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Inflow Uncertainty in Hydropower Markets

Abstract:
In order to analyse the consequences of uncertainty for prices and efficiency in a hydropower system, we apply a two-period model with uncertainty in water inflow. We study three different market structures, perfect competition, monopoly and oligopoly and stress the importance of the shape of the demand function under different distributions of water inflow. The uncertainty element creates possibilities of exercising market power depending on the distribution of uncertainty among producers. The introduction of thermal power into the hydropower market has an impact on the residual demand function, which is important for the hydropower producers' possibilities of exercising market power.

Keywords: hydropower, uncertainty, electricity, thermal power, demand functions, monopoly, duopoly

JEL classification: D40, Q11, Q41, L10

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1 Introduction

Stochastic water inflow in a hydropower system may create possibilities for exercising market power. A recent example is the winter of 2002 and 2003, when the hydro-dominated Nordic power market experienced extraordinarily high prices caused by a very dry autumn with low water inflow. To determine whether market power was actually exercised in this situation is difficult. Market participants, politicians and consumers claimed that electricity producers produced "too much" during the summer period, and that this increased prices in the winter period when the price elasticity is low. The aim of this paper is to add to the theoretical background for such a discussion.

Market power in electricity markets is a well-known phenomenon, and the exercise of market power has been extensively analysed in several places in the literature. However, most analyses do not focus on hydropower systems, but on systems with thermal power production.[1] Hydropower systems differ substantially, and the analyses of thermal systems are not necessarily suitable for markets based on hydropower production. There are three differences between hydropower systems and systems based on thermal power; the possibility of storing water (energy) that implies an opportunity cost of producing today, the variation in inflow to the reservoirs, and the low short-run production costs. These elements have some important implications for the market. The low short-run costs and the possibility of easily adjusting production gives low volatility in prices in the short-run, relative to thermal systems. However, stochastic inflow gives high volatility in prices over seasons and between years, although the possibility of storing water can reduce this volatility in price by transferring water between seasons and years. In this paper we analyse how these properties give market power to the hydropower producers, and especially how the stochastic element increases the possibility of exercising market power. To fulfil the picture, we also include an analysis with thermal power producers in a combined hydropower and thermal power market.

A number of recent articles analyse how market power can be exercised in hydropower markets. The most relevant article for our purpose is Crampes and Moreaux (2001), who apply a simple deterministic two-period model to analyse a combined hydropower and thermal power market. They assume that the market is a Cournot market, and an important result in their analysis is that in the presence of hydropower, with intertemporal opportunity cost, even the thermal power production

[1] See for example, Borenstein and Bushnell (1999), Cardell et al. (1997), Green and Newbery (1992), von der Fehr and Harbord (1993)

is determined by intertemporal considerations. Our discussion follows this tradition, and extends the analysis of uncertainty in the context of hydropower and market power. For a comprehensive discussion of a perfectly competitive market and monopoly market under certainty, see Førsund (2007), who discusses all the relevant cases with transmission constraints, maximization over many periods and reservoir constraints in hydropower market. Thermal power is also included in this analysis.

The market-power problem is also discussed in applied models. Bushnell (2003) describes a Cournot market where the producers control both hydro and thermal power generation resources. The model is used to numerically analyse the degree of competition for the western US. The model neglects uncertainty. Johnsen (2001) provides an alternative approach, with a simple two-period model that includes uncertainty and electricity trade between areas with different production technologies. The uncertainty element in this model is only used to analyse how a monopolist can exercise market power in an area with transmission constraints. The analysis includes a numerical model. Mathiesen et al. (2003) analyse the possibility of supply shortages in dry years; however, their main focus is not uncertainty, but reservoir constraints.

Articles that are more in line with our paper, with uncertainty and opportunity cost, include Garcia et al. (2001) and Bjerkholt and Olsen (1984). Garcia et al. (2001) use a two-period model with specific assumptions about the demand function. Their results depend on the assumption of a rectangular demand function with a positive probability of price equal to zero in the second-period. They show that hydropower producers with lower probability of replenishment of reservoir have a lower opportunity cost of selling power under these assumptions. Bjerkholt and Olsen (1984) discuss how uncertainty affects the social-planner investment decisions. They pinpoint that information about uncertainty and demand is crucial. They assume a convex demand curve, which turns out to be important for the results. However, they do not discuss different market structures.

None of these papers focus on how the stochastic element in water inflow, that is the element that make hydropower markets significantly different from markets based on thermal power, provides possibilities to exercise market power.

The possibility of water shortage creates a similarity between hydropower generation and production of non-renewable resources. The model we use in this paper has parallels to models in articles discussing optimal exploitation of a reserve of exhaustible natural resources, see for example Pindyck (1980) and Loury (1978) or, for a complete survey, Cairns (1990). The time horizon of the hydropower problem is two seasons, or at most two to three years, while exploitation of an exhaustible natural resource typically has a time horizon of decades. The nature of uncertainty is also very different, because hydropower producers have much more information about the production possibilities based on experience, while short-term demand fluctuations are related to known factors.

We apply a two-period stochastic model and compare it with the outcome under certainty. In addition, we compare the first-best solution with monopoly and oligopoly solutions. An important element turns out to be the shape of the demand function. Different assumptions about uncertainty, demand and market structure affect the way the producers exercise market power. To highlight the effect of uncertainty, the demand functions are initially assumed identical in both periods; we later relax this assumption. Under certainty, equal demand functions give no room for exercising market power. We use this as a benchmark that enables us to analyse how the behaviour differs between the first-best solution and outcomes under different market conditions.

The cost structure in hydropower production is very different from thermal power production, because the operating cost is close to zero, but the relevant short-term marginal production cost is the alternative value of water in future periods. We show that this opportunity cost depends on the expectation and variance of water inflow. The uncertainty distribution then affects the production decision at any time. We introduce a competitive fringe of thermal power producers, and show that the shape of the residual demand is crucial for the hydropower decision.

The paper is organized as follows. In Section 2, we present a two-period model with uncertainty. In Section 3, we determine the first-best solution, and in Section 4 we analyse a symmetric oligopoly of hydropower producers, and discuss the outcome of different demand functions in section 5. In Section 6, we introduce a competitive fringe of thermal power producers. Furthermore, in Section 7, we show how producers behave under alternative assumptions about sources of uncertainty, and, finally, in Section 8 we offer some concluding remarks.

2 A Two-Period Model with Uncertainty

We consider a two-period model with N identical hydropower producers or firms. Short-term variable costs are equal to zero. Water inflow to the reservoirs of firm i in period t is denoted q_{it} (these, as well as all other quantities, are measured in energy equivalents). Water inflow in the first-period is exogenously given and equal to $\overline{q}_{i1}$, while in the second-period inflow is stochastic; in particular, $q_{i2} = \overline{q}_{i2} + \tilde{\varepsilon}_i$, where $\overline{q}_{i2}$ is expected inflow and $\tilde{\varepsilon}_i = \varphi_i \varepsilon_i$, with ε_i being an independently and normally distributed random variable with $E\{\varepsilon_i\} = 0$, $var(\varepsilon_i) = \sigma_i$ and $cov(\varepsilon_i, \varepsilon_j) = 0$, $i \neq j$. The variance of $\tilde{\varepsilon}_i$ thus becomes $var(\tilde{\varepsilon}_i) = \varphi_i^2\sigma_i$. We let x_{it} denote output of firm i in period t. The reservoir level of firm i at the beginning of period 1 is r_{i0}, while the minimum or required level of reservoirs of firm i at the end of period 2 is r_i. The resource constraints for producer i may thus be expressed as:

$$0 \leq x_{i1} \leq r_{i0} + q_{i1}.$$

In other words, production in period 1 must be less than or equal to the sum in inflow and water initially in the reservoir. Similarly, in period 2 a water balance restriction adds up total production and inflow over the two periods:

$$0 \leq x_{i2} \leq r_{i0} + q_{i1} - x_{i1} + q_{i2} - r_i.$$

We assume $x_{i2} = r_{i0} + q_{i1} - x_{i1} + q_{i2} - r_i$ is binding (i.e., no spilling of water), and define $\bar{Q}_i = r_{i0} + q_{i1} + \overline{q}_{i2} - r_i$ as the deterministic part of available water over the two periods. The total amount of water available for production over the two periods is then $Q_i = \bar{Q}_i + \varphi_i\varepsilon_i$. It follows that $x_{i2} = \bar{Q}_i + \varphi_i\varepsilon_i - x_{i1}$. We assume that the demand function is symmetric across periods, with the inverse demand function in period t given by $p(X_t)$, with $p'(X_t) < 0$, and $X_t = \sum_{i=1}^N x_{it}$ is total electricity consumption in that period. Note that, under certainty (i.e. with $\varphi_i \equiv 0$, all i) and with the no-water-spilling constraints, market power does not affect market outcomes when demand functions are identical over time, hence the assumption of identical demand functions therefore isolates the effect of uncertainty. For simplicity, we ignore discounting.

3 First-Best

We first identify the first-best solution, defined as the allocation of output that maximizes the expected sum of consumer and producer surpluses, which here is given by the area under the demand curve. Let $\bar{Q} = \sum_{i=1}^N \bar{Q}_i$ and $\varphi\varepsilon = \sum_{i=1}^N \varphi_i\varepsilon_i$. If we normalize the variance of ε to 1, the standard deviation in inflow becomes φ. We want to solve the following maximization problem:

$$\max_{X_1, X_2} E\left\{\int_0^{X_1} p(y)\,dy + \int_0^{X_2} p(y)\,dy\right\} \quad \text{st. } X_2 = \bar{Q} + \varphi\varepsilon - X_1. \quad (1)$$

The first-order condition for this problem may be written:

$$p\left(X_1^f\right) = E\left\{p(\bar{Q} + \varphi\varepsilon - X_1^f)\right\}, \quad (2)$$

where X_t^f denotes the first-best solution in period t (t=1,2). In other words, the price in the first-period should equal the expected price in the second-period. It is immediately clear that in the special case of no uncertainty, i.e., with $\varphi \equiv 0$, symmetric demand conditions imply that production in the first-period should equal production in the second-period, i.e., $X_1^f = X_2^f = \frac{\bar{Q}}{2}$.

We consider the effect of uncertainty by examining the effect of an increase in the variation in inflow, φ. Differentiating (2) with respect to φ and solving we get:

$$\frac{dX_1^f}{d\varphi} = \frac{E\left\{\varepsilon \cdot p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right\}}{p'\left(X_1^f\right) + E\left\{p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right\}}, \tag{3}$$

where the denominator is negative, because $p' < 0$. By rewriting the numerator of (3) we find:

$$\begin{aligned} E\{\varepsilon \cdot p'\} &= E\{\varepsilon\} \cdot E\{p'\} + cov(\varepsilon, p') \\ &= cov(\varepsilon, p'), \end{aligned}$$

where the last equality follows from the assumption $E\{\varepsilon\} = 0$.

Consider first the case when the demand function is convex, i.e., $p'' > 0$. Then

$$cov\left(\varepsilon, p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right) > 0,$$

and therefore

$$\frac{dX_1^f}{d\varphi} < 0.$$

In other words, when demand is convex, an increase in the variation in inflow in the second-period, keeping the expected inflow constant, results in a lower optimal production in the first-period. Lower production implies that price increases in the first-period, and because it follows from (2) that the price in the first-period equals the expected price in the second-period, the increase in uncertainty also increases the expected price in the second-period.

It is immediately clear that in the contrary case of a concave demand function, i.e., $p'' < 0$, we have the exact opposite result:

$$cov\left(\varepsilon, p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right) < 0,$$

and therefore

$$\frac{dX_1^f}{d\varphi} > 0.$$

In this case (2) implies that prices decrease in both periods i. e. the determinisic price decrease in the first-period and the exptected price decrease in the second.

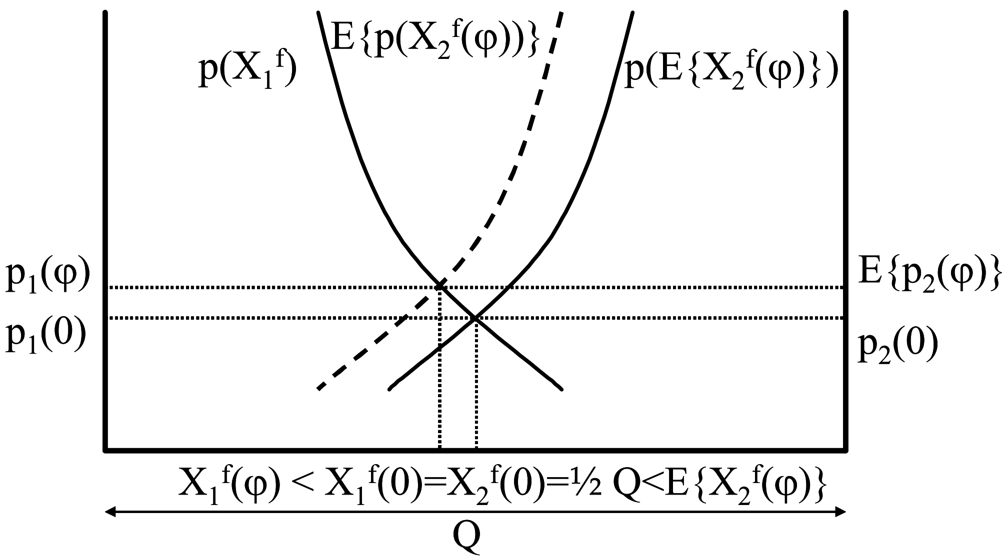

Figure 1. The optimal resource allocation for the social planner

Figure 1 shows the optimal resource allocation for a social planner under certainty and uncertainty, respectively, when demand is convex. Under certainty the producer equalizes both price and production in the two periods, $p_1(0) = p_2(0)$ and $X_1^f(0) = X_2^f(0)$. From Jensen inequality, it follows that $E\left\{p\left(X_2^f\right)\right\} \geqslant p\left(E\left\{X_2^f\right\}\right)$ when p_t is convex and X_2^f is stochastic. Under uncertainty it is therefore optimal to reduce production in the first-period relative to the level that would be optimal under certainty, by equalizing first-period price with the second-period expected price, i.e., $p_1(\varphi) = E\{p_2(\varphi)\}$.

The above analysis is summarized in the following proposition.

Proposition 1 *At the first-best allocation, we have:*

i) $\frac{dx_1^f}{d\varphi} \gtreqless 0 \Leftrightarrow p'' \lesseqgtr 0$

ii) $\frac{dp_1^f}{d\varphi} = E\left\{\frac{dp_2^f}{d\varphi}\right\} \lesseqgtr 0 \Leftrightarrow p'' \lesseqgtr 0$

4 Imperfect Competition

We now turn to the case of imperfect competition, when the production side is assumed to be concentrated so that firms choose quantities in Cournot fashion. Specifically, we assume firms first choose first-period outputs simultaneously and subsequently, having observed the realization of second-period inflows choose second-period outputs.

We maintain the assumption that there is no spillage of water, in effect assuming that in the second-period firms supply whatever output can be produced from available water. We can therefore concentrate attention on first-period behaviour, assuming that firms take into account equilibrium play in the second-period. Consequently, in the first-period firm i solves the problem of maximizing expected profit over both periods, or:

$$\max_{x_{i1}} E\left\{p(X_1)x_{i1} + p(X_2)x_{i2}\right\}, \tag{4}$$

subject to the condition:

$$x_{i2} = \bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}.$$

The first-order condition for this problem may be written as:

$$E\left\{p_1' x_{i1} + p_1 - p_2' \left[\bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}\right] - p_2\right\} = 0, \tag{5}$$

where we have dropped the argument of p. This condition may alternatively be written as:

$$p_1 + p_1' x_{i1} = E\left\{p_2 + p_2' \left[\bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}\right]\right\}. \tag{6}$$

In other words, at maximum profit, marginal revenue in the first-period equals expected marginal revenue in the second-period.

In the special case of full certainty, i.e., $\varphi_i = 0$, firms produce the same amount in both periods, i.e., $x_{i1} = x_{i2} = \frac{\bar{Q}_i}{2}$ for all i. This implies that the price will also be the same in both periods , i.e., $p_1 = p_2$. In this case, therefore, oligopoly producers are unable to exercise market power.[2]

In the Appendix A1, we demonstrate that in a symmetric market equilibrium the following condition must hold:

$$p_1 - E\left\{p_2\right\} = \frac{1}{N} E\left\{p_2' X_2 - p_1' X_1\right\}. \tag{7}$$

[2] Note that this result depends crucially on the no-spillage assumption. If firms could increase profits by spilling water, production and prices would remain constant across periods, but aggregate output would be smaller and prices would be higher compared to first best.

Holding aggregate expected output $\bar{Q} = \bar{Q}_i N$ fixed, we would expect market price to move towards the social optimum as N approaches infinity when all $\bar{Q}_i$ are equal.

We rewrite the first order condition (6) and introduce an elasticity, γ_t $(t = 1, 2)$, that in general is expected to be different in the two periods:

$$p_1 \left[1 + \frac{1}{N\gamma_1}\right] = E\left\{p_2 \left[1 + \frac{1}{N\gamma_2}\right]\right\}. \tag{8}$$

In other words, positive marginal revenues among the producers are

required for an interior solution, and $N\gamma_t < -1$ is assumed. Under negative marginal revenues the producers reduce output in one of the periods until we end up at a corner solution. If $N = 1$ that is the oligopoly market is reduced to a monopoly market, $\gamma_t < -1$ is required for an interior solution.

We now turn to the analysis of uncertainty. To simplify exposition we analyse the monopoly producer. Let $\varphi\varepsilon = \sum_{i=1}^{N} \varphi_i \varepsilon_i$ and normalize the variance of ε to 1. The first order condition for the monopoly producer can therefore be expressed as:

$$M\left(X_1^m\right) = E\left\{M\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\} \tag{9}$$

where X_1^m denotes optimal monopoly production, the left-hand side is:

$$M\left(X_1^m\right) = p\left(X_1^m\right) + p'\left(X_1^m\right) X_1^m.$$

and the right-hand side is:

$$E\left\{M\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\} = E\{p\left(\bar{Q} + \varphi\varepsilon - X_1^m\right) \\ + p'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right]\}. \tag{10}$$

In other words, the monopoly producer optimises production by equalizing the marginal revenue in the first-period with the second-period marginal revenue in the second-period. This corresponds to the first order condition for the social planner in (2), but now the marginal revenues are equalized, not the price.

We consider the effect of uncertainty on the monopoly producer by examining the effect of an increase in the variation of inflows, φ. Differentiating (9) implicitly with respect to φ and solving, and rewrite, see Appendix A.2 , equation (43), we get:

$$\frac{dX_1^m}{d\varphi} = \frac{E\left\{M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\varepsilon\right\}}{M'\left(X_1^m\right)+E\left\{M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\right\}}, \tag{11}$$

where the denominator is negative, because the second order condition for profit maximum gives us:

$$M'\left(X_1^m\right)+E\left\{M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\right\}<0,$$

where $M'\left(X_1^m\right)$ and $\left\{M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\right\}$ are given in Appendix A.2.

The numerator determines the effect of uncertainty on the optimal monopoly production. By rewriting the numerator of (11) we find:

$$E\left\{M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\varepsilon\right\} = Cov\left(M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right),\varepsilon\right).$$

Consider first the case when marginal revenue is convex, i.e.

$$E\left\{M''\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\right\} = E\left\{3p_2''+p_2'''X_2^m\right\}>0.$$

Then

$$Cov\left(M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right),\varepsilon\right)>0$$

and therefore

$$\frac{dX_1^m}{d\varphi}<0.$$

In other words, when marginal revenue is convex, an increase in the variation in inflow in the second-period results in a lower optimal production in the first-period.

It is immediately clear that in the contrary case of concave marginal revenue,

$$E\left\{M''\left(\bar{Q}+\varphi\varepsilon-X_1^m\right)\right\} = E\left\{3p_2''+p_2'''X_2^m\right\}<0.$$

Then

$$Cov\left(M'\left(\bar{Q}+\varphi\varepsilon-X_1^m\right),\varepsilon\right)<0.$$

We have the exact opposite result,

$$\frac{dX_1^m}{d\varphi}<0.$$

The above analysis is summarised in the following proposition.

Proposition 2 *At maximum profit for the monopoly producer, we have:* $\frac{dx_1^m}{d\varphi} \gtreqless 0 \Leftrightarrow M'' \lesseqgtr 0.$

We analyse whether the monopolist produces more or less than what is the case in the first-best solution by comparing (3) and (11), and to see whether

$$\frac{dX_1^f}{d\varphi} \lesseqgtr \frac{dX_1^m}{d\varphi}.$$

We look at a small increase in φ from $\varphi = 0$. For $\varphi = 0$, $\frac{dX_1^f}{d\varphi} = \frac{dX_1^m}{d\varphi} = 0$, remember that under certainty the production in the two periods are equal and $X = X_1 = X_2$, thus we analyse how the monopoly producer differs from first-best by comparing the second order derivative with respect to φ,

$$\frac{d^2X_1^f}{d^2\varphi}1 \lesseqgtr \frac{d^2X_1^m}{d^2\varphi}.$$

The second-order derivatives are derived in Appendix (A.3), and may for the first-best be written as:

$$\frac{d^2X_1^f}{d^2\varphi} = \frac{p''}{4p'}, \tag{12}$$

and for the monopoly:

$$\frac{d^2X_1^m}{d^2\varphi} = \frac{M''}{4M'} = \frac{3p'' + p'''X}{4(2p' + p''X)}. \tag{13}$$

The sign of (12) and (13) are negative for convex demand and marginal revenue, respectively, and positive for concave demand and marginal revenue, respectively.

By comparing (12) and (13) we find whether the monopoly producer reduces production more or less than the first-best solution:

$$\frac{d^2X^m}{d^2\varphi} - \frac{d^2X^f}{d^2\varphi} = \frac{p''\left[p''X - p'\right] - p'p'''X}{2p'\left(2p' + p''X\right)}. \tag{14}$$

The denominator is positive, because $2p' + p''X = M_1' = M_2'$ and $2p'$ are negative. The sign of the nominator,

$$p''\left[p''X - p'\right] - p'p'''X,$$

then decides if the monopolist produces more or less than first-best. We can summarise the conditions for the sign of (14) in the following proposition.

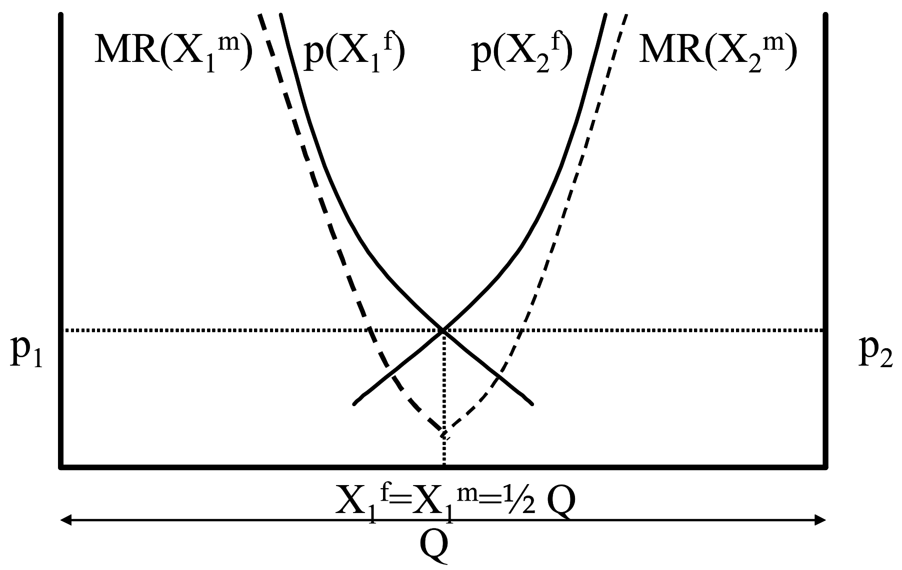

Figure 2. The first best and monopoly solution under certainty

Proposition 3 *The monopoly solution differs from first-best for a small increase in* φ *from* $\varphi = 0$;

> *i) The general result;*
> $X_1^m \gtrless X_1^f \Leftrightarrow p'' \left[p' - p''X\right] \gtrless -p'''p'X.$
>
> *ii) Special case;*
> *If* $p''' \gtrless 0$ *and* $p'' \gtrless 0 \Rightarrow X_1^m \gtrless X_1^f$

Figure 2 shows an example under certainty where the monopolist produces the first-best outcome under certainty. From equation (12) and (13) we see that the curvature of the inverse demand function and marginal revenue over the first derivative of the inverse demand and marginal revenue, respectively, decide how the producers react to uncertainty. We cannot decide how the monopoly producer differs from first-best by looking at the figure. However, Proposition (3) gives the conditions for the demand function. In the real world, there is always a probability of water shortage in the second-period, and the less water in the reservoir at the start of the second-period, the higher the probability of shortage in the second-period. In such cases there is a need to be concerned about the security of supply, because the monopolist produces more than the first-best in the first-period.

5 The Shape of Demand

We now concentrate on more specific demand functions. First, we consider the optimal solutions for two demand functions often used in the economic literature, a function with a constant price elasticity of demand and a quadratic demand function. Then, we discuss an example where the demand functions are different in the two periods.

Consider a demand function written on inverse form with a constant price elasticity:

$$p = AX^{\frac{1}{\gamma}}, \tag{15}$$

where we assume $\gamma < -1$, because we consider an interior solution.

By inserting the constant price elasticity demand function in (6), one obtains:

$$p_1 \left[1 + \frac{1}{\gamma}\right] = E\left\{p_2 \left[1 + \frac{1}{\gamma}\right]\right\}, \tag{16}$$

and it follows that:

$$p_1 = E\{p_2\}. \tag{17}$$

In other words, when output does not change the price elasticity of demand, the monopoly solution coincides with the first-best solution. This may imply that results from numerical models with a constant price elasticity of demand are biased, because the total uncertainty aspect in exercising market power is not taken into account. More general demand function should therefore be used in such analysis. The above analysis is summarised in the following proposition.

Proposition 4 *Suppose* $p_t = AX_t^{\frac{1}{\gamma}}$ *for* $t = 1, 2$ *where* $\gamma < -1$ *and* $\varphi \geqslant 0$ $\Rightarrow X^f = X^m$.

Consider a quadratic demand function that takes the form:

$$p = A + BX + CX^2, \tag{18}$$

where A, B and C are constants and demand is downward sloping, $p' = [B + 2CX_t] < 0$ for all $X_t \in [0, \bar{Q}]$ .This implies that we only get one solution, despite that we have a quadradic function. We assume that inflow is normally distributed, and has a constant variance.[3] We first derive the first-best solution, then the monopoly solution.

[3]This reinforces the fact that $\bar{Q}$-$\varphi\varepsilon$ can not be negative.

By inserting the quadratic demand function (18) into the first order condition for the first-best (2) and solving we get:

$$X_1^f = \frac{C\varphi^2}{2B + 2C\bar{Q}} + \frac{\bar{Q}}{2}. \tag{19}$$

By inserting the quadratic function into the first order condition for the monopolist (5) and solving we get:

$$X_1^m = \frac{C\varphi^2}{\frac{4}{3}B + 2C\bar{Q}} + \frac{\bar{Q}}{2}. \tag{20}$$

In other words, both the first-best and monopoly solution depend on the standard deviation in inflow and the parameters in the demand function.

When assuming a convex quadratic function, i.e $C > 0$ in (18), the production in the first-period under uncertainty is less than the production under certainty in both cases. The monopolist produces even a smaller quantum than the first-best solution, $\frac{\bar{Q}}{2} > X_1^f > X_1^m$, because the denominator in (20) is larger than the denominator in (19) under the social optimum,

$$\frac{4}{3}B + 2C\bar{Q} > 2B + 2C\bar{Q}.$$

It is clear that under contrary conditions on demand, a concave inverse quadratic demand function, i.e. $C < 0$, i.e., gives the opposite result, $\frac{\bar{Q}}{2} < X_1^f < X_1^m$.

When assuming $C = 0$, i.e., a linear demand function. We find by looking at (19) and (20) that the production is the same in the two periods for both cases, $X_1^m = X_1^f = X_2^m = X_2^f = \frac{\bar{Q}}{2}$. The monopoly solution does not deviate from the social optimum, the same result as shown under a demand fucntion with constant price elasticity. However, the produced output in the first-period is independent of the variation in inflow.

The above analysis is summarised in the following proposition.

Proposition 5 *Suppose* $p_t = A + BX_t + CX_t^2$ *for t=1,2,* $p' = [B + 2CX_t] < 0$ *for all* $X_t \in [0, \bar{Q}]$ *and* $\varphi > 0$;

i) $\frac{\bar{Q}}{2} \gtrless X_1^f \gtrless X_1^m \Leftrightarrow C \gtrless 0 \Leftrightarrow p'' \gtrless 0.$

ii) $X_1^f = X_1^m = \frac{\bar{Q}}{2} \Leftrightarrow C = 0 \Leftrightarrow p'' = 0.$

We now turn to a situation where the demand functions are different in the two periods, and analyse this under certainty. The demand functions include the same function $p(X)$ in the two periods, but we shift the function in the second-period by adding an additive constant to the function.[4] Technically the demand function in the first-period is $p(X)$ and in the second-period $k+p(X)$. An increase in k can be interpreted as an increase in the willingness to pay for a given quantum. By replacing the demand function in (2) and assume $\varphi = 0$, the first order condition for the social planner may be written as:

$$p\left(X_1^f\right) = k + p(\bar{Q} - X_1^f). \tag{21}$$

Differentiating (21) implicitly with respect to k, setting $k = 0$ and solving we get:

$$\frac{dX_1^f}{dk} = \frac{1}{2p'} < 0, \tag{22}$$

which is negative, because $p' < 0$. In other words, when k increases, production in the first-period decreases.

The first order condition for the monopoly producer (5) may be written as:

$$p\left(X_1^m\right)+p'\left(X_1^m\right)X_1^m = k+p\left(\bar{Q}-X_1^m\right)+p'\left(\bar{Q}-X_1^m\right)\left[\bar{Q}-X_1^m\right]. \tag{23}$$

Differentiating (23) implicitly with respect to k, setting $k = 0$ and solving we get:

$$\frac{dX_1^m}{dk} = \frac{1}{2\left[2p' + p''X\right]} < 0, \tag{24}$$

where the denominator is negative in the light of the second order condition for profit maximization.

In other words, when willingness to pay for a given quantum in the second-period increases, the quantum produced in the first-period is reduced.

We have shown that both the social planner and the monopoly producer decrease their production in the first-period when k increases. We compare the two expressions by substracting (22) from (24):

$$\frac{dX_1^m}{dk} - \frac{dX_1^f}{dk} = \frac{-\left(p' + p''X\right)}{2p'\left[2p' + p''X\right]}, \tag{25}$$

[4] Note that by adding a multiplicative constant to the demand function in the second period, the results under uncertainty and equal demand hold.

where the denominator is positive, because $p' < 0$ and $2p' + p''X < 0$.

The nominator is an expression of how the price elasticity changes when the price changes, $\frac{\partial El_p}{\partial p} = p' + p''X$.

Remember we analyse a case where $k = 0$, hence the demand functions are identical in the two periods. An increase in k will then make the demand functions differ, therefore the elasticities may differ in the two periods.

Consider first a function $p(X)$ where the price elasticity of demand increases when price increases, then the nominator in (25) is positive. The monopolist produces more than first-best in the first-period. In the contrary case, where the price elasticity decreases with increases in price, we have the opposite result.

In the case of uncertainty and different demand functions in the two periods, we see by comparing (25) and (14) that the effects can work in opposite directions. In a special case the uncertainty may neutralize the effect of market power caused by different demand functions in the two periods. The uncertainty has an effect on the expected price elasticity of demand. This means that in situations where the demand is different, and the uncertainty makes the expected elasticity in the second-period equal to the elasticity in the first-period, the monopoly solution coincides with the first-best solution. The above analysis is summarized in the following proposition.

Proposition 6 *Suppose $p_1 = p$ and $p_2 = k + p$ for a small increase in k from $k = 0$;*

i) $\frac{dX_1^m}{dk} - \frac{dX_1^f}{dk} \lesseqgtr 0 \Leftrightarrow \frac{\partial El_p}{\partial p} = (p' + p''X) \gtreqless 0.$

ii) For a special case when $k \neq 0$ and $\varphi > 0$, it may occur that $X_1^m = X_1^f$.

6 The Presence of Thermal Power Producers

In the discussion of the shape of the demand function, one cannot only look at the demand function for consumers. The Nordic electricity market has both hydropower and thermal power plants. In this section we analyse two cases. One with a social planner and one with a dominant hydropower producer. In both cases the thermal power producers are price takers, and there is no uncertainty about the thermal power production function. We show that in both cases the relevant demand function facing the hydropower producer is the residual demand function derived in the following section.

6.1 First-Best

We first turn to the case of a social planner. Let $c\left(x_t^t\right) > 0$ be the cost function for the thermal power producer and x_t^{th} the quantity produced in each period we have that $c'\left(x_t^{th}\right) > 0$. The hydropower producer faces a resource constraint $x_2^h = \bar{Q} + \varphi\varepsilon - x_1^h$, where $\bar{\mathrm{Q}}$ is the total expected quantity produced, and x_t^h is the quantity produced from the hydropower producer in period t. The demand and cost functions are certain. The stochastic inflow makes market price and production of thermal power uncertain in the second-period. In the social optimum, thermal power production is decided by:

$$p\left(x_t^h + x_t^{th}\right) = c'\left(x_t^{th}\right). \tag{26}$$

Thermal power production in period t increases until the marginal cost of production equals price in period t. The thermal power producers make the production decision after inflow is observed in the second-period. Given the level of hydropower production in period t, thermal power production may be written as a function of the demand and cost functions, $x_t^t = o\left(x_t^h\right)$. By inserting this into the demand function we get the residual demand function for the hydropower production, $p^r\left(x_t^h\right) = p\left(x_t^h + o\left(x_t^h\right)\right)$. We replace the demand function in (1) with the residual demand function and get:

$$\max_{x_1^h, x_2^h} E\left\{\int_0^{x_1^h} p^r(y)\,dy + \int_0^{x_2^h} p^r(y)\,dy\right\} \text{ s.t. } x_2^h = \bar{Q} + \varphi\varepsilon - x_1^h. \tag{27}$$

The first order condition for this maximization problem may be written as:

$$p^r\left(x_1^h\right) = E\left\{p^r\left(\bar{Q} + \varphi\varepsilon - x_1^h\right)\right\}.$$

In other words, the hydropower producer should produce until the price

in the first-period is equal to the expected price in the second-period. The relevant demand function is the residual demand function towards the hydropower producer. The thermal power producers produce the quantity where price equals marginal cost.

6.2 A dominant hydropower producer with a competitive fringe of thermal power producers

We turn to the case of a dominant hydropower producer with a competitive fringe of thermal power producers. The first order condition (5) for the dominant hydropower producer may be written as:

$$p\left(X_1\right) + p_1' \cdot x_1^h = E\left\{p(X_2) + p_2' x_2^h\right\}, \tag{28}$$

where

$$X_1 = x_1^h + x_1^{th} \text{ and } X_2 = x_2^h + x_2^{th}.$$

The competitive fringe of thermal power producers produces at marginal cost, $p\left(x_1^h + x_1^{th}\right) = c'\left(x_t^{th}\right)$. The optimal thermal power production may be written as $x_t^{th} = o\left(x_t^h\right)$. As in first-best the residual demand towards the hydropower producer may be written as, $p^r\left(x_t^h\right) = p\left(x_t^h + o\left(x_t^h\right)\right)$. By replacing the demand function in the first order condition, (28) with the residual demand function, the first order condition for the dominant hydropower producer may be written as:

$$p^r\left(x_1^h\right) + p^{r\prime}\left(x_1^h\right)x_1^h = E\left\{p^r\left(x_2^h\right)\right\} + E\left\{p^{r\prime}\left(x_2^h\right)x_2^h\right\}. \tag{29}$$

In other words, when hydropower production is determined by the residual demand function we have the same results for market power as discussed in Section 4. Thermal power production is decided by $x_t^{th} = o\left(x_t^h\right)$. In a mixed market with hydropower and thermal power production, not only is the shape of the demand function important for the potential to exercise market power, but the marginal cost structure of the thermal power producers is also crucial.

This section describes a dominant hydropower producer and a competitive fringe, when only the dominant hydropower producer exercises market power. Crampes and Moreaux (2001) show that in a duopoly market, the thermal power producer has an intertemporal choice of exercising market power. A crucial assumption for the Crampes and Moreaux (2001) results are that elasticities are differ in the two periods. In the case of uncertainty, the elasticities in the two periods may be interpreted as different, even if the demand functions are the same. Their results will therefore hold under uncertainty and equal demand functions.

The assumption for an interior solution is that the residual elasticity facing the oligopoly producer is less than one. Introducing a competitive fringe makes this assumption even more realistic since the residual elacatity increase in absolute value when introducinge a thermal power.

7 Uncertainty in a Duopoly Market

So far we have studied a monopoly or an oligopoly market with identical producers, now we analyse two cases of a duopoly market where two producers with equal expected inflow may differ in behaviour based on their different variance of inflow.

In the two situations the assumptions about the inflow are different. First, the inflows are perfectly correlated, but the producers' shares of the total variance differ. Later, we analyse the effect of correlation between the producers' inflow holding each producer's variance in inflow constant and equal.

We start with the situation where the production constraints may be written as:

Producer 1,

$$x_{12} = Q + \varphi\left[1-\lambda\right]\varepsilon - x_{11}. \tag{30}$$

Producer 2,

$$x_{22} = Q + \varphi[\lambda]\varepsilon - x_{21}. \tag{31}$$

We have introduced a new parameter λ, to make it possible to analyse how an increase in variance affects the aggregate output for different variances in inflow among producers. This is a simplification of each producers' variance in inflow $var\left(\varepsilon_1\right)$ and $var\left(\varepsilon_2\right)$. It follows by adding (30) and (31) that the total uncertainty in the system is $\varphi\varepsilon$, and the total variance may be written as $\varphi^2 var\left(\varepsilon\right)$, and the share of the variance is given by the parameter, $\lambda \in (0,1)$. Note that the level of λ does not affect the total variance in the system. We analyse the effects of a small increase in the total standard deviation of inflow in the system given by φ, when λ is constant. We use this in the maximization problem for each producer (5). In Appendix A.4, by totally differentiating and solving the system we get:

Producer 1,

$$\frac{dx_{11}}{d\varphi} = \frac{dX_1}{d\varphi}\frac{p_1' + p_1''x_{11} + E\left\{p_2' + p_2''x_{12}\right\}}{p_1' + E\left\{p_2'\right\}} + \frac{E\left\{\left[1+\lambda\right]p_2'\varepsilon + p''\varepsilon x_{12}\right\}}{p_1' + E\left\{p_2'\right\}}. \tag{32}$$

Producer 2,

$$\frac{dx_{21}}{d\varphi} = \frac{dX_1}{d\varphi}\frac{p_1' + p_1''x_{21} + E\left\{p_2' + p_2''x_{22}\right\}}{p_1' + E\left\{p_2'\right\}} + \frac{E\left\{\left[2-\lambda\right]p_2'\varepsilon + p''\varepsilon x_{22}\right\}}{p_1' + E\left\{p_2'\right\}}. \tag{33}$$

In other words, we see that the two producers increase in production in the first-period differ depending on the share of variance λ. The producers react differently to a change in variance depending on their share of the increase in variance. When the producers are equal, i.e. $\lambda = 0.5$, the changes in production are equal. For market efficiency the producers' share of production changes is not important, so now we look at how the distribution of variance among producers affects total output. We get the aggregate effect on output in the first-period by adding (32) and (33), and solving we get:

$$\frac{dX_1}{d\varphi} = \frac{E\{3p_2'\varepsilon + p_2''\varepsilon X_2\}}{3p_1' + 3p_2' + E\{p_1''X_1 + p_2''X_2\}}. \quad (34)$$

In other words, we see that the distribution of variance among the two producers does not affect the change in aggregate output.

In order to compare the aggregate output under duopoly with the monopoly and first-best solutions as shown in an earlier section, we examine around the point where $\varphi = 0$ and $var(\varepsilon) = 1$, and by differentiating (34), remember that with not uncertainty $X = X_1 = X_2$ and taking expectations we get the second-order derivative:

$$\frac{d^2X_1}{d\varphi^2} = \frac{4p'' + p'''X}{4[3p' + p''X]}. \quad (35)$$

It follows from (7) that the effect of uncertainty on production in a duopoly market is between the first-best given by (22) and the monopoly producer given by (24), as expected. It is not clear from (32) and (33) how the producers differ in production. To analyse this we subtract the first-order conditions for the two producers from (6) when we use (30) and (31), and solving we get:

$$x_{11} - x_{21} = \frac{\varphi[1-2\lambda]\,Cov(\varepsilon, p_2')}{p_1' + E\{p_2'\}}, \quad (36)$$

where the size of the differences in production depends on the $Cov(\varepsilon, p_2')$, and the differences in uncertainty denoted by λ, because $p_1' + E\{p_2'\} < 0$.

We have earlier shown that $Cov(\varepsilon, p_2')$ is positive for $p_2'' > 0$ and negative for $p_2'' < 0$. Consider the case where the demand function $p'' > 0 \Rightarrow Cov(p_2', \varepsilon) > 0$, and producer 2 faces no uncertainty, i.e., $\lambda = 0$. Then producer 1, with uncertainty in inflow, produces less than producer 2 in the first-period. This means that in this case, the producer facing uncertainty in production has a higher water value for a given amount of water. In the contrary case where $p'' < 0 \Rightarrow Cov(p_2', \varepsilon) < 0$ we have the opposite result, the producer with uncertainty in inflow produces more than the producer with no variance in production, in the first-period. The above analysis is summarized in the following proposition.

Proposition 7 *Suppose p_t for $t = 1, 2$, two producers i=1,2 and $Q_1 = Q_2$, but $var(\varepsilon_1) \neq var(\varepsilon_2)$;*

i) $x_{11} \gtreqless x_{21} \Leftrightarrow p'' > 0$ and $var(\varepsilon_1) \lesseqgtr var(\varepsilon_2)$.

ii) $x_{11} \lesseqgtr x_{21} \Leftrightarrow p'' < 0$ and $var(\varepsilon_1) \lesseqgtr var(\varepsilon_2)$.

iii) X_1 *is decided by* $\varphi^2 var(\varepsilon)$ *and var* $(\varepsilon_1) \lesseqgtr var(\varepsilon_2)$ *does not play any role.*

We now turn to the case where we analyse how correlation between the producers may affect the production decisions. The production constraints may be written as:

Producer 1,

$$x_{12} = Q + \varphi\varepsilon_1 - x_{11}. \tag{37}$$

Producer 2,

$$x_{22} = Q + \varphi\varepsilon_2 - x_{21}. \tag{38}$$

The uncertainty for each producer is represented by $\varphi\varepsilon_1$ and $\varphi\varepsilon_2$, where the variances of ε_1 and ε_2 are equal and normalized to 1. We introduce a correlation parameter between the producers represented by $\delta = Corr(\varepsilon_1, \varepsilon_2) = Cov(\varepsilon_1, \varepsilon_2)$. This implies that the total variance changes when the correlation changes. To analyse the effect of correlation between ε_1 and ε_2, we subtract the first-order condition for the two producers from (6), when we use (37) and (38) and solving we get:

$$x_{11} - x_{21} = \frac{\varphi\left[E\left\{p_2'\varepsilon_1\right\} - E\left\{p_2'\varepsilon_2\right\}\right]}{p_1' + E\left\{p_2'\right\}}, \tag{39}$$

where $E\left\{p_2'\varepsilon_1\right\} = E\left\{p_2'\varepsilon_2\right\}$, the covariances are equal, and it follows that

$$x_{11} - x_{21} = 0.$$

In other words, because the numerator of the right-hand side of (39) is zero, the production in the first-period for the two producers has to be equal, $x_{11} = x_{21}$. This means that when producers have the same variance, the correlation between ε_1 and ε_2 for the two producers do not make the producers differ in the first-period. However, market power may be exercised intertemporally, and the price and the aggregate production in the first-period X_1, may differ from the first-best solution when the correlation changes. In Appendix A.5 we derive the market solution from (51):

$$p_1 - E\left\{p_2\right\} = \frac{E\left\{p_2'\left[x_{11}\bar{Q}_1 + x_{21}\bar{Q}_2\right]\right\}}{x_{11} + x_{21}} - \frac{E\left\{p_2'\left[x_{11}^2 + x_{21}^2\right]\right\}}{x_{11} + x_{21}}$$

$$-p_1'\frac{x_{21}^2 + x_{21}^2}{x_{11} + x_{21}} + \frac{E\left\{p_2'\varphi\varepsilon_1\right\}x_{11} + E\left\{p_2'\varphi\varepsilon_2\right\}x_{21}}{x_{11} + x_{21}}. \tag{40}$$

We rewrite the last term of (40) using covariances and get:

$$\frac{Cov\left(p'_{2,}\varphi\varepsilon_1\right)x_{11}+Cov\left(p'_{2,}\varphi\varepsilon_2\right)x_{21}}{x_{11}+x_{21}}. \tag{41}$$

We showed above that $x_{11}=x_{21}$, and it follows that (41) may be written as:

$$\frac{\varphi Cov\left(p'_{2,}\varepsilon_1+\varepsilon_2\right)}{x_{11}+x_{21}}.$$

This term decides how the correlation affects the market solution. For

perfect negative correlation $\delta=-1$ we find that, $var\left(\varepsilon_1+\varepsilon_2\right)=var\left(\varepsilon\right)=E\left\{\varepsilon^2\right\}=E\left\{\varepsilon_1+\varepsilon_2\right\}^2=E\left\{\varepsilon_2^2\right\}+2E\left\{\varepsilon_1\varepsilon_2\right\}+E\left\{\varepsilon_2^2\right\}=1+2Cov\left(\varepsilon_1,\varepsilon_2\right)+1=2+2\delta=0$, where δ is the correlation term. $\varphi Cov\left(p'_{2,}\varepsilon_1+\varepsilon_2\right)$ is zero since $var\left(\varepsilon_1+\varepsilon_2\right)=0$. We see that if the producers have perfectly negative correlation, the last term in (40) is zero, and we end up in the case under certainty with no possibility of exercising market power.

It is clear that in the case where we consider perfect correlation, $\delta=1$, $var\left(\varepsilon_1+\varepsilon_2\right)>0$, and $\varphi Cov\left(p'_2,\varepsilon_1+\varepsilon_2\right)$ is positive or negative depending on the shape of the demand function, therefore the duopoly solution deviates from the first-best under uncertainty. The above analysis is summarized in the following proposition.

Proposition 8 *Suppose p_t for $t=1,2$, two producers i=1,2, $Q_1=Q_2$, and $var\left(\varepsilon_1\right)=var\left(\varepsilon_2\right)\Leftrightarrow x_{11}=x_{21}$ independent of the correlation between ε_1 and ε_2. However, X_1 depends of the correlation, δ.*

8 Concluding Remarks

This paper focuses on the effect of uncertainty in inflow on hydropower markets, and on mixed markets with thermal power producers. To isolate the effect of uncertainty, the demand functions are assumed to be the same in both periods. Under the assumptions made, it is impossible to exercise market power under certainty, and the prices in a market with a dominant producer are the first-best solution. Introduction of uncertainty increases the possibility to exercise market power.

The shape of the demand curve is crucial for the exercise of market power. In the world of constant elasticity of demand the exercise of market power because of uncertainty is impossible. The direction of water disposal because of the exercise of market power, which could be important because of the security of supply, is different depending on the demand function. We also show that in special cases when the demand function is different in the two periods the uncertainty element can be neutralized.

The introduction of thermal power shifts the residual demand curve for the hydropower producer, which under the same demand implies less uncertainty, and less variation in price than with only hydropower producers. However, in the case of an oligopoly market, the thermal power producers also face an intertemporal deviation caused by the uncertainty.

In a market with only two producers with the same expected production, the distribution of uncertainty among producers does not matter for efficiency. However, aggregate output differs from first-best, and the producers differ depending on each producer's variance and the shape of the demand function.

We have shown that the efficiency loss from market power depends upon the uncertainty. It is not possible to quantify this effect in a theoretic model, but it should be an important element in a numerical model. In future research of market power in the hydropower market, it is important to identify the exact demand function and how the uncertainty is distributed.

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A Appendix

A.1 N Producers

With symmetry among producers i.e all producers are equal , we get from (52) by summarize over all N producers:

$$p_1 - E\{p_2\} = \frac{E\left\{p_2' N x_{i1} \bar{Q}_i\right\}}{N x_{i1}} - \frac{E\left\{p_2' N x_{i1}^2\right\}}{N x_{i1}}$$

$$-p_1' \frac{N x_{i1}^2}{N x_{i1}} + \frac{N x_{i1} E\left\{p_2' \varphi_i \varepsilon_i\right\}}{N x_{i1}} \Leftrightarrow$$

$$p_1 - E\{p_2\} = \frac{E\left\{p_2' x_{i1} \bar{Q}\right\}}{N x_{i1}} - \frac{E\left\{p_2' N x_{i1}\right\}}{N}$$

$$-p_1' \frac{N x_{i1}}{N} + \frac{N E\left\{p_2' \varphi_i \varepsilon_i\right\}}{N} \Leftrightarrow$$

$$p_1 - E\{p_2\} = \frac{E\left\{p_2' \bar{Q}\right\}}{N} - \frac{E\left\{p_2' X_1\right\}}{N}$$

$$-p_1' \frac{X_1}{N} + \frac{E\left\{p_2' \varphi \varepsilon\right\}}{N} \Leftrightarrow$$

$$p_1 - E\{p_2\} = \frac{1}{N} E\left\{p_2' X_2 - p_1' X_1\right\}. \tag{42}$$

A.2 Marginal Revenues and the definition of $\frac{dX_1^m}{d\varphi}$

To simplify the notation we define the marginal revenues in the first-period and the expected marginal revenues in the second-period and their derivatives:

$$M\left(X_1^m\right) = p\left(X_1^m\right) + p'\left(X_1^m\right) X_1^m,$$

$$\begin{aligned} E\left\{M\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\} = & E\{p\left(\bar{Q} + \varphi\varepsilon - X_1^m\right) \\ & + p'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right]\}, \end{aligned}$$

$$M'\left(X_1^m\right) = 2p'\left(X_1^m\right) + p''\left(X_1^m\right) X_1^m,$$

$$\begin{aligned} E\left\{M'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\} = & E\{2p'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right) \\ & + p''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right]\}, \end{aligned}$$

$$M''\left(X_1^m\right) = 3p''\left(X_1^m\right) + p'''\left(X_1^m\right) X_1^m,$$

$$\begin{aligned} E\left\{M''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\} = & E\{3p''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right) \\ & + p'''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right]\}. \end{aligned}$$

For the monopolist:

$$\begin{aligned} \frac{dX_1^m}{d\varphi} & = \frac{E\left\{M'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\varepsilon\right\}}{M'\left(X_1^m\right) + E\left\{M'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\}} \\ & = \frac{N}{D}, \end{aligned} \tag{43}$$

where

$N = E\left\{2p'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right) \cdot \varepsilon\right\} + E\left\{p''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right] \cdot \varepsilon\right\}$

$D = 2p'\left(X_1^m\right) + p''\left(X_1^m\right) X_1^m + E\left\{2p'\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\}$
$\quad + E\left\{p''\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\left[\bar{Q} + \varphi\varepsilon - X_1^m\right]\right\}.$

A.3 The Second Order Derivatives

The social planner solution:

$$\frac{dX_1^f}{d\varphi} = \frac{E\{\varepsilon \cdot p_2'\}}{p_1' + E\{p_2'\}}$$

and we differensiate with respect to φ and get

$$\frac{d^2 X_1^f}{d\varphi^2} = \frac{E\{\varepsilon\varepsilon p_2'' [p_2' + p_1']\} - E\{\varepsilon p_2'\} E\{\varepsilon p_2''\}}{[p_1' + E\{p_2'\}]^2}.$$

We set $\varphi = 0$, and take expectations. The production in the two periods is equal, and therefore $p_1 = p_2 = p$, $p_1' = p_2' = p'$, $p_1'' = p_2'' = p''$,

$$\frac{d^2 X_1^f}{d^2\varphi} = \frac{p'' 2p' - p'p''}{4p'^2} = \frac{p''}{4p'}. \tag{44}$$

The monopoly solution

$$\frac{dX_1^m}{d\varphi} = \frac{E\{\varepsilon \cdot M'(X_1^m)\}}{M'(X_1^m) + E\{M'(\bar{Q} + \varphi\varepsilon - X_1^m)\}},$$

$$\frac{d^2 X_1^m}{d^2\varphi} =$$

$$\frac{E\{\varepsilon\varepsilon M''(\bar{Q} + \varphi\varepsilon - X_1^m)\}[M'(X_1^m) + EM'(\bar{Q} + \varphi\varepsilon - X_1^m)]}{[M'(X_1^m) + E\{M'(\bar{Q} + \varphi\varepsilon - X_1^m)\}]^2}$$
$$\frac{-E\{\varepsilon M'(\bar{Q} + \varphi\varepsilon - X_1^m)\} E\{\varepsilon M''(\bar{Q} + \varphi\varepsilon - X_1^m)\}}{[M'(X_1^m) + E\{M'(\bar{Q} + \varphi\varepsilon - X_1^m)\}]^2}.$$

Setting $\varphi = 0$, and taking expectations. The production in the two periods is equal, and therefore $p_1 = p_2 = p$, $p_1' = p_2' = p'$, $p_1'' = p_2'' = p''$and $p_1''' = p_2''' = p'''$,

$$\frac{d^2 X_1^f}{d^2\varphi} = \frac{M'' M'}{4M'^2} = \frac{M''}{4M'}. \tag{45}$$

A.4 Asymmetric Uncertainty in a Duopoly Market

We have two producers with the first order condition:

$$p_1' \cdot x_{i1} + p_1 = E\{[p_2' [\bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}] + p_2]\}$$

where $i = 1, 2$.

Production constraints,

Producer 1,

$$x_{12} = Q + \varphi\,[1-\lambda]\,\varepsilon - x_{11}.$$

Producer 2,

$$x_{22} = Q + \varphi\,[\lambda]\,\varepsilon - x_{21}.$$

Aggregate,

$$X_2 = Q_1 + Q_2 + \varphi\varepsilon - X_1.$$

Total differentiating the system yields

$$p_1' dX_1 + p_1'' dX_1 x_{i1} + p_1' dx_{i1} - p_2' dX_2 - p_2'' dX_2 x_{i2} - p_2' dx_{i2} = 0$$

where i=1,2 and

$$dX_2 = d\varphi\varepsilon - dX_1,$$

$$dx_{12} = d\varphi\,[1-\lambda]\,\varepsilon - dx_{11},$$

$$dx_{22} = d\varphi\,[\lambda]\,\varepsilon - dx_{21}.$$

Solving for producer 1.
Inserting and solving we get,

$$\begin{aligned} 0 = {} & dX_1\left(p_1' + p_1' x_{11} + p_2' + p_2'' x_{12}\right) + \\ & dx_{11}\left(p_1' + p_2'\right) - d\varphi\left(p_2'\varepsilon + p_2'' x_{12} + p_2'\,[1-\lambda]\,\varepsilon\right). \end{aligned}$$

Solving with respect to $\frac{dx_{11}}{d\varphi}$,

$$\frac{dx_{11}}{d\varphi} = \frac{dX_1}{d\varphi}\frac{p_1' + p_1'' x_{11} + E\left\{p_2' + p_2'' x_{12}\right\}}{p_1' + E\left\{p_2'\right\}} + \frac{E\left\{[1+\lambda]\,p_2'\varepsilon + p''\varepsilon x_{12}\right\}}{p_1' + E\left\{p_2'\right\}}. \qquad (46)$$

Solving for producer 2,

$$dX_1\left(p_1' + p' x_{21} + p_2' + p_2'' x_{22}\right) + dx_{21}\left(p_1' + p_2'\right) - d\varphi\left(p_2'\varepsilon + p_2'' x_{22} + p_2'\,[\lambda]\,\varepsilon\right) = 0.$$

Solving with respect to $\frac{dx_{21}}{d\varphi}$,

$$\frac{dx_{21}}{d\varphi} = \frac{dX_1}{d\varphi} \frac{p_1' + p_1'' x_{21} + E\{p_2' + p_2'' x_{22}\}}{p_1' + E\{p_2'\}} + \frac{E\{[2-\lambda] p_2' \varepsilon + p'' \varepsilon x_{22}\}}{p_1' + E\{p_2'\}}. \tag{47}$$

We know that $dX_1 = dx_{11} + dx_{21}$, and solving we get:

$$\frac{dX_1}{d\varphi} = \frac{E\{3p_2'\varepsilon + p_2''\varepsilon X_2\}}{3p_1' + 3p_2' + E\{p_1'' X_1 + p_2'' X_2\}}. \tag{48}$$

A.5 Market Solution: Cournot Game

To look at the Cournot market we multiply the first order condition (6) by x_i:

$$p_1' \cdot x_{i1} x_{i1} + p_1 x_{i1} = E\left\{\left[p_2' \left[\bar{Q} + \varphi_i \varepsilon_i - x_{i1}\right] + p_2\right] x_{i1}\right\}, \tag{49}$$

and summing over all N producers we get:

$$\sum_{i=1}^{N} p_1' x_{i1}^2 + \sum_{i=1}^{N} p_1 x_{i1} = \sum_{i=1}^{N} E\left\{\left[p_2' \left[\bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}\right] + p_2\right] x_{i1}\right\}. \tag{50}$$

When rearranging expression (50), equation (51) displays the difference in prices in the two periods:

$$p_1 - E\{p_2\} = \frac{E\left\{p_2' \sum_{i=1}^{N} x_{i1} \bar{Q}_i\right\}}{\sum_{i=1}^{N} x_{i1}} - \frac{E\left\{p_2' \sum_{i=1}^{N} x_{i1}^2\right\}}{\sum_{i=1}^{N} x_{i1}} \tag{51}$$

$$-p_1' \frac{\sum_{i=1}^{N} x_{i1}^2}{\sum_{i=1}^{N} x_{i1}} + \frac{\sum_{i=1}^{N} x_{i1} E\{p_2' \varphi_i \varepsilon_i\}}{\sum_{i=1}^{N} x_{i1}},$$

which is the equilibrium condition for the Cournot market with uncertainty in inflow. In the case of certainty the equilibrium condition for the Cournot market is:

$$p_1 - p_2 = \frac{p_2' \sum_{i=1}^{N} x_{i1} \bar{Q}_i}{\sum_{i=1}^{N} x_{i1}} - \frac{p_2' \sum_{i=1}^{N} x_{i1}^2}{\sum_{i=1}^{N} x_{i1}} - p_1' \frac{\sum_{i=1}^{N} x_{i1}^2}{\sum_{i=1}^{N} x_{i1}}. \tag{52}$$

We see by comparing (51) and (52) that uncertainty causes an extra term in the equilibrium condition, compared with the case of no uncertainty. When we have the same demand function $p(X)$, then $p_1 = p_2$, and under certainty there is no deviation from the social optimum. Under uncertainty this is not the case because $p_1 \neq E\{p_2\}$, and the Cournot market deviates from the social optimum.