ϖ	is the impact of unemployment on bargained wage, or
φ	is the effect of unemployment in case of no equilibrium correction
ι	Elasticity of productivity
ω	Elasticity of price wedge

Equation for the consumer price inflation (16)

ϕ	Degree of openness of the economy

Equation for the rate of unemployment (22)

c_{u0}	Constant in the reduced form expression
c_{u1}	Size of a simulated level shift in the unemployment rate
α	Degree of persistence in unemployment
ρ	Degree of feedback from the (lagged) real exchange rate
τ	Elasticity of government expenditure

Exogenous processes

g_a	Constant growth in productivity, eq. (21)
g_{pf}	Constant foreign inflation, eq. (3)
θ_e	Degree of mean reversion of the nominal exchange rate, eq. (5)

Standard deviation of shocks (residuals) to...

σ_z	...variable $z \in \{q, w, u, a, pf, e\}$

B Dynamic analysis when unemployment is targeted

The structural form of the model is given by equations (14)-(16). We want to transform the producer price q_t into the real exchange rate $re_t \equiv pi_t - q_t$, and the nominal wage w_t and the producer price q_t into the productivity corrected real wage $prw_t \equiv w_t - q_t - \iota\, a_t$. From (16) we get $p_t - q_t = (1-\phi)\, re$. By adding/subtracting variables, collecting terms and substituting the real variables for their composite nominal ones, we arrive at the following structural form equations:

$$\begin{aligned}
(1-\psi_{qw})\, re_t + \psi_{qw}\, prw_t &= (1-\psi_{qw})\, re_{t-1} + (\psi_{qw} - \theta_q)\, prw_{t-1} \\
&\quad +(1-\psi_{qw}-\psi_{qpi})\,\Delta pi_t - \psi_{qw}\,\iota\,\Delta a_t \\
&\quad +\theta_q\,(1-\iota)\, a_{t-1} + \mu_q\, u_{t-1} - (\theta_q\, m_q + c_q) - \varepsilon_{q,t}, \\
(1-\psi_{wq}-\phi\,\psi_{wp})\, re_t - prw_t &= (1-\psi_{wq}-\phi\,\psi_{wp}+\theta_w\,\omega(1-\phi))\, re_{t-1} \\
&\quad -(1-\theta_w)\, prw_{t-1} + (1-\psi_{wq}-\psi_{wp})\,\Delta pi_t \\
&\quad +\iota\,\Delta a_t + \mu_w\, u_{t-1} - (\theta_w\, m_w + c_w) - \varepsilon_{q,t}.
\end{aligned}$$

According to (17)-(20), $\mu_q = \theta_q\,\vartheta$ or ς, and $\mu_w = \theta_w\,\varpi$ or φ. After solving for re and prw, the nominal variables can be reconstructed as follows: $q_t = pi_t - re_t$, $p_t = (1-\phi)\, re_t + q_t$, and $w = prw_t + q_t + \iota\, a_t$.

The unemployment rate (22) is already a real variable. The structural form of the model with the transformed variables and unemployment can be written as a vector equation:

$$\mathbf{A}\,\mathbf{y}_t = \mathbf{B}\,\mathbf{y}_{t-1} + \mathbf{C}\,\mathbf{x}_t + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{y}_t' = (re_t, prw_t, u_t)$ is a vector of current endogenous real variables, and $\mathbf{y}_{t-1}'$is a vector of the same variables, but lagged. The vector $\mathbf{x}_t' = (\Delta pi_t, \Delta a_t,\, a_{t-1}, gs_t, 1)$ contains the autonomous/exogenous variables. $\mathbf{A}$ is a 3×3 matrix of structural coefficients for the (transformed) current endogenous variables:

$$\mathbf{A} = \begin{pmatrix} 1-\psi_{qw} & \psi_{qw} & 0 \\ 1-\psi_{wq}-\phi\,\psi_{wp} & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$\mathbf{B}$ is a 3×3 matrix of structural coefficients for the (transformed) lagged endogenous variables:

$$\mathbf{B} = \begin{pmatrix} 1-\psi_{qw} & \psi_{qw}-\theta_q & \mu_q \\ 1-\psi_{wq}-\phi\,\psi_{wp}+\theta_w\,\omega(1-\phi) & -(1-\theta_w) & \mu_w \\ -\rho & 0 & \alpha \end{pmatrix}.$$

$\mathbf{C}$ is a 2×4 matrix of structural coefficients for the exogenous variables:

$$\mathbf{C} = \begin{pmatrix} 1-\psi_{qw}-\psi_{qpi} & -\psi_{qw}\,\iota & \theta_q\,(1-\iota) & -(m_q\,\theta_q + c_q) \\ 1-\psi_{wq}-\psi_{wp} & \iota & 0 & -(\theta_w\, m_w + c_w) \end{pmatrix}.$$

Reduced form

Assuming that the matrix $\mathbf{A}$ is invertible, the reduced form is $\mathbf{y}_t = \mathbf{A}^{-1}\mathbf{B}\,\mathbf{y}_{t-1} + \mathbf{A}^{-1}\mathbf{C}\,\mathbf{x}_t + \mathbf{A}^{-1}\boldsymbol{\varepsilon}_t \equiv \mathbf{R}\,\mathbf{y}_{t-1} + \mathbf{P}\,\mathbf{x}_t + \boldsymbol{\epsilon}_t$, where

$$\underbrace{\begin{pmatrix} re_t \\ prw_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k \\ \lambda & \kappa \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ prw_{t-1} \end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & b & n & -d \\ \xi & -\iota & \beta & -\eta & \delta \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ a_{t-1} \\ u_{t-1} \\ 1 \end{pmatrix}}_{\mathbf{x}_t} - \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{prw,t} \end{pmatrix}}_{\boldsymbol{\epsilon}_t}. \tag{31}$$

The reduced form coefficients for the real exchange rate are

$$\begin{aligned}
l &= 1-\theta_w\,\omega\,\psi_{qw}\,(1-\phi)/\chi, \\
k &= (\theta_q - \theta_w\psi_{qw})/\chi, \\
e &= 1-(\psi_{qpi} + \psi_{qw}\,\psi_{wp}\,(1-\phi))/\chi, \quad = 0 \text{ if dynamic homogeneity} \\
b &= (1-\iota)\,\theta_q/\chi, \\
n &= (\mu_q + \mu_w\,\psi_{qw})/\chi, \\
d &= (m_q\,\theta_q + c_q + (m_w\,\theta_w + c_w)\,\psi_{qw})/\chi,
\end{aligned}$$

where the denominator $\chi = 1-\psi_{qw}(\phi\psi_{wp} + \psi_{wq}) > 0$.

The reduced form coefficients for the productivity corrected real wage are

$$\begin{aligned}
\lambda &= \theta_w\,\omega\,(1-\psi_{qw})(1-\phi)/\chi, \\
\kappa &= 1-(\theta_w\,(1-\psi_{qw})+\theta_q\,(1-\psi_{wq}-\phi\,\psi_{wp}))/\chi, \\
\xi &= (\psi_{wp}\,(1-\psi_{qw})(1-\phi)-\psi_{qpi}\,(1-\psi_{wq}-\phi\,\psi_{wp}))/\chi, \qquad = 0 \text{ if dynamic homogeneity} \\
\beta &= (1-\iota)\,\theta_q\,(1-\psi_{wq}-\phi\,\psi_{wp})/\chi, \\
\eta &= (\mu_w\,(1-\psi_{qw})-\mu_q\,(1-\psi_{wq}-\phi\,\psi_{wp}))/\chi, \\
\delta &= ((m_w\,\theta_w+c_w)(1-\psi_{qw})-(m_q\,\theta_q+c_q)(1-\psi_{wq}-\phi\,\psi_{wp}))/\chi.
\end{aligned}$$

The reduced form error terms are linear combinations of the contemporaneous shocks to the structural form for the nominal wage and the producer price:

$$\epsilon_{re,t} = (\varepsilon_{q,t}+\psi_{qw}\,\varepsilon_{w,t})/\chi \quad \text{and} \quad \epsilon_{prw,t} = (\varepsilon_{q,t}\,(1-\psi_{wq}-\phi\,\psi_{wp})-\varepsilon_{w,t}\,(1-\psi_{qw}))/\chi.$$

(In)stability

A necessary condition for the recursive system (31) to be asymptotically stable is that both eigenvalues of the matrix $\mathbf{R}$,

$$r = \frac{1}{2}\left(\kappa+l\pm\sqrt{(\kappa-l)^2-4\,k\,\lambda}\right) = \frac{1}{2}\left(\kappa+l\pm(\kappa-l)\sqrt{1-4\,k\,\lambda/(\kappa-l)^2}\right), \tag{32}$$

are less than one in magnitude. The system is *unstable* if $r=1$, which requires $k\,\lambda=0$ *and* $\kappa=1$ and/or $l=1$. In the no-wedge or the wage Phillips curve model the restrictions $\omega=0$ or $\theta_w=0$ makes $\lambda=0$ and $l=1$. That single unit root causes a trend in the real exchange rate. In the wage and price Phillips curve model the restrictions $\theta_q=\theta_w=0$ makes $\lambda=k=0$ and $l=\kappa=1$. The two unit roots cause a trend in both the real exchange rate and the productivity corrected real wage. Other parameter restrictions that cause a trend are listed in section 3.3: restrictions in 5 cause a trend in the productivity corrected real wage, while 6 or 7 cause a trend in the real exchange rate.

The *necessary* conditions for stability ($k\,\lambda>0$ *or* $\kappa<1$ and $l<1$) are *not sufficient* for stationarity of the real exchange rate and the productivity corrected real wage. Each period t the exogenous vector $\mathbf{x}_t$ adds an impulse to the system, which the matrix $\mathbf{P}$ distributes onto the real exchange rate and the productivity corrected real wage. While all other elements in the vector $\mathbf{x}_t$ are stationary or constant, the third element is the productivity a_{t-1}, which has a deterministic growth rate $g_a>0$. Both re_t and prw_t will inherit the deterministic trend in a_{t-1} unless $b=\beta=0 \Rightarrow \iota=0$ or $\theta_q=0$. The latter restriction implies a Phillips curve in the price setting. Without further restrictions, $r<1$ and the model is stable.

The no-wedge and Phillips curve restrictions make $r_1=\kappa$ and $r_2=l$. Both are real functions or the structural parameters. Only in the unrestricted model is it possible that the expression under the square root in (32) is negative, in which case the eigenvalues make a pair of of complex numbers $\{a+b\,i, a-b\,i\}$, where a is the real part and $b\,i$ is the imaginary part. Depending on the size of $a,b\in(0,1)$ a pair of complex conjugate eigenvalues might induce cycles in the real exchange rate and the productivity corrected real wage. Since the magnitude of a complex eigenvalue is less than 1 in this model, the oscillations will cease over time. If a and b are not large enough, the oscillations might not be visible.

Final form for the unrestricted model (WPECM)

We assume that both eigenvalues are less than one in magnitude, so that the only source of instability might be the growing productivity. If all temporary shocks are set equal to their expectation value of zero, every exogenous variable grows at a constant rate or has a constant level. Then the vector of exogenous variables can be split into a constant part and a time-varying part: $\mathbf{x}_t = \mathbf{x}_{const}+\mathbf{x}_{a(t-1)}$, where the constant part is $\mathbf{x}_{const}=(g_{pi},g_a,0,u^*,1)'$ and the time-varying part is $\mathbf{x}_{a(t-1)}=(0,0,a_{t-1},0,0)'$. If we choose the initial value $a_0=0$ we get $a_{t-1}=(t-1)\,g_a$, and $\mathbf{x}_{a(t-1)}=g_a\,(0,0,t-1,0,0)'$. With the notation $\mathbf{b}=(b,\beta)'$, we can derive the final form from the reduced form as follows

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{R}\,\mathbf{y}_{t-1}+\mathbf{P}\,\mathbf{x}_t \\
&= (\mathbf{I}-\mathbf{R}L)^{-1}\,\mathbf{P}\,\mathbf{x}_t \\
&= (\mathbf{I}-\mathbf{R})^{-1}\,\mathbf{P}\,\mathbf{x}_{const}+(\mathbf{I}+\mathbf{R}L+\mathbf{R}^2L^2+\ldots)\,\mathbf{P}\,\mathbf{x}_{a(t-1)} \\
&\equiv \mathbf{S}\,\mathbf{x}_{const}+g_a\,(\mathbf{I}+\mathbf{R}L+\mathbf{R}^2L^2+\ldots)\,(t-1)\,\mathbf{b} \\
&= \mathbf{y}_{const}+g_a\,[\mathbf{I}\,(t-1)+\mathbf{R}\,(t-2)+\mathbf{R}^2\,(t-3)+\ldots]\,\mathbf{b} \\
&= \mathbf{y}_{const}+g_a\,[(t-1)\,(\mathbf{I}+\mathbf{R}+\mathbf{R}^2+\ldots)-(\mathbf{R}+2\,\mathbf{R}^2+3\,\mathbf{R}^3+\ldots)]\,\mathbf{b}.
\end{aligned}$$

The latter parenthesis is

$$\begin{aligned}
\mathbf{R}+2\,\mathbf{R}^2+3\,\mathbf{R}^3+\ldots &= \mathbf{R}\,(\mathbf{I}+2\,\mathbf{R}+3\,\mathbf{R}^2+\ldots) \\
&= \mathbf{R}\,[(\mathbf{I}+\mathbf{R}+\mathbf{R}^2+\mathbf{R}^3+\ldots)+(\mathbf{R}+2\,\mathbf{R}^2+3\,\mathbf{R}^3+\ldots)] \\
&= \mathbf{R}\,(\mathbf{I}-\mathbf{R})^{-1}+\mathbf{R}\,(\mathbf{R}+2\,\mathbf{R}^2+3\,\mathbf{R}^3+\ldots) \\
&= \mathbf{R}\,(\mathbf{I}-\mathbf{R})^{-2},
\end{aligned}$$

and we get the result

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{y}_{const} + g_a \left[(t-1)(\mathbf{I}-\mathbf{R})^{-1} - \mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}\right]\mathbf{b} \\
&= \mathbf{y}_{const} + g_a (t-1)(\mathbf{I}-\mathbf{R})^{-1}\mathbf{b} - g_a \mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}\mathbf{b} \\
&\equiv (\mathbf{y}_{const} - g_a \mathbf{r}) + g_a (t-1)\mathbf{v} \\
&\equiv \mathbf{y}_{ss} + \Delta\mathbf{y}(t-1).
\end{aligned}$$

The final form is

$$\underbrace{\begin{pmatrix} re_t \\ prw_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} e_{ss}\, g_{pf} + b_{ss}\, g_a + n_{ss}\, u^* - d_{ss} \\ \xi_{ss}\, g_{pf} - \beta_{ss}\, g_a + \eta_{ss}\, u^* - \delta_{ss} \end{pmatrix}}_{\mathbf{y}_{ss}} + \underbrace{\begin{pmatrix} \Delta_{re} \\ \Delta_{prw} \end{pmatrix}}_{\Delta\mathbf{y}} (t-1), \tag{33}$$

where u^* is the targeted unemployment rate. The coefficients for a constant steady-state ($\mathbf{y}_{ss}$) are

$$\begin{aligned}
e_{ss} &= [\theta_q (1-\psi_{wq}-\psi_{wp}) + \theta_w (1-\psi_{qw}-\psi_{qpi})]/\Gamma && = 0 \text{ if dynamic homogeneity,} \\
b_{ss} &= \iota\,(\theta_q - \theta_w\,\psi_{qw})/\Gamma + \text{coeff}_{re}\cdot(1-\iota), \\
n_{ss} &= (\theta_q\,\mu_w + \theta_w\,\mu_q)/\Gamma = (\varpi+\vartheta)/[\omega(1-\phi)], \\
d_{ss} &= [\theta_q(c_w + m_w\,\theta_w) + \theta_w(c_q + m_q\,\theta_q)]/\Gamma, \\
\xi_{ss} &= (1-\psi_{qw}-\psi_{qpi})/\theta_q && = 0 \text{ if dynamic homogeneity,} \\
\beta_{ss} &= \iota\,\psi_{qw}/\theta_q + \text{coeff}_{prw}\cdot(1-\iota), \\
\eta_{ss} &= \mu_q/\theta_q = \vartheta, \\
\delta_{ss} &= (c_q + m_q\,\theta_q)/\theta_q,
\end{aligned}$$

with $\Gamma = \theta_q\,\theta_w\,\omega\,(1-\phi)$. Two messy expressions with structural coefficients are replaced by $\text{coeff}_{re}\cdot(1-\iota)$ and $\text{coeff}_{prw}\cdot(1-\iota)$. They are presumably of small magnitude, and anyway inconsequential since they disappear with the stability requirement $\iota = 1$.

The constant growth rates ($\Delta\mathbf{y}$) in the final form (33) are

$$\Delta_{re} = (1-\iota)\, g_a/(\omega\,(1-\phi)) \quad \text{and} \quad \Delta_{prw} = (1-\iota)\, g_a.$$

Clearly, $\iota = 1$ is a necessary condition for stability. Adding the other necessary conditions for stability, $k\,\lambda > 0$ *or* $\kappa < 1$ and $l < 1$, they are together sufficient for stability in a regime where unemployment is kept at a target rate u^*.

Assuming stationarity, $\iota = 1 \Rightarrow prw_t = w_t - q_t - a_t \equiv ws_t$: the productivity corrected producer real wage becomes the wage share. We note from (33) that a stationary wage share depends positively on the targeted unemployment level. This is a general equilibrium result, and opposite to the partial equilibrium result that follows from the reduced form equation (24) alone. The result is consistent with the sign of unemployment in the wage share expression implied by the price target equation (1). Conversely, the result is at odds with the sign of unemployment in the wage share expression implied by the wage target equation (2). Since the relative price level affects wage formation, the price target dominates the wage target as an equilibrating mechanism.

We would like the steady-state $\mathbf{y}_{ss}$ to be expressed by the same 'variables' as the steady-state equations (26) and (27), but with other expressions for the final form coefficients. The targeted unemployment rate u^* has to be exchanged with the government expenditure gs. This makes sense when the exogenous unemployment rate and endogenous government spending are both constants in a deterministic steady-state. The constant targeted unemployment rate can be written as a "function" of the government expenditure: $u^* = (c_u - \rho\, re - \tau\, gs)/(1-\alpha)$, where $\rho\, re + \tau\, gs$ is constant. If we substitute this into the constant part $\mathbf{y}_{ss}$ of the final form (33), we get

$$\mathbf{y}_{ss} = \begin{pmatrix} re \\ ws \end{pmatrix} = \begin{pmatrix} e_{\varsigma_{ss}}\, g_{pf} + b_{\varsigma_{ss}}\, g_a - \varsigma_{ss}\, gs + d_{\varsigma_{ss}} \\ \xi_{\varsigma_{ss}}\, g_{pf} - \beta_{\varsigma_{ss}}\, g_a - \zeta_{ss}\, gs - \delta_{\varsigma_{ss}} \end{pmatrix}, \tag{34}$$

which is of the same form as (26) and (27), but where the coefficients are

$$\begin{aligned}
e_{\varsigma_{ss}} &= [\theta_q (1-\psi_{wq}-\psi_{wp}) + \theta_w (1-\psi_{qw}-\psi_{qpi})]/\Upsilon && = 0 \text{ if dynamic homogeneity,} \\
b_{\varsigma_{ss}} &= (\theta_q - \theta_w\,\psi_{qw})/\Upsilon, \\
\varsigma_{ss} &= \tau\,(\varpi+\vartheta)/[\omega(1-\phi)(1-\alpha) + \rho\,(\varpi+\vartheta)], \\
d_{\varsigma_{ss}} &= \{c_u\,\theta_q\,\theta_w\,(\varpi+\vartheta)/(1-\alpha) - [\theta_q(c_w + m_w\,\theta_w) + \theta_w(c_q + m_q\,\theta_q)]\}/\Upsilon, \\
\xi_{\varsigma_{ss}} &= \left[(1-\psi_{qw}-\psi_{qpi})/\theta_q\right] \times \Lambda && = 0 \text{ if dynamic homogeneity,} \\
\beta_{\varsigma_{ss}} &= \left[\psi_{qw}/\theta_q\right] \times \Lambda \\
\zeta_{ss} &= \tau\,\vartheta/(1-\alpha+\rho\,\vartheta), \\
\delta_{\varsigma_{ss}} &= [(c_q + m_q\,\theta_q)/\theta_q - c_u\,\vartheta/(1-\alpha)] \times \Lambda,
\end{aligned}$$

with $\Upsilon = \theta_q\,\theta_w\,[\omega\,(1-\phi) + \rho\,(\varpi+\vartheta)/(1-\alpha)]$ and $\Lambda = (1-\alpha)/(1-\alpha+\rho\,\vartheta)$.

Finally, we transform final form unemployment into a 'function' of gs:

$$u = -\,\mathfrak{e}_{\varsigma_{ss}}\, g_{pf} - \mathfrak{b}_{\varsigma_{ss}}\, g_a \;-\mathfrak{c}_{\varsigma_{ss}}\, gs + \mathfrak{d}_{\varsigma_{ss}}, \tag{35}$$

where $\mathfrak{e}_{\varsigma_{ss}} = e_{ss}\,\rho/(1-\alpha)(= 0$ if dynamic homogeneity), $\mathfrak{b}_{\varsigma_{ss}} = b_{ss}\,\rho/(1-\alpha)$, $\mathfrak{c}_{ss} = (\tau - \rho\,\varsigma_{ss})/(1-\alpha) = \varsigma_{\varsigma_{ss}}$ $\omega(1-\phi)/(\varpi + \vartheta)$ and $\mathfrak{d}_{\varsigma_{ss}} = (c_u - \rho\, d_{ss})/(1-\alpha)$.

Since the model has constant steady-state levels for the re, ws and u, it follows that the steady-state nominal growth rates are the same as in the the stable unrestricted model with endogenous unemployment, see subsection 3.5.

Final form for the no-wedge model (NWM)

Imposing a no-wedge restriction makes the composite coefficients $\lambda = 0$ and $l = 1$ in the reduced form expressions (23) and (24). That makes the real exchange rate a random walk with drift, while the wage share becomes a stable autoregressive process which is independent of the real exchange rate. Ignoring the temporary shocks, so that $\Delta pi_t = g_{pf}$ and $\Delta a_t = g_a$, the steady-state equations are

$$\Delta re \;=\; -k\, ws + e\, g_{pf} + n\, u^* - d, \tag{36}$$

$$ws \;=\; \frac{\xi}{1-\kappa}\, g_{pf} - \frac{1}{1-\kappa}\, g_a - \frac{\eta}{(1-\kappa)}\, u^* + \frac{\delta}{1-\kappa} \;\equiv\; \xi'_{ss}\, g_{pf} - \beta'_{ss}\, g_a - \eta'_{ss}\, u^* + \delta'_{ss}, \tag{37}$$

(since $\iota = 1 \Rightarrow b = \beta = 0$ and $prw \equiv ws$). The equation (37) is the final form for the wage share. We note that the no-wedge restriction $\omega = 0$ changes the sign of the elasticity of unemployment from positive in the basis model (33) to negative in the no-wedge model. The no-wedge restriction breaks the system properties of the basis model, and brings the wage share more in line with the reduced form equation (24) and the partial wage-curve result of textbooks.

The steady-state growth rate for the real exchange rate and the final form for the wage share implies the following final form for the real exchange rate:

$$\begin{aligned} re_t &= re_0 + t \times \Delta re = re_0 + t \times (-k\, ws + e\, g_{pf} + n\, u^* - d) \\ &= re_0 + t \times \left[\left(e - k\,\xi'_{ss}\right)\, g_{pf} + k\,\beta'_{ss}\, g_a + \left(n + k\,\eta'_{ss}\right)\, u^* - \left(d + k\,\delta'_{ss}\right)\right], \\ &\equiv re_0 + t \times (e'_{ss}\, g_{pf} + b'_{ss}\, g_a + n'_{ss}\, u^* - d'_{ss}), \end{aligned}$$

where $\Psi = \theta_q\,(1 - \psi_{wq} - \phi\,\psi_{wp}) + \theta_w\,(1 - \psi_{qw})$ and

$$\begin{aligned} e'_{ss} &= [\theta_q\,(1 - \psi_{wq} - \psi_{wp}) - \theta_w\,(1 - \psi_{qw} - \psi_{qpi})]/\Psi && = 0 \text{ if dynamic homogeneity}, \\ b'_{ss} &= (\theta_q - \theta_w\,\psi_{qw})/\Psi, \\ n'_{ss} &= \theta_q\,\theta_w\,(\vartheta + \varpi)/\Psi, \\ d'_{ss} &= [\theta_w\,(m_q\,\theta_q + c_q) + \theta_q\,(m_w\,\theta_w + c_w)]/\Psi. \\ \xi'_{ss} &= [\psi_{wp}\,(1 - \psi_{qw})(1-\phi) - \psi_{qpi}\,(1 - \psi_{wq} - \phi\,\psi_{wp})]/\Psi && = 0 \text{ if dynamic homogeneity}, \\ \beta'_{ss} &= \chi/\Psi = [1 - \psi_{qw}(\phi\psi_{wp} + \psi_{wq})]/\Psi, \\ \eta'_{ss} &= [\theta_w\,\varpi\,(1 - \psi_{qw}) - \theta_q\,\vartheta\,(1 - \psi_{wq} - \phi\,\psi_{wp})]/\Psi, \\ \delta'_{ss} &= [(m_w\,\theta_w + c_w)(1 - \psi_{qw}) - (m_q\,\theta_q + c_q)(1 - \psi_{wq} - \phi\,\psi_{wp})]/\Psi. \end{aligned}$$

Since the real exchange rate re, unemployment u^* and government expenditure gs are all stable in the unrestricted model, u^* and gs can change roles and places in the final form equations. When re is trending in the no-wedge model — and in the Phillips curve models below —, gs_t has to trend in the opposite direction to keep u constant at u^*. Since u^* and gs are integrated of different order they cannot change place in the final form expressions.

Since $\Delta re = e'_{ss}\, g_{pf} + b'_{ss}\, g_a + n'_{ss}\, u^* - d'_{ss}$, $\Delta ws = 0$ and, from (13), $\Delta p = \phi\,\Delta q + (1-\phi)\, g_{pf}$ we get

$$\begin{aligned} \Delta q &= g_{pf} - \Delta re &&= (1 - e'_{ss})\, g_{pf} - b'_{ss}\, g_a - n'_{ss}\, u^* + d'_{ss}, \\ \Delta p &= g_{pf} - \phi\,\Delta re &&= (1 - \phi\, e'_{ss})\, g_{pf} - \phi\, b'_{ss}\, g_a - \phi\, n'_{ss}\, u^* + \phi\, d'_{ss}, \\ \Delta w &= \Delta q + \Delta a &&= (1 - e'_{ss})\, g_{pf} + (1 - b'_{ss})\, g_a - n'_{ss}\, u^* + d'_{ss}. \end{aligned}$$

Final form for the wage Phillips curve model (WPCM)

Imposing a wage Phillips curve restriction makes the composite coefficients $\lambda = 0$ and $l = 1$ in the reduced form expressions (23) and (24). The resulting steady-state equations are identical in structure to the ones in the no-wedge case above, (36)-(37), but the final form coefficients have simplified expressions:

$$\begin{aligned} e'_{ss} &= (1 - \psi_{wq} - \psi_{wp})/(1 - \psi_{wq} - \phi\,\psi_{wp}) && = 0 \text{ if dynamic homogeneity}, \\ b'_{ss} &= 1/(1 - \psi_{wq} - \phi\,\psi_{wp}), \end{aligned}$$

$$
\begin{aligned}
n'_{ss} &= \varphi/(1-\psi_{wq}-\phi\,\psi_{wp}),\\
d'_{ss} &= c_w/(1-\psi_{wq}-\phi\,\psi_{wp}),\\
\xi'_{ss} &= \xi/(1-\kappa) = [\psi_{wp}\,(1-\psi_{qw})(1-\phi)-\psi_{qpi}\,(1-\psi_{wq}-\phi\,\psi_{wp})]/\Psi' \quad = 0 \text{ if homogeneity},\\
\beta'_{ss} &= 1/(1-\kappa) = \chi/\Psi' = [1-\psi_{qw}(\phi\psi_{wp}+\psi_{wq})]/\Psi',\\
\eta'_{ss} &= \eta/(1-\kappa) = [\varphi\,(1-\psi_{qw})-\theta_q\,\vartheta\,(1-\psi_{wq}-\phi\,\psi_{wp})]/\Psi',\\
\delta'_{ss} &= \delta/(1-\kappa) = [c_w(1-\psi_{qw})-(m_q\,\theta_q+c_q)(1-\psi_{wq}-\phi\,\psi_{wp})]/\Psi',
\end{aligned}
$$

where $\Psi' = \theta_q\,(1-\psi_{wq}-\phi\,\psi_{wp})$.

The final form expressions for the steady-nominal state growth rates Δq, Δw and Δp are the same as for the no-edge model above, except that the coefficients have the expressions above (for the wage Phillips curve model).

Final form for the model with a wage and price Phillips curve (PCM)

With a wage and price Phillips curve the restrictions $\theta_w = \theta_q = 0 \Longrightarrow l = \kappa = 1$ and $k = \lambda = 0$. The reduced form equations (23) and (24) simplify to random walks with drift:

$$
\Delta re = e\,g_{pf} + n''\,u^* - d'' \quad \text{and} \quad \Delta ws = \xi\,g_{pf} - g_a - \eta''u^* + \delta'',
$$

where the apostrophes denote that the reduced form coefficients have changed relative to their unrestricted versions, but the signs have not. The final form for the trending levels are

$$
re_t = re_0 + t\times(e\,g_{pf} + n''u^* - d'') \quad \text{and} \quad ws_t = ws_0 + t\times(\xi\,g_{pf} - g_a - \eta''u^* + \delta'').
$$

Since $\theta_q = 0 \Rightarrow \mu_q = \varsigma$ and $\theta_w = 0 \Rightarrow \mu_w = \varphi$, these restrictions on the reduced form coefficients yield the following final form coefficients:

$$
\begin{aligned}
n'' &= (\varsigma+\varphi\,\psi_{qw})/\chi \quad \text{and} \quad d'' = (c_q + c_w\,\psi_{qw})/\chi\\
\eta'' &= [\varphi\,(1-\psi_{qw})-\varsigma\,(1-\psi_{wq}-\phi\,\psi_{wp})]/\chi \quad \text{and} \quad \delta'' = [c_w(1-\psi_{qw})-c_q(1-\psi_{wq}-\phi\,\psi_{wp})]/\chi.
\end{aligned}
$$

The coefficients e, ξ and χ are given in the subsection on the reduced form first in this appendix.

The Phillips curves uncouple the real exchange rate and the wage share. They become independent processes, but remain correlated through common explanatory variables, the trend in foreign inflation g_{pf} and the unemployment level u. The uncoupling does away with system properties, and the signs of the elasticities of unemployment are in line with the reduced form equations (23) and (24).

The final forms for the growth rates of the nominal variables are

$$
\begin{aligned}
\Delta q &= g_{pf} - \Delta re &&= (1-e_{ss})\,g_{pf} - n''_{ss}\,u^* + d''_{ss},\\
\Delta p &= g_{pf} - \phi\,\Delta re &&= (1-\phi\,e_{ss})\,g_{pf} - \phi\,n''_{ss}\,u^* + \phi\,d''_{ss},\\
\Delta w &= \Delta ws + \Delta q + \Delta a &&= (1-e_{ss}+\xi_{ss})\,g_{pf} - (n''_{ss}+\eta''_{ss})\,u^* + (\delta''_{ss}+d''_{ss})\\
& &&= g_{pf} + g_a - \Delta re + \Delta ws < [(1+\xi_{ss})\,g_{pf} + \delta_{ss}].
\end{aligned}
$$

All the nominal growth rates in the unstable restricted models are less than in the stable unrestricted model, and thus less than (in all models) in the regime with endogenous unemployment. In the stable model the nominal growth rates are determined by foreign inflation and productivity growth only. In the unstable models, the nominal growth rates are influenced by the unemployment rate and structural parameters too.

C Dynamic analysis when unemployment is endogenous

Reduced form

The unemployment equation (22) is already a reduced form. If we abstract from the step dummy for the simulations, simplify the constant term to c_u, and add the equation to the reduced form equations (23) and (24), we can write the reduced form model as

$$
\underbrace{\begin{pmatrix} re_t\\ ws_t\\ u_t\end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k & n\\ \lambda & \kappa & -\eta\\ -\rho & 0 & \alpha\end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1}\\ ws_{t-1}\\ u_{t-1}\end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & 0 & -d\\ \xi & -1 & 0 & \delta\\ 0 & 0 & -\tau & c_u\end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t\\ \Delta a_t\\ gs_t\\ 1\end{pmatrix}}_{\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t}\\ \epsilon_{prw,t}\\ \epsilon_{u,t}\end{pmatrix}}_{\boldsymbol{\epsilon}_t}, \qquad (38)
$$

where we have imposed $\iota = 1 \Longrightarrow b = \beta = 0$ to purge this system from trends due to deterministic growth in productivity. Expressions for the composite reduced form coefficients, and error/shock terms, are given in Appendix B. The dynamic properties of the system $\mathbf{y}_t = \mathbf{R}\,\mathbf{y}_{t-1} + \mathbf{P}\,\mathbf{x}_t + \boldsymbol{\epsilon}_t$ in (38) depend on the recursion matrix $\mathbf{R}$ and its eigenvalues. The general analytic expressions for the eigenvalues are too large and complex to be of much help. Instead, in Appendix D, we resort to numerical investigation into the question of stability, and calculate the magnitudes of the eigenvalues for a large number of combinations of parameter and coefficient values.

The recursion matrix $\mathbf{R}$ has three eigenvalues (r). They might be real and/or complex. In all parameterizations we have investigated, the eigenvalues are either all real or two of the eigenvalues are complex conjugates. The latter is a pair of complex numbers $\{a + b\,i, a - b\,i\}$, where a is the real part and $b\,i$ is the imaginary part. A pair of complex conjugate eigenvalues makes all three endogenous variables in the system cyclical. Their oscillations might be hard to see or obvious, depending on the numerical values of the elements of $\mathbf{R}$. If a real eigenvalue $r = a \geq 1$, one of the real variables will have a trend. If $r = 1$ a non-zero constant term provides the random walk with a drift. If a pair of complex conjugate eigenvalues $r = a \pm b\,i$ have magnitude $\|r\| = \sqrt{a^2 \pm b^2} < 1$, the oscillations are damped. The oscillations keep or increase in amplitude if $\|r\| \geq 1$. A model without a trend or oscillations is stable. If oscillations are (slowly) damped, the model is asymptotically stable. Model simulations displayed in the figures show that damped oscillations might dominate the dynamics for a very long time (more than 200 periods (quarters $\Rightarrow$ 50 years)).

The unrestricted recursion matrix $\mathbf{R}$ has eight nonzero elements. The six reduced form coefficients in the upper two rows are functions of nine structural parameters in the wage-price spiral, cf. Appendix B. In the bottom row are the coefficients of the unemployment equation: α determines the sluggishness unemployment and ρ represents the sensitivity of unemployment to the real exchange rate. In our numerical investigations, the dynamics and (in)stability of some models appear to be more sensitive to the value of any one of these two coefficients than to any other single structural parameter in the system. That is not surprising considering that α and ρ are reduced form coefficients that govern the behavior of the unemployment rate. A single structural parameter is always an element in the reduced form coefficient in $\mathbf{R}$. The parameter values are delimited to the unit interval [0,1]. Hence, the effect of a (non-zero) value of a structural parameter on the real exchange rate, the wage share or the unemployment rate might be diluted by the other parameters in the expressions for the reduced form coefficients in $\mathbf{R}$, cf. Appendix B.

(In)stability

Let us presume a stable system in which all three real variables are stationary. Simulations show that stability is possible in the unrestricted model, the no-wedge model and the wage Phillips curve model, but not in the wage and price Phillips curve model which has a unit root, cf. below. With constant levels of the three endogenous variables collected into the vector $\mathbf{y}$, we have the steady-state equation: $\mathbf{y} = \mathbf{R}\,\mathbf{y} + \mathbf{P}\,\mathbf{x} \Rightarrow \mathbf{y} = (\mathbf{I} - \mathbf{R})^{-1}\,\mathbf{P}\,\mathbf{x}$. A necessary although not sufficient requirement for stability is a non-zero determinant

$$\det(\mathbf{I} - \mathbf{R}) = (1 - \alpha)[(1 - l)(1 - \kappa) + k\,\lambda] + \rho\,[n\,(1 - \kappa) + k\,\eta] \neq 0,$$

where $\mathbf{R}$ is so far unrestricted. We see that the determinant is zero only when $(1-\alpha)[(1-l)(1-\kappa)+k\,\lambda] = 0$ *and* $\rho\,[n\,(1 - \kappa) + k\,\eta] = 0$. The first equality holds if $\alpha = 1$ or in any of the six unstable cases 2-6 listed in section 3.1. The second equality holds if $n\,(1 - \kappa) = k\,\eta = 0$, which is true only in the case of a Phillips curve model with $\theta_q = \theta_w = 0$. This result confirms that the system is generally unstable only in the wage and price Phillips curve model, and that the system has *no* trend in a no-wedge or a wage Phillips curve model. The steady-state coefficients can be calculated as $(\mathbf{I} - \mathbf{R})^{-1}\,\mathbf{P}$, but that leads to large and tedious calculations in the unrestricted case or in the case with the no-wedge restriction $\omega = 0$. In those two cases, we may use the steady-state results from the previous regime and save some calculations.

Steady states for the unrestricted model (WPECM) and the no-wedge model (NWM)

For simplicity we shall ignore the error terms or temporary shocks, and look at the deterministic version of the system. If we assume that the vector of exogenous variables $\mathbf{x}_t = (\Delta pi_t, \Delta a_t, gs_t, 1)$ driving the system is constant $\mathbf{x} = (g_{pf}, g_a, gs, 1)$, and that the system has converged to constant levels for the real exchange rate re, the wage share ws and unemployment u, then we may write the latter as

$$u = \mathfrak{d} - \mathfrak{l}\,re - \mathfrak{c}\,gs, \tag{39}$$

where $\mathfrak{d} = c_u/(1 - \alpha)$, $\mathfrak{l} = \rho/(1 - \alpha)$, $\mathfrak{c} = \tau/(1 - \alpha)$ and gs is the constant level of government expenditure. Substituting this into the final form equation (33) for re — now without the first trend term — yields (26), where the final form coefficients are

$$\begin{aligned}
e'_{ss} &= e_{ss}/(1 + n_{ss}\,\mathfrak{l}) = (1 - \alpha)[\theta_q(1 - \psi_{wq} - \psi_{wp}) + \theta_w(1 - \psi_{qw} - \psi_{qpi})]/(\theta_q\,\theta_w\,\Omega),\\
b'_{ss} &= b_{ss}/(1 + n_{ss}\,\mathfrak{l}) = (1 - \alpha)(\theta_q - \theta_w\,\psi_{qw})/(\theta_q\,\theta_w\,\Omega),\\
\varsigma_{ss} &= \mathfrak{c}\,n_{ss}/(1 + n_{ss}\,\mathfrak{l}) = \tau(\varpi + \vartheta)/\Omega,\\
d'_{ss} &= (\mathfrak{d}\,n_{ss} - d_{ss})/(1 + n_{ss}\,\mathfrak{l}) = (c_u\,(\varpi + \vartheta) - (1 - \alpha)[m_w + m_q + c_w/\theta_w + c_q/\theta_q])/\Omega,
\end{aligned}$$

with $\Omega = \omega\,(1-\phi)\,(1-\alpha) + \rho\,(\varpi + \vartheta)$, because $\mu_w = \theta_w\,\varpi$ and $\mu_q = \theta_q\,\vartheta$ in the unrestricted model, and $\Gamma = \theta_q\,\theta_w\,\omega\,(1-\phi)$, cf- Appendix B. We notice that these coefficients have some similarities with the corresponding coefficients in the regime with targeted unemployment, but that they also involve coefficients from the unemployment equation.

Substituting (26) into equation (39) gives the final form equation (28) for u, with the following final form coefficients:

$$
\begin{aligned}
\mathfrak{e}_{ss} &= \mathfrak{l}\,e'_{ss} = \rho\,(\theta_q(1-\psi_{wq}-\psi_{wp}) + \theta_w(1-\psi_{qw}-\psi_{qpi}))/(\theta_q\,\theta_w\,\Omega),\\
\mathfrak{b}_{ss} &= \mathfrak{l}\,b'_{ss} = \rho\,(\theta_q - \theta_w\,\psi_{qw})/(\theta_q\,\theta_w\,\Omega),\\
\mathfrak{c}_{ss} &= \mathfrak{c} - \mathfrak{l}\,\varsigma_{ss} = \tau\,\omega\,(1-\phi)/\Omega \qquad\qquad = 0 \text{ if no wedge},\\
\mathfrak{d}_{ss} &= \mathfrak{d} - \mathfrak{l}\,d'_{ss} = (c_u\,\omega\,(1-\phi) + \rho\,[m_w + m_q + c_w/\theta_w + c_q/\theta_q])/\Omega.
\end{aligned}
$$

Substituting (28) into the final form equation (33) for ws — also without the first trend term — gives (27), with the following final form coefficients:

$$
\begin{aligned}
\xi'_{ss} &= \xi_{ss} - \eta_{ss}\,\mathfrak{e}_{ss} = [\theta_w(1-\psi_{qw}-\psi_{qpi})(\omega\,(1-\phi)(1-\alpha) + \rho\,\varpi)\\
&\qquad - \vartheta\,\rho\,\theta_q(1-\psi_{wq}-\psi_{wp})]/\theta_q\,\theta_w\,\Omega,\\
\beta'_{ss} &= \beta_{ss} + \eta_{ss}\,\mathfrak{b}_{ss} = [\theta_w\,\psi_{qw}\,(\omega\,(1-\phi)(1-\alpha) + \rho\,\varpi) + \vartheta\,\rho\,\theta_q]/\theta_q\,\theta_w\,\Omega,\\
\zeta_{ss} &= \eta_{ss}\,\mathfrak{c}_{ss} = \vartheta\,\tau\,\omega\,(1-\phi)/\Omega \qquad\qquad = 0 \text{ if no wedge},\\
\delta'_{ss} &= \delta_{ss} - \eta_{ss}\,\mathfrak{d}_{ss} = [(\omega\,(1-\phi)(1-\alpha) + \rho\,\varpi)\,\theta_w\,(c_q + m_q\,\theta_q)\\
&\qquad - \vartheta\,\theta_q\,(c_u\,\theta_w\,\omega\,(1-\phi) + \rho\,(c_w + m_w\,\theta_w))]/\theta_q\,\theta_w\,\Omega.
\end{aligned}
$$

If we impose dynamic homogeneity ($\psi_{wq}+\psi_{wp} = \psi_{qw}+\psi_{qpi} = 1$) on the stable basis model, with or without a price wedge, we see from the expressions above that $e'_{ss} = \mathfrak{e}_{ss} = \xi'_{ss} = 0$. That cancels any effect of foreign inflation (g_{pf}) on the stationary level of the real exchange rate, the wage share and unemployment.

In the case of no wedge, $\omega = 0 \Rightarrow \mathfrak{c}_{ss} = \zeta'_{ss} = 0$ and $\varsigma_{ss} = \tau/\rho$. Then government expenditure gs_t has effect on the stationary level of the real exchange rate only. Hence, government expenditure cannot permanently change the steady-state unemployment rate. Neither can a permanent shift in the unemployment rate be implemented in the simulations by increasing the constant c_u in the unemployment equation (22). The constant c_u appears in a product with $\omega = 0$ in the expression for the constant term $\mathfrak{d}_{ss}$ in (28), and in the expression for the constant term δ'_{ss} in (27) as well. In the case of no wedge, a shift in steady-state unemployment is temporary, but it will cause a permanent shift in the real exchange rate.

Steady state for the wage Phillips curve model (WPCM)

Assuming a parameterization that makes the wage Phillips curve model stable (possibly after damped oscillations), the restriction $\theta_w = 0$ and $\varphi > 0$ simplifies the final form equation $\mathbf{y} = \mathbf{R}\,\mathbf{y} + \mathbf{P}\,\mathbf{x} = (\mathbf{I}-\mathbf{R})^{-1}\,\mathbf{P}\,\mathbf{x}$ to

$$
\underbrace{\begin{pmatrix} re \\ ws \\ u \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} 0 & k & -n \\ 0 & 1-\kappa & \eta \\ \rho & 0 & 1-\alpha \end{pmatrix}^{-1}}_{(\mathbf{I}-\mathbf{R})^{-1}} \underbrace{\begin{pmatrix} e & 0 & 0 & -d \\ \xi & -1 & 0 & \delta \\ 0 & 0 & -\tau & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi \\ \Delta a \\ gs \\ 1 \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} e'_{ss} & b'_{ss} & -\varsigma_{ss} & d'_{ss} \\ \xi'_{ss} & -\beta'_{ss} & 0 & -\delta'_{ss} \\ -\mathfrak{e}_{ss} & -\mathfrak{b}_{ss} & 0 & \mathfrak{d}_{ss} \end{pmatrix}}_{(\mathbf{I}-\mathbf{R})^{-1}\,\mathbf{P}} \underbrace{\begin{pmatrix} g_{pf} \\ g_a \\ gs \\ 1 \end{pmatrix}}_{\mathbf{x}}.
$$

The zeros in the first column of $(\mathbf{I}-\mathbf{R})^{-1}$, and the simplified expressions for some of the reduced form coefficients, makes the calculations practically feasible (compared to the unrestricted case). In steady-state, $\Delta pi = g_{pf}$ and $\Delta a = g_a$. The real exchange rate has the following final form coefficients:

$$
\begin{aligned}
e'_{ss} &= (1-\alpha)(1-\psi_{wq}-\psi_{wp})/(\rho\,\varphi),\\
b'_{ss} &= (1-\alpha)/(\rho\,\varphi),\\
\varsigma_{ss} &= \tau/\rho,\\
d'_{ss} &= c_u/\rho - (1-\alpha)\,c_w/(\rho\,\varphi).
\end{aligned}
$$

The wage share has the following final form coefficients:

$$
\begin{aligned}
\xi'_{ss} &= (1-\psi_{qw}-\psi_{qpi})/\theta_q - \vartheta(1-\psi_{wq}-\psi_{wp})/\varphi,\\
\beta'_{ss} &= \vartheta/\varphi + \psi_{qw}/\theta_q,\\
\delta'_{ss} &= -\vartheta\,c_w/\varphi + (m_q\,\theta_q + c_q)/\theta_q.
\end{aligned}
$$

(and $\zeta_{ss} = 0$). The unemployment rate has the following final form coefficients:

$$
\mathfrak{e}_{ss} = (1-\psi_{wq}-\psi_{wp})/\varphi, \qquad \mathfrak{b}_{ss} = 1/\varphi \qquad \text{and} \qquad \mathfrak{d}_{ss} = c_w/\varphi,
$$

(and $\mathfrak{c}_{ss} = 0$). All coefficients are non-negative, except for the constant terms d'_{ss} and δ'_{ss}. They can take negative and positive values, depending on the parameterization.

As for the basis and no-wedge model, dynamic homogeneity ($\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$) cancels any effect of foreign inflation (g_{pf}) on the stationary level of the real exchange rate, the wage share and unemployment ($e'_{ss} = \mathfrak{e}_{ss} = \xi'_{ss} = 0$). Government expenditure has effect on the stationary level of the real exchange rate only ($\varsigma_{ss} = \tau/\rho$).

Steady state for the wage and price Phillips curves model (PCM)

The lack of equilibrium correction of both the nominal wage and the producer price imposes the restrictions $\theta_q = \theta_w = 0$ and ς, $\varphi > 0$. The recursion matrix in the reduced form of the model (38) simplifies to

$$\mathbf{R}_{\mathrm{P}} = \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & -\eta \\ -\rho & 0 & \alpha \end{pmatrix}. \tag{40}$$

The matrix $\mathbf{R}_{\mathrm{P}}$ has the eigenvalues

$$r_1 = 1, \quad r_2 = \frac{1}{2}\left(1 + \alpha + \sqrt{(1-\alpha)^2 - 4\,n\,\rho}\right) \text{ and } r_3 = \frac{1}{2}\left(1 + \alpha - \sqrt{(1-\alpha)^2 - 4\,n\,\rho}\right).$$

The eigenvalue $r_1 = 1$ belongs to the eigenvector ws. The unit root makes the wage share ws a random walk process. Constant terms induce drift, which produce a visible trend in the wage share. Depending on the parameterization, it might be positive or negative.

An additional trend in the real exchange rate and/or in the unemployment rate (which would be economically less meaningful) requires $n = 0$ or $\rho = 0$. Neither complies with the model. That rules out any trend in re or u, but instability in the form of oscillations is still possible. Then the two roots r_2 and r_3 have to be complex conjugates, which requires $\alpha > 1 - 2\sqrt{n\,\rho} > 0$. The parameterizations P_3 and P_4 in Table 2 in Appendix D provide fairly 'realistic' examples, in which the feedback loops between the unemployment rate and both the wage and price growth are strong and the unemployment rate is sluggish (ς, φ, ρ and α all relatively large). Both the real exchange rate and the unemployment rate display oscillations. Oscillations with increasing amplitude is possible, but seems to require 'unrealistic' parameterization. Damped oscillations around constant (steady-state) levels is normal.

If we replace the general matrix $\mathbf{R}$ in the reduced form (38) with its restricted version $\mathbf{R}_{\mathrm{P}}$, we see from the second column of (40) that the wage share does not influence upon the real exchange rate nor unemployment. It can thus be uncoupled from the latter two. We might then split the system (38) in two. The inter-dynamics of the real exchange rate and unemployment has the following reduced form:

$$\underset{\mathbf{z}_t}{\begin{pmatrix} re_t \\ u_t \end{pmatrix}} = \underset{\mathbf{R}'_{\mathrm{P}}}{\begin{pmatrix} 1 & n \\ -\rho & \alpha \end{pmatrix}} \underset{\mathbf{z}_{t-1}}{\begin{pmatrix} re_{t-1} \\ u_{t-1} \end{pmatrix}} + \underset{\mathbf{P}'}{\begin{pmatrix} e & 0 & -d \\ 0 & -\tau & c_u \end{pmatrix}} \underset{\mathbf{x}'_t}{\begin{pmatrix} \Delta pi_t \\ gs_t \\ 1 \end{pmatrix}} + \underset{\boldsymbol{\epsilon}'_t}{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{u,t} \end{pmatrix}}, \tag{41}$$

while the trending behavior of the wage share depends on lagged unemployment and constants:

$$\Delta ws_t = -\eta\, u_{t-1} + \begin{pmatrix} \xi & -1 & \delta \end{pmatrix} \begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ 1 \end{pmatrix} + \epsilon_{prw,t}. \tag{42}$$

Simulations have shown that the subsystem (41) might be stable after damped oscillations. Presuming a stable subsystem, we can solve the steady-state equation $\mathbf{z} = \mathbf{R}'_{\mathrm{P}}\,\mathbf{z} + \mathbf{P}'\,\mathbf{x}' \Rightarrow \mathbf{z} = (\mathbf{I} - \mathbf{R}'_{\mathrm{P}})^{-1}\,\mathbf{P}'\,\mathbf{x}'$, with $\Delta pi_t = g_{pf}$ and $gs_t = gs$:

$$\underset{\mathbf{z}}{\begin{pmatrix} re \\ u \end{pmatrix}} = \underbrace{\begin{pmatrix} 0 & -n \\ \rho & 1-\alpha \end{pmatrix}^{-1}}_{(\mathbf{I}-\mathbf{R}'_{\mathrm{P}})^{-1}} \underset{\mathbf{P}'}{\begin{pmatrix} e & 0 & -d \\ 0 & -\tau & c_u \end{pmatrix}} \underset{\mathbf{x}'}{\begin{pmatrix} g_{pf} \\ gs \\ 1 \end{pmatrix}} = \underbrace{\begin{pmatrix} e'_{ss} & -\varsigma_{ss} & d'_{ss} \\ -\mathfrak{e}_{ss} & 0 & \mathfrak{d}_{ss} \end{pmatrix}}_{(\mathbf{I}-\mathbf{R}'_{\mathrm{P}})^{-1}\,\mathbf{P}} \underset{\mathbf{x}'}{\begin{pmatrix} g_{pf} \\ gs \\ 1 \end{pmatrix}}.$$

The real exchange rate equation has the final form coefficients

$$\begin{aligned} e'_{ss} &= (1-\alpha)[1 - \psi_{qpi} - \psi_{qw}(\psi_{wq} + \psi_{wp})]/[\rho\,(\varsigma + \varphi\,\psi_{qw})], \\ \varsigma_{ss} &= \tau/\rho \quad \text{and} \quad d'_{ss} = c_u/\rho - (1-\alpha)(c_q + c_w\,\psi_{qw})/[\rho\,(\varsigma + \varphi\,\psi_{qw})], \end{aligned}$$

(and $b'_{ss} = 0$). The unemployment equation has the final form coefficients

$$\mathfrak{e}_{ss} = [1 - \psi_{qpi} - \psi_{qw}(\psi_{wq} + \psi_{wp})]/(\varsigma + \varphi\,\psi_{qw}) \quad \text{and} \quad \mathfrak{d}_{ss} = (c_q + c_w\,\psi_{qw})/(\varsigma + \varphi\,\psi_{qw}),$$

(and $\mathfrak{b}_{ss} = \mathfrak{c}_{ss} = 0$). As for the the basis, no-wedge and wage Phillips curve model, dynamic homogeneity ($\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$) cancels any effect of foreign inflation (g_{pi}) on the stationary level of the real exchange rate and unemployment ($e'_{ss} = \mathfrak{e}_{ss} = 0$). Government expenditure has effect on the stationary level of the real exchange rate only ($\varsigma_{ss} = \tau/\rho$).

In a steady-state, equation (42) determines the constant growth rate of the wage share. Substituting constant growth rates for the differenced variables and zeros for the temporary shocks yields

$$\begin{aligned} \Delta ws &= \underbrace{-\eta \begin{pmatrix} -\mathfrak{e}_{ss} & 0 & \mathfrak{d}_{ss} \end{pmatrix} \begin{pmatrix} g_{pf} \\ gs \\ 1 \end{pmatrix}}_{u} + \begin{pmatrix} \xi & -1 & \delta \end{pmatrix} \begin{pmatrix} g_{pf} \\ g_a \\ 1 \end{pmatrix} \\ &= (\eta\, \mathfrak{e}_{ss} + \xi)\, g_{pf} - g_a + (\delta - \eta\, \mathfrak{d}_{ss}) \;\equiv\; \xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}, \end{aligned}$$

where

$$\xi'_{ss} = [\varphi\,(1 - \psi_{qw} - \psi_{qpi}) - \varsigma\,(1 - \psi_{wq} - \psi_{wp})]/(\varsigma + \varphi\,\psi_{qw}) \quad \text{and} \quad \delta'_{ss} = (\varsigma\, c_w - \varphi\, c_q)/(\varsigma + \varphi\,\psi_{qw}).$$

(and $\beta'_{ss} = 1$, $\zeta_{ss} = 0$). Dynamic homogeneity ($\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$) makes $\xi'_{ss} = 0$, in which case foreign inflation has no effect on the trend in the wage share. With a constant growth rate, the level of the wage share at time point $t \geq 1$ is

$$ws(t) = ws_0 + t \times \Delta ws = ws_0 + t\,(\xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}).$$

Depending on the parameterization, the wage share might have a negative or positive trend. A constant wage share ($= ws_0$) is theoretically/numerically possible, but unlikely.

D Parameterizations and simulations of the models

The analytical results in Appendix B and C reveal the long-run dynamic properties of the unrestricted model and the restricted models. The analyses only tell which model is stable or which real variable is trending. It does not say anything quantitatively about the short-run dynamic properties of the variables in the different models. We explore those properties by numerous simulations of the unrestricted and restricted models..

Each parameter and coefficient is allowed to take on a value from a wide domain. The first two rows in Table 1 delimit the domain. We select a large number of parameterizations, each being a set of parameter and coefficient values from the domain. Each model — the unrestricted, the no-wedge, the wage Phillips curve, and the wage and price Phillips curve model — is simulated with all selected parameterization. Equivalently, each parameterization is used for the simulations of all models A single parameterization is the same in all models except for the restrictions that define the restricted models, e.g. $\omega = 0$ in the no-wedge model.

Six parameterizations are selected and presented in Table 1. They are denoted by the subscripts b for basis, h for dynamic wage and price homogeneity, and $1-4$ for different values. They are selected to illustrate the range of dynamics possible in the model(s). The basis parameterization in the second row (U_b) is based econometric analysis, cf. Bårdsen et al. (2005, Ch 5), and is thus claimed to be *realistic*. The constants c_q, m_q, c_w, m_w, c_{u0} and c_{u1} are left out of the presentation since they do not influence on the dynamics of the model. The constant $m_q = 0.31$ and $m_w = 0.46$ are the same in all simulations. The constants c_q, c_w and c_{u0} are not structural, but rather 'econometric'. In each simulation their values are set so that the real exchange rate, the wage share and the unemployment rate always start at the same values, i.e. $re_1 = -2.4$, $ws_1 = -0.22$ and $u_1 = 1.38$.

All models have the same four parameterizations, with subscripts $b, h, 3, 4$. The wage Phillips curve model has two extra parameterizations, numbered 1 and 2. According to (17)-(20), in the Phillips curve parameterizations we have let $\varphi = \varpi\ \theta_w$ and $\varsigma = \vartheta\ \theta_q$, where the values of the parameters on the right hand side are the same as in the unrestricted model.

In each of the parameterized models U_b–P_4 we perform two simulations, both with the same set (row) of parameters. Each simulation generates a single time series for each variable in the model. In the first simulation the solution for each variable at each period t is perturbed by a temporary shock: $\varepsilon_{\text{variable},t} \sim IN(0, \sigma_{\text{variable}})$, where $\sigma_q = \sigma_w = 0.001$, $\sigma_a = 0.0005$, $\sigma_{pf} = 0.003$, $\sigma_e = 0.007$. In the second simulation, the parameters have the same values, but with all temporary shocks are switched off: $\varepsilon_{q,t} = \varepsilon_{w,t} = \varepsilon_{a,t} = \varepsilon_{pf,t} = \varepsilon_{e,t} = 0$ for all t. Because the shocks are additive to the linear(ized) model, the deterministic simulation approximates the mean stochastic simulation. In all figure the ragged graphs are the stochastic simulations and the smooth graphs are the steady-state simulations. The number of simulation periods is 300. In period $t = 51$ the unemployment rate is subject to a permanent positive shock of size $c_{u1} = 0.1$. The large increase in the unemployment level is chosen just to make the figures more clear and easy to grasp.

	ϑ	θ_q	ς	ψ_{qw}	ψ_{qpi}	ϖ	θ_w	φ	ω	ψ_{wq}	ψ_{wp}	ϕ	α	ρ
min	0	0	0	0	0	0	0	0	0	0	0	0	0	0
max	.65	.5	.1	.6	.7	1	.5	.2	1	.8	.5	1	1	1
U_b	.065	.13	0	.40	.40	.10	.12	0	.5	.5	.2	.6	.85	.10
U_h					.60		.				.5			
U_3	.060	.12		.35	.65	.18	.11		.4	.7	.3	.7	.95	.20
U_4	.060	.10		.30	.70	.22	.10		.3	.7	.3	.7	.98	.20
N_x									**0**					
W_z						-	**0**	.0120	-					
W_1						-	**0**	.0120	-				.80	
W_2					.60	-	**0**	.0120	-	.6	.4		.90	.20
W_3						-	**0**	.0198	-					
W_4						-	**0**	.0220	-					
P_z	-	**0**	.00845			-	**0**	.0120	-					
P_3	-	**0**	.00720			-	**0**	.0198	-					
P_4	-	**0**	.00600			-	**0**	.0220	-					

Table 1: Parameterizations of the models. In the first column: min = minimum value, max = maximum value, U = unrestricted models, N = No-wedge model, W = Wage Phillips curve model, and P = wage and price Phillips curve model. The subscripts are: b = *basis* values for the parameters, h = basis values + dynamic *homogeneity*, 1-4 = models with parameters and/or coefficients different from the basis values. To economize on space we let subscript x represent parameterization (subscripts) b, h, 3 and 4, while subscript z represents only b and h. The wage Phillips curve model has two unique parameterizations denoted (by subscripts) 1 and 2. For the restricted models (N, W and P) only the parameter and/or coefficient values different from their values in the corresponding unrestricted model (U) are shown.

The first row shows the parameters and coefficients. When unemployment is targeted, $\rho = 0$ in all models. The second and third rows delimit the domain of each parameter or coefficient. The fourth row shows the basis values. The following rows show only values that differ from the basis values.

The boldface zeros are restrictions defining the no-wedge, the wage Phillips curve and the wage and price Phillips curve model. Dashes in the ϑ-column and in the ϖ-column mark that the values are irrelevant because of the zero-restriction in the adjacent column. The non-zero basis value of ς is set equal to the product of the basis values of ϑ and θ_q. The nonzero basis value of φ is set equal to the product of the basis values of ϖ and θ_w.

Even though the parameterizations in Table 1 span a very small part of the parameter space delimited by the min and max values, numerical simulations of the models with those parameterizations display a full range of dynamics: stability, damped cycles, persistent cycles, increasing cycles, a trending real exchange rate, and a trending real exchange rate and wage share. Some of them are shown in Figure 1-4. The (in)stability analyses in Appendix B and C, and the numerical eigenvalues in Table 2 justify the limited selection of parameterizations in Table 1.

Table 2 summarizes the dynamic properties of the parameterized models in Table 1. The U_b row shows that the unrestricted model with basis values for the parameters and coefficients is stable. The U_h row refers to the same model with dynamic homogeneity imposed by increasing ψ_{wp} from 0.2 to 0.5. Note that the increase of ψ_{qpi} from 0.4 to 0.6 has no affect on $\mathbf{R}$. Its composite coefficients do not contain ψ_{qpi}. Foreign inflation affects the real exchange rate and the wage share only additively, cf. equation (38) above. Increasing ψ_{wp} increases the sensitivity of wage growth to inflation. That makes the two eigenvalues r_1 and r_2 a pair of complex conjugates, with magnitude $||r||_{\max} = 0.928 < 1$, which causes all three real variables to display damped cycles around stable levels. U_3 and U_4 are two other unrestricted models with dynamic homogeneity (of different composition) imposed, and all parameters and coefficients different from their basis values. It is not possible to see from the parameterization in Table 1 that U_3 displays damped cycles, while U_4 displays constant cycles. But Table 2 reveals it, and Figure 1 shows it clearly. Comparing model U_4 to U_3, the weaker equilibrium correction in the wage and price formation, and a more responsive unemployment rate are no longer able to stabilize the short-run dynamics over time. Due to the interdependence of the variables, variables are cyclical.

Except for the no-wedge restriction $\omega = 0$, the models N_b, N_h, N_3 and N_4 are identical to the models U_b, U_h, U_3, U_4. Imposing dynamic homogeneity (N_h) reduces nominal rigidity. In the unrestricted model that causes damped oscillations that are barely visible. But lack of a wedge counters it and keeps the model free of cycles. For the other parameterizations of the no-wedge model, its dynamics are qualitatively equal to the unrestricted model's. Figure 2 shows two simulations.

Only the lack of error correction in the wage formation, $\theta_w = 0$, with unemployment effect $\varphi > 0$, makes the models W_b, W_h, W_3 and W_4 different from their unrestricted counterparts. Lack of stabilization through error correction in opposite directions by the wage share and the wedge implies damped oscillations,

	r_1	r_2	r_3	$\|\|r\|\|_{\max}$	re	ws	u	Figure
U_b	.942	.901	.833	.942	s	s	s	1
U_h	.927+.035 i	.927-.035 i	.838	.928	d.o	d.o	d.o	
U_3	.981+.085 i	.981-.085 i	.861	.984	d.O	d.O	d.O	1
U_4	.997+.079 i	.997-.079 i	.873	1.000	O	O	O	1
N_b	.984	.877	.828	.984	s	s	s	2
N_h	.979	.893	.834	.979	s	s	s	
N_3	.987+.079 i	.987-.079 i	.855	.990	d.O	d.O	d.O	2
N_4	1.001+.077 i	1.001-.077 i	.869	1.004	i.O	i.O	i.O	
W_b	.965+.014 i	.965-.014 i	.854	.965	d.o	d.o	d.o	3
W_h	.979+.034 i	.979-.034 i	.854	.980	d.o	d.o	d.o	3
W_1	.979	.950	.805	.979	s	s	s	
W_2	.987+.065 i	.987-.065 i	.894	.989	d.O	d.O	d.O	3
W_3	1.007+.072 i	1.007-.072 i	.920	1.010	i.O	i.O	i.O	
W_4	1.017+.069 i	1.017-.069 i	.934	1.019	i.O	i.O	i.O	
P_b	1	.987	.863	1	s	t	s	4
P_h	1	.986	.864	1	s	t	s	
P_3	1	.975+.056 i	.975-.056 i	1	d.O	t+d.O	d.O	
P_4	1	.990+.057 i	.990-.057 i	1	d.O	t+d.O	d.O	4

Table 2: Dynamic properties of the models with endogenous unemployment. In the first column are the different models. Their parameterizations are shown in Table 2. The following four columns contain the eigenvalues of the recursion matrix **R** and the magnitude of the largest eigenvalue. The three next columns show the dynamic behaviour of each of the three real variables: s = stability, d.o = damped small oscillations, barely visible, d.O = damped large oscillations, clearly visible, O = persistent oscillations of constant amplitude, i.O = increasing oscillations, and t = trend. The simulated dynamics for certain models are displayed in figures in section 4. The final column in the table shows which figures display which models.

When unemployment is targeted it is effectively exogenous in the models. That reduces the dimension of the dynamic system, and simplifies the short-run dynamics. The restricted models all have a unit root corresponding with a trend in the real exchange rate. The wage and price Phillips curve model has two unit roots. The second corresponds with a trend in the wage share. Otherwise the magnitude of the eigenvalues are all less than one. There are no complex conjugate eigenvalues in the models U_b-P_4. In other more extreme parameterizations with complex eigenvalues the imaginary components are too small to cause visible cycles.

barely visible with as well as without dynamic homogeneity (W_b and W_h)..Model W_1 is equal to W_b, except for the lower value of $\alpha = 0.8$. That makes the unemployment rate more rigid, and reacting too weakly to cause cycles via the feedback loop with the real exchange rate. Increasing the sensitivity of unemployment ($\alpha = 0.9$) and imposing dynamic homogeneity ($\psi_{wq} = 0.6, \psi_{wq} = 0.6, \psi_{wp} = 0.4$) make model W_2 react to a shock with large and slowly damped oscillations. Figure 7 shows the three simulations with damped oscillations. The size or visibility of the cycles reflects the largest magnitude of the eigenvalues, $||r||_{\max}$.

Without equilibrium correction in the wage-price spiral, all Phillips curve models (P_x) are unstable due to a trend in the wage share. That is shown analytically in Appendix C. Both the unemployment rate and the real exchange rate remain stable in the basis model, without or with dynamic homogeneity. Increased parameter values implies increased sensitivity to shocks and reduced nominal rigidity. Consequently, both model P_3 and P_4 display damped oscillations in all variables. Even the wage share oscillates around its trend, as seen in Figure 8.

It is hardly feasible to establish general dynamic properties of the models analytically, as functions of the parameter and coefficient values. If we plot r_i and $||r_i||$ for $i = 1, 2, 3$, as univariate functions of any single parameter conditional on the values of the other parameters and coefficients in **R**, no general pattern emerges. In some models and parameterizations an eigenvalue and/or its magnitude is a strictly increasing or decreasing marginal function of a parameter or coefficient. In other models and/or parameterizations it displays the opposite monotonicity, or an n- or u-shaped dependence.

E Computer implementation and simulations

The model is created and simulated in the TROLL system (Hollinger (2003)). The input file `makemodel.inp` defines the model, and `parameters.inp` sets all parameters to basis values. Both files are shown below. Start values for the variables are set in an input file that is not shown.

A certain version of the general model gets simulated by a TROLL input file named `simulateXYz.inp`, where `X`∈ {`T`,`E`} distinguishes between a targeted or endogenous unemployment rate, `Y`∈ {`U`,`N`,`W`,`P`} denotes the type of model and `z`∈ {`b`,`h`,`1`,`2`,`3`,`4`} marks the parameterization: e.g. `simulateTNb.inp` simulates the no-wedge model with basis parameterization and targeted unemployment. Below we show that particular input file and one that simulates the wage and price Phillips curve model in order to demonstrate the logic and practice of the simulations. The files are commented, and shown without further explanatory text.

```
// TROLL input file "makemodel.inp".  Creates the model.
USEMOD KNmodel; MODEDIT;
ADDSYM ENDOGENOUS    // Defining endogenous variables
a     // productivity
w     // nominal wage
q     // producer price
rw    // producer real wage
rwb   // workers' real wage claim
rwf   // firms' real wage plan
ecmb  // deviation of real wage from workers claim
ecmf  // deviation of real wage firms' plan
ws    // wage share
p     // consumer price (domestic)
pq    // price wedge
pf    // export price of foreign goods in foreign currency
pim   // import price
u     // unemployment rate
v     // exchange rate
re    // real exchange rate
;
ADDSYM EXOGENOUS    // Defining exogenous variables
zw     // standard normal innovation N(0,1) in the wage growth equation
zq     // standard normal innovation N(0,1) in the producer price equation
zu     // standard normal innovation N(0,1) in the unemployment rate equation
za     // standard normal innovation N(0,1) in the productivity equation
zpf    // standard normal innovation N(0,1) in the foreign export price equation
zv     // standard normal innovation in the exchange rate equation
shift  // step dummy = 0 in t=1,...t=50, thereafter =1
;
ADDSYM PARAMETER    // Defining parameters and coefficients
mq       // mark-up price
mw       // mark-up wage
iota     // parameter for productivity in wage setting plan
omega    // parameter for wedge in wage setting plan
uq       // parameter for unemployment in producer price setting plan
uw       // parameter for unemployment in wage setting plan
tetaq    // ECM parameter for price change
tetaw    // ECM parameter for wage change
cq       // constant periodwise growth in producer price (vartheta)
cw       // constant periodwise growth in wage (varomega)
psiqw    // parameter for wage in producer price equation
psiwq    // parameter for producer price in wage equation
psiqpi   // parameter for import price in producer price equation
psiwp    // parameter for consumer price in wage equation
varfi    // parameter for unemployment in wage equation
varsigma // parameter for unemployment in price equation
fi       // parameter in consumer price equation
cu       // parameter term in unemployment equation
alfa     // parameter for lag in unemployment equation
rho      // parameter for real exchange rate in unemployment equation
ga       // constant periodwise growth in productivity
gpf      // constant periodwise growth in foreign prices
gv       // constant periodwise growth in the exchange rate
sigmazq  // standard deviation of shock zq
sigmazw  // standard deviation of shock zw
sigmazu  // standard deviation of shock zu
sigmaza  // standard deviation of shock za
sigmazpf // standard deviation of shock zw
sigmazv  // standard deviation of shock zw
ushock   // size of shock to unemployment
;
ADDEQ BOTTOM    // Defining the structural equations of the model, all z-residuals are N(0,1)
rw   :  rw   = w   - q ,
pq   :  pq   = p   - q ,
re   :  re   = pim - q ,
ws   :  ws   = rw  - a ,
rwb  :  rwb  =  mw + iota*a + omega*pq - uw*u ,
rwf  :  rwf  = -mq +      a            + uq*u ,
ecmb :  ecmb = rw - rwb ,
ecmf :  ecmf = rw - rwf ,
w    :  DEL(1:  w) = cw - tetaw*ecmb(-1) + psiwp*DEL(1:  p)
                   + psiwq*DEL(1:  q) - varfi*u(-1) + sigmazw*zw ,
q    :  DEL(1:  q) = cq + tetaq*ecmf(-1) + psiqw*DEL(1:  w)
                   + psiqpi*DEL(1:  pim) - varsigma*u(-1) + sigmazq*zq ,
p    :  p          = fi*q + (1-fi)*pim ,
// The non-negative (log) unemployment process u is 'targeted' if the parameter rho = 0
u    :  u = MAX(cu + shift*ushock + alfa*u(-1) - rho*re(-1) + sigmazu*zu , 0) ,
```

```
// Processes for autonomous or 'exogenous' variables
pim :  pim         = pf + v ,
a   :  DEL(1:  a)  = ga  + sigmaza*za ,
pf  :  DEL(1:  pf) = gpf + sigmazpf*zpf ,
v   :  DEL(1:  v)  = gv  + sigmazv*zv ;
FILEMOD; QUIT; DELSEARCH ALL; DELACCESS ALL;
```

```
// TROLL input file "parameters.inp".  Contains parameter values for basis model with endogenous
// unemployment.  In the inputfile for the simulation of a certain model version, e.g. no-wedge, the
// restriction ω = 0 is implemented by omega = 0, which overrules the assignment below.  Different
// parameterizations of the different model version are implemented the same way in each simulation
// input file.
// Values for the structural parameters in the price setting
DO cq    = 0,       // constant periodwise growth in producer price (c_q)
mq       = 0.31,    // mark-up price (m_q)
tetaq    = 0.13,    // ECM coefficient for price change (θ_q)
psiqw    = 0.4,     // coefficient for wage in producer price equation (ψ_qw)
psiqpi   = 0.4,     // coefficient for import price in producer price equation (ψ_qpi)
uq       = 0.065,   // coefficient for unemployment in price setting plan (ϑ)
varsigma = 0,       // coefficient for unemployment in price Phillips equation (ς)
// Values for the structural parameters in the wage formation
cw       = 0,       // constant periodwise growth in wage (c_w)
mw       = 0.46,    // mark-up wage (m_w)
tetaw    = 0.12,    // ECM coefficient for wage change (θ_w)
iota     = 1,       // coefficient for productivity in wage setting plan (ι)
omega    = 0.5,     // coefficient for wedge in wage setting plan (ω)
uw       = 0.1,     // coefficient for unemployment in wage setting plan (ϖ)
psiwq    = 0.5,     // coefficient for producer price in wage equation (ψ_wq)
psiwp    = 0.2,     // coefficient for consumer price in wage equation (ψ_wp)
varfi    = 0,       // coefficient for unemployment in wage Phillips equation (φ)
// Parameter value for the consumer price
fi       = 0.6,     // coefficient in consumer price equation (ϕ)
// Values for the reduced form coefficient in the equation for the unemployment rate
cu       = -0.03,   // constant term in unemployment equation (c_u)
alfa     = 0.85,    // coefficient for lag in unemployment equation (α)
rho      = 0.1,     // coefficient for real exchange rate in unemployment equation (ρ)
// Exogenous growth rates
ga       = 0.005,   // constant periodwise growth in productivity (g_a)
gpf      = 0.01,    // constant periodwise growth in productivity (g_pf)
gv       = 0,       // constant periodwise growth in the exchange rate (g_v)
// Standard deviation of temporary shocks to variables in the model
sigmazq  = 0.001,   // standard deviation of shock zq (σ_q)
sigmazw  = 0.001,   // standard deviation of shock zw (σ_w)
sigmazu  = 0.02,    // standard deviation of shock zu (σ_u)
sigmaza  = 0.0005,  // standard deviation of shock za (σ_a)
sigmazpf = 0.003,   // standard deviation of shock zpf (σ_pf)
sigmazv  = 0.007,   // standard deviation of shock zv (σ_v)
// Size of shift in the unemployment rate
ushock   = 0.1;     // size of shift to constant term in unemployment equation (c_u1)
```

```
TROLL input file "simulateTNb.inp".  Simulates no-wedge model with basis
parameterization and targeted unemployment rate.
ACCESS indata    TYPE FAME ID ../inputdata.db MODE R;
ACCESS outdata   TYPE FAME ID outputTNb.db    MODE C;
ACCESS outdatass TYPE FAME ID outputTNbss.db  MODE C;
SEARCH DATA outdata outdatass W; SEARCH FIRST indata;
USEMOD KNmodel; INPUT parameters;
// Changing parameters from their basis values
DO rho     = 0,          // targeted/exogenous unemployment
omega      = 0,          // coefficient for wedge in wage setting plan
cq         = 0,          // constant periodwise growth in producer price
cw         = -0.0570065, // constant periodwise growth in wage
cu         = 0.2075,     // constant term in unemployment equation
// Qarterly (stochastic) simulation 2001-2300
SIMULATE; SIMSTART 2001Q1; DOTIL 2300Q4; FILESIM outdata;
// Nullifying shocks for steady-state simulation
DO sigmazq = 0,          // standard deviation of shock zq
sigmazw    = 0,          // standard deviation of shock zw
sigmazu    = 0,          // standard deviation of shock zu
sigmaza    = 0,          // standard deviation of shock za
sigmazpf   = 0,          // standard deviation of shock zpf
sigmazv    = 0;          // standard deviation of shock zv
// Qarterly (deterministic) simulation 2001-2300
SIMULATE; SIMSTART 2001Q1; DOTIL 2300Q4; FILESIM outdatass;
QUIT; DELSEARCH ALL; DELACCESS ALL;
```

TROLL input file "simulateEP4.inp". Simulates the wage and price Phillips curve model with prameterization 4 and endogenous unemployment rate.

```
ACCESS indata    TYPE FAME ID ../inputdata.db MODE R;
ACCESS outdata   TYPE FAME ID outputEP4.db    MODE C;
ACCESS outdatass TYPE FAME ID outputEP4ss.db  MODE C;
SEARCH DATA outdata outdatass W; SEARCH FIRST indata;
USEMOD KNmodel; INPUT parameters;
// Changing parameters from their basis values
DO tetaq   = 0,          // ECM coefficient for price change
tetaw      = 0,          // ECM coefficient for wage change
uq         = 0.06,       // coefficient for unemployment in price setting plan
uw         = 0.22,       // coefficient for unemployment in wage setting plan
varsigma   = 0.006,      // coefficient for unemployment in price equation
varfi      = 0.022,      // coefficient for unemployment in wage equation
psiqw      = 0.3,        // coefficient for wage in producer price equation
psiqpi     = 0.7,        // coefficient for import price in producer price equation
psiwq      = 0.7,        // coefficient for producer price in wage equation
psiwp      = 0.3,        // coefficient for consumer price in wage equation
omega      = 0,          // coefficient for wedge in wage setting plan
fi         = 0.7,        // coefficient in consumer price equation
alfa       = 0.98,       // coefficient for lag in unemployment equation
rho        = 0.2,        // coefficient for real exchange rate in unemployment equation
cq         = 0.00705106, // constant periodwise growth in producer price
cw         = 0.0346039,  // constant periodwise growth in wage
cu         = -0.447384,  // constant term in unemployment equation
SIMULATE; SIMSTART 2001Q1; DOTIL 2300Q4; FILESIM outdata;
// Nullifying shocks for steady-state simulation
DO sigmazq = 0,          // standard deviation of shock zq
sigmazw    = 0,          // standard deviation of shock zw
sigmazu    = 0,          // standard deviation of shock zu
sigmaza    = 0,          // standard deviation of shock za
sigmazpf   = 0,          // standard deviation of shock zpf
sigmazv    = 0;          // standard deviation of shock zv
SIMULATE; SIMSTART 2001Q1; DOTIL 2300Q4; FILESIM outdatass;
QUIT; DELSEARCH ALL; DELACCESS ALL;
```

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