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Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data

Abstract:
This paper analyses the importance of scale economies by means of unbalanced plant-level panel data from three Norwegian manufacturing industries. Focus is on heterogeneous technologies, and unlike most previous work on micro data, the model description includes heterogeneity in both the scale properties (the slope coefficients) and the intercept term, represented by random coefficients in the production function. Three (nested) functional forms are investigated: the Translog, an extended Cobb-Douglas, and the strict Cobb-Douglas. Although constant or weakly increasing returns to scale is found for the average plant, the results reveal considerable variation across plants. Variations in both input and scale elasticities are to a larger extent due to randomness of the production function parameters than to systematic differences in the input mix.

Keywords: Panel Data. Economies of Scale. Heterogeneity. Random Coefficients

JEL classification: C23, D24, L61, L65, L73

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1 Introduction

The scale properties of production technologies are of vital importance for our understanding of market structure, productivity, and economic growth, and, within the industrial economics literature, economies of scale is put forward as a possible important barrier to entry, see *e.g.* Tirole (1989, pp. 305 – 306) and the references therein. Hence, knowing the scale properties may help us understand the evolution of industries. Although there are important exceptions, it is common in empirical analyses of the production process, applying both the primal and the dual approach, to assume a constant returns to scale technology. This is true for analyses using both micro and macro data. One explanation for this restriction is co-movements of the explanatory variables that makes it difficult to identify independently the impacts of technical change, capital stock growth, and returns to scale; cf. Morrison (1988) and Biørn, Lindquist and Skjerpen (1998). However, if the constant returns to scale restriction is false, this is likely to influence conclusions regarding technical change and productivity.

There is a growing number of articles that analyse the production process econometrically using micro data under the assumption that heterogeneity in size, age, management, employees' education, technology, etc., can be represented by a plant specific fixed or random intercept term in the production, cost, or profit function. Most likely, however, such differences will manifest themselves not only as a permanent, *i.e.*, constant, variation in efficiency across plants, but will also result in heterogeneity in scale properties. In this case, the standard modelling approach, with only fixed or random effects in intercept terms, may lead to inefficient estimation of the slope coefficients and invalid inference.

This paper chooses a more general approach and analyses the importance of scale economies by estimating a four-factor ($KLEM$) production function with *heterogeneous scale properties* and no *a priori* restrictions on the returns to scale. Our approach differs from that in the panel data literature on *frontier production functions* and efficiency measurement, dealing with deterministic or stochastic production frontiers in a framework with firm specific heterogeneity; cf. Cornwell and Schmidt (1996). In the present paper, three (nested) functional forms of the *average* production function are investigated: the Translog, an extended Cobb-Douglas, and the strict Cobb-Douglas. Heterogeneity in both the slope coefficients representing the scale properties as well as the intercept term is allowed for. To avoid overparameterization and the degrees of freedom problem, a random coefficient approach, with specific assumptions made about the distribution from which the plant specific coefficients are drawn, is applied. This is a parsimonious and easily interpretable way of representing heterogeneity. The expectation vector in this distribution represents the coefficients of an average plant, while its covariance matrix

gives readily interpretable measures of the degree of heterogeneity which is due to the random coefficient variation. In addition, the non-homotheticity of the production function allows for systematic variation in the scale elasticity, *i.e.*, variation with the input quantities. The purpose of this paper is to quantify both the random and the systematic variation of the scale elasticity.

Our primary argument for using the primal approach and not following the alternative dual approach is our focus on heterogeneity in the production function parameters rather than in the parameters of the cost or profit function. Arguments for taking the *primal approach*, even if the agents follow optimizing behaviour, have been given by, *inter alia*, Zellner, Kmenta, and Drèze (1966) and Mundlak (1996) in a Cobb-Douglas context; see also Griffiths and Anderson (1982), Mairesse (1990), Mairesse and Griliches (1990), Wan, Griffiths, and Anderson (1992), and Griliches and Mairesse (1998, section 2).

The panel data set applied in this paper is from the Norwegian manufacturing statistics data base of Statistics Norway. It is *unbalanced* and consists of plants from the Pulp and paper industries, the Chemical industries, and the Basic metals industries in Norway. We follow the recommendations in Mátyás and Lovrics (1991) and Baltagi and Chang (1994) and do not apply a balanced subsample of the original unbalanced data set. Our output measures for the three industries are in *physical units*, and are in several respects preferable to those used in other studies of production technologies, *e.g.*, deflated sales, which may be affected by measurement errors; see Klette and Griliches (1996).

The combination of a random coefficient model and unbalanced panel data which our analysis examplifies, is far from standard, at least in applied econometrics. Mixed regression models with unbalanced design, however, have, to some extent, been discussed in the statistical literature, see, *e.g.*, Amemiya (1994) and Shin (1995). Random coefficients in regression equations in econometrics are treated in the pioneering studies of Swamy (1970, 1971, 1974); see also Hsiao (1975, 1996) and Longford (1995a,b).

A major finding in this study is that substantial improvement in model fit is obtained when allowing for random coefficient heterogeneity. We find constant or weakly increasing returns to scale for a plant with an average technology, but the results reveal important variation across plants, and plants with both increasing and decreasing economies of scale are present.

2 Model and econometric method

We assume that the average plant has a four-factor technology, with capital (K), labour (L), energy (E), and materials (M) as inputs and with one output (Y). The most general

specification of the technology is assumed to be non-homothetic and is represented by a production function belonging to the Translog class, with a trend, and with some coefficients specified as random variables. This random variation represents non-systematic heterogeneity of the technology.

Below we describe the basic elements of our model, for simplicity without explicitly incorporating the unbalancedness of the panel data set. The accommodation of the model to our unbalanced panel data and the Maximum Likelihood estimation procedure is elaborated in Appendix A.

Let subscripts i and t denote the plant and the year (number) of observation, respectively. Our Translog model framework can be written as

$$y_{it} = c_i + \gamma\tau_t + \tfrac{1}{2}\gamma^*\tau_t^2 + z_{it}'\alpha_i + \tfrac{1}{2}z_{it}'Bz_{it} + z_{it}'\delta\tau_t + u_{it}, \tag{1}$$

where

$$y_{it} = \ln(Y_{it}), \qquad z_{it} = [\ln(K_{it}), \ln(L_{it}), \ln(E_{it}), \ln(M_{it})]',$$

c_i is a plant specific random intercept term, τ_t is a deterministic trend term representing the level of the technology in year t, and u_{it} is a genuine disturbance term. The vector α_i is specified as plant dependent and random, and the matrix and vector of second-order coefficients, B and δ, as constants:[1]

$$\alpha_i = \begin{bmatrix} \alpha_{Ki} \\ \alpha_{Li} \\ \alpha_{Ei} \\ \alpha_{Mi} \end{bmatrix}, \quad B = \begin{bmatrix} \beta_{KK} & \beta_{LK} & \beta_{EK} & \beta_{MK} \\ \beta_{LK} & \beta_{LL} & \beta_{EL} & \beta_{ML} \\ \beta_{EK} & \beta_{EL} & \beta_{EE} & \beta_{ME} \\ \beta_{MK} & \beta_{ML} & \beta_{ME} & \beta_{MM} \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_K \\ \delta_L \\ \delta_E \\ \delta_M \end{bmatrix}.$$

The intercept term c_i and all elements of α_i are specified as plant dependent and random in general, but in some models, we impose additional restrictions, as will be explained below.

The heterogeneity of the coefficient structure across plants is represented as follows. Let θ_i denote the column vector containing all the (random or fixed) coefficients in the model, *i.e.*,

$$\theta_i = [c_i,\ \alpha_i',\ \gamma,\ \gamma^*,\ \beta',\ \delta']', \tag{2}$$

where $\beta = \operatorname{vech} B$ is the half-vectorization of B, *i.e.*, the lower triangular part of B stacked into a column vector. We assume that all z_{it}, u_{it}, and θ_i's are mutually independent,

[1] Attempts were made to solve the Maximum Likelihood estimation problem (under normality of the random coefficients and the disturbance terms) for the specification with random and plant dependent B, γ, γ^*, and δ, but this turned out to raise numerical problems. We therefore decided to consider only models in which these second-order coefficients are constants.

with $\mathsf{E}(u_{it}) = 0$, $\mathrm{var}(u_{it}) = \sigma_{uu}$, and

$$(3) \qquad \mathsf{E}(\theta_i) = \theta = \begin{bmatrix} c \\ \alpha \\ \gamma \\ \gamma^* \\ \beta \\ \delta \end{bmatrix}, \ \mathsf{E}[(\theta_i - \theta)(\theta_i - \theta)'] = \mathbf{\Omega} = \begin{bmatrix} \omega_{cc} & \Omega'_{\alpha c} & 0 \\ \Omega_{\alpha c} & \Omega_{\alpha\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where $\alpha = \mathsf{E}(\alpha_i)$, $\omega_{cc} = \mathrm{var}(c_i)$, $\Omega_{\alpha\alpha} = \mathsf{E}[(\alpha_i - \alpha)(\alpha_i - \alpha)']$, etc., and the zero sub-matrices of Ω, representing non-random coefficients, have suitable dimensions.

We denote the model with diagonal B, $\delta = 0, \gamma^* = 0$ as the *Extended Cobb-Douglas model*, $B = 0, \delta = 0, \gamma^* = 0$ giving a strict Cobb-Douglas model. The Extended Cobb-Douglas model implies that the output elasticity of a specific input (input elasticity, for short) depends on the volume of that input, but is independent of the other inputs. This is in contrast with the more flexible Translog model, in which the input elasticities depend on all inputs. Both the Translog and the Extended Cobb-Douglas technologies are non-homothetic, unlike the strict Cobb-Douglas model, which is homothetic and has a constant scale elasticity.

The (column) vector of *input elasticities* of plant i in year t is

$$(4) \qquad \eta_{it} = [\eta_{Kit}, \eta_{Lit}, \eta_{Eit}, \eta_{Mit}]' = \frac{\partial y_{it}}{\partial z_{it}} = \alpha_i + B z_{it} + \delta \tau_t,$$

the derivative of log output with respect to the trend is

$$(5) \qquad \eta_{\tau it} = \frac{\partial y_{it}}{\partial \tau_t} = \gamma + \gamma^* \tau_t + \delta' z_{it},$$

and the *scale elasticity* of plant i in year t is

$$(6) \qquad \nu_{it} = e_4' \eta_{it} = \bar{\alpha}_i + \bar{\beta}' z_{it} + \bar{\delta} \tau_t,$$

where e_n is the n vector of ones and $\bar{\alpha}_i = e_4' \alpha_i$, $\bar{\beta}' = e_4' B$, $\bar{\delta} = e_4' \delta$. Conditionally on z_{it}, the random variation of the input elasticities and the scale elasticity is due to the randomness of the first-order coefficients in the production function, α_i, only. The expectation and variance of the scale elasticity of plant i in year t, conditionally on the input vector z_{it}, can be written as, respectively,

$$(7) \qquad \mathsf{E}(\nu_{it} | z_{it}) = \mathsf{E}(\bar{\alpha}) + \bar{\beta}' z_{it} + \bar{\delta} \tau_t,$$

$$(8) \qquad \mathrm{var}(\nu_{it} | z_{it}) = \mathrm{var}(\bar{\alpha}),$$

where $\mathsf{E}(\bar{\alpha}) = e_4'\alpha$ and $\mathrm{var}(\bar{\alpha}) = e_4'\Omega_{\alpha\alpha}e_4$ are the common expectation and variance of $\bar{\alpha}_i$. We can interpret $\mathsf{E}(\nu_{it}|z_{it})$ as representing the systematic (non-random) heterogeneity in the scale elasticity, while $\mathrm{var}(\nu_{it}|z_{it})$ represents the dispersion of its random heterogeneity. Similar expressions can be derived for the input elasticities. Using (7), (8), and the law of iterated expectations, the marginal (unconditional) expectation and variance of the scale elasticity can be written as

$$
\begin{aligned}
(9) \qquad \mathsf{E}(\nu_{it}) &= \mathsf{E}[\mathsf{E}(\nu_{it}|z_{it})] = \mathsf{E}(\bar{\alpha}) + \bar{\beta}'\mathsf{E}(z_{it}) + \bar{\delta}\tau_t \\
&= e_4'\alpha + e_4'B\mathsf{E}(z_{it}) + e_4'\delta\tau_t, \\
(10) \qquad \mathrm{var}(\nu_{it}) &= \mathsf{E}[\mathrm{var}(\nu_{it}|z_{it})] + \mathrm{var}[\mathsf{E}(\nu_{it}|z_{it})] \\
&= \mathrm{var}(\bar{\alpha}) + \bar{\beta}'\mathsf{V}(z_{it})\bar{\beta} = e_4'\Omega_{\alpha\alpha}e_4 + e_4'B\mathsf{V}(z_{it})Be_4,
\end{aligned}
$$

where $\mathsf{V}(z_{it})$ is the covariance matrix of z_{it}. Eq. (10) represents jointly the heterogeneity in the scale elasticity which is due to the stochastic variation in the first-order coefficients (the first term) and the heterogeneity which is due to the variation in the input vector across plants (the second term). Similar expressions can be obtained for the input elasticities.

Nine models, that differ with respect to functional form and the representation of the heterogeneity of the technology, are considered. As abbreviations for Translog, Extended Cobb-Douglas, and strict Cobb-Douglas we use TL, ECD, and CD, respectively – in parenthesis indicating which coefficients are treated as random in each model. The models are specified below:

Model	ω_{cc}	$\Omega_{\alpha\alpha}$	$\Omega_{\alpha c}$	B	$\delta,\ \gamma^*$
TL(c,α)	unrestricted	unrestricted	unrestricted	unrestricted	unrestricted
TL(c)	unrestricted	0	0	unrestricted	unrestricted
TL	0	0	0	unrestricted	unrestricted
ECD(c,α)	unrestricted	unrestricted	unrestricted	diagonal	0
ECD(c)	unrestricted	0	0	diagonal	0
ECD	0	0	0	diagonal	0
CD(c,α)	unrestricted	unrestricted	unrestricted	0	0
CD(c)	unrestricted	0	0	0	0
CD	0	0	0	0	0

The structure of the model tree is presented in Figure 1.

The expected coefficient vector θ and the unknown elements of the covariance matrix $\mathbf{\Omega}$, given by (3), for the different models are estimated by Maximum Likelihood, using

the PROC MIXED procedure in the SAS/STAT software [see Littell *et al.* (1996)]. Positive definiteness of the non-zero submatrix of $\mathbf{\Omega}$ (relating to the random coefficients) is imposed as an *a priori* restriction. See also Appendix A.

3 Data

We use an unbalanced plant-level panel data set that covers the period 1972 – 1993. The primary data source is the Manufacturing Statistics database of Statistics Norway. Our initial data set includes all large plants, generally defined as plants with five or more employees (ten or more employees from 1992 on), classified under the Standard Industrial Classification (SIC)-codes 341 Manufacture of paper and paper products (Pulp and paper, for short), 351 Manufacture of industrial chemicals (Chemicals, for short) and 37 Manufacture of basic metals (Basic metals, for short). Both plants with contiguous and non-contiguous time series are included.

Some minor data cleaning has been performed; *i.e.*, we have removed observations with zero production or zero inputs. This reduced the number of observations by 4 – 8 per cent in the three industries. The number of plants per year ranges from 81 to 179 in Pulp and paper, from 46 to 66 in Chemicals, and from 71 to 111 in Basic metals. There is a clear negative trend in the number of plants from the mid-seventies in all three industries. The unbalance in our data set is shown in Table 1, which gives the number of plants sorted by the number of observations. For example in Pulp and paper, 60 plants are observed in all 22 years (1972 – 1993), while 20 plants are observed in one year only.

Some remarks on why gaps occur in the time series of some plants (non-contiguous time series) seem appropriate. All large plants are obliged by law to report information on a large number of variables to Statistics Norway. Missing observations due to non-response can therefore be expected to be a minor problem. Three reasons for gaps in the series may be given: (i) Only large plants, according to the above mentioned criterion, are obliged to report. If a plant switches between being 'large' and 'small', there may be gaps in its time series. This may cause a potential endogenous selection problem, and ideally, our data set should have included these 'missing' observations. An inspection of the data revealed, however, that this was not an important cause for gaps. (ii) The plants in our sample are in general multi-output plants and are defined as belonging to a specific industry depending on their most important products. Although not very common, a plant can switch between two industries due to major shifts in output composition, and hence go into and out of our sample. With respect to identifying the technology of true Pulp and Paper, Chemicals, and Basic metals plants, these plants represent a potential

problem. (iii) Gaps may be due to dramatic events such as insolvency. If the same type of production continues at the same location after an inactive period, the plant will re-enter the data base with the same plant-number.[2] In general, plants with gaps do not seem to differ from plants with contiguous time series, and we therefore decided to include these plants in our information set. By reproducing the estimation with the non-contiguous time series removed, within a related dual approach for Chemicals, we found that such plants did not tend to 'pollute' the estimation results [cf. Biørn, Lindquist, and Skjerpen (1998)].

TABLE 1. NUMBER OF PLANTS CLASSIFIED BY NUMBER OF REPLICATIONS

p = no. of observations per plant, N_p = no. of plants observed p times,

$N = \sum N_p$, $n = \sum N_p p$

Industry	*Pulp & paper*		*Chemicals*		*Basic metals*	
p	N_p	$N_p p$	N_p	$N_p p$	N_p	$N_p p$
22	60	1320	29	638	44	968
21	9	189	0	0	2	42
20	5	100	3	60	4	80
19	3	57	0	0	5	95
18	1	18	2	36	2	36
17	4	68	4	68	5	85
16	6	96	9	144	5	80
15	4	60	6	90	4	60
14	3	42	1	14	5	70
13	4	52	3	39	3	39
12	7	84	1	12	10	120
11	10	110	2	22	7	77
10	12	120	3	30	6	60
09	10	90	2	18	5	45
08	7	56	2	16	2	16
07	15	105	2	14	13	91
06	11	66	3	18	4	24
05	14	70	3	15	5	25
04	9	36	2	8	6	24
03	18	54	3	9	3	9
02	5	10	3	6	6	12
01	20	20	7	7	20	20
Sum: N, n	237	2823	90	1264	166	2078

[2] If the plant is 'new', *i.e.*, largely retooled, it is identified by a new number.

4 Empirical results

Goodness of fit

Table 2 reports the goodness of fit of all the estimated models, expressed in terms of the log-likelihood value (LLH),[3] Akaike's Information Criterion (AIC), and Schwarz's Bayesian Criterion (SBC).[4] Within models with the same functional form, these three criteria give identical ranking with respect to the specification of heterogeneity: the models that include heterogeneity in both the intercept term and the first-order coefficients in the production function, *i.e.*, c_i and α_i, give a clearly better fit than the models with heterogeneity only in the intercept term, and a markedly better fit than the models with no coefficient heterogeneity. There is thus evidence that allowing for random heterogeneity in the α_i coefficients of the production function improves the fit to our plant panel data in comparison with more restrictive models.

Concentrating on the functional form and comparing models with the same specification of heterogeneity, we find that the picture is somewhat less clear, although the general result is that TL($\cdot$) outperforms both ECD($\cdot$) and CD($\cdot$). In some cases, however, SBC ranks CD first. This is particularly true for the models with random intercepts and α_i's. It should be remembered, though, that this criterion penalizes coefficient-rich models relatively hard. The estimates of the genuine disturbance variance, σ_{uu}, support our general conclusion with respect to model fit: it decreases strongly when more heterogeneity is allowed for and also when the flexibility of the functional form increases.

The estimated variance of the random intercept, ω_{cc} is substantially higher when the coefficient vector α_i is specified as random than when it is fixed (compare columns $1-3$ with $4-6$ in Table 2). On the other hand, the choice of functional form affects the estimated variance of the random intercept modestly.

Degree of coefficient heterogeneity

The last row of each panel of Table 2 gives a measure of the overall degree of coefficient heterogeneity (including intercept heterogeneity) in each estimated equation. The

[3] Likelihood Ratio test statistics can be easily calculated from the tables. These statistics are, however, not asymptotically χ^2-distributed under the null hypothesis of full coefficient homogeneity, because the parameters in $\mathbf{\Omega}$ then are on the border of the admissible parameter space, see Shin (1995, p. 321). Thus, for making formal inference of coefficient heterogeneity versus homogeneity, other test procedures may be needed, see the recent papers by Khuri *et al.* (1998) and Andrews (1999). We have not followed up these ideas in the present paper, however.

[4] The two latter criteria are defined, for a model, m, by, respectively, $AIC_m = l_m - q_m$ and $SBC_m = l_m - 0.5 q_m \ln(N_m)$, where l_m is the log-likelihood value of model m, q_m is its number of parameters, and N_m is its number of observations.

measure is the estimated share, φ, at the overall sample mean of the inputs,[5] of the variances of the gross disturbance $\psi_{(ip)t}$ [cf. (A.3) and (A.8)] which is due to coefficient heterogeneity. In the model with random intercept term c_i only, it is the estimated value of

$$\varphi = \frac{\omega_{cc}}{\omega_{cc} + \sigma_{uu}}.$$

In the random coefficients model, the share is the estimated value of

$$\varphi = \frac{w'\Omega_w w}{w'\Omega_w w + \sigma_{uu}},$$

where w is the 5×1 vector with 1 in the first position and the log of the overall mean of the inputs in the remaining positions, and Ω_w is the sub-matrix of $\mathbf{\Omega}$ which corresponds to the random coefficients, cf. (3). These ratios show that a very high share of the total variance is due to coefficient heterogeneity: $72 - 85$ per cent in the models with random intercept term and $82 - 91$ per cent in the models that also include random first-order coefficients.

Input and scale elasticities

The complete set of (mean) coefficient estimates in the various models is given in Tables A2 – A4 in Appendix C. Table 3 reports the derived estimates of the expected input elasticities, the expected scale elasticity, and the expected trend effect – all calculated at the overall mean of the inputs.[6] The expected scale elasticity is relatively stable across models and the results clearly indicate weakly increasing or constant returns to scale for Pulp and paper and Basic metals. The estimates for Chemicals are more variable and both Models TL(c, α) and ECD(c, α) show increasing returns to scale, with scale elasticities in the range $1.3 - 1.4$. There is no systematic pattern in the expected scale elasticity with respect to choice of functional form or specification of heterogeneity that is robust across industries.

Overall, the estimated expected input elasticities at the sample mean show larger variability across models than does the scale elasticity. Most estimates have the expected positive sign, the exception is the labour elasticity in six of the nine cases that do not include coefficient heterogeneity. This indicates that "no heterogeneity" is an inappropriate empirical specification of the average technology for our plant panel data. A weakly, although not significantly, negative[7] labour elasticity is also found in Model TL(c, α) in

[5] The overall means of the inputs are defined as the logarithms of their arithmetric means; cf. Table A1 in Appendix C.

[6] Note that the standard deviation estimates given in parenthesis refer to the uncertainty of the estimated parameters and hence is conceptually different from the standard deviation of the random parameter, *i.e.*, the square root of the diagonal elements of $\Omega_{\alpha\alpha}$.

[7] A 5 per cent significance level is used throughout.

Basic metals, however. Materials input comes out with a relatively high input elasticity, in particular in Pulp and paper and Basic metals. In Chemicals, but also to some degree in Pulp and paper, the input elasticity of capital is high according to several models.

Comparing columns $4-6$ in Table 3 with columns $1-3$, it is clear that when we allow for randomness of the α_i's, the standard deviation estimates of the expected coefficients increase substantially – in most cases to almost the double value. This seems to be a consequence of allowing a less restrictive model specification.

Trend effects

The trend variable τ_t is represented by the calendar year. Excepting the three models with no heterogeneity in Pulp and paper, the estimated (sample mean) trend coefficient η_τ, in Table 3, is significantly positive in all models. The values vary between 0.7 and 0.8 per cent in Pulp and paper, between 3 and 4 per cent in Chemicals, and around 2 per cent in Basic metals. Hence, the estimated technical progress is strongest in Chemicals and weakest in Pulp and paper.

This is consistent with industry specific R & D costs, which are available in some years. It is plausible to assume a connection between R & D costs over time and technical progress, and the Chemical industry invests much more in R & D than the other two industries, measured both in NOK and as a share of value added. For example in 1995, R & D costs were about 12 per cent of the value added in Chemicals, and only $3-4$ per cent in Pulp and paper and Basic metals.

Distribution of the random coefficients

Tables 4a – 4c, 5a – 5c and 6a – 6c all characterize, in different ways, plant heterogeneity. Tables 4a – 4c contain estimates of the covariance matrix of the random coefficients in our most general specification of plant heterogeneity. There is one table, covering all three industries, for each functional form, with variances along the main diagonal and correlation coefficients below. For each industry, the results are very robust with respect to the form of the average production function. In only one case [corr$(\alpha_{Ei}, \alpha_{Ki})$ for Chemicals], the sign switches. The majority of the correlation coefficients are negative, and in several cases, they are quite large in absolute value. Hence, a relatively high coefficient of one input is often matched with a relatively low coefficient of the other inputs, and *vice versa*.

The correlation structure of the random coefficients seems to be somewhat different for the three industries. The coefficients of capital and materials are clearly negatively correlated in both Pulp and paper (about -0.60) and Chemicals (about -0.35). The coefficients of capital and energy are clearly negatively correlated in both Pulp and

paper (about -0.40) and Basic metals (about -0.65). The correlation between the random coefficients of capital and labour is rather weak in Pulp and paper, but fairly strong in the two other industries: about -0.30 in Chemicals and about -0.55 in Basic metals. In Chemicals the highest correlation coefficient, about -0.50, is found between the coefficients of labour and energy. Turning to the coefficients of materials and energy, which is often treated as one single input in empirical studies, we find that the correlation is positive, but weak in Pulp and paper, clearly negative in Chemicals (about -0.30), and quite large in Basic metals (less than -0.60). The large negative correlation between the intercept c_i and the coefficient of labour and the positive correlation between the intercept and the coefficent of energy for all industries and models should also be noted.

Predicted input and scale elasticities. Random and systematic heterogeneity

In Appendix A we explain how plant specific random coefficients can be predicted [cf. in particular eq. (A.10)]. From such predictions we can obtain plant specific scale elasticities, ν_i, and input elasticities, $(\eta_{Ki}, \eta_{Li}, \eta_{Ei}, \eta_{Mi})$. Figures 2 – 10 exhibit all the predicted scale elasticities and input elasticities according to Models TL(c, α), ECD(c, α), and CD(c, α), evaluated at the *plant specific means* of the explanatory variables, when the plants are sorted by ascending scale elasticities. In Tables 5a – 5c and 6a – 6c we report descriptive statistics of these plant specific predictions. These figures and tables all represent both the *random* heterogeneity, *i.e.*, due to the random coefficient variation, and the *systematic* heterogeneity which is due to differences in the input mix across plants. Confer the variance expression (10) for the scale elasticity, in which the first component, var$(\bar{\alpha})$, represents the random part (as illustrated in Tables 4a – 4c) and the second component, $\bar{\beta}'\mathsf{V}(z_{it})\bar{\beta}$, represents the systematic part.

Taking Model ECD(c, α) as an example (cf. Table 5b), we see that the pairs of the lowest and highest predicted scale elasticities are $(0.42, 2.10)$, $(-0.22, 4.43)$ and $(0.72, 1.58)$ in Pulp and paper, Chemicals, and Basic metals, respectively. About two thirds of the plants in Pulp and paper and Basic metals have increasing returns to scale when evaluated at the plant specific means of the explanatory variables. The corresponding share in Chemicals is somewhat lower, about 0.55. Similiar results are obtained for the two other functional forms.

The means of the plant specific predicted elasticities are given in the first column of Tables 5a – 5c. Figures 2 – 10 show that the variability of the scale elasticities is much less than the variability of the input elasticities. This is confirmed by the coefficients of variation of the predicted elasticities in the third column of Tables 5a – 5c: for all functional forms and all industries, the coefficient of variation is smaller for the scale elasticity than for any of the input elasticities. The coefficient of variation of the scale

elasticities is uniquely higher in Chemicals than in the two other industries for any of the three functional forms. This explains why we can find clearly increasing returns to scale at the sample mean in some models in Chemicals only, although this industry has the smallest share of individual plants with increasing returns.

The last column of Tables 5a – 5c (and Figures 2 – 10) shows that a share of the predicted input elasticities are negative. The occurrence of negative input elasticities means that the predicted functions for some of the plants does not fulfill the usual regularity conditions of a production function, as representing the technically efficient combinations of inputs and output. This makes economic interpretation more difficult. Generally, this problem seems to be somewhat more pronounced for Models TL(c,α) and ECD(c,α) than for CD(c,α). Stated otherwise, increased flexibility of the functional form intensifies the problem of violation of the regularity conditions. Negativity of the predicted input elasticities occurs least frequently for materials. The occurrence of negative predicted input elasticities as well as negative estimates of average input elasticities (cf. Table 3) may suggest that some kind of constrained estimation procedure, or other distributional assumptions for the random coefficients, should have been applied. Such modifications, however, may require computer software which is presently unavailable, and/or may enhance the numerical problems. Hence, depending on the purpose of the analysis, it may be advisable to choose a relatively restrictive functional form if heterogeneity in technology is a major concern and is represented by random coefficients.

We will now compare the input and scale elasticities in Tables 5a – 5c with those in Table 3. Differences between these values reflect both random and systematic heterogeneity. The values in Table 3 are calculated at the estimated expected values of the random coefficients and the *overall means* of the inputs. For the CD functional form, the α_i's have interpretations as input elasticities. In this case there is practically no difference between the two types of measures (see Table 5c, column 1, and Table 3, column 3). However, this does not hold for the two other functional forms, in which the input elasticities depend on the input vector. In Pulp and paper, the differences are modest for the scale elasticities, the largest difference, 0.06, occurring for the energy elasticity in Model TL(c,α). For the other two industries, we find larger discrepancies. In Chemicals (compare Table 3 with Tables 5a and 5b), the estimated average scale elasticities are 1.31 and 1.43 in Models TL(c,α) and ECD(c,α), respectively, whereas the means of the predicted plant specific elasticities are lower, 1.04 and 1.09, respectively. The main contribution to these discrepancies comes from the capital elasticity. For instance in Model TL(c,α), the estimate (Table 3) is 0.52, whereas the mean of the predicted elasticity is only 0.25.

Predicted input and scale elasticities. Correlation pattern

The empirical covariance pattern of the predicted plant specific input elasticities, given in the last four columns of Tables 6a – 6c (variances along the diagonal, correlation coefficients below), shows that most of the correlations are negative. To a considerable extent this reflects the pattern in Tables 4a – 4c for the (random) first-order coefficients $(\alpha_{Ki}, \alpha_{Li}, \alpha_{Ei}, \alpha_{Mi})$. Comparing Table 6c with 4c, both of which relate to the CD model with random, but input independent input elasticities, is interesting. Since the η_i's and the α_i's coincide in this case, we get an illustration of the difference between the estimated joint *population* distribution of the α's and the *sample* distribution of the predicted empirical counterparts. The empirical variances of the predictions are far smaller than the estimated population variances of the random input coefficients.

High estimates (in absolute value) of $\text{corr}(\alpha_{mi}, \alpha_{ni})$ (m and n denoting two arbitrary inputs) seem to be accompanied by high empirical correlations between predicted input elasticities of inputs m and n. Since parameter heterogeneity accounts for an important part of the dispersion in the predicted input elasticities, this is not surprising. For instance the estimate of $\text{corr}(\alpha_{Mi}, \alpha_{Ki})$ is -0.57 for Model ECD(c, α) in Pulp and paper (Table 4b), whereas the corresponding empirical correlation coefficient between $\widehat{\eta}_M$ and $\widehat{\eta}_K$ is -0.71 (Table 6b). In both cases this is the highest correlation (in absolute value) among any pair of inputs. Reproducing the calculations in Tables 6a – 6c with the predicted plant specific coefficients replaced by their estimated means confirms that most of the dispersion reflects the randomness of the coefficients. For example, the empirical variances of the scale elasticities is reduced to less than one third.

Finally, turning to the first column of Tables 6a – 6c, we note that the predicted plant specific scale elasticity is strongly positively correlated with the predicted labour input elasticity in all the three industries (correlation coefficient 0.70 or more). Hence, plants with a high labour input elasticity tend to have a high scale elasticity. On the other hand, the predicted plant specific scale elasticity is clearly negatively correlated with the predicted energy input elasticity (correlation coefficient -0.25 or below).

5 Concluding remarks

In this paper, the importance of heterogeneity in economies of scale is analysed using an unbalanced plant-level panel data set from Norwegian Manufacturing Statistics. The plants are from Pulp and paper, Chemicals, and Basic metals industries. A random coefficient approach is chosen, and unlike most previous work on micro data, our model specification allows for heterogeneity in the slope coefficients representing the scale prop-

erties as well as in the intercept term. Nine specifications of a four-factor ($KLEM$) production function is estimated: the Translog, an Extended Cobb-Douglas and the strict Cobb-Douglas, each with three alternative representations of the heterogeneity. The three functional forms are nested, and both the Translog and the Extended Cobb-Douglas imply non-homothetic technology with input dependent scale elasticity.

We find constant or weakly increasing returns to scale for a plant with an average technology, but the results reveal considerable variation across plants, and plants with both increasing and decreasing economies of scale are present. The input elasticities at the sample mean are found to be even more variable than the scale elasticity, this is particularly true for labour. In general, the input elasticity of materials is largest, while that of energy is smallest. Variations in the input coefficients across plants seem to a larger extent to be due to randomness of the production function parameters than to systematic differences in the input mix.

Specifications that include heterogeneity in slope coefficients, in addition to heterogeneous intercept terms, improve the fit. This holds for all three functional forms. Among the models with heterogeneity in slope coefficients the fit does not seem to deviate much across functional forms, and for two of the three industries the ranking of the models depends on the choice of information criterion. However, according to the predicted input elasticities, the CD model yields plant specific production functions which to a less degree than the two other functional forms violate the regularity conditions regarding technical efficiency. Hence, it may be advisable to choose a relatively restrictive functional form if heterogeneity in technology is a major concern and is represented by random coefficients.

We expect our main finding, *i.e.*, that economies of scale properties vary substantially across plants, to be a general feature in micro data. The lesson we learn is that one should work carefully with the representation of the plant specific heterogeneity when analysing the production technology by means of micro data. This supports the findings of Mairesse and Griliches (1990), who use a simpler description of the average technology than we do. It is interesting to note, though, that, apart from one industry, the estimated scale elasticity for the average plant is very robust to the choice of the model specification. An interesting issue for future research would be to analyse the distribution of scale properties, and their aggregate implications, in more detail. This may be important since knowledge about systematic variation in characteristics of plants with either increasing or decreasing returns to scale, such as age, size, growth performance, etc., could be important for our understanding of the evolution of an industry.

Table 2. Model fitting information for the various models in the three industries[a]

	Model								
	TL(c,α)	ECD(c,α)	CD(c,α)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
Pulp and paper									
q[b]	37	26	22	23	12	8	22	11	7
LLH	-330.65	-358.77	-364.73	-725.83	-764.85	-779.22	-2250.15	-2368.11	-2409.85
AIC	-367.65	-384.77	-386.73	-748.83	-776.85	-787.22	-2272.15	-2379.11	-2416.85
SBC	-477.65	-462.06	-452.14	-817.20	-812.52	-811.00	-2337.55	-2411.81	-2437.66
σ_{uu}	0.0397	0.0406	0.0408	0.0708	0.0729	0.0734	0.2883	0.3134	0.3228
ω_{cc}	5.9590	5.7793	5.9336	0.3789	0.3821	0.4003	0[c]	0[c]	0[c]
φ[d]	0.9034	0.8986	0.9085	0.8426	0.8398	0.8450	0[c]	0[c]	0[c]
Chemicals									
q[b]	37	26	22	23	12	8	22	11	7
LLH	-1350.06	-1371.88	-1374.42	-1539.89	-1601.92	-1624.17	-2102.14	-2151.74	-2179.96
AIC	-1387.06	-1387.88	-1396.42	-1562.89	-1613.92	-1632.17	-2124.14	-2162.74	-2186.96
SBC	-1482.19	-1464.73	-1452.98	-1622.02	-1644.77	-1652.74	-2180.70	-2191.02	-2204.96
σ_{uu}	0.2926	0.3019	0.3019	0.5214	0.5711	0.6028	1.6295	1.7626	1.8431
ω_{cc}	23.6710	24.6901	25.0253	1.5646	1.5791	1.5196	0[c]	0[c]	0[c]
φ[d]	0.9082	0.8993	0.9006	0.7501	0.7307	0.7160	0[c]	0[c]	0[c]
Basic metals									
q[b]	37	26	22	23	12	8	22	11	7
LLH	-1041.70	-1076.64	-1083.00	-1223.51	-1279.51	-1284.46	-2533.11	-2622.37	-2654.60
AIC	-1078.70	-1102.64	-1099.00	-1246.51	-1291.51	-1292.46	-2555.11	-2633.37	-2661.60
SBC	-1183.02	-1175.95	-1144.11	-1311.36	-1325.35	-1315.02	-2617.14	-2664.39	-2681.34
σ_{uu}	0.0968	0.0984	0.0986	0.1409	0.1483	0.1490	0.6704	0.7306	0.7536
ω_{cc}	2.7431	3.3558	3.5973	0.6517	0.7084	0.7109	0[c]	0[c]	0[c]
φ[d]	0.8196	0.8212	0.8293	0.8223	0.8269	0.8267	0[c]	0[c]	0[c]

[a] LLH is the Log likelihood value; AIC is Akaike's Information Criterion; SBC is Schwarz's Bayesian Criterion.
[b] q is the number of parameters in the model.
[c] A priori restriction.

Table 3. Estimated scale properties. Standard errors in parentheses[a]

Parameter	Model								
	TL(c,α)	ECD(c,α)	CD(c,α)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
Pulp & paper									
η_K	0.2677	0.2448	0.2503	0.1275	0.1532	0.1717	0.1093	0.0969	0.0735
	(0.0469)	(0.0455)	(0.0344)	(0.0296)	(0.0277)	(0.0197)	(0.0272)	(0.0264)	(0.0187)
η_L	0.1485	0.1476	0.1717	0.1096	0.1330	0.1863	-0.4189	-0.3036	-0.2314
	(0.0514)	(0.0468)	(0.0381)	(0.0301)	(0.0278)	(0.0215)	(0.0270)	(0.0251)	(0.0206)
η_E	0.1339	0.1487	0.0854	0.1649	0.1582	0.0921	0.4936	0.3995	0.3333
	(0.0256)	(0.0253)	(0.0169)	(0.0189)	(0.0175)	(0.0103)	(0.0190)	(0.0166)	(0.0099)
η_M	0.5136	0.5183	0.5666	0.6367	0.6243	0.6064	0.7074	0.7586	0.7530
	(0.0421)	(0.0406)	(0.0309)	(0.0263)	(0.0239)	(0.0167)	(0.0246)	(0.0226)	(0.0160)
ν	1.0637	1.0595	1.0740	1.0386	1.0687	1.0564	0.8914	0.9514	0.9284
	(0.0426)	(0.0411)	(0.0287)	(0.0302)	(0.0291)	(0.0186)	(0.0139)	(0.0135)	(0.0095)
η_τ	0.0067	0.0065	0.0065	0.0069	0.0074	0.0084	-0.0093	-0.0005	-0.0002
	(0.0017)	(0.0013)	(0.0013)	(0.0016)	(0.0012)	(0.0012)	(0.0026)	(0.0019)	(0.0019)
Chemicals									
η_K	0.5201	0.4490	0.1270	0.5020	0.3986	0.0713	0.7839	0.7557	0.4646
	(0.1954)	(0.1905)	(0.1149)	(0.1176)	(0.1150)	(0.0667)	(0.0846)	(0.0826)	(0.0577)
η_L	0.4215	0.3457	0.3117	0.2057	0.2618	0.4711	-0.2408	-0.1379	0.0537
	(0.2245)	(0.2089)	(0.1605)	(0.1150)	(0.0978)	(0.0763)	(0.1041)	(0.0883)	(0.0733)
η_E	0.1500	0.1698	0.2156	0.1548	0.1916	0.2244	0.2504	0.2247	0.3046
	(0.1089)	(0.0999)	(0.0718)	(0.0646)	(0.0594)	(0.0368)	(0.0618)	(0.0575)	(0.0350)
η_M	0.3389	0.3478	0.3544	0.4548	0.5245	0.2530	0.2341	0.2146	0.1825
	(0.1321)	(0.1250)	(0.0968)	(0.0807)	(0.0725)	(0.0482)	(0.0693)	(0.0597)	(0.0404)
ν	1.4305	1.3123	1.0087	1.3172	1.3763	1.0199	1.0276	1.0570	1.0053
	(0.1918)	(0.1821)	(0.1062)	(0.1085)	(0.1059)	(0.0606)	(0.0435)	(0.0437)	(0.0292)
η_τ	0.0375	0.0323	0.0306	0.0329	0.0384	0.0422	0.0243	0.0253	0.0238
	(0.0067)	(0.0048)	(0.0047)	(0.0066)	(0.0046)	(0.0045)	(0.0098)	(0.0067)	(0.0068)
Basic metals									
η_K	0.1806	0.0270	0.1246	0.1180	0.0619	0.0944	0.1039	0.0103	0.1438
	(0.0830)	(0.0732)	(0.0472)	(0.0587)	(0.0461)	(0.0273)	(0.0564)	(0.0437)	(0.0280)
η_L	-0.0316	0.2400	0.2749	0.0381	0.2257	0.3073	-0.0648	0.1149	0.1629
	(0.0847)	(0.0702)	(0.0550)	(0.0602)	(0.0444)	(0.0351)	(0.0721)	(0.0484)	(0.0360)
η_E	0.4440	0.3970	0.2138	0.3010	0.1734	0.1628	0.3857	0.2478	0.1502
	(0.0635)	(0.0618)	(0.0374)	(0.0437)	(0.0330)	(0.0174)	(0.0386)	(0.0293)	(0.0190)
η_M	0.4262	0.3960	0.4928	0.5521	0.5666	0.5210	0.5411	0.6446	0.6868
	(0.0640)	(0.0598)	(0.0406)	(0.0452)	(0.0356)	(0.0235)	(0.0480)	(0.0324)	(0.0217)
ν	1.0192	1.0600	1.1061	1.0091	1.0276	1.0856	0.9660	1.0176	1.1438
	(0.0570)	(0.0535)	(0.0324)	(0.0510)	(0.0492)	(0.0271)	(0.0999)	(0.0228)	(0.0142)
η_τ	0.0156	0.0220	0.0214	0.0153	0.0215	0.0228	0.0246	0.0211	0.0220
	(0.0035)	(0.0022)	(0.0021)	(0.0034)	(0.0020)	(0.0020)	(0.0059)	(0.0036)	(0.0036)

[a] The elasticity of output with respect to a specific input j (η_j), the scale elasticity (ν) and the derivative of the log of output with respect to time (η_τ) are evaluated at the overall empirical mean and at the expectation of random coefficients.

Table 4a. The distribution of plant specific coefficients in model TL(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	5.9590				
α_{Ki}	-0.4606	0.1182			
α_{Li}	-0.7185	-0.0749	0.1539		
α_{Ei}	0.3611	-0.4387	-0.2442	0.0224	
α_{Mi}	0.4157	-0.5697	-0.4498	0.1072	0.1045
Chemicals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	23.6710				
α_{Ki}	-0.2175	0.4984			
α_{Li}	-0.8084	-0.2561	1.2501		
α_{Ei}	0.4750	-0.0832	-0.5478	0.2660	
α_{Mi}	0.1811	-0.3389	-0.2037	-0.3169	0.3743
Basic metals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	2.7431				
α_{Ki}	-0.0959	0.1496			
α_{Li}	-0.6226	-0.5912	0.1422		
α_{Ei}	0.2727	-0.6727	0.2784	0.0852	
α_{Mi}	0.1081	0.0952	-0.3612	-0.6432	0.1007

Table 4b. The distribution of plant specific coefficients in model ECD(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	5.7793				
α_{Ki}	-0.4419	0.1163			
α_{Li}	-0.7111	-0.0850	0.1501		
α_{Ei}	0.3424	-0.4025	-0.2504	0.0231	
α_{Mi}	0.3791	-0.5748	-0.4264	0.0707	0.1075
Chemicals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	24.6901				
α_{Ki}	-0.1680	0.5284			
α_{Li}	-0.7909	-0.3138	1.2903		
α_{Ei}	0.4052	0.0071	-0.5199	0.2408	
α_{Mi}	0.1818	-0.3718	-0.2111	-0.3161	0.4423
Basic metals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	3.3558				
α_{Ki}	-0.0698	0.1611			
α_{Li}	-0.6744	-0.5664	0.1753		
α_{Ei}	0.2091	-0.6226	0.2600	0.1004	
α_{Mi}	0.2153	0.0698	-0.4188	-0.6390	0.1335

Table 4c. The distribution of plant specific coefficients in model CD(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	5.9336				
α_{Ki}	-0.4512	0.1147			
α_{Li}	-0.7274	-0.0559	0.1515		
α_{Ei}	0.3968	-0.4197	-0.3009	0.0232	
α_{Mi}	0.3851	-0.6029	-0.4262	0.1437	0.1053
Chemicals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	25.0253				
α_{Ki}	-0.1666	0.5715			
α_{Li}	-0.7798	-0.3362	1.3028		
α_{Ei}	0.3933	-0.0189	-0.4965	0.2443	
α_{Mi}	0.1784	-0.3545	-0.2091	-0.3112	0.4484
Basic metals	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
c_i	3.5973				
α_{Ki}	-0.0787	0.1604			
α_{Li}	-0.6846	-0.5503	0.1817		
α_{Ei}	0.3040	-0.6281	0.1366	0.1190	
α_{Mi}	0.1573	0.1092	-0.3720	-0.6122	0.1200

Table 5a. Descriptive statistics of plant specific predicted elasticities for model TL(c,α)

Predicted elasticities	Mean	Std. error	Coef. of variation[a]	Minimum value	Maximum value	Share of values < 0[b]
Pulp and paper						
$\hat{v}$	1.0733	0.1915	0.1784	0.4231	2.1030	0.0
$\hat{\eta}_K$	0.2699	0.2523	0.9349	-0.9108	1.4421	8.0
$\hat{\eta}_L$	0.1689	0.2885	1.7079	-1.0834	1.5788	23.6
$\hat{\eta}_E$	0.0886	0.1032	1.1654	-0.4367	0.3732	16.5
$\hat{\eta}_M$	0.5459	0.2324	0.4257	-0.5641	2.6416	1.3
Chemicals						
$\hat{v}$	1.0889	0.6032	0.5540	-0.2642	4.1287	3.3
$\hat{\eta}_K$	0.2476	0.6023	2.4319	-1.4071	2.2645	37.8
$\hat{\eta}_L$	0.2902	0.8677	2.9902	-2.5113	4.0672	31.1
$\hat{\eta}_E$	0.2030	0.3982	1.9617	-0.9682	1.8736	25.6
$\hat{\eta}_M$	0.3481	0.4616	1.3259	-2.0210	1.9588	13.3
Basic metals						
$\hat{v}$	1.0557	0.1381	0.1308	0.7141	1.5756	0.0
$\hat{\eta}_K$	0.1018	0.2876	2.8253	-1.7133	0.7442	29.5
$\hat{\eta}_L$	0.1997	0.3623	1.8147	-0.5816	1.5107	30.7
$\hat{\eta}_E$	0.2220	0.2666	1.2012	-0.9719	1.2953	15.1
$\hat{\eta}_M$	0.5323	0.2696	0.5064	-0.1642	1.4767	3.0

[a] Defined as the standard error divided by the mean.
[b] In percentage of the total number of plants.

Table 5b. Descriptive statistics of plant specific predicted elasticities for model ECD(c,α)

Predicted elasticities	Mean	Std. error	Coef. of variation[a]	Minimum value	Maximum value	Share of values < 0[b]
Pulp and paper						
$\hat{v}$	1.0685	0.1906	0.1784	0.4176	2.0975	0.0
$\hat{\eta}_K$	0.2511	0.2491	0.9922	-0.9079	1.4487	8.0
$\hat{\eta}_L$	0.1667	0.2768	1.6602	-0.8692	1.6395	20.7
$\hat{\eta}_E$	0.0875	0.1088	1.2425	-0.4752	0.4150	15.6
$\hat{\eta}_M$	0.5632	0.2365	0.4199	-0.5824	2.6413	1.3
Chemicals						
$\hat{v}$	1.0381	0.5750	0.5539	-0.2242	4.4304	3.3
$\hat{\eta}_K$	0.1595	0.5753	3.6073	-1.4167	2.0343	40.0
$\hat{\eta}_L$	0.3065	0.8805	2.8730	-2.4537	4.3556	31.1
$\hat{\eta}_E$	0.2106	0.3708	1.7605	-0.9477	1.8410	21.1
$\hat{\eta}_M$	0.3615	0.4989	1.3800	-2.1927	2.0873	15.6
Basic metals						
$\hat{v}$	1.0687	0.1315	0.1230	0.7210	1.5817	0.0
$\hat{\eta}_K$	0.0956	0.3085	3.2250	-1.6831	0.8331	34.9
$\hat{\eta}_L$	0.2665	0.2906	1.0904	-0.4398	1.5265	12.7
$\hat{\eta}_E$	0.2238	0.2687	1.2007	-0.9921	1.3472	15.7
$\hat{\eta}_M$	0.4828	0.2631	0.5451	-0.2515	1.5322	3.6

[a] Defined as the standard error divided by the mean.
[b] In percentage of the total number of plants.

Table 5c. Descriptive statistics of plant specific predicted elasticities for model CD(c,α)

Predicted elasticities	Mean	Std. error	Coef. of variation[a]	Minimum value	Maximum value	Share of values < 0[b]
Pulp and paper						
$\hat{v}$	1.0740	0.1933	0.1799	0.4024	2.1530	0.0
$\hat{\eta}_K$	0.2503	0.2489	0.9946	-1.0200	1.4549	8.0
$\hat{\eta}_L$	0.1717	0.2777	1.6171	-0.9296	1.6471	20.7
$\hat{\eta}_E$	0.0854	0.0927	1.0854	-0.4532	0.3246	12.7
$\hat{\eta}_M$	0.5666	0.2352	0.4151	-0.5906	2.5597	1.3
Chemicals						
$\hat{v}$	1.0087	0.5347	0.5301	-0.3507	4.2879	1.1
$\hat{\eta}_K$	0.1270	0.5540	4.3622	-1.3238	2.1714	36.7
$\hat{\eta}_L$	0.3117	0.8872	2.8464	-2.5227	4.4403	30.0
$\hat{\eta}_E$	0.2156	0.3726	1.7281	-0.9524	1.8507	21.1
$\hat{\eta}_M$	0.3544	0.5044	1.4233	-2.2047	2.1518	15.6
Basic metals						
$\hat{v}$	1.1061	0.1079	0.0976	0.8579	1.6006	0.0
$\hat{\eta}_K$	0.1246	0.2970	2.3840	-1.6788	0.8409	27.1
$\hat{\eta}_L$	0.2749	0.2927	1.0647	-0.3374	1.4944	12.0
$\hat{\eta}_E$	0.2138	0.2563	1.1990	-1.0058	1.4276	14.5
$\hat{\eta}_M$	0.4928	0.2449	0.4970	-0.1662	1.4471	1.8

[a] Defined as the standard error divided by the mean.
[b] In percentage of the total number of plants.

Table 6a. The distribution of plant specific predicted elasticities from Model TL(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.037				
$\hat{\eta}_K$	0.170	0.064			
$\hat{\eta}_L$	0.777	-0.031	0.083		
$\hat{\eta}_E$	-0.309	-0.512	-0.318	0.011	
$\hat{\eta}_M$	-0.189	-0.679	-0.426	0.252	0.054
Chemicals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.364				
$\hat{\eta}_K$	0.316	0.363			
$\hat{\eta}_L$	0.724	-0.236	0.753		
$\hat{\eta}_E$	-0.531	-0.085	-0.584	0.159	
$\hat{\eta}_M$	-0.010	-0.373	-0.121	-0.347	0.213
Basic metals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.019				
$\hat{\eta}_K$	-0.086	0.083			
$\hat{\eta}_L$	0.701	-0.558	0.131		
$\hat{\eta}_E$	-0.366	-0.567	0.043	0.071	
$\hat{\eta}_M$	0.023	0.200	-0.432	-0.628	0.073

Table 6b. The distribution of plant specific predicted elasticities from Model ECD(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.036				
$\hat{\eta}_K$	0.162	0.062			
$\hat{\eta}_L$	0.778	-0.009	0.077		
$\hat{\eta}_E$	-0.294	-0.447	-0.336	0.012	
$\hat{\eta}_M$	-0.140	-0.707	-0.380	0.167	0.056
Chemicals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.331				
$\hat{\eta}_K$	0.233	0.331			
$\hat{\eta}_L$	0.689	-0.325	0.775		
$\hat{\eta}_E$	-0.474	0.042	-0.556	0.137	
$\hat{\eta}_M$	0.019	-0.341	-0.182	-0.356	0.249
Basic metals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.017				
$\hat{\eta}_K$	-0.003	0.095			
$\hat{\eta}_L$	0.562	-0.663	0.084		
$\hat{\eta}_E$	-0.276	-0.685	0.356	0.072	
$\hat{\eta}_M$	0.165	0.258	-0.411	-0.750	0.069

Table 6c. The distribution of plant specific predicted elasticities from Model CD(c,α). Variances along the main diagonal and correlation coefficients below

Pulp and paper	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.037				
$\hat{\eta}_K$	0.160	0.062			
$\hat{\eta}_L$	0.794	0.021	0.077		
$\hat{\eta}_E$	-0.296	-0.495	-0.384	0.009	
$\hat{\eta}_M$	-0.168	-0.757	-0.400	0.341	0.055
Chemicals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.286				
$\hat{\eta}_K$	0.103	0.307			
$\hat{\eta}_L$	0.746	-0.354	0.787		
$\hat{\eta}_E$	-0.488	0.027	-0.534	0.139	
$\hat{\eta}_M$	-0.005	-0.386	-0.185	-0.346	0.254
Basic metals	$\hat{\nu}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
$\hat{\nu}$	0.012				
$\hat{\eta}_K$	-0.216	0.088			
$\hat{\eta}_L$	0.720	-0.689	0.086		
$\hat{\eta}_E$	-0.250	-0.708	0.361	0.066	
$\hat{\eta}_M$	0.103	0.257	-0.420	-0.729	0.060

Figure 1.The model tree. Random coefficients in parentheses

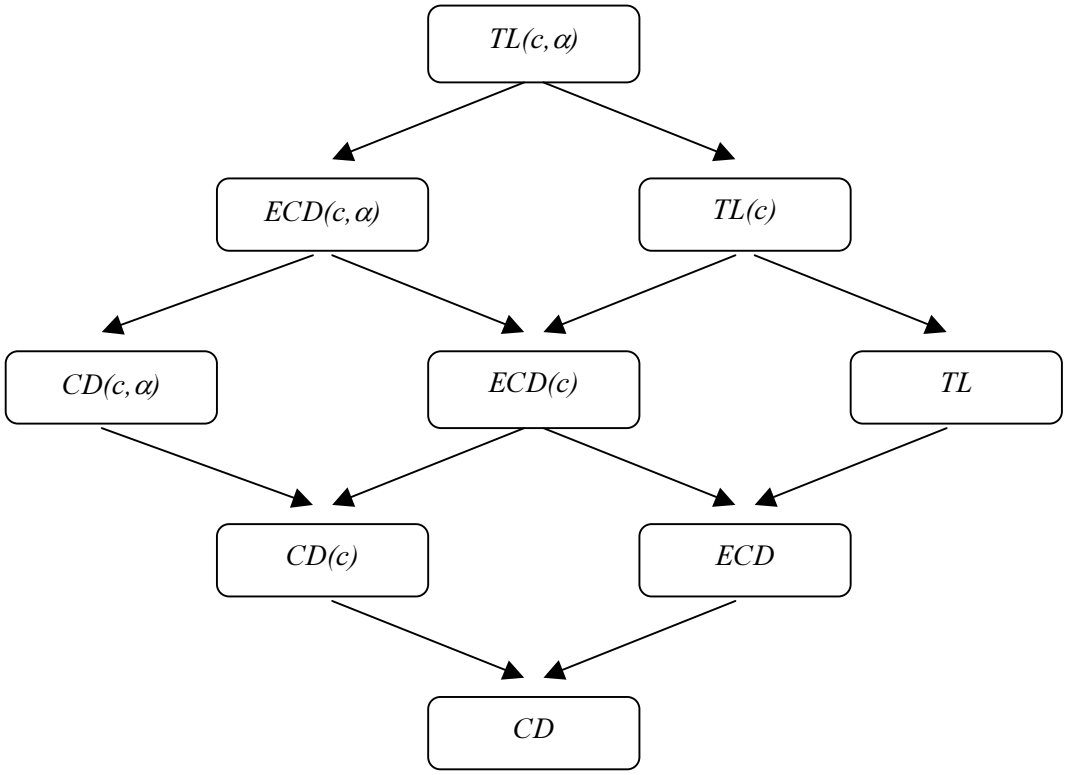


Figure 2. Predicted input and scale elasticities from Model TL(c,α) for the plants in Pulp and paper

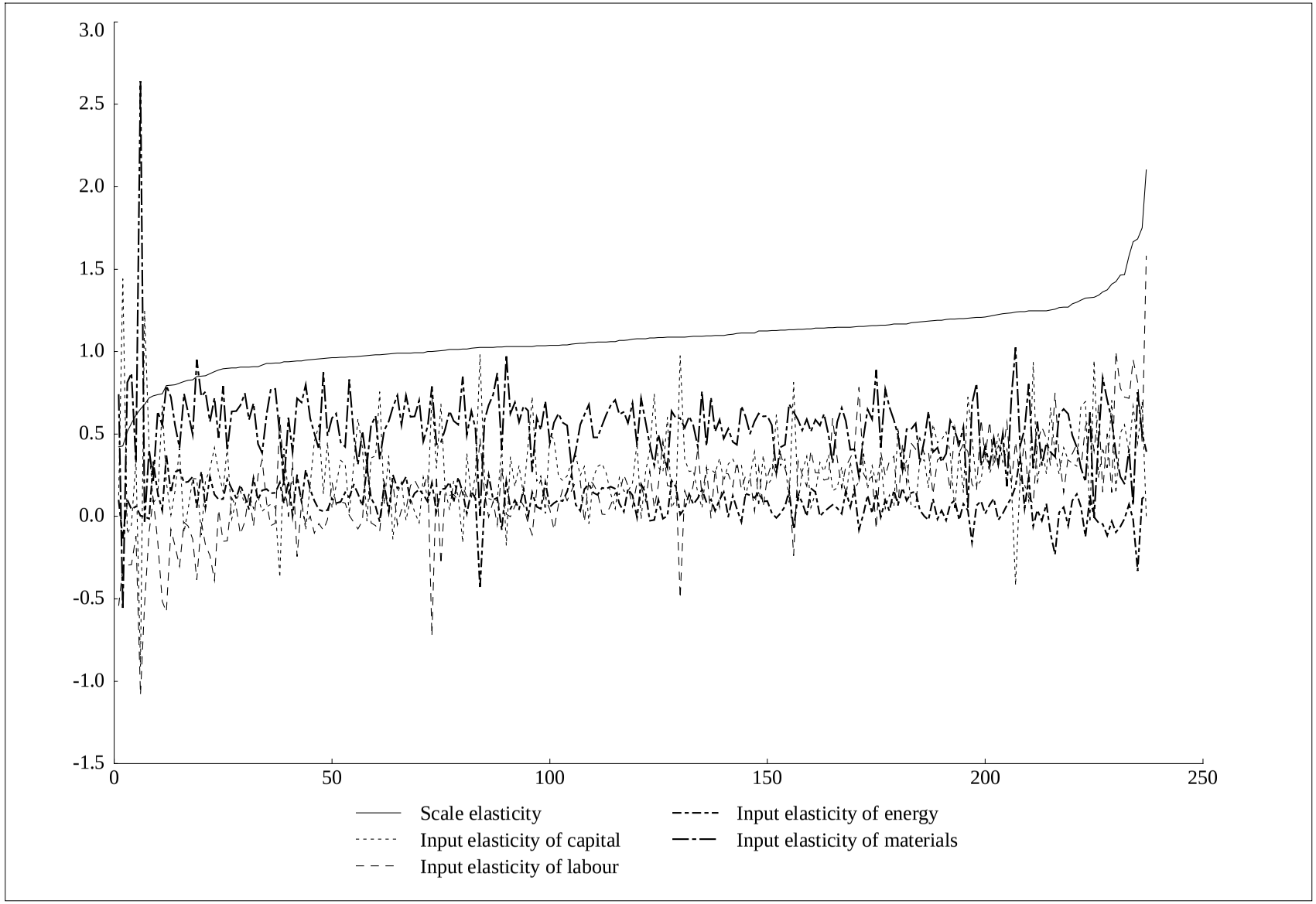


Figure 3. Predicted input and scale elasticities from Model TL(c,α) for the plants in Chemicals

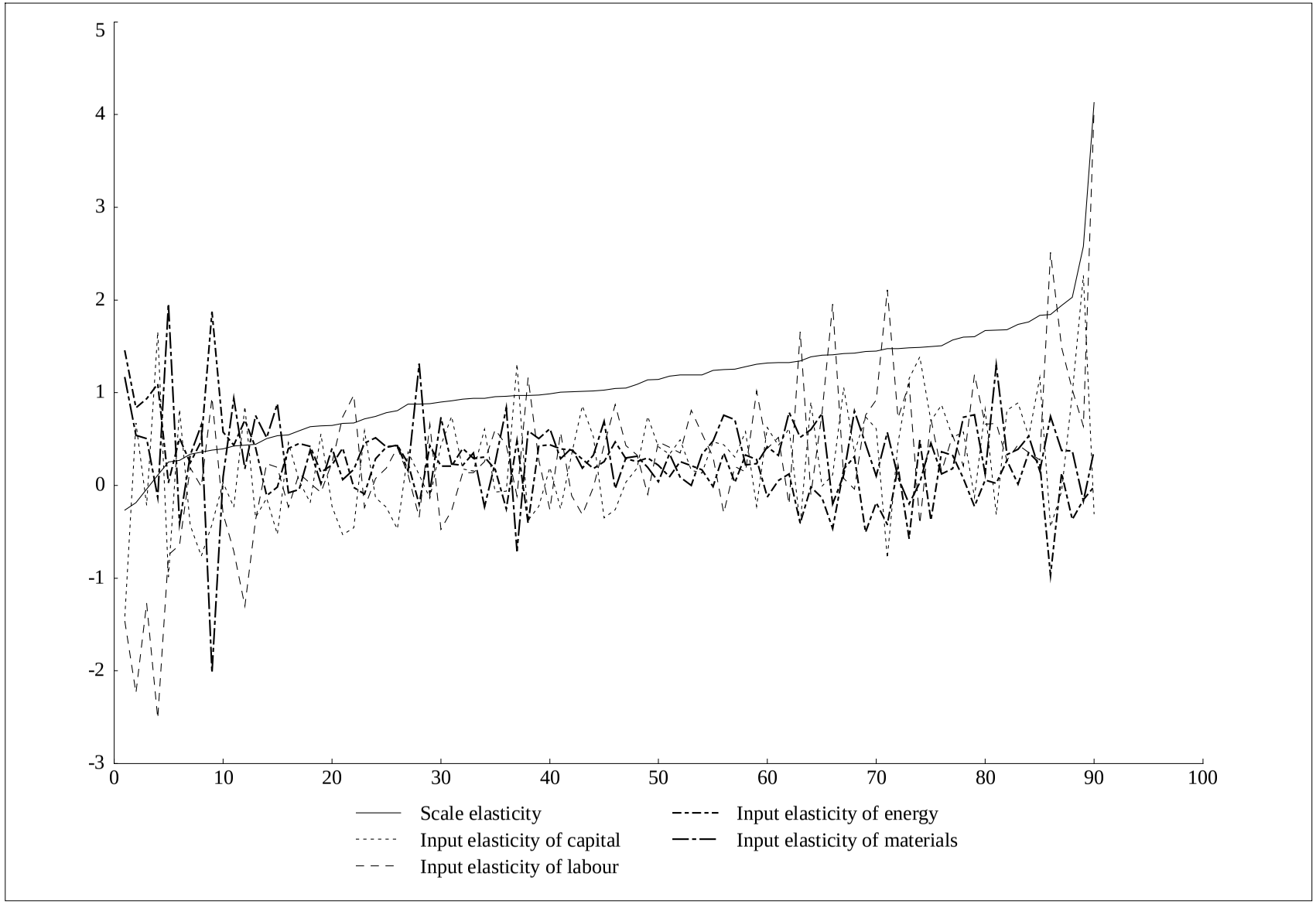


Figure 4. Predicted input and scale elasticities from Model TL(c,α) for the plants in Basic metals

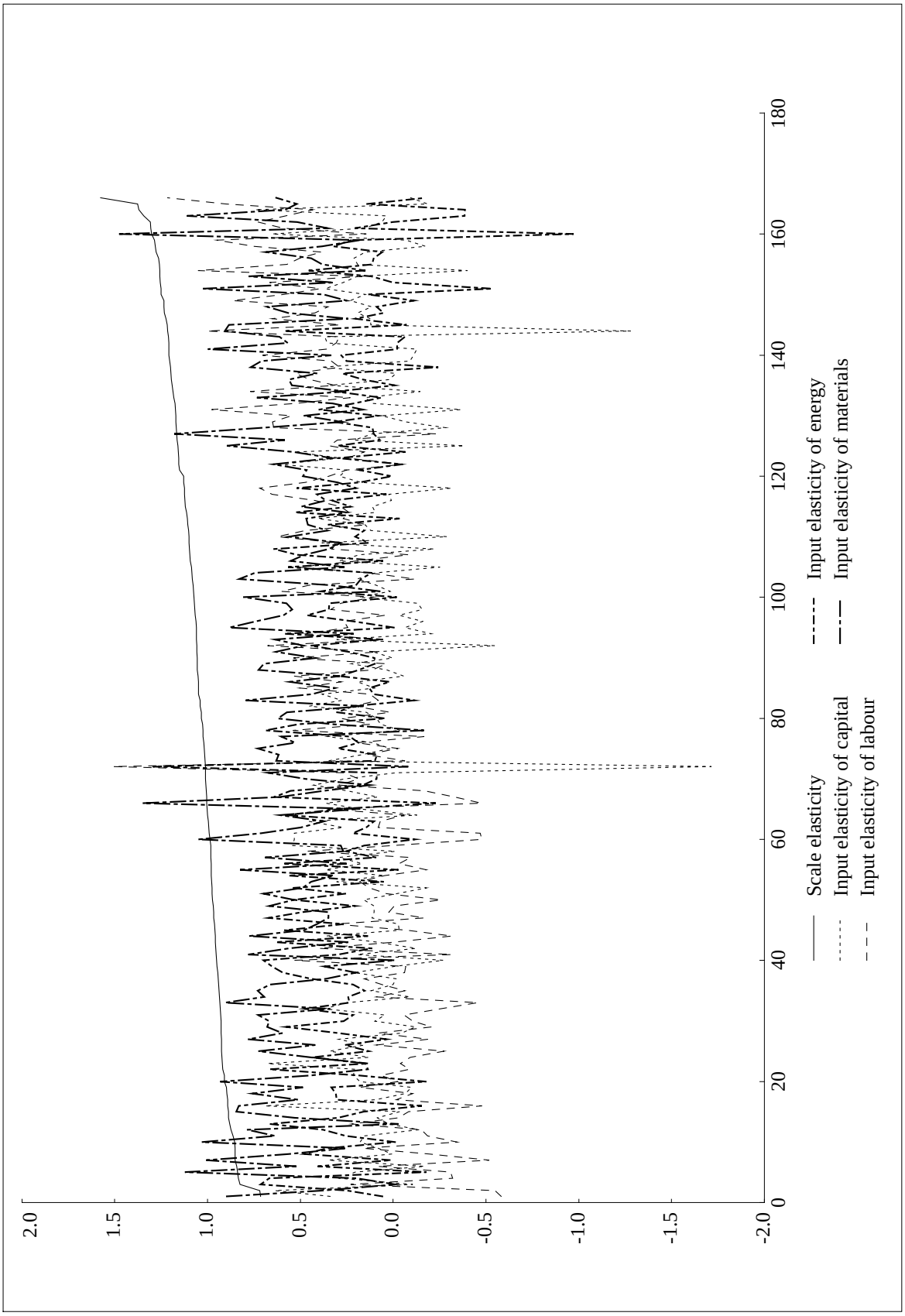


Figure 5. Predicted input and scale elasticities from Model ECD(c,α) for the plants in Pulp and paper

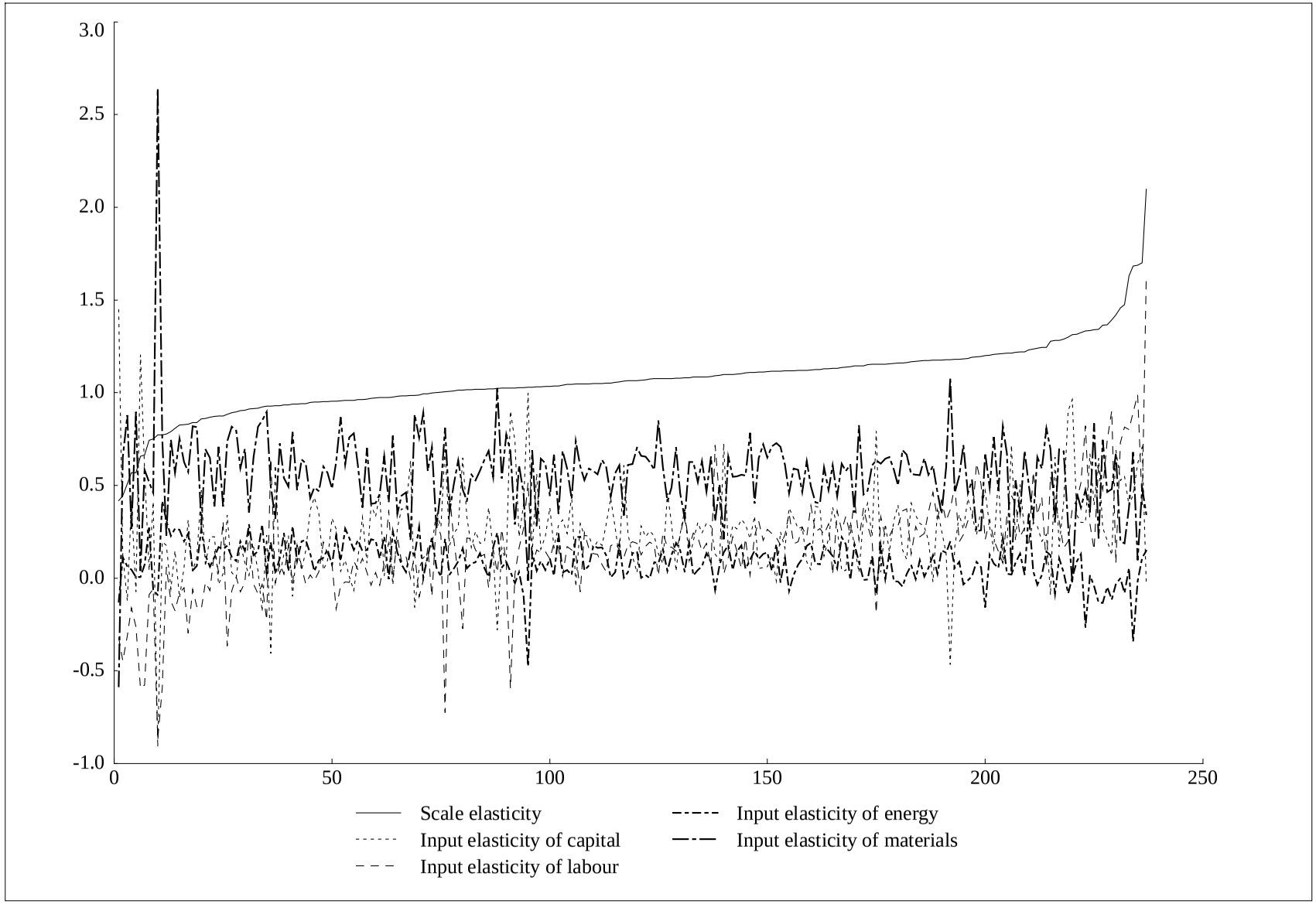


Figure 6. Predicted input and scale elasticities from Model ECD(c,α) for the plants in Chemicals

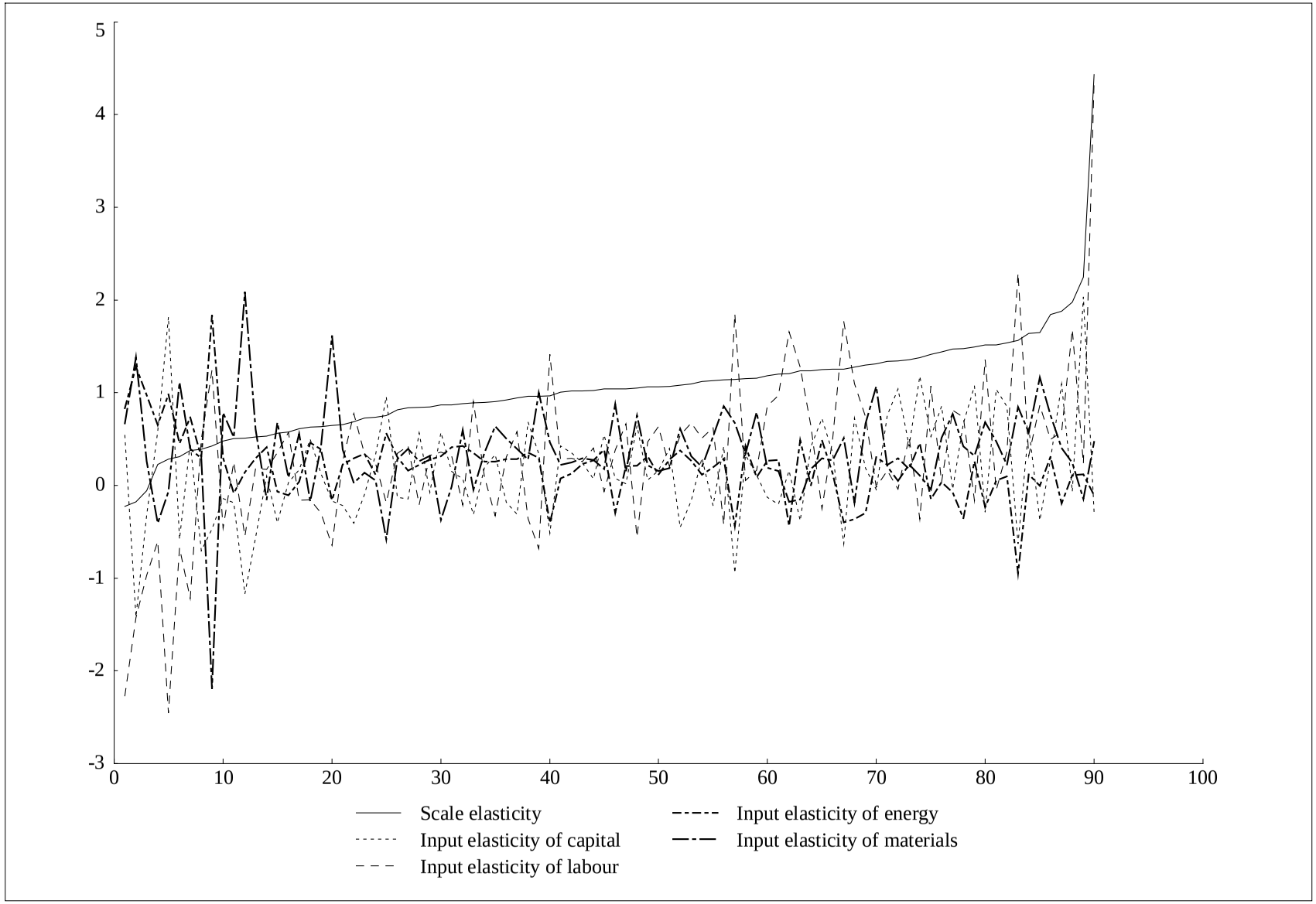


Figure 7. Predicted input and scale elasticities from Model ECD(c,α) for the plants in Basic metals

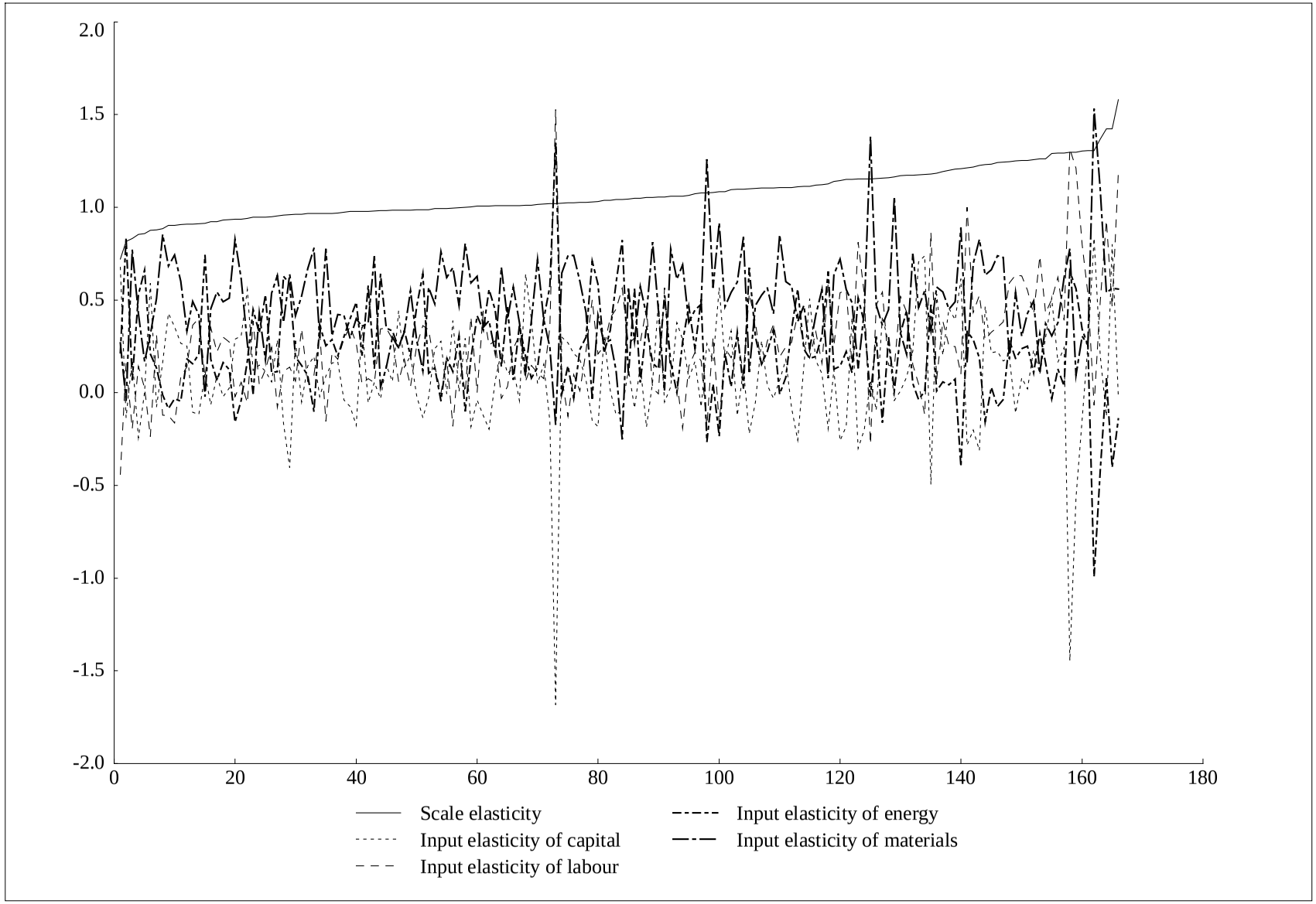


Figure 8. Predicted input and scale elasticities from Model CD(c,α) for the plants in Pulp and paper

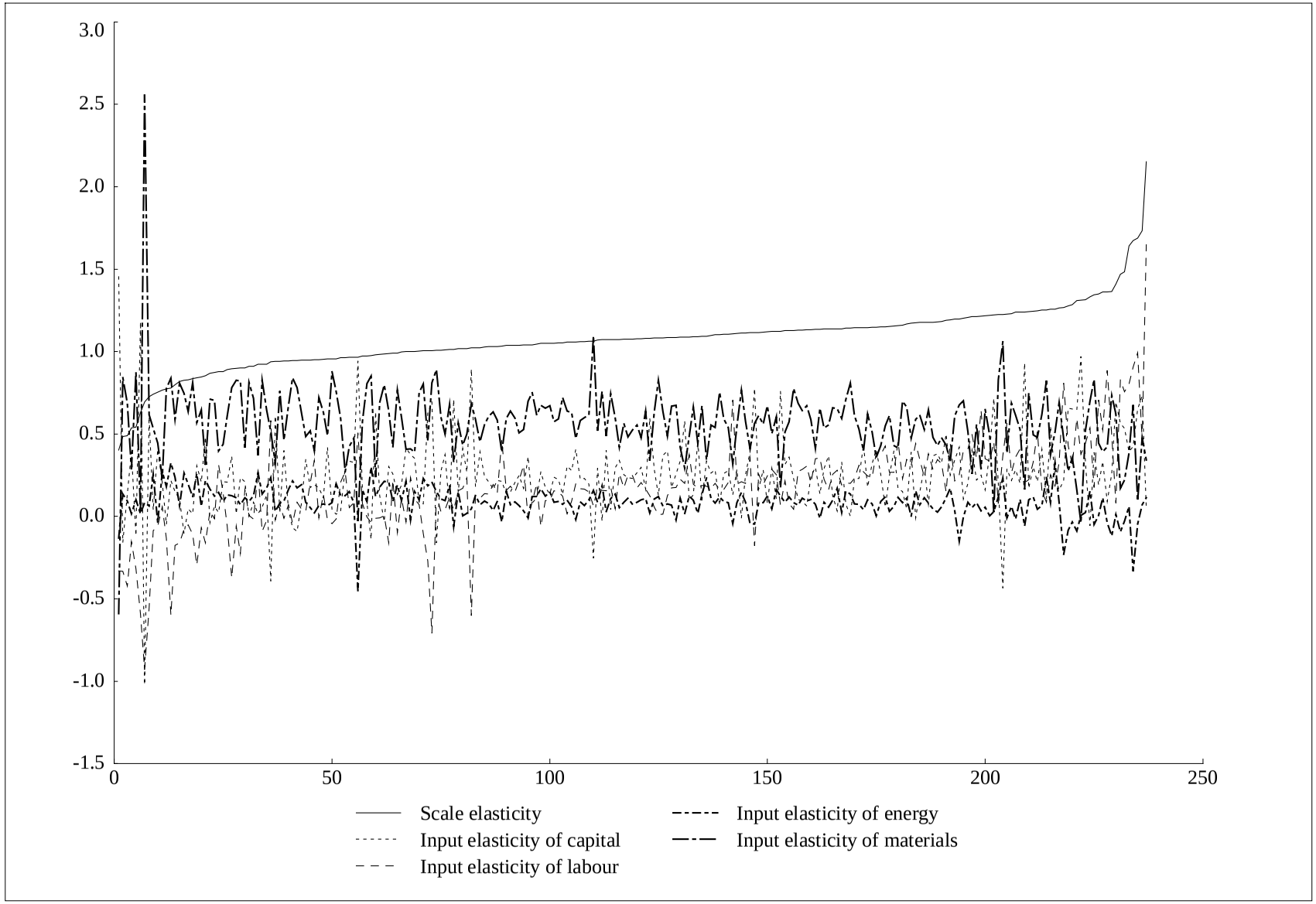


Figure 9. Predicted input and scale elasticities from Model CD(c,α) for the plants in Chemicals

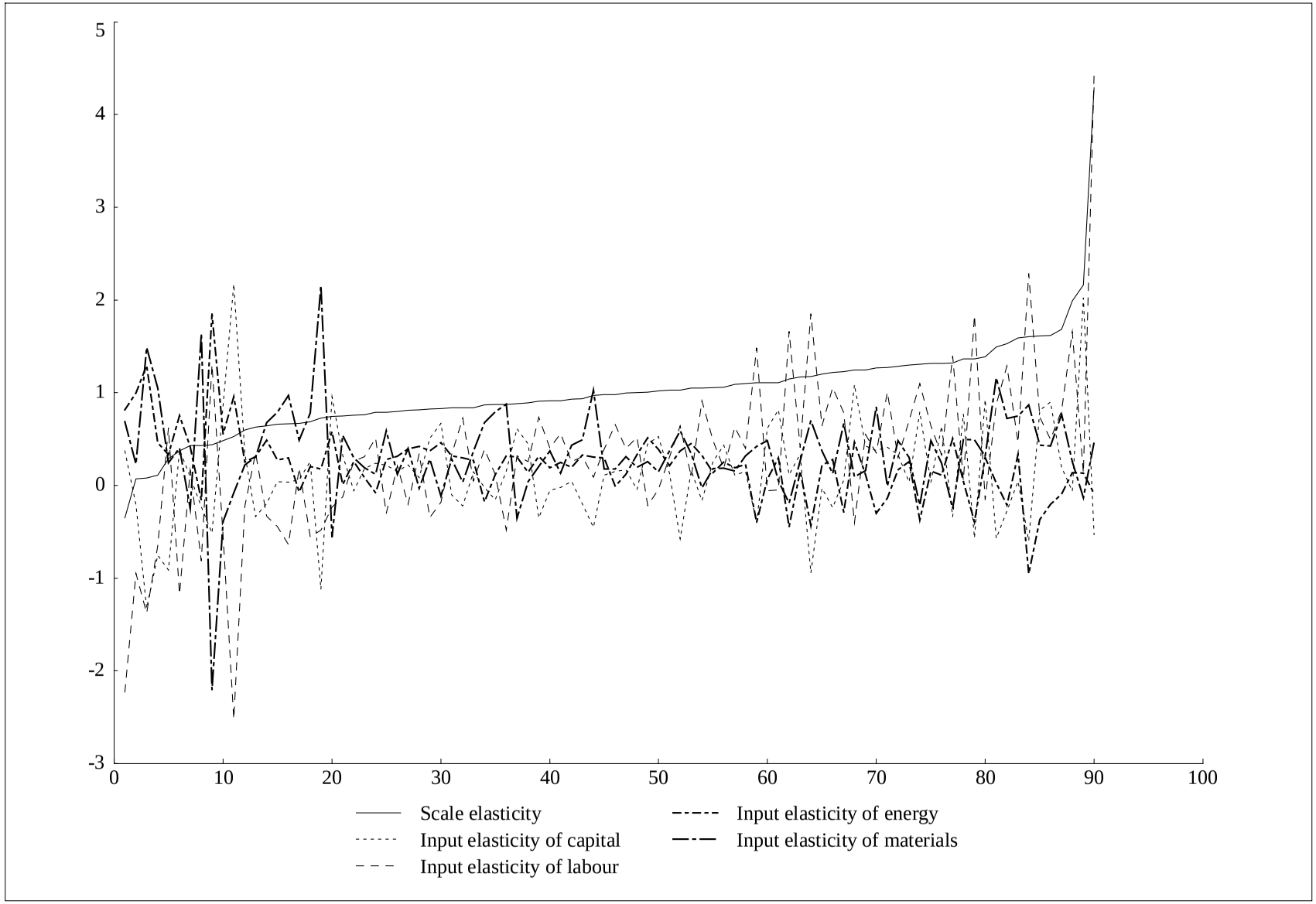
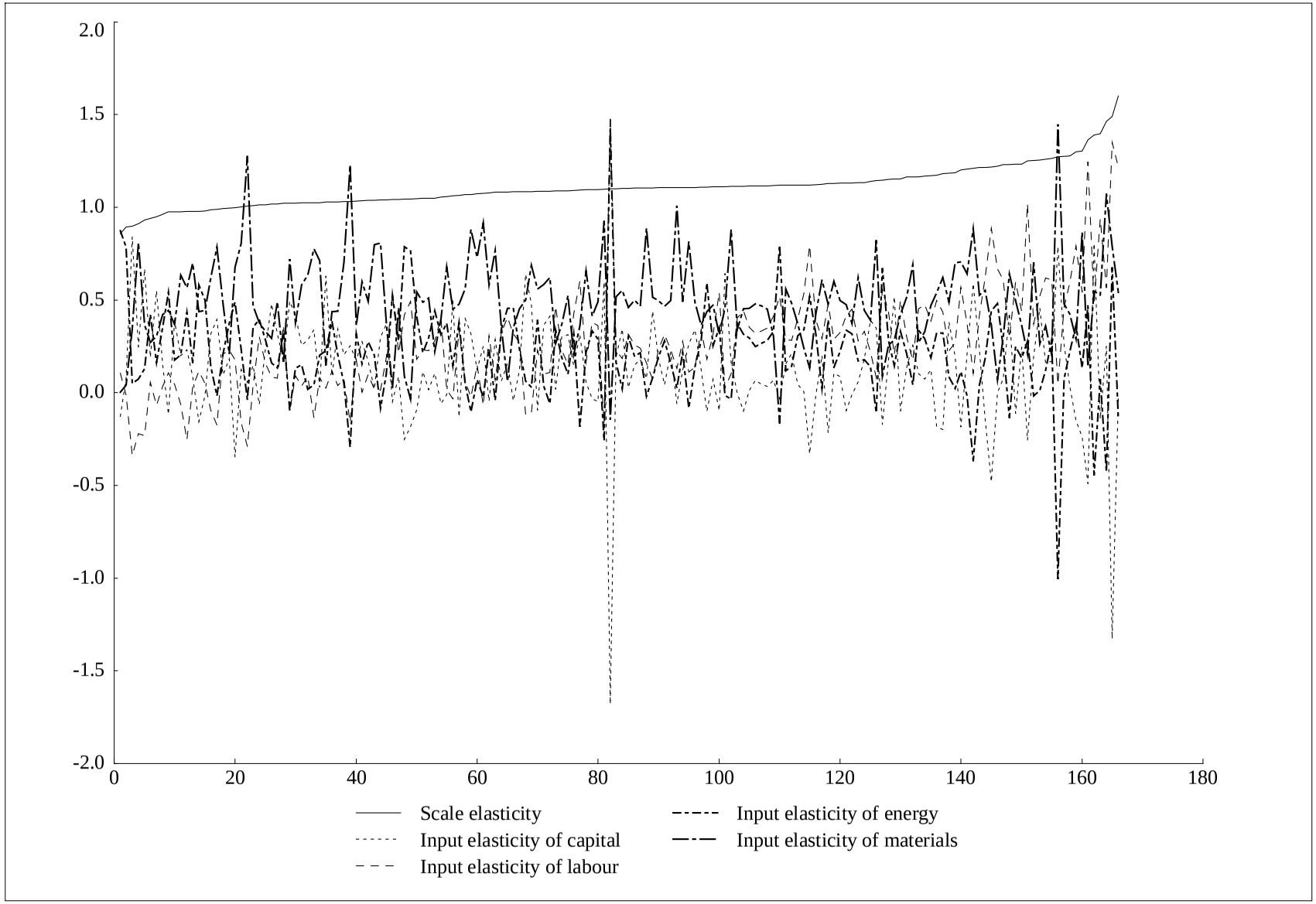


Figure 10. Predicted input and scale elasticities from Model CD(c,α) for the plants in Basic metals



APPENDIX A: Details on estimation method and coefficient prediction

Consider a data set from an unbalanced panel, in which the plants are observed in at least 1 and at most P years. We assume that the selection rules for the unbalanced panels are ignorable, *i.e.*, the way in which the plants enter or exit is not related to the endogenous variables in the model; see Verbeek and Nijman (1996, section 18.2). The plants are arranged in groups according to the number of years the plants are observed. Let N_p be the number of plants which are observed in exactly p years (not necessarily the same and not necessarily consecutive), let (ip) index the i'th plant among those observed in p years ($i = 1, \ldots, N_p$; $p = 1, \ldots, P$), and let t index the observation number ($t = 1, \ldots, p$). The total number of plants in the panel is $N = \sum_{p=1}^{P} N_p$ and the total number of observations is $n = \sum_{p=1}^{P} N_p p$. The regression equation, *i.e.*, the production function (1), can be written compactly as

$$(A.1) \qquad y_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\theta}_{(ip)} + u_{(ip)t}, \qquad p = 1, \ldots, P;\ i = 1, \ldots, N_p;\ t = 1, \ldots, p,$$

where $\boldsymbol{\theta}_{(ip)}$ is the coefficient vector of plant (ip). The regressand of plant (ip), observation t is $y_{(ip)t}$, the corresponding $(1 \times H)$ regressor vector is $\boldsymbol{x}_{(ip)t}$, and the disturbance is $u_{(ip)t}$. The $(H \times 1)$ coefficient vector of plant (ip), cf. (2), is

$$(A.2) \qquad \boldsymbol{\theta}_{(ip)} = \boldsymbol{\theta} + \boldsymbol{\epsilon}_{(ip)},$$

where $\boldsymbol{\theta}$ is the common expectation vector of $\boldsymbol{\theta}_{(ip)}$ for all plants, and $\boldsymbol{\epsilon}_{(ip)}$ is a zero mean random vector specific to plant (ip). Inserting (A.2) in (A.1), we get

$$(A.3) \qquad y_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\theta} + \psi_{(ip)t}, \qquad \psi_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\epsilon}_{(ip)} + u_{(ip)t},$$

where we interpret $\psi_{(ip)t}$ as a 'gross disturbance'. We assume that

$$(A.4) \qquad \boldsymbol{x}_{(ip)t},\ u_{(ip)t},\ \boldsymbol{\epsilon}_{(ip)} \text{ are all independent,}$$

$$(A.5) \qquad u_{(ip)t} \sim \mathsf{IIN}(0, \sigma_{uu}), \qquad \boldsymbol{\epsilon}_{(ip)} \sim \mathsf{IIN}(\mathbf{0}, \boldsymbol{\Omega}),$$

where IIN signifies independently, identically, normally distributed. The matrix $\boldsymbol{\Omega}$ is singular, reflecting that some of the coefficients are fixed, cf. (3).

We stack the p realizations from plant (ip) in

$$\boldsymbol{y}_{(ip)} = \begin{bmatrix} y_{(ip)1} \\ \vdots \\ y_{(ip)p} \end{bmatrix}, \quad \boldsymbol{X}_{(ip)} = \begin{bmatrix} \boldsymbol{x}_{(ip)1} \\ \vdots \\ \boldsymbol{x}_{(ip)p} \end{bmatrix}, \quad \boldsymbol{u}_{(ip)} = \begin{bmatrix} u_{(ip)1} \\ \vdots \\ u_{(ip)p} \end{bmatrix}, \quad \boldsymbol{\psi}_{(ip)} = \begin{bmatrix} \psi_{(ip)1} \\ \vdots \\ \psi_{(ip)p} \end{bmatrix},$$

which have dimensions $(p \times 1)$, $(p \times H)$, $(p \times 1)$, and $(p \times 1)$, respectively. Then we can write (A.3) as

$$(A.6) \qquad \boldsymbol{y}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\theta} + \boldsymbol{\psi}_{(ip)}, \qquad \boldsymbol{\psi}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\epsilon}_{(ip)} + \boldsymbol{u}_{(ip)}.$$

It follows from (A.3), (A.4), and (A.5) that

(A.7) All $\boldsymbol{\psi}_{(ip)}|\boldsymbol{X}_{(ip)}$ are independent and $\boldsymbol{\psi}_{(ip)}|\boldsymbol{X}_{(ip)} \sim \mathtt{N}(\mathbf{0}, \boldsymbol{\Omega}_{(ip)})$,

where

$$\boldsymbol{\Omega}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\Omega}\boldsymbol{X}'_{(ip)} + \sigma_{uu}\boldsymbol{I}_p. \tag{A.8}$$

The joint log-density function of plant (ip), *i.e.* of $\boldsymbol{y}_{(ip)}$ conditional on $\boldsymbol{X}_{(ip)}$, is

$$\mathcal{L}_{(ip)} = -\frac{p}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{\Omega}_{(ip)}| - \frac{1}{2}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}]'\boldsymbol{\Omega}^{-1}_{(ip)}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}],$$

so that by utilizing the ordering of the observations in the P groups, we can write the log-likelihood function of all observations on the $\boldsymbol{y}$'s conditional on all observations on the $\boldsymbol{X}$'s as

$$\begin{aligned}\mathcal{L} = \sum_{p=1}^{P}\sum_{i=1}^{N_p} L_{(ip)} &= -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}\ln|\boldsymbol{\Omega}_{(ip)}| \\ &\quad - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}]'\boldsymbol{\Omega}^{-1}_{(ip)}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}].\end{aligned} \tag{A.9}$$

The *Maximum Likelihood (ML)* estimators of $(\boldsymbol{\theta}, \sigma_{uu}, \boldsymbol{\Omega})$ are obtained by maximizing $\mathcal{L}$ with respect to (the unknown elements of) these parameter matrices. The solution conditions may be simplified by concentrating $\mathcal{L}$ over $\boldsymbol{\theta}$ and maximizing the resulting function with respect to σ_{uu} and the unknown elements of $\boldsymbol{\Omega}$. For a further discussion, see Biørn (1999).

The coefficient vector of plant (ip), $\boldsymbol{\theta}_{(ip)}$, can be predicted as follows:

$$\boldsymbol{\theta}^*_{(ip)} = \widehat{\boldsymbol{\theta}} + \widehat{\boldsymbol{\Omega}}\boldsymbol{X}'_{(ip)}(\boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\Omega}}\boldsymbol{X}'_{(ip)} + \widehat{\sigma}_{uu}\boldsymbol{I}_p)^{-1}(\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\theta}}), \tag{A.10}$$

where $\widehat{\boldsymbol{\theta}}$ is the ML (strictly, the Feasible GLS) estimator of the expected coefficient vector $\boldsymbol{\theta}$ [cf. Lee and Griffiths (1979, section 4) and Hsiao (1986, p. 134)], and $\widehat{\boldsymbol{\Omega}}$ and $\widehat{\sigma}_{uu}$ are the corresponding estimates of $\boldsymbol{\Omega}$ and σ_{uu}. Apart from the fact that $\boldsymbol{\Omega}$ and σ_{uu} have been estimated, this is the best linear unbiased predictor (BLUP) of $\boldsymbol{\theta}_{(ip)}$. It can be shown that this expression can be rewritten as a matrix weighted average of the overall estimator of $\boldsymbol{\theta}$ and the OLS estimator of $\boldsymbol{\theta}_{(ip)}$, based on observations from plant (ip), *i.e.*, $\widehat{\boldsymbol{\theta}}_{(ip)} = (\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)})^{-1}(\boldsymbol{X}'_{(ip)}\boldsymbol{y}_{(ip)})$, in the following way

$$\boldsymbol{\theta}^*_{(ip)} = \left[\widehat{\boldsymbol{\Omega}}^{-1} + \widehat{\sigma}^{-1}_{uu}\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)}\right]^{-1}\left[\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\boldsymbol{\theta}} + \widehat{\sigma}^{-1}_{uu}\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\theta}}_{(ip)}\right], \tag{A.11}$$

cf. Judge *et al.* (1985, pp. 540 – 541). The latter expression, however, is only valid when $p > H$, since otherwise $\widehat{\boldsymbol{\theta}}_{(ip)}$ does not exist.

APPENDIX B: Data description

Our unbalanced plant-level panel data set is collected from the Manufacturing Statistics database of Statistics Norway. The Manufacturing Statistics follow the Standard Industrial Classification (SIC) and gives annual data for large plants at the 5-digit code. Until 1992, plants with at least 5 employees were defined as large, while from 1992 on the limit is 10 employees. In 1993, the activity classification was revised according to EU's NACE Rev. 1 and UN's SIC Rev. 3, while previously based on UN's SIC Rev. 2. While the revision of the activity classification does not cause inconsistency problems in our data, the change in the definition of large plants causes a break in the time series for plants with $5-9$ employees in 1992. Our data set includes all industries classified under SIC-codes 341 Manufacture of paper and paper products (Pulp and paper), 351 Manufacture of industrial chemicals (Chemicals) and 37 Manufacture of basic metals (Basic metals). In addition, to achieve consistency after the revision of the activity classification, a few plants belonging to other SIC industries are included.

Most variables are observed directly. The exceptions are materials input and capital stock, which are calculated from available information. In the description below, MS indicates that the data are from the Manufacturing Statistics, and the data are plant specific. NNA indicates that the data are from the Norwegian National Accounts. In this case, the data are identical for all plants classified in the same National Account industry. While the plants in our unbalanced panel mainly are collected from 18 different industries at the 5-digit SIC-code level, the plants are classified in 14 different National Accounts industries. Data in value terms are in 100 000 Norwegian kroner (NOK).

Y: Output, 100 tonnes (MS)

$K = KB + KM$: Total capital stock (buildings/structures plus machinery/transport equipment), 100 000 1991-NOK (MS,NNA)

L: Labour input, 100 man-hours (MS)

E: Energy input, 100 000 kWh, electricity plus fuels (excl. motor gasoline) (MS)

$M = CM/QM$: Input of materials (incl. motor gasoline), 100 000 1991-NOK (MS,NNA)

CM: Total material cost (incl. motor gasoline) (MS)

QM: Price of materials (incl. motor gasoline), 1991=1 (NNA).

Output: The plants in the Manufacturing Statistics are in general multi-output plants and report output of a number of products measured in both NOK and primarily tonnes or kg. The classification of products follows The Harmonized Commodity Description and Coding System (HS), and assigns a 7-digit number to each specific commodity. For each plant, an aggregate output measure in tonnes is calculated.

Capital stock: The calculations of capital stock data are based on the perpetual inventory method assuming constant depreciation rates. We combine plant data on gross investment with fire insurance values for each of the two categories Buildings and structures and Machinery and transport equipment from the Manufacturing statistics. The data on investment and fire insurance are deflated using industry specific price indices of investment goods from the Norwegian National Accounts (1991=1). The depreciation rate for Buildings and structures is 0.020 in all industries. For Machinery and transport equipment, the depreciation rate is set to 0.040 in Pulp and paper and Basic metals, and 0.068 in Chemicals. For further documentation of the capital stock calculations, see Biørn, Lindquist and Skjerpen (2000, Section 4).

Other inputs: From the Manufacturing Statistics we get the number of man-hours used, total electricity consumption in kWh, the consumption of a number of fuels in various denominations, and total material costs in NOK for each plant. The different fuels, such as coal, coke, fuelwood, petroleum oils and gases, and aerated waters, are transformed to the common denominator kWh by using estimated average energy content of each fuel [Statistics Norway (1995, p. 124)]. This enables us to calculate aggregate energy use in kWh for each plant. For most plants, this energy aggregate is dominated by electricity. Total material costs is deflated by the price index (1991=1) of material inputs (incl. motor gasoline) from the Norwegian National Accounts. This price is identical for all plants classified in the same National Account industry.

APPENDIX C. Estimation results for the three industries

Table A1. Overall mean and standard deviation of basic variables

Industry	Pulp and paper			Chemicals			Basic metals		
Variable	Mean	Log of mean	Std.dev.	Mean	Log of mean	Std.dev.	Mean	Log of mean	Std.dev.
ln (Y)	4.117		2.079	4.750		2.444	3.586		2.658
Y		5.697			7.351			5.861	
ln (K)	6.691		1.787	7.217		1.942	6.643		2.244
K		7.971			8.777			8.512	
ln(L)	6.836		1.297	6.886		1.496	7.060		1.694
L		7.568			7.889			8.277	
ln(E)	4.417		2.599	5.332		2.481	4.808		2.807
E		6.575			7.440			7.809	
ln(M)	5.393		1.792	5.452		2.014	5.404		2.254
M		6.603			6.983			7.285	
τ	9.940		6.205	10.952		6.188	10.794		6.259

Table A2. Coefficient estimates for models in Pulp and paper. Standard errors in parentheses

Coef.	TL(α,c)	ECD(α,c)	CD(α,c)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
c	-4.1716	-2.8340	-2.3021	-2.2214	-2.7811	-2.1214	0.3804	-2.2919	-0.3244
	(0.7319)	(0.5774)	(0.2279)	(0.4249)	(0.3387)	(0.1331)	(0.4592)	(0.3002)	(0.0842)
γ	0.0234	0.0065	0.0065	0.0074	0.0074	0.0084	0.0124	-0.0005	-0.0002
	(0.0108)	(0.0013)	(0.0013)	(0.0099)	(0.0012)	(0.0012)	(0.0162)	(0.0019)	(0.0019)
α_K	0.5379	0.2716	0.2503	0.0666	0.2172	0.1717	-0.6000	0.0534	0.0735
	(0.1984)	(0.1298)	(0.0344)	(0.1223)	(0.0742)	(0.0197)	(0.1462)	(0.0540)	(0.0187)
α_L	0.9479	0.2753	0.1717	0.3524	0.4145	0.1863	0.1129	0.4711	-0.2314
	(0.2446)	(0.1619)	(0.0381)	(0.1547)	(0.0879)	(0.0215)	(0.1910)	(0.1045)	(0.0206)
α_E	-0.0255	0.0078	0.0854	0.3113	0.0215	0.0921	1.2893	0.1829	0.3333
	(0.1035)	(0.0291)	(0.0169)	(0.0708)	(0.0188)	(0.0103)	(0.0901)	(0.0244)	(0.0099)
α_M	-0.0083	0.6871	0.5666	0.3912	0.5705	0.6064	0.0368	0.7081	0.7530
	(0.1771)	(0.0780)	(0.0309)	(0.1135)	(0.0382)	(0.0167)	(0.1332)	(0.0405)	(0.0160)
β_{KK}	0.0081	-0.0034	0[a]	0.0657	-0.0080	0[a]	0.1072	0.0055	0[a]
	(0.0421)	(0.0194)		(0.0262)	(0.0114)		(0.0317)	(0.0087)	
β_{LL}	-0.1565	-0.0169	0[a]	-0.0361	-0.0372	0[a]	-0.1352	-0.1024	0[a]
	(0.0603)	(0.0244)		(0.0403)	(0.0136)		(0.0515)	(0.0153)	
β_{EE}	0.0269	0.0214	0[a]	0.0447	0.0208	0[a]	0.1648	0.0329	0[a]
	(0.0125)	(0.0064)		(0.0089)	(0.0046)		(0.0129)	(0.0054)	
β_{MM}	-0.1298	-0.0256	0[a]	-0.0353	0.0081	0[a]	-0.1025	0.0077	0[a]
	(0.0391)	(0.0148)		(0.0229)	(0.0078)		(0.0300)	(0.0081)	
γ*	0.0013	0[a]	0[a]	0.0019	0[a]	0[a]	0.0027	0[a]	0[a]
	(0.0003)			(0.0003)			(0.0006)		
β_{LK}	-0.0841	0[a]	0[a]	-0.0088	0[a]	0[a]	0.0518	0[a]	0[a]
	(0.0399)			(0.0230)			(0.0332)		
β_{EK}	-0.0072	0[a]	0[a]	-0.0419	0[a]	0[a]	-0.0797	0[a]	0[a]
	(0.0208)			(0.0134)			(0.0185)		
β_{MK}	0.0567	0[a]	0[a]	-0.0093	0[a]	0[a]	0.0030	0[a]	0[a]
	(0.0309)			(0.0173)			(0.0246)		
δ_K	-0.0026	0[a]	0[a]	-0.0060	0[a]	0[a]	-0.0033	0[a]	0[a]
	(0.0025)			(0.0022)			(0.0032)		
β_{EL}	0.0256	0[a]	0[a]	-0.0351	0[a]	0[a]	-0.1632	0[a]	0[a]
	(0.0229)			(0.0155)			(0.0188)		
β_{ML}	0.1431	0[a]	0[a]	0.0516	0[a]	0[a]	0.1773	0[a]	0[a]
	(0.0385)			(0.0253)			(0.0317)		
δ_L	-0.0058	0[a]	0[a]	-0.0010	0[a]	0[a]	-0.0019	0[a]	0[a]
	(0.0026)			(0.0023)			(0.0037)		
β_{ME}	-0.0275	0[a]	0[a]	0.0203	0[a]	0[a]	-0.0012	0[a]	0[a]
	(0.0180)			(0.0122)			(0.0152)		
δ_E	0.0028	0[a]	0[a]	0.0025	0[a]	0[a]	-0.0001	0[a]	0[a]
	(0.0012)			(0.0011)			(0.0017)		
δ_M	0.0025	0[a]	0[a]	0.0029	0[a]	0[a]	-0.0010	0[a]	0[a]
	(0.0020)			(0.0018)			(0.0028)		

[a] A priori restriction.

Table A3. Coefficient estimates for models in Chemicals. Standard errors in parentheses

Coef.	TL(α,c)	ECD(α,c)	CD(α,c)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
c	1.7184	1.1441	-1.9302	2.0138	-1.4777	-2.1141	1.5094	-3.0740	-1.8519
	(2.4210)	(2.0642)	(0.7378)	(1.4086)	(1.2103)	(0.4501)	(1.2633)	(0.9433)	(0.2715)
γ	-0.0739	0.0323	0.0306	-0.1371	0.0384	0.0422	-0.0896	0.0253	0.0238
	(0.0361)	(0.0048)	(0.0047)	(0.0348)	(0.0046)	(0.0045)	(0.0518)	(0.0067)	(0.0068)
α_K	-1.1664	-0.7696	0.1270	-0.4862	-0.6916	0.0713	-1.7504	-0.7604	0.4646
	(0.5810)	(0.4388)	(0.1149)	(0.3404)	(0.2656)	(0.0667)	(0.3709)	(0.2060)	(0.0577)
α_L	0.5126	0.1298	0.3117	-0.8201	1.1386	0.4711	0.2165	1.6863	0.0537
	(0.8271)	(0.6375)	(0.1605)	(0.5009)	(0.3502)	(0.0763)	(0.5862)	(0.3704)	(0.0733)
α_E	0.1581	0.2866	0.2156	-0.0413	0.2348	0.2244	1.0083	0.4592	0.3046
	(0.3527)	(0.1281)	(0.0718)	(0.1997)	(0.0685)	(0.0368)	(0.2217)	(0.0851)	(0.0350)
α_M	0.4553	0.3957	0.3544	1.4507	-0.0694	0.2530	1.1574	-0.0270	0.1825
	(0.4244)	(0.1922)	(0.0968)	(0.2830)	(0.0813)	(0.0482)	(0.2860)	(0.0933)	(0.0404)
β_{KK}	0.0843	0.1388	0[a]	-0.2988	0.1242	0[a]	-0.3551	0.1727	0[a]
	(0.1365)	(0.0658)		(0.0827)	(0.0400)		(0.1020)	(0.0292)	
β_{LL}	-0.1132	0.0274	0[a]	0.3316	-0.1111	0[a]	-0.3064	-0.2312	0[a]
	(0.2235)	(0.0952)		(0.1425)	(0.0511)		(0.1814)	(0.0524)	
β_{EE}	-0.0323	-0.0157	0[a]	-0.0915	-0.0058	0[a]	0.0576	-0.0315	0[a]
	(0.0436)	(0.0237)		(0.0306)	(0.0141)		(0.0337)	(0.0167)	
β_{MM}	-0.0007	-0.0069	0[a]	0.2396	0.0850	0[a]	0.2967	0.0346	0[a]
	(0.0623)	(0.0352)		(0.0394)	(0.0172)		(0.0471)	(0.0183)	
γ*	0.0034	0[a]	0[a]	0.0070	0[a]	0[a]	0.0039	0[a]	0[a]
	(0.0013)			(0.0014)			(0.0023)		
β_{LK}	0.0398	0[a]	0[a]	0.1574	0[a]	0[a]	0.5802	0[a]	0[a]
	(0.1398)			(0.0866)			(0.1119)		
β_{EK}	-0.0056	0[a]	0[a]	0.1516	0[a]	0[a]	0.1142	0[a]	0[a]
	(0.0625)			(0.0360)			(0.0467)		
β_{MK}	0.0344	0[a]	0[a]	0.1299	0[a]	0[a]	-0.0166	0[a]	0[a]
	(0.0698)			(0.0411)			(0.0512)		
δ_K	0.0397	0[a]	0[a]	0.0305	0[a]	0[a]	0.0310	0[a]	0[a]
	(0.0074)			(0.0065)			(0.0095)		
β_{EL}	0.0770	0[a]	0[a]	-0.0342	0[a]	0[a]	-0.1987	0[a]	0[a]
	(0.0784)			(0.0523)			(0.0584)		
β_{ML}	0.0034	0[a]	0[a]	-0.4104	0[a]	0[a]	-0.2288	0[a]	0[a]
	(0.0941)			(0.0630)			(0.0717)		
δ_L	-0.0131	0[a]	0[a]	0.0136	0[a]	0[a]	-0.0052	0[a]	0[a]
	(0.0093)			(0.0088)			(0.0130)		
β_{ME}	-0.0349	0[a]	0[a]	-0.0219	0[a]	0[a]	-0.0979	0[a]	0[a]
	(0.0484)			(0.0291)			(0.0372)		
δ_E	-0.0075	0[a]	0[a]	-0.0028	0[a]	0[a]	0.0057	0[a]	0[a]
	(0.0043)			(0.0037)			(0.0056)		
δ_M	-0.0165	0[a]	0[a]	-0.0373	0[a]	0[a]	-0.0289	0[a]	0[a]
	(0.0055)			(0.0052)			(0.0080)		

[a]A priori restriction.

Table A4. Coefficient estimates for models in Basic metals. Standard errors in parentheses

Coef.	TL(α,c)	ECD(α,c)	CD(α,c)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
c	-3.4141	-3.9143	-3.1177	-3.0541	-4.0191	-3.0379	-4.7863	-4.9276	-3.1912
	(0.7874)	(0.6481)	(0.2702)	(0.5717)	(0.4271)	(0.2055)	(0.7134)	(0.4355)	(0.1424)
γ	0.0900	0.0220	0.0214	0.0695	0.0215	0.0228	0.1410	0.0211	0.0220
	(0.0172)	(0.0022)	(0.0021)	(0.0163)	(0.0020)	(0.0020)	(0.0294)	(0.0036)	(0.0036)
α_K	0.3304	0.3012	0.1246	-0.0671	0.1638	0.0944	0.5409	0.4690	0.1438
	(0.2201)	(0.1508)	(0.0472)	(0.1368)	(0.0917)	(0.0273)	(0.1730)	(0.0847)	(0.0280)
α_L	-0.2718	0.3731	0.2749	-0.0761	0.6303	0.3073	-0.4586	0.4271	0.1629
	(0.3211)	(0.2040)	(0.0550)	(0.2219)	(0.1183)	(0.0351)	(0.3155)	(0.1598)	(0.0360)
α_E	0.2200	0.0285	0.2138	0.5352	0.1668	0.1628	0.4398	0.0803	0.1502
	(0.1712)	(0.0628)	(0.0374)	(0.1145)	(0.0361)	(0.0174)	(0.1623)	(0.0465)	(0.0190)
α_M	0.9901	0.6632	0.4928	0.8175	0.4217	0.5210	1.0896	0.7338	0.6868
	(0.1712)	(0.0886)	(0.0406)	(0.1190)	(0.0493)	(0.0235)	(0.1418)	(0.0602)	(0.0217)
β_{KK}	-0.0072	-0.0322	0[a]	-0.0020	-0.0120	0[a]	0.0206	-0.0539	0[a]
	(0.0452)	(0.0233)		(0.0257)	(0.0146)		(0.0360)	(0.0134)	
β_{LL}	0.2832	-0.0161	0[a]	0.2183	-0.0489	0[a]	0.4834	-0.0377	0[a]
	(0.0866)	(0.0291)		(0.0611)	(0.0168)		(0.0886)	(0.0228)	
β_{EE}	0.0812	0.0472	0[a]	0.0895	0.0008	0[a]	0.1040	0.0214	0[a]
	(0.0286)	(0.0131)		(0.0205)	(0.0076)		(0.0271)	(0.0082)	
β_{MM}	0.0880	-0.0367	0[a]	0.1340	0.0199	0[a]	0.2391	-0.0122	0[a]
	(0.0375)	(0.0166)		(0.0251)	(0.0095)		(0.0336)	(0.0110)	
γ*	-0.0024	0[a]	0[a]	-0.0029	0[a]	0[a]	-0.0047	0[a]	0[a]
	(0.0006)			(0.0006)			(0.0012)		
β_{LK}	-0.0583	0[a]	0[a]	0.0045	0[a]	0[a]	-0.1131	0[a]	0[a]
	(0.0495)			(0.0319)			(0.0486)		
β_{EK}	0.0499	0[a]	0[a]	-0.0115	0[a]	0[a]	0.0397	0[a]	0[a]
	(0.0288)			(0.0152)			(0.0224)		
β_{MK}	0.0024	0[a]	0[a]	0.0345	0[a]	0[a]	-0.0009	0[a]	0[a]
	(0.0353)			(0.0225)			(0.0301)		
δ_K	-0.0013	0[a]	0[a]	0.0003	0[a]	0[a]	0.0020	0[a]	0[a]
	(0.0039)			(0.0034)			(0.0056)		
β_{EL}	-0.0732	0[a]	0[a]	-0.0691	0[a]	0[a]	-0.0914	0[a]	0[a]
	(0.0385)			(0.0262)			(0.0379)		
β_{ML}	-0.1313	0[a]	0[a]	-0.1608	0[a]	0[a]	-0.2356	0[a]	0[a]
	(0.0429)			(0.0312)			(0.0390)		
δ_L	-0.0073	0[a]	0[a]	-0.0018	0[a]	0[a]	-0.0198	0[a]	0[a]
	(0.0045)			(0.0042)			(0.0072)		
β_{ME}	-0.0267	0[a]	0[a]	-0.0301	0[a]	0[a]	-0.0597	0[a]	0[a]
	(0.0255)			(0.0160)			(0.0226)		
δ_E	-0.0032	0[a]	0[a]	-0.0040	0[a]	0[a]	-0.0011	0[a]	0[a]
	(0.0021)			(0.0018)			(0.0033)		
δ_M	0.0065	0[a]	0[a]	0.0028	0[a]	0[a]	0.0124	0[a]	0[a]
	(0.0024)			(0.0021)			(0.0037)		

[a] A priori restriction.

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