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Empirical errors and predicted errors in fertility, mortality and migration forecasts in the European Economic Area

Abstract:
We analyse empirical errors observed in historical population forecasts produced by statistical agencies in 14 European countries since 1950. The focus is on forecasts for three demographic variables: fertility (Total Fertility Rate - TFR), mortality (life expectancy at birth), and migration (net migration). We inspect forecast bias and forecast accuracy in the historical forecasts, as well as the distribution of the errors. Finally, we analyse for each of the three variables correlation patterns in forecast errors across countries and, for mortality, the correlation between errors for men and women.

In the second part of the report we use time series model to construct prediction intervals to 2050 for the TFR, the life expectancy for men and women, and net migration in 18 European countries. GARCH models are used for fertility and mortality, while net migration is modelled as an autoregressive process

Keywords: stochastic population forecast, empirical forecast errors, prediction intervals, GARCH-models, TFR, life expectancy, net migration, EEA

JEL classification: C22, J11.

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Preface

This report contains the main findings for Work Packages 2 and 3 of the research project "Changing Population of Europe: Uncertain Future", abbreviated as UPE (Uncertain Population of Europe). This is a collaborative project with participants from Finland (University of Joensuu and Statistics Finland), the Netherlands (Netherlands Interdisciplinary Demographic Institute and Statistics Netherlands), and Norway (Statistics Norway).

The major goal of the UPE-project is to develop and implement stochastic population forecasts for the countries of the European Economic Area (EEA). Switzerland is also included in the project, but Liechtenstein was omitted. Hence the following 18 countries are covered: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

The current report contributes to the project, through the following analyses:

- An analysis of errors in past forecasts. This task is known as Work Package 2 (WP2) in the project.
- Model-based estimates of errors. This is UPE's Work Package 3 (WP3).

Both WP2 and WP3 will contribute to the formulation of assumptions that are required for stochastic forecasts in the 18 countries, together with the results from Work Package 4, in which expert views on future fertility, mortality, and migration in the 18 countries are elicited. WP4 will be reported by Statistics Netherlands and the Netherlands Interdisciplinary Demographic Institute.

1. The need for stochastic population forecasts

The demographic future of any human population is uncertain, but some of the many possible trajectories are more probable than others. So attempts to forecast demographic aspects of a population, such as its size by a given year, should include two elements: a range of possible outcomes, and a probability attached to that range. Together, these elements constitute the ‘prediction interval’ for the population variable concerned.

There is a clear trade-off between greater certainty (higher odds) and better precision (narrower intervals). For instance, in 2001 we estimated that the odds were two to one (a 67% chance) that Norway’s population, at that time at 4.5 million, would be between 3.9 million and 6 million in the year 2050 (Keilman et al 2001). Odds of 19 to 1 (a 95% chance) resulted in a wider prediction interval: 3.2 million to 7.3 million.

Demographers have become increasingly concerned about the accuracy of their forecasts, in part because the rapid fall in fertility in Western countries in the 1970s came as a surprise. Forecasts made in those years predicted birth rates that were up to 80% too high and too many young children. The rapid reduction in mortality after the Second World War was also not foreseen; life-expectancy forecasts were too low by 1–2 years; and the predicted number of elderly, particularly the oldest people, was far too low.

Those who use forecasts should be informed about the accuracy of historical predictions. But even more important is the expected accuracy of the current forecast. Statistical agencies traditionally deal with the uncertainty of forecasting population variables by producing two or more predictions of fertility or mortality (or both), and then calculating a range of predictions. For instance, Statistics Norway expects the number of children aged 6–12 in Norway in 2010 to be between 413,000 and 427,000, depending on whether fertility is low or high — that is, on whether women have an average of 1.5 or 2.1 children, respectively, in 2010 — and migration is low or high — an annual net immigration flow of 6,000 or 20,000 persons. The agency attaches no probability to this interval. Yet those who are planning provisions for education need to know whether the likelihood of this scenario is roughly 30%, 60% or even 90%.

So, during the 1990s, demographers and statisticians developed methods for making probabilistic population forecasts, the aim of which is to calculate prediction intervals for every variable of interest.

These forecasts comprised prediction intervals for variables such as age structure, average number of children per woman and immigration flow in the respective countries.

There are three main methods of probabilistic forecasting: time-series extrapolation; expert judgement; and extrapolation of historical forecast errors. Time-series methods rely on statistical models that are fitted to historical data. These methods, however, seldom give an accurate description of the past. If many of the historical facts remain unexplained, uncertainty is large and thus time-series methods result in excessively wide prediction intervals when used for long-term forecasting. Judgemental methods can be used to correct or constrain such broad prediction intervals. Expert judgement is also used when expected values and corresponding prediction intervals are hard to obtain by formal methods. In short, the expert is asked to indicate the probability that a key parameter (such as the average number of children per woman, or life expectancy) in some future year falls within a certain pre-specified range. A weakness of this approach is that experts, often being unduly confident, tend to give overoptimistically narrow prediction intervals (Armstrong 1985). When the forecasts are later compared with actual data, the intervals turn out to fit the observed trends much less frequently than the probabilities suggested. Finally, empirical errors observed for past forecasts may be extrapolated to predict the expected errors for the current forecast. A problem here is that forecasts prepared in the 1960s or earlier were poorly documented, so data on historical errors do not stretch back as far as one would like.

Elements of the three methods are often used in combination. For instance, time-series methods involve some degree of subjectivity, perhaps in choosing the extrapolation model or the length of the historical data series. These decisions may strongly influence the prediction intervals. And the intervals, whether obtained by time-series methods or expert opinion, are frequently checked against historical error patterns.

2. The UPE-project

The research project "Changing Population of Europe: Uncertain Future", abbreviated as UPE (Uncertain Population of Europe), was defined to develop and implement stochastic population forecasts for the countries of the European Economic Area (EEA). Switzerland is also included in the project, but Liechtenstein was omitted. Hence the following 18 countries are covered: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

An important goal of the UPE project is to use a common methodology to compute stochastic population forecasts for the 18 countries, combining the three methods mentioned in Chapter 1. One task in the project is to specify predictive distributions for the parameters of the population forecast model for each country. We have selected the total fertility rate for fertility, the expectation of life at birth for mortality, and the net migration surplus for international migration. For these three indicators the following types of assumptions are needed:

1. The form of the probability distribution of the forecast errors. Initially we will assume normality in the log-scale, so the predictive distribution depends on the first and the second moments of the three components.
2. The first moments. These will be specified based on empirical data and judgement.
3. Second moments. Variances and autocovariances of forecast errors in fertility, mortality and migration have to be specified.
4. Correlations between components. Forecast errors of female and male mortality are likely correlated. Correlations between fertility, mortality, and migration are expected to be of secondary importance.
5. Correlations across countries. Forecast errors of fertility, mortality, and migration in different countries may be correlated.

The current report contributes to all types of assumptions, through the following analyses:

1. An analysis of errors in past forecasts. Historical error patterns in fertility, mortality and migration are analysed for 14 of the 18 countries.
2. Model-based estimates of errors. We use time series models for fertility, mortality and migration indicators in all 18 countries.

In brief, the objective of the historical error analysis is to trace systematic patterns in errors observed for historical population forecasts, specifically differences in forecast errors between fertility, mortality, and migration. We analyse for each of the three indicators their accuracy and bias, the statistical distribution of the errors, and the cross-country correlations. For the life expectancy, we also analyse the correlation between men and women. These results are reported in Chapter 4.

We have restricted ourselves to population forecasts produced by statistical agencies. An important reason for this choice is that all of these were made with a single methodology, namely the cohort-component method of population forecasting - indeed, this is the standard methodology for such forecasts (Keilman and Cruijsen 1992). In addition, the forecasts were produced in stable institutional settings. These two factors imply that we have obtained a relatively homogeneous data set, which provides a meaningful basis for error analysis.

The objective of the time series analysis is to use time-series models to compute prediction intervals for the three indicators. These intervals are derived from autocovariances identified by means of time series of observed data. Covariances across countries, and across sexes in the case of mortality, are also investigated. Chapter 5 contains the findings.

3. Data

Our time series models (Chapter 5) are estimated using observed annual values for the TFR, the life expectancy and the immigration surplus. Long time series are desirable in order to analyse the variability of the three indicators over time. Our ambition has been to estimate these models, to the extent this was possible, on time series that cover the whole of the 20th century. For fertility and mortality this turned out to be possible for the majority of the countries, see below. For migration we had to work with much shorter series.

Forecast errors are obtained by comparing forecast variables with their corresponding observed values, see Chapter 4. We have limited the information on forecasts to the period after World War II. Therefore observed TFR- and life expectancy values were taken from the same sources as those used in Chapter 5. For migration, these sources had to be complemented with national data.

3.1 Observed values for the TFR, the life expectancy, and net migration

Observed values for the TFR, the life expectancy, and net migration were taken, to the extent possible, from international sources. We have put more emphasis on international than on national data sources in order to increase international comparability. We have used the following sources.

- For the TFR: Chesnais (1992) and Council of Europe (2002).
- For the life expectancy at birth: Council of Europe (2002) and the Human Mortality Database of the University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on 16 June 2003).
- For net migration: Council of Europe (2002).

In some cases the international sources had to be supplemented with national sources. Table 3.1 gives an overview of these sources, together with the years they covered. In most cases the additional data were found in official publications from the national statistical agencies. Other sources have been used in a few cases.

For some years, there were minor differences across the data sources. These have been ignored.

Table 3.1. Data sources for observations on the TFR, the life expectancy at birth (by sex), and net immigration

Country	TFR	Life expectancy	Net immigration
Austria	1: 1951-1959; 3: 1960-2000.	2: 1948-1959; 3: 1960-2000	3: 1960-2000
Belgium	1: 1946-1959; 3: 1960-2000.	3: 1960-2000; 11:1892-1959	3: 1960-2000
Denmark	1: 1911-1959; 3: 1960-2000	2: 1921-1959; 3: 1960-2000	3: 1960-2000
Finland	1: 1866-1959; 3: 1960-2000; 4: 1776-1865	2: 1941-1959; 3: 1960-2000	3: 1960-2000
France	1: 1855-1959; 3: 1960-2000	2: 1900-1959; 3: 1960-2000; 9: 1806-1899.	3: 1960-2000
Germany	3: 1960-2000	3: 1963-2000	3: 1960-2000
FRG	1: 1925-1959; 3: 1960-2000	2: 1956-1959; 3: 1960-2000	3: 1960-2000
Greece	3: 1960-2000	3: 1960-2000	3: 1960-2000
Iceland	3: 1960-2000; 5: 1855-1959	3: 1970-2000	3: 1960-2000
Ireland	3: 1960-2000	3: 1985-2000	3: 1960-2000
Italy	1: 1930-1959; 3: 1960-2000	2: 1906-1959; 3: 1960-2000	3: 1960-2000
Luxembourg	3: 1960-2000; 6: 1950-1959	3: 1970-2000; 14:1901-1969	3: 1960-2000
Netherlands	1: 1901-1959; 3: 1950-2000	3: 1960-2000; 10:1900-1959	3: 1960-2000
Norway	3: 1960-2000; 7: 1845-1959	3: 1960-2000; 12.1846-1959	3: 1960-2000
Portugal	3: 1960-2000	3: 1960-2000	3: 1960-2000
Spain	1: 1922-1959; 3: 1960-2000	3: 1965-2000	3: 1965-2000
Sweden	1. 1855-1959; 3: 1960-2000	2: 1861-1959; 3: 1960-2000	3: 1960-2000
Switzerland	1: 1932-1959; 3: 1960-2000; 8: 1861-1931	2: 1876-1959; 3: 1960-2000	3: 1960-2000
England and Wales	1. 1911-1959; 3: 1960-2000	2: 1841-1959; 3: 1960-2000	3, 13: 1960-2000

Notes: 1. Chesnais (1992). 2. Human Mortality Data Base. 3. Council of Europe (2002). 4. Turpeinen (1979). 5. Icelandic Historical Statistics (Statistics Iceland, 1997). 6. Jean Langers (personal communication 2003). 7. Brunborg and Mamelund (1994). 8. Swiss Federal Statistical Office (1998). 9. Meslé and Vallin (1989). 10. Tabeau et al (1994) and Van Poppel (personal communication 2003). 11. Veys (1981). 12. Mamelund and Borgan (1996). 13 Data apply to the United Kingdom. 14. Trausch (1997).

3.2 Forecast data

Information on *forecast* values was taken from national sources[1]. We sent a request for information on details from national population forecasts to national statistical agencies in the 18 countries in October 2001, with a reminder to non-responding countries in May/June 2002. We received useful information from a total of 14 countries. Appendix 1 gives a detailed account of the sources.

Concerning *observed* variables, we only used annual time series. In contrast, *forecast* variables were not available as annual series in many cases, but only in the form of values for certain selected years, for instance every tenth year. In those cases we interpolated linearly between known values. This may

[1] Thus observed values are taken from *international* sources, and forecast values from *national* sources. This may have increased the variance in the forecast errors, but we do not know by how much.

have reduced the variability in the forecast errors somewhat, but not very much, because all forecast variables are smooth extrapolations of current trends.

In many cases, *variant* assumptions were used in a specific forecast. For example, the 1990 forecast of Norway includes a low, a medium, and a high assumption for fertility. Variant assumptions were also frequently made for the components of mortality and migration. In that case, we included all variants in our data set, because very few of the forecast reports contained a clear advice as to which of the variants the statistical agency considered as the most probable one at the time of publication[2]. Hence, it was left to the user to pick one of the variants. We may assume that all variants have been used, although the middle one probably more often than the high or the low one (in case there were three variants)[3].

3.3 Geographical coverage

Concerning *geographical coverage*, there were problems with Germany and with the United Kingdom.

Germany:
Observed trends: Long time series with annual values for the three variables of interest do not exist for Germany; see Table 3.1. For the territory of the former Federal Republic of Germany (FRG), however, we have annual TFR values since 1925 (Chesnais 1992), annual life expectancy values since 1956 (Human Mortality Data Base), in addition to annual net immigration numbers since 1960 (Council of Europe 2002). Therefore we decided to construct time series models for the former FRG, instead of Germany as a whole, assuming that the variability in and predictability of demographic variables would not differ much between the two countries.
Forecasts: For the forecasts produced before 1990, we only dispose of information for the FRG, not the German Democratic Republic (GDR). Thus errors in forecasts of the TFR, the life expectancy or the immigration surplus from these forecasts apply to the FRG only. The forecasts published by Statistics Germany beginning in 1990 comprise both countries. However, in many cases distinct assumptions were formulated for the old FRG and GDR separately, and the available information on fertility and mortality does not allow for a simple aggregation into indicators for the whole of Germany. For international migration, assumed net immigration into Germany for the period 1990-

[2] The 1980-based forecast of the Netherlands is one exception.

[3] For some countries, we had enough data to check the implications of this choice. For Norway, the standard deviation in the observed TFR-errors based on all forecast variants was very close to that based on main variants only. For Sweden, the all-variants standard deviations were approximately 10 per cent higher than those based on main variants.

2000 in the forecasts of 1990, 1993, and 1998 could be compared with actual immigration into Germany. The resulting errors were combined with those for migration assumptions into the FRG in forecasts before 1990.

United Kingdom:

Also for the United Kingdom (England, Wales, Scotland, and Northern Ireland) the available time series for the observed values of the TFR, the life expectancy, and the immigration surplus are rather short. The situation is a lot better for England and Wales: annual TFR-values are available since 1911 from Chesnais, and life expectancy values since 1841 from the Human Mortality Data Base. Thus we have analysed fertility and mortality for England and Wales, again assuming that these countries could be representative for the whole of the United Kingdom with respect to variability and predictability. Immigration was analysed for the UK on the basis of time series starting in 1960; see Table 3.1.

4. Empirical errors

4.1 Measurement

Measures

We have used different types of measures to characterize the errors. Assume that the interest is in a forecast variable from a forecast with a certain base year at a certain future year. Write that variable as *F*, and the corresponding observed value as *O*. The signed error *E* is defined as *F*-*O*. Hence for positive values of *E*, the forecast was too high. A negative value of *E* indicates too low a forecast.

When we have several forecast values at our disposal (across time, across countries), there is a series of errors E_1, E_2, …, E_n of length *n*. We have computed several summary measures for the errors.

1. The mean error (*ME*) is the average value of the series E_1, E_2, …, E_n. It reflects the tendency to over- or under-predict the variable of interest. Positive errors will cancel negative ones. Therefore, the mean error is not a good indicator of accuracy, but it yields useful information on *bias*.
2. The standard deviation in the signed errors (*SDE*). It measures the variability in the errors around their mean value. It reflects uncertainty in the variable appropriately, provided that its expected value is predicted correctly.
3. The latter assumption may be relaxed by inspecting the Root Mean Squared Error (*RMSE*), which adds a bias component to the standard deviation.

One disadvantage of the *SDE* and the *RMSE* is that they give relatively much weight to extreme values, as a consequence of the squared terms. Error measures based on absolute errors do not have this property.

4. The mean absolute error (*MAE*), computed as the mean value of the errors disregarding sign.
5. In one particular case (see below) we also needed the standard deviation of the absolute error (*SDAE*).

Under certain conditions, the summary measures for absolute errors can be computed when the summary measures for the signed errors are known. We will discuss two particular cases: 1. the signed errors E_i are normally distributed; 2. the signed errors E_i are exponentially distributed with parameter α for positive values of E_i, and the errors ($-E_i$) are exponentially distributed with the same parameter α for negative E_i-values. In this case, the density of the signed errors E_i is symmetrical around zero.

1. When the signed errors E_i are drawn from a normal distribution with expectation μ_S and standard deviation σ_S, one can show that the expected value of the *absolute* errors $|E_i|$ equals

$$\mu_A = \mu_S + \sigma_s \sqrt{\frac{2}{\pi}} \exp\left(-\frac{\mu_S^2}{2\sigma_S^2} \right). \tag{4.1}$$

The median value of the absolute errors equals $\mu_S + 0.675\sigma_S$. Furthermore, since $(E_i)^2=|E_i|^2$, the second moments of the absolute errors and the signed errors are equal, i.e. $\mu_S^2 + \sigma_S^2$. Then the standard deviation of the absolute errors follows immediately.

2. Shlyakhter and Kammen (1993) and Shlyakhter et al. (1994) studied the error distributions for forecasts of population size, sea-level rise, and energy demand. They found that the commonly assumed normal distribution underestimates the frequency of extreme errors. Instead they proposed that absolute errors $|E_i|$ are exponentially distributed. This situation arises when signed errors E_i are exponentially distributed with parameter α for positive values of E_i, and the errors $-E_i$ are exponentially distributed with the same parameter α for negative E_i-values. In this case, both the expectation and the standard deviation of the absolute errors are equal to $1/\alpha$, while the standard deviation of the signed errors equals $\sqrt{2}/\alpha$. The expected value of the signed errors is zero by construction.

Three dimensions

For a given country and a given indicator, there are three dimensions in the data set of forecast errors: the starting year of the forecast, the calendar year for which the error is computed, and the duration of the forecast. These three dimensions are not independent: once we know two of them, the third one follows immediately. In general:

starting year + forecast duration = calendar year.

For instance, the error in the assumed TFR-value of the Norwegian forecast with starting year 1993 at duration of four years applies to the calendar year 1997.

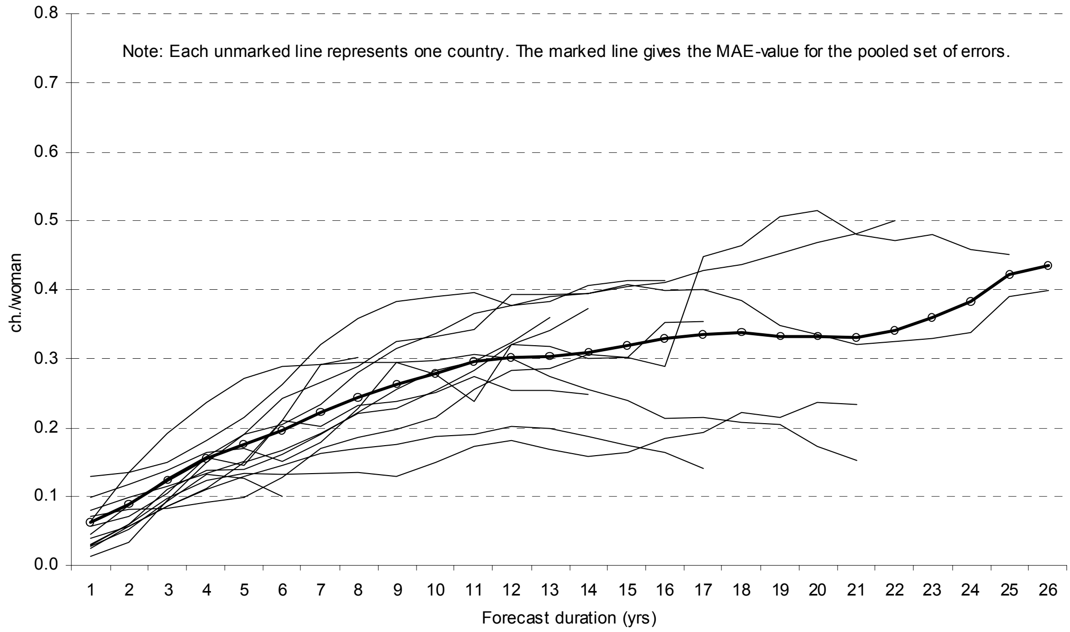
Most of our findings will be presented with forecast duration as the dimension of interest. The reason is that we want to use the historical errors as a benchmark for assessing the predictions derived from time series models. Neither the calendar years nor the starting years of those two types of forecasts overlap, but they do have forecast duration in common. In a wider sense, the historical errors can be used to assess the quality of current forecasts. One may argue that population forecasters nowadays produce forecasts with smaller errors than in the past, due to better data, methods, and theoretical understanding of the underlying mechanisms.[4] Whether or not this is true, the historical errors are useful as a benchmark for the predictability of current forecasts. This then leads naturally to a focus on forecast duration.

4.2 TFR

4.2.1 Accuracy and bias

Figure 4.1 plots the mean of the absolute errors in the TFR (*MAE*) for14 countries. For each country, the mean is computed across several forecast rounds, controlling for forecast duration. Each line represents one country. For long forecast durations we have fewer observations than for short ones. Therefore, in order to avoid too much randomness, we plotted, for each country, only those *MAE*-values that were based upon at least 10 observations. The marked solid line gives the *MAE*-value for the pooled set of errors, i.e.the errors for all countries and forecast rounds combined, including those observations that were left out from the country-specific means. The latter curve is based on 295 observations in the first forecast year and 203, 94, and 16 observations at durations of 10, 20, and 30 years ahead, respectively.

Figure 4.1. Mean of observed absolute TFR errors in 14 countries



[4] Yet there is no empirical evidence that supports this assumption (NRC 2000).

The pattern that emerges is that of slowly increasing errors. Long-term forecast accuracy is less than short-term accuracy, because the chances that conditions that affect fertility may have changed are relatively large for remote forecast years. Across all countries and all forecasts, the mean absolute error increases from 0.06 children per woman in the first year of the forecast, to 0.3 for a forecast horizon of 15 years ahead, and 0.4 children per woman 25 years ahead. Thus, measured by this indicator, the growth is slightly slower than linear. Although the patterns for the individual countries vary rather strongly around the mean of the pooled errors, they are roughly consistent with the overall picture.

Figure 4.2. Mean of observed TFR errors in 14 countries

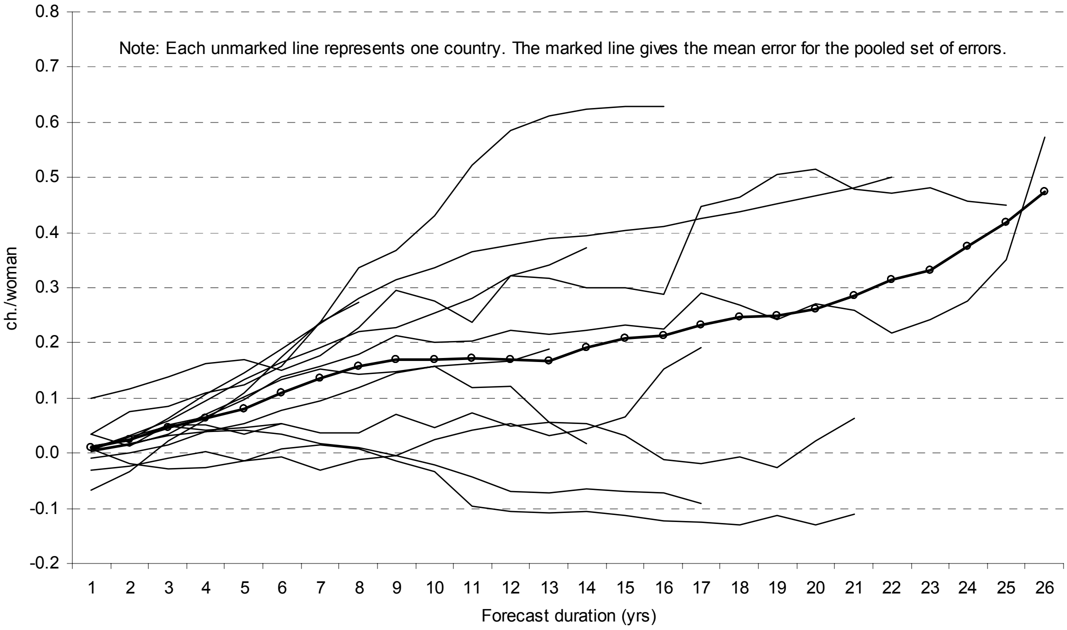
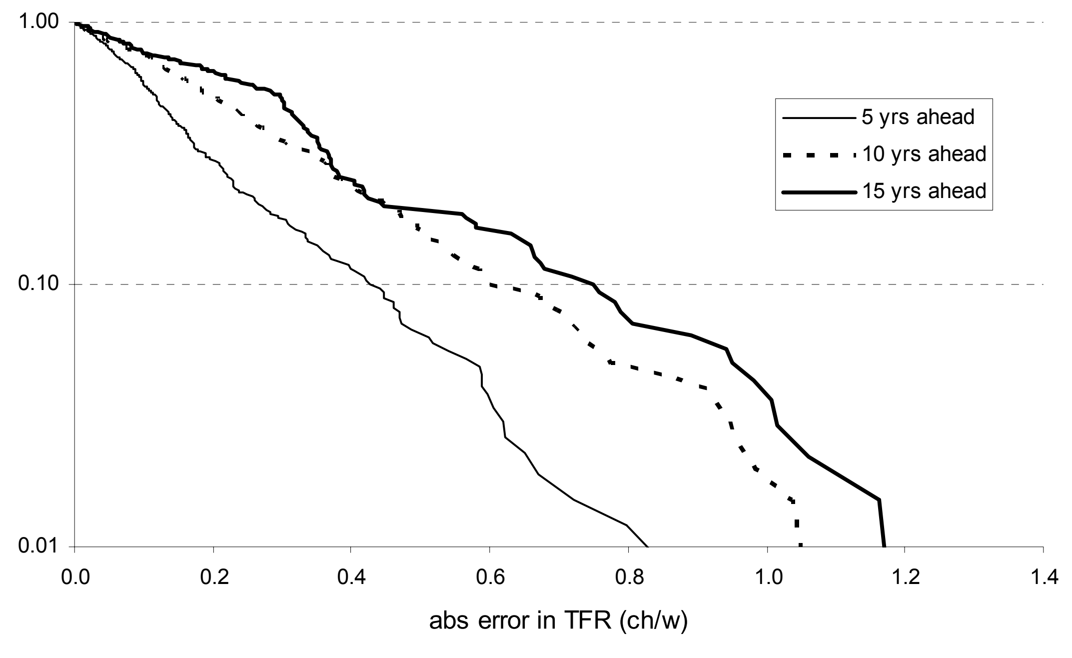


Figure 4.2 shows that TFR-forecasts after World War II in the EEA-countries were biased upwards: the overall (that is, on average across countries and forecast rounds) mean error (*ME*) in the TFR is negligible in the first forecast year, but it grows regularly to a little over 0.4 children per woman 25 years ahead. This pattern reflects the well-known fact that fertility was overpredicted in the late 1960s and the 1970s, when actual numbers fell rapidly in the region. Indeed - the mean error is only slightly lower than the mean absolute error in Figure 4.1, reflecting the fact that almost all errors were positive. Two countries, viz. Finland and Denmark, have underpredicted their TFR by 0.1 children per woman 15-20 years ahead. The reason is that the observation period for these countries starts when fertility already was at a low level: 1972 in Finland and 1974 in Denmark.

4.2.2 Error distributions

Figure 4.3 plots the relative frequency curve for absolute errors in the pooled data set at forecast horizons of five, ten, and fifteen years. The curves show the probability that the absolute error in the TFR exceeds a certain value. For instance, at a forecast duration of 10 years, there is a 10 per cent chance for an error of at least 0.6 children per woman. Not surprisingly, the graph shows that large errors are more frequent for longer forecast durations.

Figure 4.3 Probability that absolute error in TFR exceeds a certain value



If the absolute errors would be normally distributed, the curves in the semi-logarithmic plot would be quadratic; exponentially distributed errors would imply straight lines. Since the ratio of the *MAE* to the *SDAE* is only around 0.9 for durations 5, 10, and 15 years ahead, a normal distribution with its indefinite left tail is unlikely. Indeed, the patterns in Figure 4.3 do not very closely resemble quadratic curves, except perhaps for large error values at a forecast horizon of 15 years. A straight line seems to fit better, in any case in the probability range 5-100 per cent.

Assume that the absolute errors Y stem from an exponential distribution with parameter α:

$F(y) = \Pr\{Y \leq y\} = 1 - \exp(-\alpha y), \quad \alpha, y \geq 0.$[5]

[5] Figure 4.3 plots 1-$F(y)$.

When α increases, the curve in Figure 4.3 becomes steeper, and extreme errors become less likely. Since the expected value μ_A of the error equals $1/\alpha$, α can be estimated as the inverse of the observed mean error. This gives 5.7, 3.6, and 3.1 women per child as estimated parameter values for the error distributions at durations 5, 10, and 15 years. Indeed, long durations imply small values of α, and thus more likely extreme error values. We can correct for this duration effect, by inspecting the *normalized* error – in other words αY, which is the same as Y/μ_A. Since the standard deviation of an exponentially distributed variable is equal to its expectation, the same result is obtained when the errors are divided by the standard deviation.

Figure 4.4 Probability that absolute error in TFR exceeds a certain value (errors scaled by MAE)

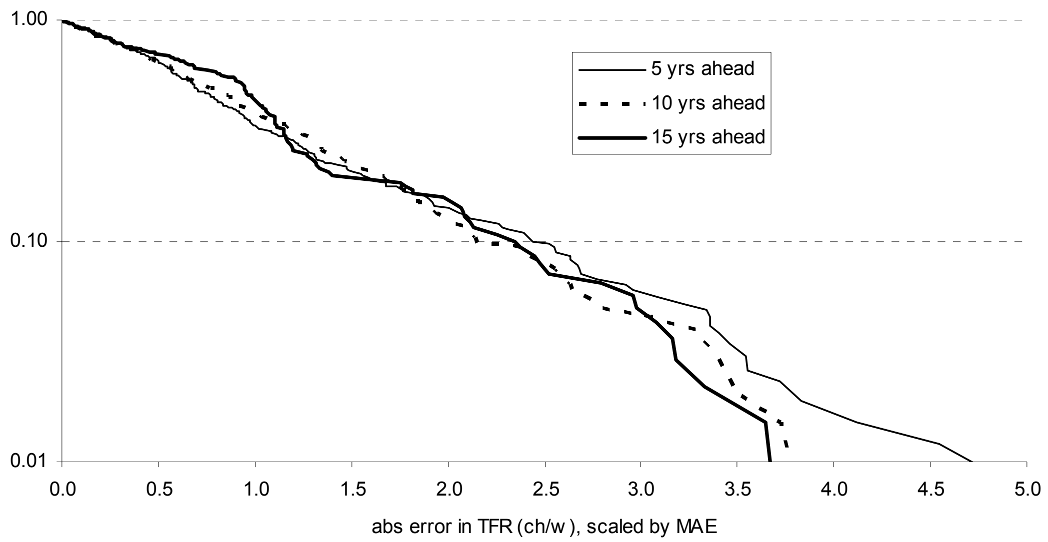


Figure 4.4 is the frequency plot for the normalized absolute errors. For each forecast horizon, the normalized error was computed as the observed error divided by the corresponding *MAE*. The three curves coincide quite well, except for very large errors with probability less than five per cent. There is a 10-per cent chance that the absolute error in the TFR exceeds 2.3 times its mean value – in other words that it exceeds 0.40, 0.64, or 0.73 children per woman at durations 5, 10, and 15 years ahead. Alternatively we can say that the chance is 10 per cent that the absolute error in the TFR exceeds 2.3 times its standard deviation. We inspected the value of the *MAE* as a ratio of the standard deviation of the absolute errors. For exponentially distributed absolute errors, this ratio should be one. In our case it turned out to be between 0.96 and 1.09 for forecast durations 4-20 years ahead. The ratio was much lower than one for some very short and very long durations.

As a further check on the form of the error distribution we used expression (4.1) to predict the expected value of the absolute errors, assuming normally distributed signed errors. The expected value and the standard deviation of the signed errors were estimated by the *ME* and the *SDE*. For all forecast durations, however, the predicted value was much larger than the empirical value (*MAE*), even larger than the upper bound of the 95 per cent confidence interval of the expected value of the absolute errors. Thus we conclude that a normal distribution is unlikely for the signed errors.

4.2.3 Cross-country correlations

Do large errors in one country tend to go together with large errors in another country, or rather with smaller errors? Tables 4.1-4.3 present correlations across countries based on absolute errors in the TFR. We selected forecast horizons of 5, 10, and 15 years. The number of observations is given in parentheses for each country. For each pair of countries with numbers of observations equal to n and m, we tested whether the correlation would be significantly different from zero (two-tailed, $\alpha = 0.05$), based on a number of degrees of freedom equal to min(n,m)-2. Correlations that appear significant are marked.

Table 4.1 Cross-country correlations of absolute errors in TFR, 5 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	Pt	S	CH	UK
Austria (18)	1													
Belgium (11)	0.293	1												
Denmark (39)	0.267	0.154	1											
Finland (32)	0.112	-0.322	0.214	1										
France (13)	-0.097	-0.274	0.672	0.474	1									
WGermany (14)	0.469	0.573	0.005	0.022	-0.080	1								
Italy (15)	0.420	0.082	0.507	0.162	0.309	-0.126	1							
Luxembourg (16)	0.564	0.206	0.377	0.154	0.118	-0.045	0.634	1						
Netherlands (18)	0.386	0.144	0.669	0.715	0.153	0.006	0.405	0.406	1					
Norway (28)	0.324	0.199	0.156	-0.065	-0.202	0.342	0.305	0.357	0.301	1				
Portugal (7)	0.076	-0.036	0.934	0.658	0.712	-0.595	0.400	0.354	0.490	-0.640	1			
Sweden (23)	0.050	-0.252	0.245	0.215	0.347	-0.312	0.168	-0.051	0.349	-0.351	0.904	1		
Switzerland (18)	0.007	0.070	0.492	0.576	0.195	-0.124	0.414	0.173	0.625	0.261	0.328	0.212	1	
UK (19)	0.322	0.167	0.697	0.634	0.280	0.061	0.542	0.596	0.787	0.480	0.502	0.240	0.728	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.2 Cross-country correlations of absolute errors in TFR, 10 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	Pt	S	CH	UK
Austria (13)	1													
Belgium (5)	0.659	1												
Denmark (25)	0.273	0.541	1											
Finland (24)	-0.144	-0.135	0.173	1										
France (13)	0.390	0.522	0.195	0.434	1									
WGermany (13)	0.107	-0.469	-0.373	-0.053	-0.095	1								
Italy (9)	0.414	0.856	0.526	0.024	0.590	-0.673	1							
Luxembourg (13)	-0.002	0.296	0.002	-0.341	-0.066	0.306	0.090	1						
Netherlands (13)	0.405	0.934	0.553	0.300	0.568	-0.047	0.607	-0.229	1					
Norway (22)	0.623	0.666	0.432	0.401	0.553	0.252	0.486	0.201	0.587	1				
Portugal (4)	-0.966	-0.682	-0.789	0.499	-0.077	0.831	-0.863	-0.086	-0.516	-0.960	1			
Sweden (20)	-0.187	-0.017	-0.107	0.105	0.154	0.222	-0.091	0.004	0.400	-0.111	0.684	1		
Switzerland (13)	0.718	-0.238	0.026	-0.102	0.294	0.347	-0.058	-0.145	0.219	0.353	-0.741	0.277	1	
UK (16)	0.708	0.092	0.499	0.332	0.521	0.247	0.290	-0.067	0.628	0.639	-0.575	0.433	0.663	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.3. Cross-country correlations of absolute errors in TFR, 15 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	Pt
Austria (9)	1										
Denmark (15)	0.415	1									
Finland (16)	-0.109	-0.027	1								
France (10)	-0.015	-0.585	0.265	1							
WGermany (11)	-0.594	-0.104	-0.266	-0.189	1						
Luxembourg (8)	0.489	0.511	-0.082	-0.136	-0.182	1					
Netherlands (8)	0.478	0.145	0.545	0.467	-0.426	-0.169	1				
Norway (18)	0.705	0.226	-0.259	0.210	0.089	0.410	0.548	1			
Sweden (14)	-0.295	0.294	-0.200	-0.652	0.383	0.236	-0.202	0.034	1		
Switzerland (13)	0.526	0.172	-0.164	-0.039	0.096	0.034	0.052	0.364	-0.475	1	
UK (13)	0.738	0.276	-0.287	0.278	-0.283	0.155	0.661	0.713	-0.251	0.440	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

The correlation pattern across countries that emerges from Tables 4.1-4.3 is not stable. The only significant correlation that appears in all three tables is that between Norway and the United Kingdom, in strength varying between +0.48 and +0.71. The correlations 15 years ahead are seldom significant, because we have so few observations. When we restrict ourselves to forecasting horizons of five and ten years, there seems to be a group of countries that move together: Denmark, Norway, Netherlands, Switzerland, and the UK. Their correlations are on average 0.67 for a forecast horizon of five years, and 0.57 ten years ahead. These five countries experienced the strong fertility decline at approximately the same time. The correlations for Finland fit also in here, provided we restrict ourselves to five-year forecasts. The correlations for other countries are less systematic.

4.2.4 Conclusion

The TFR forecasts in 14 countries indicate that the absolute TFR-errors have a distribution that is close to an exponential distribution. The commonly assumed normal distribution fits the data somewhat less well. Thus the absolute errors in the TFR in our data set can be said to be characterized by two features: a set of normalization factors that increase with forecast duration, as plotted in Figure 4.1, combined with an exponential distribution for the probability that the normalized error exceeds a given value, as plotted in Figure 4.4. These two characteristics are to be used for forecast durations up to approximately 20 years and for probabilities not lower than about five per cent. For example, the probability is approximately 20 per cent that the absolute error 6 years ahead exceeds 1.5*0.2=0.3 children per woman. The value pair (20%, 1.5 children per woman) is read off from Figure 4.4, whereas the normalized value 0.2 children per woman at a forecast duration of 6 years is found in Figure 4.1. At 12 years ahead, the normalized value is 0.3. Hence there is a 20-per cent chance that the absolute error in the TFR will exceed 1.5*0.3=0.45 children per woman at that horizon. There is a tendency that Denmark, Finland, Netherlands, Norway, Switzerland, and the UK move together: large (small) absolute errors in the TFR in one country tended to coincide with large (small) errors in the other countries in this group.

4.3 The life expectancy at birth

4.3.1 Accuracy and bias

Figures 4.5 and 4.6 plot the means of the absolute errors in the life expectancy at birth of men and women (*MAE*) for14 countries. Some countries had fewer than 10 observed errors, even at the first forecast duration. There is no individual line for these countries, but their errors are included in the mean of the pooled errors. The figures show a slightly accelerating growth in inaccuracy by forecast horizon, with errors increasing by 0.2 years per year for forecast horizons 10-25 years, and somewhat slower rising errors for shorter durations. The patterns are very similar for men and women.

Figure 4.5 Mean of observed absolute life expectancy errors for men in 14 countries

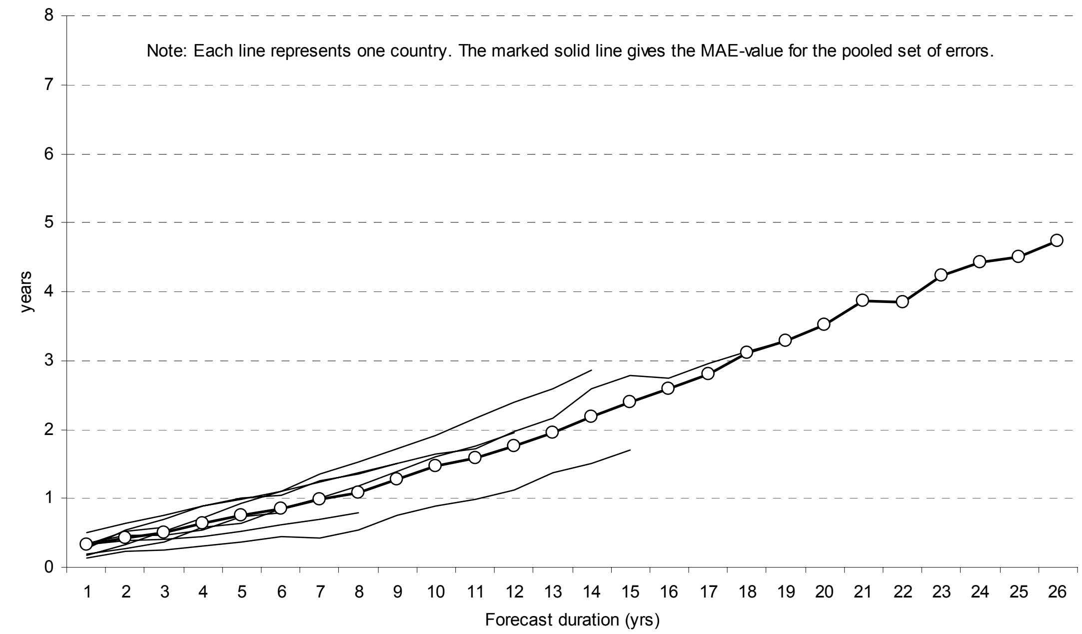
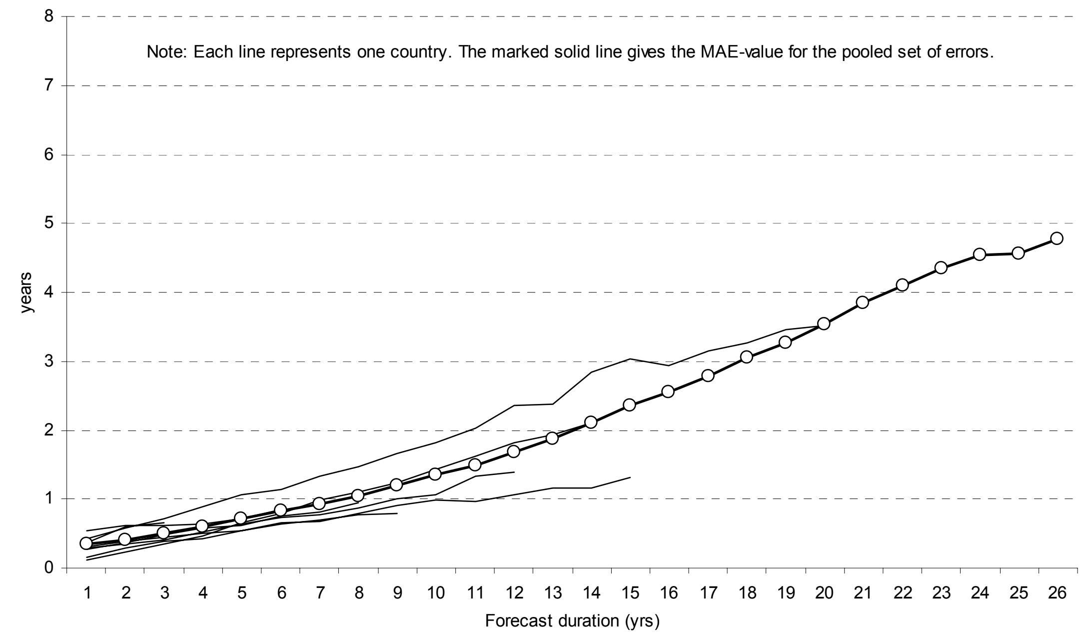
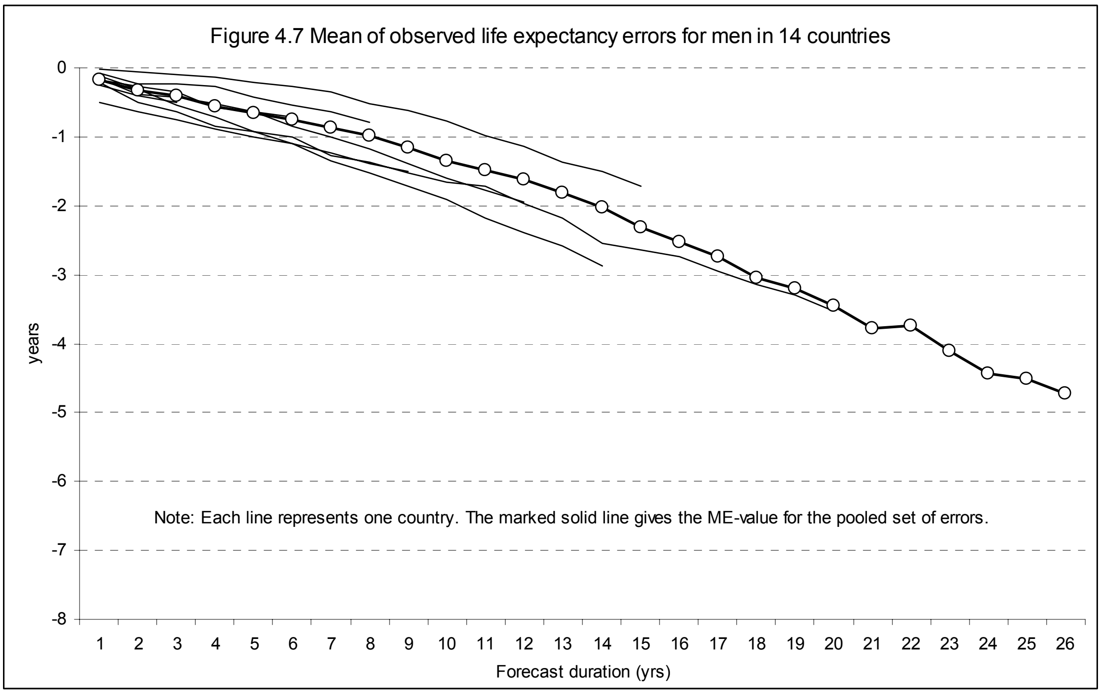
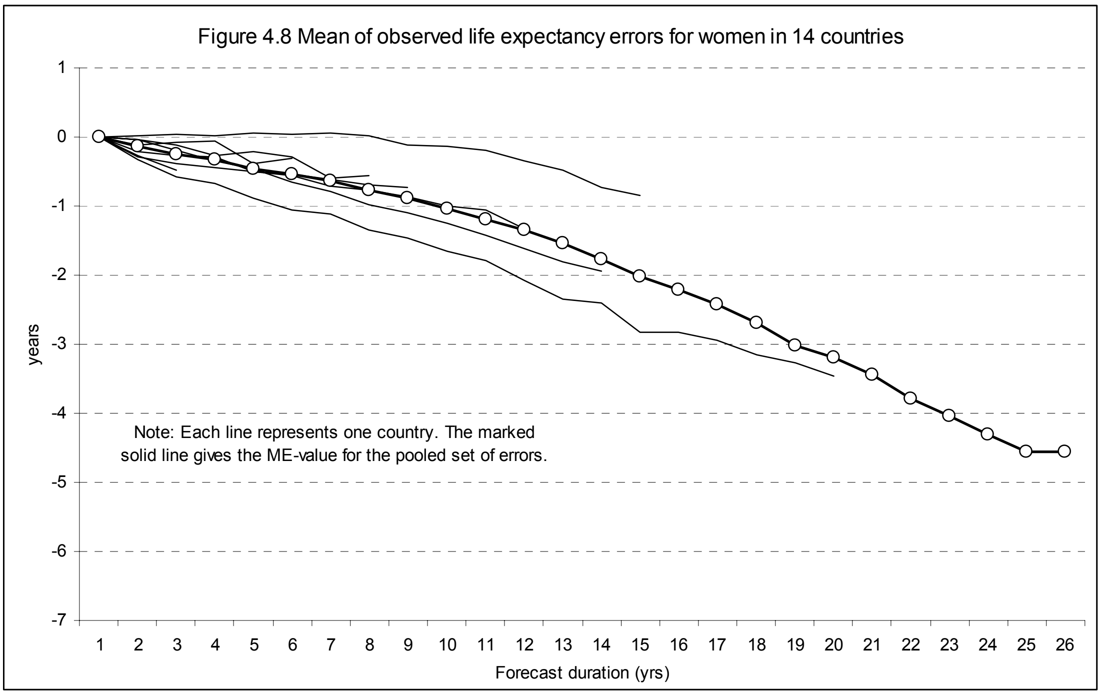


Figure 4.6 Mean of observed absolute life expectancy errors for women in 14 countries



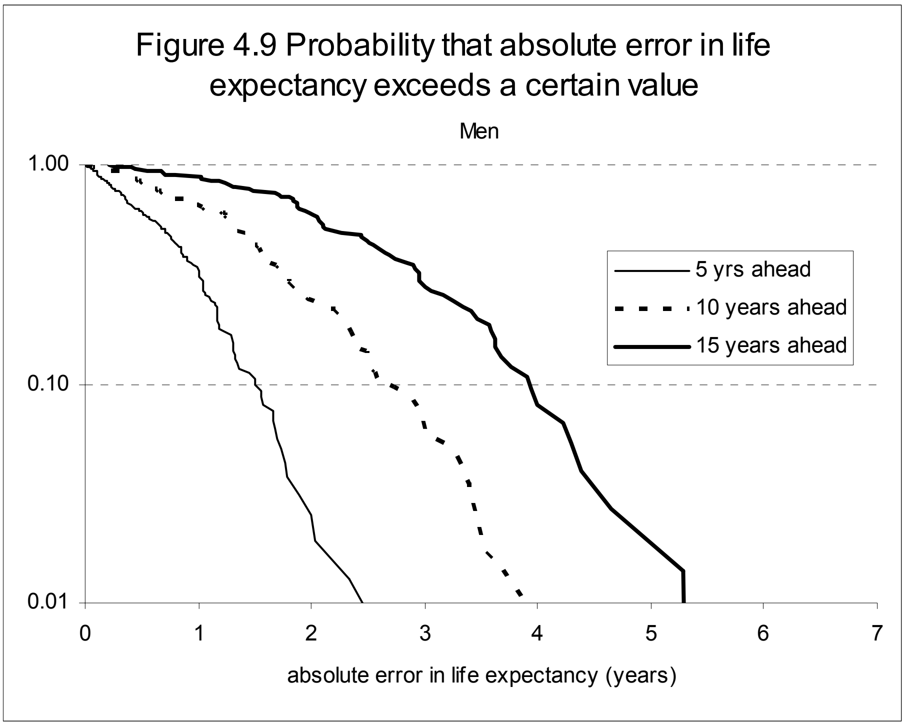
Figures 4.7 and 4.8 indicate that life expectancy forecasts have been too low on average. The underprediction amounted to 1.0-1.3 and 3.2-3.4 years of life expectancy at forecast horizons of 10 and 20 years ahead, respectively. This confirms earlier findings for selected industrialized countries (Keilman 1997).



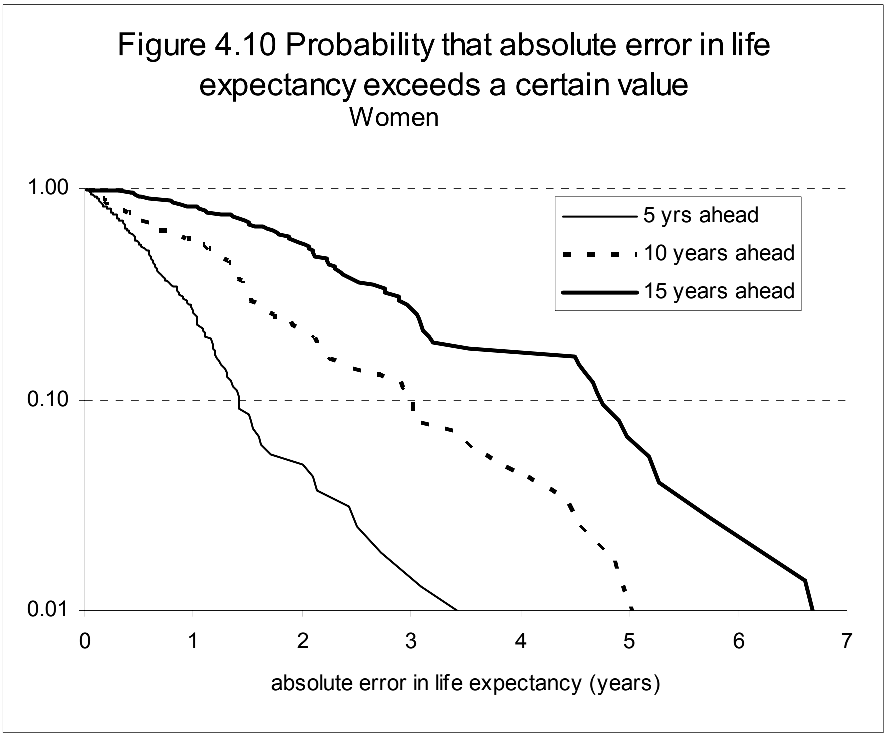


4.3.2 Error distributions

Figures 4.9 and 4.10 suggest that the distributions of the absolute errors for forecast durations of 5, 10, and 15 years ahead are close to a normal one, in particular for men. One explanation for the normal distribution is that the errors are caused by a gradual improvement in life expectancy, which was not picked up by population forecasters.[6]



[6] The errors in the TFR were caused by sudden trend shifts, and hence they became more extreme than those for the life expectancy. This explains why their distribution is exponential, rather than normal.



This implies that the distributions for life expectancy errors can be described by two parameters, as opposed to only one parameter for the TFR-errors. Figures 4.11 and 4.12 plot the distribution of the standardized errors, i.e. errors obtained after having subtracted the mean and divided by the standard error. Assuming a normal distribution for the absolute errors at each forecast duration, we can use Figure 4.13, combined with Figures 4.11 and 4.12, for statements about the likelihood of errors of a certain magnitude in life expectancy forecasts. For example, the probability is 20 per cent that the standardized absolute error in female life expectancy will exceed 0.6 years (Figure 4.12). Figure 4.13 gives the two parameters that are necessary to recompute standardized errors to errors in the original scale. Since there was very little difference between the sexes (compare Figures 4.5 and 4.6 for the means), we plotted the curves for only one sex. At 10 years into the future, Figure 4.13 gives a mean error of 1.3 years and a standard deviation of 1.0 year. Thus the standardized error of 0.6 years mentioned earlier will translate into an unstandardized error of 1.3 + 1.0*0.6 = 1.9 years. In other words, there is a 20 per cent chance that the ten-year ahead life expectancy forecast for women will be wrong by at least 1.9 years.

Figure 4.11 Probability that absolute error in life expectancy exceeds a certain value (standardized errors)

Men

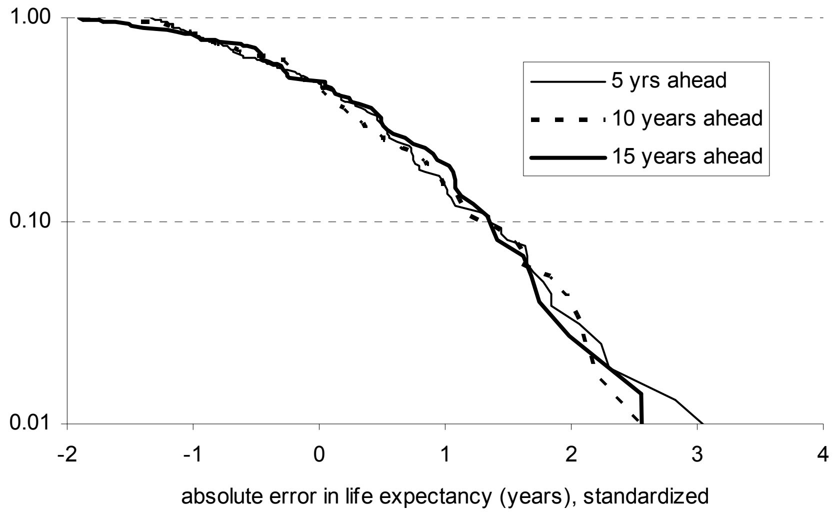
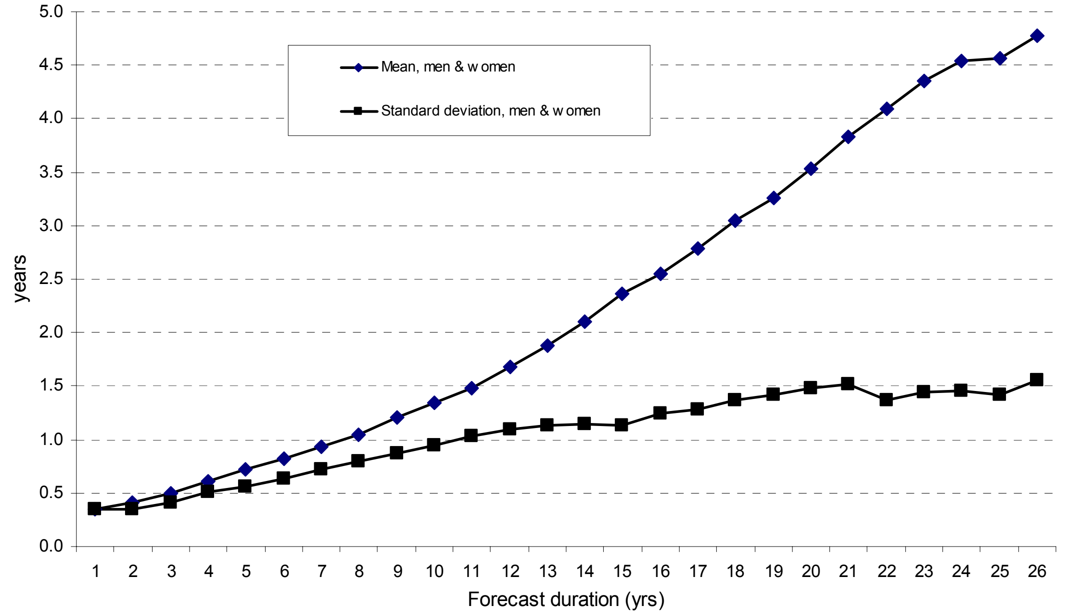


Figure 4.12 Probability that absolute error in life expectancy exceeds a certain value (standardized errors)

Women

Figure 4.13 Means and standard deviations of observed absolute life expectancy errors for men and women in 14 countries



4.3.3 Cross-country correlations

Tables 4.4-4.9 report correlations across countries in absolute errors of life expectancy forecasts for men and women for selected forecast horizons.

Table 4.4. Cross-country correlations of absolute errors in life expectancy of men, 5 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	S	CH	UK
Austria (17)	1												
Belgium (6)	0.616	1											
Denmark (14)	-0.643	-0.480	1										
Finland (26)	0.195	-0.904	-0.298	1									
France (7)	-0.753	0.147	-0.744	-0.265	1								
WGermany (9)	0.468	0.625	-0.369	0.033	-0.377	1							
Italy (5)	-0.454	-0.363	0.560	0.135	-0.058	-0.593	1						
Luxembourg (7)	-0.001	0.680	-0.395	-0.534	0.491	0.149	-0.124	1					
Netherlands (20)	0.346	0.595	-0.708	0.315	0.426	0.602	-0.563	0.606	1				
Norway (15)	-0.485	-0.797	0.080	0.151	0.311	-0.574	0.530	-0.440	-0.296	1			
Sweden (17)	-0.070	0.000	0.591	-0.456	-0.641	-0.253	0.448	-0.123	-0.605	-0.053	1		
Switzerland (10)	0.149	0.472	-0.119	-0.521	0.254	-0.076	-0.131	-0.040	-0.235	-0.053	-0.038	1	
UK (7)	0.746	-0.213	0.779	-0.116	-0.692	-0.145	-0.279	-0.382	-0.668	-0.336	0.936	0.108	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.5 Cross-country correlations of absolute errors in life expectancy of men, 10 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	S	CH	UK
Austria (13)	1												
Belgium (5)	-0.538	1											
Denmark (9)	-0.044	-0.676	1										
Finland (16)	0.664	-0.452	-0.638	1									
France (7)	-0.654	0.207	-0.686	0.129	1								
WGermany (8)	0.146	-0.730	-0.046	-0.053	-0.164	1							
Italy (4)	-0.516	0.043	-0.082	-0.281	0.246	0.161	1						
Luxembourg (6)	-0.695	0.701	-0.619	-0.085	0.648	-0.433	0.604	1					
Netherlands (15)	0.092	-0.409	-0.616	0.315	0.614	0.612	0.172	-0.087	1				
Norway (7)	0.125	0.344	0.179	0.305	-0.113	-0.893	0.129	0.265	-0.790	1			
Sweden (14)	-0.238	-0.794	0.832	-0.477	-0.299	0.160	0.592	-0.285	-0.280	-0.055	1		
Switzerland (6)	-0.102	0.812	-0.369	0.072	-0.053	-0.861	-0.161	0.348	-0.724	0.782	-0.921	1	
UK (4)	0.492	0.152	0.063	0.126	-0.421	-0.455	-0.950	-0.659	-0.392	0.070	-0.792	0.458	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.6 Cross-country correlations of absolute errors in life expectancy of men, 15 years ahead

	A	Dk	SF	F	FRG	Lux	Nl	N	S	CH
Austria (9)	1									
Denmark (5)	0.326	1								
Finland (11)	0.661	0.009	1							
France (6)	-0.729	0.214	-0.492	1						
WGermany (6)	0.029	0.590	-0.552	0.143	1					
Luxembourg (5)	-0.214	0.278	0.388	0.371	-0.215	1				
Netherlands (10)	-0.123	0.450	0.186	0.812	0.667	0.084	1			
Norway (6)	0.370	0.860	0.407	0.106	0.183	0.667	0.204	1		
Sweden (8)	-0.310	0.423	-0.520	-0.068	0.737	0.632	-0.135	0.311	1	
Switzerland (6)	0.011	-0.891	0.374	-0.365	-0.779	-0.298	-0.716	-0.664	-0.624	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.7. Cross-country correlations of absolute errors in life expectancy of women, 5 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	S	CH	UK
Austria (17)	1												
Belgium (9)	0.693	1											
Denmark (14)	-0.716	-0.569	1										
Finland (26)	0.160	-0.181	-0.377	1									
France (7)	-0.598	-0.219	0.479	0.931	1								
WGermany (9)	0.239	0.594	-0.566	0.008	-0.328	1							
Italy (5)	-0.120	-0.374	0.815	0.326	0.320	-0.707	1						
Luxembourg (7)	0.370	0.703	0.024	0.114	0.237	0.088	-0.180	1					
Netherlands (20)	0.498	0.622	-0.494	0.389	-0.083	0.749	-0.721	0.600	1				
Norway (15)	0.233	0.221	-0.168	0.673	0.826	0.252	0.779	0.533	0.387	1			
Sweden (17)	0.319	-0.107	-0.277	0.439	0.257	-0.007	0.243	0.390	0.373	0.613	1		
Switzerland (10)	0.451	0.624	-0.088	0.246	0.292	0.338	0.309	0.521	0.155	0.704	0.108	1	
UK (7)	-0.546	-0.541	0.690	0.015	0.191	-0.297	0.086	-0.040	0.051	0.100	0.682	-0.367	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.8. Cross-country correlations of absolute errors in life expectancy of women, 10 years ahead

	A	B	Dk	SF	F	FRG	I	Lux	Nl	N	S	CH	UK
Austria (13)	1												
Belgium (5)	0.395	1											
Denmark (9)	-0.224	-0.308	1										
Finland (16)	0.664	-0.135	-0.495	1									
France (7)	0.432	-0.678	0.064	0.791	1								
WGermany (8)	-0.046	-0.432	-0.881	0.150	-0.162	1							
Italy (4)	-0.057	0.757	-0.259	-0.921	-0.981	-0.141	1						
Luxembourg (6)	0.375	0.515	0.307	0.509	0.066	-0.754	0.918	1					
Netherlands (15)	0.116	0.289	-0.687	0.360	-0.345	0.560	0.569	0.044	1				
Norway (7)	0.779	0.473	-0.139	0.794	0.582	-0.991	0.070	0.720	-0.120	1			
Sweden (14)	0.647	0.299	-0.091	0.544	-0.002	0.077	0.816	0.843	0.481	0.478	1		
Switzerland (6)	0.908	0.724	0.536	0.397	0.211	-0.886	0.370	0.628	-0.410	0.870	0.381	1	
UK (4)	-0.055	0.942	-0.609	-0.653	-0.886	0.019	0.798	0.511	0.703	0.058	0.328	0.417	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

Table 4.9. Cross-country correlations of absolute errors in life expectancy of women, 15 years ahead

	A	Dk	SF	F	FRG	Lux	Nl	N	S	CH
Austria (9)	1									
Denmark (5)	-0.920	1								
Finland (11)	0.484	-0.053	1							
France (6)	-0.210	0.649	0.802	1						
WGermany (6)	-0.608	0.336	-0.810	-0.590	1					
Luxembourg (5)	0.802	-0.735	0.638	-0.047	-0.574	1				
Netherlands (10)	0.208	-0.361	0.318	-0.339	0.443	0.291	1			
Norway (6)	0.765	-0.710	0.699	0.378	-0.928	0.718	-0.164	1		
Sweden (8)	0.562	-0.569	0.500	0.021	-0.461	0.751	0.263	0.501	1	
Switzerland (6)	0.895	-0.820	0.541	0.115	-0.799	0.679	-0.050	0.947	0.411	1

Note: Number of observations in parentheses; correlations that are significantly different from zero (two-sided; α=0.05) are marked.

In a number of cases, women in Western Germany and in Denmark correlate negatively with women in other countries. For Denmark, this is explained by the slow improvement in female life expectancy in recent decades. This led to small errors in Danish forecasts, in a time when life expectancy forecasts in other forecasts showed much larger errors. Indeed, the MAE for Denmark in Figure 4.1 (not indicated) is well below the average. For Germany the explanation is less clear.

There seems to be a group of countries that move together: significantly positive correlations are found in at least two of three cases (5, 10, or 15 years ahead) for *female* life expectancy errors in Austria, Finland, France, Norway, and Switzerland. These countries are among those with the highest female life expectancy in Europe, at least since 1980. Future life expectancies have been underestimated in all five. The average values of the significant correlations in this group of five countries are 0.80 (5 years ahead), 0.86 (10 years) and 0.92 (15 years). It would be natural to add Sweden to this group.

For *men*, the cross-country correlations are not stable. Denmark and Sweden correlate positively at five and ten years ahead, with estimated coefficients equal to 0.59 and 0.83, respectively. All other correlations are less systematic. Austria and Finland also correlate positively at ten and fifteen years ahead, but negatively at five years ahead. We conclude that there is no systematic cross-correlation in male life expectancy errors.

4.3.4 Correlation across the sexes

The empirical correlations across the sexes in absolute errors of life expectancy forecasts turned out to be 0.67, 0.74, and 0.67 at forecast durations of 5, 10, and 15 years, respectively. These values are highly significant, as they are based on 163, 116, and 76 observations, respectively.

4.3.5 Conclusion

Absolute errors in life expectancy forecasts increase by 0.2 years per year for forecast horizons 10-25 years, and somewhat slower for shorter durations. The patterns are very similar for men and women. The forecasts have been too low on average. The underprediction amounts to 1.0-1.3 and 3.2-3.4 years of life expectancy at forecast horizons of 10 and 20 years ahead, respectively. The distributions of the absolute errors are close to a normal one, in particular for men. The means and standard deviations increase with forecast lead time, as plotted in Figure 4.13. For example, there is a 20 per cent chance that the ten-year ahead life expectancy forecast for women will be wrong by at least 1.9 years. Absolute errors in life expectancy forecasts are correlated across sexes with a correlation coefficient of about 0.7. Cross-country correlations for men are not systematic, but for women in Austria, Finland, France, Norway, Sweden, and Switzerland they increase with forecast lead time, from 0.8 for five years ahead, to over 0.9 for 15 years ahead.

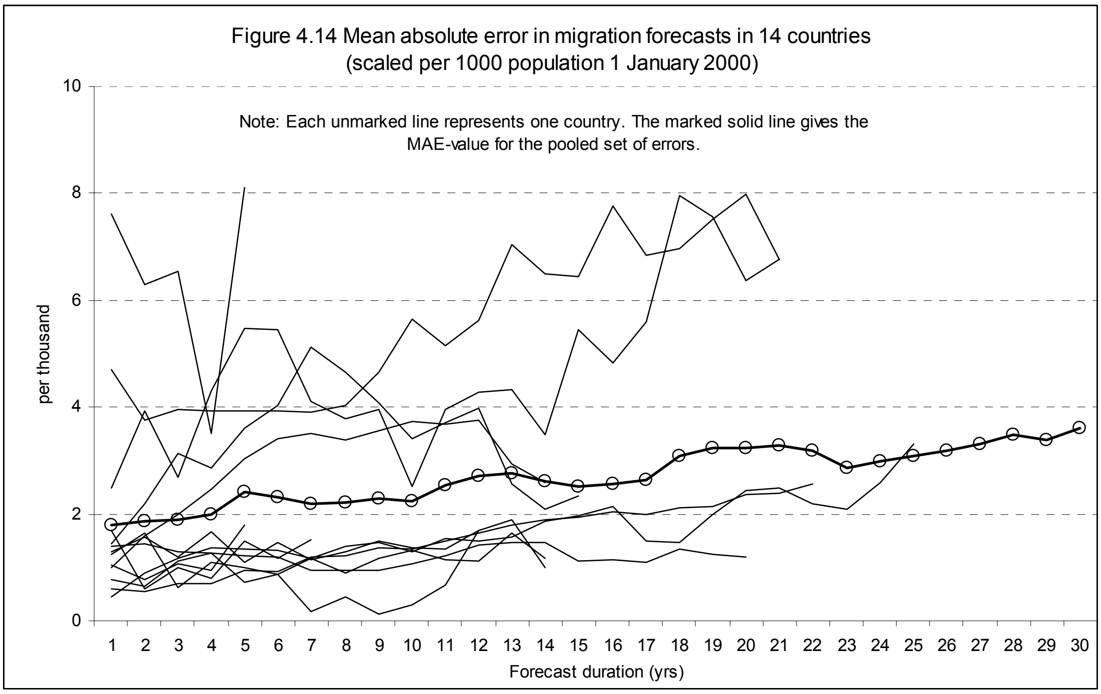
4.4 Net migration

Net migration is defined, for a certain year, as the number of immigrants minus the number of emigrants. To facilitate comparison across countries, we have scaled all national migration numbers by the national population size as of 1 January 2000. Thus the unit of measurement is "net migration per 1000 population". Averages and standard deviations were computed based on these scaled numbers.

Many historical projections have ignored migration. We have assumed that the implicit assumption for those projections was a net migration level of zero. In those cases, the signed error was simply equal to minus the observed level of net migration. The reason for this choice is the fact that many users will have interpreted these projections as proper forecasts, reflecting plausible future demographic developments.

4.4.1 Accuracy and bias

Figure 4.14 shows that the mean absolute migration error for all countries taken together rises slowly from just under two per thousand in the first forecast years, to about three to four per thousand at forecast durations of twenty years or more. For a country like Germany (82 million inhabitants in 2000) the regional average would imply an absolute error of 164,000-328,000 migrants. At the other end of the spectrum we find Luxembourg with a population of 436,000 persons in 2000, resulting in an error of only approximately 900-1700 migrants.[7]



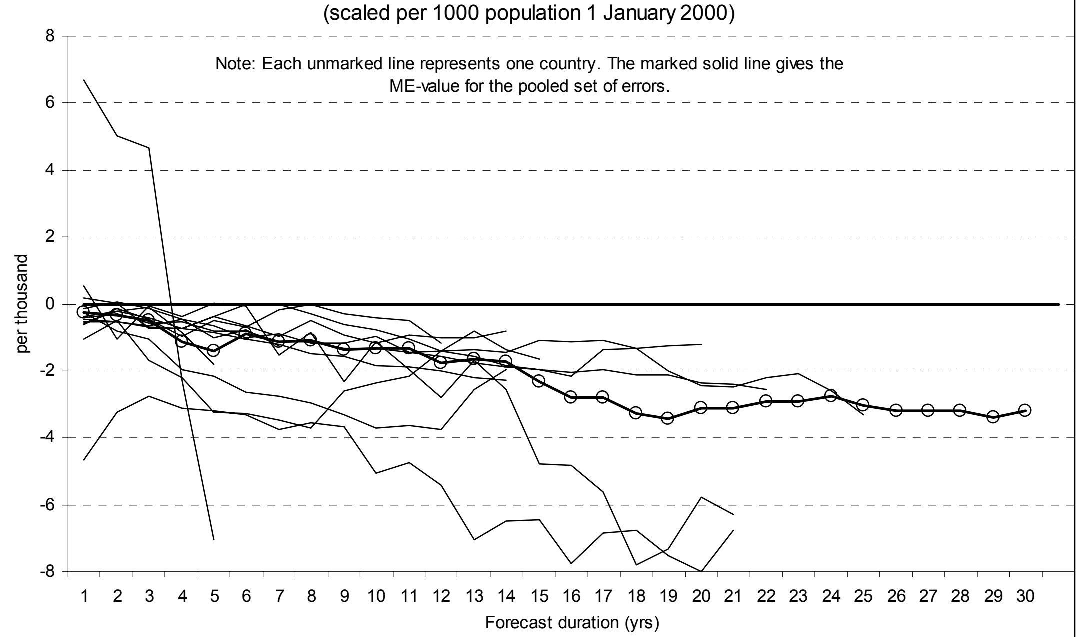
There are two distinct groups of countries. One group, consisting of Austria, West Germany, Luxembourg, Portugal, and Switzerland have mean errors well above the average for the pooled data set. The forecasts of Austria, Germany, and, to some extent, Switzerland were less accurate than the average, because of large immigration flows after the fall of the Berlin Wall in 1989. Luxembourg is a small country in which the level of migration in itself is high. Hence large migration forecast errors occur frequently. The large errors for Portugal are explained by the fact that migration statistics are not as reliable as those in other EEA countries. For instance, the 2002 issue of "Recent demographic developments in Europe" reports "observed" net migration to Portugal in multiples of 1000 for each year since 1992 (Council of Europe 2002). The 1998-issue reported net migration for the years 1991-1997 even in multiples of 5000. For the years 1993-1997, there is little agreement between the two time series of net migration numbers.

[7] Iceland is the smallest country in terms of population among the 18 (279,000 inhabitants on 1 January 2000); however, forecast errors for this country are not included in the data set in this Chapter – see also Appendix 1.

The other group, consisting of Belgium, Denmark, Finland, France, Italy, Netherlands, Norway, Sweden, and the UK show much smaller errors in their migration forecasts.

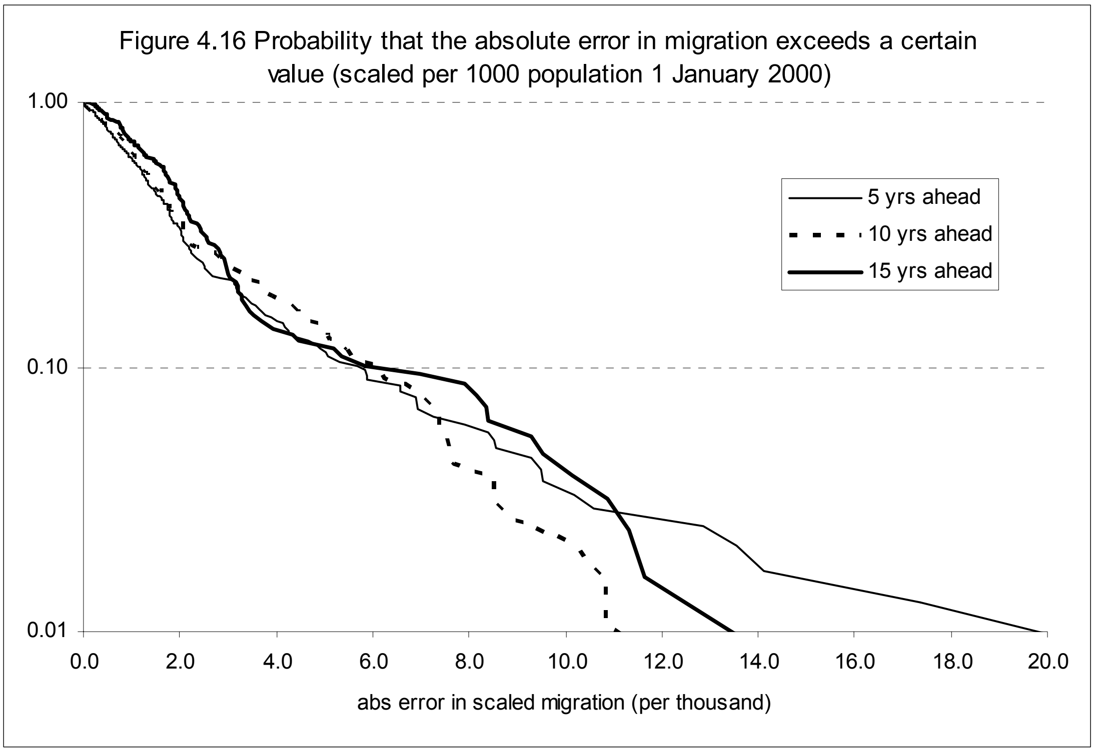
Migration has been consistently underestimated in historical forecasts. In a number of cases, the error is negative simply because migration was omitted, and the actual migration level was positive, see above. In other cases, the assumption was just too low. Figure 4.15 shows that the mean error in scaled migration falls regularly to minus 3 per thousand after 20 years and next it stabilizes around that level.

Figure 4.15 Mean error in migration forecasts in 14 countries
(scaled per 1000 population 1 January 2000)



4.4.2 Error distributions

There is a general belief among demographers that of the three components of population change at the national level, migration is the least predictable (Cruijsen and Keilman 1992, 332). The consequences for migration flows to industrialized countries brought about by economic, political, and demographic developments are much more difficult to foresee than those for births or deaths. This explains our finding that extreme migration errors are more likely than an exponential distribution would predict. Figure 4.16 plots the empirical probability distributions for absolute errors in migration assumptions at forecast durations of five, ten, and fifteen years ahead. The pattern is that of a straight line for probabilities between 10 and 100 per cent. This suggests an exponential distribution. However, in the range between 0 and 10 per cent, the empirical pattern deviates from this straight line, in particular for the short term. In other words, the probability for extreme errors is larger than an exponential distribution would predict.



Between 10 and 100 per cent, the three empirical curves in Figure 4.16 are rather close. At the same time, Figure 4.14 shows that the mean absolute values for the pooled set of errors for five, ten, and fifteen years ahead are also very close: 2.4, 2.2, and 2.5 per thousand, respectively[8]. These two facts combined make it unnecessary to compute normalized distributions.

4.4.3 Cross-country correlations

We checked whether large errors in one country tend to go together with large errors in other countries. However, no stable (across forecast duration) interpretable correlation patterns were found.

4.4.4 Conclusion

We have investigated forecast errors for scaled net migration in the 14 countries, that is, migration as a fraction of national population size (population 1 January 2000). Migration forecasts for Austria, West Germany, Luxembourg, Portugal, and Switzerland were clearly less accurate than the average for the

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[8] A linearly smoothed curve predicts values of 2.2, 2.4 and 2.7, respectively.

14 countries, for different reasons: large unforeseen immigration flows after the fall of the Berlin Wall (Germany, Austria), small population size with large migration flows that are inherently difficult to predict (Luxembourg), or simply inaccurate migration statistics (Portugal). Migration has been consistently underpredicted in historical forecasts. The mean error in scaled migration falls regularly to minus 3 per thousand after 20 years, and for longer forecast durations it stabilizes around that level. The error distribution of the absolute error in scaled migration is exponential, except for low probabilities. For probabilities less than ten per cent the errors are more extreme than an exponential distribution would predict. There seems to be no systematic pattern in cross-country correlations.

5. Predicted errors

5.1 Time series models

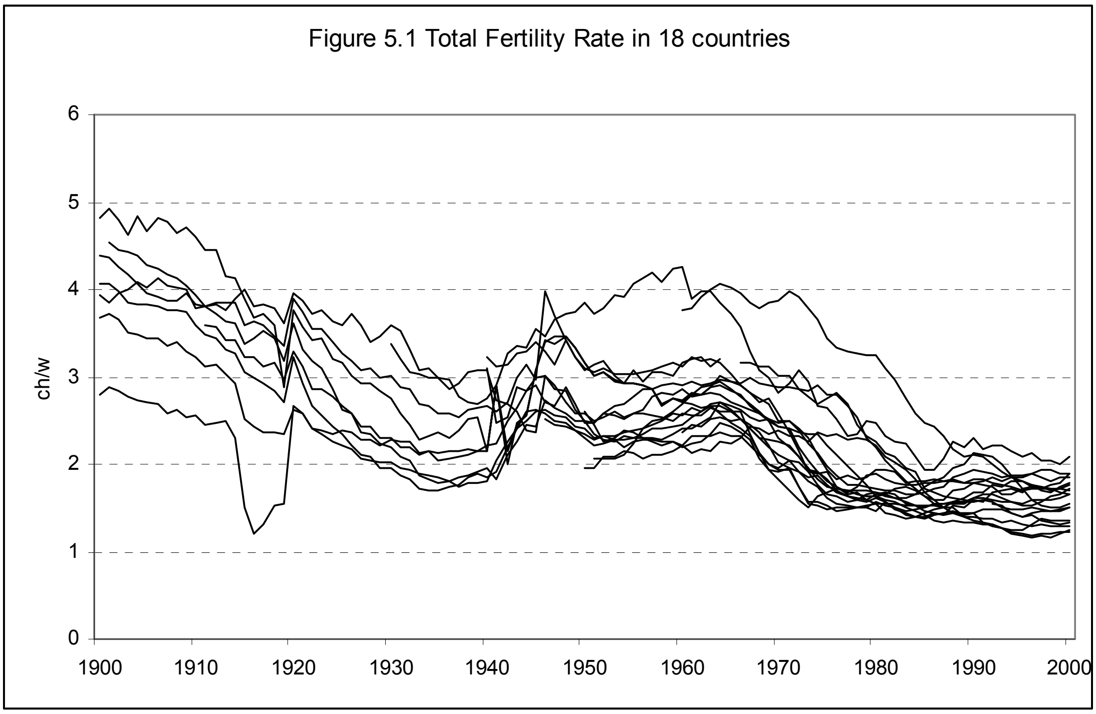
A number of recent stochastic population forecasts have used some form of time series analysis for one or more key indicators, when assessing the expected accuracy of predicted values for these indicators. Time series models were used to predict the TFR in stochastic forecasts prepared for the US (Lee and Tuljapurkar 1994), Finland (Alho 1998), the Netherlands (De Beer and Alders 1999), and Norway (Keilman et al. 2001). One attractive property of time series models is that they not only give a prediction of future values of the variable in question, but also allow us to compute prediction intervals.

A common finding with TFR- time series in industrialized countries is that these are non-stationary. As a consequence, long run prediction intervals, when unchecked, may become extremely wide. Therefore, adjustments are necessary. For instance, Lee and Tuljapurkar (1994) introduced upper and lower bounds to the TFR by a generalized logit-transformation. This way they constrained TFR-predictions to between 0 and 4 children per woman on average. Alho (1998) found that time-series based TFR-prediction intervals 50 years ahead were 15 per cent wider than those obtained based on the volatility in the historical TFR-observations, and he decided to rely on the latter type of intervals. De Beer and Alders (1999) initially found a 95-per cent prediction interval for the TFR in 2050 equal to [0.6 – 2.8] based on time series models. Next, an analysis of fertility by birth order led them to suggest that an interval of [1.1-2.3] would be more appropriate. Keilman et al. (2001) simulated predicted TFR-values, and rejected TFR-simulations that would fall outside the interval [0.5 – 4] in any year up to 2050.

In this chapter we present time series models for the TFR, the life expectancy, and net immigration in the 18 countries. An important aim of this modelling exercise was to obtain prediction intervals that were not excessively wide, even on the long run.

5.2 TFR

Figure 5.1 plots the Total Fertility Rate for the 18 countries. The data sources are listed in Table 3.1.



TFR-plots for distinct countries will be presented later in this chapter. Here the interest is in the overall trend. The countries show a similar pattern in the TFR, which reflects the demographic transition, followed by the effects of the economic recession in the 1930s and the baby boom in the 1950s and 1960s. In the 20th century, many countries show a tendency towards lower variability in the TFR. Major events, such as the First World War, and the occurrence of the Spanish Influenza in 1918/1919 are clearly reflected in the series for most countries.

An important question is how much of the data one should use in the modelling. Several issues are at stake here. First, Box and Jenkins (1970, 18) suggest at least 50 observations for ARIMA-type of time series models, although annual models (in contrast to monthly time series) probably need somewhat shorter series. Second, the quality of the data is better for the 20th century than for earlier years. This is particularly true for the denominators of the fertility rates, i.e. the annual numbers of women by single years of age. Third, one may question the relevance of data as long back as the mid-1800s. Current childbearing behaviour is very different from that of women in the 19th century. Fourth, our ultimate goal is to compute long-term predictions of some 50 years ahead, which necessitates a long series.

The ultimate choice is necessarily a subjective one, which includes a good deal of judgement and arbitrariness. We believe that we strike a reasonable balance between conflicting goals by selecting the 20th century as the basis for our models. An analysis based on the last 50 years, say, would be unfortunate: it would include the baby boom of the 1950s and early 1960s, but not the low fertility of the 1930s, to which the boom was a reaction, at least partly. A base period stretching back into the 19th century would be hampered by problems of data quality, and it would also unrealistically assume that the demographic behaviour over such a long period could be captured by one and the same model. In a sensitivity analysis for Denmark, Finland, Norway, and Sweden we also experimented with base period 1945-2000. For Norway and Finland we found 95 per cent prediction intervals that were smaller (by 1.4 and 0.5 children per woman on average, respectively) than those that we have accepted for further analysis (see Figure 5.2 below). For Denmark and Sweden they were larger (by 0.8 and 1.2 children per woman, respectively).

We have used observed annual TFR data for the period 1900 to 2000 for the following countries (see Table 3.1): Finland, France, Iceland, Netherlands (series starting in 1901), Norway, Sweden, and Switzerland. To increase comparability across countries, we used a TFR time series starting in 1900 for Denmark and for England and Wales as well, in spite of the fact that TFR-observations start in 1911 only. Danish and English TFR-values for the years 1900-1910 were estimated on the basis of observed Crude Birth Rate values for these countries; see Appendix 2. Thus we have estimated time series models for the TFR based on a whole century of data for nine countries. Time series models for the remaining nine countries were estimated based on annual TFR-data for the years 1950-200. This was the case for Austria, Belgium, Germany, Greece, Ireland, Italy, Luxembourg, Portugal, and Spain.

Traditional time series models of the ARIMA type assume homoscedasticity, i.e. constant residual variance. Given the tendency towards less variability in the TFR in recent decades, such traditional models could not be used. The Autoregressive Conditional Heteroscedastic (ARCH) model introduced in Engle (1982) combines time-varying variance levels with an autoregressive process. This model and its generalizations (generalized, integrated, and exponential ARCH models, to name a few) have gained popularity in recent years (Bollerslev 1986). The model has already proven useful in analysing economic phenomena such as inflation rates, volatility in macroeconomic variables, and foreign exchange markets; see Bollerslev (1986) for a review. Application to demographic time series is less widespread. Yet, given the varying levels of volatility in the TFR during the 20th century, an ARCH-type of model is an obvious candidate.

Let Z_t be the logarithm of the TFR in year t. Then the model is

(5.1)
$$\begin{aligned} Z_t &= C + \phi Z_{t-1} + v_t + \eta_1 U_{1,t} + \eta_2 U_{2,t} + \eta_3 U_{3,t} + \eta_4 U_{4,t} + \eta_5 U_{5,t} \\ v_t &= \psi_1 v_{t-1} + \psi_2 v_{t-2} + \ldots + \psi_m v_{t-m} + \varepsilon_t \\ \varepsilon_t &= \left(\sqrt{h_t}\right) e_t \\ h_t &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \end{aligned}$$

where $e_t \sim$ N(0,1). This is the AR(m)-ARCH(q) model. The outliers caused by the two world wars and by the Spanish Influenza are handled by between two (Denmark, Iceland, Netherlands, Sweden) and five (Switzerland) dummy variables $U_{i,t}$. In addition we have $\omega > 0$ and $\alpha_i \geq 0$.

The maximum number of terms m included in the autoregressive expression of v_t was set equal to 10, but few of the ψ -estimates turned out to be significantly different from zero. In practice, m was restricted to 2. Similarly, estimates for α_i suggested that the order (q) of the CH-part of the model could be restricted to one. We will first present results for the nine countries with long TFR-time series (section 5.3.1), and next results for all 18 countries based on observed TFR data for the period 1950-2000 (section 5.3.2).

5.2.1 A time series model for the TFR in nine countries, 1900-2000

Table 5.1 specifies the AR(m)-ARCH(q) models with dummy variables for the nine countries with long TFR-time series.

Table 5.1. ARCH models with dummy variables for Z_t = log(TFR) in nine countries, 1900-2000

	Dummy variables	AR: v_t =	CH: h_t =
Denmark	1920, 1942	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Finland	1919,1920,1940,1941	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
France	1915, 1920, 1940, 1946	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Iceland	1920, 1942	ε_t	
Netherlands	1920, 1946	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Norway	1915, 1919, 1920, 1946	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Sweden	1920, [1942-45][1]	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Switzerland	1915, 1920, 1941, 1942, 1943	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
England and Wales	[1915-17][1], [1919-20][1], [1942-44][1]	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$

Note 1: one common dummy variable for each year in the period.

Table 5.2 lists the estimation results, with t-values in parentheses. None of the C-estimates were significantly different from zero. Yet the constant was retained in the model: trial calculations without a constant resulted in implausibly low point predictions for the TFR in 2050, ranging from a high 1.66 children per woman in Iceland, to a low 1.21 children per woman in Sweden. The predictions for 2050 based on the model with constant are in the interval from 1.95 (Netherlands) to 1.34 (Switzerland). We tested the residuals for normality, independence, and constant variance.

Table 5.2. Parameter estimation results for the models in Table 5.1; t-values in parentheses

	η_1	η_2	η_3	η_4	η_5	C	ϕ	ψ_1	ψ_2	ω	α_1
Denmark	0.149	0.090				0.016	0.970	0.367		9E-4	0.005
	(2.1)	(2.8)				(0.9)	(48.2)	(3.1)		(6.1)	(0.0)
Finland	-0.208	0.291	-0.109	0.156		0.005	0.981		0.214	7E-4	0.708
	(-2.1)	(1.7)	(-6.4)	(7.5)		(0.5)	(98.0)		(2.4)	(4.2)	(3.5)
France	-0.295	0.536	-0.045	0.247		0.018	0.970	0.240		5E-4	0.576
	(-13.6)	(7.1)	(-3.1)	(3.4)		(0.8)	(39.2)	(1.9)		(4.3)	(2.7)
Iceland	0.098	0.123				-0.003	0.995				
	(2.4)	(3.0)				(-0.2)	(59.1)				
Netherlands	0.178	0.333				0.023	0.967		0.347	3E-4	0.604
	(5.4)	(7.3)				(2.0)	(86.6)		(3.9)	(3.6)	(2.9)
Norway	-0.068	-0.053	0.183	0.102		0.011	0.975		0.477	5E-4	0.363
	(-2.3)	(-2.4)	(8.8)	(2.3)		(0.6)	(48.1)		(5.41)	(5.2)	(2.5)
Sweden	0.226	0.075				0.022	0.959	0.579		7E-4	0.000
	(10.4)	(4.5)				(1.2)	(48.7)	(6.5)		(6.1)	(0.0)
Switzerland	-0.121	0.139	0.108	0.079	0.037	0.003	0.983		0.560	4E-4	0.000
	(-2.1)	(12.2)	(3.5)	(5.2)	(2.1)	(0.2)	(57.0)		(5.9)	(5.7)	(0.0)
England and Wales	-0.062	0.233	0.091			0.004	0.982	0.250		7E-4	0.481
	(-6.9)	(11.2)	(6.0)			(0.2)	(42.4)	(2.6)		(5.8)	(2.5)

Note the high α_1-estimates for Finland, France, and the Netherlands. They reflect the large variability in the data in these three countries. The α_1-estimates for Denmark, Sweden, and Switzerland are close to zero – however, omitting the ARCH(1) part from the model (which essentially boils down to assuming constant variance) would lead to rejecting the null hypothesis of constant variance in the residuals, as trial calculations showed.

Note also that all ϕ-estimates are close to one. Indeed, with the exception of the Netherlands, they do not differ significantly from one. For this reason we could have used a more parsimonious model by selecting ϕ = 1. However, this would have increased the width of the prediction intervals, in particular the long rune ones. For example, the 95% prediction interval for the TFR of Norway 2050 equals

- [0.77-2.96] with $\phi = \hat{\phi} = 0.975\,(\text{s.e.e.} = 0.0203)$,
- [0.85-4.23] with ϕ = 1 (and all other parameter estimates unchanged).

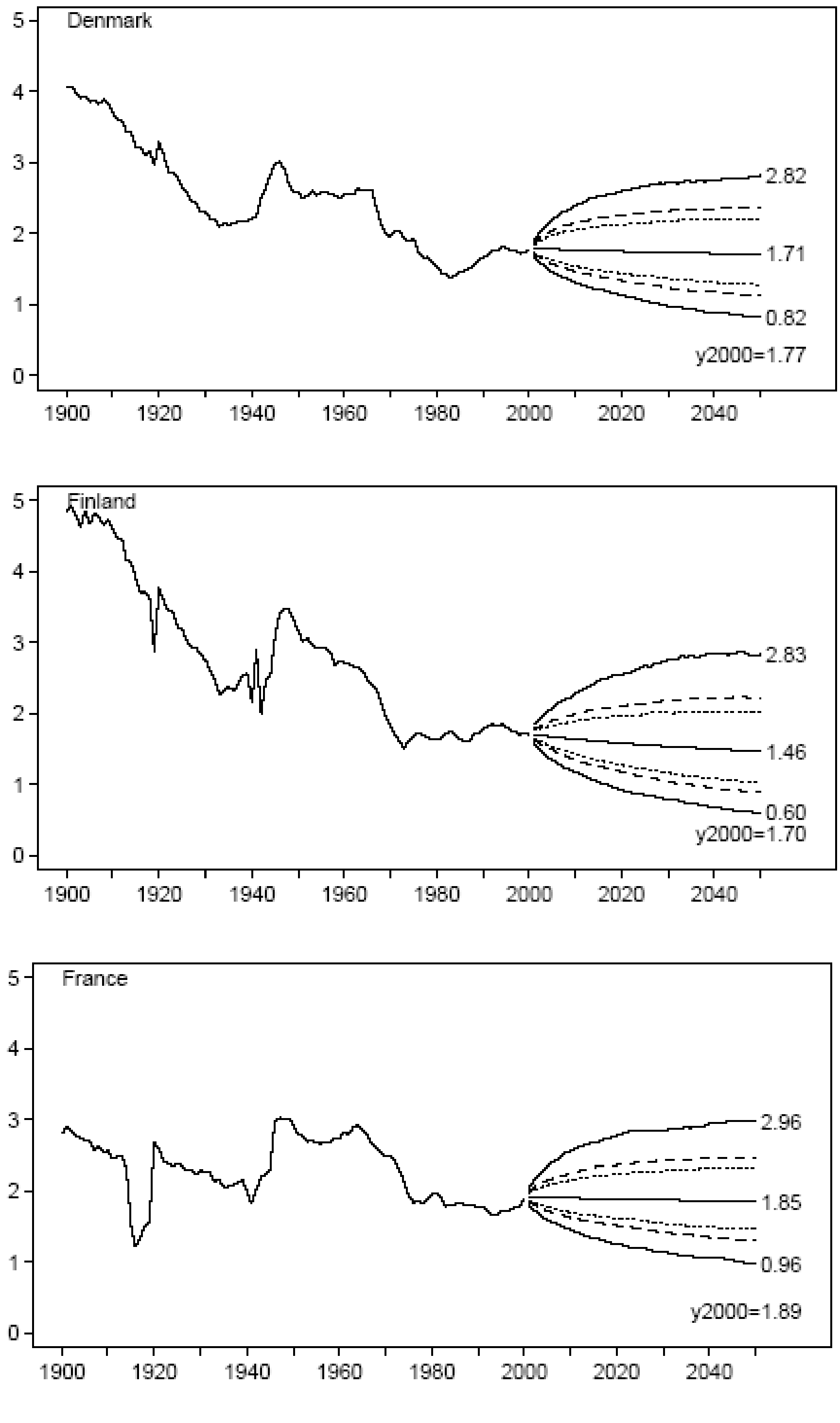
The overall impression is that model (5.1) is a useful device to capture the TFR-trends in the nine countries during the past century. We used the model to compute prediction intervals for the future TFR up to 2050. Since we cannot be certain that the estimated coefficients are equal to the real ones, we used simulation to obtain these intervals. In each of the 5000 simulation runs, parameter values were drawn from a multivariate normal distribution, with expectation equal to the parameter estimates in Table 5.2, and with corresponding covariance matrix as estimated earlier. The possibility that a pandemic as bad as the Spanish Flu, or a war with consequences as catastrophic as WWI or WWII could occur during the prediction period, was included in these simulations. For each of the two dummy variables, we first drew a random number from the binomial distribution with a probability of "catastrophe" equal to 1/101. Next, the starting year for the catastrophe was determined on the basis of a random draw from the uniform distribution on the interval [2001, 2050]. Finally, the appropriate η-value was drawn from its estimated distribution.

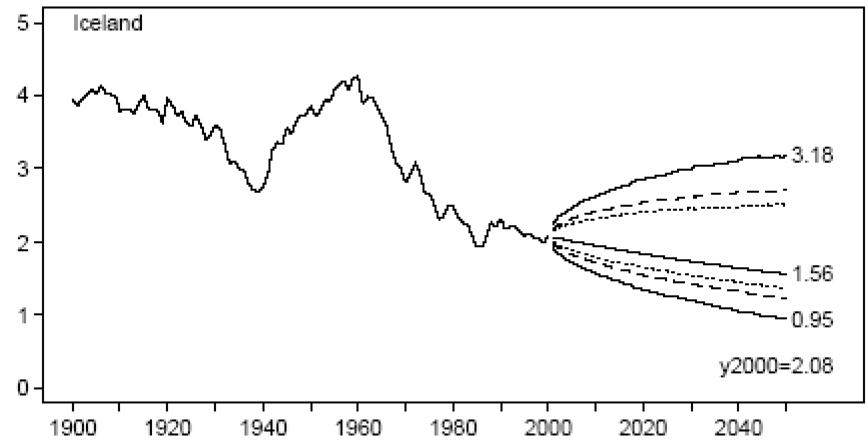
The simulations resulted in a few ϕ-values equal to or larger than one. These were rejected, and ϕ was redrawn until 5000 admissible values had been obtained for each country.

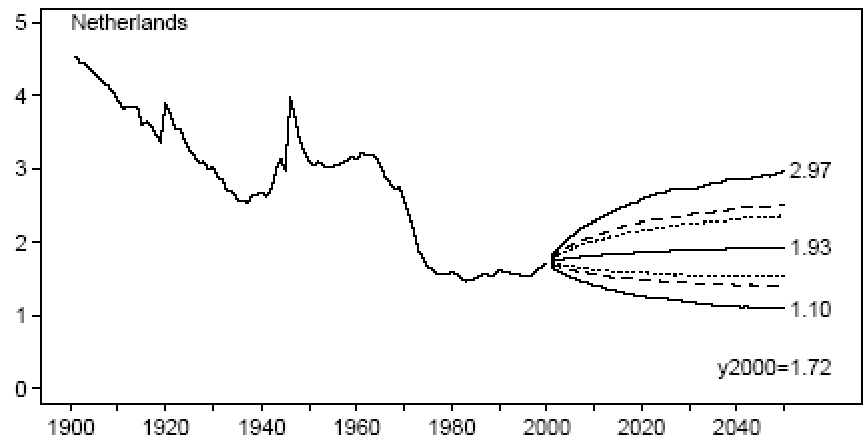
Figure 5.2 plots point forecasts (i.e. expected values) and prediction intervals up to 2050. Long-range (50 years ahead) 95 per cent prediction intervals are between 1.7 (Switzerland) and 2.3 (Iceland) children per woman wide, and ten years ahead the width is only 0.7 to 1.2 children per woman.

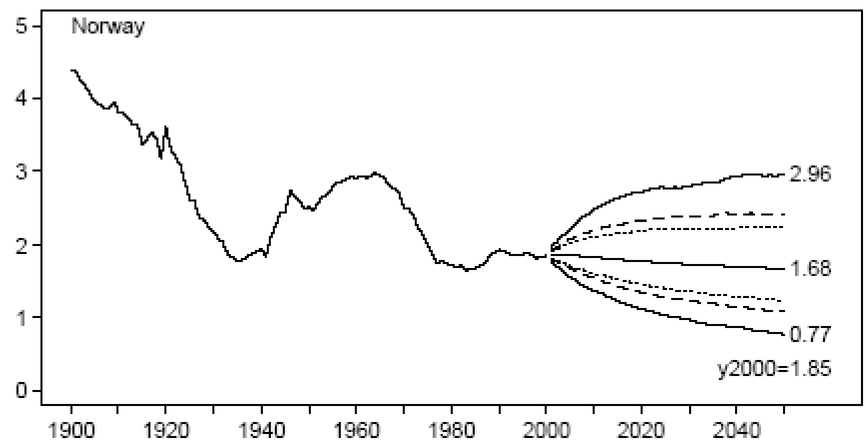
The time-series model predicts an expected TFR-value in 2050 in the range 1.3 (Switzerland) to 1.9 (Netherlands) children per woman. Although these values are not entirely unrealistic, the result for Finland (1.5) is much lower than that in official population forecasts, for instance forecasts prepared by Statistics Finland (1.75, see Council of Europe 2001) or by the United Nations (1.85, see http://www.un.org/esa/population/publications/wpp2002/ wpp2002annextables.PDF). In this report, we focus primarily on the width of the predictive distribution, much less on its central tendency. In an actual fertility prediction, all long-term point predictions would have to be examined critically, for example by inspecting the mean number of children born to women in successive birth generations.

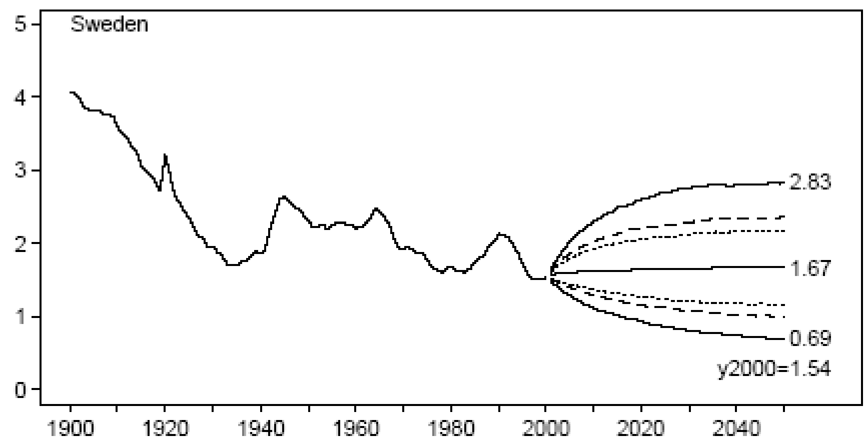
Figure 5.2. Forecasts and 67%, 80%, and 95% prediction intervals for the TFR. Data 1900-2000. Observed TFR-values for the year 2000 are indicated as "y2000"

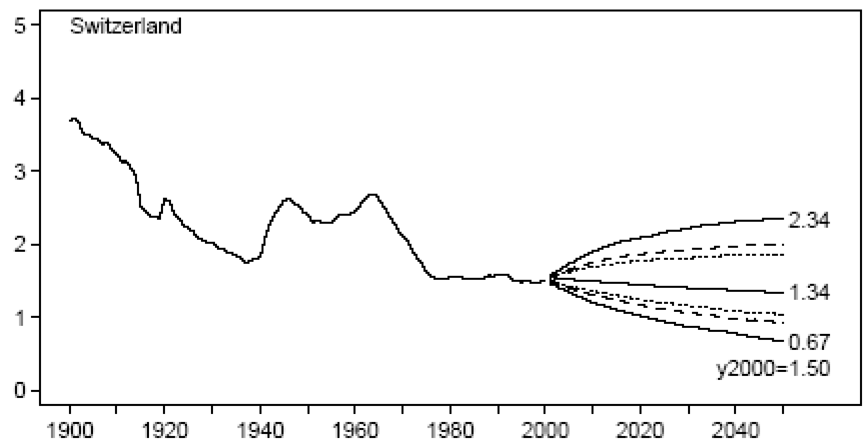


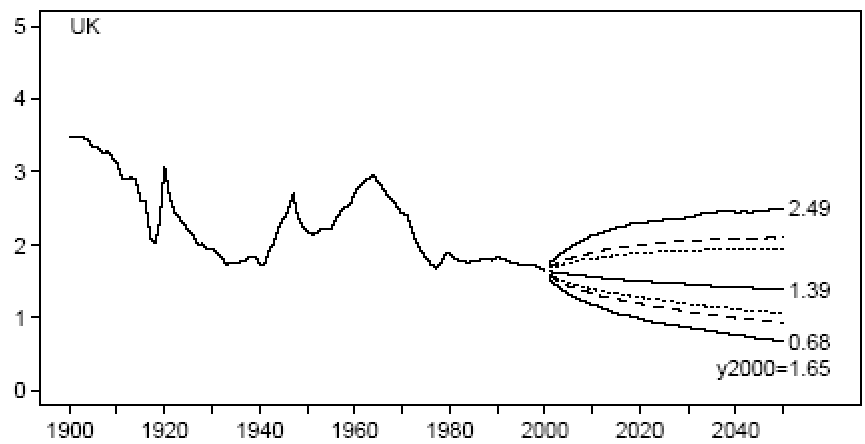

Iceland
3.18
1.56
0.95
y2000=2.08


Netherlands
2.97
1.93
1.10
y2000=1.72


Norway
2.96
1.68
0.77
y2000=1.85







In order to assess the robustness of the prediction intervals obtained in the previous section, we have experimented with several other time series models for Z_t:

- a pure AR(m)-model
- an AR(m)-CH(1) model
- an AR(m)-model with dummy variables

The experiments were restricted to the cases of Denmark, Finland, Norway, and Sweden. The results can be summarized as follows.

Fitting an AR(m)-model or an AR(m)-CH(1) model
A purely autoregressive model for Z_t (with maximum lags equal to 5 for Norway, 2 for Denmark, 1 for Sweden, and 2 for Finland) indeed indicated non-constant variance: using a Portmanteau Q-test and a Lagrange Multiplier (LM-) test, a hypothesis of homoscedastic residuals had to be rejected at the five per cent level for Norway, Sweden, and Finland. For Denmark, such a hypothesis was not rejected at the ten per cent level. When we introduced a CH(1)-part to the model in order to account for heteroscedastic residuals, the situation improved considerably for Denmark and Sweden, but for Norway and Finland, there were still some signs of heteroscedasticity at lag 1. A Kolmogorov-Smirnov test hypothesis for normality could only be accepted for Norway and Denmark (at the five per cent level), not for the other two countries.

Fitting an AR(m)-model with dummy variables
Can the non-constant residual variance be captured by introducing dummy variables to the AR(m)-model? This is the case for three countries: Finland was the exception. For the other three countries, dummy variables for the periods around 1918, 1944, and 1970 were introduced. A hypothesis of homoscedastic residuals at all lags could not be rejected (5%) for all three countries. The 95% prediction intervals in 2050 turned out to be 2.7 (Norway), 3.0 (Denmark), and 3.5 (Sweden) children per woman wide - much wider than the intervals in Figure 5.2.

Based on these sensitivity tests we conclude that the ARCH-model in expression (5.1) gives a useful and reliable description of the development in the TFR in the four countries in the previous century. Given the similarity of trends, we assume that this is also the case for the other countries.

5.2.2 A time series model for the TFR in 18 countries, 1950-2000
In this section we present results for 18 countries based on observed TFR for the years 1950-2000.

The period 1950-2000 is characterized by much less variability in the TFR than the first half of the century. The post-WWII baby boom is the only major irregularity. In contrast, the years 1900-1950 witnessed a sharp overall decline in TFR, the depression of the 1930s, the recovery of the TFR by the end of the 1930s, and the peak fertility levels in 1946 and 1947. This explains our finding why an ARCH-type of model was not necessary except for three cases. To begin with, the ARCH-model in expression (5.1) was estimated for all 18 countries, but the α-estimates turned out to be significant in none of these cases. Next, the CH-part of the model, represented by the expression for h_t, was removed, the model was re-estimated, and the residuals were tested with respect to constant variance. The hypothesis of constant variance could not be rejected in 15 of the 18 countries. The exceptions were Belgium, Germany, and England and Wales. Dummy variables were not necessary for any country.

Table 5.3 specifies the models, and Table 5.4 gives the parameter estimates. The constant was omitted from the expression for Z_t for Greece, Ireland, Italy, Portugal, and Spain. Trial calculations resulted in negative estimates for the constant, caused by the recent strong fall in fertility in these five countries. In turn, the negative constant led to unrealistically low TFR predictions in 2050: between 0.6 (Greece) and 1.1 (Ireland) children per woman. A model without a constant gave more realistic TFR-predictions: between 1.1 (Greece, Italy, and Spain) and 1.5 (Ireland) children per woman in 2050. All models as specified and estimated in Tables 5.3 and 5.4 had independent, normally distributed residuals, with constant variance.

Table 5.3. Time series models for Z_t = log(TFR) in 18 countries, 1950-2000

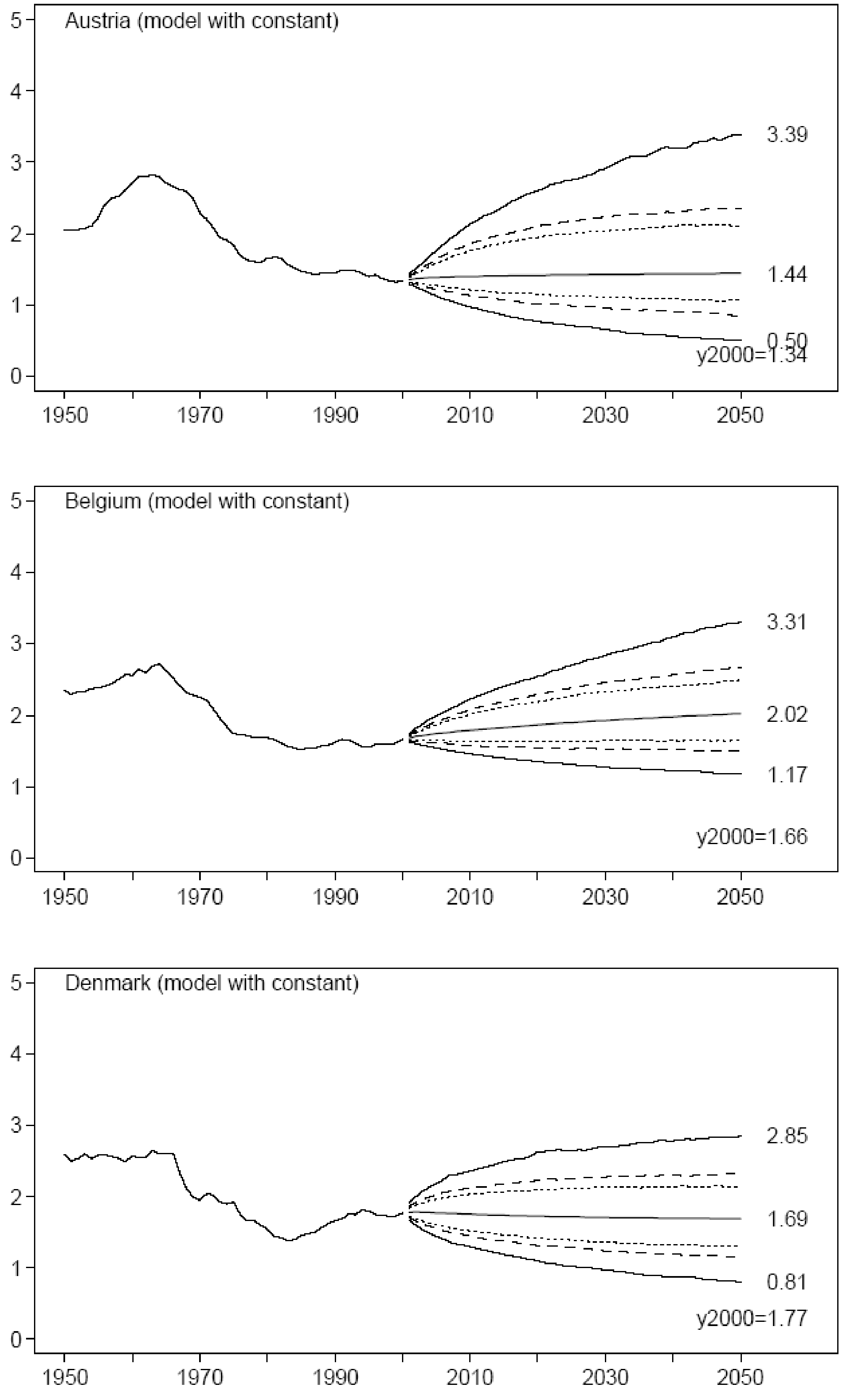
	$Z_t =$	$v_t =$	$h_t =$
Austria	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Belgium	$C+\phi Z_{t-1}+v_t$	$\psi_2 v_{t-2}+\varepsilon_t$	$\omega+\alpha_1\varepsilon_{t-1}^2$
Denmark	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Finland	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
France	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Germany	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\psi_2 v_{t-2}+\varepsilon_t$	$\omega+\alpha_1\varepsilon_{t-1}^2$
Greece	$\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Iceland	$C+\phi Z_{t-1}+v_t$	ε_t	
Ireland	$\phi Z_{t-1}+v_t$	$\psi_2 v_{t-2}+\varepsilon_t$	
Italy	$\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Luxembourg	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Netherlands	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Norway	$C+\phi Z_{t-1}+v_t$	$\psi_2 v_{t-2}+\varepsilon_t$	
Portugal	$\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Spain	$\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Sweden	$C+\phi Z_{t-1}+v_t$	$\psi_1 v_{t-1}+\varepsilon_t$	
Switzerland	$C+\phi Z_{t-1}+v_t$	$\psi_2 v_{t-2}+\varepsilon_t$	
England and Wales	$C+\phi Z_{t-1}+v_t$	$\psi_2 v_{t-2}+\varepsilon_t$	$\omega+\alpha_1\varepsilon_{t-1}^2$

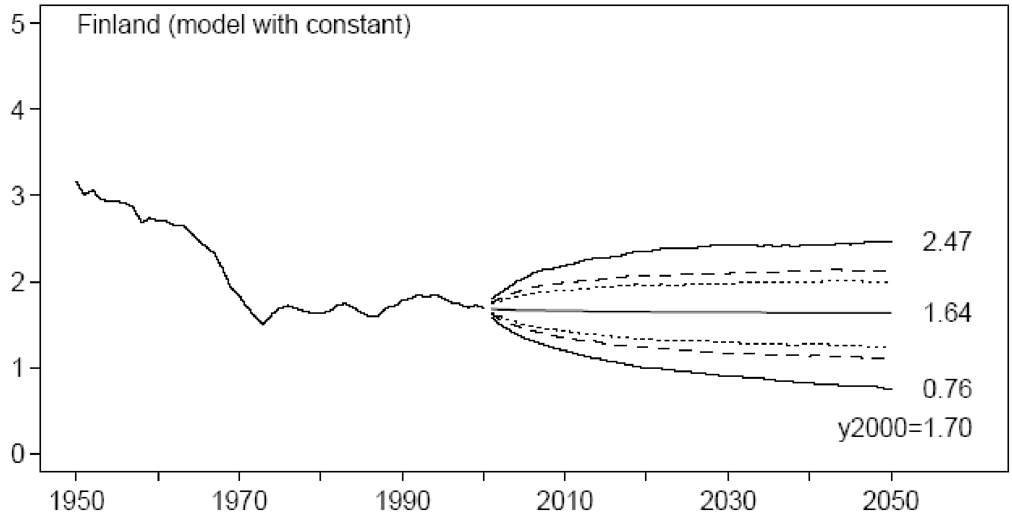
Table 5.4 Parameter estimation results for the models in Table 5.3; t-values in parentheses

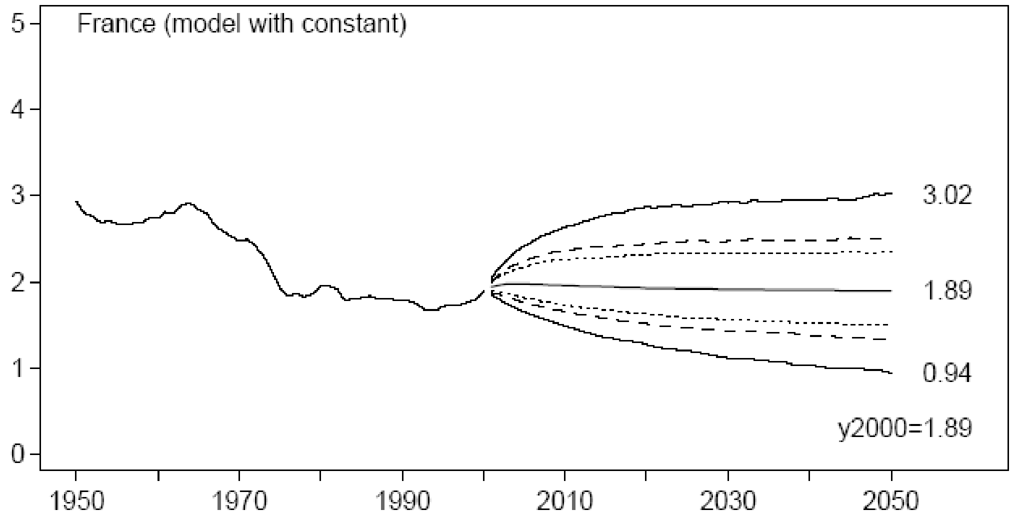
	C	ϕ	ψ_1	ψ_2	ω	α_1
Austria	0.011	0.971	0.690			
	(0.3)	(18.4)	(5.1)			
Belgium	0.017	0.979		0.433	3E-4	0.395
	(0.7)	(32.9)		(2.4)	(2.7)	(1.0)
Denmark	0.021	0.959	0.383			
	(0.8)	(26.0)	(2.7)			
Finland	0.029	0.941	0.434			
	(1.2)	(29.2)	(3.2)			
France	0.033	0.948	0.508			
	(1.0)	(24.5)	(3.5)			
Germany	0.014	0.964	0.349	0.271	9E-4	0.007
	(0.4)	(16.0)	(2.0)	(1.4)	(3.6)	(0.04)
Greece	0	0.981	0.259			
		(115)	(1.9)			
Iceland	0.001	0.988				
	(0.04)	(41.4)				
Ireland	0	0.991	0.359			
		(155)	(2.7)			
Italy	0	0.977	0.619			
		(79.6)	(5.4)			
Luxembourg	0.031	0.933	0.283			
	(1.2)	(20.8)	(1.7)			
Netherlands	0.024	0.954	0.678			
	(0.8)	(24.6)	(5.4)			
Norway	0.014	0.975		0.528		
	(0.5)	(25.6)		(3.7)		
Portugal	0	0.986	0.251			
		(145)	(1.8)			
Spain	0	0.986	0.612			
		(83.1)	(5.3)			
Sweden	0.088	0.856	0.688			
	(1.0)	(6.2)	(3.4)			
Switzerland	0.011	0.969		0.620		
	(0.4)	(25.7)		(4.5)		
England and Wales	0.009	0.976		0.416	7E-4	0.163
	(0.3)	(22.3)		(3.1)	(5.2)	(1.0)

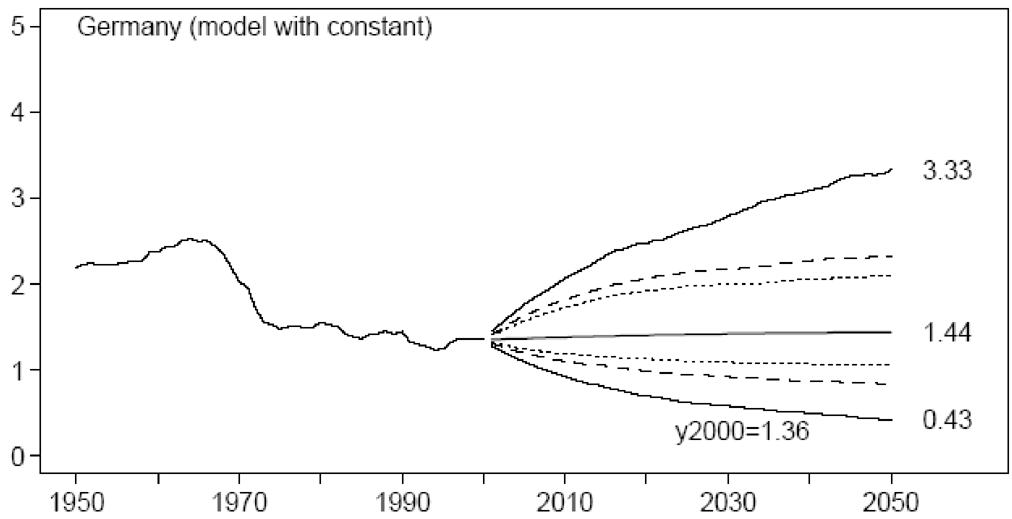
Figure 5.3 plots expected values and prediction intervals up to 2050. The width of long-range 95 per cent prediction intervals varies a great deal, ranging from a low 1.0 (Greece) and 1.3 (Portugal), to a high 2.9 (Austria, Germany) and 3.4 (Sweden). A possible explanation for the large uncertainty in Sweden is the baby boom around 1990, caused by a change in legislation for maternity leave. Greece had a baby boom in the 1960s and 1970s that was much less pronounced than that in most other countries. Fertility fell more or less regularly after 1950, and hence the narrow interval in 2050. Ten years ahead 95 per cent intervals are 0.6 (Greece) to 1.2 (Sweden) children per woman wide.

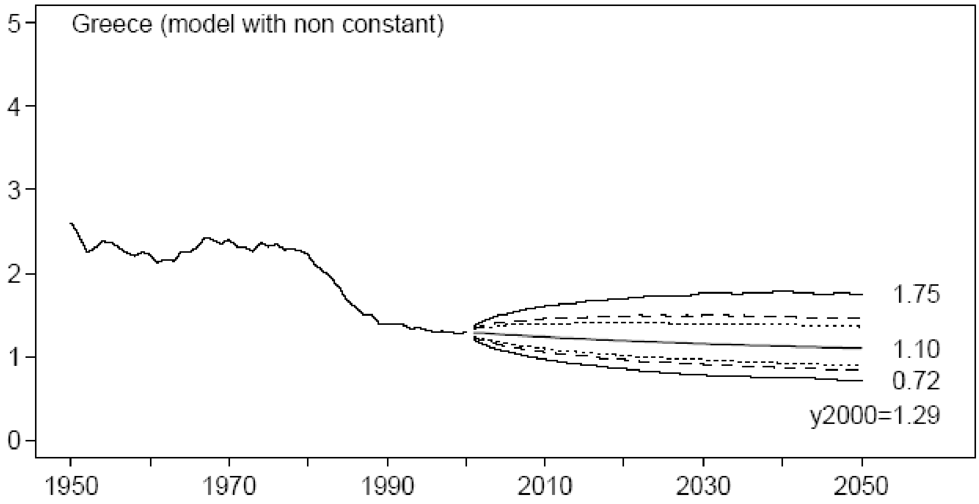
Figure 5.3 Forecasts and 67%, 80%, and 95% prediction intervals for the TFR. Data 1950-2000

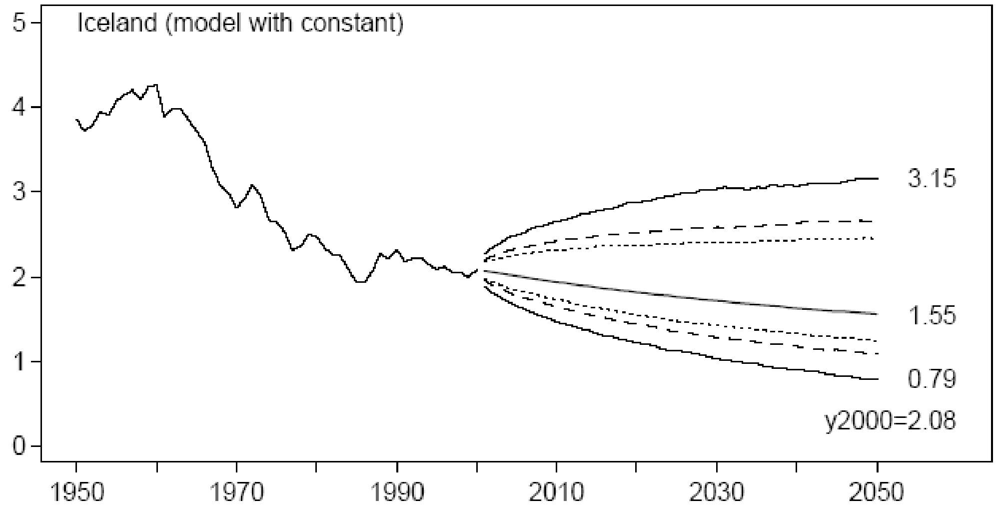


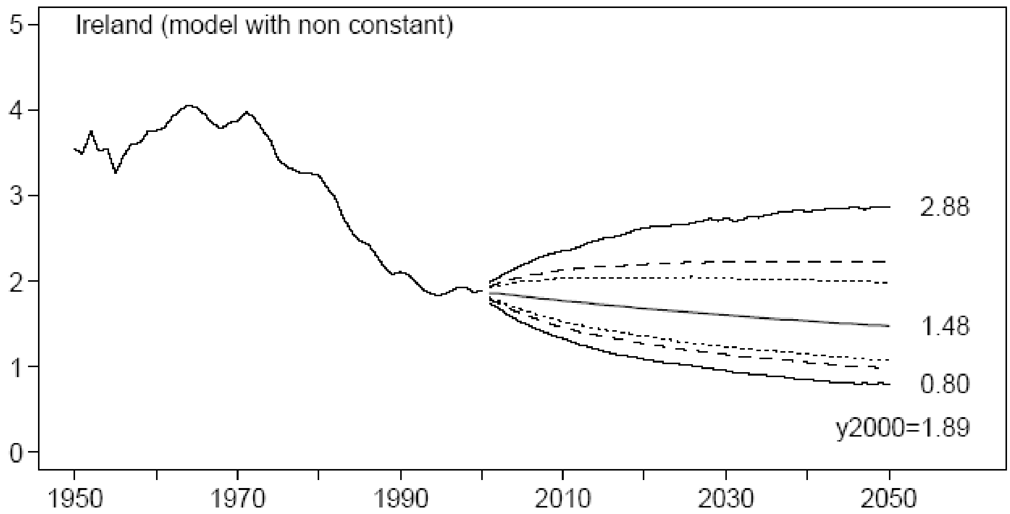

Finland (model with constant)
2.47
1.64
0.76
y2000=1.70

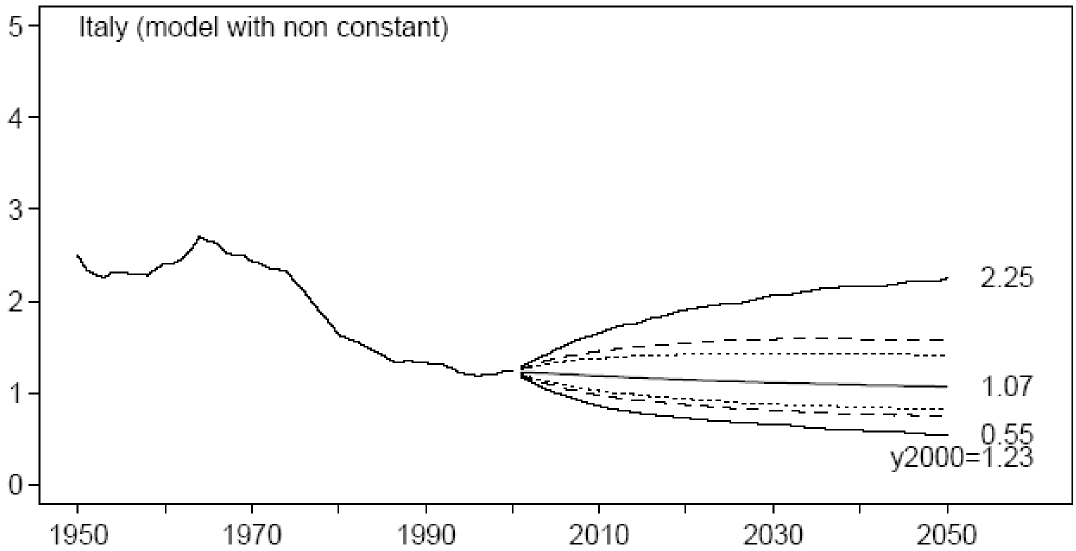

France (model with constant)
3.02
1.89
0.94
y2000=1.89

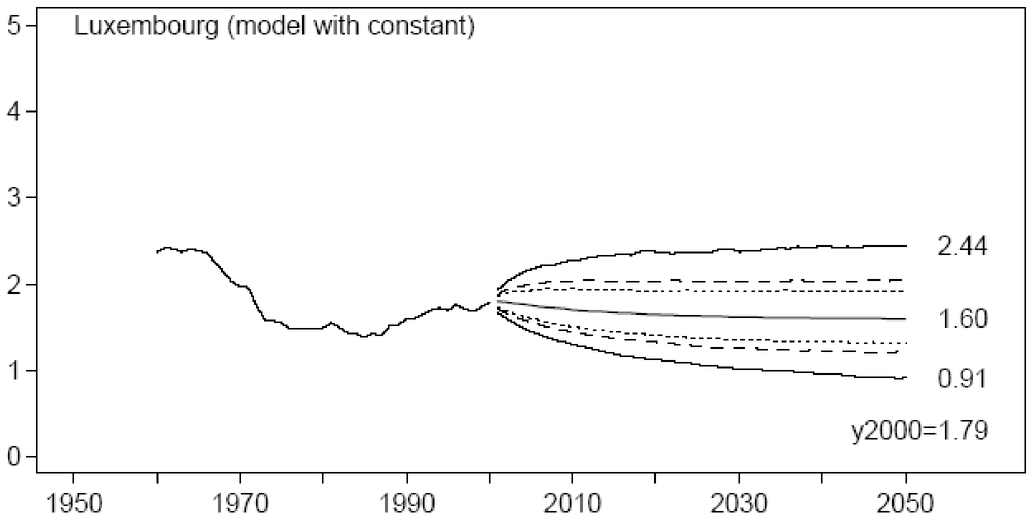

Germany (model with constant)
3.33
1.44
0.43
y2000=1.36

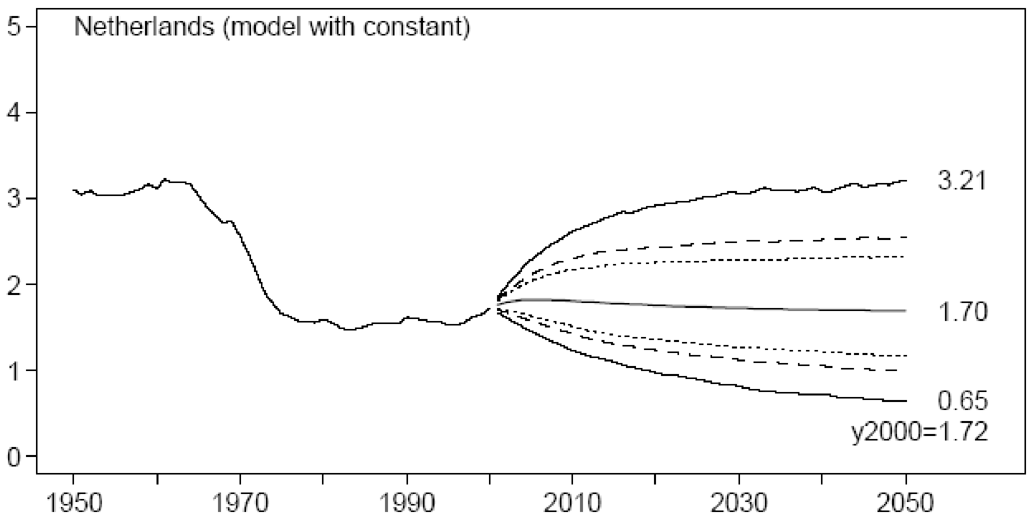

Greece (model with non constant)
5
4
3
2
1
0
1.75
1.10
0.72
y2000=1.29
1950
1970
1990
2010
2030
2050

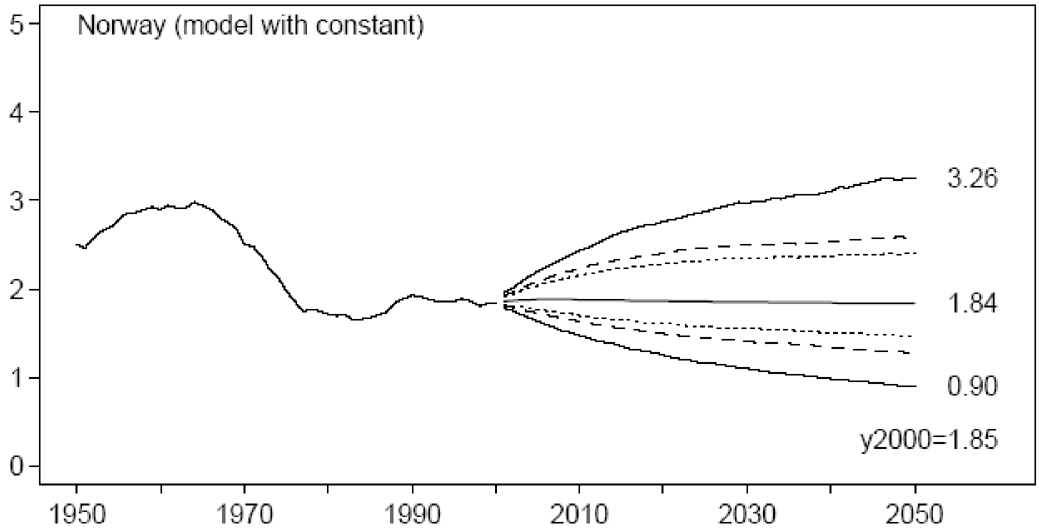

Iceland (model with constant)
5
4
3
2
1
0
3.15
1.55
0.79
y2000=2.08
1950
1970
1990
2010
2030
2050

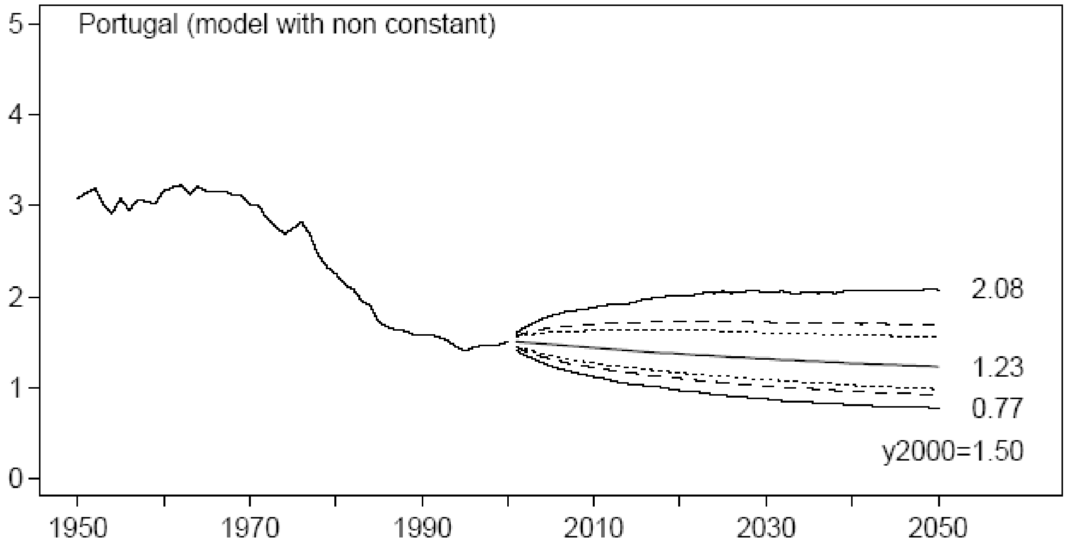

Ireland (model with non constant)
5
4
3
2
1
0
2.88
1.48
0.80
y2000=1.89
1950
1970
1990
2010
2030
2050

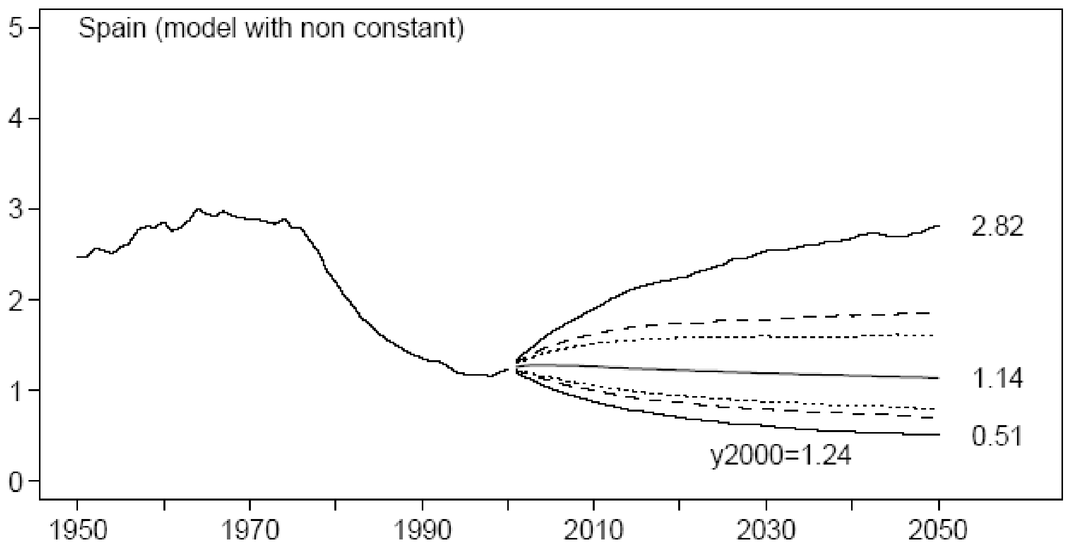

Italy (model with non constant)
2.25
1.07
0.55
y2000=1.23
0
1
2
3
4
5
1950
1970
1990
2010
2030
2050

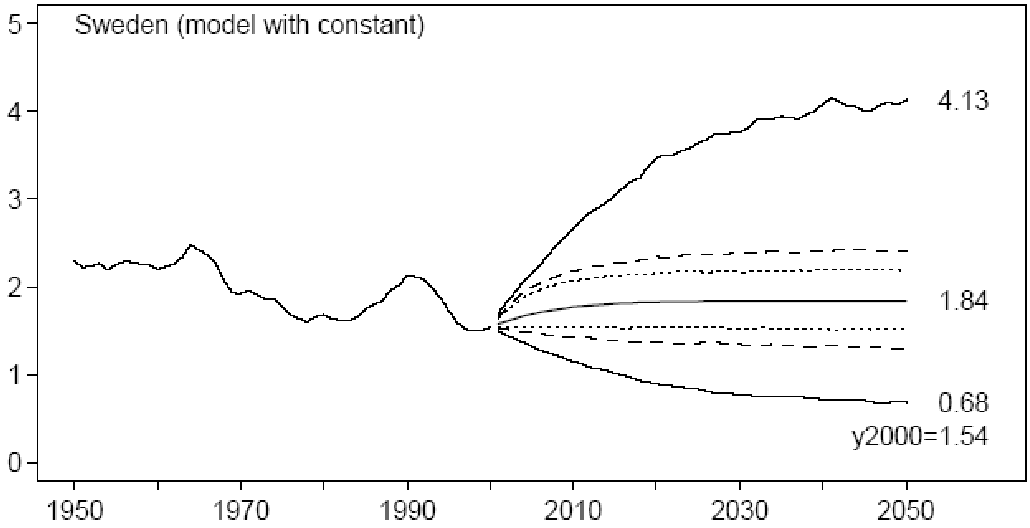

Luxembourg (model with constant)
2.44
1.60
0.91
y2000=1.79
0
1
2
3
4
5
1950
1970
1990
2010
2030
2050


Netherlands (model with constant)
3.21
1.70
0.65
y2000=1.72
0
1
2
3
4
5
1950
1970
1990
2010
2030
2050

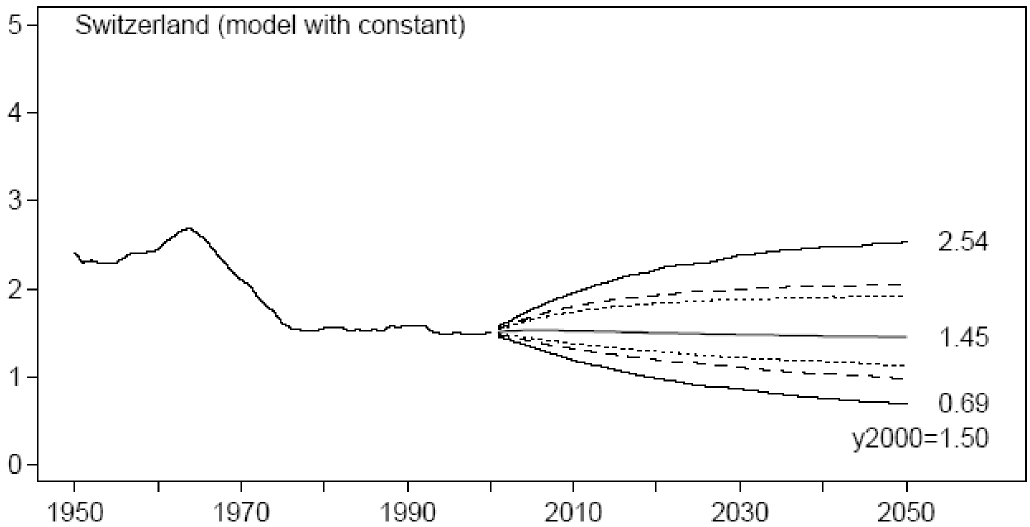

Norway (model with constant)
3.26
1.84
0.90
y2000=1.85
5
4
3
2
1
0
1950
1970
1990
2010
2030
2050


Portugal (model with non constant)
2.08
1.23
0.77
y2000=1.50
5
4
3
2
1
0
1950
1970
1990
2010
2030
2050

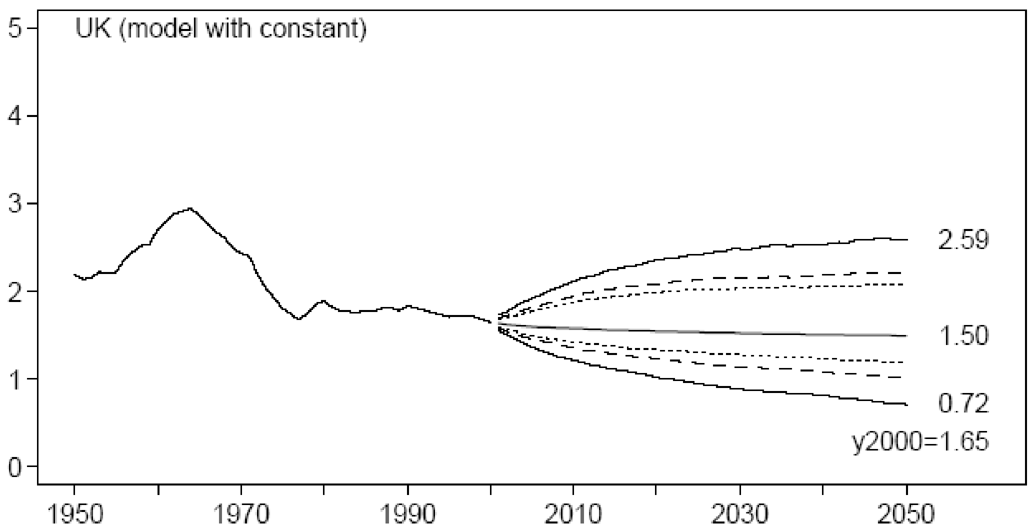

Spain (model with non constant)
2.82
1.14
0.51
y2000=1.24
5
4
3
2
1
0
1950
1970
1990
2010
2030
2050



Sweden (model with constant)
5
4
3
2
1
0
4.13
1.84
0.68
y2000=1.54
1950
1970
1990
2010
2030
2050

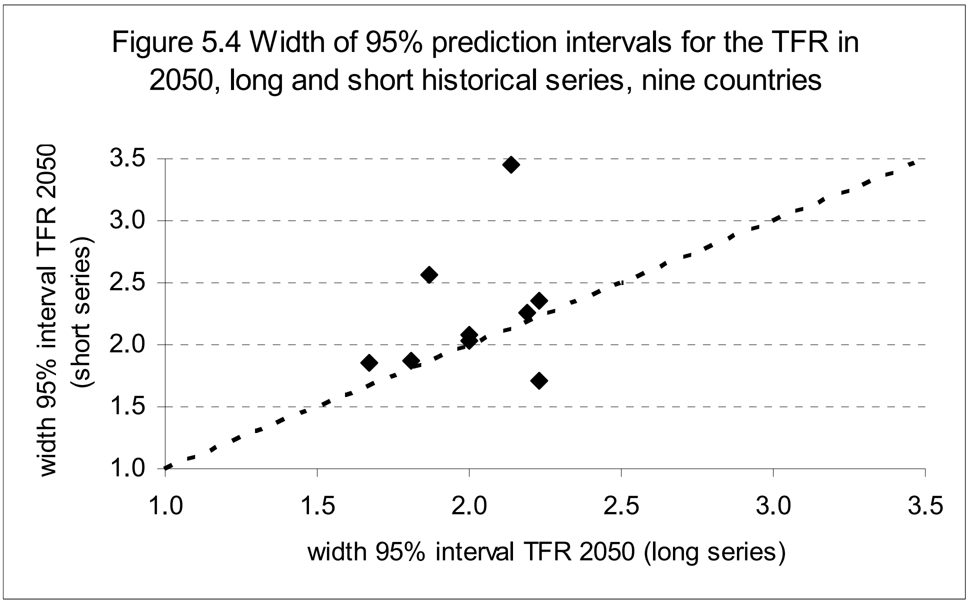


Switzerland (model with constant)
5
4
3
2
1
0
2.54
1.45
0.69
y2000=1.50
1950
1970
1990
2010
2030
2050



UK (model with constant)
5
4
3
2
1
0
2.59
1.50
0.72
y2000=1.65
1950
1970
1990
2010
2030
2050

How do the prediction intervals derived from time series models based on short series (1950-2000) compare to those based on long series (1900-2000)? Figure 5.4 plots the width of the 95 per cent intervals in 2050 for the nine countries for which we have a long series, i.e. Denmark, Finland, France, Iceland, Netherlands, Norway, Sweden, Switzerland, and England and Wales. We see that with one exception (Finland), the intervals based on short series are at least as wide, if not wider, than those computed on the basis of long series. The difference is large for Sweden and the Netherlands.



5.2.3 Cross-country correlations

We have computed correlations across countries between residuals from the time series models. Table 5.5 shows the 9x9 correlation matrix for countries for which we have a long series of observations (1900-2000), and Table 5.6 contains the 18x18 correlation matrix for all countries, based on data for the period 1950-2000.

Table 5.5. Correlations for residuals across countries, TFR time series models, data 1900-2000

	Dk	SF	N	S	F	Is	Nl	CH	E&W
Denmark	1								
Finland	0.291	1							
Norway	0.392	0.092	1						
Sweden	0.409	0.352	0.349	1					
France	0.262	0.055	0.247	0.270	1				
Iceland	-0.036	0.036	0.029	0.076	0.048	1			
Netherlands	0.024	-0.043	0.255	0.100	0.030	0.146	1		
Switzerland	0.375	0.280	0.311	0.357	0.503	0.072	0.141	1	
England & Wales	0.357	0.162	0.337	0.350	0.313	0.060	0.251	0.363	1

Table 5.6. Correlations for residuals across countries, TFR time series models, data 1950-2000

	A	B	Dk	SF	F	D	EL	Is	IRL	I	Lux	Nl	N	P	E	S	CH
Austria	1																
Belgium	0.333	1															
Denmark	0.247	0.086	1														
Finland	0.250	0.148	0.230	1													
France	0.408	0.540	0.192	-0.051	1												
Germany	0.419	0.437	0.105	0.378	0.291	1											
Greece	0.034	0.047	-0.105	0.250	-0.128	0.081	1										
Iceland	0.205	0.166	0.348	0.126	0.102	0.117	0.069	1									
Ireland	0.134	0.430	0.087	-0.032	0.440	0.242	-0.011	0.119	1								
Italy	0.136	0.271	0.122	0.283	0.226	0.180	0.239	0.263	0.231	1							
Luxembourg	0.525	0.476	0.293	0.400	0.342	0.531	0.104	0.124	0.106	0.151	1						
Netherlands	0.325	0.444	0.032	0.173	0.331	0.488	-0.040	0.048	0.305	0.084	0.253	1					
Norway	0.438	0.335	0.405	0.148	0.289	0.230	-0.107	0.436	0.140	0.272	0.327	0.294	1				
Portugal	0.161	0.321	0.005	0.095	0.095	0.250	0.103	0.153	0.176	0.162	0.279	0.17	-0.076	1			
Spain	0.152	0.262	0.181	0.211	0.096	0.195	0.247	0.121	0.336	0.440	-0.008	0.154	0.055	0.579	1		
Sweden	0.262	0.373	0.382	0.348	0.337	0.247	0.112	0.248	0.128	0.236	0.363	0.227	0.343	0.094	0.260	1	
Switzerland	0.535	0.633	0.299	0.240	0.573	0.364	-0.078	0.206	0.366	0.239	0.474	0.230	0.508	0.064	0.249	0.451	1
England & Wales	0.347	0.435	0.277	0.103	0.277	0.329	-0.001	0.284	0.400	0.122	0.303	0.331	0.467	-0.014	0.038	0.390	0.592

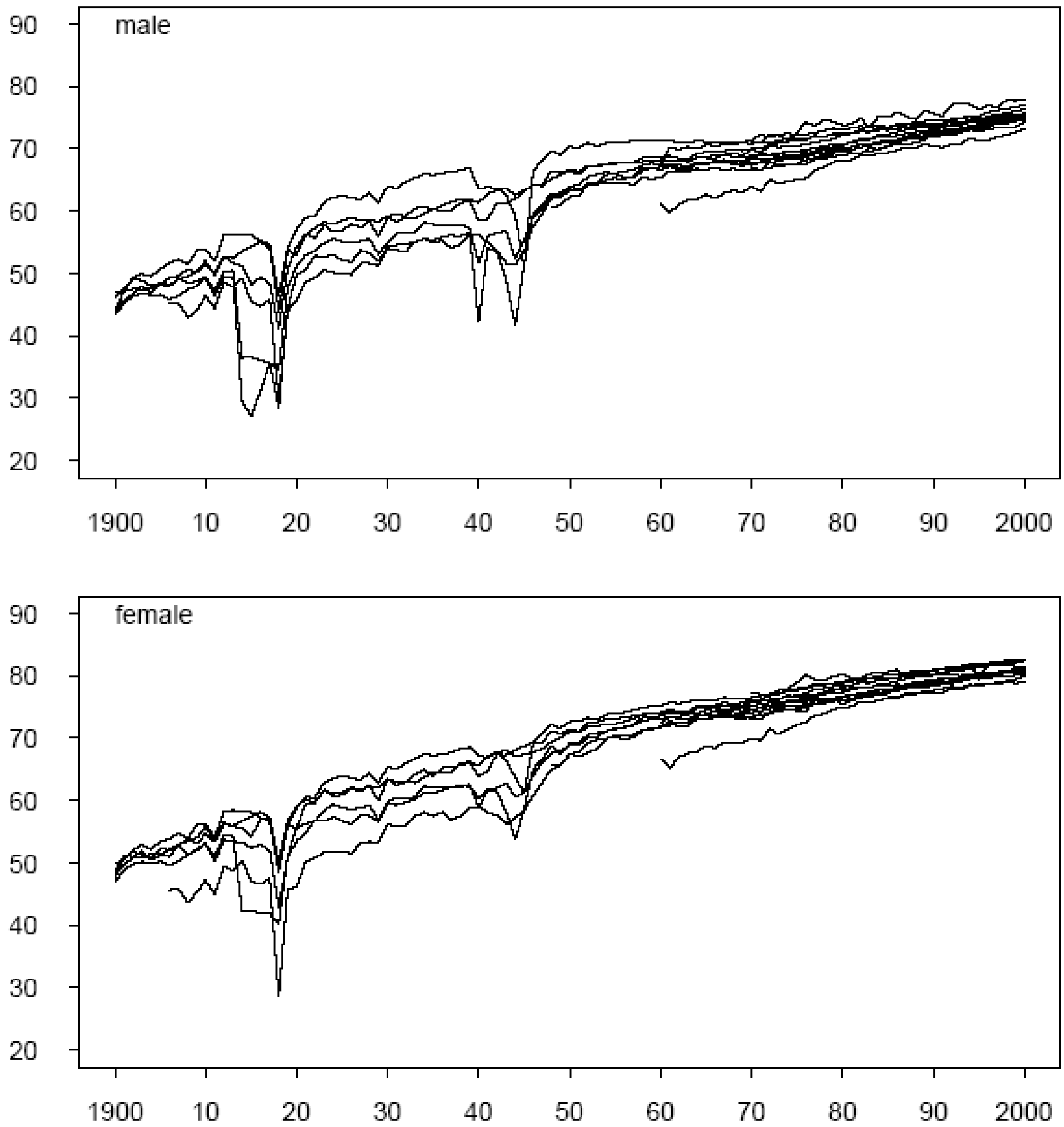
The critical level with 101 observations is 0.196 (α=0.05, two-sided). Thus we see that Iceland is unrelated to the other countries. Otherwise the countries are moderately correlated (although Netherlands only with Norway and England & Wales), with an average correlation coefficient equal to 0.33. The correlations in the observed data (not shown here) were 0.45 or higher – in most cases larger than 0.7. Thus the time series model has removed a large part of the original correlation between the countries.

For the shorter time series (51 data points, data for 1950-2000), the critical value of the correlation is 0.27. Table 5.6 shows a large number of moderately strong correlations. The average value of those correlations that exceed the critical value is 0.39.

5.3 The life expectancy at birth

Figure 5.5 plots the life expectancy at birth for men and women in the 18 countries. Major interruptions caused by the First World War, the Spanish Influenza, and the Second World War are clearly visible. The time series show less variability in the second half of the twentieth century than in the first half.

Figure 5.5. Life expectancy at birth in 18 countries



As Table 3.1 shows, the time series for the life expectancy at birth vary a great deal in length across countries. The longest series is that for France, which starts in 1806. At the other end of the spectrum we find Ireland: an uninterrupted series with annual life expectancy values exists since 1985 only, although there are observations for 1960-61, 1970-71, and 1980-81 also (probably in connection with population censuses held in those years).

We have attempted to construct time series models based on time series of maximum length. However, for reasons of data quality and mortality behaviour, the time series could not be "too" long.

In practice, we started with the year 1900. Time series with annual observations since that year for male and female life expectancies are available for seven countries: Belgium, England and Wales, France, Netherlands, Norway, Sweden, and Switzerland. But also for Denmark, Finland, Italy, and Luxembourg, rather long series are available, these series varying in length from 100 data points for Luxembourg to 60 data points for Finland. These eleven countries constitute a group for which we have estimated Generalized Autoregressive Conditional Heteroscedasticity (GARCH)-models, i.e. models that are slightly more general than the ARCH-models employed for fertility in the previous section. A second set of estimates was computed for all 18 countries, based on data for the period 1960-2000. All models were estimated for men and women separately.

5.3.1 A time series model for the life expectancy in eleven countries, 1900-2000

Let $e_{0,t}$ represent the life expectancy at birth in year t, and define $\nabla e_{0,t}$ as $e_{0,t}$ - $e_{0,t-1}$. The model is

$$
\begin{aligned}
&\nabla e_{0,t} = C + \phi \nabla e_{0,t-1} + v_t + \sum_j \eta_j U_{j,t} \\
&v_t = \psi_1 v_{t-1} + \psi_2 v_{t-2} + \ldots + \psi_m v_{t-m} + \varepsilon_t \\
(5.2) \qquad &\varepsilon_t = \left(\sqrt{h_t}\right) e_t, \text{ where } e_t \sim N(0,1), \text{ and} \\
&h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \gamma_j h_{t-j}
\end{aligned}
$$

This is the AR(m)-GARCH(p,q) regression model. Tables 5.7 and 5.8 specify the models for men and women for the eleven countries with long life expectancy time series.

Table 5.7. ARCH-GARCH models for differences in life expectancy at birth, data 1900-2000

		$\nabla e_{0,t} = C +$ dummies $+$	$v_t =$	$h_t =$
Belgium	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_2 \varepsilon_{t-2}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Denmark	men	$\phi\nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Finland	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
France	men	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_2 \varepsilon_{t-2}^2$
	women	v_t	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$
Italy	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \psi_4 v_{t-4} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \psi_4 v_{t-4} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Luxembourg	men	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Netherlands	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \psi_3 v_{t-3} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Norway	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Sweden	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Switzerland	men	$\phi\nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
England & Wales	men	$\phi\nabla e_{0,t-1} + v_t$	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi\nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$

As the model estimates in Table 5.9 show, we have retained a number of non-significant estimates. The reason is that these insignificant parameters guaranteed independence, normality, and constant variance of the residuals, similar to the case for fertility.

Table 5.8. Dummy variables for the models in Table 5.7

	men	women
Belgium	1914 1919 1949 1941 [1943-44] [1945-46]	1914 1919 [1943-44] [1945-46]
Denmark	none	none
Finland	1944 1945 1946 [1950-51]	1944 1945 1946 [1950-52]
France	1914 [1919-20] 1940 1941 [1943-44] [1945-46]	1918 1919 1940 [1943-44] [1945-46]
Italy	1918 1946	1918 1919
Luxembourg	1915 [1917-18] 1920 [1943-44] 1946	1918 [1919-20] [1944-45] 1946
Netherlands	1918 1919 1940 [1943-44] 1945 1946	1918 [1919-20] [1943-44] 1946
Norway	1918 [1919-20] [1940-41] [1945-46]	1918 [1919-20] [1944-45]
Sweden	1918 1919 1920	1918 1919 1920 1941
Switzerland	1918 [1919-21]	1918 [1919-20]
England & Wales	1915 1918 1919 1940 1942	1915 1918 1940 1942

Table 5.9. Estimates of model (5.2). Data 1900-2000

	Men		Women	
	Estimate	t-value	Estimate	t-value
		Belgium		
C	0.176	6.6	0.178	6.4
η_1	-13.200	-13.2	-11.536	-6.2
η_2	12.673	2.5	11.379	7.7
η_3	-5.727	-1.9	-1.034	-2.0
η_4	4.729	2.8	1.696	2.3
η_5	-2.296	-8.3		
η_6	3.008	2.0		
φ	0.229	4.7	0.198	3.0
ψ_1	-0.525	-6.6	-0.393	-2.7
ω	1.E-08	255.7	1.E-04	0.0
α_2	0.140	2.3	0.179	2.7
γ_1	0.849	18.0	0.820	17.3
		Denmark		
C	0.130	3.7	0.161	4.6
φ	-0.137	-1.0	-0.155	-1.7
ω	0.007	0.9	0.013	1.3
α_1	0.355	1.7	0.448	2.1
γ_1	0.627	4.5	0.537	3.4
		Finland		
C	0.150	4.6	0.167	5.4
η_1	-13.109	-16.9	-3.125	-4.6
η_2	17.971	19.8	2.853	5.1
η_3	-2.610	-2.6	1.975	5.5
η_4	1.634	6.3	0.620	5.0
φ	0.341	6.1	0.268	4.8
ψ_1	-0.791	-8.1	-0.614	-5.7
ω	0.095	4.6	1.E-08	49.2
α_1	0.000	0.0	0.113	0.9
γ_1	0.001	0.5	0.875	7.3

Table 5.9 (continued)

		France		
C	0.256	21.9	0.248	7.7
η_1	-22.564	-21.0	-10.393	-7.6
η_2	9.067	3.2	9.093	11.6
η_3	-15.079	-74.0	-3.038	-20.8
η_4	12.000	28.5	-4.096	-3.4
η_5	-5.933	-29.1	5.046	16.1
η_6	9.043	25.7		
ψ_1	-0.551	-7.7	-0.383	-4.0
ψ_2	-0.361	-2.9		
ω	1.E-08	1769.0	0.045	2.2
α_1			1.054	3.2
α_2	0.551	3.1	0.419	2.5
γ_1	0.559	6.5		
		Italy		
C	0.375	7.3	0.213	7.4
η_1	-19.291	-17.0	-18.252	-4.0
η_2	2.423	2.1	18.189	9.6
φ	-0.424	-6.0	0.171	2.9
ψ_1	0.400	3.6	-0.430	-3.3
ψ_4	-0.174	-1.7	-0.144	-1.5
ω	1.E-08	13.6	3.E-04	0.2
α_1	0.245	1.7	0.230	2.0
γ_1	0.742	6.8	0.756	9.1
		Luxembourg		
C	0.257	5.8	0.265	5.6
η_1	1.969	2.1	-8.257	-2.1
η_2	-3.637	-6.6	4.985	3.7
η_3	10.343	4.6	-2.612	-3.0
η_4	-9.096	-17.1	6.697	5.9
η_5	20.843	5.1		
ψ_1	-0.480	-4.6	-0.478	-4.5
ψ_2	-0.361	-3.3	-0.363	-3.6
ω	1.E-08	6537.0	1.E-08	6838.0
α_1	0.148	1.8	0.133	1.4
γ_1	0.842	13.4	0.858	10.5

Table 5.9 (continued)

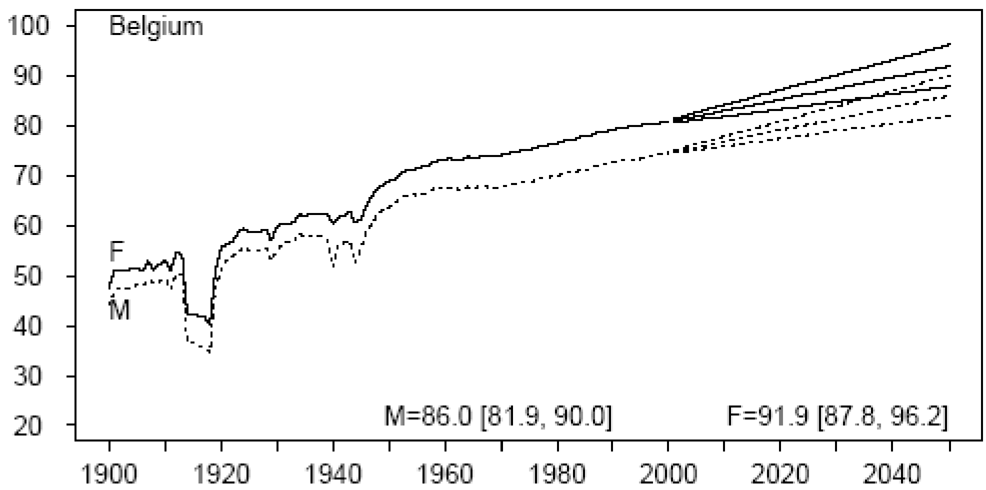
		Netherlands		
C	0.121	5.2	0.102	4.2
η_1	-9.367	-6.9	-6.268	-9.2
η_2	9.845	4.1	5.069	5.3
η_3	-3.467	-5.5	-2.428	-8.2
η_4	-2.395	-11.5	7.429	3.2
η_5	-8.841	-3.1		
η_6	17.036	12.8		
φ	0.150	3.0	0.302	3.8
ψ_1	-0.590	-5.2	-0.706	-4.8
ψ_2			-0.303	-2.3
ψ_3	0.207	2.0		
ω	0.001	0.4	0.004	0.9
α_1	0.202	1.9	0.234	2.1
γ_1	0.783	8.9	0.746	9.5
		Norway		
C	0.151	4.9	0.140	5.4
η_1	-6.177	-20.8	-6.389	-19.8
η_2	4.582	16.8	4.224	18.2
η_3	-1.341	-4.9	0.638	2.2
η_4	2.126	2.7		
φ	0.054	0.8	0.045	0.8
ψ_1	-0.617	-4.7	-0.513	-4.4
ω	1.E-08	60.8	1.E-08	62.2
α_1	0.089	2.4	0.130	2.2
γ_1	0.900	26.4	0.855	20.3
		Sweden		
C	0.187	7.5	0.192	7.1
η_1	-9.364	-2.1	-8.438	-2.7
η_2	6.881	3.9	5.591	5.0
η_3	2.735	3.4	3.222	7.0
η_4			1.024	3.0
φ	0.074	1.2		
ψ_1	-0.329	-2.5	-0.359	-3.3
ω	1.E-08	40.3	0.004	1.0
α_1	0.166	2.0	0.222	1.8
γ_1	0.827	13.1	0.764	6.9
		Switzerland		
C	0.329	8.8	0.338	5.7
η_1	-11.328	-5.8	-9.340	-7.0
η_2	3.945	9.4	1.048	2.2
φ	-0.429	-10.2	-0.543	-16.4
ψ_1			0.347	2.5
ω	0.003	0.5	0.001	0.3
α_1	0.290	1.8	0.174	1.4
γ_1	0.713	5.3	0.813	7.0

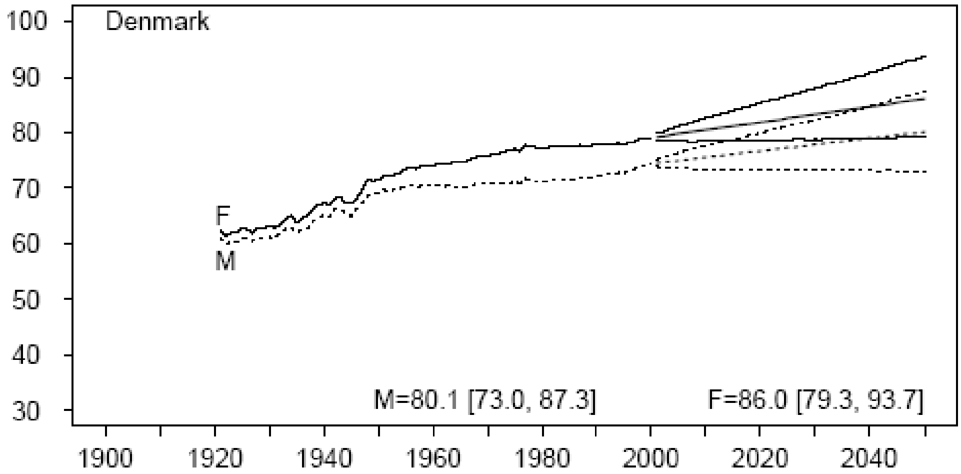
Table 5.9 (continued)

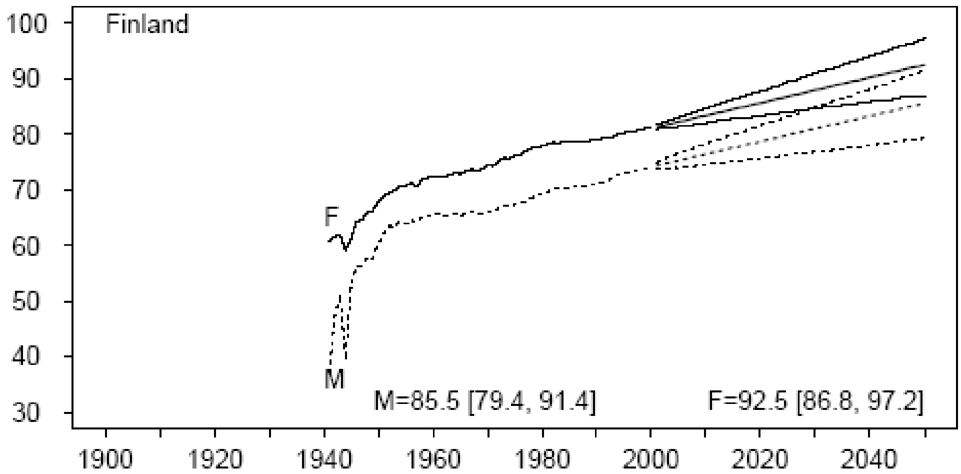
		England and Wales		
C	0.176	7.5	0.258	9.6
η_1	-2.298	-1.9	-2.550	-2.2
η_2	-8.286	-1.4	-8.971	-12.0
η_3	11.955	5.9	-2.462	-16.9
η_4	-3.119	-3.1	2.731	4.0
η_5	2.326	3.6		
φ	0.233	4.9	-0.482	-10.3
ψ_1	-0.612	-6.1		
ω	1.E-08	2.9	1.E-08	0.2
α_2	0.174	1.7	0.264	2.5
γ_1	0.814	10.2	0.740	10.3

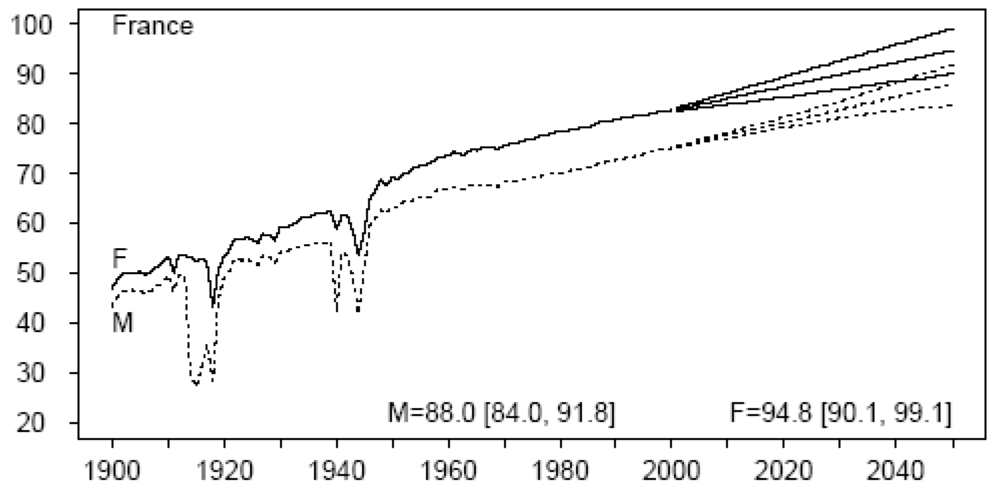
The models were used to simulate prediction intervals for future life expectancy values of men and women. Residuals, estimated coefficients, and the occurrence of dummy variables were all drawn from their respective distributions. The set of figures numbered as Figure 5.6 gives the prediction results.

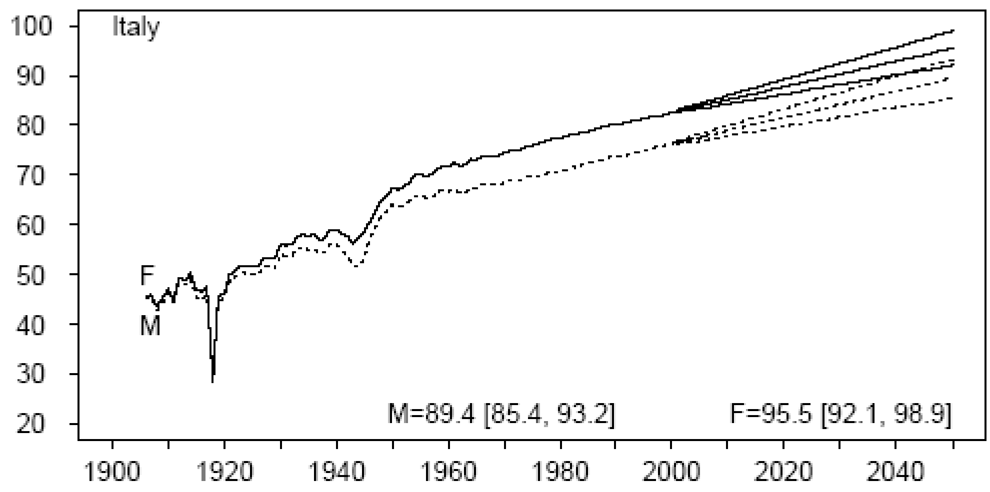
Figure 5.6. Forecasts and 95% prediction intervals for the life expectancy at birth for men (M) and women (F). Data 1900-2000. Values written in each graph indicate the predicted life expectancy for 2050, and the 95% prediction interval bounds in 2050 in brackets

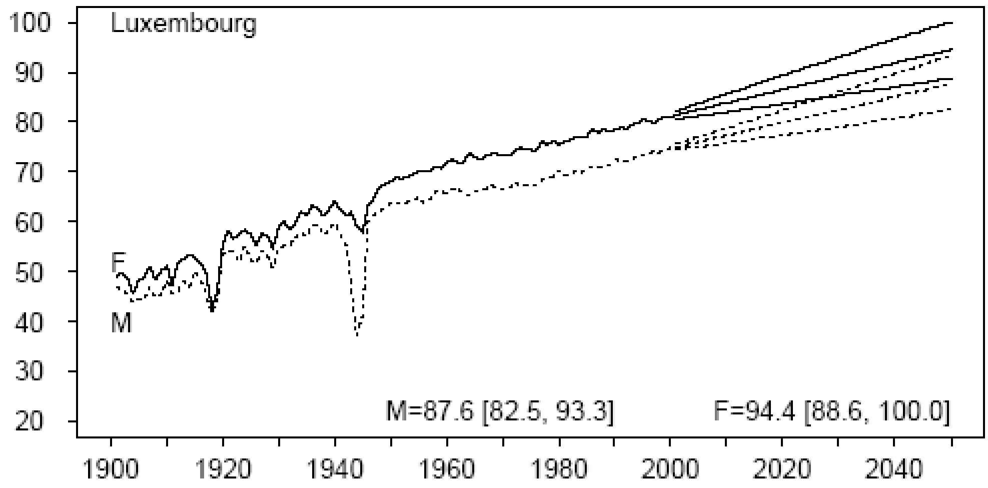


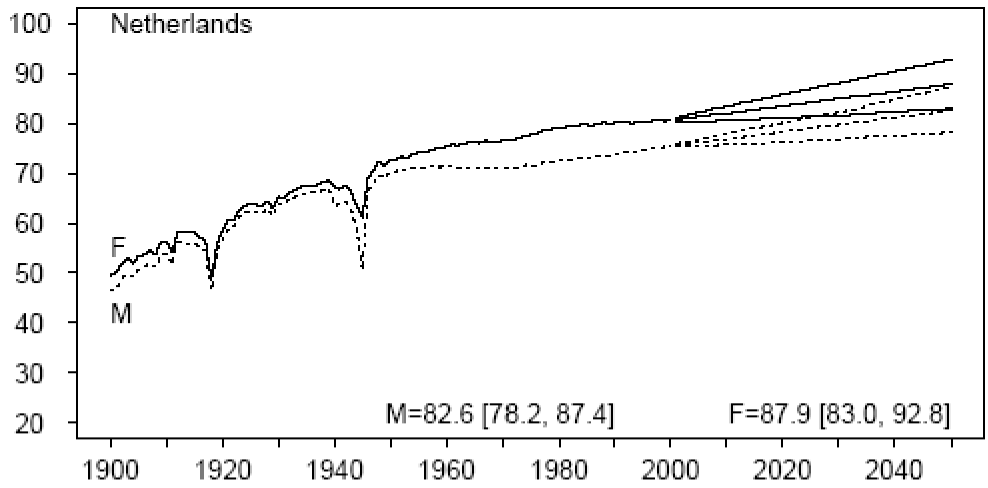

Denmark
F
M
M=80.1 [73.0, 87.3]
F=86.0 [79.3, 93.7]

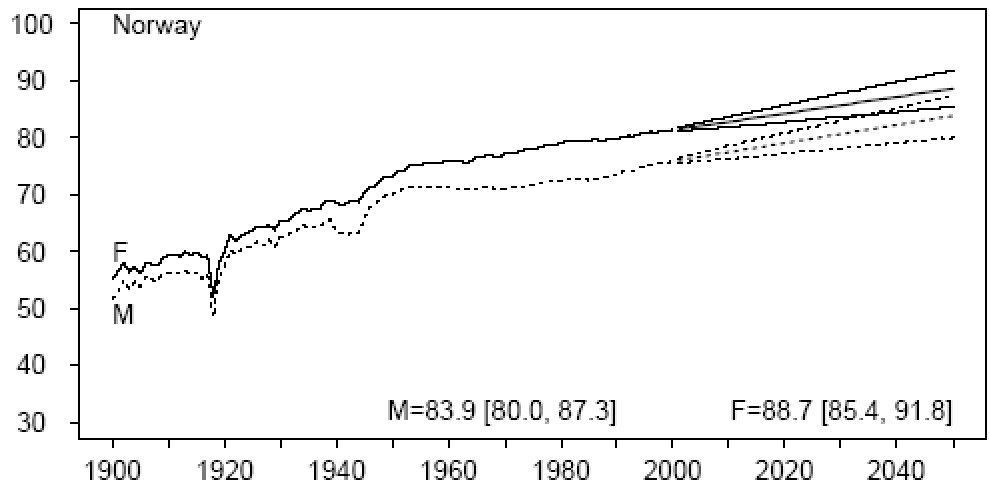

Finland
F
M
M=85.5 [79.4, 91.4]
F=92.5 [86.8, 97.2]

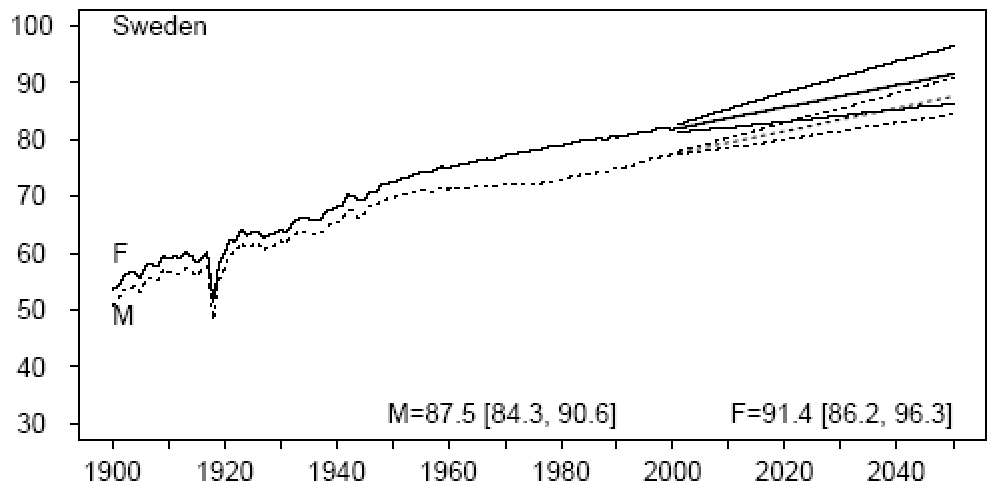

France
F
M
M=88.0 [84.0, 91.8]
F=94.8 [90.1, 99.1]

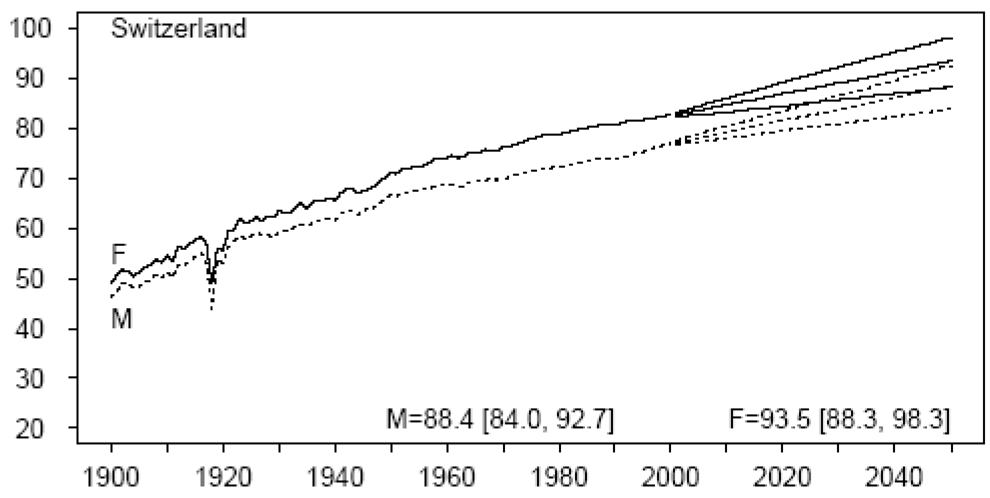

Italy
F
M
M=89.4 [85.4, 93.2]
F=95.5 [92.1, 98.9]

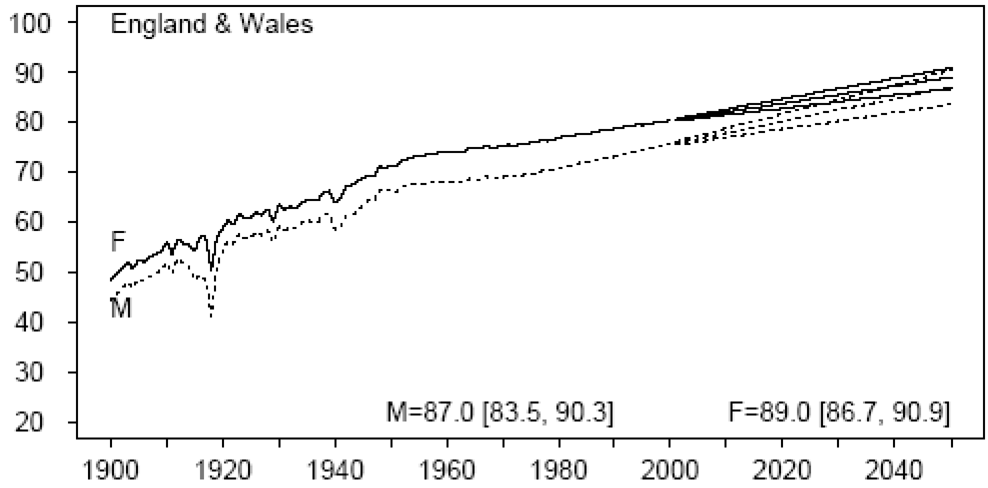

Luxembourg
F
M
M=87.6 [82.5, 93.3]
F=94.4 [88.6, 100.0]
100
90
80
70
60
50
40
30
20
1900
1920
1940
1960
1980
2000
2020
2040


Netherlands
F
M
M=82.6 [78.2, 87.4]
F=87.9 [83.0, 92.8]
100
90
80
70
60
50
40
30
20
1900
1920
1940
1960
1980
2000
2020
2040


Norway
F
M
M=83.9 [80.0, 87.3]
F=88.7 [85.4, 91.8]


Sweden
F
M
M=87.5 [84.3, 90.6]
F=91.4 [86.2, 96.3]


Switzerland
F
M
M=88.4 [84.0, 92.7]
F=93.5 [88.3, 98.3]



Between 2000 and 2050, life expectancy at birth for men and women is expected to rise by between 6 and 13 years. Across countries and sexes, the average annual increase amounts to 0.2 years. This is in line with historical developments (0.23 years for record life expectancy 1840-2000, Oeppen and Vaupel, 2002; 0.21 years for 21 industrialized nations 1955-1995, White, 2002), but it reflects a much stronger increase than that projected for 21 OECD countries to 2050 (0.09 years , Dang et al. 2001) .

The relatively small increase for men and women in Denmark (6 and 7 years, respectively) is a direct consequence of the stagnation in Danish life expectancy in recent decades. This led to negative estimates for φ (Table 5.9), and next to small annual increments in predicted life expectancy. Long-range (fifty years) 95 per cent prediction intervals are 4-14 years wide, with Swedish men and women from England and Wales at the lower end of the spectrum, and Danish men and women at the upper end. Ten years ahead 95 per cent intervals are between one (men in France, women in England and Wales) and four (Denmark, both men and women) years wide. Denmark was the only country for which we could omit dummy variables, and nonetheless obtain constant variance in the residuals. This explains the rather wide intervals for that country.

5.3.2 A time series model for the life expectancy in 18 countries, 1960-2000

This section presents time series predictions for all 18 countries based on observed life expectancy data since 1960. For a few countries, the necessary historical data were missing: Germany (1960, 1961, 1962), Iceland (1961-1965, 1967-1969), Ireland (1962-1969, 1972-1979, 1981-1984), and Spain (1961-1964); see Table 3.1. For Germany, data for the years 1960-1962 are available for the former GDR and the former FRG, but not for Germany as a whole. In view of the small differences in life expectancy between the two countries, we estimated the missing data for Germany by computing the simple average of the GDR and the FRG data. Missing values of the life expectancy at birth for men and women for the other countries were estimated by using data of a neighbour country. We used the life expectancies in England and Wales, Norway, and Portugal as independent variables in a regression model to predict the missing life expectancy data for Ireland, Iceland, and Spain, respectively. Appendix 2 gives details.

The life expectancy time series model estimated on the basis of the short data series (observations from 1960 to 2000) is of the form as given in expression (5.2), but without dummy variables. Table 5.10 specifies the models for men and women, and Table 5.11 gives parameter estimates. As in previous cases, a number of non-significant estimates were retained in the models, to ensure normally distributed independent residuals with constant variance.

Between 2000 and 2050, life expectancy at birth for men and women is expected to rise by about ten years. This is the average increase across countries and sexes, and it is the same as the one found on the basis of long time series for eleven countries in Section 5.3.1. Life expectancy stagnated or increased very little in Denmark and the Netherlands in recent decades. This led to small estimates for the constant C (see Table 5.11), and next to modest life expectancy increases for the period until 2050: 5-6 years for men in the two countries, and 6 years for women. Long-range (fifty years) 95 per cent prediction intervals are typically 9 years wide, and ten-year ahead 95 per cent intervals on average 3 years. Ireland and Italy (both sexes) have narrow long-range intervals: less than 6 years.

Table 5.10. ARCH-GARCH models for differences in life expectancy at birth, data 1960-2000

		$\nabla e_{0,t} = C +$	$v_t =$	$h_t =$
Austria	men	v_t	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Belgium	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Denmark	men	v_t	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Finland	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
France	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Germany	men	v_t	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	$\psi_3 v_{t-3} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Greece	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Iceland	men	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	
	women	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	
Ireland	men	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	$\psi_1 v_{t-1} + \psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Italy	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \gamma_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2$
Luxembourg	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	$\psi_2 v_{t-2} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Netherlands	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	$\psi_3 v_{t-3} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Norway	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
Portugal	men	v_t	$\psi_1 v_{t-1} + \varepsilon_t$	
	women	v_t	$\psi_1 v_{t-1} + \varepsilon_t$	
Spain	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	v_t	$\psi_1 v_{t-1} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$
Sweden	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	
Switzerland	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
England & Wales	men	$\phi \nabla e_{0,t-1} + v_t$	ε_t	$\omega + \alpha_1 \varepsilon_{t-1}^2$
	women	$\phi \nabla e_{0,t-1} + v_t$	$\psi_3 v_{t-3} + \varepsilon_t$	$\omega + \alpha_1 \varepsilon_{t-1}^2$

Table 5.11 Estimates of model (5.2) as specified in Table 5.10

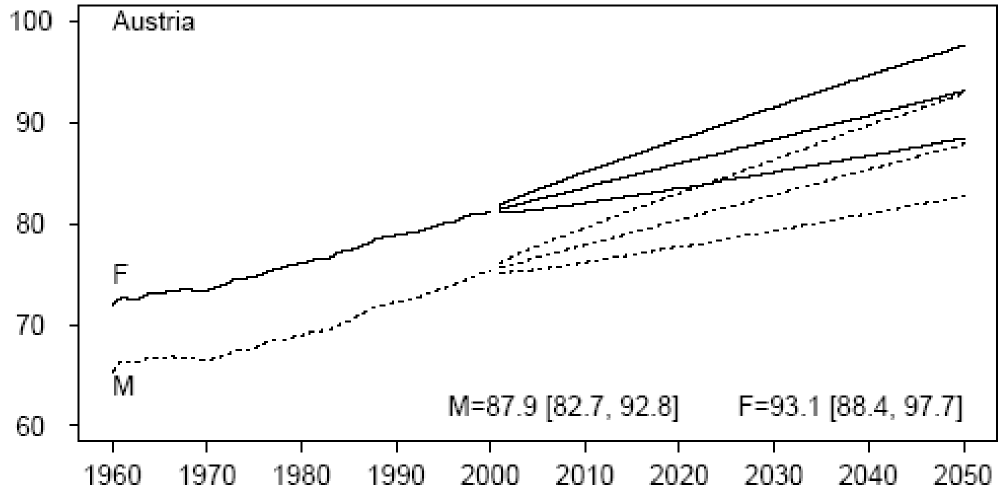
	Men			Women	
	Estimate	t-value		Estimate	t-value
			Austria		
C	0.249	6.4		0.238	6.9
ω	0.061	5.4		0.031	2.9
α_1	0.000	0.0		0.441	1.4
			Belgium		
C	0.253	5.1		0.274	6.5
ω	-0.374	-2,2		-0.382	-2.3
α_1	0.277	1.0		0.266	0.7
			Denmark		
C	0.091	2.9		0.168	3.9
φ				-0.315	-1.5
ω	0.035	2.3		0.045	2.6
α_1					
			Finland		
C	0.266	4.8		0.289	5.3
φ	-0.235	-1.5		-0.319	-2.1
			France		
C	0.312	6.8		0.340	6.7
φ	-0.450	-2.7		-0.416	-2.8
ω	0.019	2.9		0.036	3.3
α_1	0.742	1.9		0.356	1.2
			Germany		
C	0.211	5.7		0.238	5.5
ψ_3				0.311	1.9
ω	0.041	4.0		0.039	4.1
α_1	0.113	0.4		0.022	0.1
			Greece		
C	0.188	3.8		0.262	5.7
φ	-0.268	-3.5		-0.548	-3.8
ω	0.059	2.0		0.048	2.1
α_1	0.087	0.2		0.561	1.5
			Iceland		
C	0.168	2.2		0.155	2.7
ψ_1	-0.463	-2.9		-0.315	-2.0
ψ_2	-0.246	-1.6		-0.263	-1.6
			Ireland		
C	0.155	7.7		0.180	10.0
ψ_1	-0.561	-3.1		-0.777	-4.6
ψ_2	-0.369	-1.8		-0.333	-1.7
ω	0-043	2.8		0.053	1.5
α_1	0.192	0.4		0.155	0.3

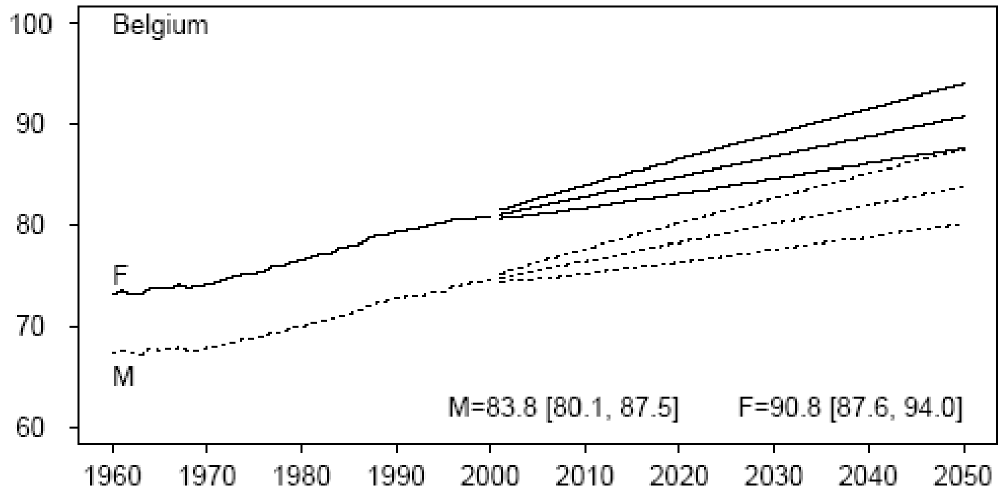
Table 5.11 (cont.)

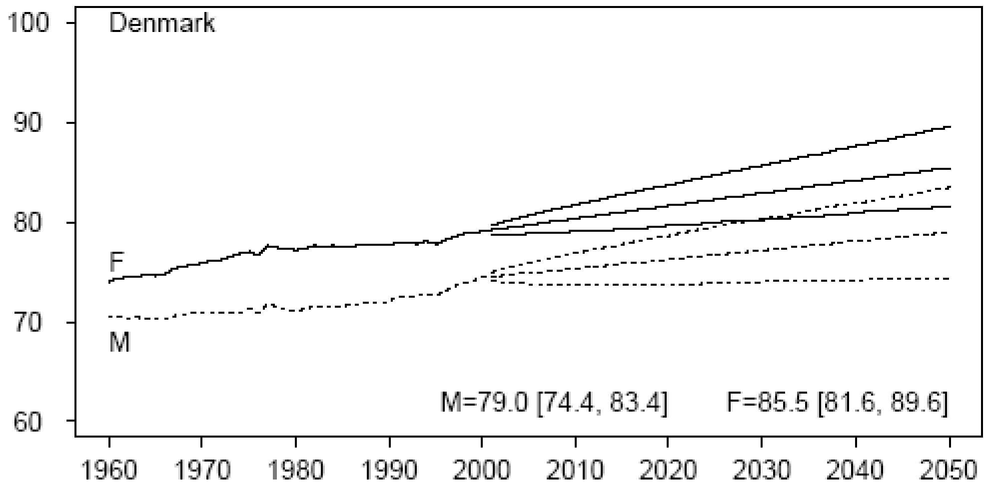
		Italy		
C	0.344	6.6	0.376	7.7
φ	-0.358	-2.2	-0.500	-3.1
ω	1E-8	718	0.003	0.43
α_1	0.208	1.2	0.298	1.1
γ_1	0.783	5.3	0.649	2.0
		Luxembourg		
C	0.308	3.5	0.374	6.8
φ	-0.371	-2.2	-0.705	-5.5
ψ_2			-0.506	-3.7
ω	0.200	2.2	0.224	3.9
α_1	0.175	0.5	0.000	0.0
		Netherlands		
C	0.131	3.7	0.165	2.7
φ	-0.285	-1.7	-0.254	-1.5
ψ_3			0.543	3.5
ω	0.050	4.5	0.033	3.1
α_1	0.000	0.0	0.000	0.0
		Norway		
C	0.133	3.2	0.178	4.4
φ	-0.199	-1.3	-0.290	-1.9
		Portugal		
C	0.314	6.2	0.350	7.6
ψ_1	-0.641	-5.2	-0.687	-5.9
		Spain		
C	0.340	7.0	0.239	6.4
φ	-0.608	-5.2		
ψ_1			-0.423	-2.4
ω	0.076	3.9	0.062	3.8
α_1	0.000	0.0	0.571	2.2
		Sweden		
C	0.173	4.5	0.231	5.5
φ	-0.227	-1.5	-0.413	-2.8
		Switzerland		
C	0.261	5.4	0.309	6.8
φ	-0.276	-1.5	-0.510	-2.6
ω	0.045	3.8	0.040	2.5
α_1	0.201	0.8	0.309	1.0
		England and Wales		
C	0.265	6.1	0.254	8.6
φ	-0.276	-1.9	-0.720	-5.3
ψ_3			0.329	3.0
ω	0.030	2.3	1E-8	99.7
α_1	0.411	0.9	0.846	2.2
γ_1			0.340	2.4

The set of figures numbered as Figure 5.7 gives the prediction results.

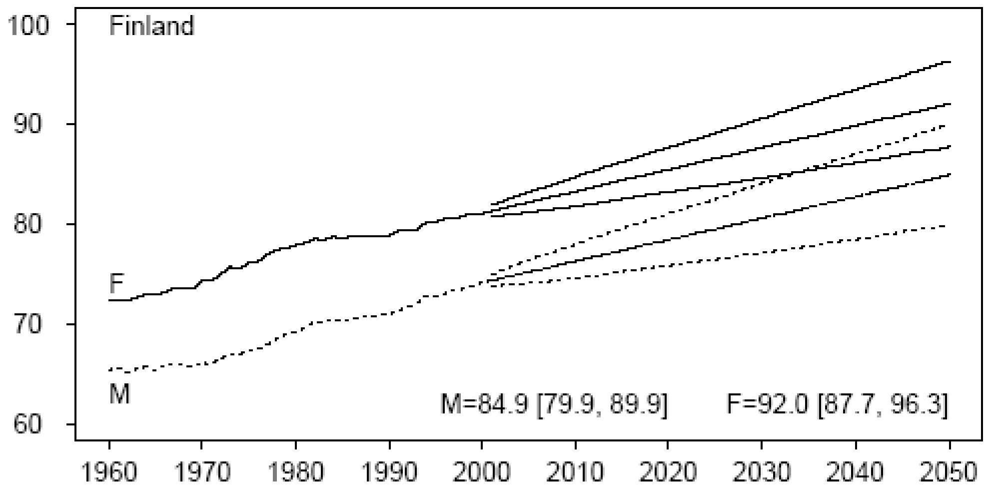
Figure 5.7 Forecasts and 95% prediction intervals for the life expectancy at birth for men (M) and women (F). Data 1960-2000. Values written in each graph indicate the predicted life expectancy for 2050, and the 95% prediction interval bounds in 2050 in brackets



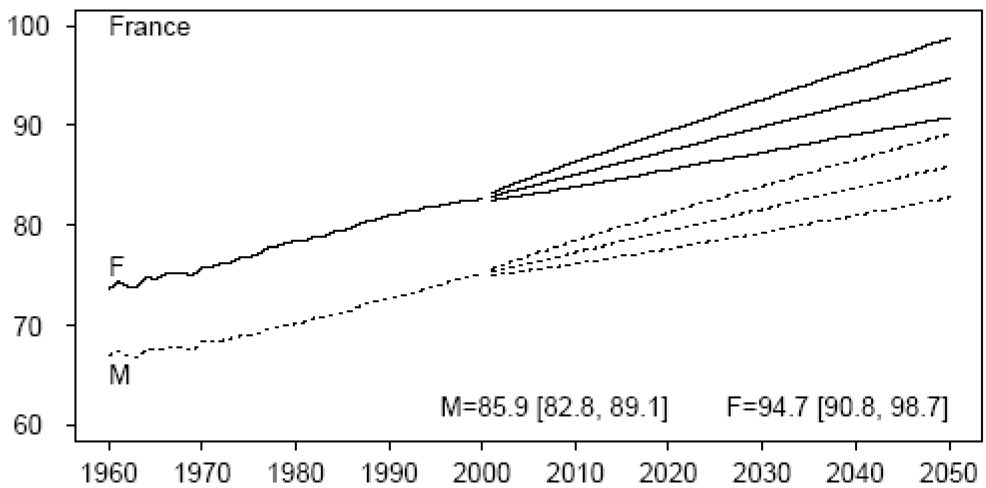




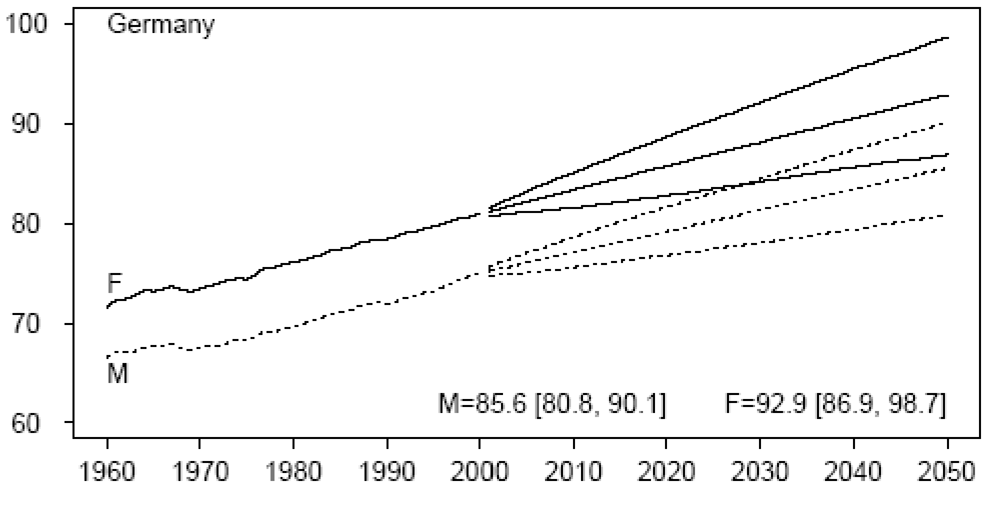
Denmark
100
90
80
70
60
F
M
M=79.0 [74.4, 83.4]
F=85.5 [81.6, 89.6]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

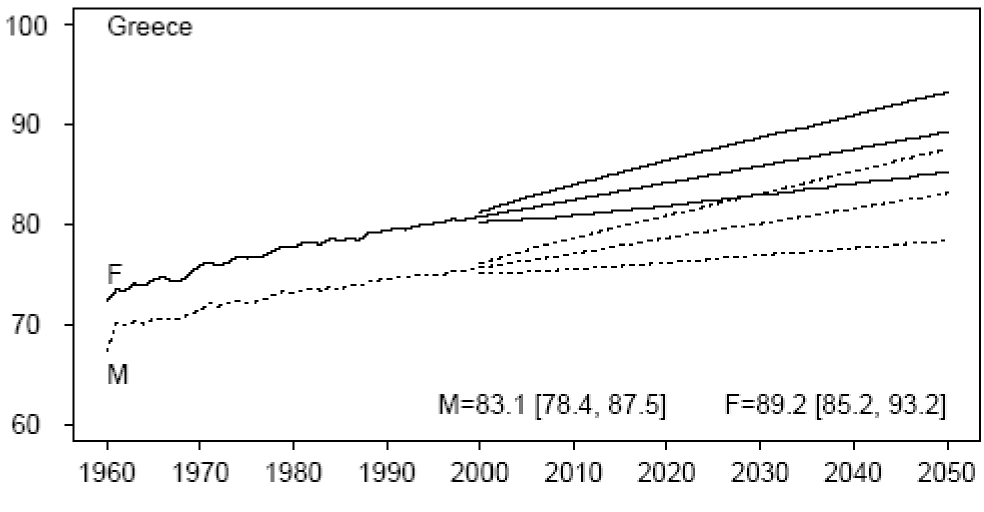


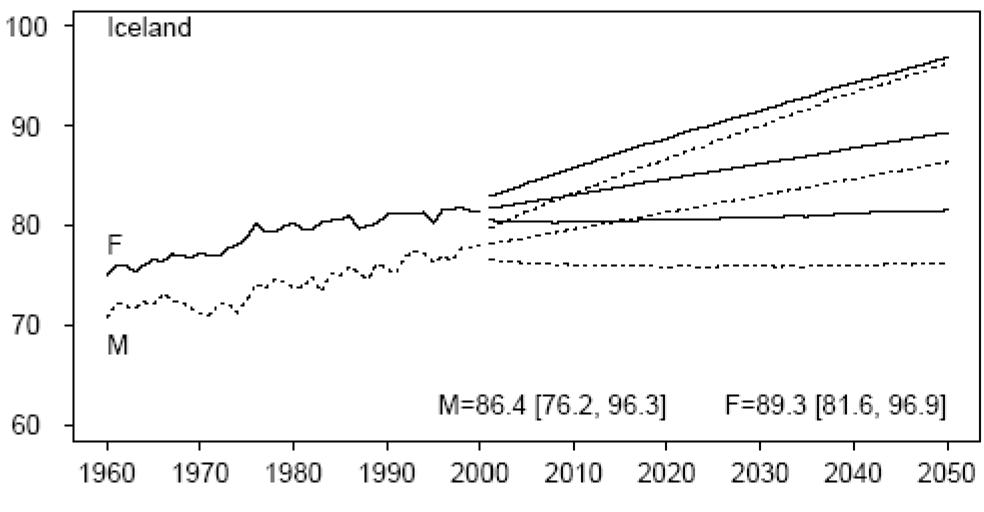
Finland
100
90
80
70
60
F
M
M=84.9 [79.9, 89.9]
F=92.0 [87.7, 96.3]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

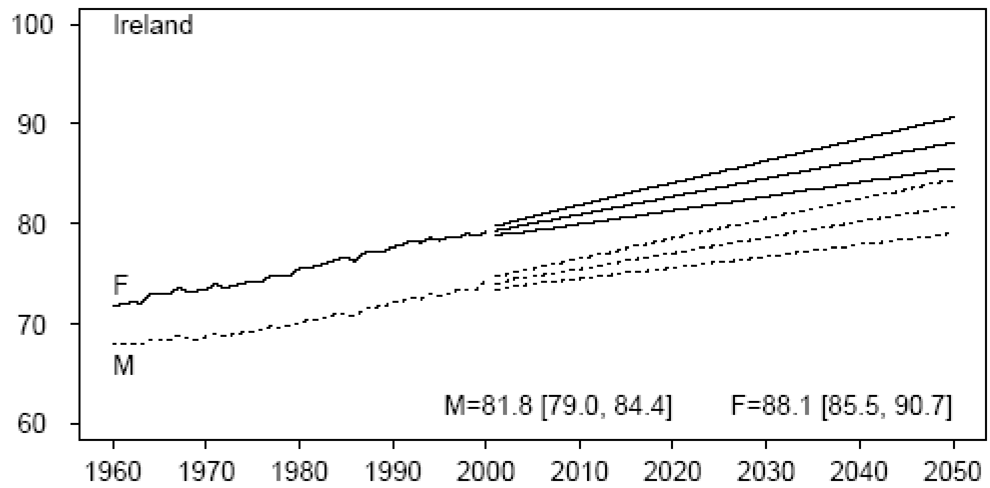


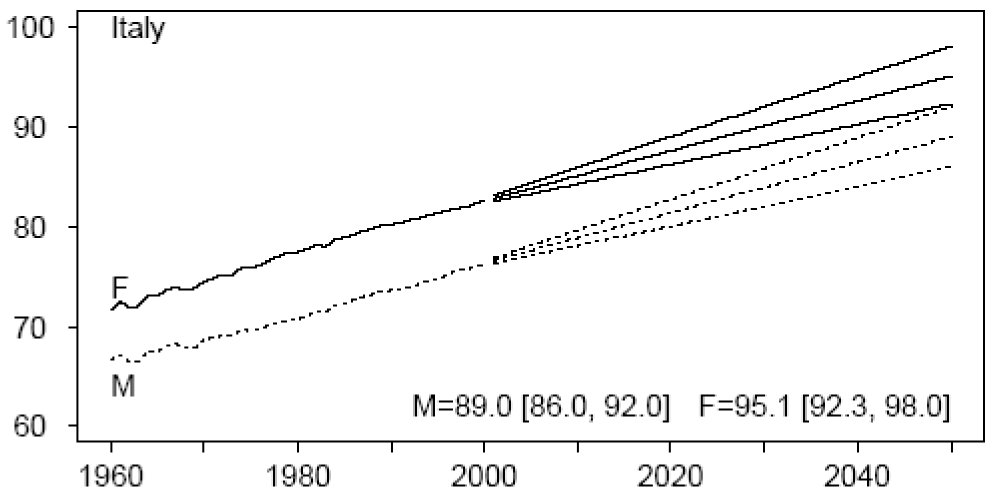
France
100
90
80
70
60
F
M
M=85.9 [82.8, 89.1]
F=94.7 [90.8, 98.7]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

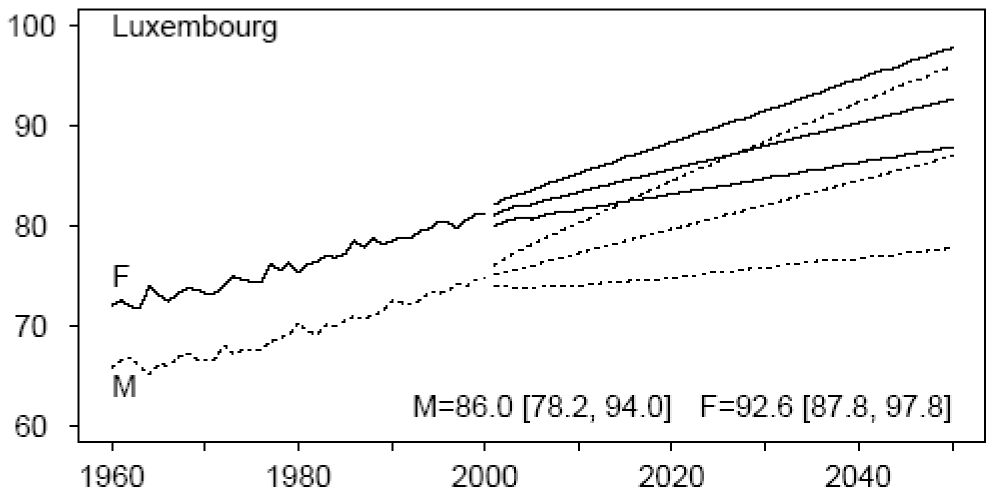

Germany
100
90
80
70
60
F
M
M=85.6 [80.8, 90.1]
F=92.9 [86.9, 98.7]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

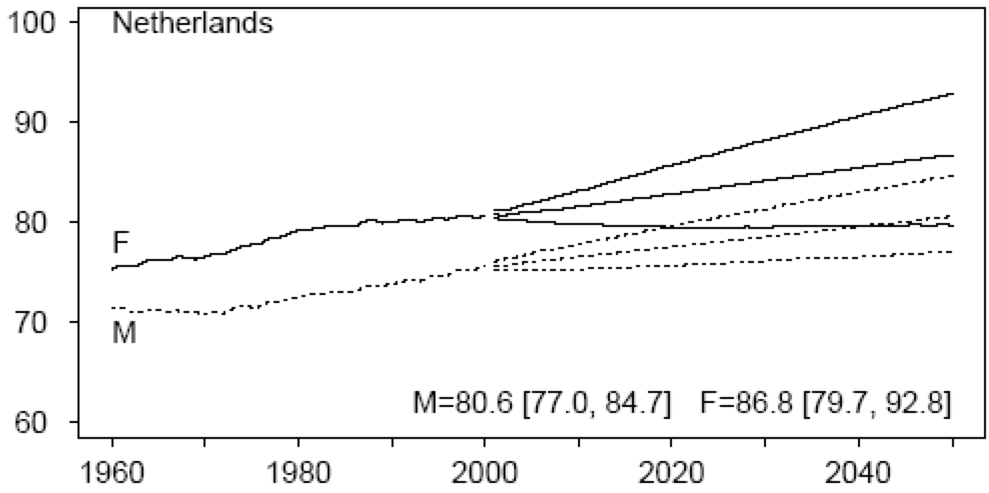

Greece
100
90
80
70
60
F
M
M=83.1 [78.4, 87.5]
F=89.2 [85.2, 93.2]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

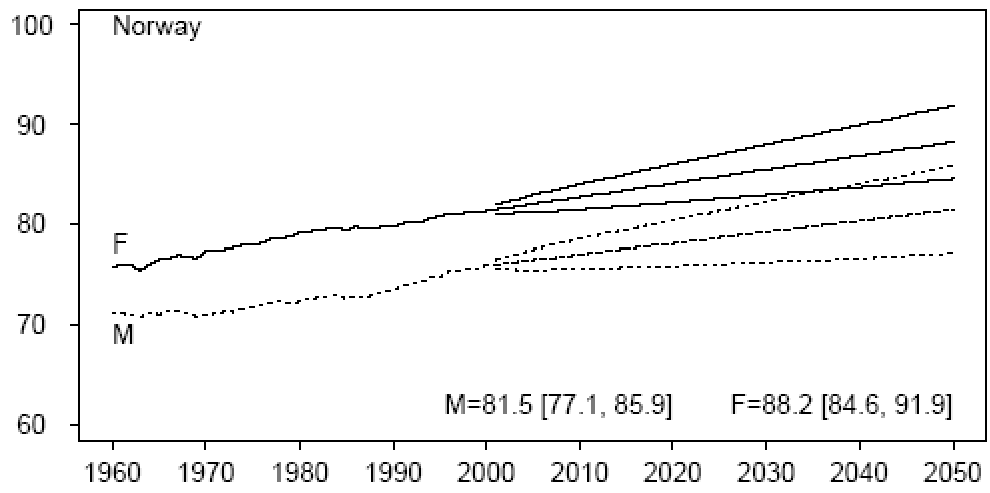

Iceland
100
90
80
70
60
F
M
M=86.4 [76.2, 96.3]
F=89.3 [81.6, 96.9]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

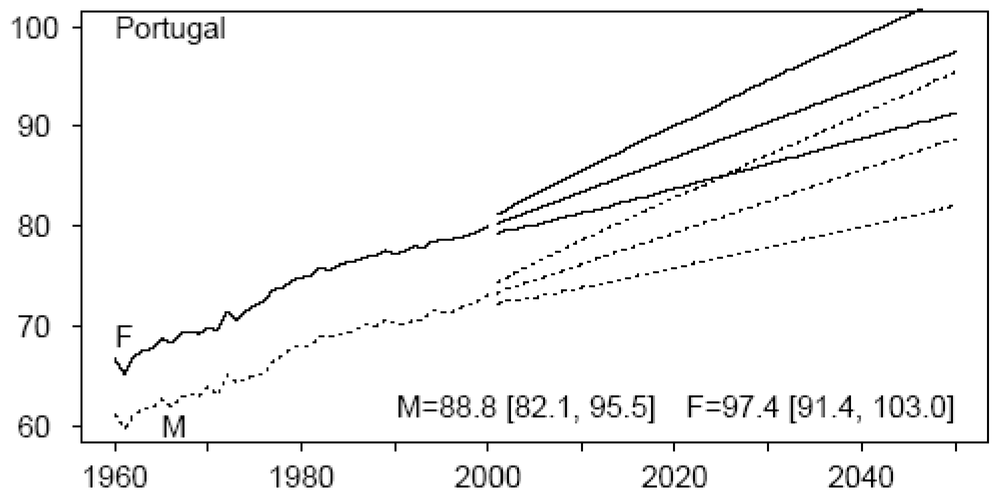

Ireland
100
90
80
70
60
F
M
M=81.8 [79.0, 84.4]
F=88.1 [85.5, 90.7]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

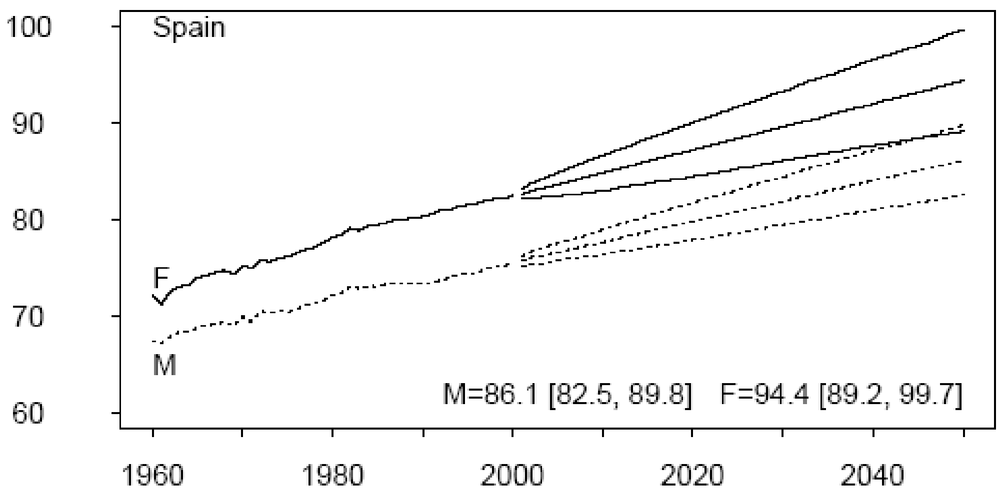

Italy
100
90
80
70
60
F
M
M=89.0 [86.0, 92.0]
F=95.1 [92.3, 98.0]
1960
1980
2000
2020
2040

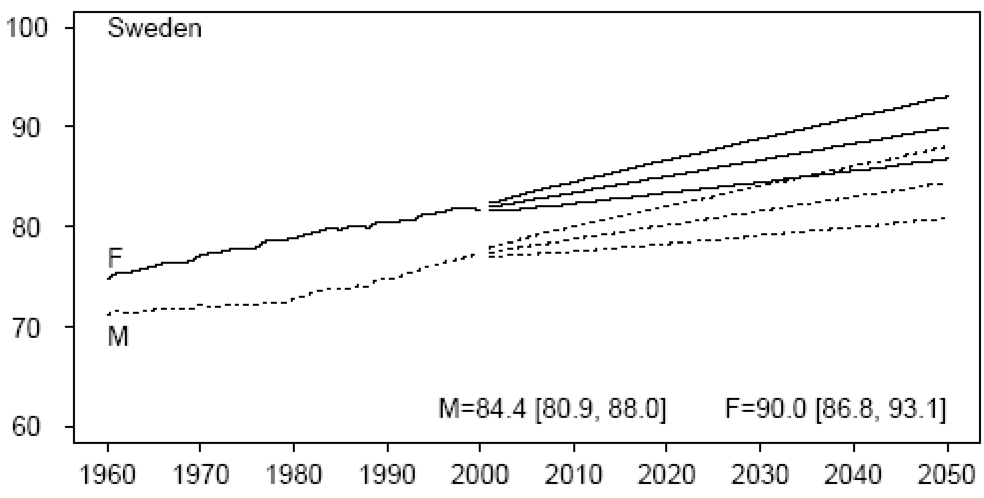

Luxembourg
100
90
80
70
60
F
M
M=86.0 [78.2, 94.0]   F=92.6 [87.8, 97.8]
1960
1980
2000
2020
2040

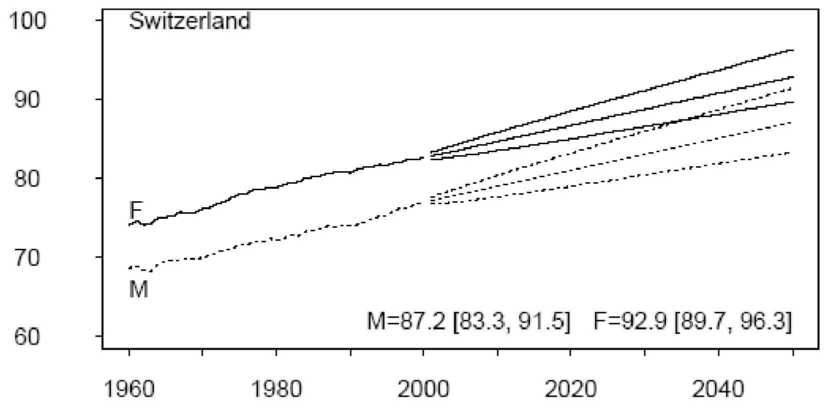

Netherlands
100
90
80
70
60
F
M
M=80.6 [77.0, 84.7]   F=86.8 [79.7, 92.8]
1960
1980
2000
2020
2040


Norway
100
90
80
70
60
F
M
M=81.5 [77.1, 85.9]   F=88.2 [84.6, 91.9]
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Portugal
F
M
M=88.8 [82.1, 95.5]
F=97.4 [91.4, 103.0]
100
90
80
70
60
1960
1980
2000
2020
2040


Spain
F
M
M=86.1 [82.5, 89.8]
F=94.4 [89.2, 99.7]
100
90
80
70
60
1960
1980
2000
2020
2040


Sweden
F
M
M=84.4 [80.9, 88.0]
F=90.0 [86.8, 93.1]
100
90
80
70
60
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050



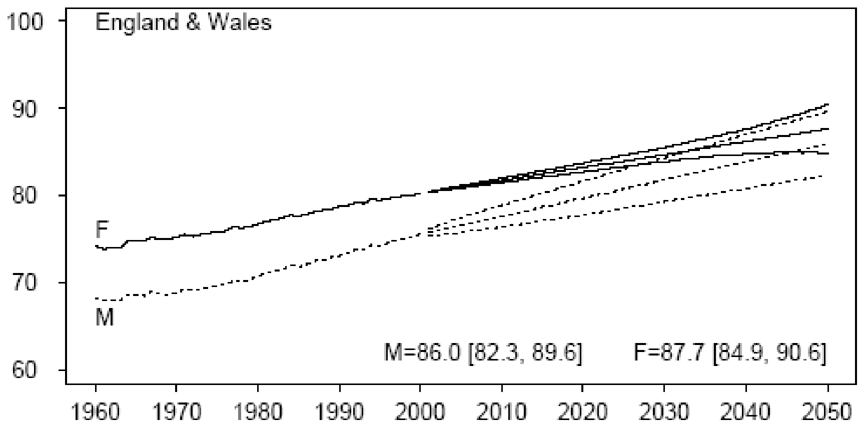
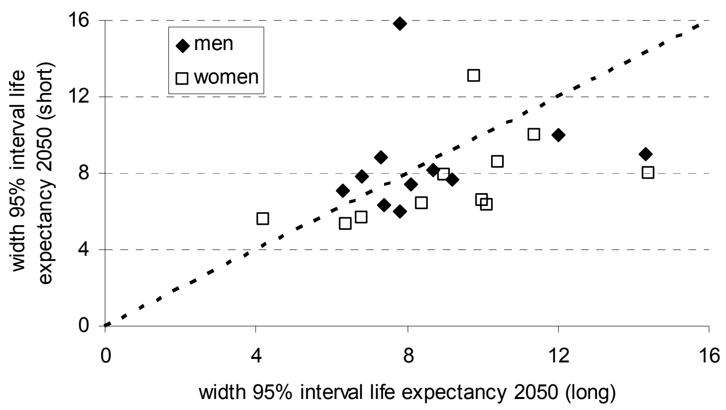


Figure 5.8 shows that interval widths in this section, i.e. based on data for the years 1960-2000, tend to be close to interval widths based on data for the years 1900-2000 presented for eleven countries in Section 5.3.1. For women there is a weak tendency for the intervals based on long series to be wider than those based on short series.

Figure 5.8 Width of 95% prediction intervals for the life expectancy in 2050, long and short historical series, eleven countries



5.3.3 Cross-country correlations

We have computed correlations across countries between residuals from the time series models. Table 5.12 shows the 11x11 correlation matrix for countries for which we have a long series of observations (1900-2000), and Table 5.13 contains the 18x18 correlation matrix for all countries, based on data for the period 1960-2000.

Table 5.12. Correlations for residuals across countries, life expectancy time series models, data 1900-2000

	B	Dk	SF	F	I	Lux	Nl	N	S	CH	E&W
men											
Belgium	1										
Denmark	0.146	1									
Finland	0.209	0.250	1								
France	0.530	0.162	0.290	1							
Italy	0.194	0.103	0.239	0.402	1						
Luxembourg	0.210	-0.037	0.003	0.261	0.131	1					
Netherlands	0.525	0.285	0.276	0.412	0.178	0.047	1				
Norway	0.316	0.273	0.353	0.236	0.218	0.144	0.391	1			
Sweden	0.143	0.439	0.459	0.215	0.274	-0.081	0.342	0.506	1		
Switzerland	0.414	0.298	0.333	0.289	0.336	0.000	0.347	0.367	0.245	1	
England & Wales	0.482	0.171	0.260	0.314	0.304	0.051	0.500	0.452	0.413	0.281	1
women											
Belgium	1										
Denmark	0.248	1									
Finland	0.193	0.292	1								
France	0.632	0.312	0.208	1							
Italy	0.333	0.186	0.204	0.388	1						
Luxembourg	0.343	0.127	0.058	0.282	0.305	1					
Netherlands	0.456	0.358	0.285	0.403	0.287	0.152	1				
Norway	0.188	0.419	0.347	0.169	0.338	0.076	0.395	1			
Sweden	0.106	0.563	0.496	0.174	0.333	-0.047	0.300	0.571	1		
Switzerland	0.436	0.525	0.370	0.503	0.276	0.229	0.317	0.274	0.250	1	
England & Wales	0.413	0.215	0.213	0.416	0.198	0.119	0.399	0.293	0.276	0.385	1

In the long run, ten of the eleven countries reported in Table 5.12 seem to have moderately positively correlated errors in life expectancy. The exception is Luxembourg.[9] The average value of the significant correlations equals 0.35. Given the fact that the correlations in the observed data are 0.9 or higher (data not shown), we conclude that the time series model removes a large part of the original correlation, but not all.

[9] The shortest time series is the one for Finland with 60 data points. The critical value for the correlation coefficient is 0.25 in that case (two-sided; α=0.05). Most countries have 101 data points, with a critical value of 0.196.

Table 5.13a Correlations for residuals across countries, life expectancy time series models, data 1960-2000. Men

	A	B	Dk	SF	F	D	EL	Is	IRL	I	Lux	Nl	N	P	E	S	CH
Austria	1																
Belgium	0.326	1															
Denmark	0.114	0.348	1														
Finland	0.289	0.315	0.340	1													
France	0.477	0.712	0.287	0.374	1												
Germany	0.477	0.611	0.080	0.161	0.601	1											
Greece	-0.238	-0.028	0.025	-0.099	0.046	-0.044	1										
Iceland	0.453	-0.039	0.021	0.355	0.048	0.211	-0.100	1									
Ireland	0.100	0.527	0.287	0.241	0.482	0.414	-0.111	-0.001	1								
Italy	0.280	0.569	0.143	0.355	0.632	0.511	0.034	0.218	0.253	1							
Luxembourg	-0.015	0.077	0.056	0.062	-0.064	-0.200	-0.123	0.015	-0.010	-0.181	1						
Netherlands	0.315	0.665	0.288	0.315	0.678	0.549	-0.029	-0.219	0.539	0.295	0.002	1					
Norway	0.192	0.460	0.311	0.499	0.387	0.305	-0.046	0.176	0.528	0.324	0.096	0.371	1				
Portugal	-0.211	-0.040	-0.017	0.346	-0.060	-0.180	-0.036	0.087	-0.023	-0.021	0.216	0.024	-0.12	1			
Spain	0.004	-0.069	0.063	0.453	0.084	0.119	-0.001	0.140	-0.001	0.035	0.012	-0.029	0.013	0.621	1		
Sweden	0.203	0.451	0.295	0.508	0.497	0.459	-0.097	0.142	0.471	0.470	0.001	0.303	0.451	0.057	0.180	1	
Switzerland	0.335	0.495	0.259	0.384	0.441	0.414	-0.215	0.270	0.247	0.727	-0.048	0.311	0.371	0.076	0.019	0.339	1
England & Wales	-0.019	0.537	0.320	0.177	0.454	0.383	-0.02	-0.152	0.811	0.259	-0.047	0.579	0.527	0.032	0.008	0.395	0.289

Table 5.13b Correlations for residuals across countries, life expectancy time series models, data 1960-2000. Women

	A	B	Dk	SF	F	D	EL	Is	IRL	I	Lux	Nl	N	P	E	S	CH
Austria	1																
Belgium	0.440	1															
Denmark	0.099	0.128	1														
Finland	0.190	0.257	0.315	1													
France	0.603	0.642	0.311	0.141	1												
Germany	0.618	0.509	0.256	0.233	0.606	1											
Greece	-0.067	-0.114	-0.059	0.037	0.079	-0.109	1										
Iceland	-0.026	-0.122	0.030	0.083	0.104	-0.026	-0.089	1									
Ireland	0.086	0.442	0.095	-0.018	0.414	0.220	-0.152	0.103	1								
Italy	0.396	0.558	0.164	0.102	0.719	0.460	0.257	-0.139	0.232	1							
Luxembourg	0.331	0.507	0.104	0.114	0.250	0.158	-0.130	-0.041	-0.006	0.216	1						
Netherlands	0.450	0.510	0.399	0.169	0.544	0.472	-0.088	-0.042	0.394	0.316	0.235	1					
Norway	0.068	0.171	0.240	0.366	0.443	0.191	0.012	0.274	0.344	0.305	0.199	0.218	1				
Portugal	-0.142	0.012	-0.069	0.419	-0.127	-0.042	-0.070	-0.177	-0.017	-0.022	-0.057	0.073	-0.043	1			
Spain	-0.071	0.065	-0.160	0.350	0.010	0.002	0.014	-0.108	0.237	-0.005	-0.195	0.067	0.269	0.742	1		
Sweden	0.105	0.105	0.335	0.390	0.364	0.427	0.265	-0.120	0.117	0.307	-0.085	0.127	0.462	0.006	0.126	1	
Switzerland	0.291	0.579	0.450	0.455	0.555	0.483	-0.198	0.165	0.367	0.500	0.299	0.532	0.424	0.128	0.080	0.372	1
England & Wales	0.096	0.460	0.147	0.202	0.312	0.231	-0.087	-0.080	0.727	0.207	-0.028	0.338	0.343	0.113	0.294	0.233	0.374

For the shorter time series, the critical value is 0.31. In this case, Greece is unrelated to all other countries, while Luxembourg and Iceland are positively correlated with no more than two. Otherwise the correlations are somewhat stronger than those for long time series, with an average value of 0.46, which is higher by 0.11 than the average correlation based on long time series. However, part of this increase is due to the fact that there are fewer observations, which in turn leads to a higher critical value (0.31 instead of 0.20-0.25).

5.3.4 Life expectancy correlations across sexes

Are model errors predicted by the life expectancy models correlated across the sexes? In other words, do large model residuals for men tend to go together with large residuals for women? The answer can be found by inspecting correlations across sexes. Table 5.14 gives, for each country, two types of correlations. One is based on long time series (only eleven countries), the other on short time series (all 18 countries).

Table 5.14. Correlations for residuals across sexes, life expectancy time series models, data 1900-200 and 1960-2000

	Data 1900-2000	Data 1960-2000
Austria		0.777
Belgium	0.813	0.808
Denmark	0.726	0.580
Finland	0.707	0.661
France	0.491	0.760
Germany		0.801
Greece		0.658
Iceland		0.318
Ireland		0.693
Italy	0.817	0.772
Luxembourg	0.349	0.067 (sic)
Netherlands	0.594	0.593
Norway	0.700	0.478
Portugal		0.860
Spain		0.772
Sweden	0.792	0.530
Switzerland	0.628	0.636
England & Wales	0.784	0.782

Cross-sex correlations in the raw data were 0.9 or higher (data not shown here). Thus the table demonstrates that, with the exception of Luxembourg and Iceland, a large part of this correlation is still present in the residuals. A multivariate GARCH-time series model for men and women simultaneously could possibly capture this correlation (Engle and Kroner 1995, Kroner and Ng 1998). However, a recent test of the available software for such multivariate GARCH models revealed considerable differences in the resulting parameter estimates across four software packages (Brooks et al. 2003). Since a reliable benchmark data set is not available, the authors conclude that much work remains to be done before this class of models is to be reliably used in practice.

5.4 Net migration

Net migration poses a greater challenge than total fertility or life expectancy, for two reasons:

- the observed trends are strongly volatile, due to political and economic developments, and changes in legislation;
- the data situation is problematic – time series of observed net migration are rather short, and the data quality may be questioned in some cases, cf. the comment for Portugal in Section 4.4.

5.4.1 Three time series models

Data on net migration to the 18 countries for the years 1960-2000 are available from the Council of Europe, see Table 3.1. Spain is the only country with a shorter time series, which starts in 1965. We have estimated three very different models on the basis of the time series for each country, namely a linear trend model, a random walk with drift, and an autoregressive process of order 1 (AR(1)). This way we hope to cover particular national trends and developments in the 18 countries. (Tests for constant variance indicated that GARCH-type of models were not necessary in the case of migration.) Dummy variables are included in order to capture outliers caused by special events.

Let Z_t be the net migration in year t. The three models are as follows.

Linear trend (LT)

(5.3) $$Z_t = \eta_1 U_{t_1} + \ldots + \eta_n U_{t_n} + C + \beta t + \varepsilon_t \,,$$

where U_{t_i} is the dummy variable for year t_i, C is the constant, and β is the slope of the trend.

Random walk with drift (RWD)

(5.4) $$Z_t = \gamma_1 U_{t_1} + \ldots + \gamma_n U_{t_n} + \mu + Z_{t-1} + a_t \,,$$

where U_{t_i} is the dummy variable for year t_i, and μ is the drift.

Autoregressive $AR(1)$ *process (AR(1))*

(5.5) $$Z_t = \lambda_1 U_{t_1} + \ldots + \lambda_n U_{t_n} + K + \phi Z_{t-1} + v_t$$

where U_{t_i} is the dummy variable for year t_i, K is a constant, and the autoregressive coefficient ϕ < 1.

We have tested the residuals ε_t, a_t, and v_t for independence, normality, and constant variance.

Table 5.15 Calendar years for dummy variables, net migration

	Linear trend	Random walk with drift	AR(1)
Austria	1989 1990 1991 1992	1989	1974 1982 1989
Belgium		1962 1988	1961 1970 1971
Denmark	1974 1975 1995	1974 1995	1974 1995
Finland	1969 1971 1972 1973	1964 1969 1971 1975 1976	1964 1969 1971 1975 1976
France	1961 1962 1969 1970 1970	1962 1963 1968 1969	1962 1963 1968 1969
Germany	1969 1989 1990 1991 1992	1970	1970
Greece	1975 1976 1978 1990 1991		
Iceland	1987 1988 1995		
Ireland	1974 1975 1985 1987 1989	1985	1985
Italy	1971 1992 1993 1994 1996	1992	1961 1971 1992 2000
Luxembourg	1973 1974 1981 1983 1984	1971 1976	1971 1976
Netherlands		1967 1975	1967 1976
Norway	1987 1989 1990 1995	1987 1989	1987 1989 1991 2000
Portugal	1966 1969 1974 1975	1974 1975 1976	1965 1969 1974 1975 1976
Spain	1998 1999 2000	1970 1971 1979 1980 2000	1970 1971 1979 1980 1981 2000
Sweden	1969 1972 1973 1989	1969 1971 1974	1969 1971
Switzerland	1962 1975 1976 1977	1961 1975	1961 1970 1975
UK	1960 1961 1962 1981	1962 1971 1972 1973 1983	1971 1972 1973 1998

Table 5.15 reports the calendar years for which dummies were included for the 18 countries. In many cases, there were major political or economic events in those years, for instance large immigration flows in the years 1989-1992 connected to the reunification of Germany, immigrants from Eastern Europe into Austria in 1989 after the fall of the Iron Curtain, Algeria gaining independence from France in 1962, the end of the Portuguese junta in 1974 and the following independence of Angola and Mozambique in 1975, and the independence of the Dutch colony of Surinam in 1975; see Jennissen (2003). But in other cases, we suspect that dummies are simply a result of discontinuities in migration measurement in connection with a population census, for instance in France in 1968, or in Spain in 2000: when census results reveal larger numbers of inhabitants born outside the country than expected from migration statistics, these statistics may be adjusted in the Census year or the years before.

The estimation results for models (5.3)-(5.5) show that the slope β was significant for twelve of the 18 countries, the drift μ in only one case, the constant K in the AR(1) model in seven cases, and the AR(1)-coefficient ϕ for all countries (with the exception of Italy, Portugal, and Spain; see below). Table 5.16 reports the estimates for the three models and 18 countries.

For Iceland, Ireland, Italy, Portugal, and Spain, the estimated constants K were negative. Since the equilibrium value equals $K/(1-\phi)$, where ϕ is the AR(1) coefficient of which the estimate was less than one in each case, negative values for predicted net migration resulted for those countries. Such negative values were judged unrealistic for Italy, Portugal, and Spain. The negative values appear sooner for low ϕ-values (close to zero) than for large ϕ-values (close to, but still less than one). We avoided negative values for these three countries by selecting ϕ-values equal to 0.95-0.96. The constant and the dummy coefficients were estimated from the data.

Table 5.16 Parameter estimates for models (5.3), (5.4), and (5.5)

	Linear trend model (5.3)			Random walk with drift (5.4)			AR(1)-process (5.5)		
	Parameter	Estimate	t-value	Parameter	Estimate	t-value	Parameter	Estimate	t-value
Austria							λ_{74}	-47988	-3.5
							λ_{82}	-48288	-3.5
	η_{89}	47383	3.2	γ_{89}	35310	1.9	λ_{89}	34676	2.6
	η_{90}	63191	4.3						
	η_{91}	78929	5.3						
	η_{92}	73345	4.9						
	C	8722	3.6				K	5688	2.3
							φ	0.7569	8.6
Belgium							λ_{61}	-50959	-4.1
				γ_{62}	59229	3.5			
							λ_{70}	-43211	-3.5
							λ_{71}	27210	2.0
				γ_{88}	37197	2.2			
	C	9925	3.8				K	7819	3.2
							φ	0.3867	3.1
Denmark	η_{74}	-10929	-2.8	γ_{74}	-19051	-4.1	λ_{74}	-16134	-4.3
	η_{75}	-13274	-3.4						
	η_{95}	19253	4.8	γ_{95}	18105	3.9	λ_{95}	20195	5.4
	C	-463777	-4.4	μ	201	0.3	K	2364	3.1
	β	237	4.5				φ	0.5851	6.4
Finland				γ_{64}	-12559	-3.1	λ_{64}	-13607	-3.6
	η_{69}	-31107	-4.2	γ_{69}	-26016	-6.4	λ_{69}	-28121	-7.3
	η_{71}	20550	2.8	γ_{71}	48255	11.8	λ_{71}	42997	10.1
	η_{72}	20134	2.8						
	η_{73}	18920	2.6						
				γ_{75}	-10127	-2.5	λ_{75}	-9163	-2.4
				γ_{76}	-8441	-2.1	λ_{76}	-8894	-2.4
	C	-1218044	-6.3	μ	511	0.8	K	406	0.6
	β	613	6.3				φ	0.8526	15.2
France	η_{61}	123635	3.8						
	η_{62}	803888	24.6	γ_{62}	680253	26.1	λ_{62}	710017	25.7
				γ_{63}	-725500	-27.8	λ_{63}	-549827	-6.6
				γ_{68}	90170	3.5	λ_{68}	83786	3.3
	η_{69}	94328	2.9	γ_{69}	49051	1.9	λ_{69}	62008	2.5
	η_{70}	122529	3.8						
	η_{71}	85243	2.6						
	C	59980	11.2				K	9620	1.2
							φ	0.7855	7.8

Table 5.16 (continued)

Germany	η_{69}	420853	2.1						
				γ_{70}	-836238	-3.8	λ_{70}	-711831	-3.4
	η_{89}	602379	3.0						
	η_{90}	512467	2.5						
	η_{91}	458864	2.3						
	η_{92}	632698	3.1						
	C	143699	4.3				K	101580	2.5
							φ	0.5997	4.7
Greece	η_{75}	68151	3.2						
	η_{76}	63326	2.9						
	η_{78}	109063	5.1						
	η_{90}	48544	2.2						
	η_{91}	62508	2.9						
	C	-4248343	-7.5	μ	1359	0.3	K	3680	0.8
	β	2146	7.5				φ	0.7517	7.2
Iceland	η_{87}	1235	1.9						
	η_{88}	1473	2.2						
	η_{95}	-1547	-2.3						
	C	-38612	-2.2	μ	49	0.4	K	-43	-0.4
	β	19	2.2				φ	0.4979	3.1
Ireland	η_{74}	26060	1.8						
	η_{75}	24016	1.6						
	η_{85}	-31701	-2.2	γ_{85}	-23855	-2.1	λ_{85}	-25483	-2.5
	η_{87}	-37008	-2.5						
	η_{89}	-28609	-1.9						
	C	-1042945	-2.7	μ	1166	0.7	K	-188	-0.1
	β	524	2.7				φ	0.7156	7.7
Italy							λ_{61}	-62878	-1.8
	η_{71}	-108463	-2.6				λ_{71}	-54827	-1.5
	η_{92}	123899	2.9	γ_{92}	175586	4.4	λ_{92}	174567	4.9
	η_{93}	118154	2.8						
	η_{94}	85549	2.0						
	η_{96}	76839	1.8						
							λ_{2000}	73760	2.1
	C	-9702741	-8.3	μ	2165	0.4	K	3392	0.6
	β	4900	8.3				φ	0.95	see text
Luxembourg				γ_{71}	3979	4.2	λ_{71}	3733	4.1
	η_{73}	2932	2.2						
	η_{74}	3060	2.3						
				γ_{76}	-2474	-2.6	λ_{76}	-2271	-2.5
	η_{81}	-2131	-1.6						
	η_{83}	-2491	-1.9						
	η_{84}	-2328	-1.8						
	C	-136658	-4.0	μ	39	0.3	K	488	2.0
	β	70	4.1				φ	0.8134	9.5

Table 5.16 (continued)

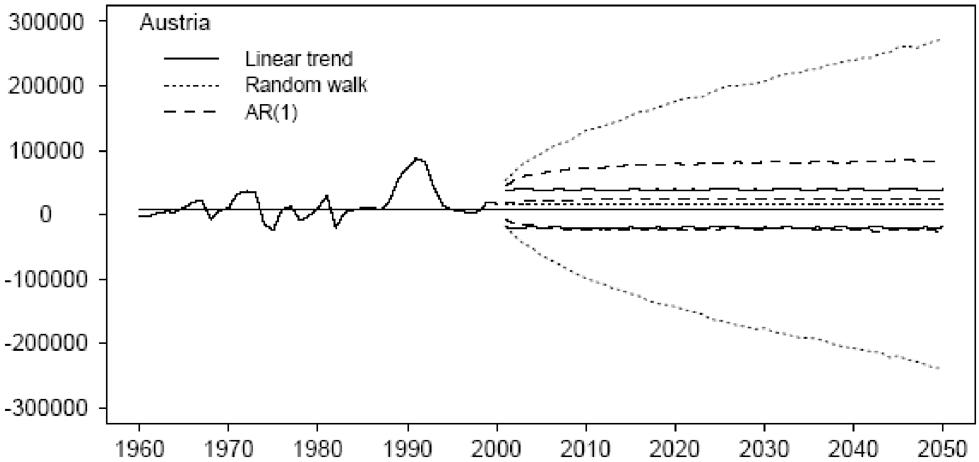
Netherlands				γ_{67}	-33766	-2.2	λ_{67}	-37336	-2.9
				γ_{75}	37357	2.4			
							λ_{76}	-40702	-2.8
	C	-1577412	-3.8	μ	1650	0.7	K	11365	3.2
	β	809	3.8				φ	0.6769	5.4
Norway	η_{87}	6104	2.7	γ_{87}	5857	2.0	λ_{87}	6034	2.5
	η_{89}	-9971	-4.4	γ_{89}	-12388	-4.2	λ_{89}	-11949	-4.9
	η_{90}	-6813	-3.0						
							λ_{91}	5322	2.2
	η_{95}	-3852	-1.7						
							λ_{2000}	-8543	-3.1
	C	-663145	-11.1	μ	462	1.0	K	966	1.7
	β	337	11.1				φ	0.9077	9.6
Portugal							λ_{65}	-49396	-1.9
	η_{66}	-96436	-2.2						
	η_{69}	-134473	-3.0				λ_{69}	-78483	-3.0
	η_{74}	230602	5.2	γ_{74}	258089	8.7	λ_{74}	250876	9.3
	η_{75}	399442	9.1	γ_{75}	172210	5.8	λ_{75}	252890	6.8
				γ_{76}	-335932	-11.3	λ_{76}	-323897	-12.5
	C	-7259326	-6.2	μ	279	0.1	K	2123	0.5
	β	3649	6.1				φ	0.94	see text
Spain				γ_{70}	184723	5.1	λ_{70}	177775	7.6
				γ_{71}	-219715	-6.0	λ_{71}	-217074	-9.2
				γ_{79}	-86643	-2.4	λ_{79}	-89444	-3.8
				γ_{80}	142222	3.9	λ_{80}	135442	5.8
							λ_{81}	-149946	-6.4
	η_{98}	94651	1.8						
	η_{99}	158493	3.0						
	η_{2000}	327727	6.1	γ_{2000}	164146	4.5	λ_{2000}	168838	7.2
	C	-3905508	-2.1	μ	7058	1.1	K	12007	2.8
	β	1971	2.1				φ	0.95	see text
Sweden	η_{69}	27102	2.3	γ_{69}	32614	3.2	λ_{69}	30593	3.0
				γ_{71}	-45950	-4.5	λ_{71}	-37095	-3.4
	η_{72}	-29649	-2.5						
	η_{73}	-29057	-2.4						
				γ_{74}	20252	2.0			
	η_{89}	27070	2.3						
	C	17123	8.8				K	5407	2.1
							φ	0.7084	5.9
Switzerland									
				γ_{61}	77673	4.9	λ_{61}	78284	6.6
	η_{62}	59613	2.7						
							λ_{70}	-37991	-3.2
	η_{75}	-79797	-3.6	γ_{75}	-59503	-3.7	λ_{75}	-65498	-5.5
	η_{76}	-76003	-3.4						
	η_{77}	-44831	-2.0						
	C	21950	6.1				K	6523	2.9
							φ	0.6812	10.7

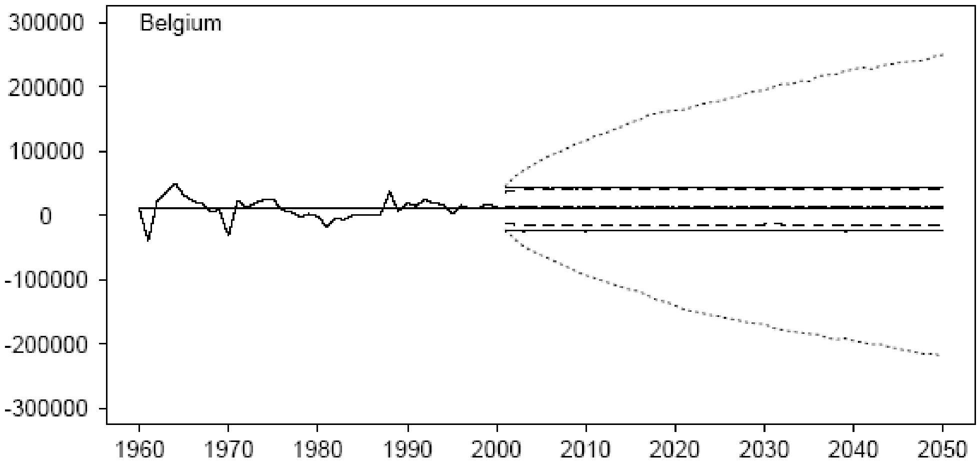
Table 5.16 (continued)

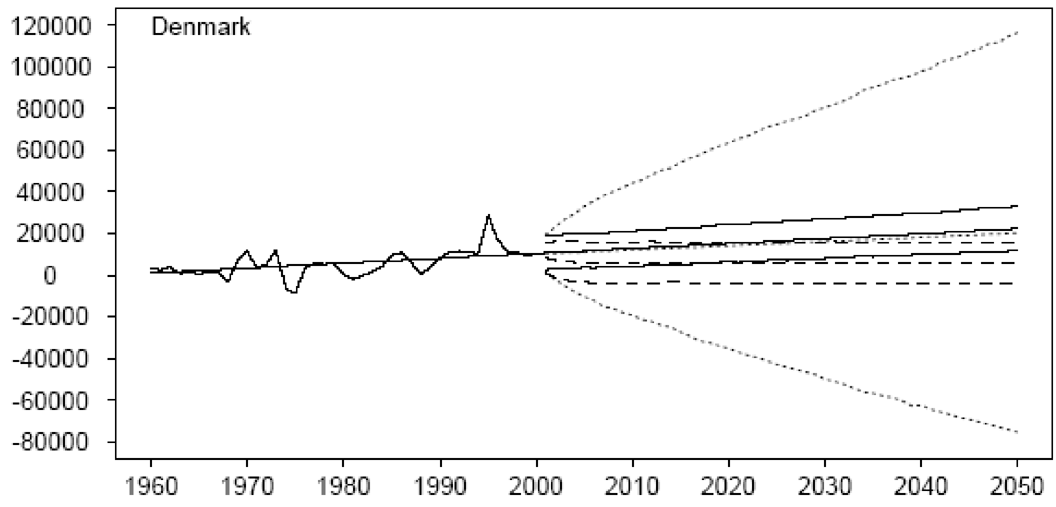
UK	η_{60}	200514	5.2						
	η_{61}	233999	6.0						
	η_{62}	143823	3.7	γ_{62}	-87855	-2.7			
				γ_{71}	-76473	-2.4	λ_{71}	-78990	-2.2
				γ_{72}	113492	3.5	λ_{72}	100395	2.8
				γ_{73}	-88881	-2.7	λ_{73}	-85156	-2.4
	η_{81}	-91389	-2.5						
				γ_{83}	71450	2.2			
							λ_{98}	71116	2.0
		-							
	C	10771972	-10.0	μ	3129	0.6	K	5395	0.9
	β	5451	10.1				φ	0.8558	10.3

All 3x18 = 54 models reported in Table 5.16 passed tests for independence and constant variance in the residuals (α=0.05), but the residuals were not normal in a number of cases: Austria (RWD), France (all three models), Ireland (RWD), Italy (RWD and AR(1)), the Netherlands (RWD), Portugal (RWD and AR(1)), Spain (all three models), Sweden (LT and RWD), and Switzerland (LT).

Figure 5.9. Forecasts and 95% prediction intervals for net migration. Data 1960-2000






Denmark
120000
100000
80000
60000
40000
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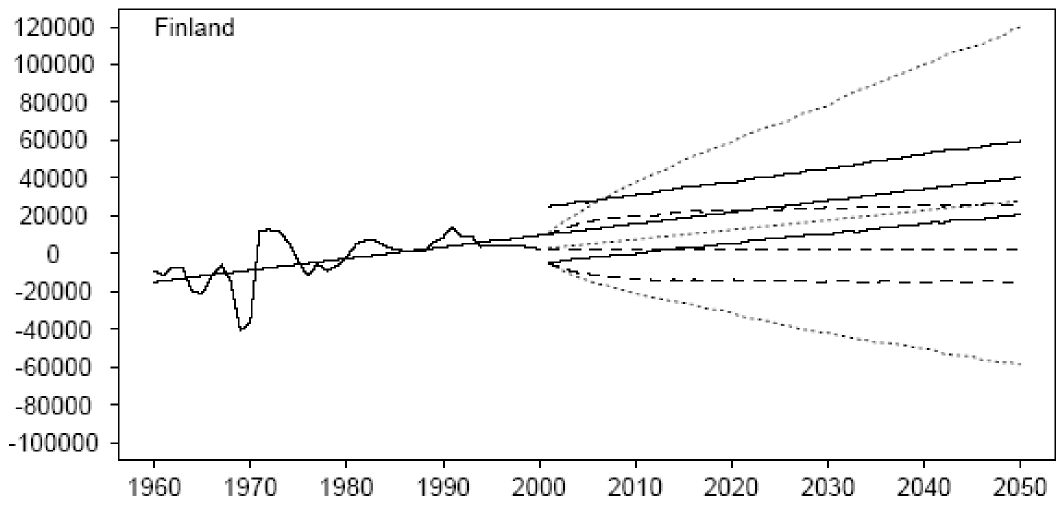
-40000
-60000
-80000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Finland
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80000
60000
40000
20000
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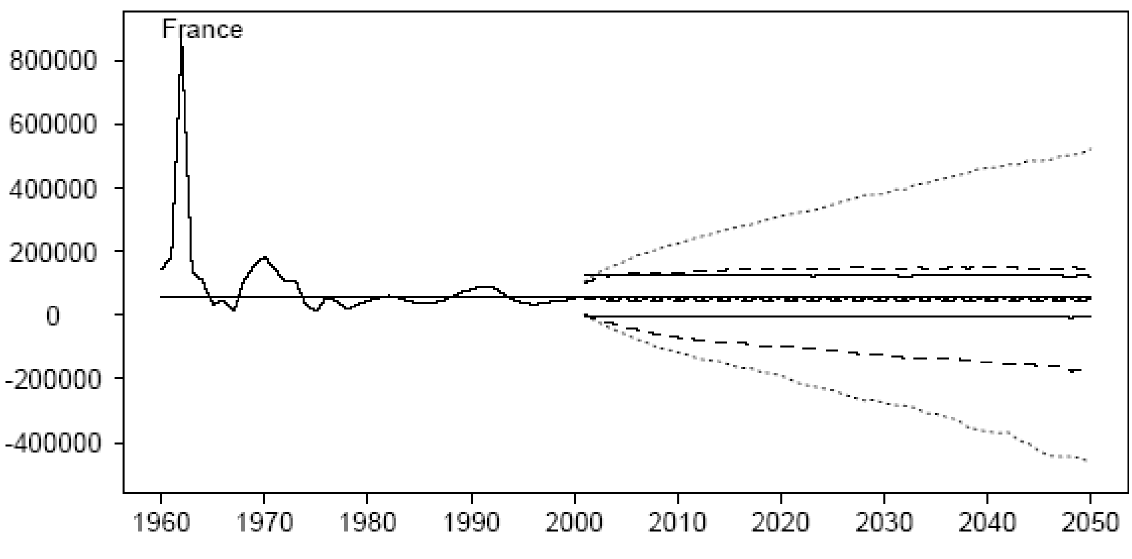
-20000
-40000
-60000
-80000
-100000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


France
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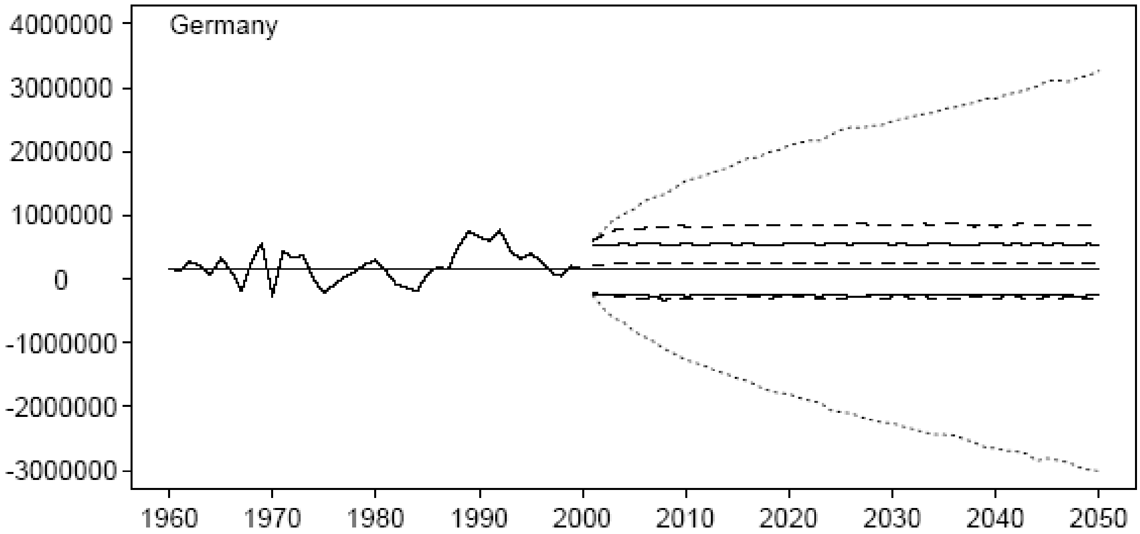
-400000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Germany
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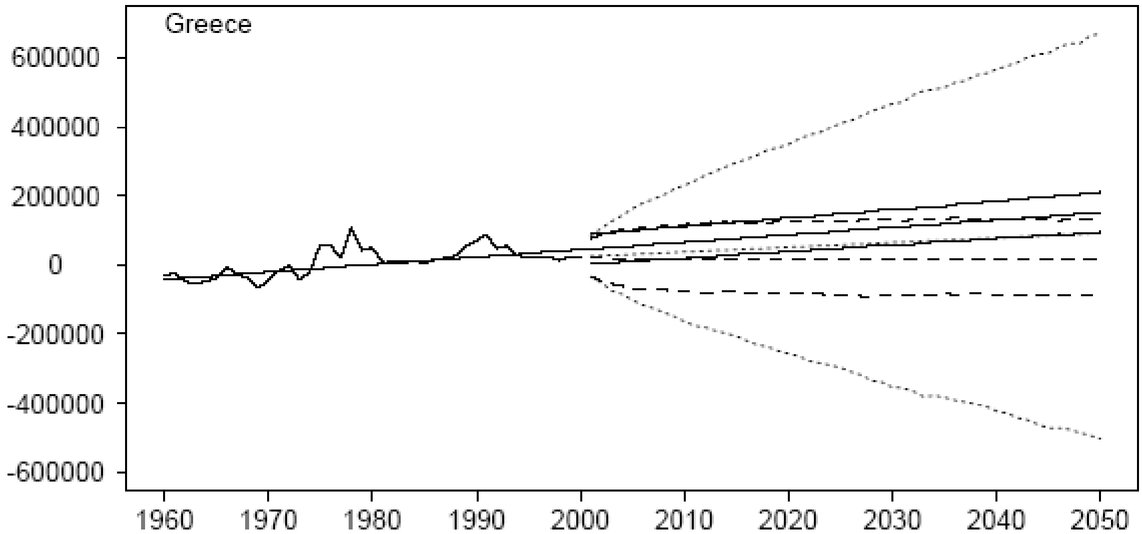
-2000000
-3000000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Greece
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1990
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2020
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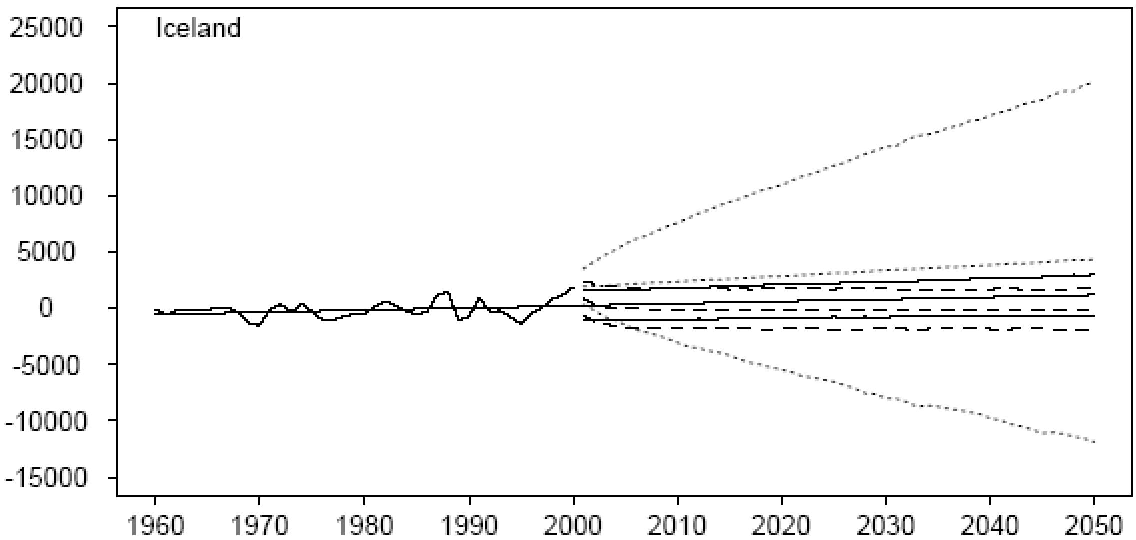
2040
2050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-15000
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1990
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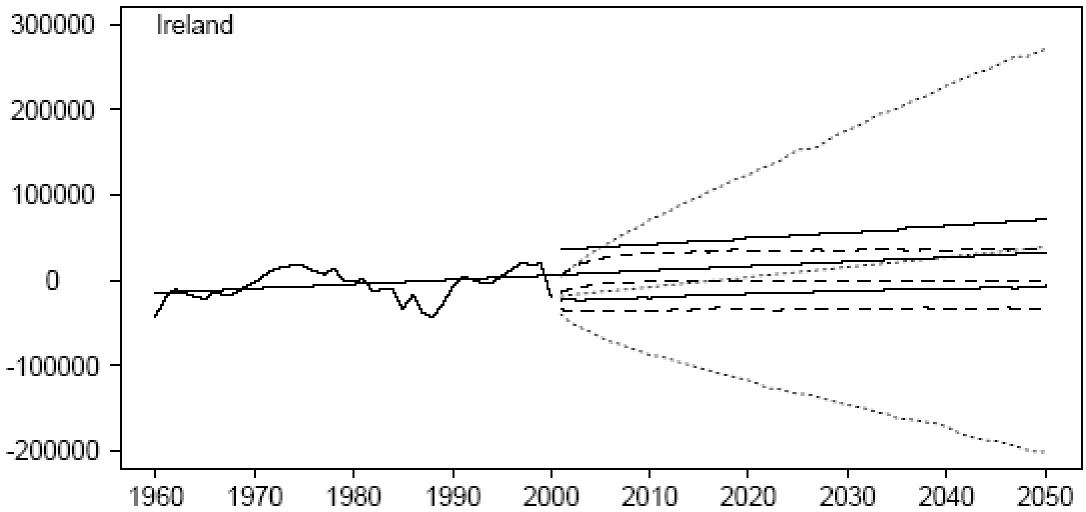
2020
2030
2040
2050


Ireland
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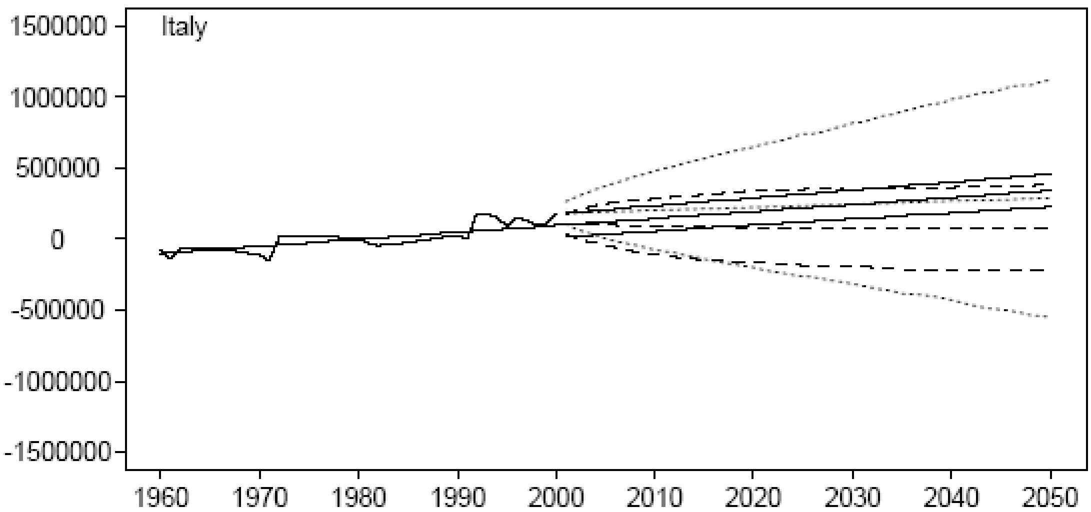
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Italy
1500000
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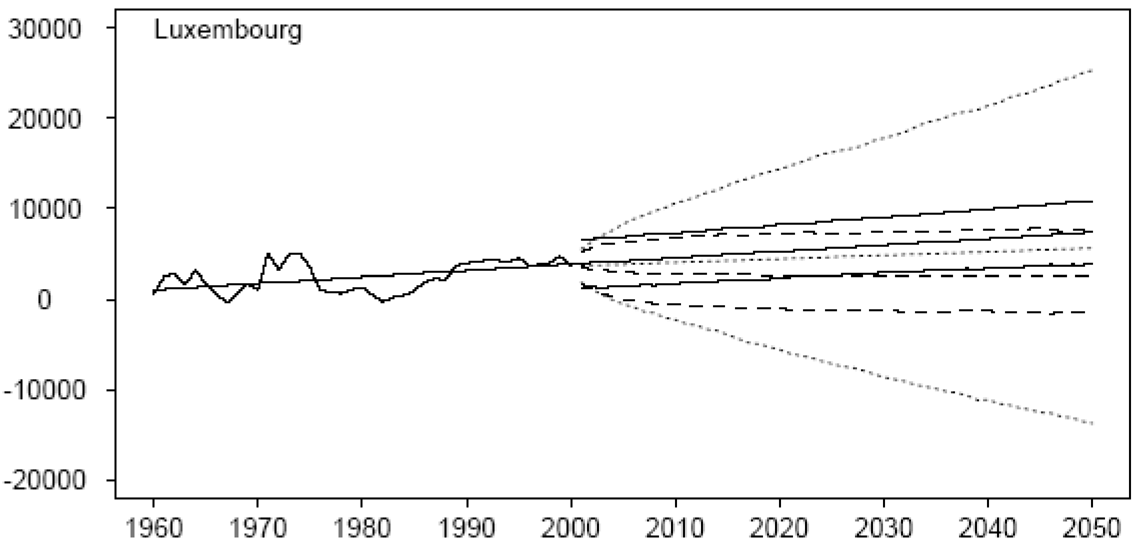
500000
0
-500000
-1000000
-1500000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Luxembourg
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1960
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1990
2000
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2020
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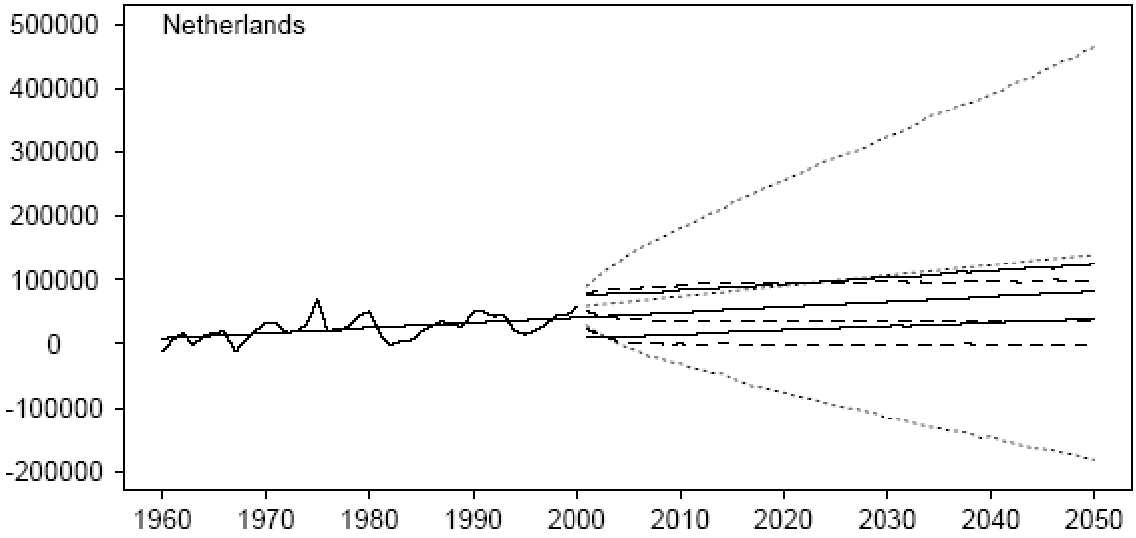
2040
2050


Netherlands
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300000
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1960
1970
1980
1990
2000
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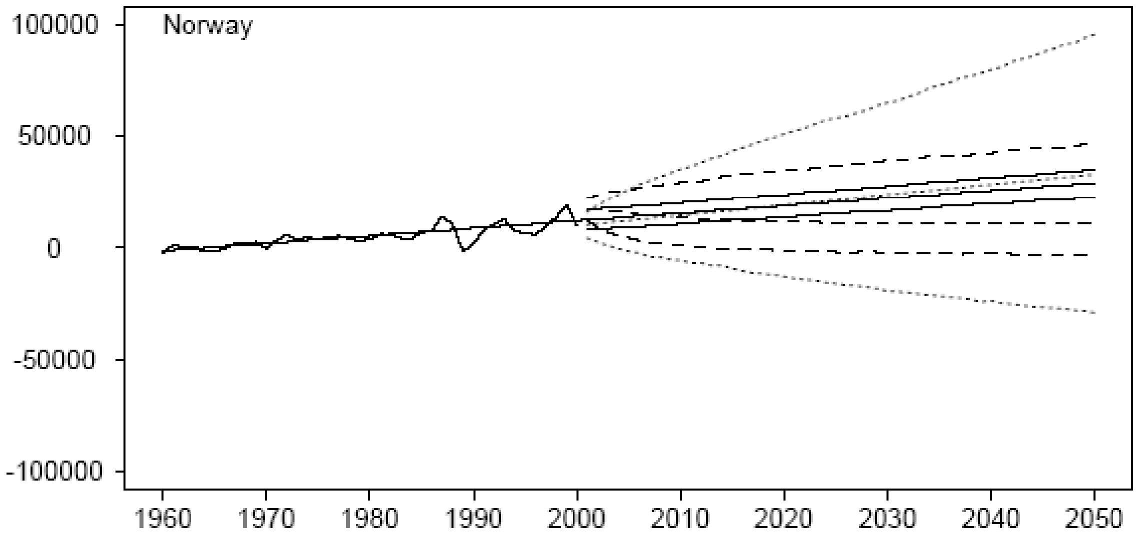
2020
2030
2040
2050


Norway
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50000
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-100000
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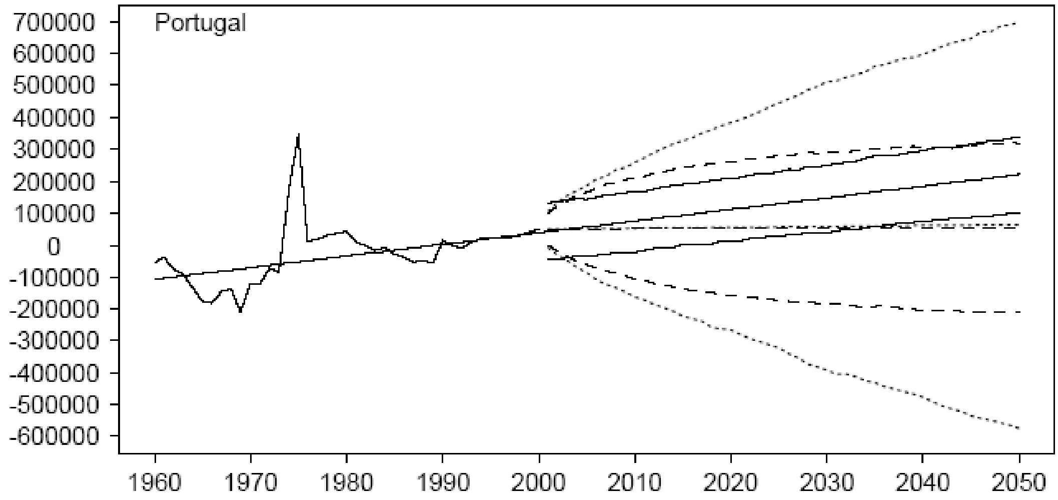
1970
1980
1990
2000
2010
2020
2030
2040
2050


Portugal
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400000
300000
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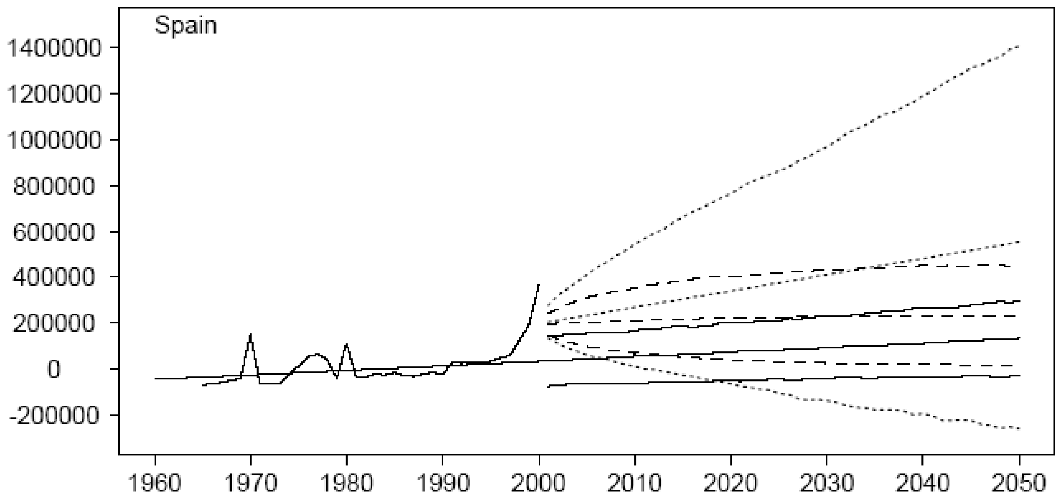
100000
0
-100000
-200000
-300000
-400000
-500000
-600000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050


Spain
1400000
1200000
1000000
800000
600000
400000
200000
0
-200000
1960
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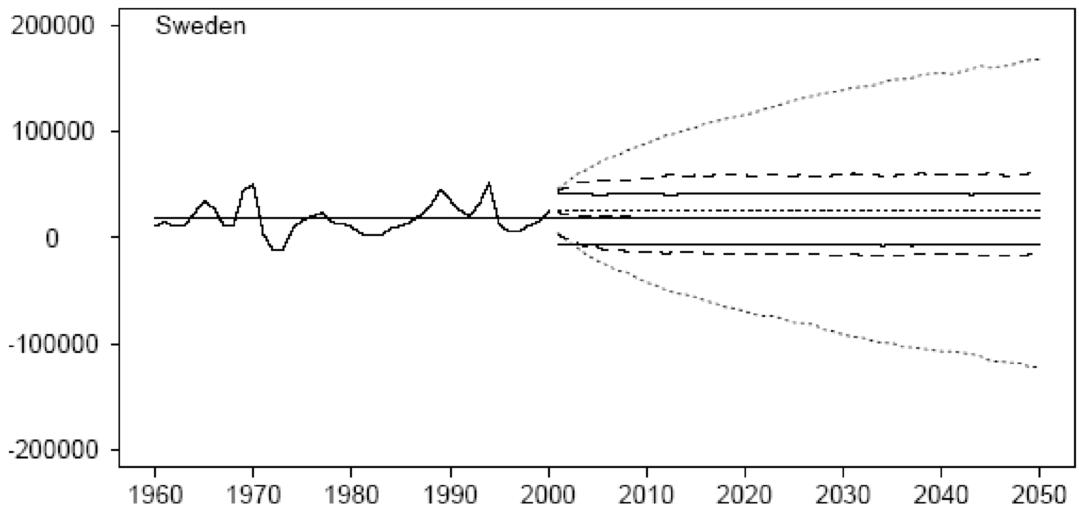
2030
2040
2050


Sweden
200000
100000
0
-100000
-200000
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

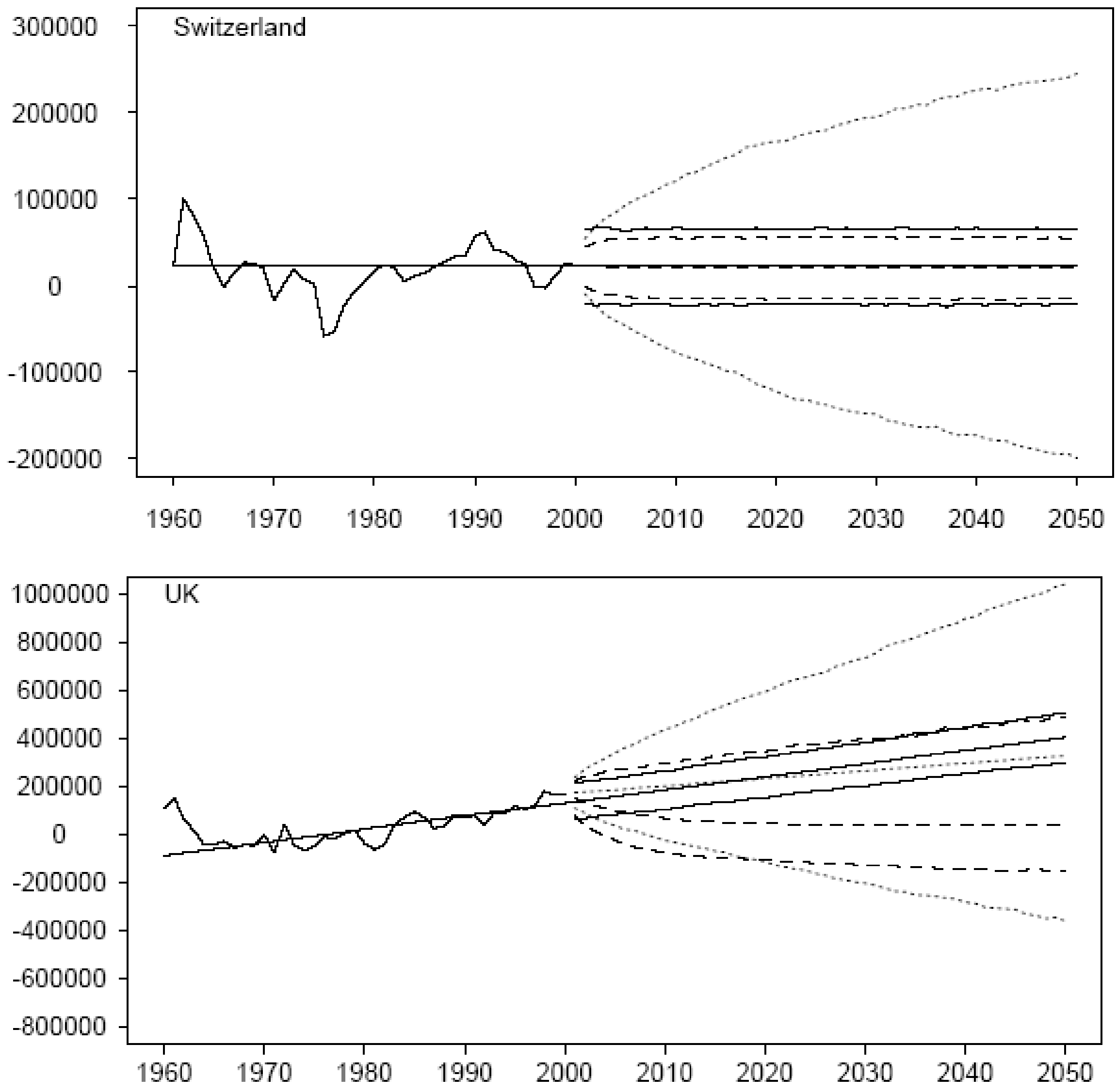
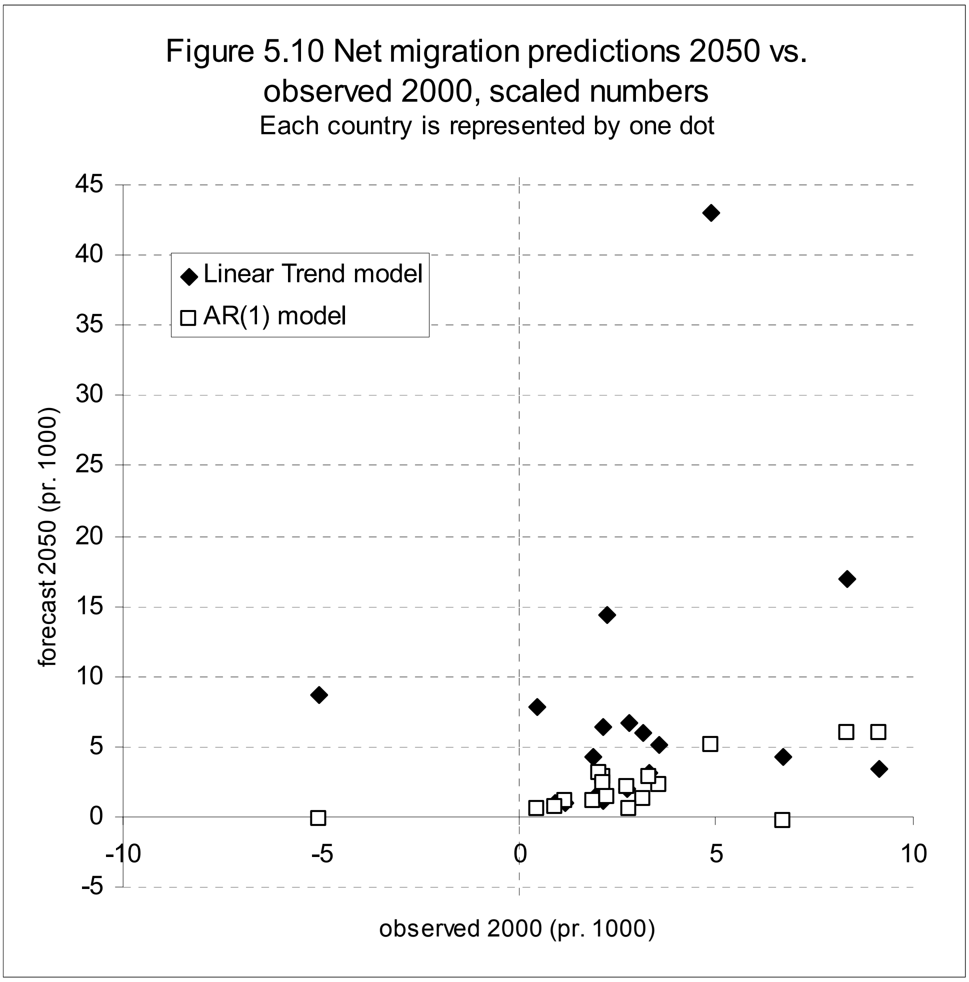


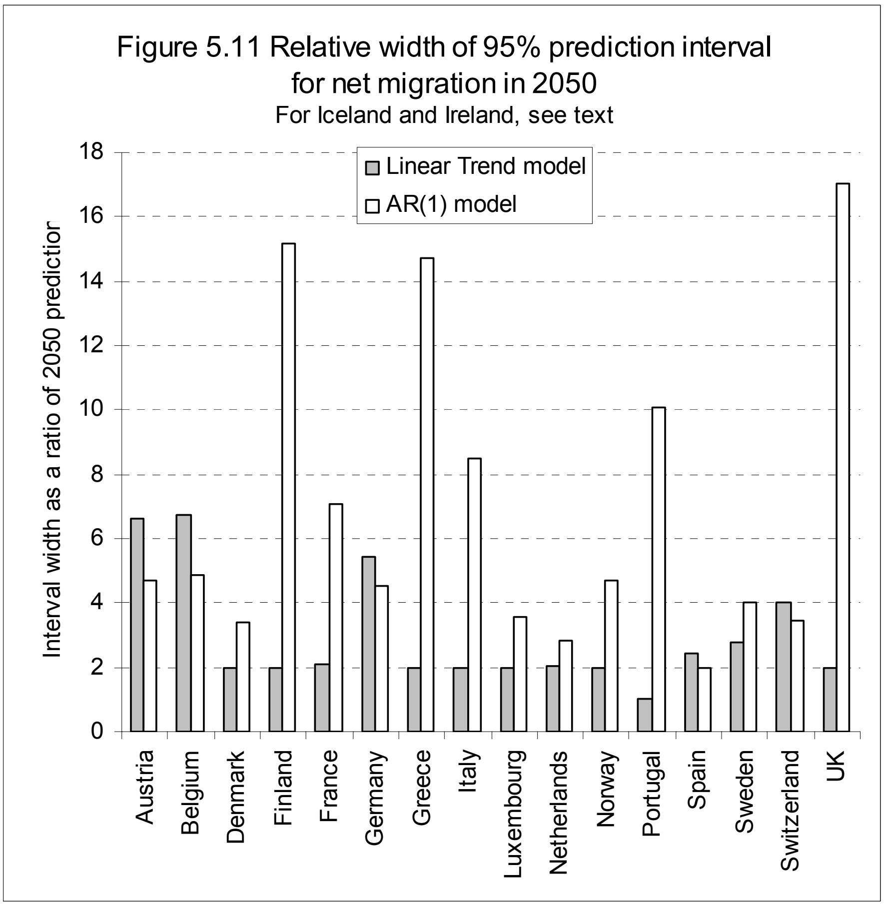
Figure 5.9 shows point predictions and prediction intervals for the three models in each country. The graphs as such are difficult to compare across countries, because absolute numbers for net migration differ widely internationally. Yet Figure 5.9 shows that the Random Walk with Drift model predicts much wider intervals than the historical data suggest. In addition, as noted above, RWD-residuals are heteroscedastic for eight countries. For these two reasons we conclude that the RWD-model is not appropriate for our purposes.

When we inspect the results for the other two models, the intervals for the AR(1)-model tend to be somewhat wider than those for the LT-model. The reason is that prediction uncertainty for multiple steps ahead forecasts cumulates for AR(1), as opposed to LT, in which each forecast step starts from the trend line. Although the AR(1)-model fails to predict the trend in a number of cases (Finland, Greece, Italy, Luxembourg, Norway, Portugal, Spain, the UK), its prediction intervals look more reasonable than those for LT.

In order to compare the forecasts across countries, we have scaled net migration in each country by the national population size as of 1 January 2000. Figure 5.10 plots the scaled LT forecasts and AR(1) forecasts in 2050 as a function of the scaled observed migration in 2000. The figure reveals that the AR(1) model predicts 2050 values that are closer to observed 2000 values than LT does. AR(1) predictions for 2050 are generally in the order of magnitude of up to 5 per thousand, whereas half the LT predictions exceed this level. The reason is again the linear trend in the historical data, which is not picked up by AR(1).



A second cross-country comparison concerns the width of the prediction intervals. We have plotted the width of the 95 per cent prediction interval in 2050, relative to the predicted value for 2050, for the LT and the AR(1) model; see Figure 5.11.



Iceland and Ireland are not included in Figure 5.11, because the 2050 predictions from the AR(1) model were just below zero: -0.31 per thousand for Iceland, and –0.18 per thousand for Ireland. Both numbers are expressed as predicted net migration in 2050 relative to population size in 2000. These small negative numbers produced huge negative relative widths for the intervals.

Figure 5.11 shows that LT generally produces narrower intervals than AR(1). There are two reasons. First, LT-intervals for some countries are narrow because the model picks up a linear trend. The estimated trend is moderate for Denmark, Italy, Luxembourg, Netherlands, Norway, and Spain, while Finland, Greece, Portugal, and the UK show a strong trend. Second, AR(1)-intervals are wide because the *K*-estimate has a large standard error. This is the case for Finland, France, Greece, Italy, Portugal, and the UK; see Table 5.16.

5.4.2 Migration forecasts for the European Economic Area plus Switzerland

When net migration numbers are added across countries, the result is the net migration for the whole area. Migration forecasts for the European Economic Area as a whole are of particular interest for European policy makers. In this brief section we present such forecasts for the EEA+, which is to be understood as the European Economic Area (including Liechtenstein), plus Switzerland. As before, we have fitted a Linear Trend model, a Random Walk with Drift, and an AR(1) process. We have observed migration for the period 1960-2000, except for Spain, for which the observations start in 1965. Figure 5.12 plots the observed net migration for the EEA+, while Figure 5.13 gives the forecasts and the prediction intervals. Model estimates are contained in Table 5.17. A dummy variable for the period 1960-1964 captures the effect of missing data for Spain for these years.

Figure 5.12. Net migration to the European Economic Area plus Switzerland (EEA+)

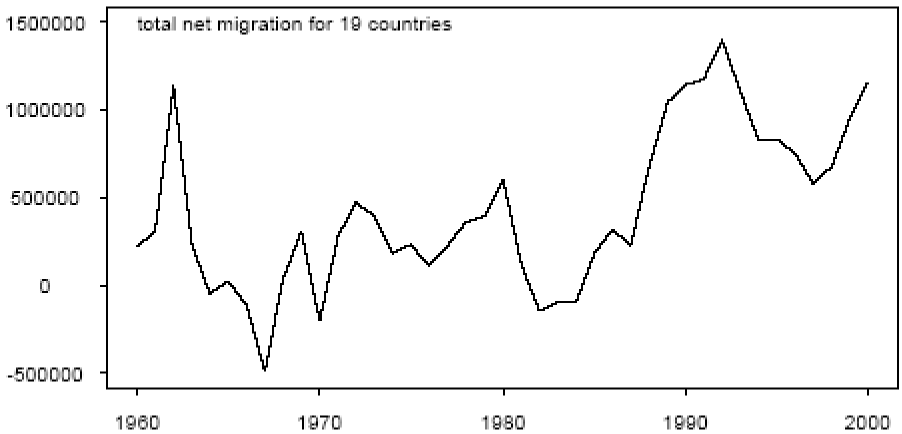


Figure 5.13. Forecasts and 95% prediction intervals for net migration to EEA+

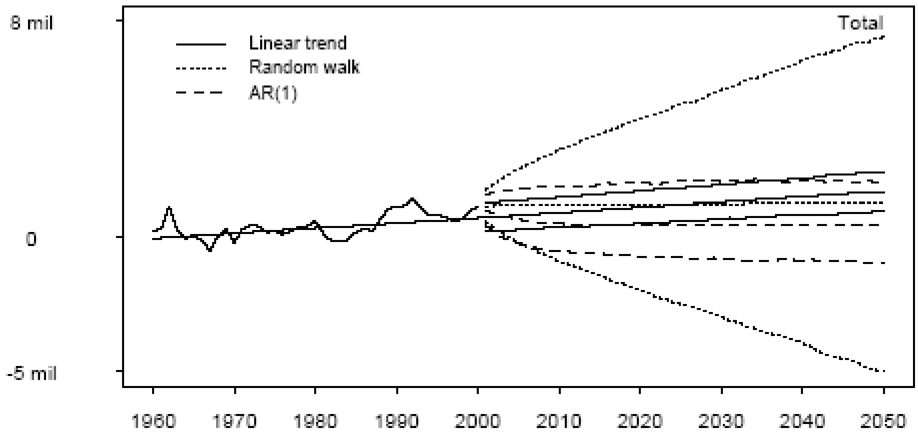


Table 5.17 Parameter estimates for models (5.3), (5.4), and (5.5), EEA+

	Linear trend model (5.3)			Random walk with drift (5.4)			AR(1)-process (5.5)		
	Parameter	Estimate	t-value	Parameter	Estimate	t-value	Parameter	Estimate	t-value
EEA+									
	η_{Spain}	300719	2.0	γ_{Spain}	-400199	-2.4	λ_{Spain}	-375659	-2.4
	η_{62}	956762	3.5	γ_{62}	1196406	3.8	λ_{62}	1153652	3.8.8
	η_{67}	-484435	-1.9						
	η_{89}	527194	2.1						
	η_{90}	597602	2.4						
	η_{91}	607404	2.4						
	η_{92}	806258	3.2						
	η_{93}	495702	2.0						
	C	-46647826	-5.5	μ	33322	0.7	K	110010	1.9
	β	23715	5.7				φ	0.8097	8.5

The large migration flow from Algeria into France in 1962 dominates the pattern in the early 1960s in the case of the EEA+, too. Also, consequences for European migration of the fall of the Berlin Wall and the reunification of Germany are clearly visible. The effects continued until 1993, at least as measured as a systematic deviation from a linear trend.

The three models predict very different levels of net migration in 2050, ranging from 580 000 for AR(1) to 1.96 million for LT and 2.82 million for RWD. To compare: the simple sums across countries of the point forecasts in 2050 are 842 000 (AR(1)), 1.73 million (LT), and 1.89 million (RWD). The random walk model for the EEA+ predicts much higher net migration than the 18 country-specific random walk models do. The reason is that the estimated drift for EEA+ (33 000) is almost twice the sum of the country-specific drifts (18 000). The high point forecast from the RWD model (2.82 million in 2050) together with the wide prediction interval (from –3.2 million to 8.7 million in 2050; 95%) lead to the conclusion that the RWD model is not very useful for predicting net migration to the EEA+. In practice, the linear trend model excludes the possibility for a net emigration from the EEA+ in 2050: the 95% prediction interval around the predicted value of 1.96 million ranges from 1.22 to 2.73 million. For the AR(1) model the interval bounds are -0.55 million and 3.21 million.

The cumulated net migration flows for the period 2000-2050 are 31.4 million (0.9 to 120.3 million for the 95% bounds) for the AR(1) model, and 69.4 million (50.4 to 88.5 million) for LT.

5.4.3 Cross-country correlations

The fall of the Berlin Wall in 1989 induced large migration flows from Eastern Europe into EEA-countries, in particular Germany and Austria. The war in the former Yugoslavia in the 1990s led to large numbers of refugees from Bosnia, who settled into Western and Nordic countries, such as the

Netherlands, Norway and Denmark. These two examples illustrate that migration flows may be correlated across countries. This is also shown in empirical data, see Table 5.18. Next, the question is how large the residual correlation is after we fitted the migration time series models in this section.

Table 5.18 Correlations across countries for observed net migration, data 1960-2000

	A	B	Dk	SF	F	D	EL	Is	IRL	I	Lux	Nl	N	P	E	S	CH
Austria	1																
Belgium	0.239	1															
Denmark	0.304	-0.135	1														
Finland	0.381	0.267	0.147	1													
France	0.347	-0.191	0.140	-0.276	1												
Germany	0.706	0.422	0.321	0.245	0.307	1											
Greece	0.285	0.060	0.100	0.437	-0.421	0.215	1										
Iceland	0.063	0.316	-0.294	0.471	-0.236	-0.024	0.064	1									
Ireland	-0.090	0.017	0.157	0.158	0.024	-0.147	0.151	-0.197	1								
Italy	0.194	0.265	0.482	0.457	-0.348	0.278	0.504	0.017	0.360	1							
Luxembourg	0.390	0.504	0.386	0.477	0.174	0.547	0.157	0.048	0.405	0.536	1						
Netherlands	0.243	0.219	0.023	0.133	0.025	0.308	0.516	0.048	0.271	0.336	0.538	1					
Norway	0.085	0.098	0.332	0.475	-0.209	0.096	0.318	0.527	0.059	0.613	0.310	0.254	1				
Portugal	-0.249	0.064	-0.160	0.385	-0.505	-0.248	0.569	0.068	0.461	0.484	0.286	0.533	0.292	1			
Spain	-0.061	-0.343	0.348	-0.134	0.005	-0.180	0.344	-0.255	0.383	0.420	0.112	0.408	0.287	0.350	1		
Sweden	0.213	0.045	0.071	-0.504	0.214	0.273	0.084	-0.346	-0.371	0.013	0.027	0.238	-0.169	-0.188	0.243	1	
Switzerland	0.630	0.123	0.268	0.371	0.157	0.644	0.088	0.313	-0.414	0.120	0.193	-0.046	0.246	-0.346	-0.255	0.147	1
UK	0.239	-0.013	0.679	0.363	-0.186	0.220	0.395	0.055	0.010	0.642	0.350	0.187	0.527	0.143	0.406	0.121	0.311

Table 5.19. Correlations for residuals across countries from fitted AR(1) models for net migration

	A	B	Dk	SF	F	D	EL	Is	IRL	I	Lux	Nl	N	P	E	S	CH
Austria	1																
Belgium	0.07	1															
Denmark	0.23	0.00	1														
Finland	0.57	-0.16	0.06	1													
France	0.38	-0.09	0.31	0.24	1												
Germany	0.20	0.10	0.06	-0.14	0.36	1											
Greece	-0.12	0.05	-0.24	-0.02	-0.14	-0.08	1										
Iceland	0.20	0.21	-0.27	0.14	-0.13	-0.22	0.11	1									
Ireland	0.16	-0.05	0.16	0.08	-0.14	0.09	0.23	-0.30	1								
Italy	-0.08	0.33	0.14	-0.11	-0.04	-0.14	0.09	0.03	-0.11	1							
Luxembourg	0.00	0.04	0.36	-0.23	0.11	0.51	-0.16	-0.25	0.23	0.09	1						
Netherlands	-0.11	0.29	0.02	-0.14	-0.01	0.01	0.37	0.03	-0.01	0.21	0.14	1					
Norway	0.30	-0.06	0.05	0.23	-0.10	0.02	0.05	0.30	0.21	-0.05	0.10	0.13	1				
Portugal	-0.03	0.24	0.25	-0.08	0.08	-0.08	0.14	-0.10	0.13	0.12	-0.17	0.29	0.01	1			
Spain	-0.23	0.12	0.28	-0.09	0.00	-0.26	0.21	-0.05	0.12	0.27	0.15	0.27	0.04	0.17	1		
Sweden	-0.21	0.25	0.09	-0.26	0.12	0.06	0.12	-0.07	-0.29	0.21	0.19	0.23	-0.19	0.06	0.14	1	
Switzerland	0.22	0.06	-0.13	0.38	0.02	0.17	0.23	0.28	0.01	-0.29	-0.12	0.12	0.30	0.31	-0.24	-0.03	1
UK	-0.15	-0.12	0.26	-0.07	0.25	0.03	0.23	-0.23	0.04	0.04	0.13	-0.02	-0.15	0.13	0.21	0.13	-0.21

Correlations larger than 0.5 in absolute value are marked in Table 5.18. (All correlations larger than 0.25 are significant at the 5 per cent level. However, marking those values would not have led to an interpretable regional pattern. The threshold level of 0.5 is selected more or less arbitrarily.) We note a cluster of Central European countries (Austria, Switzerland, and Germany) and the Benelux countries, a South European cluster (Portugal, Italy, and Greece), and a North-West European cluster (Denmark, Iceland, Norway, and the UK). The countries in these three clusters are positively correlated. There are also two clusters (Finland/Sweden and France/Portugal) with negative correlations. Probably this is explained by counter cyclical economic developments, which induce labour migration between the two countries in each pair. When we inspect in Table 5.19 the residuals from the AR(1) model, the time series model that we judged as giving the most realistic prediction intervals, there are still two correlations that exceed 0.5, namely those between Germany and Luxembourg (0.51), and between Finland and Austria. The latter one is difficult to interpret. Of the correlations that still exceed the critical value of 0.26, we mention the neighbouring countries Germany/France (0.36), Spain/Italy (0.27) and Belgium/Netherlands (0.29) with a moderate positive correlation, and the negative correlations for Finland/Sweden (-0.26), Denmark/Iceland (-0.27), and Italy/Switzerland (-0.29), which probably reflect labour migration.

6. Conclusion

This report contains the main findings of Work Packages 2 and 3 of the EU-funded research project "Changing Population of Europe: Uncertain Future", abbreviated as UPE (Uncertain Population of Europe). The major goal of the UPE-project is to develop and implement stochastic population forecasts for 18 European countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. In this report we focus on observed and expected forecast errors in total fertility, the life expectancy at birth, and the level of net migration in each country.

We have analysed observed forecast errors in historical forecasts for the three indicators in 14 countries. The TFR forecasts in 14 countries made since the 1960s indicate that TFR-predictions have been wrong by 0.3 children per woman for forecasts 15 years ahead, and 0.4 children per woman 25 years ahead. Our results confirm earlier findings that the TFR-predictions were too high on average, mainly as a result of the sharp fall in fertility in the 1960s and 1970s. The absolute TFR-errors have a distribution that is close to an exponential distribution. The commonly assumed normal distribution fits the data somewhat less well. Absolute TFR-errors in the following countries were moderately correlated: Denmark, Finland, Netherlands, Norway, Switzerland, and the UK, but the correlation pattern is not stable across forecast duration.

Absolute errors in life expectancy forecasts for men and women increase by 0.2 years per year for forecast horizons 10-25 years, and somewhat slower for shorter durations. The forecasts have been too low on average: forecasters in the 14 countries have been too pessimistic in the past, and predicted too slow increases in the life expectancy. The underprediction amounts to 1.0-1.3 and 3.2-3.4 years of life expectancy at forecast horizons of 10 and 20 years ahead, respectively. The distributions of the absolute errors are close to a normal one, in particular for men. Errors for men and women are strongly correlated, with a correlation coefficient around 0.7. Correlations across countries are different for men and women. For women, low-mortality countries move together, with a correlation coefficient equal to 0.8-0.9. For men, there is no systematic correlation pattern across countries.

Migration forecasts for Austria, West Germany, Luxembourg, Portugal, and Switzerland were clearly less accurate than the average for the 14 countries, for different reasons: large unforeseen immigration flows after the fall of the Berlin Wall (Germany, Austria), small population size combined with large migration flows that are inherently difficult to predict (Luxembourg), or simply inaccurate migration statistics (Portugal). Migration has been consistently underpredicted in historical forecasts. The error distribution of the absolute error in migration is generally exponential, although for low probabilities the errors are more extreme than an exponential distribution would predict. There is no systematic pattern in cross-country correlations.

We have used ARCH and GARCH time series model to construct prediction intervals to 2050 for fertility and mortality in all 18 countries. TFR-predictions in 2050 show 95% prediction intervals with a width that ranges from a low 1.5 (Luxembourg) and 1.7 (Finland, Switzerland), to a high 2.9 (Austria, Germany) and 3.4 children per woman (Sweden). Sweden seems to be an outlier in this regard, since the interval width for none of the other countries exceeds 3 children per woman. Ten years-ahead 95 per cent intervals are 0.6 (Greece) to 1.2 (Sweden) children per woman wide. Models estimated on the basis of long data series (1900-2000) tended to result in prediction intervals in 2050 that on average were slightly narrower than those for short data series (1950-2000).

The time series models indicate that between 2000 and 2050, life expectancy at birth for men and women is expected to rise by between 6 and 13 years. Across countries and sexes, the average annual increase amounts to 0.2 years. This is in line with historical developments. Long-range (fifty years) 95 per cent prediction intervals are 4-14 years wide, with Swedish men and women from England and Wales at the lower end of the spectrum, and Danish men and women a the upper end. Ten years ahead 95 per cent intervals are between 1 (men in France, women in England and Wales) and 4 (Denmark, both men and women) years wide.

Net migration was modelled as an autoregressive process, which led to reasonable prediction intervals. An alternative model, i.e. a linear trend model, showed relatively narrow intervals. The predictions indicate that net migration to the total of the 18 countries involved may increase to between 600 000 and 2 million in the year 2050, depending on the particular time series model one adopts. Cumulated immigration for the period 2000-2050 could amount to between 30 and 70 million persons.

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Appendix 1

Data sources for forecast variables

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Estimation of TFR values for Denmark and England and Wales for the years 1900-1911

For Denmark and for England and Wales, Chesnais (1992) tabulates annual TFR-values starting in 1911, and isolated data points for 1903 and 1908. On the other hand, his Table A1.5 lists pre-1911 values for the Crude Birth Rate (CBR) in the two countries. We have estimated TFR-values for the years 1900-1910 based on a linear regression between TFR and CBR. A plot of TFR and CBR showed a near linear relationship for the years 1911-1940. We have used this relationship and "backcasted" the TFR for the years 1900-1910. This way, we obtained the values and 95 per cent prediction intervals as given in Table A2.1. The TFR for the period 1911-2000 for the two countries (see Figure 5.2) and the corresponding first difference do not reveal any apparent anomalies in the predictions for the years 1900-1910.

Table A2.1. Backcasts and 95 % prediction interval bounds for the TFR of Denmark and England and Wales, 1900-1910

	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
Denmark											
Prediction	4.07	4.07	3.99	3.91	3.94	3.86	3.88	3.83	3.89	3.83	3.71
L95	4.01	4.01	3.93	3.85	3.88	3.80	3.81	3.77	3.83	3.77	3.65
U95	4.12	4.12	4.04	3.96	3.99	3.91	3.93	3.88	3.94	3.88	3.77
E & W											
Prediction	3.49	3.47	3.47	3.47	3.42	3.34	3.33	3.26	3.28	3.19	3.12
L95	3.27	3.25	3.25	3.25	3.20	3.14	3.14	3.07	3.09	3.01	2.95
U95	3.71	3.68	3.68	3.68	3.62	3.54	3.53	3.44	3.46	3.36	3.27

For Denmark, Chesnais (1992, 545) reports a TFR in 1908 equal to 3.84, and in 1903 equal to 4.04. Our estimate for 1908 is close to Chesnais' value, while the 1903-estimate is somewhat lower. Chesnais' figures for England and Wales are 3.14 (1908) and 3.40 (1903). Our 1908-estimate is a bit higher, while the 1903-estimate is close.

A2.2 Estimation of life expectancy values for Iceland, Ireland, and Spain

We used simple univariate linear regression models to estimate missing life expectancy values for the three countries based on observed life expectancies in Norway, England and Wales, and Portugal, respectively. Table A2.2 gives prediction results.

Table A2.2. Predictions and 95 % prediction interval bounds for the life expectancy of men and women in Iceland, Ireland, and Spain

		Ireland									
		1974	1975	1976	1977	1978	1979	1981	1982	1983	1984
Men	Prediction	69.1	69.3	69.3	69.7	69.6	69.7	70.4	70.5	70.7	71.0
	L95	69.0	69.2	69.2	69.6	69.5	69.6	70.3	70.4	70.6	71.0
	U95	69.2	69.4	69.4	69.8	69.7	69.8	70.5	70.6	70.8	71.1
Women	Prediction	74.0	74.3	74.3	74.8	74.7	74.9	75.6	75.8	76.0	76.5
	L95	73.9	74.3	74.1	74.7	74.6	74.7	75.5	75.6	75.9	76.4
	U95	74.2	74.4	74.4	75.0	74.9	75.0	75.8	75.9	76.2	76.6

		Iceland							
		1961	1962	1963	1964	1965	1967	1968	1969
Men	Prediction	72.2	71.9	71.7	72.3	72.1	72.4	72.3	71.7
	L95	71.7	71.4	71.1	71.8	71.5	71.9	71.8	71.1
	U95	72.7	72.5	72.3	72.8	72.6	72.9	72.8	72.3
Women	Prediction		76.0	75.4	76.0	76.5	77.1	76.9	76.7
	L95		75.5	74.8	75.5	76.1	76.7	76.5	76.3
	U95		76.5	76.0	76.6	77.0	77.5	77.3	77.2

	Spain							
	Men				Women			
	1961	1962	1963	1964	1961	1962	1963	1964
Prediction	67.1	67.9	68.4	68.4	71.2	72.6	73.0	78.2
L95	66.8	67.7	68.1	68.2	70.9	72.3	72.8	72.9
U95	67.3	68.1	68.5	68.5	71.5	72.8	73.2	73.4

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