

Discussion Paper

Central Bureau of Statistics, P.B. 8131 Dep, 0033 Oslo 1, Norway

No.24

August 1987

STOCHASTIC PROPERTIES AND FUNCTIONAL FORMS IN LIFE CYCLE

MODELS FOR TRANSITIONS INTO AND OUT OF EMPLOYMENT

BY

JOHN K. DAGSVIK

AND

ROLF AABERGE

Abstract

The paper discusses a justification for a particular econometric framework for analysing transitions into and out of employment in an intertemporal context with uncertainty. The analysis extends the models found in the literature by introducing a discrete choice variable that is unobservable to the econometrician.

Not to be quoted without permission from author(s). Comments welcome.

1. Introduction

Econometric models for labor supply in an intertemporal setting have been considered by a number of authors. See for instance MaCurdy (1982), (1983), Killingsworth (1984) and the references therein. Most of these studies, however, do not discuss the properties of the latent variables that enter the model. That is, the choice of probability distributions for the unobservables is made ad hoc and, apart from statistical diagnostic checks rarely any theoretical arguments to support these choices are provided.

The main focus of the present paper is the problem of characterizing the stochastic properties of dynamic models for labor supply. Our theoretical point of departure is the theory for labor supply in a life cycle context with uncertainty as developed by MaCurdy (1983) and Blundel and Walker (1986). Within this framework the individual is assumed to choose his/her lifetime labor supply, consumption and savings profiles by maximizing an expected lifetime utility function subject to uncertain wages and interest rates. Specifically, we extend the approach taken by Blundel and Walker (1986) by introducing a latent choice variable which we call "match". This notion is similar to Tinbergen's (1956) concept of "job". A match characterizes a certain "position" relative to labor market opportunities as well as to non-market opportunities. A match is specified by the tasks performed, associated with a certain market or non-market activity, as well as the qualifications required to perform these tasks. Thus within our extended decision framework the individual is supposed to maximize utility with respect to hours of work, consumption, savings as well as with respect to the matches, subject to the economic budget constraint and the (unobservable) set of feasible matches. Although the match variable is latent it serves as a fruitful starting point for a theoretical justification of the stochastic properties of the model. Some of these properties follow from a set of plausible assumptions about the preferences. Other important assumptions are only maintained for the sake of tractability but they can, to a certain extent, be tested empirically. One example of such an assumption is the Markov property. Our theory does not imply that the

class of econometric models that govern the transitions into and out of employment necessarily has the Markov property although it includes the Markov model as a special case. However, provided panel data are available this property can be tested.

Our econometric framework seems convenient for empirical estimation and policy simulations. This is due to the particular simple structure of the transition probabilities. Except for two parameters these probabilities can be expressed by the marginal probabilities of being employed or not at the respective ages.

The organisation of the paper is as follows: In Section two the individual's decision model in discrete time and with uncertainty is presented and in Section 3 the econometric model is developed. Section 4 extends the model to account for taxes and in Section 5 we discuss some theoretical implications of the model for the elasticities with respect to the demand for labor. In Section 6 we present some empirical results. Unfortunately, these results are based on the assumption of a one period budget constraint. This is due to lack of information about savings in our data. Accordingly, the estimation results are not fully consistent with the theoretical framework developed in the preceding sections and they are only useful in this context to the extent that the intertemporal allocation of wealth between periods plays a minor role. Section 6 also contains simulation results for the effect on transitions into or out of employment from changes in the mean marginal wage, level of schooling and number of children.

2. Theoretical assumptions

The present approach extends the traditional set up by taking into account the heterogeneity of the labor market with respect to wages and non-pecuniary attributes of the jobs as well as with respect to individuals choice opportunities. Although the heterogeneity is unobserved by the econometrician it has nevertheless importance for the choice of functional forms in the model. In particular, the assumption about the unobservables become important when the purpose is to estimate and test assumptions about structural parameters of the model.

The econometric model presented below is based on the plausible assumption that some of the unobservables are choice variables. Examples of such variables are type of job, schooling, household production and sport activities. Our point of departure is that the choice of the observable variables, like consumption and hours of work, are not necessarily made independently of the choice of the latent variables.

Similarly as in Dagsvik (1987) we associate with jobs and non-market opportunities the attributes and the type of tasks performed as well as the skills needed to perform certain tasks. The individual's set of feasible latent opportunities depends on personal abilities and degree of qualifications. The individual operate in an environment where future wage rates, interest rates, prices as well as the set of latent choice variables are uncertain. Let $U_t(L_t, C_t, z_t)$ be the individual's utility as of period t where L_t is annual hours of leisure, C_t is consumption and z_t is an enumeration of the latent choice variable henceforth denoted "match". It is called match because it identifies the tasks performed with certain jobs or non-market activities as well as it identifies the particular qualifications needed to perform the respective tasks. For a more general interpretation we refer the readers to Dagsvik (1987).

The budget constraints that must hold in each period are given by

$$(2.1) \quad C_t + S_t = h_t W_t(z_t) + A_t, \quad t \leq T,$$

$$(2.2) \quad A_t = (1 + r_t)S_{t-1}, \quad t \leq T, \quad A_T = 0$$

and

$$(2.3) \quad z_t \in B_t, \quad t \leq T$$

where r_t is the interest rate, B_t is the latent set of feasible matches in period t , $W_t(z_t)$ is the real wage rate specific to the match z_t , S_t is total real savings in period t , $h_t = M - L_t$ is annual hours of work, $h_t \leq M$, and A_t is the wealth in the beginning of period t . The set B_t is a function of the personal qualifications.

The individual is supposed to maximize expected lifetime utility given by

$$(2.4) \quad \tilde{E}_t \left\{ \sum_{j=t}^T (1 + \rho)^{t-j} U_j(L_j, C_j, z_j) \right\}$$

subject to the budget constraints where ρ is the rate of time preference and \tilde{E}_t denotes the subjective expectation operator conditional upon information prior to $t + 1$. Implicit in (2.4) is the assumption that utility is additively separable over time.

The optimizing problem above can be reformulated as a two stage budgeting decision process as discussed by Blundel and Walker (1986). The budget constraints can be reformulated as

$$(2.5) \quad C_t + L_t W_t(z_t) = \mu_t, \quad t \leq T$$

and

$$(2.6) \quad \sum_{j=t}^{T-1} R_j (\mu_j - MW_j(z_j)) = R_t A_t$$

where

$$\mu_t = A_t - S_t + MW_t(z_t)$$

and

$$R_j = \prod_{i=1}^j \frac{1}{1 + r_i}$$

is the discount factor that converts income in period t to its equivalent in period one. The individual's optimizing problem is clearly equivalent to the following two stage decisions: In stage one μ_t is kept fixed and he maximizes utility, $U_t(L_t, C_t, z_t)$, with respect to (L_t, C_t, z_t) subject to (2.5) and (2.3). Because of the time separa-

bility assumption this is an optimizing problem that only depends on the within period preferences, total incomes and wages, provided the choice sequence $\{B_j\}$ does not depend on past realizations of the choice process and provided we rule out the possibility of a corner solution at the upper bound on hours of work. Here we assume that B_t is independent of realizations of the choice variables outside period t and that no one prefers to work the full amount of available time.

In stage two the individual determines the sequence $\{\mu_j\}$, $j \geq t$, that maximizes expected (remaining) lifetime utility subject to (2.6).

Let $\hat{U}_t(w, \mu, z)$ be the indirect utility function that corresponds to the within period decision problem, i.e.,

$$\hat{U}_t(w, \mu, z) = \max_{L \leq M} U_t(L, \mu - Lw, z).$$

If we assume that the utility function is quasiconcave in (L, C) , increasing in C and in L it follows that $U_t(w, \mu, z)$ is convex in w , increasing in μ and the labor supply function conditional on (μ, z) is given by Roy's identity. Let

$$(2.7) \quad \hat{U}_t(\mu) = \max_{z \in B_t} \hat{U}_t(W_t(z), \mu, z).$$

and define

$$(2.8) \quad V_t(\underline{\mu}_t) = \tilde{E}_t \left\{ \sum_{j=t}^{\infty} (1+\rho)^{t-j} \hat{U}_j(\mu_j) \right\}.$$

where

$$\underline{\mu}_t = (\mu_t, \mu_{t+1}, \dots, \mu_{T-1}).$$

Thus in stage two the problem consists of maximizing (2.7) with respect to $\underline{\mu}_t$ subject to (2.6). Note that this poses a non-trivial maximization problem because $\hat{U}_j(\mu)$ is not necessarily continuous in μ . This is so because

$$\hat{U}_j(\mu) = \hat{U}_j(W_j(\bar{z}_j), \mu, \bar{z}_j)$$

where $\bar{z}_j = \bar{z}_j(\mu)$ is the optimal choice of $z \in B_j$ conditional on μ . When μ changes then for certain values of μ the optimal value of z jumps since the set B_j is discrete.

In the next section we shall specify a random utility function, $\hat{U}_j(\mu)$, that is continuously differentiable apart from on a subset that has probability measure zero (See appendix 1). As discussed below the rationale for the randomness is that in addition to individual uncertainty there may be a number of variables that influence the individual choices but are unobservable to the analyst. Consequently the utility function above is equivalent to a continuously differentiable utility function in the sense that their respective probability laws are the same..

Let $\hat{V}_t(A_t)$ denote the indirect utility function that corresponds to the maximization of $V_t(\mu_t)$ subject to (2.6)¹. Provided $\hat{U}_t(\mu)$ is continuously differentiable with probability one it follows from MaCurdy (1983) that A_t is determined by

$$(2.9) \quad \hat{V}_t(\bar{A}_t) = \frac{1}{1+q} \tilde{E}_t \{ (1 + r_{t+1}) \hat{V}_{t+1}(\bar{A}_{t+1}) \}.$$

Moreover

$$(2.10) \quad \hat{V}'_t(\bar{A}_t) = \hat{U}'_t(\bar{\mu}_t)$$

where the bar denotes optimum values. Eq. (2.10) means that the individual determines the level of the periods dissavings so that the marginal utility of wealth in period t equals the expected value of the discounted marginal utility of the next period's wealth where the discount factor is $(1 + r_{t+1})/(1 + q)$.

3. A structural probability model for transitions into and out of employment

The theoretical framework we have presented above differs from other approaches in the literature in that we have introduced an

¹In order to do that the individual must of course solve all the within period optimizing problems, since \hat{V}_t depends on $\hat{U}_j(\mu)$ for $j \leq t$.

unobservable choice variable. Although this latent variable leaves the theoretical analysis above essentially unchanged it has nevertheless important bearings for the stochastic properties of the econometric model.

The model we develop here is a dynamic random utility model where the randomness is attributable to variables that are known or uncertain to the decision-maker but unobservable to the econometrician. Even if the decision-makers were operating in an environment of perfect certainty the observed choice process would still be perceived as random by the observer due to these latent choice variables.

The indirect utility function as of period t is assumed to have the following structure

$$(3.1) \quad \hat{U}_t(w, \mu, z) = v_t(w, \mu, T(z))\varepsilon(z), \quad z \in B_t$$

where $\{T(z), \varepsilon(z)\}$ represents an enumeration of the points in the Poisson process on $(0, \infty) \times (0, \infty)$ with intensity measure $\lambda(dt) \cdot \bar{x}^{1/a-1} dx/a$ for some finite measure λ . The function $v_t(w, \mu, \tau)$ is a deterministic function that is increasing in w, μ , and is convex in w for fixed τ . These properties follow from the fact that $\varepsilon(z)$ does not depend on (w, μ) . The variables $\{T(z), \varepsilon(z)\}$ account for the variation in the value of the unobservable matches that are not captured through the observable variables that affect the preferences.

The specification (3.1) may be justified by theoretical assumptions as demonstrated in Dagsvik (1983) and (1987). One set of assumptions that can be applied relies on a version of Yellott's (1977) axiom "invariance under uniform expansions of the choice set". To explain the axiom let z_1, z_2, \dots be an enumeration of the alternatives of B and assume that the corresponding utilities $u_i = U_t(L, C, z_i)$, $i = 1, 2, \dots, k_t$, are independent draws from the same distribution for given t, h and C . Now expand the choice set B_t by adding $mk_t - m$ new alternatives, $z_{k+1}, z_{k+2}, \dots, z_{km}$, that have utilities, $u_{k+1}, u_{k+2}, \dots, u_{km}$, that are independent draws from the same distribution as the original utilities. Let B_t^* denote the expanded choice set. Then the axiom states that the distribution² of the stochastic process

²By distribution mean the finite dimensional marginal distributions.

$$\{\max_{z \in B_t^*} U_t(L, C, z)\}$$

equals the distribution of

$$\{\max_{z \in B_t} U_t(L, C, z)\}$$

apart from a linear transform. In Dagsvik (1983) it is demonstrated that this assumption implies that the joint distribution at arbitrary points in time of the process

$$\{\max_{z \in B_t} U_t(L, C, z)\}$$

is of the multivariate extreme value type. (See Galambos, 1978). The class of stochastic processes with this property is called the class of max-stable processes (cf. de Haan, 1984). de Haan (1984) demonstrates that $U_t(L, C, z)$ (provided $\{\max_z U_t(L, C, z)\}$ is continuous in probability) admits the representation

$$(3.2) \quad U_t(L, C, z) = \tilde{U}_t(L, C, T(z)) \varepsilon(z)$$

which implies (3.1) where v_t is an indirect utility function that corresponds to the maximization of \tilde{U}_t subject to the within period budget constraints.

The behavioral interpretation of the axiom is that when the choice sets of the latent opportunities are expanded uniformly the average number of persons that rank the combination (L_1, C_1) above (L_i, C_i) , $i = 2, 3, \dots$, (say) remains unchanged. This is a reasonable assumption because it is an acknowledgement of the fact that uniform expansion does not alter the "aggregate" relative preference evaluations of the observed alternatives (L_i, C_i) provided the population is "large".

Next let us consider the specification of the systematic term, $v_t(w, \mu, \tau)$, of (3.1). We shall assume that

$$(3.3) \quad v_t(w, \mu, \tau) = \beta_t(\mu) \gamma(w, \tau) K_t(w, \mu, \tau)$$

where

$$K_t(w, \mu, \tau) = \begin{cases} 1 & \text{when } (w, \mu, \tau) \in D_t \\ 0 & \text{otherwise} \end{cases}$$

and $\beta_t(\mu)$ and $\gamma(w, \tau)$ are suitable functions. The presence of the function K_t implies that the individual has zero utility assigned to matches for which

$$(W(z), \mu, T(z)) \in D_t.$$

The behavioral interpretation of the set D_t is that the individual only make utility comparisons from the subset of opportunities with a "suitable" combination of wages $W(z)$, total income, μ , and the matching variable, $T(z)$. The set D_t is of course unobservable and is treated as a random variable. When $(w, \mu, \tau) \in D_t$ the structure (3.3) implies that the (match-specific) indirect utility is multiplicatively separable in wage and total income. This assumption is made for mathematical convenience but it is less restrictive than it appears at first glance because the indicator K_t is not separable in τ , w and μ .

Now define the match-specific reservation wage, $W_t^*(\mu, \tau)$, by

$$(3.4) \quad v_t(W_t^*(\mu, \tau), \mu, \tau) = \tilde{U}_t(M, \mu, \tau).$$

The match-specific reservation wage classifies the feasible matches in two categories, namely

$$B_{1t} = \{z \mid z \in B_t, K_t(W(z), \mu_t, T(z)) = 1, W(z) > W_t^*(\mu_t, T(z))\}$$

and

$$B_{2t} = \{z \mid z \in B_t, K_t(W(z), \mu_t, T(z)) = 1, W(z) \leq W_t^*(\mu_t, T(z))\}.$$

Note that the particular specification (3.3) implies that $W_t^*(\mu_t, T(z)) = W^*(T(z))$, i.e., $W_t^*(\mu_t, T(z))$ does not depend on t nor on μ_t . The set B_{1t} is the set of all matches the individual considers and which he finds suitable for market activity. Similarly, B_{2t} is the set of matches which are found unsuitable for work. This set also contains the pure non-market matches, i.e., matches for which the wage rates are zero.

Also define

$$(3.5) \quad Z_j(t) = \max_{z \in B_{jt}} v_t(W(z), T(z)) \varepsilon(z), \quad j = 1, 2.$$

The stochastic processes, $\{Z_j(t)\}$, $j = 1, 2$, are clearly independent since $B_{1t} \cap B_{2t} = \emptyset$ (the empty set) and it can be demonstrated that they belong to the class of max-stable processes. $Z_1(t)$ is the indirect utility of working at time t while $Z_2(t)$ is the utility of not working.

Let $\{J(t)\}$ be the process that describes the individual's choice history, i.e., $J(t) = 1$ if the individual (woman) is employed in period t (age) and $J(t) = 2$ if the individual is not employed in period t . We assume that there are no transition costs nor true state dependence effects, i.e., we assume that the preferences are not influenced by past labor market history. Accordingly, the probability law of $\{J(t)\}$ is completely determined by the processes $\{Z_j(t)\}$ since $J(t) = j$ if and only if $Z_j(t) = \max_k Z_k(t)$ for $j = 1, 2$. In general the process $\{J(t)\}$ will not have the Markov property. However, provided the opportunity sets, B_{jt} , are nondecreasing³ then $\{J(t)\}$ becomes an inhomogeneous Markov chain (See Dagsvik, 1987). Recall that the Markov property is expressed as

$$\begin{aligned} & P\{J(t_n) = i_n \mid J(t_{n-1}) = i_{n-1}, \dots, J(t_1) = i_1\} \\ &= P\{J(t_n) = i_n \mid J(t_{n-1}) = i_{n-1}\} \end{aligned}$$

where $t_1 < t_2 < \dots < t_n$ are arbitrary points in time and $i_k = 1, 2$, $k = 1, 2, \dots, n$. It is in fact possible to test the Markov property even if only observations at two points in time are available, because the joint probabilities of being in state i in period t_1 and in state j in period t_2 , $i, j = 1, 2$, take a particular form when $\{J(t)\}$ is generated from max-stable processes. Here we shall simply assume that $\{B_{jt}\}$ is nondecreasing and examine the implications of this assumption. One reason for this assumption is that our data does not contain all the relevant variables that are needed for a rigorous test of the Markov

³ It is possible to relax this assumption.

property.

Let $p_j(t)$ and $p_{ij}(s,t)$ be the marginal probability of being in state j at time t and the transition probability of being in state j at time t given that state i was occupied at time s , respectively, i.e.,

$$p_j(t) = P\{J(t) = j\} = P\{Z_j(t) = \max_{k=1,2} Z_k(t)\},$$

$$p_{ij}(s,t) = P\{J(t) = j \mid J(s) = i\}$$

$$= P\{Z_j(t) = \max_{k=1,2} Z_k(t) \mid Z_i(s) = \max_{k=1,2} Z_k(s)\}.$$

Let $G_{1t}(w)$ be the expected fraction of feasible market matches for which the wage rate $W(z)$ satisfies $W^*(T(z)) < w(z) \leq W$ and let

$$u_{1t}(w) = E\{v_t(W(z), \mu_t, T(z))^a \mid W(z) = w, w > W^*(T(z)), z \in B_t\},$$

$$u_{2t} = E\{v_t(W^*(T(z)), \mu_t, T(z))^a \mid W(z) \leq W^*(T(z)), z \in B_t\},$$

and assume that the density, $g_{1t}(w)$, of $G_{1t}(w)$ exists. Moreover, let $p_1(t, w)$ denote the joint density of realized wage and labor market state one in period t . By Dagsvik (1987) it follows that

$$(3.8) \quad p_1(t, w) = \frac{u_{1t}(w) g_{1t}(w) g_{0t}}{g_{0t} \int_{y>0} u_{1t}(y) g_{1t}(y) dy + g_{2t} u_{2t}}$$

$$(3.9) \quad p_2(t) = \frac{g_{2t} u_{2t}}{g_{0t} \int_{y>0} u_{1t}(y) g_{1t}(y) dy + g_{2t} u_{2t}}$$

The interpretation of u_{jt} , g_{jt} is as follows: g_{0t} is the probability that a randomly selected match has positive wage rate. We may interpret g_{0t} as a demand parameter because it is the fraction of available matches that are job-opportunities. g_{2t} is the probability that a match is feasible but unsuitable for work where unsuitable means that the wage is less than the corresponding reservation wage (match-specific). Let

$$g_{1t} = G_{1t}(\infty)$$

and

$$u_{1t} = \int g_{1t}(w) u_{1t}(w) dw.$$

u_{1t} is the mean indirect utility across all feasible and suitable market matches while u_{2t} is the mean indirect utility across all feasible matches that are unsuitable for work. Obviously we have

$$g_{2t} = 1 - g_{0t}g_{1t}$$

so that

$$(3.10) \quad p_1(t) = \frac{g_{0t}g_{1t}u_{1t}}{u_{2t} + (u_{1t} - u_{2t})g_{0t}g_{1t}}.$$

Several cases of textbook models emerge as special cases of (3.8) and (3.9). When $a \rightarrow 0$ then $u_{1t}(w) \rightarrow 1$ and $u_{2t} \rightarrow 1$ so that (3.10) reduces to $g_{0t}g_{1t}$. Similarly (3.9) reduces to g_{2t} which is the probability that the match-specific reservation wage is higher than the corresponding wage rate. Thus this case corresponds to the case where only the wage rate associated with a match matters. Another special case is obtained when the wage rate distribution is degenerate. Then (3.8) becomes

$$p_1^*(t, w) = \frac{u_{1t}(w)g_t^*}{u_{1t}(w^*)g_t^* + (1-g_t^*)u_{2t}}$$

when $w = w^*$ and zero otherwise, where

$$g_t^* = P\{w^* > W^*(T(z)), z \in B_t\}$$

and $p_1^*(t, w)$ is the probability of being in state one in period t when all the matches have the same wage rate (w^*). It is reasonable to interpret

$$\frac{p_1^*(t, w^*)}{g_t^*} = \frac{u_{1t}(w^*)}{u_{1t}(w^*)g_t^* + (1-g_t^*)u_{2t}}$$

as the effect of the heterogeneity due to non-pecuniary attributes on the propensity towards work. Specifically, it expresses the probability of working when all the wages are equal to w^* given that

$w^* > W^*(T(z))$ and that $z \in B_t$. In other words it is the conditional probability of working given that the matches have wages that are acceptable for work. In the textbook case $p^*_1(t, w^*)/g^*_t$ reduces to one while in the general case it may be close to zero because the nonpecuniary attributes of the non-market matches may be significantly more attractive than the corresponding attributes of the market matches.

Note that the framework above permits that μ_t be correlated with the preferences through the "matching" variable $T(z)$. When μ_t and $T(z)$ are independent then g_{2t} and $g_{1t}(w)$ are independent of μ_t . Otherwise they depend on μ_t .

By applying the results of Dagsvik (1983) it can be proved that (see appendix 2)

$$(3.11a) \quad p_{ij}(s, t) = p_j(t) \left(1 - \frac{b_j(s)}{b_j(t)} e^{\eta(s) - \eta(t)} \right), \quad i \neq j$$

and

$$(3.11b) \quad p_{ji}(s, t) = 1 - p_{ij}(s, t), \quad j \neq i$$

where $\eta(t)$ is a positive function of t and

$$b_1(t) = g_{0t} g_{1t} u_{1t}, \quad b_2(t) = g_{2t} u_{2t}.$$

In order to secure non-negative transition probabilities, $\eta(t)$ must satisfy the condition

$$\log b_j(t) + \eta(t) \geq \log b_j(s) + \eta(s)$$

for $j = 1, 2$, and all $s \leq t$. In continuous time the corresponding transition intensities are given by

$$(3.12a) \quad \lambda_{ij}(t) = p_j(t) \left(\eta'(t) + \frac{b'_j(t)}{b_j(t)} \right), \quad i \neq j$$

$$(3.12b) \quad \lambda_{ji}(t) = -\lambda_{ij}(t), \quad i \neq j.$$

At this point we like to focus on a number of remarkably tractable properties of the model. First, consider the model in a pure probabilistic context. Recall that the choice process $\{J(t)\}$ is an inhomogeneous Markov chain. In principle, an (inhomogeneous) Markov chain is fully characterized and specified through the transition intensities. From the transition intensities the transition probabilities for transitions between time s and time t can be

calculated by means of the Kolmogorov differential equations. However, in practical empirical work this is in general a formidable task because the solution of the Kolmogorov equations as functions of general transition intensities is very complicated. (See Singer (1982)). Now consider (3.11) and (3.12). Notice first that a parametrization that is equivalent to specifying the transition intensities, $\lambda_{ij}(t)$, is to specify $b_j(t)\exp\eta(t)$, $j = 1, 2$, because there is a unique correspondence between $b_j(t)\exp\eta(t)$, $\lambda_{12}(t)$ and $\lambda_{21}(t)$. In fact we have

$$b_j(t)\exp\eta(t) = \int_0^t \lambda_{ij}(x)e^{\Lambda(x)} dx + c_j, \quad i \neq j$$

where

$$\Lambda(x) = \int_0^x (\lambda_{12}(y) + \lambda_{21}(y)) dy$$

and c_1, c_2 are constants such that $c_1 + c_2 = 1$. Consequently, this reparametrization represent no restriction of the model⁴. The reparametrized version also admits an appealing structural interpretation in terms of the individual's decision rule.

Now assume that

$$\eta(t) = \theta t$$

where $\theta > 0$ is a constant. Then it follows from (3.12) that when $b_j(t)$ does not depend on t then the chain $\{J(t)\}$ becomes homogeneous, because the transition intensities are then constant. The autocorrelation function of $\{Z_j(t)\}$ can be proven to be an increasing function of

$$\frac{b_j(s)e^{-\theta(t-s)}}{b_j(t)}$$

⁴ This is only true in two state case. In the multistate case the corresponding version implies strong restrictions.

at time s and t , $s \leq t$. Thus, when θ is large the temporal variation in the unobservables that influence utility is large and the transition probabilities are high. Conversely, when θ is close to zero and $b(t)$ changes little over time, then the utility processes $\{Z_j(t)\}$ are strongly autocorrelated and the transition probabilities are close to zero. Consequently, θ is a parameter that measures the temporal stability in the unobservables that affect utility. In accordance with Heckman (1981) we may call θ a habit persistence parameter. Thus when θ is large there is little habit persistence while when θ is small the habit persistence is strong.

4. Extension to account for progressive taxes

MaCurdy (1983) and Blomquist (1985) have shown how the intertemporal labor supply model can be extended to cover the case where the tax function has a particular structure. Let Y_t denote the tax function and assume that

$$(4.1) \quad Y_t = Y(h_t W(z_t), r_t \bar{S}_t)$$

The important feature of (4.1) is that it does not depend on earnings in periods outside period t . Moreover, we shall assume that Y is differentiable. Let

$$m_t(h_t) = W(\bar{z}_t) - \partial_1 Y(h_t W(\bar{z}_t), r_t \bar{S}_t)$$

be the marginal wage rate and

$$I_t^*(h_t) = I_t + h_t W(\bar{z}_t) - m_t(h_t) h_t - Y(h_t W(\bar{z}_t), r_t \bar{S}_t)$$

where ∂_1 denotes the partial derivative with respect to the first component. Conditional on the optimal values \bar{z}_t and $r_t \bar{S}_t$ the within period t constraint is

$$(4.2) \quad C_t = h_t W(\bar{z}_t) + I_t - Y(h_t W(\bar{z}_t), r_t \bar{S}_t).$$

As is well known the within period optimizing problem subject to (4.2) is equivalent to the optimizing problem subject to the linearized version

$$(4.3) \quad C_t + L_t m_t(h_t) = \mu^*_t(h_t), \quad t \leq T$$

where

$$\mu^*_t(h_t) = I^*_t(h_t) + Mm_t(h_t).$$

Thus the within period constraint (4.3) is completely analogous to (2.5) with $W(z_t)$ and μ_t replaced by m_t and μ^*_t , respectively.

5. Theoretical implications for the elasticities with respect to changes in the demand

The model framework developed above implies that we can derive some properties of the model without having estimated the parameters. In particular we shall study the effect on $p_j(t)$ and $p_{ij}(t-1, t)$ from changing the mean wage and the demand parameter g_{0t} . From (3.10) we get

$$(5.1) \quad \frac{\partial \log p_1(t)}{\partial \log g_{0t}} = \frac{p_2(t)}{g_{2t}} = \frac{u_{2t}}{u_{2t} + (u_{1t} - u_{2t})g_{0t}g_{1t}}.$$

This expression implies that

$$(5.2) \quad \frac{u_{2t}}{u_{1t}} < \frac{\partial \log p_1(t)}{\partial \log g_{0t}} < 1 \quad \text{when} \quad u_{1t} > u_{2t}$$

$$(5.3) \quad \frac{u_{2t}}{u_{1t}} > \frac{\partial \log p_1(t)}{\partial \log g_{0t}} > 1 \quad \text{when} \quad u_{1t} < u_{2t}.$$

The inequalities (5.2) and (5.3) tell us that the relative impact from increasing the demand for labor is greatest when the mean utility for work is less than the mean utility for leisure.

From (3.11a) we get

$$(5.4) \quad \frac{\partial \log p_{21}(t-1, t)}{\partial \log g_{0t}} = \frac{\partial \log p_1(t)}{\partial \log g_{0t}} - 1 + \frac{p_1(t)}{p_{21}(t-1, t)}.$$

Eq. (5.4) implies that

$$(5.5) \quad \frac{\partial \log p_{21}(t-1, t)}{\partial \log g_{0t}} > \frac{\partial \log p_1(t)}{\partial \log g_{0t}}$$

because $p_1(t) > p_{21}(t-1,t)$. Similarly it is straight forward to show that

$$(5.6) \quad 0 > \frac{\partial \log p_2(t)}{\partial \log g_{0t}} > \frac{\partial \log p_{12}(t-1,t)}{\partial \log g_{0t}}$$

Thus by (5.5) and (5.6) the relative impact on $p_{ij}(t-1,t)$ from a change in g_{0t} is always greater (in absolute value) than the corresponding impact on $p_{ij}(t)$. Eq. (5.4) also implies that when $\theta \rightarrow 0$ then the right hand side of (5.4) tends towards infinity. Note that this result depends crucially on the assumption that θ does not depend on g_{0t} . However, we have not been able to provide theoretical arguments to support this assumption. When θ is small then there is little variation over time in the unobservables that affect the utility processes, $\{Z_j(t)\}$, $j = 1,2$ (habit persistence). Thus (5.4) tells us that the relative impact on transitions is large when the habit persistence is strong.

6. Empirical results for the case with one period budget constraints.

In this section we present some empirical results based on data from the norwegian labor force survey ,1979-1980 , and the level of living survey, 1980. Unfortunately, these data contains no information on savings and we are therefore unable at this moment, to estimate a full life cycle model. The estimates can at most, be interpreted within a one period budget framework. Since our data are insufficient in order to estimate the life cycle model we have chosen to estimate a special case of (3.11) since our purpose is to focus on the general approach rather than rigorous estimation and testing.

From (3.11) it follows that an alternative expression for the transition probabilities is

$$(6.1a) \quad p_{ij}(s,t) = p_j(t) - p_j(s)\xi(s,t) \quad , \quad i \neq j$$

and

$$(6.1b) \quad p_{ii}(s,t) = p_i(t) - p_i(s)\xi(s,t) + \xi(s,t)$$

where

$$\xi(s,t) = \frac{(b_1(s)+b_2(s))e^{-\eta(t)+\eta(s)}}{b_1(t)+b_2(t)}$$

The special case we consider is obtained by letting $a \rightarrow 0$ (see page 12) which corresponds to the textbook case in which utility is concave in leisure and consumption. In this case

$$u_{1t} = u_{2t} = 1, \quad p_1(t) = g_{0t}g_{1t}, \quad p_2(t) = g_{2t}$$

and the transition probabilities reduce to

$$(6.1a) \quad p_{ij}(s,t) = p_j(t) - e^{-\theta(t-s)} p_j(s), \quad i \neq j$$

$$(6.1b) \quad p_{ii}(s,t) = p_i(t) - e^{-\theta(t-s)} p_i(s) + e^{-\theta(t-s)}$$

where θ must satisfy

$$\theta > \theta g_{2t} + \frac{dg_{2t}}{dt} > 0$$

in order to secure positive transition intensities.

The estimation is performed in two steps and two data sets are used. In the first step a cross-section of 1205 married women aged 16-67 years from the level of living survey is applied to estimate the probability of working, $p_1(t)$.

The probability $p_1(t) = g_{0t}g_{1t}$ is specified as a logit function of age, age squared, number of children less than six years, number of children between 6 and 16 years, husbands income minus tax and the marginal wage rate at zero hours of work.

The wage rate is predicted from an estimated wage equation depending on years of schooling, age and age squared.

The estimation and data are reported in another study (Dagsvik et al (1986)) but for the sake of completeness we also present the estimates here in table 1 below. Standard errors are given in parenthesis.

Table 1. Estimates of the parameters of the wage equation and the probability of working

(Survey of level of living 1980)

Variable	Estimate
Constant	2.1090 (0.2582)
Years of schooling	0.0662 (0.0088)
Age	0.0302 (0.0106)
Age squared/100	- 0.0329 (0.0128)
Constant	-8.894 (1.367)
Age	0.3045 (0.0448)
Age squared/100	-0.3876 (0.0534)
Number of children 0 - 16 years old	-0.2219 (0.0910)
Marginal wage	1.580 (0.251)

The remaining parameter to be estimated is θ . In order to estimate θ we need individual observations for at least two points in time. We have applied data from the labor force surveys at two successive years (1979 and 1980) for this purpose. Unfortunately the labor force survey provides no information on wage and income variables nor number of children less than six years. We have, therefore, estimated a reduced form version on the basis of the labor force surveys and compared it with the corresponding reduced form version obtained in step one by applying the level of living survey. The parameter estimates are displayed in table 2.

Table 2. Comparison of estimates of the (model) parameters based on data from the survey of level of living and the labor force survey, respectively.

Variable	Estimate survey of level of living	Estimate labor force survey
Years of schooling	0.105 (0.020)	0.167 (0.030)
Age	0.352 (0.045)	0.296 (0.021)
$\frac{\text{Age squared}}{100}$	- 0.440 (0.054)	- 0.357 (0.028)
Number of children 0-16 years	- 0.222 (0.062)	- 0.290 (0.050)
Habit Per- sistence (θ)	- 0.333 (0.026)	- 0.332 (0.027)

First we have estimated reduced form coefficients except θ on the basis of the level of living survey and we have estimated θ from the labor force survey data. (First column).

In the second column we report the results from estimating all the parameters by the maximum likelihood procedure on the basis of the labor force survey data. We can conclude from the two data sets that the parameter estimates are not significantly different. In order to check whether or not the habit persistence parameter depend on individual characteristics we have also estimated a version where θ is specified as a linear function of age, number of children and years of schooling. However, the estimation results indicated no dependence on these variables. These results must, nevertheless, be interpreted with caution because one important variable, namely husbands income, is not included here.

In order to illustrate the heterogeneity across different person groups we report the predicted probabilities of working in table 3. In table 4 and 5 we present predicted one year transition probabilities (reduced form) for transitions into and out of

employment. These results are obtained by using (6.1) and keeping the explanatory variables (except age) constant throughout the year.

Table 3. The Probability of being employed by age, education, number of children and husbands income.

Years of schooling:		9						18				
Number of children 0-16 years	Age	20	25	30	35	40	45	25	30	35	40	45
	Husband income (NOK)											
0	50	.49	.67	.78	.83	.85	.83	.84	.90	.93	.93	.93
	100	.39	.58	.70	.77	.79	.77	.78	.86	.89	.90	.90
	150	.29	.47	.61	.68	.71	.68	.69	.80	.85	.86	.85
	200	.19	.34	.48	.56	.59	.56	.57	.70	.76	.78	.76
1	50	.43	.62	.74	.80	.82	.80	.81	.88	.91	.92	.91
	100	.34	.52	.66	.73	.75	.73	.74	.83	.87	.88	.87
	150	.25	.41	.55	.63	.66	.63	.64	.76	.81	.83	.82
	200	.16	.29	.42	.50	.53	.50	.52	.65	.72	.74	.72
2	50	-	.57	.70	.76	.76	.76	.77	.85	.89	.90	.89
	100	-	.47	.60	.67	.70	.68	.69	.80	.85	.86	.85
	150	-	.36	.50	.58	.61	.58	.59	.72	.78	.80	.78
	200	-	.25	.37	.45	.48	.45	.46	.60	.68	.70	.68
3	50	-	.51	.65	.72	.74	.72	-	.82	.87	.88	.87
	100	-	.41	.55	.63	.66	.63	-	.76	.81	.83	.81
	150	-	.31	.44	.52	.55	.52	-	.67	.74	.76	.74
	200	-	.21	.32	.40	.42	.40	-	.54	.63	.65	.63

Table 4. One year transition probabilities for transitions into employment when no change in the observable characteristics (except age) occurs.

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of schooling Age	9	18	9	18	9	18
	20	.17	-	.14	-	.11	-
25	.21	.25	.19	.24	.16	.22	
30	.23	.26	.21	.25	.19	.24	
35	.24	.26	.22	.25	.20	.24	
40	.24	.26	.22	.25	.20	.24	
45	.23	.26	.21	.25	.19	.23	
2	25	.18	.24	.16	.22	.12	.19
	30	.21	.25	.19	.24	.16	.21
	35	.22	.25	.20	.24	.17	.21
	40	.22	.25	.20	.24	.17	.21
	45	.21	.25	.19	.23	.16	.21

Table 5. One year transition probabilities for transitions out of employment when no change in the observable characteristics (except age) occurs.

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of schooling Age	9	18	9	18	9	18
0	20	.11	-	.14	-	.17	
	25	.07	.03	.10	.05	.13	.07
	30	.05	.02	.07	.03	.10	.05
	35	.04	.02	.06	.03	.08	.04
	40	.04	.02	.06	.03	.08	.04
	45	.05	.02	.07	.03	.10	.05
2	25	.10	.05	.13	.07	.16	.09
	30	.07	.03	.10	.05	.13	.07
	35	.06	.03	.08	.05	.11	.07
	40	.06	.03	.08	.05	.11	.07
	45	.07	.03	.10	.05	.13	.07

In table 6 we have computed elasticities of the transition probabilities into employment with respect to the mean marginal wage. The table shows that the impact of wage changes is largest for young, low educated women where the husbands earnings is 150 000 NOK. Conversely, the effect is small for women age 30-45 with 18 years of schooling and husbands income equal to 50 000 NOK.

Table 7 shows the effect of schooling. In our model schooling enter the model through the marginal wage function and it is assumed that the preferences are not influenced by the length of schooling. As would be expected, table 7 show a similar picture as table 6. For example, the proportion of 25 year old women with 9 years of schooling and husbands income equal to 100 000 NOK that enters employment is predicted to increase by 0.14 when the woman takes an additional year of schooling.

Table 6. The elasticities of the one year probabilities for transitions into employment with respect to the mean marginal wage.

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of school- ing Age	9	18	9	18	9	18
0	20	2.3	-	2.7	1.2	3.1	-
	25	1.7	0.9	2.1	1.2	2.5	1.6
	30	1.2	0.5	1.6	0.8	1.7	1.1
	35	0.9	0.4	1.3	0.8	1.7	0.9
	40	0.9	0.4	1.2	0.8	1.9	0.9
	45	1.0	0.4	1.3	0.8	1.9	0.9
2	25	2.1	1.2	2.5	1.6	3.0	2.0
	30	1.6	0.8	2.0	1.1	2.5	1.3
	35	1.3	0.6	1.7	0.9	2.3	1.3
	40	1.2	0.6	1.7	0.9	2.3	1.3
	45	1.4	0.6	1.9	0.9	2.5	1.3

Table 7. The relative change in the one year probabilities for transitions into employment when the woman gets one year of additional schooling.

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of schooling Age	9	18	9	18	9	18
0	20	.16	-	.18	-	.21	-
	25	.11	.06	.14	.08	.17	.11
	30	.08	.03	.11	.05	.13	.07
	35	.06	.03	.09	.05	.11	.06
	40	.06	.03	.08	.05	.11	.06
	45	.07	.03	.09	.05	.13	.06
2	25	.15	.08	.17	.11	.20	.13
	30	.11	.05	.13	.07	.17	.09
	35	.09	.04	.11	.06	.15	.09
	40	.08	.04	.11	.06	.15	.09
	45	.09	.04	.13	.06	.17	.09

Table 8 shows the effect of childbearing. This effect is largest for highly educated women with husbands income equal to 50 000 NOK and it is lowest for low educated women with husbands income equal to 150 000 NOK.

Table 8. The relative change in the one year probabilities for transitions out of employment when the woman gets one child during the year.

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of school- ing Age	9	18	9	18	9	18
		20	.50	-	.36	-	.25
0	25	.72	1.03	.58	.91	.44	.76
	30	.81	1.00	.70	.93	.57	.83
	35	.79	.91	.71	.87	.61	.81
	40	.73	.82	.67	.79	.66	.74
2	25	.57	-	.43	-	.31	-
	30	.69	.92	.57	.83	.44	.71
	35	.70	.86	.61	.80	.50	.72
	40	.66	.79	.58	.74	.49	.68

In table 9 we have computed

$$\frac{\partial \log p_{21}(t-1,t)}{\partial \log g_{0t}} - \frac{\partial \log p_1(t)}{\partial \log g_{0t}}$$

for some selected values of the explanatory variables by using (5.4).

Table 9. Elasticities of the transitions probabilities, $p_{21}(t-1,t)$, with respect to the demand g_{0t} compared to the elasticities of $p_1(t)$

Number of children 0 - 16 years	Husbands income (NOK)	50.000		100.000		150.000	
	Years of schooling Age	9	18	9	18	9	18
0	20	2.9	-	2.8	-	2.8	-
	25	3.3	3.8	3.2	3.5	3.0	3.3
	30	3.5	4.5	3.4	3.6	3.3	3.5
	35	3.6	4.0	3.6	4.6	3.4	3.8
	40	3.7	4.0	3.6	5.1	3.7	4.0
	45	3.7	4.0	3.6	5.1	3.8	3.8
2	25	3.1	5.1	3.0	5.1	3.0	4.5
	30	3.4	4.6	3.2	4.6	3.2	3.7
	35	3.4	3.8	3.4	3.8	3.5	3.7
	40	3.4	3.8	3.6	3.8	3.7	3.7
	45	3.7	3.8	3.8	3.8	3.8	3.7

7. Final Remarks.

The model we have discussed above possesses several tractable properties. First, it allows an interpretation of the parameters in terms of an underlying behavioral theory for the individual. Second, the relationship between the transition intensities and the transition probabilities for arbitrary time intervals is simple, even in the general inhomogeneous case where the parameters depend on time (age). For general Markov chain models the transition probabilities are complicated functions of the transition intensities that can often not be expressed on closed form. The Markov chain model proposed here merits, therefore, particular interest even if the viewpoint is solely practicality and technical convenience.

In the process of developing and testing theories of individual behavior the present framework offers a natural class of models as the point of departure.

A well known problem in the analysis of longitudinal data is

to discriminate between unobserved heterogeneity and structural state dependence (Heckman, 1981). Structural state dependence is a notion for the effect of the previous labor market career on the current preferences. Unobserved heterogeneity enters at the level of the observer because he does not observe all the variables that influences the individual's preferences. From the observer's point of view both structural state dependence effects as well as unobserved heterogeneity may lead to dependence on previous labor market career at the aggregate level. The analysis above assumes that no structural state dependence effects are present nor is there any unobserved transition costs. However, such effects could be incorporated into the model by letting, for example, the systematic part of the reservation wage function depend on labor market experience, transition cost parameters, etc. Unfortunately, such extensions will in general break down the simple relationship between transition intensities (given the previous career) and the state/transition probabilities.

The empirical part of the paper is somewhat insatisfactory because we have not estimated the model rigorously from combined time series/cross-section data that includes the proper economic variables such as savings. In our model essentially all the parameters, apart from one, θ , have been estimated from a single cross-section.

Finally, we like to stress that, although we have advocated a strategy for integrating the random and the systematic parts of the model into a unified framework justified by theory there are however essential shortcomings in our approach. Unfortunately, many and possibly important assumptions have been made for mathematical convenience and are typically unjustified from a behavioral point of view. To strengthen the theoretical underpinnings and remove some of the ad hoc assumptions remains an important challenge for future research.

Appendix 1

In this appendix we demonstrate that the utility function, $\hat{U}_t(\mu)$, specified by (2.8), (3.1) and (3.3) is continuously differentiable in μ with probability one. We have

$$(A.1) \quad \hat{U}_t(\mu) = \beta_t(\mu) \max_z \gamma(W(z), T(z)) K_t(W(z), \mu, T(z)) \varepsilon(z).$$

If we assume that $\beta_t(\mu)$ is continuously differentiable in μ we only have to prove that

$$(A.2) \quad \hat{U}_t^*(\mu) = \max_z \gamma(W(z), T(z)) K_t(W(z), \mu, T(z)) \varepsilon(z)$$

is continuously differentiable. The distribution of the derivative, $f(\mu, x)$, is defined by (if it exists)

$$f(\mu, x) = P\left\{ \lim_{\mu' \rightarrow \mu} \left[\frac{U_t^*(\mu') - U_t^*(\mu)}{\mu' - \mu} \right] \leq x \right\}.$$

Let $\bar{z}(\mu)$ be the value of z at which the right hand side of (A.2) attains its maximum. Assume that for some neighbourhood, $N(\mu)$, of μ we have

$$P\{K_t(W(\bar{z}(\mu)), \mu', T(\bar{z}(\mu))) = 1\} = 1$$

for $\mu' \in N(\mu)$. This assumption implies that $U_t^*(\mu)$ is continuous in probability and that

$$\frac{U_t^*(\mu') - U_t^*(\mu)}{\mu' - \mu} = 0$$

with probability one. Accordingly we get that

$$U_t'(\mu) = \beta_t'(\mu) U_t^*(\mu)$$

with probability one, which completes the demonstration.

Appendix 2

In this appendix we shall outline a proof of (4.11). Let

$$Z_j^*(t) = \log Z_j(t)$$

where $Z_j(t)$ is defined by (3.5). According to de Haan (1984), $\{Z_j^*(t)\}$ is a max-stable process and it is easily verified that

$$P\{Z_t^* \leq x\} = \exp(-\exp(\frac{d_j(t)-x}{a}))$$

where

$$d_1(t) = \log(g_{0t}g_{1t}u_{1t}), \quad d_2(t) = \log(g_{2t}u_{2t}).$$

By assumption the sets B_{jt} are nondecreasing. Let

$$\Delta_j(s,t) = \max_{z \in B_j(t) - B_j(s)} [\log v_t(W(z), T(z)) + \log \varepsilon(z)]$$

then it follows that

$$Z_j^*(t) = \max(Z_j^*(s), \Delta_j(s,t))$$

and that $\Delta_j(s,t)$ has distribution of the type

$$\exp(-\exp(\frac{\delta_j(s,t) - x}{a}))$$

for a suitable $\delta_j(s,t)$.

Moreover since $(B_j(t) - B_j(s)) \cap B(s) = \emptyset$, $\Delta(s,t)$ is independent of $Z_j^*(s)$. But this means that $\{Z_j^*(t)\}$ belongs to the subclass of max-stable processes called inhomogeneous extremal processes. Now Theorem 13 of Dagsvik (1983) yields that $\{J(t)\}$ is a Markov chain with transition probabilities given by

$$p_{ij}(s,t) = p_j(t)(1 - e^{\frac{d_j(s) - d_j(t) + n(s) - n(t)}{a}}), \quad i \neq j$$

and

$$p_{ij}(s,t) = 1 - p_{ij}(s,t), \quad i \neq j$$

which coincide with (4.11). This completes the proof.

REFERENCES

- Blomquist, S.N. (1985): "Labour Supply in a Two-Period Model: "The effect of a Nonlinear Progressive Income Tax". Rev. Economic Studies, 52, 515-524.
- Blundel, R. and I. Walker (1986): "A Life-Cycle Consistent Empirical Model of Family Labour Supply Using Cross-Section Data." Rev. Economic Studies, 53, 539-558.
- Dagsvik, J.K. (1983): "Discrete Dynamic Choice: An Extension of the Choice Models of Thurstone and Luce." J. Math. Psychology, 27, 1-43.
- Dagsvik, J.K., O. Ljones, S. Strøm and R. Aaberge (1986): "Labour Supply, Taxes and Income Distributions." Central Bureau of Statistics, Report. (In norwegian only)
- Dagsvik, J.K., (1987): "A General Econometric Framework for Analysing Static Models for Labor Supply in the Presence of Nonlinear Budget Constraints". Central Bureau of Statistics, Working Paper.
- Galambos, J. (1978): "The Asymptotic Theory of Extreme Order Statistics." Wiley, New York.
- de Haan, L. (1984): "A Spectral Representation for Max-Stable Processes". Ann. Probability, 12, 1194-1204.
- Heckman, J.J. (1981): "Statistical Models for the Analysis of Discrete Panel Data." In C.F. Manski and D. McFadden (Eds.). Structural Analysis of Discrete data. Cambridge, MIT Press.
- Killingsworth, M.R. (1983): "Labor Supply". Cambridge Univ. Press, New York.
- MaCurdy, T.E. (1982): "Interpretating Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty". Mimeo, Stanford University.
- MaCurdy, T.E. (1983): "A Simple Scheme for Estimating an Intertemporal Model of Labor Supply and Consumption in the Presence of Taxes and Uncertainty". Int. Economic Rev. 24, 265-289.
- Singer, B. (1984): "Aspects of Nonstationarity". J. Econometrics. 18, 169-170.
- Tinbergen, J. (1956): "On the Theory of Income Distribution". In Weltwirtschaftliches Archiv, 77, 155-175.
- Yellott, J.I. Jr. (1977): "The Relationship between Luce's Choice Axiom, Thurstone's Theory of Comparative Judgment and the Double Exponential Distribution." J. Math. Psychology, 15, 109-146.

ISSUED IN THE SERIES DISCUSSION PAPER

- No. 1 I. Aslaksen and O. Bjerkholt: Certainty Equivalence Procedures in the Macroeconomic Planning of an Oil Economy.
- No. 3 E. Biørn: On the Prediction of Population Totals from Sample surveys Based on Rotating Panels.
- No. 4 P. Frenger: A Short Run Dynamic Equilibrium Model of the Norwegian Production Sectors.
- No. 5 I. Aslaksen and O. Bjerkholt: Certainty Equivalence Procedures in Decision-Making under Uncertainty: an Empirical Application.
- No. 6 E. Biørn: Depreciation Profiles and the User Cost of Capital.
- No. 7 P. Frenger: A Directional Shadow Elasticity of Substitution.
- No. 8 S. Longva, L. Lorentsen, and Ø. Olsen: The Multi-Sectoral Model MSG-4, Formal Structure and Empirical Characteristics.
- No. 9 J. Fagerberg and G. Sollie: The Method of Constant Market Shares Revisited.
- No.10 E. Biørn: Specification of Consumer Demand Models with Stochastic Elements in the Utility Function and the first Order Conditions.
- No.11 E. Biørn, E. Holmøy, and Ø. Olsen: Gross and Net Capital, Productivity and the form of the Survival Function . Some Norwegian Evidence.
- No.12 J. K. Dagsvik: Markov Chains Generated by Maximizing Components of Multidimensional Extremal Processes.
- No.13 E. Biørn, M. Jensen, and M. Reymert: KVARTS - A Quarterly Model of the Norwegian Economy.
- No.14 R. Aaberge: On the Problem of Measuring Inequality.
- No.15 A-M. Jensen and T. Schweder: The Engine of Fertility - Influenced by Interbirth Employment.
- No.16 E. Biørn: Energy Price Changes, and Induced Scrapping and Revaluation of Capital - A Putty-Clay Approach.
- No.17 E. Biørn and P. Frenger: Expectations, Substitution, and Scrapping in a Putty-Clay Model.
- No.18 R. Bergan, R. Cappelen, S. Longva, and N. M. Stølen: MODAG A - A Medium Term Annual Macroeconomic Model of the Norwegian Economy.
- No.19 E. Biørn and H. Olsen: A Generalized Single Equation Error Correction Model and its Application to Quarterly Data.

- No.20 K. H. Alfsen, D. A. Hanson, and S. Glomsrød: Direct and Indirect Effects of reducing SO₂ Emissions: Experimental Calculations of the MSG-4E Model. ²
- No.21 J. K. Dagsvik: Econometric Analysis of Labor Supply in a Life Cycle Context with Uncertainty.
- No.22 K. A. Brekke, E. Gjelsvik, B. H. Vatne: A Dynamic Supply Side Game Applied to the European Gas Market.
- No.23 S. Bartlett, J. K. Dagsvik, Ø. Olsen and S. Strøm: Fuel Choice and the Demand for Natural Gas in Western European Households.
- No.24 J. K. Dagsvik and R. Aaberge: Stochastic Properties and Functional Forms in Life Cycle Models for Transitions into and out of Employment.