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Tout est au mieux dans ce meilleur des ménages possibles
The Pangloss critique of equivalence scales

Abstract:
A common approach to defining equivalence scales is to consider a household modelled as if it maximizes a single utility function. This may be founded on an assumption of the household maximizing a welfare function of individual utilities. For a positive analysis of the household, this may be appropriate, but it is argued that basing inter-household comparisons of welfare on this approach is generally not valid. The household will generally put different weight on the utility of the various household members, and this weighting does not necessarily correspond to society's aggregation of utility. This complication is called the Pangloss problem. An alternative definition of equivalence scales taking this into account is introduced and discussed.

Keywords: Equivalence scales, household, welfare function, Pangloss problem, intra-household distribution, children

JEL classification: D11, D12, D31, D63

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1 Introduction

Households require different incomes to obtain the same level of material well-being. This is because various individuals have differing needs and because larger households may benefit from returns to scale. To take this into account when performing comparisons of welfare for both practical policy making and descriptive studies, household incomes are often scaled by equivalence scales. Although the most common scales are based on supposed needs and normative budgets, efforts have been made to estimate equivalence scales from observed household behaviour. These approaches normally postulate that the household maximizes a household utility function, and then use observed market behaviour to estimate the corresponding expenditure function. Since equivalence scales are defined as the ratio between the expenditure functions, it may seem straightforward to estimate the equivalence scales once the expenditure functions are known.

Nevertheless, it is well-known that there are several problems of identification associated with the procedure above. First, only ordinal utility functions are identifiable from market behaviour. Consequently, almost any equivalence scale is compatible with every observed market behaviour (Blundell and Lewbel 1991). Furthermore, the definition of equivalence scales takes the structure of the household as given. This is a strong assumption since both marriage and fertility to a large extent are both controllable and voluntary. This means that it is a more or less rational decision to get children or to get married. Consequently, the household should not be modelled as choosing only between different consumption bundles but combinations of commodity bundles and household compositions (Pollak and Wales 1979). From this point of view, the whole concept of equivalence scales is to some extent meaningless. When it comes to the "cost of children", most parents' major expenditure on their children is time (see Bittman and Goodin (2000) for an approach attempting to taking this explicitly into account). This means that the labour supply decision, and hence income, should not be seen as exogenous. Nevertheless, by adopting a short run view, the last two problems may to a large extent be ignored.

In what follows, the term "utility" will be used to denote individual utility, interpreted as material well-being. Moreover, unless stated otherwise, utility is assumed to be interpersonally comparable ("ordinal level comparability" in the words of Sen (1977), see Blackorby and Donaldson (1991) for an extended discussion). Although interpersonal comparability of utility is controversial, we shall assume that it holds throughout the present work. See *inter alia* Hammond (1991) and the reference therein for further details on the debate. The aggregate utility of a household or a society will be denoted by the term "welfare" since it is unclear how a group of individuals can feel utility. An exception is the "household utility function", a term that appears so frequently in the literature that I prefer to keep it. The term is given a more precise meaning below. Inter-household comparisons of welfare does not necessarily follow from interpersonal comparability; conditions for this to hold are discussed below.

The approach to defining equivalence scales mentioned above is based on the existence of a household utility function, an approach that is seldom problematized. As brought up above, though, it is quite problematic to give this function a welfare interpretation since only individuals experience utility. Although some contributions mention Samuleson's (1956) seminal work to justify the existence of a household utility function, thorough discussions of the problem are rare. An exception is the excellent discussion by Blackorby and Donaldson (1993).

In most of their study, Blackorby and Donaldson (1993) assume that the household maximizes a welfare maximin welfare function which implies that all households members are equally well off when there is local non-satiation. Nevertheless, empirical studies indicates that the income share of different household members influences aggregate consumption (Duflo 2000; Lundberg et al. 1997). Furthermore, several studies reveal that boys frequently get more resources than girls (Behrman 1992). Hence, it may be unrealistic to assume that the household welfare function is of the Leontief type.

Actually, Blackorby and Donaldson's (1993) study does not require the household welfare function to take this particular functional form. Nevertheless, a new problem may arise when this assumption is relaxed: How is the welfare function determined? This function may reflect an unequal intra-household distribution of resources by putting more weight on some household members than others. Particularly, children cannot choose which household to live in, and hence cannot protect themselves against being given a small weight. Thus, to give a measure of the aggregate welfare of the household, a different function may be required. Treating the household's own welfare function as the ideal utility aggregator will give rise to in a problem that will be labelled the Pangloss problem. Ignoring this problem may lead to absurd conclusions. For instance, we shall see that there is a tendency that households with a more unequal intra-household distribution would be considered more productive in generating welfare. A similar argument is made by Bojer (1998), but the scope of her analysis is more limited. Haddad and Kanbur (1990) also discuss related problems when they show that ignoring intra-household inequality will give a downward bias in measures of inequality. Moreover, Apps and Rees (1988) give a thorough discussion of the intra-household distribution of resources in a similar model of the household decision making process, but the aim of their study is different from that of the present work.

We shall argue that a more sensible definition of equivalence scales than the one presented above is possible. If there is a social planner with an ethically sound way of averaging the utility of the household members, this new welfare function may be used to evaluate the aggregate welfare of the household given that it distributes resources so as to maximize its own welfare function. This definition has certain difficulties, but escapes from some of the problems of the traditional definition.

The paper is organized as follows: Section 2 introduces the model and defines some key concepts. The Pangloss problem is introduced and discussed in Section 3. The alternative definition of equivalence scales is introduced in Section 4, and Section 5 discusses interpretations and possible problems related to it. Section 6 concludes.

2 From the household utility function to the household welfare function

To formulate the Pangloss problem in a precise manner, we shall begin by outlining a model of household behaviour closely related to Blackorby and Donaldson's (1993) framework. Households are assumed to behave as if they maximize a welfare function depending on the individual members' utilities. Although the aggregate behaviour of the household may be described by a household utility function, the present framework makes discussions of household welfare and individual utility clearer.

We shall consider an economy populated with agents belonging to one out of a finite number of types or groups. These types may be simply adults and children or a more detailed decomposition. A household consists of individuals each belonging to one of the groups. The demographic composition of a household will be described by a vector z, where the g'th element of z denotes the number of household members belonging to type g. Let ψ denote a vector of other household characteristics, such as cultural attributes, education, area of residence etc. We shall assume that ψ is observable. For our purposes, a household is now completely described by a vector $\phi := (z', \psi')' \in \Phi$ where Φ is the space of household types.

In the traditional approach to definition and estimation of equivalence scales from household expenditure data, the household is assumed to behave as if it maximizes a *household utility function*

$$U : \mathcal{Q} \times \Phi \to \mathbb{R}. \tag{1}$$

Here, $\mathcal{Q}$ is the consumption set, in most cases a subset of Euclidean space, which for simplicity is assumed to be identical for all households and individuals. When the household faces a price vector $p \in \mathcal{P}$, duality theory gives us the household expenditure function $C : \mathcal{P} \times \mathbb{R} \times \Phi \to \mathbb{R}$, which is the lowest income necessary for a household ϕ facing prices p to reach a welfare level $\mathcal{W}$. An *equivalence scale* gives the ratio between the income a household ϕ needs to reach a welfare level $\mathcal{W}$ relative to that of a reference household ϕ_0. Most of the time, ϕ_0 is assumed to be a household consisting of a single agent, but it may be any other household as well. Formally, an equivalence scale is a function $L : \mathcal{P} \times \mathbb{R} \times \Phi^2 \to \mathbb{R}$ defined as

$$L(p, \mathcal{W}, \phi, \phi_0) := \frac{C(p, \mathcal{W}, \phi)}{C(p, \mathcal{W}, \phi_0)}. \tag{2}$$

If the reference household derives welfare $\mathcal{W}$ from an income y_0, $y_0 L(p, \mathcal{W}, \phi, \phi_0)$ is said to be the *equivalent income* of household ϕ given prices p.

The existence of the household utility function (1) is not trivial. First, as already mentioned, it is only meaningful to say that individuals experience utility. This implies that it is dubious to compare the value of the function U for different households consisting of more than one person. Furthermore, the aggregate demand of a group of individuals each maximizing their own utility function is generally not rationalizable by a common utility function. Unless all individuals have preferences satisfying Gorman's (1953) polar form, the distribution of income among the individuals have to satisfy a certain optimality condition. Particularly, for aggregation over household members to be permissible for a general set of individual utility functions, Chipman and Moore (1979) show that the income distribution has to be rationalizable as the maximum of a *Bergson-Samuelson welfare function* (see also the seminal paper by Samuelson (1956)).

Let an agent belonging to group g derive utility from consumption given by a function $u_g : \mathcal{Q} \to \mathbb{R}$. These utility functions are assumed to be interpersonally comparable, continuous, and strictly concave. To construct a model that is useful for discussing equivalence scales, it is crucial to model returns to scale within the household in a reasonable way. To achieve this, we employ a model formulation from production theory. For a given household consumption, the set of possible intra-household allocations of resources is given by the *allocation-possibility correspondence* $\Omega_\phi : \mathcal{Q} \to \mathcal{Q}^{n_\phi}$, where n_ϕ denotes the number of household members in a household with characteristics ϕ. To clarify the model, consider a couple of particular cases. The case of no returns to scale implies that each unit of household consumption can be consumed by one and only one household member. Consequently, when we assume free disposal, the sum of individual consumption vectors has to remain below total household consumption, which means that $\Omega_\phi(q) = \left\{(q_1, \ldots, q_{n_\phi}) : \sum_i q_i \leq q\right\}$. In a world with only pure public goods, every household member has access to the aggregate consumption vector. In this case, we have $\Omega_\phi(q) = \{(q_1, \ldots, q_{n_\phi}) : q_i \leq q \text{ for all } i\}$. Obviously, some goods may be private, some goods may be purely public and some goods may be semi-public. We shall assume that there is no true production of goods within the households in the sense that if $(q_1, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$, then $q_i \leq q$ for all i. This implies that returns to scale arises solely from the possibility of sharing certain goods. To assure the existence of an optimal intra-household allocation, we shall assume that $\Omega_\phi(q)$ is compact for all q.

Formally, the welfare functions may be defined as a class of increasing functions $W_\phi : \mathbb{R}^{n_\phi} \to \mathbb{R}$ for all $\phi \in \Phi$. W_ϕ takes as arguments the individual utilities of each household member and aggregates them to a common measure of household welfare. For a given household consumption $q \in \mathcal{Q}$, the individual household members will have access to a consumption vector

$$(q_1, \ldots, q_{n_\phi}) = \arg\max_{q_i \in \mathcal{Q}} \left\{ W_\phi \left(u_{\gamma(1)}(q_1), \ldots u_{\gamma(n_\phi)}(q_{n_\phi}) \right) : (q_1, \ldots, q_{n_\phi}) \in \Omega_\phi(q) \right\}, \qquad (3)$$

where $\gamma : \mathbb{N} \to \mathbb{N}$ is the function that assign to each household member its agent-type. We may now use this to give a rationale for the household utility function (1) by defining

$$U(q, \phi) = \max_{q_i \in \mathcal{Q}} \left\{ W_\phi \left(u_{\gamma(1)}(q_1), \ldots u_{\gamma(n_\phi)}(q_{n_\phi}) \right) : (q_1, \ldots, q_{n_\phi}) \in \Omega_\phi(q) \right\}. \qquad (4)$$

To assure continuity of U, we assume that Ω_ϕ has a closed graph for all ϕ. Now, we may also derive *individual indirect utility functions* given by $v_i(p, y, \phi) := u_{\gamma(i)}(q_i)$, where q is determined as the maximum of U given the budget constraint $p'q \leq y$ and q_i is determined by (3) given q. v_i gives the utility of individual i in a household ϕ when the household has income y and faces prices p. Furthermore, the definitions of the cost function and the equivalence scales given above still make sense for this new interpretation of U.

For (2) to be a meaningful expression, welfare levels between households have to be comparable in the sense that we are able to say that two households with different characteristics are at the same level of welfare. To compare the welfare of two households, it is necessary to normalize the welfare function in some way. One normalization of W_ϕ that may make these inter-household comparisons of welfare more meaningful is the concept of agreeing, which is based on Aczél and Roberts (1989). A household welfare function W_ϕ is said to satisfy the *agreeing property* (AG) if for all $u \in \mathbb{R}$ we have $W_\phi(u, \ldots, u) = u$. It is then seen that when AG holds, for all $\phi \in \Phi$ such that $n_\phi = 1$, W_ϕ is the identity function, so the welfare of a one-person household is simply the utility of the person. A welfare function satisfying the AG property corresponds to Blackorby and Donaldson's (1993, 338) concept of "equally-distributed-equivalent utility functions." Since we, for any intra-household distribution of utility $u_1, \ldots, u_{n_\phi}$, may find a utility level $\tilde{u}$ such that

$W_\phi\left(u_1,\ldots,u_{n_\phi}\right)=W_\phi\left(\tilde{u},\ldots,\tilde{u}\right)$, inter-household comparisons of the level of W_ϕ is sensible if individual utility is interpersonally comparable.

Another concept that will prove useful is the notion of *anonymity* (AN) introduced by May (1952). It states that the name (and hence agent type) of an individual is irrelevant for her weight in the household welfare function. AN is a strong property that is not necessarily satisfied by all household welfare functions. Formally, a household welfare function W_ϕ satisfies the anonymity property if for every vector of utility levels $u\in\mathbb{R}^{n_\phi}$ and every permutation of u, u^*, we have $W_\phi\left(u\right)=W_\phi\left(u^*\right)$.

A related property may hold for the allocation-possibility correspondences. A household allocation-possibility correspondence Ω_ϕ will be said to satisfy *permutational symmetry* (PS) if for every q and every $\left(q_1,\ldots,q_{n_\phi}\right)\in\Omega_\phi\left(q\right)$ we also have $\left(q_1^*,\ldots,q_{n_\phi}^*\right)\in\Omega_\phi\left(q\right)$, where $\left(q_1^*,\ldots,q_{n_\phi}^*\right)$ is any permutation of $\left(q_1,\ldots,q_{n_\phi}\right)$. PS implies that the feasibility of an intra-household allocation does not depend on the recipient.

Before embarking on a discussion of the Pangloss problem, we shall prove a result that will become handy in what follows and which shows the usefulness of the concepts introduced above.

Proposition 1: *If a household ϕ maximizes a quasi-concave AN welfare function W_ϕ, all household members have an identical concave utility function u and Ω_ϕ is PS and convex for all q, then there is a solution to the welfare maximization problem such that all household members get the same utility level for any household consumption vector $q\in\mathcal{Q}$.*

Proof: Since W_ϕ is increasing and quasi-concave, and u is concave, $\left(q_1,\ldots,q_{n_\phi}\right)\to W_\phi\left[u\left(q_1\right),\ldots,u\left(q_{n_\phi}\right)\right]$ is quasi-concave. Furthermore, if $\left(q_1,q_2,q_3,\ldots,q_{n_\phi}\right)\in\Omega_\phi\left(q\right)$, then it follows from PS that $\left(q_2,q_1,q_3,\ldots,q_{n_\phi}\right)\in\Omega_\phi\left(q\right)$. Hence, if we define $\tilde{q}:=\frac{1}{2}\left(q_1+q_2\right)$, convexity of Ω_ϕ implies that $\left(\tilde{q},\tilde{q},q_3,\ldots,q_{n_\phi}\right)\in\Omega_\phi\left(q\right)$. Assume now that $\left(q_1,q_2,\ldots,q_{n_\phi}\right)$ maximizes household welfare for a given household consumption. Then $\left(\tilde{q},\tilde{q},q_3,\ldots,q_{n_\phi}\right)$ is also feasible. Furthermore, since $\left(q_1,\ldots,q_{n_\phi}\right)\to W_\phi\left[u\left(q_1\right),\ldots,u\left(q_{n_\phi}\right)\right]$ is quasi-concave and AN , $W_\phi\left[u\left(q_1\right),u\left(q_2\right),u\left(q_3\right),\ldots,u\left(q_{n_\phi}\right)\right] = W_\phi\left[u\left(q_2\right),u\left(q_1\right),u\left(q_3\right),\ldots,u\left(q_{n_\phi}\right)\right] \leq W_\phi\left[u\left(\tilde{q}\right),u\left(\tilde{q}\right),\ldots,u\left(q_{n_\phi}\right)\right]$. Consequently, if we have $q_1\neq q_2$ in one optimum, then there is another optimum where $q_1=q_2$. By a renaming of individuals, an analogue line of reasoning implies that there is an optimum where $q_1=q_2=\ldots=q_{n_\phi}$.■

Remark: If either W_ϕ is strictly quasi-concave or u is strictly concave and W_ϕ is strictly increasing in at least one of its arguments, then there is a unique solution to the welfare maximization problem where everybody get the same consumption vector.

3 The Pangloss critique

Although the household utility function as defined in (4) may be suitable for positive analyses of household behaviour, we shall argue that it is not when it comes to normative implications. Particularly, the function W_ϕ is not generally appropriate to aggregate the utility levels of the household members. Moreover, ethically questionable conclusions may easily arise when using this function uncritically.

To see the fundamental problem, assume that W_ϕ satisfy AG for all $\phi \in \Phi$ and that the reference household ϕ_0 consists of a single agent, say agent number one. If this agent receives an income y_0, she reaches a level of utility $v_1(p, y_0, \phi_0) =: w_0$. A household ϕ's equivalent income is $y_\phi^* = y_0 L^*(p, w_0, \phi, \phi_0)$, giving agent i a level of utility $v_i\left(p, y_\phi^*, \phi\right) =: w_i$. From the definition of equivalent income, it follows that

$$W_\phi\left(w_1, \ldots, w_{n_\phi}\right) = w_0. \tag{5}$$

Furthermore, from the AG property, we have

$$W_\phi\left(w_0, \ldots, w_0\right) = w_0. \tag{6}$$

Consequently, the intra-household allocation of utility $\left(w_1, \ldots, w_{n_\phi}\right)$ gives the same total household welfare as an allocation giving every household member a utility level w_0. However, this statement is certainly non-trivial. Its validity depends crucially on W_ϕ being in some sense the correct way of aggregating the individual utility levels to an aggregate measure of welfare. Even for AG welfare functions, there is a myriad of possible welfare functions giving different results when the intra-household distribution of utility is unequal. To claim that the function the household itself uses to evaluate different intra-household distributions is the best, is certainly "Panglossian", referring to Voltaire's (1990) character Dr. Pangloss, whose doctrine was "*tout est au mieux dans ce meilleur des mondes possibles*",[1] and I will refer to this problem as the "Pangloss-problem". To rephrase the problem, if we consider W_ϕ to be the best way of aggregating utility levels into an aggregate measure of welfare, we claim that what is, is best.

If power is unequally distributed within the household, it is likely that the utility of some of the household members influences the objective function W_ϕ more than that of other household members. The utmost case is probably a household welfare function equalling the utility level of one household member. In this case, the utility allocations $(u, 0, \ldots, 0)$ and $(u, u, \ldots, u)$ would be judged equal, which is rather unintuitive. In this case, it is probable that a social planner would aggregate the utilities of the household members by another welfare function than W_ϕ.

[1] This label was suggested by Muellbauer (quoted in Pollak 1981) within a resembling framework.

To see one of the implications of the Pangloss problem, consider the case where all agents have the same utility function and the regularity conditions of Proposition 1 hold. If the one-person reference household ϕ_0 receives y_0, then the equivalent income of a household ϕ with a AN and AG welfare function is y^*_ϕ defined implicitly as

$$v_1 (p, y_0, \phi_0) = v_i \left(p, y^*_\phi, \phi\right) \tag{7}$$

since Proposition 1 implies that every household member will get the same level of utility, and this is equal to that of the reference household by definition of equivalent income. Hence the equivalence scale is y^*_ϕ/y_0. This equal intra-household distribution will generally not take place if W_ϕ is not AN, since this normally implies that some individuals will get a higher share of total income than others. Nevertheless, since W_ϕ is AG, the equal utility-allocation will still give a welfare level equal to that of the reference household. Since this allocation is not chosen, it implies that the household manages to reach a higher level of welfare with a different allocation of resources. Consequently, the equivalent income to y_0 is less than y^*_ϕ, and a household that does not pursue equality in the distribution of income will have an equivalence scale below y^*_ϕ/y_0. Furthermore if for some $\phi \in \Phi$, we have for all $u \in \mathbb{R}^{n_\phi}$ that $W_\phi (u) = u_i$ for some i, that is, the utility of agent i determines the welfare of the household, then maximizing household welfare equals maximizing agent i's utility. If agent i and the one-person reference household has the same utility of money for some price-regime $p \in \mathcal{P}$, then the equivalence scale equals unity.

This shows that when using W_ϕ to evaluate household welfare, there is a tendency that it is more expensive to run a household where resources are distributed evenly than one with unequal intra-household distribution. This is because a welfare function putting much weight on a small group of individuals will result in a high household welfare if that particular group receives most of the income. If all agents count equally, on the other hand, the resources have to be spread among a larger number of individuals.

4 Towards a solution

One way to solve the problem above is to retain the model of household behaviour presented above, but aggregate individual utilities by a different welfare function. Since only individuals experience utility, a social planner should take the individual utility levels of all the citizens as a starting point for social welfare calculations. For an evaluation of a policy change, this approach is certainly desirable. Nevertheless, such a welfare function will get complicated unless severe simplifying assumptions are imposed. Furthermore, construction of an aggregate measure of a household's welfare will normally require less data that calculation of all the individual utilities. Particularly, most statistical surveys use the household as the unit of analysis. Hence it is occasionally convenient to consider a social welfare function depending on the welfare level of the individual households, or at least to determine which of two households is "best off". Although it may be doomed nonsensical to state that two households are "equally well off" in this case, it appears frequently in everyday speech. This indicates that we are able to perform such judgements. Furthermore, to claim that no comparisons of household welfare is possible is almost as absurd as the claim that all interpersonal comparisons of utility are impossible. Nevertheless, the construction of a complete transitive ranking of the welfare of every household in a society may be more complicated. In any case, the computation of a household's welfare level should be in accordance with the social planner's evaluation of the household's welfare. One approach may be to assume that the social planner assigns a welfare level to the household equal to the utility level of the least well off member. This may be sensible in some cases, but may underestimate the total welfare of a household where resources are distributed unequally. Consequently, it may be cases where other welfare functions are more sensible. If, as above, the household is assumed to behave as if it maximizes a welfare function, this particular function is only usable if it corresponds to the social planner's welfare function. In the general case, these functions are not equal. Consequently, a new set of equivalence scales corresponding to the new measure of welfare will have to be derived.

Consider a new class of welfare functions $W_\phi^S : \mathbb{R}^{n_\phi} \to \mathbb{R}$ for all $\phi \in \Phi$, that corresponds to the social planners aggregation of the individual utilities. To make inter-household comparisons of welfare, we will normally require W_ϕ^S to satisfy AG. Further, it is normally ethically appealing to let everybody count equally, which implies that the welfare function should be AN in most cases. The social planner's expenditure function for a household ϕ is a function $C^S : \mathcal{P} \times \mathbb{R} \times \Phi \to \mathbb{R}$ that gives the minimum income the household requires to obtain a given level of welfare as measured by W_ϕ^S when income is distributed within the household according to (3). Formally, we may define the cost of reaching welfare level $\mathcal{W}$ given prices p as

$$C^S(p, \mathcal{W}, \phi) = \min \left\{ y \in \mathbb{R} : W_\phi^S \left[v_1(p, y, \phi), \ldots, v_{n_\phi}(p, y, \phi) \right] \geq \mathcal{W} \right\}. \tag{8}$$

From this definition, we may also define the social planner's equivalence scales

$$L^S(p, \mathcal{W}, \phi, \phi_0) := \frac{C^S(p, \mathcal{W}, \phi)}{C^S(p, \mathcal{W}, \phi_0)}. \tag{9}$$

It was argued above that in most cases, L will decrease when more weight is put on a small group of household members. Since this conclusion may seem rather unethical, we can hope that L^S does not share this property. Assume as above that all agent types have identical utility functions and that the regularity conditions of Proposition 1 hold. Moreover, we shall assume that W_ϕ^S is AN. If the household's own welfare function W_ϕ is also AN, then every household member get the same level of utility, and the aggregate welfare is identical whether measured by W_ϕ or W_ϕ^S. Let y_ϕ^* denote household ϕ's equivalent income to the reference household ϕ_0 receiving y_0. If W_ϕ is not AN, then the intra-household allocation of resources is not optimal when measured by W_ϕ^S, which implies that

$$W_\phi^S \left[v_1 \left(p, y_\phi^*, \phi \right), \ldots, v_{n_\phi} \left(p, y_\phi^*, \phi \right) \right] < v_1 \left(p, y_0, \phi_0 \right). \tag{10}$$

Consequently, the equivalent income corresponding to y_0 is above y_ϕ^* when judged by the social planner's welfare function.

Hence, when using a AG-AN function W_ϕ^S to aggregate individual utilities, a household distributing resources equitably within the household is the "cheapest" household to run. Distributing resources unevenly is inefficient since the marginal gain to increasing household welfare of making an agent that is well off is less than that of an agent that is less well off.

It should be remarked that if utility functions differ among household members, the conclusions become less clear cut. If there are important differences in the "productivity of utility" between individuals, a household with an AN welfare functions putting little weight on equality between household members may result in distributions further away from the ideal distribution of a social planner putting much weight on equality than certain non-AN welfare functions would. Nevertheless, it is unclear what we should mean by large differences in productivity, so the conclusions above will probably hold as an approximation in most reasonable circumstances.

This line of reasoning means that if we have two households with identical demographic composition, but where the ψs differ, so that the households have different welfare functions, the household with the most uneven intra-household distribution of wellbeing needs a higher income than the one with a more even distribution for the two households to be at the same level of welfare as seen by the social planner. This may seem counter-intuitive and even unfair. Nevertheless, this is not as much a problem of the approach considered herein as a general problem of unfair decision mechanisms within the households. Furthermore, in cases where some households have an unfair way of distributing resources, a pure transfer of income may be a poor instrument. Instead, subsidies of particular goods may be judged superior.

It may also be argued that imposing W_ϕ^S to evaluate household welfare is paternalistic. Most normative studies have a non-paternalistic approach considering the agents' own utility functions as the "correct" way of calculating their utility in a given situation. It may very well be argued that this should apply to the welfare function as well. As mentioned introductorily, households choose to some extent both consumption and composition. Especially, the formation of couples is normally based on voluntariness from both parties. This means that all household members should prefer W_ϕ to any other possible welfare function when they choose to enter a household of type ϕ. However, all household members do not choose which household to belong to. Probably most important, children do not decide where to be born. In most households, children's consumption is to a large extent determined by the parents, both because children may have a perceived individual utility function that differs from what is to their own benefit by for instance putting much weight on sweets, and because the adults are the main income earners. Consequently, it is natural to model the children's impact on household welfare through the adults' utility function. For instance, an adult may have an altruistic utility function depending on her own individual utility and the utility of her children. Then the household welfare function may depend on these function for each adult. From this, we may derive a household welfare function as in Section 2. Nevertheless, even if the welfare function is in some sense correct for the adults, the children's position is not necessarily correct as seen by the social planner. Public provision of e.g. schools and kindergartens indicates that the social planner does not agree with the intra-household allocation taking place, and hence provides subsidies to influence the household's behaviour. A similar argument is made by Del Boca and Flinn (1995); see also e.g. Bojer (2000) or Levison (2000) and the references therein for further discussion of problems related to children in models of households.

5 Discussion

Hitherto, W_ϕ has been regarded mainly as a "black box". This is probably not particularly useful to get a grasp of the welfare implications of the decision mechanism that are necessary for a discussion of the Pangloss problem. Some more intuitively appealing justifications of this particular household decision structure may include:

1. A "household council" where all household members are present makes consensually the consumption and distribution decisions. This does not necessarily imply an equal intra-household distribution of utility. For instance, maximization of household income may imply inequality, cf. Pitt et al. (1990); see also Glaeser (1992) for a more amusing example.

2. W_ϕ corresponds to the utility function of a more or less altruistic head of household who

determines the intra-household distribution according to needs. This interpretation is in accordance with e.g. Becker's (1974) "Rotten kid"-theorem.

3. The household's consumption and distribution decisions are in reality the outcome of a bargaining process. For instance, W_ϕ may be a Nash-product where the outside opportunity is assumed to be independent of prices. In this case, different weights on the household members may correspond to differences in bargaining power. Such a bargaining procedure may be given an ethical content although the issue is controversial. Roemer (1996, ch. 2) provides a discussion of some of the points of view.

Evidently, most real-world household will be characterized by a mixture of these, where for instance the "household council" is only composed of the adults, and where there is some degree of bargaining, but also a large extent of consensus.

If the household is seen as a "locus of gender, class, and political struggle" (Hartmann 1981), it is normally quite clear that the observed household welfare function W_ϕ is inappropriate to aggregate household utility in an ethically appealing way. On the other hand, if household decision making is seen as mainly based on agreement, it may seem superfluous to introduce a separate social planner's welfare function. Nevertheless, even though unanimity may seem to prevail, in the sense that there is no apparent use of power, there may be latent conflicts cf. Lukes' (1974) definition of power. He presents a critique of the behavioural focus of more traditional definitions of powers, and argues in favour of including control of issues and potential uses, as well as a distinction between real and expressed interests.

Hitherto, the functions u_i has implicitly been considered as known to all the household members as well as the social planner. The household being a close-knit group, it is probably reasonable to assume that the household members know each others' utility functions. The social planner, and indeed an economist, does not necessarily have such knowledge. Hence, the definition of equivalence scales in equation (9) is not operationalizable without further information making estimation of the individual utility functions possible. Since utility should be interpersonally comparable, it is far from obvious how to do this. Consequently, estimation of household equivalence scales will probably have to be based on a more restrictive definition taking the definition (9) as a basis. Bourguignon (1999) and Pitt (1997) make some steps towards achieving this.

In their discussion of equivalence scales, Blackorby and Donaldson (1991, 1993) argues for using the Equivalence scales exactness (ESE - also known as independence of base) procedure. If the households' and the social planner's welfare functions are similar, this is an excellent way to identify equivalence scales if one believes in the identifying restrictions. Nevertheless, if the social planner has a different way of aggregating individual utilities than the household, estimation is more complicated. If the household expenditure function satisfies the ESE conditions, this does not imply that the social planner's equivalence scales are independent of the household welfare level. On the other hand, if we want the social planner's equivalence scales to satisfy ESE, then this does impose restrictions on household behaviour, but these restrictions are different from the ordinary ESE-restrictions.

Remark also that (8) may be generalized. Among others, Alderman et al. (1995) argue that the unitary approach to household decision making is too restrictive both from a theoretical and an empirical point of view. Alternatives include the general approach of Bourguignon, Browning, Chiappori, and Lechêne (Browning et al. 1994; Browning and Chiappori 1998), as well as a wide range of cooperative and non-cooperative game theoretic approaches (Lundberg and Pollak 1996). Assume that we can construct a class of functions $\xi_i : \mathcal{P} \times \mathbb{R} \times \Phi \to \mathcal{Q}$, such that for all i, $\xi_i(p, y, \phi)$ gives agent i's consumption vector. Such an allocation rule is compatible with a wide variety of household decision mechanisms, also the one presented above. It is reasonable that a particular agent's consumption vector depends on the income share of the different household members. To take this into account, we may let some of the elements of ψ contain information on own income shares. Furthermore, ψ may also contain information on the household members' bargaining power. Consequently, it may be restrictive to assume ϕ fixed across time. Since the present analysis is restricted to the static case, this will not imply any difficulties. Now individual i has an individual indirect utility function

$$v_i(p, y, \phi) = u_{\gamma(i)}(\xi_i(p, y, \phi)). \tag{11}$$

Consequently, the social planner's expenditure function is still given by (8) and the equivalence scales by (9) if we use the new definition of v_i.

6 Conclusion

In most approaches to estimating equivalence scales from household expenditure data, the household is modelled as maximizing a common household utility function. Because households normally consist of more than one individual, this approach is dubious both in its positive and normative implications. The former may be resolved by considering the household utility function as a reduced form of a household welfare function. Nevertheless, the welfare significance of this function is indeed questionable. A problem labelled the Pangloss problem arises since this use of the household welfare function implies accepting that the observed intra-household distribution of resources and hence utility is optimal from a social point of view. Moreover, this implies in most cases that households distributing resources unevenly are judged more efficient in generating welfare than households distributing resources evenly.

The equivalence scales exactness-method to estimate equivalence scales is clearly vulnerable to this criticism since the scales are defined as the ratio of expenditure functions. Since both the Engel and Rothbarth methods are also derived from household utility functions, their validity is also doubtful. The subjective scales of the Leyden school (see e.g. van Praag and van der Sar 1988) may seem to escape from the problem. Nevertheless, the evaluation of a household's welfare may depend on the person answering the income evaluation question. If the respondent is also in charge of intra-household distribution, this answer may be misleading in the same way as the household welfare function for judging the true welfare of the household.

To attempt to solve the Pangloss problem, it was suggested to keep the model of household behaviour, but to introduce a new welfare function corresponding to the ethical preferences of the social planner to aggregate the utility level of the household members. This new measure has more appealing properties than the traditional measure. A difficulty is obviously that this welfare function should be determined politically, which may not be straightforward.

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