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Sectoral Labor Supply, Choice Restrictions and Functional Form

Abstract:
In this paper we discuss a general framework for analyzing labor supply behavior in the presence of complicated budget- and quantity constraints of which some are unobserved. The individual's labor supply decision is viewed as a choice from a set of discrete alternatives (jobs). These jobs are characterized by attributes such as hours of work, sector specific wages and other sector specific aspects of the jobs. We focus in particular on the theoretical justification of functional form assumptions and properties of the random components of the model.

The labor supply model for married women is estimated on Norwegian data. Wage elasticities and the outcome of a tax reform analysis show that overall labor supply is moderately elastic, but these modest overall responses shadow for much stronger inter-sectoral changes. Our structural model, with a detailed specification of job opportunities, is compared empirically with a model in which the utility is approximated with a series expansion. It turns out that the performance of our model is at least as good as the labor supply model with flexible preferences.

Keywords: Labor supply, non-convex budget sets, non-pecuniary job-attributes, sector-specific wages.

JEL classification: J22, C51

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1. Introduction

In this paper we discuss a particular framework for modeling labor supply behavior in the presence of complicated budget sets, qualitative job attributes and restrictions on hours of work, and apply this framework to analyze data on workers' observed choice of sector and hours in the labor market. Compared to earlier attempts, our framework for estimating labor supply allows for a more complete empirical strategy in circumstances where job choices are not adequately summarized by hours and wages.

In the 1970s and 1980s labor supply studies applied the traditional textbook model for labor supply, extended to allow for convex and smooth tax functions (cf. contributions such as Rosen (1976), Wales and Woodland (1979), Nakamura and Nakamura (1981), Kohlase (1986) and Ransom (1987)). However, in most western countries the tax system and social benefit rules imply a non-convex budget set.[1] Fixed costs of working and tax deductions if working contribute to these non-convexities. Attempts to take the non-convexity properties of the tax structure into account include Burtless and Hausman (1978), Blomquist (1983), (1992), Arrufat and Zabalza (1986), Hausman (1980), (1981), (1985), and Hausman and Ruud (1984). In principle it is possible to apply the "Hausman approach" to account for nonlinear and non-convex budget sets. That approach, however, is rather cumbersome when there are more than one adult in the household or when complicated social benefit- and tax deduction rules are present. In contrast, the particular approach advocated in our paper, and which we shall describe in a moment, has the advantage that it becomes simple to handle complicated nonlinear tax and transfer systems as well as non-standard opportunity sets. This is also the case for many-persons households.

In the studies mentioned above the mathematical structure of the modeling framework rests upon the assumption that the fundamental choice variables of the household in this context are "consumption" (composite) and "leisure" (hours of work), which can be chosen freely subject to the economic budget constraint. Yet, it seems apparent that hours of work and income are only two out of several job-related attributes, which are important for individual behavior in the labor market. "Type of work", and other "non-pecuniary job attributes", do often matter a great deal and may even be more important than hours of work. An extreme example of the latter phenomenon is found among scientists, artists and government bureaucrats for whom specific work-activities represent major means for self-realization. Another characteristic of the labor market is that hours of work are fixed for many

[1] In recent years the tax and benefit system has been simplified in many countries. Most budget sets are, however, still non-convex.

types of jobs. Thus, if an individual wishes to change his or her workload he or she would in this case have to change job.[2] This property is consistent with the findings of Altonji and Paxson (1988).

In view of the arguments above it may be more appropriate to consider labor supply behavior as the outcome of households choosing from a finite set of job "packages", each of which is characterized by an offered wage rate, offered hours of work, and non-pecuniary attributes. The individual specific choice sets of job opportunities may be thought of as being determined by employers or in negotiation between employers and unions and they are thus exogenous to the individuals. The qualitative job attributes are often unobservable, or at most only partly observable to the analyst. This is the point of departure taken in this paper. Specifically, the choice environment is assumed to consist of a latent, individual specific set of jobs. A job is characterized by a combination of fixed hours of work, wage rate and non-pecuniary job-attributes (such as type of work and working conditions). The notion of individual specific choice sets is important for our modeling of choice constraints. In our setup there are thus two sources of unobserved heterogeneity, unobserved heterogeneity in tastes and in opportunities.

In Dagsvik (1994) a general framework for modeling this type of settings was developed. Simplified versions of this framework have been applied by Anderson et al. (1988), Aaberge, Dagsvik and Strøm (1990), (1995), Dagsvik and Strøm (1997) and Aaberge, Colombino and Strøm (1999) to analyze labor supply behavior. In contrast, our paper is more theoretical in that it focuses on a detailed discussion and interpretation of underlying assumptions of the framework in the context of modeling labor supply behavior, and relates the present approach to previous ones. In particular, we discuss how functional form and the probability law of unobservables can be justified from behavioral arguments, and we investigate empirically the properties of alternative functional form specifications.

Previous attempts to take (quantity) constraints on the choice set into account have been restricted solely to one job attribute, namely hours of work. Contributions by Ilmakunnas and Pudney (1990), Kapteyn, Kooreman and van Soest (1990), Dickens and Lundberg (1993), and van Soest (1994) emphasize the inability of standard empirical labor supply models to account for observed peaks in the hours of work distribution at part-time and full-time hours. They have discussed approaches to take account of this type of constraints in the econometric modeling of labor supply. These approaches are, however, different from the one developed in our paper. Below, we will show that these previous studies can be considered as a special case of our model.

In all of these recent labor supply contributions the individuals are assumed to have the same wage across jobs. Thus, in previous labor supply studies it is assumed that an individual has a fixed wage rate, and the possibility of job-specific wages are ignored. Labor market theories, like the

[2] Alternatively, the worker may have to change the content of his current job.

theories of efficient wages and trade unions, suggest that wages may differ across jobs, see for example Krueger and Summers (1988) and Edin and Zetterberg (1992). In the labor supply literature there are approaches that allow offered job-specific wage rates to vary systematically with hours worked (Moffitt, 1984).

A serious problem with most structural econometric models is the lack of theoretical support for the choice of functional form and distributional assumptions of the unobservables. In this paper we propose a combined theoretical and empirical approach to this end. As is well known, one can apply the "Independence from Irrelevant Assumption" (IIA) to motivate the distribution of the stochastic error terms in random utility models. In this paper we apply a completely analogous assumption proposed by Dagsvik (1994). Under this assumption and a particular Poisson process representation of the distribution of the latent choice sets of jobs, the implied distribution of realized hours and wage rates turns out to be analogous to the continuous logit model introduced by Ben-Akiva, Litinas and Tsunokawa (1985). Similarly, it is an important challenge to provide a justification for the choice of *functional form* of the deterministic components in the probability model of realized hours and wage rates. In this respect the attitude among economists seems to be a general resignation: It is believed to be a hopeless task to achieve useful results on a purely theoretical basis, that is, from first principles. As a consequence, the functional form problem is “solved” by selecting a convenient parametric or semi-parametric mathematical structure, see for example Blomquist and Newey (2002) and van Soest, Das and Gong (2002). Data and statistical methods are applied to choose between competing candidates. Unfortunately, without theoretical principles almost any form is *a priori* possible and the correct one is difficult to determine because of unobserved variables and measurement errors. Within the field of psychology and psychophysics there is a tradition where functional forms are justified on the basis of invariance principles. These principles are similar to certain invariance principles applied in physics, which typically are invariant under uniform translation and rotation of the coordinate system. In this paper we discuss how results in Falmagne (1985) apply in our context and in our opinion lead to a plausible justification of functional forms.

The empirical part of the paper deals with labor supply among married females in Norway in 1994, who can choose between jobs within the public and the private sector of the economy. Other authors that analyze agents’ choice of sector are Magnac (1991) and Heckman and Sedlacek (1990). Magnac also allows for rationing in the sense that workers face costs of entry into a sector. However, neither Heckman and Sedlacek nor Magnac consider workers’ choice of hours.

The estimated model is used to simulate the impact on labor supply of introducing a flat tax reform, given that tax revenue is kept constant. Both the wage elasticities and the tax reform analysis indicate that overall labor supply is moderately elastic, but these moderate responses shadow for much stronger inter-sectoral responses.

Recently, van Soest, Das and Gong (2002) have proposed a labor supply model where the agent faces no restrictions other than the budget constraint, and the "menu" of feasible hours of work is discrete. Moreover, they represent the structural part of the utility by a polynomial in hours and consumption. We use our data to estimate several versions of the model proposed by van Soest, Das and Gong (2002), and we make a detailed comparison with our model.

This paper is organized as follows. In the next section we present the model, which includes a characterization of the stochastic properties of the unobserved variables and the functional form of the deterministic part of the utility function of the agents. In Section 3 we discuss the relationship between previous labor supply models and our model. In Section 4 we discuss empirical applications based on our model as well as on the model of van Soest, Das and Gong (2002).

2. The modeling framework

As alluded to above, the choice environment of a worker is assumed to consist of a set of latent job- and non-market opportunities. Each job is characterized by fixed observed attribute variables that represent the contract (wage rate) and unobserved attributes that describe the job-type. We shall first discuss the case where qualitative attributes are latent. Later, we extend the framework to accommodate sector-specific jobs (public sector versus private sector).

2.1. Preferences and choice sets

Let U(C,h,z) be the (ordinal) utility function of the household where C denotes household consumption, z indexes the market and non-market opportunities, or job-types, and h is hours of work of the married female. Let positive indices, $z = 1,2,\dots,$ refer to market opportunities (jobs) and $z = 0$ refer to the non-market alternative. To a market opportunity z, there are associated hours of work, H(z), and wage rate, W(z). How these are determined will be discussed later. The opportunity index z in the utility function accommodates the notion that workers may have preferences over job-types (which includes preferences for working in specific sectors of the economy) in addition to income and hours of work. For given hours and wage rate, h and w, the economic budget constraint is represented by

$$C = f(hw, I) \tag{2.1}$$

where I is non-labor income, which includes the income of the husband, and $f(\cdot)$ is the function that transforms gross income into after-tax household income. Here, the income of the husband is treated as given. The function $f(\cdot)$ will capture all details of the tax and benefit system. The price index for the composite good, C, is set equal to one. Our first assumption concerns the structure of the preferences.

Assumption 1

The utility function has the structure

$$U(C,h,z) = v(C,h)\,\varepsilon(z)$$

for $z = 0,1,2,...,$ *where* $v(\cdot)$ *is a positive deterministic function and* $\varepsilon(z)$ *is a positive random taste-shifter.*

The random taste-shifter is assumed to account for the unobservable individual characteristics and non-pecuniary job-type attributes that affect utility. For notational simplicity we will use the notation

$$\psi(h,w;I) \equiv v\big(f(hw,I),h\big). \tag{2.2}$$

The term $\psi(h,w;I)$ is the representative utility of jobs with hours of work h and wage rate w, given non-labor *income* I. In addition to (2.1), there are restrictions on the set of feasible market opportunities a specific worker faces because there are job-types for which the worker is not qualified and there may not be jobs available for which she is qualified.

2.2. A simplified description of the model

Since the general model is based on a somewhat abstract representation of the unobserved heterogeneity in choice restrictions and preferences we shall first consider a simplified version. In this simplified version unobserved heterogeneity in the choice sets is ignored. Furthermore, it is assumed that the agent only can choose among a finite set of jobs. Let D(h,w) denote the agent's set of available jobs with hours of work, $H(z) = h$, and wage rate $W(z) = w$. Let m(h,w) be the number of jobs in D(h,w). For the non-market alternative, $m(0,0) = 1$. The random error terms $\{\varepsilon(z)\}$, are assumed independent of offered hours and wages.

Assumption 2

The choice of an alternative (job, or the non-market alternative) from a given set of alternatives, satisfies the Independence form Irrelevant Alternatives property, (IIA).

Recall that the underlying intuition of the IIA assumption is, loosely speaking, that the agent's ranking of job opportunities from a subset B (say), within the choice set of feasible jobs with given job-specific hours of work and wage rates, does not change if the choice set of feasible jobs is altered. Recall also that the stochastic formulation of IIA adopted by the psychologists, means that this property only is claimed to hold *on average,* for an agent that is exposed to a large number of repeated choice experiments, cf. Thurstone (1927).

In Appendix A we demonstrate that IIA in fact seems to be less restrictive than the implications for constrained demand that follow from standard consumer theory. The reason why it is commonly believed that IIA is a very strong assumption is because it is rejected in some empirical tests[3]. Typically, these tests depend crucially on additional ad hoc hypotheses about functional form and parameters that are equal across the sample. Thus, what is typically tested are joint hypotheses about a combination of functional forms, equal parameters across individuals *and* IIA. For serious empirical tests of IIA it is therefore required to have detailed stated preference type of data at the individual level (which is beyond the scope of our article). It is in our view natural to use IIA as a basic postulate of individual probabilistic rationality. Recall that "probabilistic" may be related to the view that behavior is stochastic at the individual level in the sense that if an agent is exposed to repeated choice experiments he or she may choose different alternatives each time; see for example Tversky (1969). (Alternatively, "probabilistic" may be due to the fact that researchers are not able to observe all factors that affect the choices of the individuals). It is of crucial importance to be able to pin down a theoretically justified *individual* model, because otherwise we surely will not be able to identify the possible variation of model parameters across the sample. If we cannot separate the individual model structure from unobserved population heterogeneity we cannot test several interesting hypotheses about behavior and choice constraints. As mentioned in the introduction, in our empirical model specification (Section 4) we shall allow for random effects, which means that in the empirical application below IIA only is assumed to hold conditional on the random effect[4].

Consider now the choice model. As is well known (Strauss, 1979), Assumption 2 implies that the random error terms $\{\varepsilon(z)\}$ can, with no loss of generality, be assumed independent with c.d.f.

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[3] The fact that there is are few workers that are observed working with low hours of work may be also be interpreted as a result of fixed costs of working, as, van Soest, Das and Gong (2002) do. Fixed costs of working may imply that it is more attractive not to work than working few hours. Of course fixed costs of working may be of some importance, but in unionized economies like the Norwegian economy it is also a fact that jobs offered with few hours are rare.)

[4] A famous example often used to demonstrate how restrictive IIA is, is the so-called red bus-blue bus example (Debreu, 1960). However, this example is rather extreme because two of the alternatives are practically identical. In most cases alternatives are fortunately different in a more essential way. It is true though that the alternatives in many cases can differ in varying degree with respect to common aspects, which may weaken the plausibility of IIA. Nevertheless, it is still not evident that different degree of substitutability between alternatives will yield violation of IIA on the individual level. If the stochastic elements of the utility function to a large extent represent bounded rationality, they can be viewed as pure "noise", and therefore may not reflect substitution relations in a systematic way.

$\exp(-x^{-1}), x > 0$[5]. Note that the agent will be observed working h hours at wage rate w if she chooses a job within D(h,w). Let $\varphi(h,w;I)$ denote the probability that the agent shall choose a particular job with offered hours h and wage rate w, when her non-labor income is I. From standard results in discrete choice theory (McFadden, 1984) it now follows that the probability that a specific job z (say) within D(h,w) shall be the feasible job with the highest utility equals

$$P\left(\psi(h,w;I)\varepsilon(z) = \max_{x,y} \max_{k\in D(x,y)} \left(\psi(x,y;I)\varepsilon(k)\right)\right) = \frac{\psi(h,w;I)}{\sum_{x,y}\sum_{k\in D(x,y)} \psi(x,y;I)} = \frac{\psi(h,w;I)}{\sum_{x,y}\psi(x,y;I)m(x,y)}.$$

The probability of choosing *any* job within D(h,w) is thus obtained by summing over all jobs in D(h,w), which yields

(2.3)
$$\begin{aligned}\varphi(h,w;I) &= \sum_{z\in D(h,w)} P\left(\psi(h,w;I)\varepsilon(z)\right) = \max_{x,y} \max_{k\in D(x,y)} \left(\psi(x,y;I)\varepsilon(k)\right)\\ &= \sum_{z\in D(h,w)} \frac{\psi(h,w;I)}{\sum_{x,y}\sum_{z\in D(x,y)} \psi(x,y;I)} = \frac{\psi(h,w;I)m(h,w)}{\sum_{x,y}\psi(x,y;I)m(x,y)}.\end{aligned}$$

The resulting expression is a choice model that is analogous to a multinomial logit model with representative utility terms $\{\psi(h,w;I)\}$ weighted with the frequencies of feasible jobs, $\{m(h,w)\}$. Unfortunately, the frequencies $\{m(h,w)\}$ are not directly observable, but under specific assumptions one can identify m(h,w) and $\psi(h,w;I)$ and estimate their parameters. We shall return to this issue below.

2.3. The general case with stochastic choice sets

In the simplified model version described above the choice sets, represented by $\{D(h,w)\}$, were assumed to be equal across observationally identical agents. It is however, desirable to allow choice sets also to vary across agents due to unobserved heterogeneity in opportunities. In this subsection we shall present a modeling framework that accommodates the notion of both observed and unobserved heterogeneity in opportunities and preferences.

[5] The c.d.f. $\exp(-x^{-1}), x > 0$, occurs in this context because of the multiplicative specification of the utility function in Assumption 1. This c.d.f. is completely equivalent to the more familiar c.d.f $\exp(-e^{-x})$ that occurs in the corresponding additive formulation. Recall that the standard type I distribution function has the form exp(-1/y), y>0, cf. Resnick (1987). In the statistical literature there is some confusion where some authors call this distribution type III.

To this end we turn to discuss a particular representation of random choice sets through the set of triples $\{(\mathrm{H}(z), \mathrm{W}(z), \varepsilon(z)), z = 1, 2, ...\}$ associated with the respective jobs. We shall assume that these triples are independent and randomly scattered in some suitable set Ω. This conforms with the intuition that due to the researcher's lack of information the locations of points in Ω appears to be independent. A convenient formal representation of this type of stochastic "process" is obtained through the multidimensional non-homogeneous spatial Poisson process. Recall that a Poisson process on Ω has the properties that its points are randomly scattered throughout Ω. If the Poisson process is homogeneous the points are randomly but evenly dispersed on Ω. In contrast, the non-homogeneous Poisson process allows for uneven distribution of points in the sense that there are, on average, higher concentration of points in some parts of Ω than in other parts. This allows us to account for the possibility that there may be fewer jobs available with high values of the taste shifters $\{\varepsilon(z)\}$ than jobs with low values of $\{\varepsilon(z)\}$, and the possibility that there may be more jobs with full-time hours available than jobs with other hours of work. To facilitate exposition it may be convenient to explain the multidimensional Poisson process representation by first starting with the more familiar one-dimensional Poisson process, namely the one that represents the taste-shifters $\{\varepsilon(z)\}$. Subsequently, we extend the process to higher dimensions by "assigning" further attributes (job-specific hours and wage rates) to the one-dimensional Poisson points.

Assumption 3

The taste-shifters $\{\varepsilon(z), z = 1,2,...\}$ *are realizations of a non-homogeneous Poisson process on* $(0,\infty)$ *with intensity* $\kappa(\varepsilon)$*. Moreover* $\{\varepsilon(z), z = 1,2,...\}$ *are distributed independently of* $\varepsilon(0)$*.*

As mentioned above the points (taste-shifters) of the Poisson process are independent and randomly scattered, and the probability that there is a job with $\varepsilon(z) \in (\varepsilon, \varepsilon + d\varepsilon)$ is available to the individual equals $\kappa(\varepsilon)d\varepsilon$. Moreover, the probability that there are more that one job with taste-shifters within $(\varepsilon, \varepsilon + d\varepsilon)$ is negligible.

Assumption 4

The hours of work and wage rates $(H(z), W(z)), z = 1,2,...,$ *are i.i.d. with joint density g(h,w), where* $0 < h < \bar{h}$ *is an upper bound on hours of work. Moreover,* $\{(H(z), W(z)), z = 1,2,...,\}$ *are independent of* $\{\varepsilon(z), z = 0,1,2,...\}$.

From Assumptions 3 and 4 we realize that the points $\{H(z), W(z), \varepsilon(z)\}$ can be interpreted as realizations from a three-dimensional non-homogeneous Poisson process on Ω with intensity $g(h,w)\kappa(\varepsilon)$. A formal proof of this is found in Resnick (1987), p.135. This intensity is a complete representation of the Poisson process in the sense that it governs the corresponding probability distribution of the Poisson points in Ω.

There are two different interpretations of the notion of random choice sets. In the first interpretation the randomness is viewed as a result of unobserved heterogeneity in choice opportunities across the population. That is, while the individual agent is viewed as having complete information about his choice set and the utilities of every feasible alternative, the researcher cannot observe this heterogeneity. He can at most account for heterogeneity in the choice set due to observed individual characteristics. Given the set of feasible jobs, $g(h,w)dhdw$ can be interpreted as the probability (from the researcher's point of view) that, for a randomly selected agent, there is a job with hours of work $H(z) \in (h, h + dh)$ and $W(z) \in (w, w + dw)$ that is feasible. The corresponding "empirical" counterpart is the fraction of the feasible jobs that have hours of work and wage rates within $(h, h + dh) \times (w, w + dw)$.

The second interpretation is related to bounded rationality in the sense that the agent has difficulties with assessing the precise value to him of the respective alternatives and also has limited capacity to identify and take into account the exact choice set. The actual choice set she takes into account may therefore to some extent be a randomly "selected" (by the agent) subset from the "objective" choice set of feasible alternatives. This means that in replications of a choice experiment the agent may make different choices due both to differences in utility evaluations and differences in the subjectively "selected" choice set. Psychologists have often emphasized this second interpretation. The interpretation of $g(h,w)dhdw$ in this setting is as the probability that the agent both consider a job with hours of work and wages that lies within $(h, h + dh) \times (w, w + dw)$, and perceive this job as feasible. Most likely, the actual heterogeneity in the choice sets will be a combination of bounded rationality and unobserved heterogeneity. To which extent bounded rationality play a role can usually not be identified by the researcher.

From Assumptions 1, 3 and 4 it is now possible to derive choice probabilities of observed labor market choices. However, without further restrictions on the intensity of the Poisson process the expression for the choice probabilities will be rather general and complicated analytically. It thus is desirable to add further behavioral assumptions to restrict the modeling framework. Similarly to the simplified setup in Section 2.2 we shall assume IIA as given in Assumption 2.

We are now ready to express the probability distribution of *realized* hours and wages, including the probability of not working. Let Φ(h,w;I) be the joint cumulative distribution of realized hours and wages that follow from utility maximizing behavior, i.e.,

$$\text{(2.4)} \qquad \Phi(\mathrm{h},\mathrm{w};\mathrm{I}) \equiv \mathrm{P}\left(\max_{\substack{z \\ \mathrm{H(z)\le h,W(z)\le w}}} \left(\psi\left(\mathrm{H(z)},\mathrm{W(z)};\mathrm{I}\right)\varepsilon(\mathrm{z})\right) = \max_{z}\left(\psi\left(\mathrm{H(z)},\mathrm{W(z)};\mathrm{I}\right)\varepsilon(\mathrm{z})\right)\right).$$

Equation (2.4) defines the probability that the chosen opportunity (i.e. job) has hours of work less than or equal to h and wage rate less than or equal to w.

Theorem 1

Assume that Assumptions 1 to 4 hold. Assume furthermore that $\kappa(\varepsilon)$ *is continuous and that*

$$\int_x^\infty \kappa(\varepsilon)d\varepsilon < \infty,$$

for $x > 0$ *. Then* $\varepsilon(0)$ *is type I extreme value distributed and the intensity* $\kappa(\varepsilon)$ *has the form* $\theta\varepsilon^{-2}$ *, for positive* ε *where* θ *is a positive constant. Moreover, the probability density* $\varphi(h,w;I)$ *is given by*

$$\text{(2.5)} \qquad \varphi(h,w;I) = \frac{\psi(h,w;I)\,g(h,w)\theta}{\psi(0,0;I)+\theta\int\int_D \psi(x,y;I)\,g(x,y)dx\,dy}$$

for $h > 0, w > 0,$ *and*

$$\text{(2.6)} \qquad \varphi(0,0;I) = \frac{\psi(0,0;I)}{\psi(0,0;I)+\theta\int\int_D \psi(x,y;I)\,g(x,y)dx\,dy},$$

for $h = w = 0,$ *where* $D = \left[0,\bar{h}\right]\times\left(0,\infty\right).$

The proof of Theorem 1 is given in Appendix B.

Recall that when ε(0) has type I extreme value c.d.f.., then

$$\mathrm{P}\left(\varepsilon(0) \le \mathrm{x}\right) = \exp\left(-\mathrm{x}^{-1}\right),$$

for positive x. Furthermore, the parameter θ that appears in the intensity $\kappa(\varepsilon)$ is a measure of job-availability since a high value of θ means that there is a high probability that there is an available job with taste-shifters belonging to $(\varepsilon, \varepsilon + d\varepsilon)$ for any positive ε.

To explain θ further, we may consider the particular case where the agent perceives no constraints on behavior and the agent is, on average, indifferent between working and not working. This means that his utility of either alternative is governed by i.i.d. random terms. Clearly, in this case the probability of working should be equal to 0.5. The case with totally random preferences corresponds to the case with $\psi(h, w; I) = \psi(0, 0; I)$ for all h and w. Hence, in this case (2.6) reduces to $\varphi(0, 0; I) = \frac{1}{1+\theta}$. Thus, for $\varphi(0, 0; I)$ to be equal to 0.5, θ must be equal to 1. Thus, 1 is the upper bound on θ, and θ equal to 1 can be interpreted as a reference case with no "quantity" restrictions on behavior. This case may cover that no involuntary unemployment is present.

So far, we have demonstrated that the formulation above allows for a particular type of quantity constraints, which typically are rather difficult to account for by means of the econometric formulations used in previous labor supply studies.

Although we have assumed that the agent's taste-shifters are (stochastically) independent of offered hours and wage rates, the opportunity density will depend on the *distribution* of the preferences due to equilibrium conditions. In other words, the market forces that regulate the balance between supply and demand, be it a market clearing regime or not, are assumed to operate solely on an aggregate level. The opportunity density will consequently depend on the production technology of the firms as well as of the contract and wage setting policies of the unions and the firms. It is beyond the scope of this paper to discuss fully how the opportunity density $\theta g(\cdot)$, through market equilibrium processes, depend on the systematic part of the utility function, $\psi(\cdot)$. This means that the estimated model only can be applied to simulate behavior conditional on the opportunity density. This parallels the assumptions made in previous labor supply models, in which it is assumed that the agents take wages as given and that the agent can freely choose between any hours of work. In Dagsvik (2000), it is discussed how an explicit equilibrium model version can be specified and how the opportunity density depends on workers' preferences and firms' technologies, and we shall use that framework to justify that the opportunity density is multiplicatively separable in hours and wage rates, which means that the offered hours and wage rates are independent. In Appendix C we show how this property follows from the setup in Dagsvik (2000), combined with the assumption of constant returns to scale production functions of the firms.

2.4. Identification

Identification of the model given in (2.5) and (2.6) were considered in Dagsvik and Strøm (1997) under the assumption that the structural term of the utility function is multiplicatively separable in consumption and hours of work (leisure). However, here the assumption of a multiplicatively separable utility function will be relaxed. With a reference to the arguments given in Appendix C, we will instead consider identification when

$$g(h,w)=g_1(w)g_2(h) \tag{2.7}$$

which means that hours and wage rates $\{H(z)\}$ and $\{W(z)\}$ offered by the firms are independent.

Let (C_0,h_0,I_0), $h_0>0$, be *fixed* levels of consumption, hours of work and non-labor income, and let (C,h) be arbitrary values of consumption and hours. Moreover, let $f^{-1}(C,I)$ be the function determined by

$$f\left(f^{-1}(C,I),I\right)=C.$$

That is, $f^{-1}(C,I)$ is the level of labor income that corresponds to consumption C and non-labor income I when the budget constraint (2.1) holds. Let w_0 and h be determined by

$$w_0=f^{-1}(C_0,I_0)/h_0$$

and

$$h=f^{-1}(C,I_0)/w_0 .$$

Then it follows from (2.5), (2.7) and (2.2) that

$$\frac{\varphi(h,w_0;I_0)}{\varphi(h_0,w_0;I_0)}=\frac{\psi(h,w_0;I_0)g_2(h)}{\psi(h_0,w_0;I_0)g_2(h_0)}=\frac{v\left(f(hw_0,I_0),h\right)g_2(h)}{v\left(f(h_0w_0,I_0),h_0\right)g_2(h_0)}=\frac{v(C,h)g_2(h)}{v(C_0,h_0)g_2(h_0)}. \tag{2.8}$$

Eq. (2.8) demonstrates that for $h>0$ one can identify $v(C,h)g_2(h)$ up to a multiplicative constant $\left(v(C_0,h_0)g_2(h_0)\right)$. This multiplicative constant is of course irrelevant since it does not affect comparisons of utility levels. Since $v(C,h)g_2(h)$ is identified it follows from (2.2) and (2.5) that

$$\frac{g_1(w)}{K(I)}=\frac{\varphi(h,w;I)}{\psi(h,w;I)g_2(h)} \tag{2.9}$$

where K(I) denotes the denominator of the right side of (2.5) divided by θ. By integrating both sides of (2.9) with respect to w, we get for $h > 0$ that K(I) is determined by

$$\frac{1}{K(I)} = \int \frac{\varphi(h,w;I)dw}{\psi(h,w;I)g_2(h)}. \tag{2.10}$$

Hence $g_2(w)$ is uniquely determined by (2.9) and (2.10). Finally, since $g_1(w)$ and $\psi(h,w;I)g_2(h)$ are identified for $h > 0$ and K(I) is determined by (2.10) it follows from (2.5 and (2.6) that $v(C,0)/\theta$ is identified because

$$\frac{\psi(0,0;I)}{\theta} = \frac{v(f(0,I),0)}{\theta} = \varphi(0,0;I)K(I). \tag{2.11}$$

If $v(f(0,I),0)$ is normalized to be equal to one for a suitable value of I we realize that also θ can be determined from (2.11).

We have thus demonstrated that $v(C,h)g_2(h), \theta$ and $g_1(w)$ are non-parametrically identified.

2.5. Functional form

Current quantitative economic research often suffers from the lack of theoretical principles on which assumptions about functional form can be made. While elaborate and sophisticated theoretical models of behavior exist, such models are often not detailed enough to be useful for purposes other than qualitative predictions. The standard approach in empirical analyses is either to "let the data determine" functional forms within ad hoc selected parametric classes (including so-called flexible functional forms). This is clearly unsatisfactory in the context of structural modeling. In the preceding sections we have insisted on a theoretical foundation for the stochastic properties of our model based on IIA. These properties led to a particular representation of the labor supply choice probabilities ((2.5) and (2.6)) in terms of functions that represent preferences and opportunities. However, unless we also are able to justify the choice of functional form of the systematic part of the utility function and the opportunity distribution, the implications may, as regards structural empirical analyses, be ambiguous. This is due to the fact that the class of a priori admissible opportunity distributions and utility functions is very large[6]. In this section we shall discuss some interesting implications from the theory of psychophysical measurement and dimensionality analysis. The point of departure taken and

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[6] The following passage by Frisch (1932) is a good example of a balanced view about the merits and limitations of a purely statistical approach:.. "No statistical technique, however refined, will ever be able to do such a thing (solve the problem of testing "significance"). The ultimate test of significance must consist in a network of conclusions and cross checks where theoretical economic consideration, intimate and realistic knowledge of the data and refined statistical technique concur."

exploited in some of the literature of psychophysical measurement is that numerical representations of sensory perceptions and physical stimuli can only be measured up to a scale[7]. For example, if the relevant stimuli are quantities or money, this type of variables are measured on a ratio scale. There is by now a considerable literature that addresses the issue of meaningfulness and dimensional invariance of scientific laws. We shall apply a typical approach within the field of psychophysics, as presented in Falmagne (1985), to restrict the class of functional forms for the systematic part of the utility function.

To this end, consider the particular case with an opportunity distribution that has all mass in two points (h, w_1) and (h, w_2), with probability mass equal to 0.5 in either point. (Since preferences are assumed independent of opportunities, the analyst is, for the sake of interpretation and theoretical analysis, free to select any opportunity distribution he finds suitable for a specific purpose while keeping the function $v(\cdot)$ unchanged). Let L denote leisure defined as total time available for work minus hours of work. Then from (2.5) it follows that we can write

$$\text{(2.12)} \qquad \frac{v(C_1, h_1)}{v(C_1, h_1) + v(C_2, h_2)} \equiv \tilde{\varphi}(C_1, L_1; C_2, L_2)$$

where $\tilde{\varphi}(C_1, L_1; C_2, L_2)$ is the probability that (C_1, L_1) is preferred to (C_2, L_2). It is understood that consumption in this context means disposable income minus subsistence expenditure and leisure means leisure minus subsistence leisure.

Assumption 5

Suppose C_1, C_2, C_1^*, C_2^*, L_1, L_2, are such that

$$\tilde{\varphi}(C_1, L_1; C_2, L_2) \le \tilde{\varphi}\left(C_1^*, L_1; C_2^*, L_2\right).$$

Then

$$\tilde{\varphi}(r\,C_1, L_1; r\,C_2, L_2) \le \tilde{\varphi}\left(r\,C_1^*, L_1; r\,C_2^*, L_2\right)$$

for any positive r.

[7] Recall that the scale types are: Ordinal scale, Ratio scale, Interval scale and Logarithmic interval scale, cf. Falmagne (1985).

Assumption 5 states that if the fraction of workers that prefer jobs that yield $\left(C_1, L_1\right)$ to jobs that yield $\left(C_2, L_2\right)$ is less than the fraction of workers that prefer jobs that yield $\left(C_1^*, L_1\right)$ to jobs that yield $\left(C_2^*, L_2\right)$, then the same is true when the respective consumption levels are scale transformations of the original levels. Recall that the non-pecuniary characteristics of the jobs are represented by random terms that are independent of the systematic terms $\left\{v\left(C,h\right)\right\}$. Assumption 5 captures the notion that once basic needs (subsistence) are fulfilled, then the absolute levels of quantities tend not to be essential, rather the individuals relate to relative consumption levels. Note, however, that Assumption 5 does *not* claim that $\tilde{\varphi}\left(rC_1, L_1; rC_2, L_2\right)$ is independent of r. It only expresses that if the number of individuals that prefer $\left(C_1^*, L_1\right)$ to $\left(C_2^*, L_2\right)$ is greater than the number of individuals that prefer $\left(C_1, L_1\right)$ to $\left(C_2, L_2\right)$, this inequality remains true when consumption levels are increased or decreased by the same factor. For the sake of understanding the limitation of Assumption 5, we can think of two objections against this assumption. One objection is that the individual's perception about his personal subsistence level may be somewhat vague and may not be identified by a single fixed amount. Rather it may vary from one moment to the next according to fluctuations in his mood and state of mind. Another objection is related to satiation. If satiation is present and rC_1^* and rC_2^* are close to satiation levels for sufficiently large r and $L_1 = L_2 = L$ (which means that the deterministic part of the utility approaches a constant), the second inequality in Assumption 5 may be reversed because $\tilde{\varphi}\left(rC_1^*, L; rC_2^*, L\right)$ will be close to 0.5, independent of the levels of C_1^*, C_2^* and L. In the absence of satiation it seems to us to be highly unreasonable that the inequality $\tilde{\varphi}\left(C_1, L_1; C_1, L_2\right) \le \tilde{\varphi}\left(C_1^*, L_1; C_2^*, L_2\right)$ should be reversed if the consumption levels are rescaled; i.e., that Assumption 5 is violated.

The notion that relative stimuli levels matter rather than absolute ones is supported by numerous stated preference experiments, see for example Stevens (1975).

Assumption 6

Suppose $L_1, L_2, L_1^*, L_2^*, C_1, C_2,$ *are such that*

$$\tilde{\varphi}\left(C_1, L_1; C_2, L_2\right) \le \tilde{\varphi}\left(C_1, L_1^*; C_2, L_2^*\right).$$

Then

$$\tilde{\varphi}\left(C_1, rL_1; C_2, rL_2\right) \leq \tilde{\varphi}\left(C_1, rL_1^*; C_2, rL_2^*\right)$$

for any positive r.

We realize that Assumption 6 is completely analogous to Assumption 5 and thus the motivation is similar.[8]

Theorem 2

Assume that (2.12), Assumptions 5 and 6 hold with v(C,h) continuous and strictly increasing in C and strictly decreasing in h. Then

(2.13) $$\log v\left(C,h\right) = \beta_1 \frac{\left(C^{\alpha_1} - 1\right)}{\alpha_1} + \beta_2 \frac{\left(L^{\alpha_2} - 1\right)}{\alpha_2} + \beta_3 \frac{\left(C^{\alpha_1} - 1\right)\left(L^{\alpha_2} - 1\right)}{\alpha_1 \alpha_2}$$

where $\left\{\alpha_j\right\}$ *and* $\left\{\beta_j\right\}$ *are constants with* $\beta_j > 0$, $j = 1,2$, β_3 *can be negative.*

A proof of Theorem 2 is given in Appendix B.

If one imposes the stronger assumption that $\tilde{\varphi}\left(rC_1, L_1; rC_2, L_2\right)$ is *independent* of $r > 0$, it can easily be demonstrated that this implies that $\alpha_1 = 0$. If one imposes the assumption that $\tilde{\varphi}\left(rC_1, sL_1; rC_2, sL_2\right)$ is *independent* of r and s, for all $r > 0$, $s > 0$, it is easily seen that this yields that $\alpha_1 = \alpha_2 = \beta_3 = 0$ so that (2.13) reduces to

(2.14) $$\log v\left(C,h\right) = \beta_1 \log C + \beta_2 \log L$$

which is the well known Stone Geary utility function. (Recall that C and L are defined as consumption and leisure minus the respective subsistence levels.)

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[8] Although Luce (1959b) derived the power law as the functional relation between subjective continua and physical continua from the assumption of dimensional invariance, his approach nor Steven's empirical method do not apply directly to aggregate relations. Recall that the challenge faced here is to characterize choice probabilities, or equivalently, the mathematical and stochastic structure of a random utility function. If only the approach discussed by Luce (1959b) was available, then we would not be able to discriminate between specifications such as for example $v_1(C) = \beta C^{\alpha}$ and $v_1(C) = m\left(C^{\alpha}\right)$, where $m(\cdot)$ is an increasing function since in our context, utility, U(C,h,z), is ordinal and only determined up to a monotone transform. Thanks to the approach developed by Falmagne and Narens (cf. Falmagne, 1985, ch. 14) we are, however, able to get rather sharp results as demonstrated above.

There is a number of studies in experimental psychophysics that are concerned with the measurement of the utility of income. Consistent with the result of Theorem 2, Stevens (1975) and his followers have found that a power function fits the data well,[9] cf. Stevens (1975), p. 246. See also Breault (1981).

The result of Theorem 2 does not depend on the particular form of $\tilde{\varphi}(\cdot)$ given in (2.12). In fact, it can easily be verified that it is enough to assume that $\tilde{\varphi}$ has the form

$$\tilde{\varphi}(C_1, L_1; C_2, L_2) = F\left(\frac{v(C_1, h_1)}{v(C_2, h_2)}\right) \tag{2.15}$$

where F is any strictly increasing c.d.f. on R_+. This is so because the proof does not depend on the form of $F(\cdot)$.

2.6. Extension to several sectors

An essential motivation for the framework discussed in this paper is that it is particularly convenient for modeling workers' choice among jobs with observable non-pecuniary job attributes. In general, jobs in different sectors may differ with respect to job non-pecuniary attributes, such as job-security (with the government sector at one extreme, and private export industries at the other) and the nature of the tasks to be performed.

In this section we shall outline how the model can be extended to a multi-sectoral setting. To this end, we now suppose that the agent can choose among n sectors. The utility function in this case is assumed to have the structure

$$U(C, h, j, z) = v_j(C, h)\varepsilon_j(z) \tag{2.16}$$

where j indexes sector, $j = 0, 1, ..., n$, and $j = 0$ represents "not working", and $v_j(C, h)$ is the representative utility of consumption and hours of work (C,h) and working in sector j. In this setting Assumption 3 is extended to involve sector specific densities $g_j(h,w)$ where $g_j(h,w)$ represents the joint distribution of the feasible hours of work and wage rates in sector j. Also the Poisson process representation of the job opportunities is allowed to be sector-specific with sector-specific parameter θ_j associated with the intensity of the taste-shifters of the Poisson processes. Thus, $g_j(h, w)dh\,dw$ is the mean fraction of feasible jobs in sector j with offered hours of work and wage rates within

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[9] Stevens and others have observed the power law in innumerable experiments. Sinn (1983) has compressed the content of Stevens' Psychophysical power law into the following statement: "Equal relative changes in stimulus intensity bring about equal relative changes in sensation intensity".

$\left(h,h+dw\right)\times\left(w,w+dw\right)$. Let $\psi_j\left(h,w;I\right)=v_j\left(f\left(hw,I\right),h\right)$, $\psi(0,0;I)=v(f(0,I),0)$, and let $\varphi_j\left(h,w\right)dh\,dw$ denote the probability of choosing a job in sector j with hours of work and wage rate within $\left(h,h+dw\right)\times\left(w,w+dw\right)$. Similarly to Theorem 1 it follows that

(2.17) $$\varphi_j\left(h,w;I\right)=\frac{\psi_j\left(h,w;I\right)\theta_j g_j\left(h,w\right)}{\psi\left(0,0;I\right)+\sum_{k=1}^{n}\theta_k\iint_D\psi_k\left(x,y;I\right)g_k\left(x,y\right)dx\,dy}$$

for $h>0, w>0$, and

(2.18) $$\varphi\left(0,0;I\right)=\frac{\psi\left(0,0;I\right)}{\psi\left(0,0;I\right)+\sum_{k=1}^{n}\theta_k\iint_D\psi_k\left(x,y;I\right)g_k\left(x,y\right)dx\,dy}.$$

3. Relation to studies with discrete choice and latent constraints on hours of work

For the sake of comparison with some recent studies in labor supply econometrics that discuss modeling strategies for dealing with constraints, consider for a moment the following setting: The agent has a utility function $\widetilde{U}(C,h,\varepsilon)$ where ε is a random taste-shifter (independent of (C,h)). The budget constraint is given by (2.1) and the offered wage rate is fixed for each agent. Assume that hours of work take values in a finite set B (say). Let

(3.1) $$V(h,w,\varepsilon)=\widetilde{U}\left(f(hw,I),h,\varepsilon\right).$$

Then it follows that the probability density of hours, conditional on the wage rate and the set B, is given by

(3.2) $$\hat{\varphi}\left(h\,|\,w,B\right)\equiv P\left(V(h,w,\varepsilon)=\max_{x\in B}V(x,w,\varepsilon)\right).$$

Suppose now that B is unobserved by the analyst and can take any value in the set $\left\{B_1,B_2,...,B_K\right\}$. For example B could consist of the options "full-time", "part-time" and "not working", or of "part-time" and “not working”. To account for this, assume that B is random. Let q_j be the probability that $B = B_j$. The unconditional probability density that corresponds to the data the analyst has at hand therefore equals[10]

$$\hat{\varphi}\left(h \mid w\right) \equiv E_B\, \hat{\varphi}\left(h \mid w, B\right) = \sum_{B_j \supset h} q_j \, P\left(V(h,w,\varepsilon) = \max_{x \in B_j} V(x,w,\varepsilon) \right). \tag{3.3}$$

In (3.3) it is the quantity “hours of work” that is rationed, whereas in our model, presented in section 2 above, a latent choice variable, “job opportunity”, is introduced. In the model developed in section 2, possible rationing of hours may occur because there may be few or no feasible jobs with the desired hours of work. The models developed by Ilmakunnas and Pudney (1990), and Dickens and Lundberg (1993) fall within the framework represented by (3.3).

In contrast, our notion of unobservable job opportunities introduced in section 2 allows for the interpretation that the outcome of an agent's labor supply decision is the result of the agent maximizing utility over “job-packages” with several attributes of which hours of work is only one of them. Second, our framework is convenient for dealing with latent opportunity sets, while the type of formulation represented by (3.3) is a mixture of multinomial logit type densities and becomes rapidly intractable when K increases.

Van Soest (1994) on the contrary, assumes that the choice set consists of a finite (given) number of hours of work options and he specifies a model, which is a mixture of logit-type models across unobservable taste-shifters. He interprets the observed concentrations of hours of work as resulting from agents having strong preferences for “full-time” and/or “part-time” hours of work. In van Soest, Das and Gong (2002) they use a similar framework as van Soest (1994) but with different functional form assumptions[11]. The model of van Soest et al. (2002) can in fact be interpreted as a version of the simplified modeling framework described in Section 2.2, extended to include a particular random effect. To realize this, consider formula (2.3). Note that when the set D(h,w) of feasible jobs with hours and wage rates equal to (h,w) is independent of h, we obtain that the conditional choice model, given that the wage rate of the chosen job is equal to w, becomes

$$\varphi\left(h,w;I\right) = \frac{\psi\left(h,w;I\right) m(w)}{\sum_{x,y} \psi\left(x,y;I\right) m(y)}. \tag{3.4}$$

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[10] The notation $B_j \supset h$, means that the summation takes place across all j for which $B_j \supset h$.

[11] Van Soest (1994) argues that one may assume that the peak at full time hours are due to preferences, since possible constraints on hours are unobserved. This argument will in general be flawed, because if in fact there are restrictions on hours of work then this may have important implications for the structural model. That it is not evident how one should deal with choice constraints is illustrated by the other contributions mentioned in this section.

where, according to Section 2.2, m(w) is the number of jobs with wage rate w. The interpretation is that all hours of work are equally feasible in the market. Hence, the conditional density of hours, given the wage rate of the chosen job, $\tilde{\varphi}(h;w,I)$, equals

$$\tilde{\varphi}(h;w,I) \equiv \frac{\varphi(h,w;I)}{\sum_x \varphi(x,w;I)} = \frac{\psi(h,w;I)}{\sum_x \psi(x,w;I)}. \tag{3.5}$$

Due to Weierstrass theorem any continuous function can be approximated arbitrarily closely by polynomials. This result motivates van Soest, Das and Gong (2002) to approximate logv(C,h) by a polynomial of a suitable degree in consumption and hours of work. Other details of their empirical specification will be discussed below when we compare the results of the empirical analysis on Norwegian data based on our model versus the model used by van Soest, Das and Gong (2002).

4. An empirical application

4.1. Empirical specification

In this application we only consider the case with two sectors, private and public sector, and for the following reasons. For women with higher level of education there are more job opportunities in the public than in the private sector. Moreover, in the Norwegian public sector more emphasis has been put on facilitating combination of work and childcare, and thus one is more likely to find a job with a subsidized day-care center in the public than in the private sector. The public sector is more unionized than the private sector. Wages are more compressed and hours are more constrained. However, the job security tends to be higher in the public sector than in the private sector. Finally, some job types are almost only found in the public sectors (such as hospitals, colleges and universities) while others are found solely in the private sector (a large number of manufacturing firms). Thus, there are important differences between the private and the public sector that could influence the labor supply decisions of married women. Some of these differences are observed (like wages) while others are not. The modeling framework appropriate for this application is the one outlined in subsection 2.6, where sector one is the public sector and sector two is the private sector and $n = 2$.

As outlined above and justified in Appendix C, we will assume that offered hours and offered wages are independent, i.e.

$$g_j(h,w) = g_{j1}(w)g_{j2}(h) \tag{4.1}$$

for $j = 1,2$. Although offered wages and hours may vary across jobs, our assumption is that hours are *set* independent of wages. The justification for this assumption is that offered wages, in the unionized part of economy, are set in yearly wage settlements. Normal working hours, on the other side, are determined more infrequently, typically once or twice every decade. The density of offered hours, $g_{j2}(h)$, is assumed uniform except for peaks at full-time and part-time hours. Recall that uniformly distributed offered hours corresponds to the notion of a perfect competitive economy. Thus, the full-time peak in the hours distribution captures institutional restrictions and technological constraints and hence market imperfections in the economy. We allow the sizes of the full-time and part-time peaks to vary across sectors. The rationale is that the public sector is more regulated than the private sector, also because the private sector is more heterogeneous and less unionized. Thus we expect the full-time peaks associated with the public sector to be higher than the full-time peak associated with the private sector. Note also that normal working hours may vary across jobs according to how strenuous the jobs are considered to be. For example nurses, fire-workers and police officers have typically lower normal working hours than the average worker.

To facilitate estimation, we have assumed that the choice set of hours is discrete. For each sector we have specified 7 hours of work intervals. The medians of the intervals range from 420 annual hours in the first interval to 2808 in the 7th interval. For each sector the full-time peak occurs in the 5th interval where the median is 1950 annual hours. The part-time peak is related to the 3rd interval with a median equal to 1040 annual hours. These intervals correspond to the most common agreements of what constitutes full time and half time annual hours of work.

In section 2.3 we postulated particular invariance properties that allowed us to characterize the functional form of the structural part of the utility function. Unfortunately, we have not been able to provide similar principle to characterize the functional form of $g_{j1}(w)$. Recall that $g_{j1}(w)$ is the subjective density of offered wage rates, as perceived by the agent. We shall therefore, in the present application, abandon the specification and estimation of $g_{j1}(w)$, which implies that we can only estimate the marginal density of chosen hours of work and sector.

Assume next that

(4.2)
$$v_j(C,h) = v(C,h)\mu_j$$

where μ_j is a positive term that represents the pure average utility of working in sector j. Hence, we can write $\psi_j(h,w;I) = \psi(h,w;I)\mu_j$.

Let $\bar{w}_j$ be the subjective mean in the offered wage rate distribution in sector j, i.e., (4.3) $$\bar{w}_j = \int_{y>0} y g_{j1}(y)dy .$$

By the mean value theorem we have that

(4.4) $$\int_{y>0} \psi(h, y; I) g_{j1}(y)dy \cong \psi(h, \bar{w}_j; I) .$$

The approximation in (4.3) is good if the variance in the subjective opportunity density $g_{j1}(w)$ is small.

To allow for unobserved heterogeneity in the opportunity densities we assume that

(4.5) $$\bar{w}_j = w_j^* \eta_j$$

for $j = 1, 2$, where $\{\eta_j\}$ are *random effects*. We assume that $\log \eta_j$, $j = 1, 2$, are independent and normally distributed, $N(0, \sigma_j)$.

The systematic term of the subjective mean wage rate, w_j^*, is assumed to vary across sectors and $\log w_j^*$ is assumed to be a linear function of length of schooling, experience and work experiences squared. Let $b_j \equiv \mu_j \theta_j$. Unless we assume that $\mu_j = 1$ we cannot identify θ_j. The restriction $\mu_j = 1$ means that, ceteris paribus, the average value of non-market and market non-pecuniary attributes do not differ across sectors and the non-market state. We have also experimented with random effects associated with the terms b_j, and with the coefficient associated with leisure in the systematic part of the utility function.

Thus, when accounting for the unobserved heterogeneity, it follows from (2.12), (2.13) and (4.2) to (4.5) that the resulting choice probabilities that correspond to our observations are

(4.6) $$\bar{\varphi}_j(h; w_1^*, w_2^*, I) = E\left[\frac{\psi(h, w_j^* \eta_j; I) g_{j2}(h) b_j}{\psi(0,0;I) + b_1 \sum_{x>0} \psi(x, w_1^* \eta_1; I) g_{12}(x) + b_2 \sum_{x>0} \psi(x, w_2^* \eta_2; I) g_{22}(x)}\right],$$

for $h > 0$, $j = 1, 2$, and

(4.7) $$\varphi(0; w_1^*, w_2^*, I) = E\left[\frac{\psi(0,0;I)}{\psi(0,0;I) + b_1 \sum_{x>0} \psi(x, w_1^* \eta_1; I) g_{12}(x) + b_2 \sum_{x>0} \psi(x, w_2^* \eta_2; I) g_{22}(x)}\right],$$

where expectation is taken with respect to the random effects.

For many reasons, most women are working in the service branch of the economy and thus for women there are more feasible jobs available in firms that provide services than elsewhere. In Norway, most of the services are provided by the public sector (health services, education etc) and many of the jobs here require higher education, while the services provided in the private sector say, in retail sale, are typically based on low-skill labor. Thus it is reasonable to assume that θ_j may depend on education. Also μ_j may depend on education. We will expect that the higher the education is, the higher is the number of feasible jobs in the public sector. To this end we have assumed that

(4.8) $$\log b_j = f_{j1} + f_{j2} S$$

where S is the length of education.

We have chosen $\log v(\cdot)$ to be of the form given in Theorem 2.

(4.9)
$$\log v(C,h) = \alpha_2 \left(\frac{[10^{-4}(C-C_0)]^{\alpha_1} - 1}{\alpha_1} \right) + \left(\frac{(L-L_0)^{\alpha_3} - 1}{\alpha_3} \right) \left(\alpha_4 + \alpha_5 \log A + \alpha_6 (\log A)^2 + \alpha_7 CU6 + \alpha_8 CO6 \right)$$
$$+ \alpha_9 \left(\frac{[10^{-4}(C-C_0)]^{\alpha_1} - 1}{\alpha_1} \right) \left(\frac{(L-L_0)^{\alpha_3} - 1}{\alpha_3} \right)$$

where A, is the age of the married woman, CU6 and CO6 are the number of children less than 6 and above 6 years, C is given by $f(hw, I)$, L is leisure, defined as

(4.10) $$L - L_0 = 1 - h/3640,$$

and α_j, $j = 1,2,...,8$, are unknown parameters. Observe that we have subtracted from total annual hours a "subsistence" level, amounting to 5120 hours, which allows for sleep and rest. This corresponds to about 14 hours per day reserved for sleep and rest.

Consistent with psychophysical evidence, we have also introduced a subsistence threshold level, C_0 for consumption in the $v(\cdot)$ function. We have chosen C_0 to be close to the official estimate of a subsistence level in Norway (NOK 60 000). C is measured as the sum of the annual wage income of the woman and her husband after tax, household capital income after tax and child allowances. The tax functions and the child allowances are given in Appendix F.

If $\alpha_1 < 1$, $\alpha_3 < 1$, $\alpha_2 > 0$, and

(4.11) $$\alpha_4 + \alpha_5 \log A + \alpha_6 (\log A)^2 + \alpha_7 CU6 + \alpha_8 CO6 > 0,$$

α_9 is positive, or if negative, sufficiently small numerically, then log $v(C, h)$ is increasing in C, decreasing in (h) for fixed C and strictly concave in (C, h).

To facilitate the estimation procedure we have estimated the wage equation (regressed log w_j^* against the observed covariates mentioned above) in a first step by applying a version of the two stage Heckman approach to control for selectivity, see Appendix D. Conditional on these estimates the remaining parameters of the model are estimated by the maximum likelihood procedure.

As regards the random effects associated with b_1 and b_2 we have tried two several types of specifications. First we tried to specify the pair (f_{11}, f_{21}) in (4.8) as distributed according to a discrete binomial distribution, which can take two values with probabilities q and 1–q (say). The second type is based on the assumption that (f_{11}, f_{21}) is normally distributed. In addition, we have also estimated a specification where α_4 in (4.9) is assumed to be a normally distributed random effect.

4.2. Data

Data on the labor supply of married women in Norway used in this study consists of a merged sample from "Survey of Income and Wealth, 1994", Statistics Norway (1994) and "Level of living conditions, 1995", Statistics Norway (1995). Data covers married couples as well as cohabiting couples with common children. The age of the spouses ranges from 25 to 64. None of the spouses are self-employed and none of them are on disability or other type of benefits. All taxes paid are observed and in the assessment of disposable income, at hours not observed, all details of the tax system are accounted for. Observed hours of work are related to main job as well as possible side jobs. In 1994 the unemployment rate was rather low by international standard. Thus, for that reason it may be of little importance to employ desired hours (which we don't have) instead of actual hours. The size of the sample used in estimating the labor supply model is 824. Wage rates above NOK 350 or below NOK 40 are not utilized when estimating the wage equations. The wage rates are computed as the ratio of annual wage income to hours worked. When computing annual wage income we take into account that some women have multiple jobs.

In Table 1 we report the summary statistics for the sample used in estimating the labor supply model.

Table 1. Summary statistics for married women in the sample, Norway 1994

	Not working		Public sector		Private sector	
	Mean	Std.	Mean	Std.	Mean	Std
Age in years	40.44	9.92	41.24	8.68	38.68	9.08
Education in years	11.02	2.01	12.36	2.34	10.88	1.62
No of children, 0-6	0.73	0.83	0.44	0.74	0.63	0.81
No of children, 7-17	1.00	1.01	0.75	0.86	0.53	0.80
Annual hours of work	0	0	1641	489	1570	571
Disposable household income, NOK per year	322 131	200 684	329 064	122 616	331 354	125 754
Wage rate, NOK per hours			104.30	28.52	100.56	30.46
Number of observations	66		405		353	
Fractions	0.080		0.492		0.428	

From Table 1 we observe that the participation rate is rather high, 92 percent. In the first place the overall participation rate among married women is very high in Norway, actually in 1994 it was the highest in the world. Second, as mentioned above, we have made a selection with respect to age, health status, etc, which contribute to increase the participation rate further. We note that the mean wage is slightly higher in the public sector than in the private sector, whereas the wage dispersion tends to be higher in the private sector than in the public sector.

4.3. Estimation results for the wage equations

In this section we report estimates of the wage equation.

Table 2. Estimates of wage equations. single and married women, Norway 1994

Variables	Public sector		Private sector	
	Estimates	t-values	Estimates	t-values
Constant	3.37	13.5	3.70	25.2
Experience in years/100	3.21	6.0	2.55	5.1
(Experience in years/100)^2	-4.75	-5.3	-3.80	-4.2
Education in years/100	5.57	4.9	5.26	4.2
Log (Probability of working in the chosen sector)	-0.12	-2.0	0.06	0.9
Variance	0.059	18.6	0.075	17.0
No of observations	691		580	
R^2	0.14		0.08	

The wage equation is specified in a conventional way, i.e., the logarithm of observed wage rates, $\log W_k$, $k = 1, 2$, is assumed to depend linearly on experience, experience squared, education level. Experience is defined as age minus years of schooling minus six. To control for the possibility of selectivity we have used a simpler model than the structural model developed above. The main reason for this is that our sample used to estimate the wage equations consists of married and unmarried women, while the model above is only postulated for married women. From the simplified modeling approach adopted in this section it can be demonstrated that one can control for selectivity bias by applying $\log P_j$ as an additional independent variable where P_j is the probability of being in sector j, $j = 0, 1, 2$, (where $j = 0$ means not working). The details of this approach are given in Appendix D. Estimates of the wage equations are given in Table 2, and we observe that on the margin workers get slightly better paid for experience and education in the public sector than in the private sector. However, the differences in returns across sectors are not significant. On a much larger sample Barth and Røed (2001) reports similar results for 1995.

Judged by R^2 the explanatory power of the wage equations is low. Thus, it seems important to account for unobservables in the wage equation when estimating the structural model. In Section 4.1 we have explained how we account for the unobservables in the wage equations. We note that the estimated variance related to the random effects in wages tend to be higher in the private than in the public sector.

4.4 Estimates of labor supply probabilities

Estimates of the parameters in the structural choice model are given in the two first number columns of Table 3. Both exponents (α_1 and α_3) are significantly below 1 and the estimates thus imply that the deterministic part of the utility function is quasi-concave. We note that the parameter associated with the interaction term between consumption and leisure is significantly different from zero and negative. Hence, we can reject the hypothesis that the deterministic part of the utility function is additively separable in consumption and leisure.

Marginal utilities of consumption and leisure (for all relevant age of the women) are both positive. The latter depends significantly on age and number of children. Marginal utility of leisure is a convex function of age, with marginal utility increasing with age after 31-32 years of age. Marginal utility of leisure is positively affected by number of children. We observe that the number of young and “old” children has the same impact on the marginal utility of leisure. It is interesting to note that when the women are young and have children this reduces their incentive to participate in labor market activities, and when they are older and without children the age effects gradually also reduces their incentives to participate in labor market activities.

The exponent α_1 is significantly different from zero, which means that agents do not only care about relative consumption levels (beyond subsistence). Absolute levels also matter. The exponent α_3 is almost not significantly different from zero (at a 5 per cent level or less), which means that the deterministic part of the utility function is close to being log-linear in leisure.

The estimates of the opportunity density confirm the conjecture that there are more jobs available in the public sector for higher educated women than for women with lower education. This means that if length of schooling is increased while wage rates are kept fixed, participation in the public sector will increase. In the private sector education does seem to have the opposite effect. At first glance this seems to indicate that highly educated women tend to be "overqualified" in the private sector. However, this may also be due to a preference effect: If preferences depend on schooling such that highly educated women tend to prefer jobs in the public sector (university, etc.) this would imply that b_2 is decreasing in S.

Moreover, the full-time peak is more distinct in the public sector than in the private. As mentioned above, this may be due to the fact that the public sector is more unionized than the private one.

It turned out to be very difficult to estimate the random effects associated with b_1 and b_2. We have tried a discrete binomial-and the normally distributed random effects specifications and both yielded degenerate results. Also, we were not able to estimate specifications base on discrete distribution with more that two support points. Although we cannot rule out the possibility that other types of random effects may exist, we nevertheless think our estimation result indicate that it is likely that any random effect is negligible.

Analogous to van Soest et al. (2002) we have also tried to include a random effect in the leisure term of the utility function. Similarly to van Soest et al. we find that the t-value of the random effect is practically equal to zero.

The number of random draws for each random component in the wage equations equals 50. Thus, because there are two random components the total number of draws used in the simulations to compute the likelihood function equals 2500.

Table 4 compares observed and predicted aggregates, and we note that the model predicts these aggregates pretty well. In Figure 1 we show the observed fractions and predicted probabilities across the different hours of work categories (category 1, is not working, categories 2-8 are public sector and 9-15 are private sector). We observe that our model predicts very well the observed frequencies.

As noted above, we can in general only identify the product $v(C,h)g_{j2}(h)$ non-parametrically. To disentangle $v_2(h)$ from $g_2(h)$ we have assumed that the clustering of hours of work at part-time and full-time work is due to technological organizational constraints and/or regulation of hours introduced by unions and/or the government. The term $g_{j2}(h)$ is meant to capture this aspect of the labor market in the highly unionized Norwegian economy. Thus, through parametric identification our model implies that observed concentration of hours of work around part-time and full-time work arise because there are institutional constraints in the labor market rather than because individuals have strong preferences for full-time and part-time hours of work. If the parameters of the utility function are robust with respect to our assumption, then our empirical model may be applied also to simulate the impact of a change in the institutional constraints on available working hours in the market.

To contrast our approach with the more familiar one with uniformly distributed offered hours we have re-estimated the model under the assumption that *offered* hours is uniformly distributed. It goes without saying that the parameters attached to the leisure term will be affected when we force the clustering of hours to be explained solely by preferences. Of greater interest is how the estimates of the other parameters of the utility function are affected when offered hours are assumed to be uniformly distributed. In the third and fourth number columns of Table 3 the estimates for this case are reported. We note that the parameters in the sector- specific opportunity densities remain unchanged. The exponent of the leisure term is significantly different from zero and negative when offered hours are uniformly distributed. The numerical value is higher when offered hours are uniformly distributed compared to the case with full-time and part-time peaks. This is natural, because when the clustering of hours is due to spikes in the distribution of offered hours, labor supply responses to marginal wage changes becomes weak, given a full time job. As in the preceding case marginal utility of leisure is positive for all relevant ages and marginal leisure is at a minimum for the same age as before, around 32 years of age. It is important to note that if offered hours are not uniformly distributed, which there is good reason to believe, then a change in this institutional constraint will be misinterpreted as a shift in the preferences of leisure in the model based on the assumption of uniformly distributed offered hours.

Table 3. Estimation results for the parameters of the labor supply probabilities

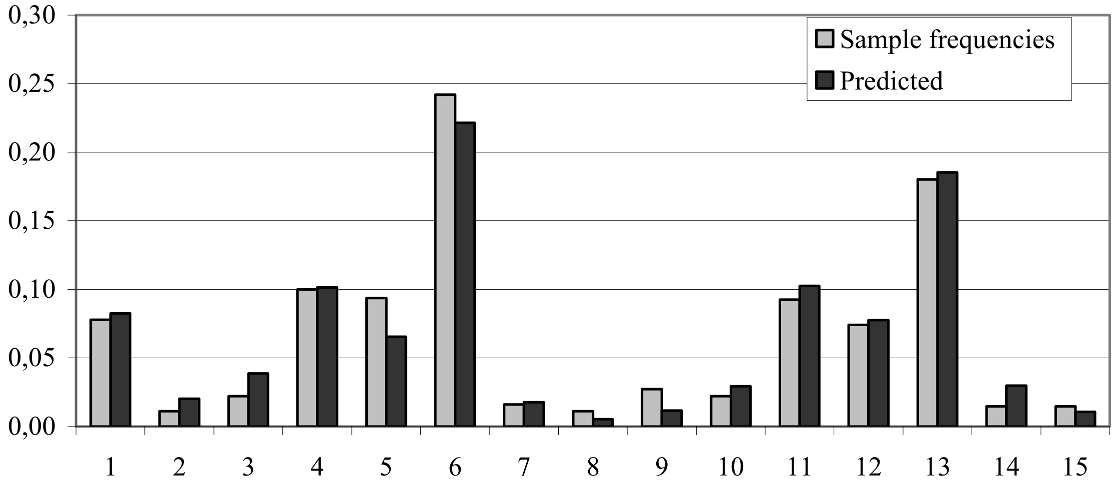
		Uniformly distributed offered hours with part-time and fulltime peaks		Uniformly distributed offered hours	
Variables	Parameters	Estimates	t-values	Estimates	t-values
Preferences:					
Consumption:					
Exponent	α_1	0.64	7.6	0.54	6.0
Scale 10^{-4}	α_2	1.77	4.2	3.16	4.0
Subsistence level C_0 in NOK per year		60 000		60000	
Leisure:					
Exponent	α_3	-0.53	-2.1	-1.88	-5.1
Constant	α_4	111.66	3.2	40.92	2.5
Log age	α_5	-63.61	-3.2	-22.50	-2.5
$(\log \text{age})^2$	α_6	9.20	3.3	3.23	2.5
# children 0-6	α_7	1.27	4.0	0.43	2.9
# children 7-17	α_8	0.97	4.1	0.30	2.7
Consumption and Leisure, interaction	α_9	-0.12	-2.7	-1.52	-1.9
Subsistence level of leisure in hours per year		5120		5120	
The parameters b_1 and b_2; $\log b_j = f_{j1} + f_{j2}S$					
Constant, public sector (sector 1)	f_{11}	-4.20	-4.7	-4.87	-5.4
Constant, private sector (sector 2)	f_{21}	1.14	1.0	-0.02	-0.1
Education, public sector (sector 1)	f_{12}	0.22	2.9	0.24	3.1
Education, private sector (sector 2)	f_{22}	-0.34	-3.3	-0.26	-2.8
Opportunity density of Offered hours, $g_{k2}(h)$, k=1,2					
Full-time peak, public sector (sector 1)*	$\log\left(g_{12}(h_{Full})/g_{12}(h_0)\right)$	1.58	11.8		
Full-time peak, private sector (sector 2)	$\log\left(g_{22}(h_{Full})/g_{22}(h_0)\right)$	1.06	7.4		
Part-time peak, public Sector	$\log\left(g_{12}(h_{Part})/g_{12}(h_0)\right)$	0.68	4.4		
Part-time peak, private Sector	$\log\left(g_{22}(h_{Part})/g_{22}(h_0)\right)$	0.80	5.2		
# observations		824		824	
Log likelihood		-1760.9		-1862.0	

* The notation h_0 refers to an arbitrary level of hours of work different from full-time and part-time hours.

Table 4. Observed and predicted aggregates, married women, Norway 1994

Variables	Not working		Public sector		Private sector	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
Choice probabilities	0.080	0.079	0.492	0.483	0.428	0.438
Annual hours	0	0	1641	1585	1570	1632
McFadden's ρ^2			0.211			
McFadden's adjusted ρ^2			0.195			

Figure1. Probabilities and frequencies :
1 not working, 2-8 working in public sector; 9-15 working in private sector



4.5. Wage elasticities

The mean utility, $\psi_j(.)$, is the utility concept that comes closest to the one often used by others in the calculation of elasticities. To calculate these elasticities one has to assume that the labor supply of the mean sample household can be simulated by maximizing the deterministic part of the utility function under the constraint represented by a linearized version of the budget constraint. Of course, this approach is rather crude since it implies that the stochastic structure of the model is ignored.

Another set of elasticities arises when we consider how the mean in the *distribution* of labor supply is affected by changes in say, wage levels. We denote these elasticities as *aggregate* ones since they take into account unobserved and observed heterogeneity in the population. Moreover, they also account for the non-convexity of the budget constraint due to taxation and restrictions on hours, and are thus consistent with the structure of the model.

In Table 5 we report what we have called aggregate uncompensated elasticities. They are calculated as follows: The model is used to simulate the labor supply for each female under the current regime and when the wage rates in each sector, and in both sectors, respectively, are increased by one per cent. The aggregate elasticity of female labor supply is obtained by calculating the relative change in the mean female labor supply (over all females in the sample) that results from a one per cent wage increase for the females, ceteris paribus.

Table 5. Aggregate uncompensated wage elasticities, married women, Norway 1994

Variables	Choice probabilities and mean hours before wage changes	Elasticities with respect to changes in wage rates		
		Public sector	Private sector	Both sectors
Participation probabilities:				
Working	0.921	0.15	0.16	0.28
Working in public sector	0.483	1.52	-1.40	0.09
Working in private sector	0.438	-1.29	1.80	0.48
Mean annual hours of work, conditional on sector:				
Public sector	1585	0.36	0.03	0.37
Private sector	1632	-0.09	0.17	0.34
Mean total annual hours of work:				
Unconditional	1480	0.34	0.37	0.65

Elasticities are numerically small apart from the case when sector specific wage levels are changed. The reasons why overall responses are small are due to high overall labor market participation among married Norwegian women and a regulated and rigid market of offered hours of work. However, we note that when the wage level in the public sector is increased by 1 per cent, the participation probability in this sector increases by as much as 1.52 per cent. Most of this increase comes at the expense of a decrease in the participation probability in the private sector of minus 1.29 per cent. Overall participation increases by a minor 0.15 per cent. A similar pattern emerges when the wage level in the private sector is raised by 1 per cent. The probability of working in the private sector increases by 1.80 per cent and as in the preceding case, most of the increase comes from a reduction of the probability of working in the other sector, the public sector (minus 1.40 per cent). A similar pattern emerges for annual hours of work, conditional on sector, but the impact is smaller. From the last row in Table 5 we notice that an overall increase of 1 per cent is estimated to increase supplied hours in our sample by 0.65 per cent, with an almost equal split on increased participation and increased supply of hours, conditional on working.

In Table 6 we report wage elasticities across deciles in the distribution of wage rates (predicted). We distinguish between the bottom 10 per cent, the middle of 80 per cent and the top 10 per cent in the distribution of wage rates in the population. The purpose of doing this is to see whether the wage elasticities are declining with the wage rate, and hence with potential income, of the female. If so, the potential well off women will be less responsive with regard to economic incentives than women with less potential income, see Røed and Strøm (2002) for a discussion of this topic and for references to previous studies that have reported this result.

From Table 6 we observe that the probability of participation is increasing with the wage rate. In the bottom decile of the wage distribution it equals 85.8 percent, while in the top decile it is 97.3 percent. We observe that the probability of working in the public sector is increasing with the wage rate, whereas the opposite is the case for the private sector. Expected hours of work, conditional on working in a specific sector, is increasing with the wage rate in both sectors. It is interesting to note that hours of work tend to be significantly higher in the private sector than in the public, in the top decile it is close to 10 per cent higher. Also, the ratio of hours worked in the top decile relative to the bottom decile is higher in the private sector (17 per cent higher) than in the public sector (10 per cent higher). Apparently, this may reflect the fact that hours are more restricted in the public than in the private sector making it possible for the higher educated woman, working in the private sector, to extend working hours beyond hours related to so-called full-time jobs. It should also be emphasized that the variance in the wage rate distribution is larger in the private than in the public sector. Thus the perceived chance of obtaining a higher wage rate is higher in the private sector than in the public. Hence, less regulated hours and a chance of obtaining a higher wage rate may explain why working hours are higher in the private than in the public sector.

Due to the differences in participation probabilities and working hours, given the sector, across deciles in the wage rate distribution, the unconditional expected hours of work in the population is much higher (21 per cent) in the top deciles of the wage distribution than in the bottom deciles.

With some interesting exceptions the wage elasticities are declining with the wage rate. Thus, we get the result that labor supply of a woman with a potential high wage rate and hence potential high income, (given hours of work) is less responsive than for a woman with lower potential wage rate. The exceptions are own wage and cross wage elasticities related to participation in the private sector. Here it seems that a higher wage in the private sector increases the probability of working in the private sector, and more so among those with the highest potential wage rate. Again, the combination of less regulation of hours worked and the possibility of obtaining higher wages may be the reason. The numerical values of the cross wage elasticities in the private sector (the elasticities of participation in the private sector with respect to the wage in the public sector) are increasing with the wage rate.

Table 6. Aggregate uncompensated wage elasticities across deciles in the distribution of wage rates

Variables	Choice probabilities and mean hours before wage changes	Elasticities with respect to changes in wage rates		
		Public sector	Private sector	Both sectors
Participation probabilities:				
Working:				
10 percent lowest wage	0.858	0.17	0.26	0.39
80 percent in the middle	0.917	0.08	0.16	0.29
10 percent highest wage	0.973	0.10	0.06	0.15
Working in public sector:				
10 percent lowest wage	0.354	1.96	-1.63	0.20
80 percent in the middle	0.452	1.63	-1.46	0.11
10 percent highest wage	0.726	0.81	-1.01	0.01
Working in private sector:				
10 percent lowest wage	0.504	-1.08	1.60	0.53
80 percent in the middle	0.464	-1.27	1.74	0.45
10 percent highest wage	0.247	-2.00	3.20	0.88
Mean annual hours of work, conditional on sector:				
Public sector:				
10 percent lowest wage	1477	0.41	0.03	0.45
80 percent in the middle	1562	0.34	0.03	0.37
10 percent highest wage	1629	0.30	0.03	0.32
Private sector:				
10 percent lowest wage	1509	0.03	0.34	0.37
80 percent in the middle	1606	0.04	0.30	0.33
10 percent highest wage	1774	0.06	0.24	0.31
Mean total annual hours of work				
Unconditional:				
10 percent lowest wage	1283	0.37	0.54	0.81
80 percent in the middle	1454	0.34	0.37	0.65
10 percent highest wage	1622	0.30	0.24	0.49

The overall elasticities are low, which reflect that labor market participation is high among married women in Norway and regulations on working hours impose restrictions on choices. But this low labor supply elasticities clearly shadow for high sector specific elasticities. This implies that with sector specific changes in wage rates one should expect a considerable increase in the mobility across sectors. The Norwegian economy is a highly unionized economy with coordinated wage settlements supported by the unions and the governments. Hence, sector specific changes in wage rates have been the exception rather than the rule. However in recent years, the unions for the higher educated as well as the employers associations in the public and the private sector have pressed for more local wage settlements and with a stronger emphasize on providing higher returns to their education than what the present compressed wage structure in Norway implies. The elasticities above then indicates that such sector (or even job-specific) wage rate increases may give rise to a much higher mobility across sector and jobs than what has been observed recently in the Norwegian economy. It is likely that local wage settlements focusing on increased returns to education first will take place in the private sector. Our model then predicts that an increase in private sector real wages, say of 5 per cent, relative to the wages in the public sector, and restricted to those belonging to the top decile in the wage distribution, will increase the probability of working in the private sector for these women from 24.7 per cent to 28,7 percent and reduce the probability of working in the public sector from 72.6 per cent to 68.9 per cent. These changes may not be viewed as large by say, US standards, but they are sizeable by Norwegian standards. We observe that the impact on overall labor market participation from such a change in wage rates will be negligible (elasticity equals 0.06), the reason being that the participation rate among these women is very high, close to 100 per cent. Expected hours of work, given the sector affiliation, will increase in both sectors. A labor supply elasticity of 0.24 in the private sector for the women belonging to the top decile, implies that on average expected hours will increase by 22 hours a year when wage rates for the women in the top decile, working in the private sector are raised by 5 per cent. In the public sector the cross elasticity equals 0.03, which implies that expected hours increase by 2.5 hours a year. The reason why it is positive (although negligible) is that those moving from the public to the private sector, when the private sector wage is increased, work less hours in the public sector than those remaining in the public sector.

4.6 A flat tax reform

In Table 7 below we show the result of replacing the tax system of 1994 by a flat tax rate, which is determined so as to keep the tax revenue (for married couples) unchanged. In doing so, we account for the fact that the change in the tax system will also change after-tax wage income of the husbands. The tax of 28 per cent on capital income is kept constant. We find that a tax rate of approximately 29 per cent charged from the very first Krone earned brings in the same tax revenue as the 1994 tax rules do. It is interesting to note that this tax rate is close to the tax rate charged on capital income. From Table 7 we note that this potential tax reform increases overall labor supply, participation as well as hours worked, but not by much. The overall participation probability increases by merely 0.07 percentage points, that is, from 92.1 to 92.8 per cent. Expected hours of work in the total population is increased by 108 hours, altering annual hours of work from 1480 to 1588. These changes shadow for stronger responses across sectors and deciles in the wage distribution.

The flat tax reform implies a flow of workers, away from the public sector and towards the private sector. The overall probability of working in the public sector is reduced by 2.65 percent, from 48.3 to 45.6, whereas the private sector participation probability increases from 43.8 to 46.5. These flows increase with the wage rate. Women who belong to the top decile are predicted to increase their participation probability in the private sector by as much as 7.56 percentage points, that is, from 24.7 percent to 32.3, most of this increase is due to a reduction among "well-paid" women working in the public sector.

Table 7. A flat tax reform with same tax revenue as with the 1994 tax rules. Percentage points changes in probabilities and changes in annual hours. Flat tax relative to 1994 rules. The flat tax rate is 29 percent

Variables	Women's wage, first deciles	Women's wage, middle deciles	Women's wage, top deciles	All
	Percentage points			
Probability of working	0.57	-0.14	1.13	0.07
Probability of working in public sector	-0.87	-2.44	-6.43	-2.65
Probability of working in private sector	1.44	2.30	7.56	2.72
	Annual hours (Changes in percent)			
Mean annual hours of work conditional on sector:				
Public sector	46 (3.1)	80 (5.1)	118 (7.2)	80 (5.1)
Private sector	99 (6.5)	148 (9.2)	216 (12.2)	126 (7.8)
Mean total annual hours of work, unconditional:	76 (5.9)	105 (7.2)	169 (10.4)	108 (7.4)

The reasons for these responses are; first that this tax reform forces women in the lowest deciles to work harder (they have to pay higher taxes on the margin as well as a per cent of total income). Second, the women at the top of the wage rate distribution get a substantial reduction in the marginal tax rates. This incentive to work more dominates over the fact that the average tax, taxes paid relative to household income, also is reduced. Our calculations of income elasticities (not shown here) indicate that the income effects are not strong.

The women in the middle of the wage rate distribution get almost no change in their marginal tax rate, but they enjoy a reduction in taxes paid relative to household income (remember that the tax rate on the labor income of their spouse is also changed). As shown in Table 7 there is a minor reduction in the probability of working among women in the middle deciles. Finally, a flat tax of 29 percent makes is more attractive to increase hours of work, in particular among the women with the highest earning potential and also, especially in the private sector, where the regulation of hours is less than in the public sector.

4.7. Comparison between our model and a van Soest type model specification

In this section we discuss the results of estimating the flexible preference model (FPM) of van Soest, Das and Gong (2002). The estimates are reported in Appendix E. First we have estimated a specification where the utility function is approximated by a polynomial of degree 5. Fixed costs of working is included as well as a random effect associated with leisure in a completely similar way as in van Soest, Das and Gong (2002). The random effect in the wage equation is however included in a similar way as in our model described above. From Table E.1 we observe that most of the coefficients are not sharply determined with the exception of the coefficients attached to the variables that are assumed to affect the fixed costs of working. Similarly to van Soest, Das and Gong (2002) we obtain the unreasonable result that fixed cost of working decreases with the number of children. By comparing the likelihood values of Table E.1 and Table 3 we realize that our model explains data much better than the FPM. Recall that our model differs from the FPM not only in the specification of the functional form of the utility function, also the specification of the opportunity sets are very different. A very disturbing fact with the estimates of the FPM reported in Table E.1 is that for most of the households the fixed costs of working exceed after tax wage income, given positive hours, yielding negative disposable incomes.

Second, we have estimated the FPM without a representation of fixed costs of working. The empirical estimates for a 3rd and 5th order polynomial utility specifications are given in Table E.2. Again, our model explains data much better than the van Soest, Das and Gong (2002) model. In Figures E.1 and E.2 we give the estimated indifference curves for an average woman and we observe that the estimated utility function is not quasi-concave.

Finally, in Table E.3, we report estimates based on combining a 3rd order polynomial utility specification with our specification of opportunity sets. Measured by McFadden's adjusted ρ^2 (see Ben-Akiva and Lerman, 1985) the goodness of fit of this combined model is the same as for our model, and Figure E.3 also indicates that the utility function is quasi-concave in the central area of consumption and leisure combinations. However, from Figures E.3 we note that the indifference curves for the polynomial utility specification implies the unreasonable property that the needed compensation (in terms of consumption) for working one more hours is almost the same irrespective of hours worked. In contrast, the corresponding indifference curves for the Box Cox utility specification displayed in Figure E.4 shows a more reasonable pattern, namely that the needed compensation for working one hour more is sharply increasing with hours of work for high levels of hours of work.

5. Conclusion

In this paper we have discussed a particular approach for labor supply modeling, with special reference to the following issues:

(i) Households may have preferences over jobs, characterized by job- and sector-specific non-pecuniary attributes, hours of work and wage rates.

(ii) Exact representation of complicated and non-convex budget constraints, and convenient representations of the set of feasible job attributes,

(iii) Justification of the functional form of the utility function and the distribution of unobserved variables.

We have demonstrated that the framework presented proves to be practical for dealing with (i) and (ii). A more fundamental theoretical issue is the problem of characterizing the functional form of the empirical model on the basis of theoretical principles. By drawing on the recent literature in measurement theory and theoretical psychophysics, we have shown that it is possible to apply invariance principles to constrain and justify the class of admissible functional forms for the utility function.

An empirical version of the model has been estimated on a sample of Norwegian married women. The estimated model turns out to reproduce the data quite well. It is demonstrated that it is of empirical importance to distinguish between job opportunities across sectors of the economy. In terms of wage elasticties the model yields moderate responses in overall female labor supply in contrast to stronger inter-sector mobility. We also report the impact of labor supply of replacing the 1994 tax rules by a revenue neutral flat tax reform. It is shown that this reform stimulates labor supply, in particular among the women with the highest wage rates. Moreover, this tax reform provides incentives to switch from jobs in the public sector to jobs in the private sector, in particular among women with potentially high wage rates.

Apparently, the rich households will benefit from having the progressive tax system (as of 1994) replaced by a flat tax system without any tax breaks at low incomes. The elasticities set out in Table 6 indicate that a tax reform that focuses on providing stronger incentives to work among married women with lower potential wage rate and hence income may improve efficiency of the economy more than a pure flat tax reform. Cutting marginal taxes for those with the lowest potential income, keeping total tax revenue fixed, may also give these households welfare gains instead of the potential rich households. However, it is beyond the scope of the present article to estimate the gains and losses in utility terms across households and to analyze the impact on efficiency and equity of various tax systems. This is left for future research.

We have also investigated the modeling framework with flexible preferences (FPM), proposed by van Soest, Das and Dong (2002). Although more experience with this framework is needed before a definite conclusion can be reached, the empirical results we have obtained indicate that this framework does not seem to be promising as a general research strategy for labor supply analysis. This is due to, (i) the inability of the pure FPM to accommodate institutional restrictions on hours of work and the sets of feasible jobs, (ii) the failure of the polynomial utility representation to fulfill global quasi-concavity and monotonicity properties. Although these properties are not required for the existence of a utility maximum in the discrete case, they are certainly plausible from an economic point of view.

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Constrained demand and IIA

Here the purpose is to show that the traditional theory of consumer behavior yields restrictions that appear (in some sense) more restrictive than IIA.

Consumer i has quasi-concave, increasing utility function $U_i(x)$. The utilities vary randomly across consumers due to unobserved heterogeneity in tastes. Let B be the choice set, i.e. a set of quantity restrictions on x in addition to the budget constraint, and let $x_i^*(p,y,B)$ denote the constrained demand function of price p, income y and choice set B. Let $A \subset B$, and define

$$P(A,B) = P\left(x_i^*(p,y,B) \in A\right).$$

The empirical counterpart to P(A,B) is the fraction of consumers that choose a vector of consumption quantities within A, conditional on quantity constraints represented by B. Now it follows from quasi-concavity and monotonicity of utility that for two choice sets, B_1 and B_2, that $P(A,B_1) = P(A,B_2)$ provided A belongs to the interior $B_1 \cap B_2$. This is due to the fact that only the point of tangency between the budget line and the indifference curve matter for the determination of the demand. In other words, when A belongs to the interior of the choice set the theory predicts that the choice probability P(A,B) is independent of B as long as A belongs to the interior of B. Thus, the conventional theory yields restrictions that are similar to IIA and appear even more restrictive than IIA, since IIA only predicts that $P(A_1,B)/P(A_2,B)$ is independent of B, where $A_1, A_2 \subset B$. An equivalent statement of IIA is that $P(A_1,B_1)/P(A_1,B_2) = P(A_2,B_1)/P(A_2,B_2)$, which of course is a weaker condition than $P(A,B_1) = P(A,B_2)$.

Appendix B

Proof of Theorem 1

By Assumption 1 and (2.2)

(B.1)
$$U(z) = \psi\big(H(z), W(z)\big)\varepsilon(z),$$

where we for simplicity have suppressed non-labor income I in the notation. The proof is completely analogous to the proof of Theorem 7 in Dagsvik (1994), but for the sake of completeness we give an alternative proof here.

From Assumptions 2 and 3, and Proposition 3.8, p. 135 in Resnick (1987) it follows that $\{(H(z), W(z), \varepsilon(z)), z = 1, 2, ...\}$ are realizations of a Poisson process on $(0, \bar{h}] \times R_+^2$ with intensity measure $d\lambda(h, w, \varepsilon)$ defined by

(B.2)
$$d\lambda(h, w, \varepsilon) = g(h, w)\kappa(\varepsilon) dh\, dw\, d\varepsilon$$

for $h > 0, w > 0, \varepsilon > 0$. Let A be a Borel set in $(0, \bar{h}] \times R_+$, and define

(B.3)
$$U_A = \max_z \big(\psi\big(H(z), W(z)\big)\big) \text{ subject to } \big(H(z), W(z)\big) \in A .$$

U_A is the highest utility the agent can attain, subject to $\big(H(z), W(z)\big) \in A$. Let

$$B = \{(h, w, \varepsilon) : \psi(h, w) \in A, \varepsilon > 0\}$$

and let N(B) be the number of Poisson process points within B. By the Poisson law

(B.4)
$$P\big(N(B) = n\big) = \frac{\Lambda(B)^n}{n!} \exp\big(-\Lambda(B)\big)$$

where $\Lambda(B) = EN(B)$, and is given by

(B.5)
$$\Lambda(B) = \int_B d\lambda(x, y, \varepsilon) = \int_{(x,y)\in A,} \int_{\psi(x,y)\varepsilon > u} \kappa(\varepsilon) g(x, y) d\varepsilon\, dx\, dy = \int_A M\left(\frac{u}{\psi(x, y)}\right) g(x, y) dx\, dy$$

where

$$M(\varepsilon) = \int_\varepsilon^\infty \kappa(v) dv .$$

This is so because $d\lambda(h,w,\varepsilon)$ is the probability that there is a Poisson point within $(h,h+dh)\times(w,w+dw)\times(\varepsilon,\varepsilon+d\varepsilon)$ and $\int_B d\lambda(x,y,\varepsilon)$ is the "sum" of probabilities of all possible Poisson point locations within B.

Note moreover that for any A and C that are disjoint Borel sets on $(0,h]\times R_+$, U_A and U_C will be independent. This property is an immediate consequence of the fact that the points of the Poisson process $\{(H(z),W(z),\varepsilon(z)),z=1,2,...\}$ are independently scattered. Now consider a choice setting where the agent has the choice of "not working" or to choose a job with $(H(z),W(z))\in A\cup C$, where A and C are two disjoint Borel sets in $(0,\bar{h}]\times R_+$. For IIA to be satisfied it must be the case that the probability of choosing a job with hours and wage rate within A must have the form

(B.6) $$P\big(U_A>\max\big(U_C,U(0)\big)\big)=\frac{V_A}{V_A+V_C+V(0)}$$

where V_A, V_C and V(0) are suitable scale parameters associated with the respective "alternatives" A, C and "not working". From Yellott (1977) we know that this can only happen if the utilities have the form

(B.7) $$\log U_A=\log V_A+\log\eta_A$$

and similarly for the other alternatives, where $\log\eta_A$, $\log\eta_C$ and $\log\eta(0)$ are i.i. type III exteme value distributed, i.e.,

$$P\big(\log\eta_A\le y\big)=\exp\big(-e^{-y}\big).$$

But this means that $U_A=V_A\eta_A$, where

(B.8) $$P\big(\eta_A\le u\big)=\exp\big(-u^{-1}\big).$$

Consequently, it follows from (B.4), (B.5), (B.7) and (B.8) that

(B.9)
$$P\big(U_A\le u\big)=P\big(\text{There are no points of the Poisson process in B}\big)$$
$$=P\big(N(B)=0\big)=\exp\big(-\Lambda(B)\big)=\exp\left(-\int_A M\left(\frac{\psi(x,y)}{u}\right)g(x,y)\,dx\,dy\right)=P\big(V_A\eta_A\le u\big)=\exp\big(-u^{-1}V_A\big)$$

which yields

(B.10) $$\frac{V_A}{u} = \int_A M\left(\frac{u}{\psi(x,y)}\right) g(x,y)\,dx\,dy\,.$$

Let $A = [h_1, h] \times [w_1, w],\ 0 < h_1 < h < \bar{h},\ 0 < w_1 < w$. By differentiating (B.10) with respect to h and w we obtain that

(B.11) $$\frac{\tilde{V}(h,w)}{u} = M\left(\frac{u}{\psi(h,w)}\right) g(h,w)$$

for some suitable function $\tilde{V}(h,w)$. Eq. (B.11) implies that M(ε) has the form $\theta\varepsilon^{-1}$ where $\theta > 0$ is a constant, so that

$$\kappa(\varepsilon) = \theta\varepsilon^{-2}.$$

Recall that we already established above that $\eta(0) = \varepsilon(0)$ is standard type I extreme value distributed. With $\kappa(\varepsilon) = \theta\varepsilon^{-2}$, we get from (B.10) that

(B.12) $$V_A = \theta\int_A \psi(x,y) g(x,y)\,dx\,dy\,.$$

Let $\hat{z}$ denote the chosen job. Hence, with $D = C \cup A$, we get from (B.6) that

(B.13) $$\Phi(h,w) = P\left(H(\hat{z}) \le h, W(\hat{z}) \le w\right) = \frac{\theta\int_A \psi(x,y)g(x,y)dxdy}{\psi(0,0) + \theta\int_D \psi(x,y)g(x,y)dxdy}\,.$$

From (B.13) it follows that for $h > 0, w > 0$, the corresponding density is given by

(B.14) $$\varphi(h,w) = \frac{\theta\psi(h,w)g(h,w)}{\psi(0,0) + \theta\int_D \psi(x,y)g(x,y)dx\,dy}$$

which yields (2.5).

Q.E.D.

Proof of Theorem 2

Assume first that Assumption 5 holds. In this case leisure L is kept fixed, and we shall for simplicity drop it in the notation, i.e., we write $v(C,h) = v(C)$ and $\tilde{\varphi}(C_1, L; C_2, L) = \tilde{\varphi}(C_1, C_2)$. In this case we see from (2.12) that we can write

(B.15) $$\tilde{\varphi}(C_1, C_2) = F\big(v(C_1)/v(C_2)\big)$$

where

(B.16) $$F(y) = \frac{y}{1+y}$$

for $y > 0$. Recall also that the input stimuli (consumption C), is measured on a ratio scale. Hence, Theorem 14.19 in Falmagne, p. 338, (see also his discussion on an application following the theorem) implies that

(B.17) $$\tilde{\varphi}(C_1, C_2) = F^*\left(\frac{\beta_1\left(C_1^{\alpha_1}-1\right)}{\alpha_1} - \frac{\tilde{\beta}_1\left(C_2^{\alpha_1}-1\right)}{\alpha_1}\right)$$

where $\beta_1 > 0$, $\tilde{\beta}_1 > 0$, and α_1 are constants, and F^* is a strictly increasing continuous function.[12] Recall, however, that α, β_1 and $\tilde{\beta}_1$ may depend on L. Since C_1 and C_2 can attain any positive value and can vary independently, it follows that the domain of F^* must be R. Also the balance condition

$$\tilde{\varphi}(C_1, C_2) + \tilde{\varphi}(C_2, C_1) = 1$$

must hold. In particular with $C_1 = C_2 = C$, we obtain that $\tilde{\varphi}(C, C) = 0.5$, for all C, which by (B.17) implies that $\beta_1 = \tilde{\beta}_1 \equiv \beta_1$. Let $M(x) = F^{-1}\left(F^*(x)\right)$. Since F^* is continuous, and F is continuous and strictly increasing, it follows that M(x) is continuous and strictly increasing. Thus, from (B.16) and (B.17) we get that

(B.18) $$M\left(\frac{\beta_1\left(C_1^{\alpha_1}-1\right)}{\alpha_1} - \frac{\beta_1\left(C_2^{\alpha_1}-1\right)}{\alpha_1}\right) = \frac{v(C_1)}{v(C_2)}.$$

Let

$$x = \frac{\beta_1\left(C_1^{\alpha_1}-1\right)}{\alpha_1} - \frac{\beta_1\left(C_2^{\alpha_1}-1\right)}{\alpha_1}$$

and

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[12] From Falmagne's Theorem 14.19 it follows that $\alpha \geq 0$. It is, however, easy to verify that the proof of the theorem also applies when α is negative. Note that the above results do not depend on the particular structure of the function $F(\cdot)$ given by (2.12). Also it is not required that F and v are differentiable. It is sufficient that this function is strictly increasing and continuous and (2.10) holds.

$$y = \frac{\beta_1\left(C_2^{\alpha_1} - 1\right)}{\alpha_1},$$

With C_1 and C_2 set equal to one, alternatively, we obtain that

$$v\left(C_1\right) = v(1)M\left(x + y\right), \text{ and } v\left(C_2\right) = v(1)M\left(-y\right),$$

so that (B.18) yields

(B.19) $$M(x) = M\left(x + y\right)M\left(-y\right).$$

With $x = 0$, it follows from (B.18) and (B.19) that

(B.20) $$M(y)M(-y) = M(0) = 1.$$

Although y is only defined for $y \geq -\beta_1/\alpha_1$, we realize that due to the symmetry of (B.20), (B.19) must hold for all real x and y. Hence, (B.19) and (B.29) imply the following functional equation

(B.21) $$M(x)\,M(y) = M(x + y)$$

which must hold for $x, y \in R$. Now recall that (B.21) is the well-known Cauchy equation which solution is the exponential function, cf. Theorem 3.2 and Remark 3.3 in Falmagne (1985), p. 82. Consequently, it follows from (B.21) that

(B.22) $$\log v(C) = \gamma_1 + \beta_1\left(C^{\alpha_1} - 1\right)/\alpha_1$$

where γ_1 is a constant. We now need to re-enter L into the notation. Thus, (B.22) will now be written as

(B.23) $$\log v\left(C, 1 - L\right) = \gamma_1(L) + \beta_1(L)\left(\frac{C^{\alpha_1(L)} - 1}{\alpha_1(L)}\right),$$

since α_1 and β_1 may depend on L, and where total number of feasible hours is normalized to one. In a completely analogous way we get from Assumption 6 that

(B.24) $$\log v(C,1-L)=\gamma_2(C)+\beta_2(C)\left(\frac{L^{\alpha_2(C)}-1}{\alpha_2(C)}\right)$$

where $\gamma_2(C)$, $\beta_2(C)$ and $\alpha_2(C)$ are unspecified functions of C. To simplify the exposition the remaining part of the proof will be divided into 7 cases.

Case (i): $\alpha_1(L)=\alpha_2(C)=0$.

In this case (B.23) and (B.24) imply that

(B.25) $$\gamma_1(L)+\beta_1(L)\log C=\gamma_2(C)+\beta_2(C)\log L,$$

which must hold for all positive C and L. We can write $\beta_1(L)=\beta_1^*+\tilde{\beta}_1(L)$ and $\beta_2(C)=\beta_2^*+\tilde{\beta}_2(C)$ where β_1^* and β_2^* are constants and $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are zero or depend on L and C respectively. This implies that

(B.26) $$\gamma_1(L)+\beta_1^*\log C+\tilde{\beta}_1(L)\log C=\gamma_2(C)+\beta_2^*\log L+\tilde{\beta}_2(C)\log L.$$

Evidently, (B.26) implies that $\gamma_1(L)=\beta_2^*\log L$, $\gamma_2(C)=\beta_1^*\log C$, $\tilde{\beta}_1(L)=\beta_3\log L$ and $\tilde{\beta}_2(C)=\beta_3\log C$ where β_3 is a constant. Hence

(B.27) $$\log v(C,1-L)=\beta_2^*\log L+\beta_1^*\log C+\beta_3\log L\log C.$$

Case (ii): $\alpha_1(L)\neq 0, \alpha_2(C)\neq 0, \gamma_1(L)$ and $\gamma_2(C)$ are not constants.

In this case (B.23) and (B.24) imply a relation of the form

(B.28) $$\gamma_1(L)+\beta_1^*C^{\alpha_1(L)}+\tilde{\beta}_1(L)C^{\alpha_1(L)}=\gamma_2(C)+\beta_2^*L^{\alpha_2(C)}+\tilde{\beta}_2(C)L^{\alpha_2(C)},$$

which must hold for all non-negative L and C. Similarly to the above case, we now have written $\beta_1(L)/\alpha_1(L)=\beta_1^*+\tilde{\beta}_1(L)$, where $\tilde{\beta}_1(L)$ is zero or depend on L. Furthermore, we have also written $\beta_2(C)/\alpha_2(C)=\beta_2^*+\tilde{\beta}_2(C)$, where $\tilde{\beta}_2(C)$ is zero or depend on C. Also $\gamma_1(L)$ and $\gamma_2(C)$ are zero or depend on L and C, respectively. Note that the left hand side of (B.28) contains an additive term, $\gamma_1(L)$, that depends solely on L (or equals zero). This implies that the right hand side of (B.28) must also contain an additive term that depends solely on L. The only term on the right hand side of (B.28) that can vary solely with L is the term $\beta_2^*L^{\alpha_2(C)}$, and this can happen only when $\alpha_2(C)$ is a constant,

α_2. Similarly, since $\gamma_2(C)$ depends on C it follows that $\alpha_1(L)$ must be a constant, α_1. This implies that

$$\gamma_1(L) = \beta_2^* L^{\alpha_2},$$

$$\gamma_2(C) = \beta_1^* C^{\alpha_1}$$

and

$$\tilde{\beta}_1(L) C^{\alpha_1} = \tilde{\beta}_2(C) L^{\alpha_2}$$

which imply that

$$\tilde{\beta}_1(L) C^{\alpha_1} = \tilde{\beta}_2(C) L^{\alpha_2} = \beta_3 C^{\alpha_1} L^{\alpha_2}$$

where β_3 is a constant.

Case (iii): $\alpha_1(L) \neq 0, \alpha_2(C) \neq 0, \gamma_1(L)$ is a constant, $\gamma_2(C)$ is not a constant.

Since $\gamma_1(L)$ is a constant we can without loss of generality set it equal to zero. As a result (B.28) reduces to

(B.29) $$\beta_1^* C^{\alpha_1(L)} + \tilde{\beta}_1(L) C^{\alpha_1(L)} = \gamma_2(C) + \beta_2^* L^{\alpha_2(C)} + \tilde{\beta}_2(C) L^{\alpha_2(C)}$$

Since $\gamma_2(C)$ is not a constant and $\alpha_2(C)$ is different from zero, then (B.29) can only hold if $\alpha_1(L)$ is a constant, α_1, and $\gamma_2(C) = \beta_1^* C^{\alpha_1}$. Hence, (B.29) reduces to

(B.30) $$\tilde{\beta}_1(L) C^{\alpha_1} = \left(\beta_2^* + \tilde{\beta}_2(C)\right) L^{\alpha_2(C)}.$$

Note that (B.30) is multiplicative separable in L and C. Hence, (B.30) can thus only be true if $\alpha_2(C)$ is a constant, α_2, so that

$$\tilde{\beta}_1(L) = \beta_3 L^{\alpha_2}$$

for some constant β_3. But then $\beta_2^* = 0$ and

$$\tilde{\beta}_2(C) = \beta_3 C^{\alpha_1}.$$

Case (iv): $\alpha_1(L)\neq 0, \alpha_2(C)\neq 0, \gamma_1(L)$ is not a constant, $\gamma_2(C)$ is a constant.
This case is completely analogous to Case (iii) and is therefore omitted.

Case (v): $\alpha_1(L)\neq 0, \alpha_2(C)\neq 0, \gamma_1(L)$ and $\gamma_2(C)$ are constants.
In this case we can set $\gamma_1 = 0$. Then (B.29) becomes

(B.31) $$\beta_1^* C^{\alpha_1(L)} + \tilde{\beta}_1(L) C^{\alpha_1(L)} = \gamma_2 + \beta_2^* L^{\alpha_2(C)} + \tilde{\beta}_2(C) L^{\alpha_2(C)}.$$

Evidently, (B.31) implies that $\gamma_2 = 0$. Moreover, by taking the logarithm mapping on both sides of (B.31) we obtain

(B.32) $$\log\left(\beta_1^* + \tilde{\beta}_1(L)\right) + \alpha_1(L)\log C = \log\left(\beta_2^* + \tilde{\beta}_2(C)\right) + \alpha_2(C)\log L.$$

Evidently, (B.32) can only hold provided

$$\log\left(\beta_1^* + \tilde{\beta}_1(L)\right) = \alpha_2(C)\log L + \text{constant}$$

in which case $\alpha_2(C)$ must be a constant, and $\beta_1^* = 0$, so that $\tilde{\beta}_1(L) = L^{\alpha_2}$. Similarly, it follows that $\alpha_1(L)$ is a constant, α_1 and $\tilde{\beta}_2(C) = C^{\alpha_1(L)}$.

In the case where both $\tilde{\beta}_1(L)$ and $\tilde{\beta}_2(C)$ are equal to zero, (B.32) implies that $\alpha_1(L) = \log L$ and $\alpha_2(C) = \log C,$ so that

(B.33) $$\log v\left(C, 1-L\right) = \beta_3 \log L \cdot \log C.$$

Thus this case coincides with a special case of *Case (i)* above.

The remaining cases are *Case (vi)*: $\alpha_1(L) = 0, \alpha_2(C)\neq 0$ and *Case (vii)*: $\alpha_1(L)\neq 0, \alpha_2(C) = 0$. The analysis of these cases is completely analogous to the previous cases and is therefore omitted. We therefore conclude that all the cases considered above yield the functional form that is claimed in the theorem.

Q.E.D.

Appendix C

Equilibrium considerations

How the opportunity density θg(h,w) is determined has been analyzed in Dagsvik (2000), and we refer to this paper for a detailed analysis. In the present paper we shall only draw on some results obtained by Dagsvik (2000) to justify the structure of the opportunity density.

Dagsvik (2000) considers a particular matching equilibrium setting, which is analogous to Crawford and Knoer (1981), see also Roth and Sotomayor (1990). In this setup suppliers have preferences over firms (jobs) including contracts (such as wage rates) and they search to obtain a suitable match among the potential firms. The set of contracts is discrete. Similarly, firms have preferences over suppliers and contracts and they search to obtain a suitable match among the potential suppliers. Assume that a firms' conditional profit function of hiring a worker at contract (h,w), given technology (indexed by t), has the form

$$\tilde{b}(h,w,t)\eta(h,w,t)^{1/\gamma}$$

where $\{\eta(h,w,t)\}$ are positive random variables with standard type I extreme value distribution that that are i.i.d. over all combinations of hours, wage rates and technologies. These errors account for unobserved heterogeneity in firms' preferences. The term $\tilde{b}(h,w,t)$ is positive and deterministic and $\gamma > 0$ is a positive parameter that governs the dispersion of the random error terms. Maximizing the conditional profit function above is of course equivalent to maximizing the function $\tilde{b}(h,w,t)^{\gamma}\eta(h,w,t) = b(h,w,t)\eta(h,w,t)$, (say). For simplicity, we assume in this exposition that (h,w,t) belong to a finite set. Supposed next that, on average, the firms' production functions have the constant returns to scale property. Then it follows, conditional on hours, that the average conditional profit function has the form $\tilde{b}(h,w,t) = hQ_t - hw$, so that

$$b(h,w,t) = h^{\gamma}(Q_t - w)^{\gamma},$$

where Q_t is a positive term that depends on output, and input prices (apart from the wage rate). Due to the properties of the extreme value distribution it follows that

$$\text{(C.1)} \qquad \text{Max}_t\left(b(h,w,t)\eta(h,w,t)\right) = \eta(h,w)b(h,w) = \eta(h,w)h^{\gamma}\sum_{Q_t \geq w}(Q_t - w)^{\gamma}$$

where $\{\eta(h,w)\}$ are random variables that also have type I standard extreme value c.d.f. In the Scandinavian economies the hours of work schedules are highly regulated, as discussed in the text above. Thus, if hours of work in the short run can be considered as fixed like other fixed inputs, the firm's maximization of the conditional profit function is equivalent to maximizing $\eta(h,w)b(w)$, where

(C.2) $$b(w) = \sum_{Q_t \geq w} (Q_t - w)^\gamma .$$

The error terms $\{\eta(h,w)\}$, as well as b(w), depend on the respective suppliers, but for simplicity we have suppressed this in the notation. Thus, since hours of work here can be viewed as a given attribute of the firm it follows from Dagsvik (2000), Section 6.2 that $\eta(h,w)b(w)$ is actually the proper representation of the firms' preferences in this case. Moreover, similarly to eq. (6.11) in Dagsvik (2000) the opportunity density can be expressed as

(C.3) $$\theta g(h,w) = b(w)K(h)$$

where K(h) (equal to M_h/B_h in the notation of Dagsvik, 2000 when j is replaced by h) is a positive factor that is determined by equilibrium conditions that are discussed in Dagsvik (2000). The factor K(h) will therefore depend on the structural terms of the utility functions and on $\{b(w)\}$.

Note that even if offered hours are not fixed in the short run we still realize from (C.1) that the implied offered hours and wage rates become independent.

Selectivity bias of the wage equation

In this appendix we discuss a simple method that corrects for selectivity bias in the wage equation. Let U_j denote the utility of working in sector j and U_0 the utility of not working. Assume that $U_j = v_j + \varepsilon_j$, $j = 0,1,2$, where v_1, v_2, v_0, are suitable deterministic terms and ε_1, ε_2 and ε_0 are random variables that are independent of the deterministic terms and have extreme value c.d.f. $\exp(-\exp(0.5772 - x))$. This particular version of the extreme value distribution is convenient here because it implies that the error terms have zero mean. It follows that the probability of working in sector j, P_j, is given by

(D.1) $$P_j = P\left(U_j = \max_k U_k\right) = \frac{\exp(v_j)}{\exp(v_0) + \exp(v_1) + \exp(v_2)}.$$

Let the wage equation of sector j be given by

(D.2) $$\ln W_j = X\beta_j + \eta_j$$

where η_j is a zero mean random variable that is independent of $X\beta_j$. The random variable ε_j may be correlated with η_j, due for example to a common latent ability factor. To accommodate this possibility assume that

(D.3) $$\eta_j = \eta_j^{\bullet} + \rho_j \varepsilon_j,$$

where ρ_j is an unknown parameter and $\eta_j^{\bullet}$ is a random variable that is independent of ε_j for all j. We assume however that η_j is independent of ε_k for $k \neq j$. Recall that when the random error terms are extreme value distributed then the distribution of $\max_k U_k$ is independent of which alternative maximizes utility (see for example Strauss, 1979), and has the same c.d.f. as ε_j, apart from a shift in the mean. Recall also that

(D.4) $$E \max_k U_k = \log\left(\exp(v_0) + \exp(v_1) + \exp(v_2)\right).$$

Hence, for $j = 1,2$,

$$\begin{aligned}
&E\left(\eta_j \middle| U_j = \max_k U_k\right) = \rho_j E\left(\varepsilon_j \middle| U_j = \max_k U_k\right) = \rho_j E\left(U_j \middle| U_j = \max_k U_k\right) - \rho_j v_j \\
\text{(D.5)} \qquad &= \rho_j E\left(\max_k U_k \middle| U_j = \max_k U_k\right) - \rho_j v_j \\
&= \rho_j E \max_k U_k - \rho_j v_j = \rho_j \log\left(\exp(v_0) + \exp(v_1) + \exp(v_2)\right) - \rho_j v_j \\
&= \rho_j \log\left\{\left(\exp(v_0) + \exp(v_1) + \exp(v_2) \middle/ \exp(v_j)\right)\right\} = -\rho_j \log P_j .
\end{aligned}$$

Similarly, it follows that

$$\begin{aligned}
&\operatorname{Var}\left(\eta_j \middle| U_j = \max_k U_k\right) = \rho_j \operatorname{Var}\left(\varepsilon_j \middle| U_j = \max_k U_k\right) + \operatorname{Var}\eta_j^* \\
\text{(D.6)} \qquad &= \rho_j \operatorname{Var}\left(\max_k U_k \middle| U_j = \max_k U_k\right) + \operatorname{Var}\eta_j^* = \rho_j \operatorname{Var}\left(\max_k U_k\right) + \operatorname{Var}\eta_j^* \\
&= \rho_j \operatorname{Var}\varepsilon_j + \operatorname{Var}\eta_j^* = \operatorname{Var}\eta_j .
\end{aligned}$$

Consequently, we get from (D.3), (D.5) and (D.6) that we can estimate the parameters of the wage equation for sector j consistently and asymptotically efficient by OLS on the sub-sample of women that work in sector j by means of the regression equation

$$\text{(D.7)} \qquad \log W_j = X\beta_j - \rho_j \log P_j + e_j ,$$

where e_j is a random term with conditional expectation, given that the woman works in sector j, equals zero and with variance equal to $\operatorname{Var}\eta_j$.

Utility specifications with polynomial functional form

Table E.1. A full van Soest et al (2002) model, 5th order polynomial, with fixed costs of working and random effects on leisure

Variables	Estimates	t-values
Fixed costs of working:Constant	-40.6826	-2.2
Log (age)	29.0781	2.5
$[\log(\text{age})]^2$	-4.6136	-2.7
Children:0-6	-0.3251	-2.3
Children:7-17	-0.9004	-3.3
Leisure: constant	141.0824	0.4
Log (age)	-253.0138	-2.7
$[\log(\text{age})]^2$	33.5090	2.7
Children 0-6	1.2267	1.6
Children 7-17	0.5144	0.3
C	6.1113	0.7
L^2	1515.7400	1.6
C^2	3.0905	0.7
LC	-26.9973	-0.5
L^3	-2996.6716	-1.7
C^3	0.8905	0.6
CL^2	68.9719	0.5
C^2L	-3.8594	-0.3
L^4	2702.8606	1.9
C^4	-0.0586	-0.3
L^3C	-113.2003	-0.8
L^2C^2	-14.5633	1.0
LC^3	-4.0904	1.2
L^5	-911.2676	-2.0
C^5	-0.0177	-0.7
L^4C	70.8748	7.8
L^3C^2	14.9191	1.0
L^2C^3	1.7597	1.2
LC^4	-0.2744	-0.9
σ, Std, random effect leisure	0.8541	0.1
Loglikelihood	-1870.887	

C = (disposable household income-60000)10^{-6} for h = 0, and C = (disposable household income-60000)10^{-6}-Fixed costs of working for h>0. L = 1-h/3640

Table E.2. A van Soest et al (2002) model, 3^{trd} and 5^{th} order polynomial, without fixed costs of working, but with random effects on leisure

Variables	3^{trd} order polynomial		5^{th} order polynomial	
	Estimates	t-values	Estimates	t-values
Leisure: constant	777.6131	2.4	5931.5283	4.9
Log age	-345.8362	-2.0	-2585.6396	-5.1
$[\log age]^2$	48.7884	2.0	366.0253	5.2
Childr. 0-6	5.1129	2.0	41.8854	5.7
Childr. 7-17	4.3902	2.1	35.9334	6.1
C	92.2601	1.6	479.4819	0.3
L^2	-232.4857	-6.0	-2777.0132	-2.1
C^2	-135.8726	-1.1	-921.8157	-0.2
LC	-81.8930	-1.6	-623.8350	-0.2
L^3	98.7196	7.8	2620.7424	1.5
C^3	32.9911	0.4	1421.1451	0.2
CL^2	34.8355	3.3	1006.5788	0.4
C^2L	14.7679	0.2	-1074.9296	-0.2
L^4			-1373.2603	-1.1
C^4			-891.2262	-0.1
L^3C			-482.2893	-0.2
L^2C^2			718.9698	0.2
LC^3			926.3572	0.1
L^5			322.7279	0.9
C^5			177.3576	0.1
L^4C			90.2804	0.1
L^3C^2			-261.7103	-0.3
L^2C^3			-230.1019	-0.2
LC^4			-189.9599	-0.1
σ: SD of random effect leisure	8.8129	1.5	79.4479	6.3
Log likelihood	-1912.000		-1884.091	

C = (disposable household income-60000)10^{-6}, L = 1-h/3640

In two figures below we give the indifference curves for an average woman in the sample, aged 40, with 11.6 years of education, with 0.49 children between 0 and 6 and with 0.68 children between 7 and 17.

Figure E.1. **3rd order polynomial utility function. Consumption along the vertical axis and leisure along the horizontal axis**

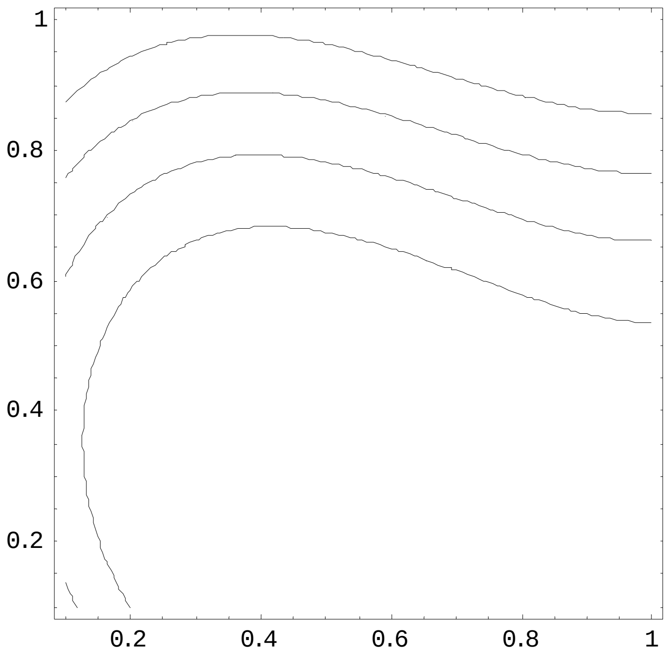

Figure E.2. **5th order polynomial utility function. Consumption along the vertical axis and leisure along the horizontal axis**

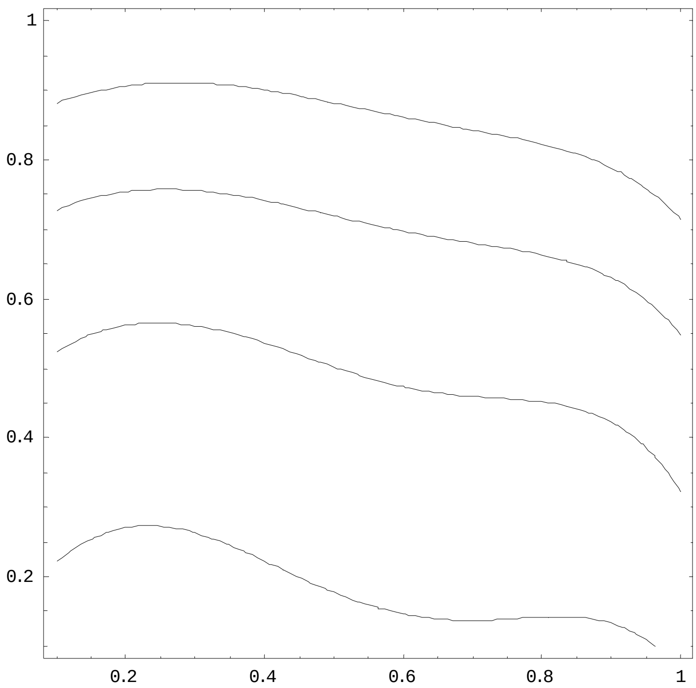

Figure E.3. **3rd order polynomial utility function, combined with ours specification of opportunities. Consumption along the vertical axis and leisure along the horizontal axis**

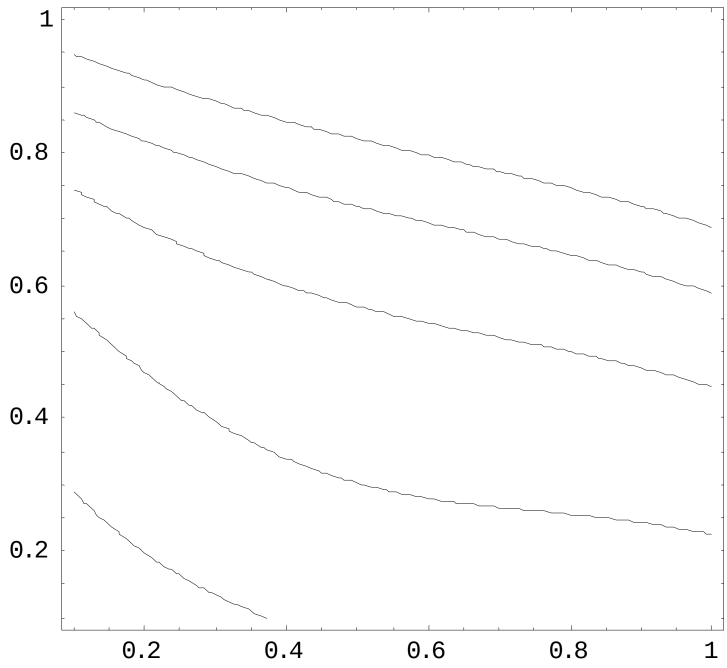

Figure E.4. **Box –Cox utility function, with ours specification of opportunities. Consumption along the vertical axis and leisure along the horizontal**

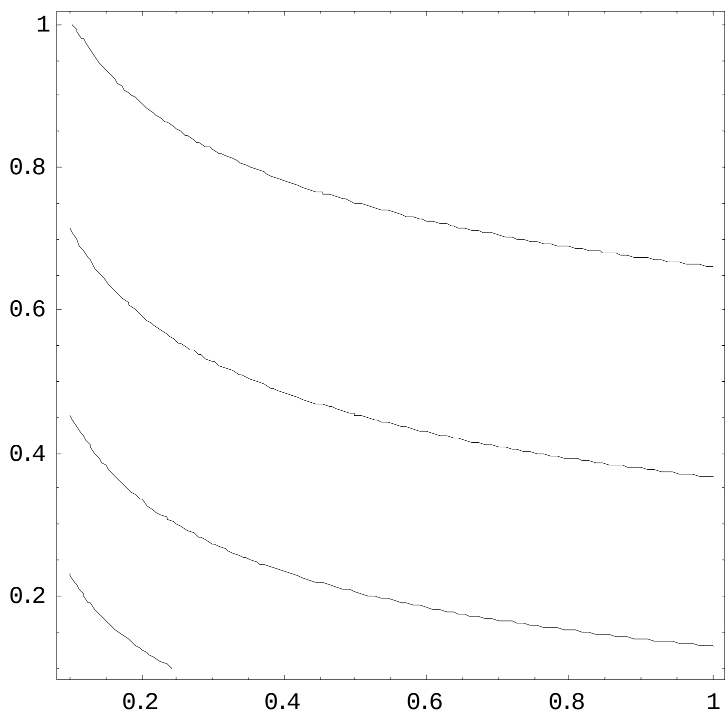

Table E.3. Third degree polynomial representation of utility function (van Soest et al utility function) combined with our specification of opportunities

Variables	Estimates	t-values
Job-opportunities: Constant, public sector, f_{11}	-4.4692	-4.2
Constant, private sector, f_{21}	0.5206	0.4
Education, public sector,f_{12}	0.2993	3.7
Education, private sector	-02124	-2.0
Offered hours: Full time peak, public sector	1.4835	8.9
Full-time peak, private sector	1.081	5.9
Part-time peak, public sector	0.7610	3.9
Part-time peak, private sector	0.8641	4.4
Leisure: Constant	346.4613	5.1
Log(age)	-142.9121	-3.9
$[\log(\text{age})]^2$	20.6204	4.1
Children 0-6	2.6414	5.9
Children 7-17	2.1786	6.3
C	137.6408	2.7
L^2	-112.8891	-4.3
C^2	-200.8554	-2.1
LC	-120.8604	-2.3
L^3	43.8384	3.1
C^3	151.6915	2.4
CL^2	55.6665	3.2
C^2L	103.3133	2.0
Log likelihood	-1751.853	
McFadden's ρ^2	0.215	
McFadden's ρ^2	0.195	

C = (disposable household income-60000)10^{-6} and L = 1-h/3640

Tax functions and child allowances, Norway 1994

Table F.1. Tax function in 1994 for a married non-working woman, husband is working NOK 1994

Wage income for the male, Y	Tax, T
0-41907	0
41907-140500	0.302Y-12656
140500-252000	0.358Y-20524
252000-263000	0.453Y-44464
263000-	0.495Y-55510

Table F.2. Tax function in 1994 for a married working woman, or working man NOK 1994

Wage income, Y	Tax ,T
0-20954	0
20954-140500	0.302Y-6328
140500-208000	0.358Y-14196
208000-236500	0.453Y-33956
236500-	0.495Y-43889

Net capital income is taxed at the rate of 28 percent.

Child allowances are given according to the following rules:

1 Child: NOK 10 416

2 Children: NOK 21 336

3 Children: NOK 33 696

4 Children: NOK 46 692

5 Children and more: NOK 60 084

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