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Testing the Multiproduct Hypothesis on Norwegian Aluminium Industry Plants

Abstract:
Although most production activities involve multiple outputs, econometric models of production or cost functions normally involve only one single homogeneous output. The aim of this paper is to test the hypothesis that a multiproduct specification is superior to a model with a single homogenous product. To do this, we use a Multiproduct Symmetric Generalized McFadden (MSGM) cost function. This functional form is globally concave and flexible in the sense that it provides a second order differentiable approximation of any arbitrary cost function which is twice continuously differentiable and linear homogenous in input prices. In an empirical application on a panel data from ten Norwegian primary aluminium plants, we find support for our hypothesis. We present estimates of price elasticities, returns to scale and scope, and product specific demand elasticities. Our results indicate economies of scope, i.e. it is more profitable to produce more than one output, and show sensitivity of factor demand when the product mix changes.

Keywords: Cost function, Multiple output, Global concavity, Returns to scale, Economies of scope, Price elasticity, Output elasticity, Panel data, Primary aluminium industry.

JEL classification: C33, D21, L61.

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1 Introduction

Although production activities of most plants are associated with multiple outputs, most econometric models of production or cost functions involve only one single homogenous output. The use of a single output model is based on the assumption that the transformation function is separable in outputs and inputs, but such a strong *a priori* assumption may lead to wrong empirical conclusions. A cost function framework described below can, for example, test the difference between a single output and a multiproduct approach.

One of the most problematic aspects of estimating cost functions is to maintain conditions implied by economic theory. Quite often, estimated cost functions do not satisfy the global concavity in prices conditions required for a well-behaved cost function. Diewert (1971) defined a flexible functional form for a cost function as one that could provide a second order differentiable approximation to an arbitrary twice continuously differentiable cost function that satisfies the linear homogeneity in prices property at any point in an admissible price domain. The most popular forms in empirical studies are the Translog (Christensen, Jorgensen and Lau, 1971) and the Generalized Leontief (Diewert, 1971). One problem of these functional forms, though, is that the conditions required by economic theory may not be fulfilled. Violations of monotonicity and concavity conditions are common in empirical studies, although it is often possible to avoid these problems by imposing restriction on the functional forms. However, this leads to significant loss of flexibility. Diewert and Wales (1987) proposed a flexible functional form in which the curvature conditions could be tested, *the Symmetric Generalized McFadden cost function* (SGM). An advantage of the SGM over other flexible functions is that the curvature conditions required by economic theory can easily be imposed on the parameters of the cost function without limiting the flexibility of the model.

The purpose of this paper is to analyse the Norwegian primary aluminium industry with the use of the Multiproduct Symmetric Generalized McFadden (MSGM) cost function, developed by Kumbhakar (1994). This functional form is an extension of the single product SGM cost function, introduced by Diewert and Wales (1987). Our model is also modified to include capital as a quasi-fixed input, in the same way as done by Kumbhakar (1989). A similar model approach has also been used by Peeters and Surry (2000), but on time series. The MSMG model allows us to independently test important economic relationships, which characterise the production processes. This includes a test of whether a single or multiproduct functional form is the most appropriate, and also provides measures of economies of scale and scope. It also allows us to test and impose the required concavity condition globally if the unconstrained model does not meet them. For these reasons, this flexible form is attractive for analyses of plant level production processes.

The paper is organised as follows: In next the section, we provide a theoretical description of the MSGM model. In Section 3, we present a description of the data. Section 4 considers an application of the model to test a single versus a multiproduct form for the cost function. Furthermore, we estimate the appropriate elasticities, overall returns to scale, product-specific output elasticities and economies of scope for ten Norwegian primary aluminium plants, each plant producing more than one commodity. In the last section, we summarise and discuss our results.

2 The multiproduct symmetric generalized McFadden cost function

2.1 The basic model

Assume that the production technology of a plant is represented by $y=F(v, k, t)$ where y is a $(m\mathrm{x}1)$ vector of outputs, v is a $(n\mathrm{x}1)$ vector of inputs, and k is a $(l\mathrm{x}1)$ vector of quasi-fixed variable capital in period t. Since capital is assumed to be quasi-fixed, this function can be regarded as the short run production possibilities. Under certain regularity conditions, the true cost function in period t, which is the dual to the production function, can be written as $C^*(y,w,k,t)$, given the a positive vector w, denoting the prices of the variable inputs. Thus, $C^*(y,w,k,t)$ is the solution to the following problem:

$$(2.1) \qquad \begin{aligned} C^*(y,w,k,t) = \quad & \min(w'v) \\ & s.t.\, y = F(v,k,t) \end{aligned}$$

The cost function C^* will satisfy various conditions depending on what restrictions we impose on the production function F^*. The most important requirements for C^* are the linearity constraint, the assumption of a homogeneous function and the concavity restriction in w. We also assume that the function is continuous and twice differentiable with respect to its arguments. Since the function C^* is unknown, our problem is to find an approximation for the cost function, C, which has similar characteristics as the general form of the cost function. In order to apply the multiproduct symmetric generalized McFadden (MSMG) framework, we require that the cost function is linear homogeneous and concave in w.

Consider the following cost function, C, which we interpret as an approximation to the true function C^*:

$$
\begin{aligned}
C(w,y,k,t) = \quad & g(w)\left\{\sum_{r=1}^{m}\beta_r y_r\right\} + \sum_{i=1}^{n}\alpha_i w_i + \sum_{i=1}^{n}\alpha_{it} w_i t + \sum_{i=1}^{n}\alpha_{ik} w_i k + \sum_{i=1}^{n}\alpha_{ii} w_i \left\{\sum_{r=1}^{m}\beta_r y_r\right\} + \\
(2.2) \qquad & \sum_{i=1}^{n}\beta_{ik} w_i k\left\{\sum_{r=1}^{m}\beta_r y_r\right\} + \sum_{i=1}^{n}\beta_{it} w_i t\left\{\sum_{r=1}^{m}\beta_r y_r\right\} + \sum_{i=1}^{n}\ \sum_{r=1}^{m}\sum_{s=1}^{m}\beta_{irs} w_i y_r y_s + \\
& \left\{\sum_{i=1}^{n}\alpha_{itt} w_i t^2\right\}\left\{\sum_{r=1}^{m}\beta_r y_r\right\} + \left\{\sum_{i=1}^{n}\alpha_{ikk} w_i k^2\right\}\left\{\sum_{r=1}^{m}\beta_r y_r\right\}
\end{aligned}
$$

where n is the number of variable inputs and m is the number of outputs. The $g(w)$ function is defined as:

$$(2.3) \qquad g(w) = w'Sw/2\theta' w\ ,$$

with S is a nxn symmetric negative semidefinite matrix, with $s_{ij} = s_{ji}$. θ is a vector of predetermined non-negative constants and not all zero. The following restrictions are made: One of the parameters in β_r $(r=1\ldots m)$ is normalised to unity. We also need some restrictions of the elements of S. These are: $S'w^* = 0$ for some w^*, where $w_i^* > 0$, for all i. For example, if w^* is chosen to be a unit vector (the normalising point) and $S'w^* = 0$, then $\sum_{j=1}^{n} s_{ij} = 0$ for all i. If the estimated S matrix is negative semidefinite, then C defined in (2.2) and (2.3) will be globally concave in input prices w.

On the other hand, even if the estimated S matrix is not negative semidefinite, it can be imposed on the cost function, without destroying its flexibility by applying a correction method. To do this, we follow the technique used by Wiley, Smith and Brambel (1973). We reparameterise S as $S = -\Gamma\,\Gamma'$, where Γ is an $N-1$ by $N-1$ lower triangular matrix:

$$
(2.4) \qquad
\begin{aligned}
&\tilde{S} = -\Gamma\Gamma', \qquad \Gamma = \left[\gamma_{ij}\right] \qquad (i,j = 1,\ldots.,n-1) \\
&\gamma_{ij=0} \qquad \text{for } i > j
\end{aligned}
$$

Using Shephard's lemma, the conditional demand functions are derived

$$
v_i = \frac{\partial C}{\partial w_i} = \left[\sum_{r=1}^{m} \beta_r y_r\right] \left[\frac{\sum_{j=1}^{n} s_{ij} w_j}{\sum_{j=1}^{n} \theta_j w_j} - \frac{\theta_i}{2} \frac{\sum_{j=1}^{n} s_{ij} w_i w_j}{\left(\sum_{j=1}^{n} \theta_j w_j\right)^2} \right] + \alpha_i + \alpha_{it} t + \alpha_{ik} k +
$$

$$
\alpha_{ii} w_i \left[\sum_{r=1}^{m} \beta_r y_r\right] + \beta_{ik} k \left[\sum_{r=1}^{m} \beta_r y_r\right] + \beta_{it} t \left[\sum_{r=1}^{m} \beta_r y_r\right] + \tag{2.5}
$$

$$
\sum_{r=1}^{m} \sum_{s=1}^{m} \beta_{irs} y_r y_s + \alpha_{ikk} k^2 \left[\sum_{r=1}^{m} \beta_r y_r\right] + \alpha_{itt} t^2 \left[\sum_{r=1}^{m} \beta_r y_r\right]
$$

2.2 Testing of hypotheses

As Kumbhakar (1994) points out, the MSGM cost function defined above is general enough to include some interesting cases. The MSGM cost function represents the unrestricted model, and we can test the multiproduct hypothesis by restricting the product specific parameters β_r and β_{irs}.

In our data, we have divided the output into three product categories: (I) primary aluminium plus products directly connected to production of this good, (II) alloys and castings, and (III) products made of aluminium. More about data is given in next section. Our model enables us to test different aggregation levels. First, we can test the hypothesis that the production can be characterised by a single product SGM cost function. Then we have to make the following restrictions on the parameters of the MSGM cost function:

(2.6) $\beta_r = 1$ for all r, and
β_{irs} = constant for all r and s.

Secondly, we test the hypothesis that the production consists of two product groups. We have chosen to test the aggregation of category (II) and (III). This implies that the following restrictions on the parameters are made:

(2.6') $\beta_2 = \beta_3 = 1$, and
$\beta_{i2s} = \beta_{i3s}$ for all s.

2.3 The economies of scale and scope

The traditional concept of scale economies under a single product plant refers to the behaviour of total costs as output expands. Formally, economies of scale are measured by the relationship of average cost to marginal cost. Economies of scale is said to exist if the marginal cost lies below the average cost:

$$(2.7) \qquad RTS = \frac{AC}{MC} = \frac{C(y)}{y\, dC/dy} > 1$$

The multiproduct generalisation of this concept is the overall returns to scale (see for example Baumol et al., 1982). It is defined as the elasticity of output with respect to cost measured along a ray in the output space:

$$(2.8) \qquad ORTS = \frac{C(\bullet)}{\sum_i y_i MC_i}$$

where MC_i is the marginal cost of product *i.* ORTS measures the responsiveness of costs to a scale change, while composition of output remains fixed. The presence of scale economies (ORTS>1) would imply revenue to fall short of costs.

The shortcoming of the economies of scale assumption is that the product mix is unlikely to stay constant when total output increases. A measure of the effect of a change in the output mix is the estimate of economies of scope, suggested by Baumol et al. (1982), and Bailey and Friedlander (1982), which is defined as

$$(2.9) \qquad ESCP = \frac{\sum_i C(0,\ldots,y_i,..,0)}{C(y)}$$

If economies of scope are present, for a given output mix, when a plant that produces all the outputs will have lower costs than the sum of costs for single output plants.

The elasticity of variable costs with respect to an increase in the capital stock is given by:

$$(2.10) \qquad \varepsilon_k = \frac{\partial C}{\partial K}\frac{K}{C}$$

To be well behaved, the cost function should be non-increasing and convex in levels of each fixed factor (see Brown and Christensen, 1981).

The own and cross-price elasticities are defined as:

$$(2.11) \qquad \varepsilon_{ij}^{sr} = \frac{\partial v_i}{\partial w_j} \frac{w_j}{v_i}$$

The last elasticity to be defined is the demand elasticity with respect to output:

$$(2.12) \qquad \eta_i^{r} = \frac{\partial v_i}{\partial y_r} \frac{y_r}{v_i}$$

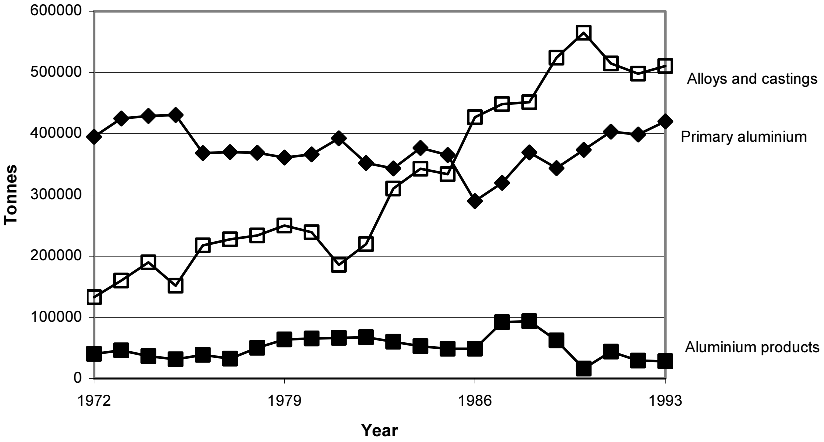
3 Data description

The empirical results are based on a panel of annual observations from ten Norwegian primary aluminium plants. It is an unbalanced panel that covers the years 1972 to 1993. The panel is unbalanced since not all plants are observed all years. Seven plants are observed for the whole period, while the other three are observed for three to eleven years. This data set is an extension of the data material used in Chapter 3 in this thesis. In total 173 observations are used.

Even though the primary aluminium industry in Norway is one of the most homogeneous industry branches, the ten aluminium plants in the industry, in fact produce up to ten different products. Each product demands different amount of inputs. For estimation purposes, we have divided outputs into three categories; (i) primary aluminium plus products directly connected to production of this good, (ii) aluminium alloys and aluminium castings and (iii) products made out of aluminium. All categories are measured in produced tonnes.

As we can see in Figure 3.1 below, primary aluminium was the main product in this industry branch until the mid 1980s. The production has been stable at 400 million tonnes per year. But during the 1980s strong growth in the production of alloys and castings, made this the largest product group. The third group, comprising aluminium products, was a small and stable product group during the period.

Figure 3.1. The production of aluminium in Norway 1972-1993



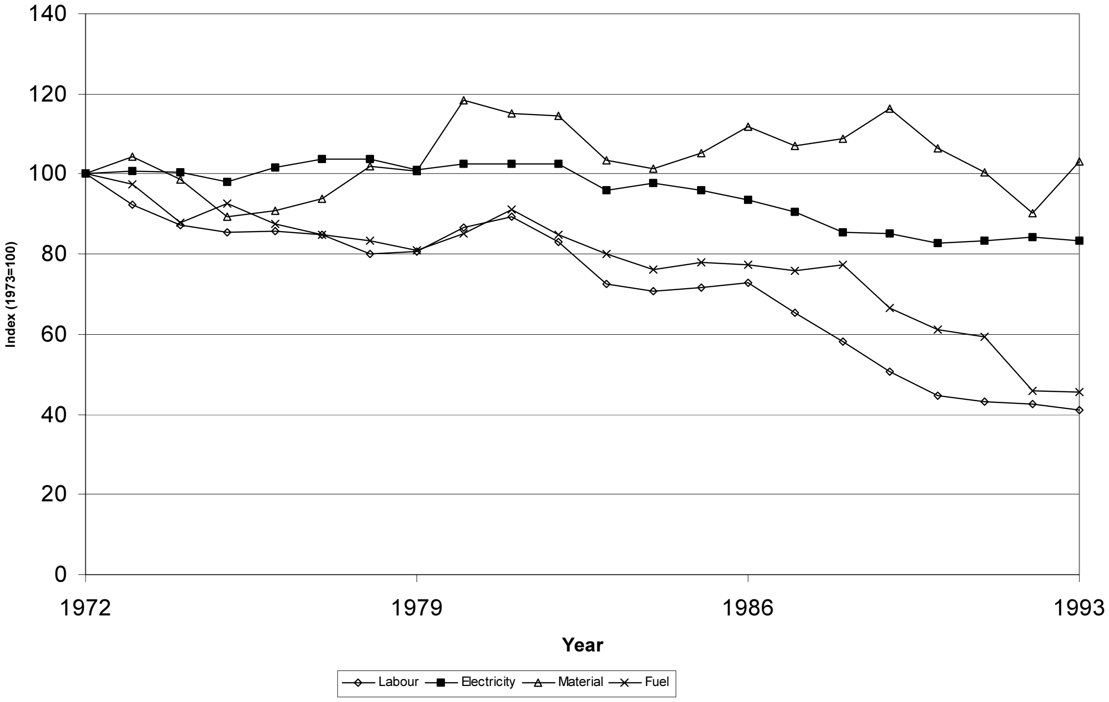
The data are obtained from the Manufacturing Statistics database of Statistics Norway, supplemented to a minor extent by data from the Norwegian National Accounts. The endogeneous inputs are (*L)* labour measured in (1000) hours worked, (*E*) electricity measured in kWh, (*F*) other fuels measured in kWh, and (*M*) other intermediate inputs measured in constant 1991-prices. The capital stock is measured in 1991-prices. Capital stock is not directly observed, but data is constructed by aggregating investment from a benchmark by the so-called "*perpetual inventory method".* Chapter 3 in this thesis reports more details of the data. The time trend *t* is assumed to capture the level of technology. Summary statistics are reported in Table 3.1. As we can see, all the plants do not produce all products. There are also differences in size between the different plants. The smallest plants only existed for a limited time.

Table 3.1 Summary statistics of aluminium production in Norway 1972-1993

Variables	Mean	Std Dev	Min	Max
Costs (in mill. NOK)	2157	1569	27	6378
Working hour (1000 h)	1263	722	5	3004
Electricty (Mwh)	1638	890	0.061	3545
Fuel (Mwh)	79	7	0.07	307
Material (in mill NOK, 1991 year prices)	519	300	1.5	1239
Capital (in mill NOK, 1991 year prices)	1261	922	.5	5340
Prime aluminium (1000 tonnes)	48	39	0	146
Alloys and castings (1000 tonnes)	41	15	0	172
Aluminium products (1000 tonnes)	6	15	0	60
Wages index (1991=1)	0.52	0.29	0.13	1.17
Price electricity index (1991=1)	0.57	0.31	0.12	1.67
Price fuel index (1991=1)	0.61	0.32	0.07	1.28
Price material index (1991=1)	0.73	0.24	0.29	1.07

Figure 3.2, shows the variable input coefficients. The material input coefficient was relatively stable during the observation period, while the use of labour and fuel showed a strongly downward trend. The labour input per tonne produced was reduced by more than 50 per cent during the observation period, from 22 to nine hours per produced tonne. The use of electricity also declined somewhat, but not as much as the use of labour and fuel. In the beginning of the period, the input coefficient of electricity was stable. However, a decline during the last part of the period is observed.

Figure 3.2 Variable input coefficients in Norwegian aluminium production 1972-1993



4 The estimation procedure and empirical results

Our MSGM cost function (2.2) presented in Section 2, in the most general form, is applied with four conditional input demand functions (2.5) derived by Shephard's lemma and with three product groups.

Since we have panel data, one may expect autocorrelated disturbances due to the existence of unobserved characteristics over time. We therefore include an autoregressive AR(1) coefficient ρ_i, specified for each equation. We also use a dummy coefficient μ_f for each plant to capture the heterogeneity in the cost function. The error term of the cost function is then specified as a gross disturbance term, $v_{if,t}$, defined as

(4.1) $$v_{if,t} = u_{if,t-1} + \mu_t$$

where $u_{if,t}$ is the net disturbance term, μ_t capture the plant specific effect,. We assume the plant effect to be fixed over time and across equations. The net disturbance term, $u_{if,t}$, is defined as

(4.2) $$u_{if,t} = \rho_i u_{if,t-1} + \eta_{if,t}$$

where subscript i is represent the equations, $f = 1, \ldots., 10$, and $t = 1, \ldots, 23$ are indices for plant and time (year), respectively. The error term η_{ift} is white noise and we also assume that it has a probability distribution that is invariant over time. We also assumes the covariance E[$\eta_{if,t} \eta j_{f,t}$] =0 when $i \neq j$.

With the above modifications, we have estimated the system by using the full information maximum likelihood regression technique (FIML) in SAS MODEL *procedure*. In Table 4.1, we summarise the values for the unrestricted three product-groups specification, defined in (2.6) and (2.6'). We have tested the restricted models against the unrestricted model with a likelihood ratio test. Since the data set only consists of 10 plants, we apply a small sample correction, defined as:

(4.3) $$\chi^2(r) = -\frac{2(T-k-1+\frac{r}{2})}{T}(L_R - L_U),$$

where L_R and L_U are the log-likelihood value for the restricted and the unrestricted model, respectively. T is the sample size, k the number of parameters in the unrestricted model and r is the number of restrictions (see Mizon, 1977).

Table 4.1 Summary statistics for the estimation of the models

Model	Three products	Two products	Single product
Maximum likelihood-value	1239.41	1215.16	1196.92
Test statistic		33.71	57.48
No. of restrictions		13	22
R^2_l-adj	0.992		
R^2_l-adj	0.969	0.993	0.993
R^2_e-adj	0.988	0.967	0.965
R^2_f-adj	0.959	0.988	0.987
R^2_m-adj	0.923	0.928	92.3
ρ_c	-0.897	-0.93	-0.92
ρ_l	-0.859	-0.93	-0.93
ρ_e	-0.786	-0.83	-0.82
ρ_f	-0.946	-0.92	-0.92
ρ_m	-0.808	-0.71	-0.68

The χ^2-test statistics for the likelihood ratio test reject the restricted two-product model and the single product form against the more general multiproduct specification at the 1 % level (the critical values are 27.7 and 40.3 respectively). The goodness of fit is high, with all adjusted R^2 over 0.92. The ρ-values are negative and high, but presumably these corrections will avoid autocorrelations in the estimates. There is a trade-off between autocorrelation and the loss in explanatory power of the system.

To test whether there is still autocorrelation in the system after adjusting with the ρ-values, we have used the Godfrey Lagrange multiplier test (Godfrey, 1978a and 1978b). According to the test statistics, the null-hypothesis of no autocorrelation cannot be rejected at the five per cent level, except for the cost equation in the single product model, see Table 4.2.

Table 4.2 Autocorrelation test

Equation	3 Products	2 Products	1 Product
Cost	2.420	2.876	4.651
	(0.120)	(0.09)	(0.03)
Labour	1.842	0.666	0.949
	(0.175)	(0.41)	(0.33)
Electricity	0.226	0.067	0.008
	(0.635)	(0.80)	(0.93)
Fuel	0.181	0.001	0.009
	(0.671)	(0.97)	(0.92)
Material	1.145	0.210	0.207
	(0.284)	(0.65)	(0.65)

(Significance probability for autocorrelation tests in parentheses)

The parameter estimates of all models are reported in Appendix.

Next, we examine several economically relevant characteristics. We begin with the multiproduct overall returns to scale (ORTS), defined in (2.8). The overall returns to scale (ORTS) seem to be increasing over time, see Figure A1 in Appendix. But variations in the estimates are very high, so no definitive conclusions can be drawn from these results (see Table 4.3). Even though the primary aluminium industry is characterised by large-scale production, the estimates of the returns to scale are remarkably high. The variation may be a result of the characteristics of the data set, where some small units are not observed during the entire period.

If economies of scope are present for a given output mix, a plant producing all the outputs will face lower costs than the sum of costs for plants producing only one of the products. In most of the studies of the primary aluminium industry, the assumption is that only one homogeneous product is produced, but in this study the hypothesis of a single homogeneous output is rejected. The result from our model shows significant and stable economies of scope for the plants over time. The standard deviation in Table 4.3 shows that the estimates of ESCP are more significant than the estimation of the ORTS. A value of economies of scope around 2 during the whole period means that the costs of producing only one product is twice as high as producing all products. The precise estimates of the scope elasticity in contrast to the large variability of the estimation of scale elasticity indicate that it is very important to apply a multiproduct estimation framework that takes the economy of scope into account.

Table 4.3 Estimate of technical progress, economies of scale and scope, 1991

Elasticities	3 goods		2 goods		1 good	
	Value	Std. dev.	Value	Std. dev.	Value	Std. dev.
ORTS	2.62	2.1	2.58	2.95	2.70	3.33
ESCP	2.14	0.36	2.06	0.24	-	-

In Table 4.4, own-price and cross-price elasticities are reported. The price elasticities are defined as in (2.9). According to our estimates, all own-price elasticities are negative and smaller than one in absolute terms. This is in line with the findings of Lindquist (1995), who used a dynamic translog approach on same data set for the Norwegian primary aluminium industry. Our estimates are, however, higher than those in Larsson (2003) made on similar same data. The main reason for the differences between the estimation results is that here we have applied a multiproduct framework. Moreover, in this study we have used an extended data set, and a different approach dealing with capital, which also can affect the results.

Table 4.4 Price elasticities, 1991

Elasticities	3 goods		2 goods		1 good	
	Value	Std. dev.	Value	Std. dev.	Value	Std. dev.
ε_{LL}	-0.33	0.08	-0.29	0.09	-0.44	0.17
ε_{EE}	-0.20	0.05	-0.19	0.05	-0.23	0.11
ε_{FF}	-0.23	0.10	-0.25	0.12	-0.20	0.14
ε_{MM}	-0.06	0.10	-0.05	0.07	-0.21	0.51
ε_{LE}	0.15	0.07	0.15	0.07	0.08	0.11
ε_{LF}	-0.05	0.03	-0.04	0.03	-0.05	0.03
ε_{LM}	0.02	0.03	0.02	0.03	0.04	0.04
ε_{EL}	0.11	0.04	0.11	0.04	0.06	0.09
ε_{EF}	0.05	0.03	0.04	0.03	0.09	0.08
ε_{EM}	-0.04	0.03	-0.02	0.03	-0.09	0.07
ε_{FL}	-1.05	0.82	-0.81	0.92	-1.10	0.99
ε_{FE}	1.79	1.26	1.62	1.47	2.12	1.97
ε_{FM}	0.04	0.49	0.09	0.52	0.76	0.93
ε_{ML}	-0.02	0.03	-0.02	0.02	-0.06	0.18
ε_{ME}	0.02	0.02	0.02	0.02	0.06	0.07
ε_{MF}	0.02	0.04	0.04	0.02	0.02	0.22

The only cross-price elasticities above 1 are between fuel and labour, and fuel and electricity. With respect to differences in performance between the models, we cannot detect any significant differences in our elasticity estimates across the models. However, the standard deviations in the single output model seem to be higher at an average. Our conclusion to this is that the multiproduct models give more efficient estimates.

We now discuss how an increase in product *i* will affect the demand for each input factor. The substitution effects caused by shifts in the product mix may explain the shift in factor use. The partial demand elasticities with respect to output are only calculated for the unrestricted model. The results are reported in Table 4.5. A change in output of primary aluminium has the greatest effect on demand. One per cent change in output leads to 0.6 per cent change in labour demand.

The effect on labour inputs of a change in output can also be seen in the development over time, as illustrated in Figures A6-A9 in Appendix. Figure 2.2 shows that the labour input has been reduced in connection to the shift in production from primary aluminium to alloys and castings. On the other hand, the response to a change in the production of alloys and castings is greatest on the demand for fuel and material. The graphs in Figure A8 and A9 in Appendix show that the trend for these elasticities is increasing.

Table 4.5 Product specific output elasticities w.r.t inputs

Variable\Output	Prime aluminium	Alloys and castings	Aluminium products
Labour	0.628	0.183	0.020
Electricity	0.201	0.282	0.034
Fuel	0.333	0.664	0.092
Material	0.348	0.557	0.040

5 Conclusions

In this paper, we use the Multiproduct Symmetric Generalized McFadden (MSGM) cost function on the primary aluminium industry in Norway. The main advantage of this functional form is that global concavity can be imposed on the cost function without destroying the flexibility of the model, and that it permits zero values on one or more outputs. We have tested three different specifications of the MSGM cost function, and our hypothesis that the multiproduct specification is superior to the one with single output, is then clearly accepted.

Our results also clearly support the hypothesis of economics of scope. It is more profitable to produce more than one product. Our estimates of economics of scope are much more significant than the estimation of the economies of scale. The precise estimates of the scope elasticity in contrast to the large variability of the estimation of scale elasticity, indicates that it is very important to apply a multiproduct estimation framework that takes the economy of scope into account.

Our estimates indicate that input demand is not sensitive to factor prices, except for the cross price elasticities between fuel and labour, and fuel and electricity. The elasticity estimates are robust between the three model specifications. However, higher standard error, at an average, for the single product specification, indicates that the multiproduct approaches are more efficient.

The production mix has a considerable influence on the factor demand. Plants have changed their production from primary aluminium to alloys and castings. These changes in output mix have lead to a less labour intensive production and more material and fuel intensive outputs. These results could not have been detected in a model with one homogeneous good.

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Appendix

Table A1 Parameter estimates

Model	Three products		Two products		Single product	
Parm.	Estimate	t-value	Estimate	t-value	Estimate	t-value
β_1	0.800	2.59	0.933	14.27	1*	-
β_2	0.867	2.63	1*	-	1*	-
λ_{ll}	-2.470	-3.85	-2.371	-5.73	-3.404	-11.00
λ_{le}	3.745	3.30	3.675	6.83	2.372	7.09
λ_{lf}	-0.146	-0.57	-0.197	-0.83	-0.016	-0.09
λ_{ee}	1.221	0.71	0.875	0.64	-1.456	-2.92
λ_{ef}	0.653	2.36	0.712	4.13	0.687	5.17
λ_{ff}	0*	-	0*	-	0*	-
α_l	1.009	2.92	1.135	3.62	1.330	4.42
α_e	0.093	0.53	0.124	0.70	0.163	0.83
α_f	-0.030	-0.29	-0.050	-0.55	-0.079	-0.89
α_m	0.156	0.58	0.079	0.36	0.059	0.30
α_{ll}	0.019	0.14	0.013	0.11	-0.038	-0.34
α_{ee}	27.890	2.34	25.202	6.47	21.011	6.84
α_{ff}	0.052	0.04	0.053	0.04	-0.277	-0.30
α_{mm}	9.446	1.58	8.648	2.32	6.243	2.10
α_{lk}	0.975	2.33	0.907	2.32	0.863	2.42
α_{ek}	0.293	0.90	0.282	1.06	0.350	1.44
α_{fk}	-0.070	-0.98	-0.066	-1.01	-0.076	-1.12
α_{mk}	0.226	0.69	0.166	0.61	0.148	0.61
α_{lt}	-0.045	-2.28	-0.048	-2.68	-0.053	-3.01
α_{et}	-0.012	-0.75	-0.017	-1.12	-0.022	-1.47
α_{ft}	0.005	1.01	0.005	1.35	0.006	1.40
α_{mt}	-0.019	-0.92	-0.017	-1.00	-0.017	-1.08
α_{lkk}	-8.332	-1.15	-6.987	-1.39	-6.106	-1.48
α_{ekk}	-3.736	-0.62	-4.488	-1.23	-5.059	-1.56
α_{fkk}	1.071	0.78	0.919	1.08	0.938	1.19
α_{mkk}	-4.218	-0.76	-3.469	-0.89	-3.094	-1.02
α_{ltt}	0.005	0.01	-0.018	-0.05	0.128	0.44
α_{ett}	-0.021	-0.05	0.030	0.09	0.121	0.43
α_{ftt}	0.053	0.59	0.051	0.81	0.044	0.85
α_{mtt}	0.209	0.49	0.268	0.91	0.333	1.56

Table A1 Parameter estimates Cont.

Model	Three products		Two products		Single product	
Parm.	Estimate	t-value	Estimate	t-value	Estimate	t-value
β_{11l}	28.394	0.98	24.580	0.99	44.247	1.84
β_{11e}	-36.799	-1.33	-44.379	-1.86	-27.447	-1.24
β_{11f}	0.145	0.03	0.292	0.05	-0.628	-0.11
β_{11m}	2.239	0.09	-3.554	-0.15	1.853	0.09
β_{12l}	55.242	1.71	50.555	1.93	*	-
β_{12e}	-27.196	-0.82	-29.264	-1.11	*	-
β_{12f}	-0.453	-0.07	-0.228	-0.04	*	-
β_{12m}	9.635	0.35	4.102	0.18	*	-
β_{22l}	43.700	1.21	41.097	1.54	*	-
β_{22e}	-40.110	-1.01	-43.706	-1.53	*	-
β_{22f}	0.849	0.13	1.075	0.17	*	-
β_{22m}	9.025	0.31	0.610	0.02	*	-
β_{13l}	31.200	0.70	*	-	*	-
β_{13e}	-37.652	-0.62	*	-	*	-
β_{13f}	-3.241	-0.34	*	-	*	-
β_{13m}	-1.663	-0.03	*	-	*	-
β_{23l}	76.941	1.49	*	-	*	-
β_{23e}	-21.746	-0.45	*	-	*	-
β_{23f}	6.296	0.72	*	-	*	-
β_{23m}	10.443	0.19	*	-	*	-
β_{33l}	109.622	1.19	*	-	*	-
β_{33e}	-100.781	-0.62	*	-	*	-
β_{33f}	0.418	0.03	*	-	*	-
β_{33m}	-6.357	-0.09	*	-	*	-
α_{lkk}	0.394	0.36	0.544	0.74	0.363	0.69
α_{ekk}	1.149	0.93	1.435	2.28	1.360	2.61
α_{fkk}	-0.227	-0.88	-0.186	-1.39	-0.168	-1.55
α_{mkk}	0.509	0.49	0.455	0.74	0.371	0.80
α_{ltt}	-0.001	-0.03	0.000	0.03	-0.004	-0.42
α_{ett}	-0.006	-0.44	-0.005	-0.51	-0.008	-0.89
α_{ftt}	-0.003	-0.76	-0.003	-1.31	-0.002	-1.42
d_1	-0.001	-0.10	-0.009	-0.20	0.030	0.22
d_2	-0.019	0.32	0.009	0.12	0.007	0.04
d_3	0.013	0.26	0.067	1.23	0.045	0.33
d_4	0.052	0.74	0.056	0.76	0.048	0.31
d_5	0.042	0..84	-0.004	-0.06	0.079	0.55
d_6	0.013	0.13	-0.098	-1.56	0.055	0.30
d_7	-0.039	-0.39	-0.012	-0.09	-0.205	-0.98
d_8	-0.048	-0.60	0.004	0.03	0.031	0.07
d_9	0.013	0.06	0.019	0.09	-0.049	-0.21

*=Restricted parameters

Figure A1 Economies of scale (ORTS) and economies of scope (ESCP)

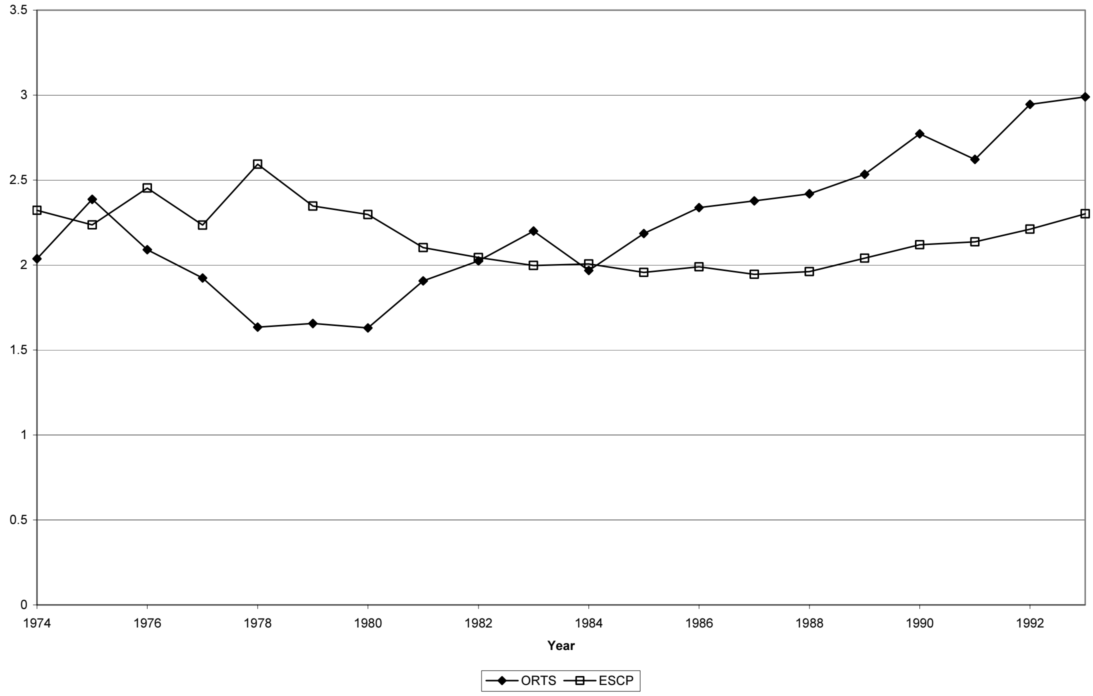


Figure A2 Price elasticities labour w.r.t changes in wages (ELFL), electricity prices (ELFE) fuel prices (ELFF) and prices of other intermediate inputs (ELFM).

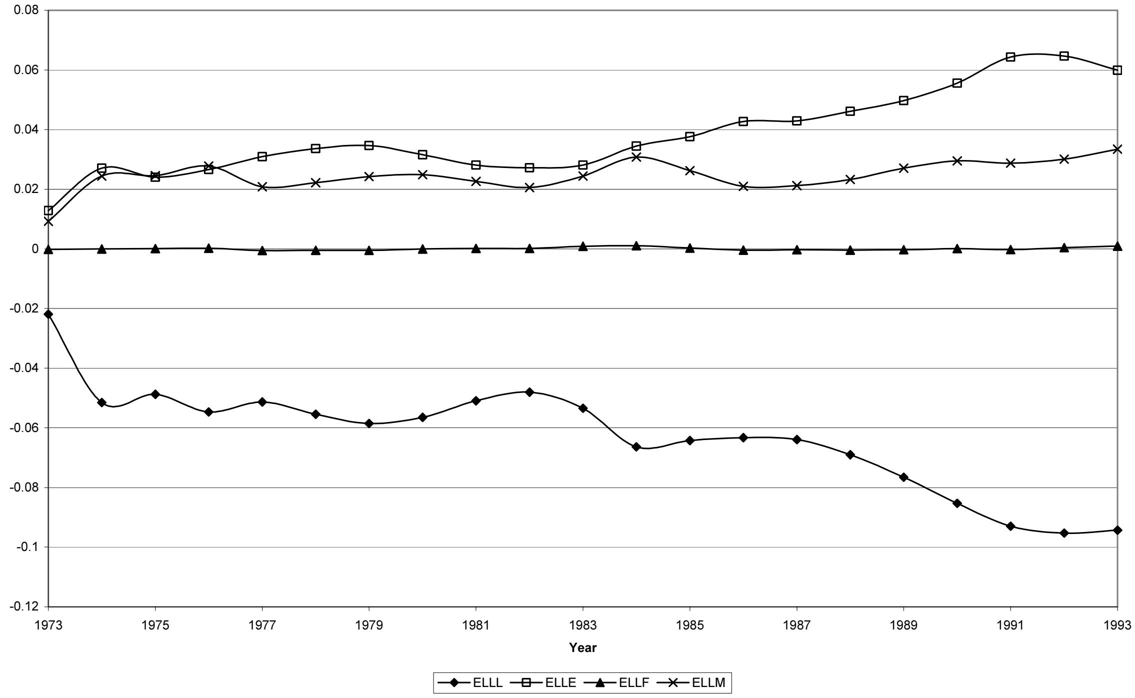


Figure A3 **Price elasticities electricity w.r.t changes in wages (ELFL), electricity prices (ELFE) fuel prices (ELFF) and prices of other intermediate inputs (ELFM).**

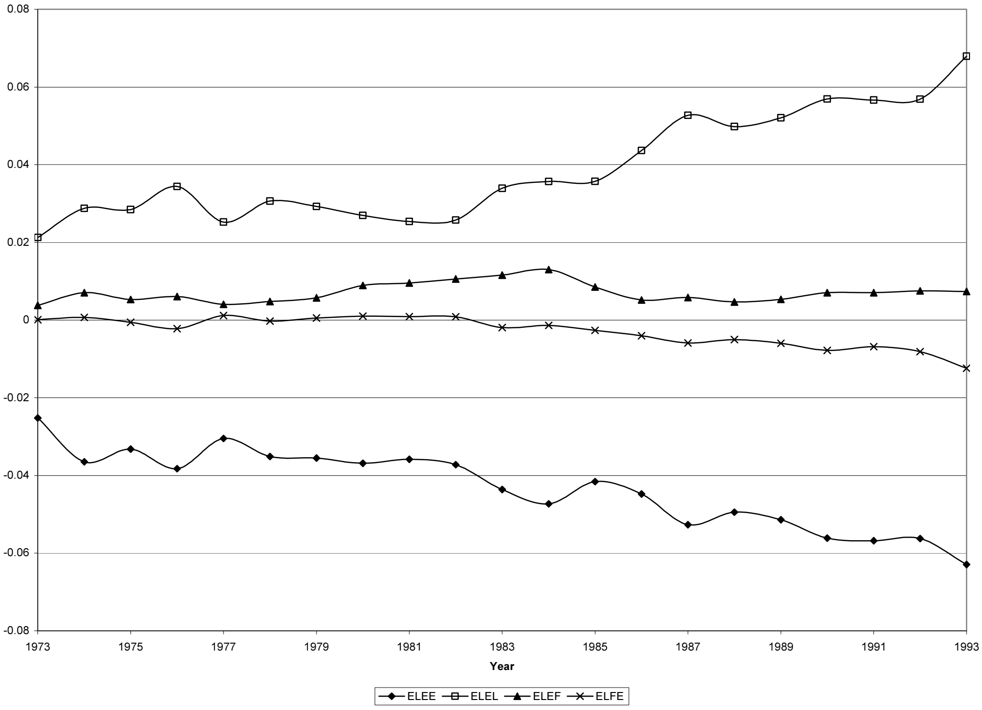


Figure A4 **Price elasticities fuel w.r.t changes in wages (ELFL), electricity prices (ELFE) fuel prices (ELFF) and prices of other intermediate inputs (ELFM).**

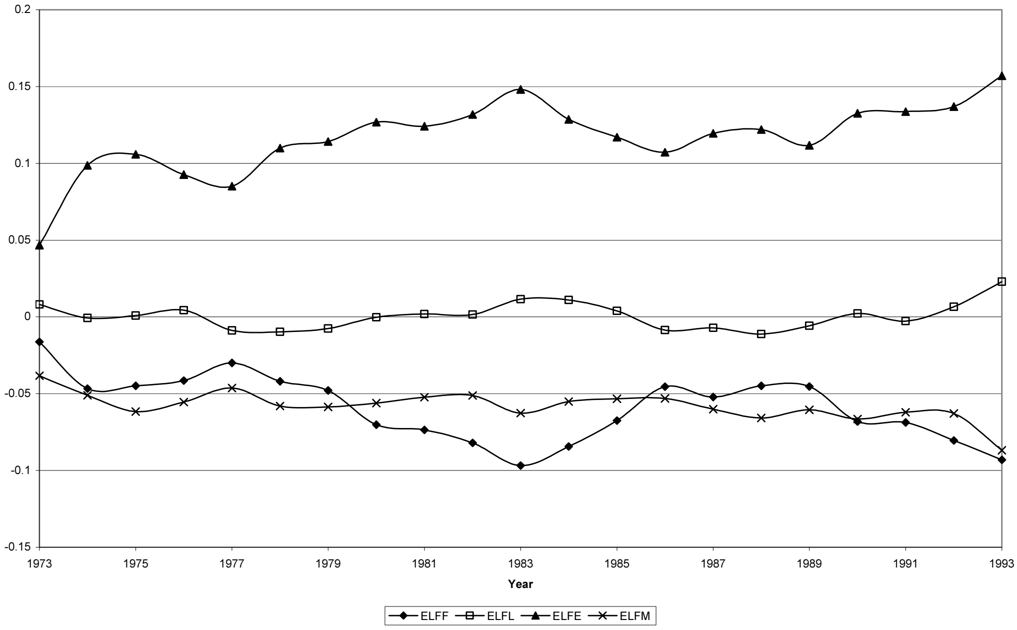


Figure A5 **Price elasticities material w.r.t changes in wages (ELFL), electricity prices (ELFE) fuel prices(ELFF) and prices of other intermediate inputs (ELFM)**

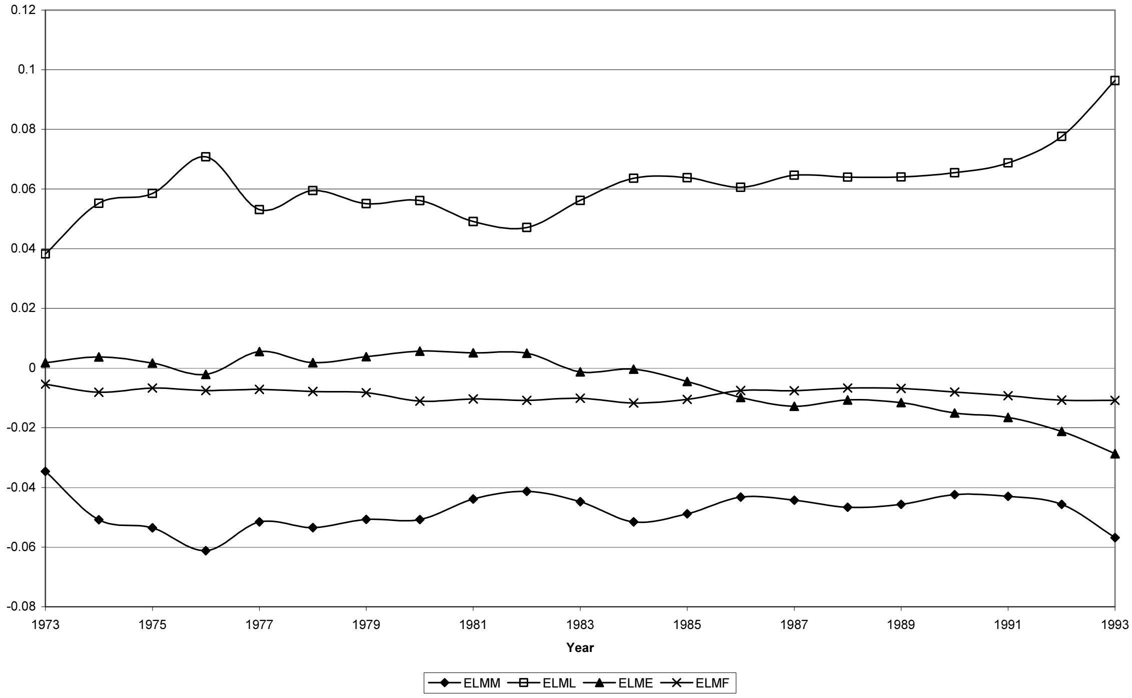


Figure A6. **Product specific output elasticities w. r. t. labour: Primary aluminium (ELLY1), aluminium alloy (ELLY2), aluminium products (ELLY3)**

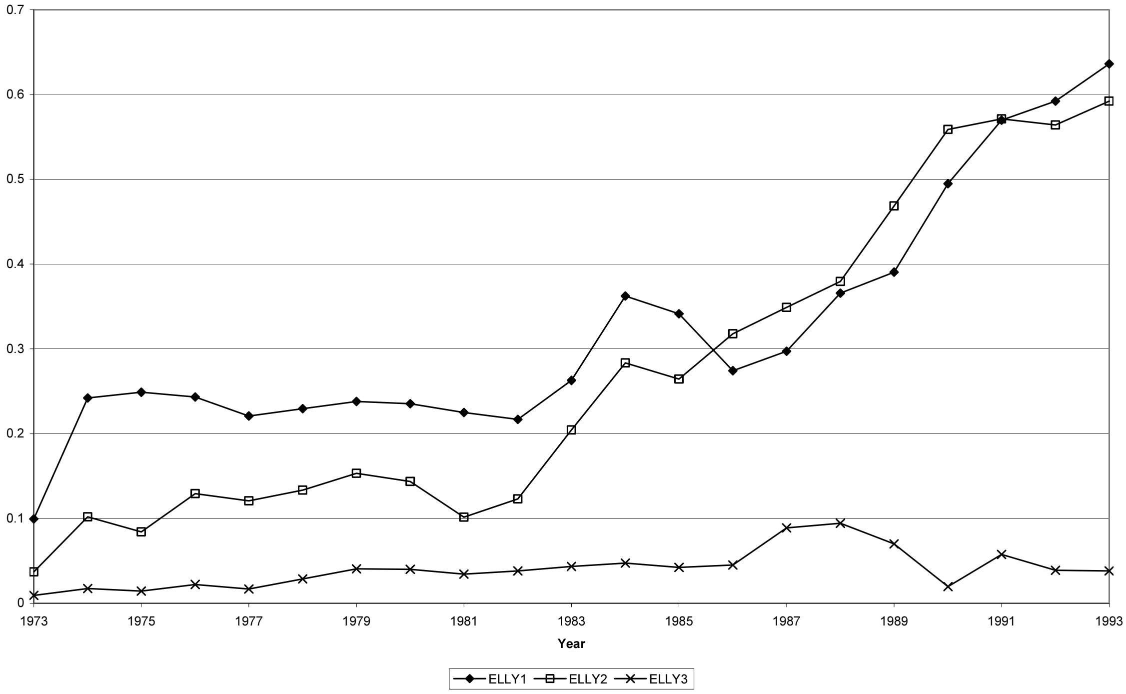


Figure A7. **Product specific output elasticities w. r. t. electricity: Primary aluminium (ELLY1), aluminium alloy (ELLY2), aluminium products (ELLY3)**

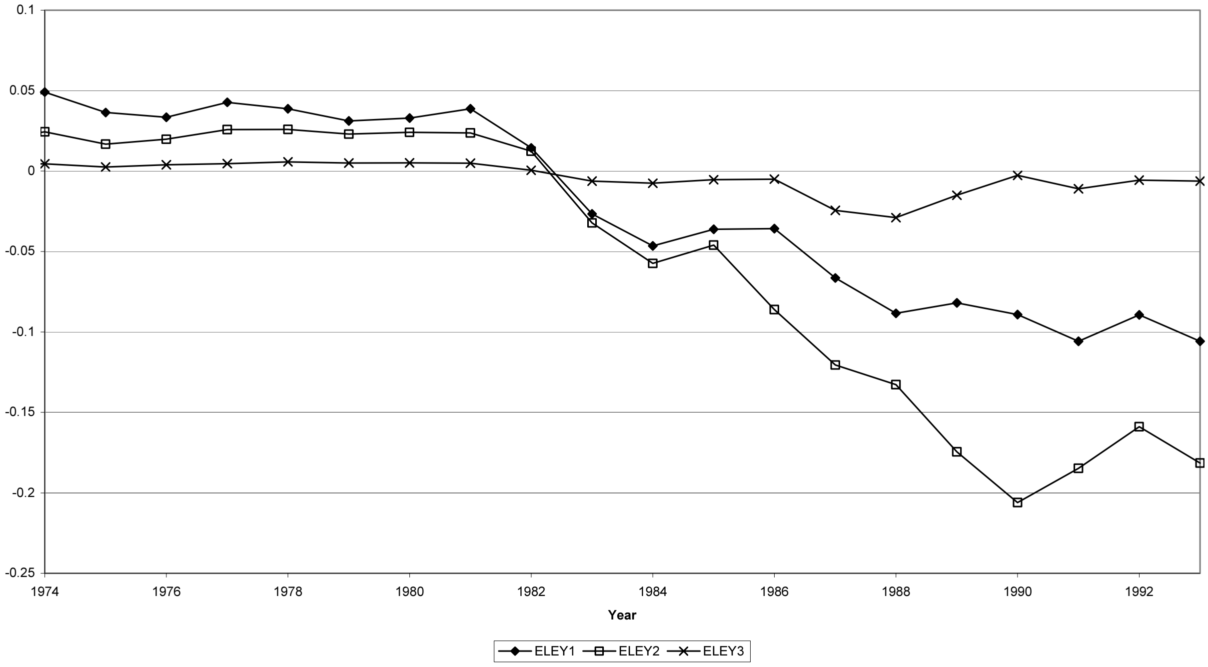


Figure A8 **Product specific output elasticities w. r. t. fuel: Primary aluminium (ELLY1), aluminium alloy (ELLY2), aluminium products (ELLY3)**

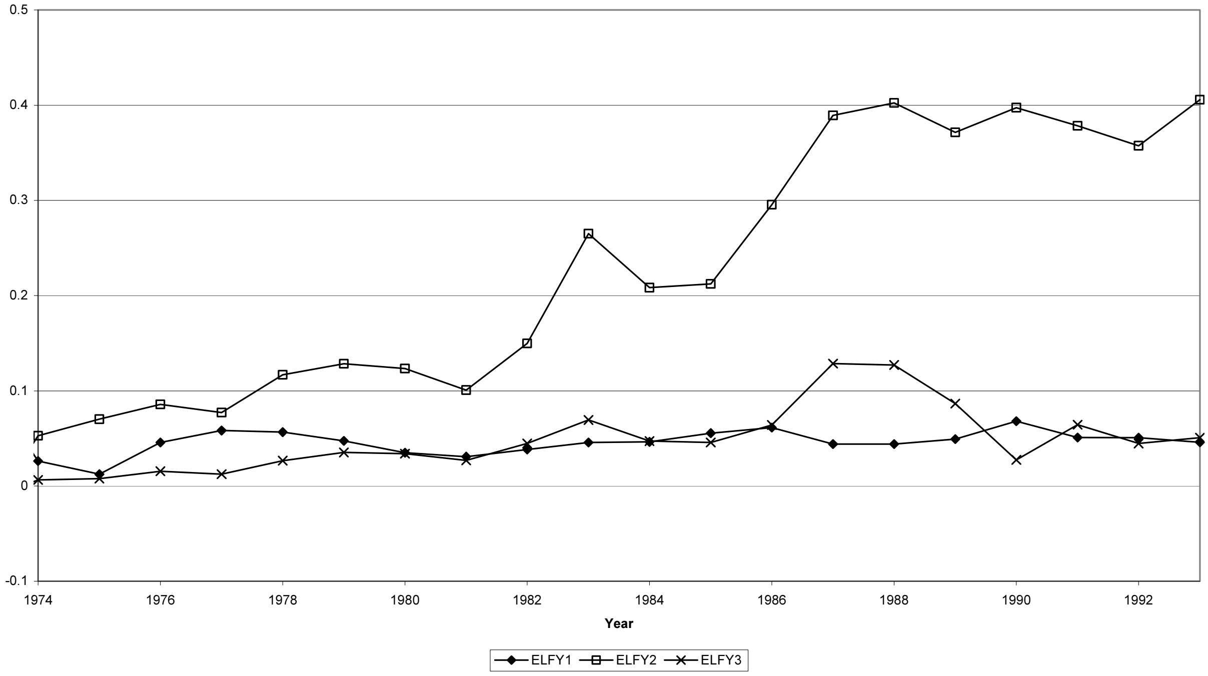
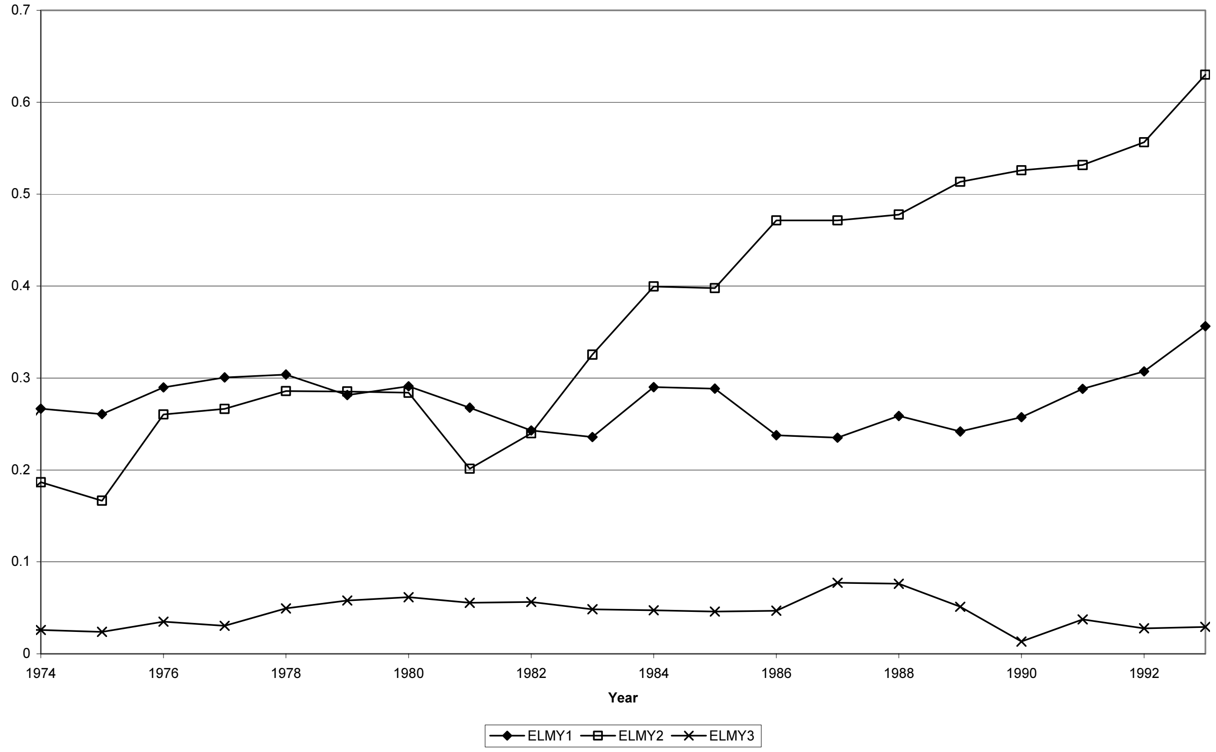


Figure A9 Product specific output elasticities w. r. t. other intermediate materials: Primary aluminium (ELLY1), aluminium alloy (ELLY2), aluminium products (ELLY3)



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