Discussion Papers No. 420, May 2005
Statistics Norway, Research Department

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Constructing Panel Data Estimators by Aggregation: A General Moment Estimator and a Suggested Synthesis

Abstract:
A regression equation for panel data with two-way random or fixed effects and a set of individual specific and period specific `within individual' and `within period', estimators of its slope coefficients are considered. They can be given Ordinary Least Squares (OLS) or Instrumental Variables (IV) interpretations. A class of estimators, obtained as an arbitrary linear combination of these `disaggregate' estimators, is defined and an expression for its variance-covariance matrix is derived. Nine familiar `aggregate' estimators which utilize the entire data set, including two between, three within, three GLS, as well as the standard OLS, emerge by specific choices of the weights. Other estimators in this class which are more robust to simultaneity and measurement error bias than the standard aggregate estimators and more efficient than the `disaggregate' estimators, are also considered. An empirical illustration of robustness and efficiency, relating to manufacturing productivity, is given.

Keywords: Panel data. Aggregation. Simultaneity. Measurement error. Method of moments. Factor productivity

JEL classification: C13, C23, C43.

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1 Introduction

A primary reason for the substantial growth in the availability and use of panel data in econometrics during the last thirty years is the opportunity that such data give for identifying and controlling for *unobserved heterogeneity* which may affect the estimation of slope coefficients and other parameters of interest from cross-section data, time-series data, or repeated (non-overlapping) cross-sections. It is well known [see *e.g.*, Baltagi (2001, chapters 2 and 3), Balestra (1996), and Mátyás (1996)] (i) that the potential nuisance created by *fixed (additive) individual* heterogeneity in OLS estimation can be eliminated by measuring all variables from their individual means or taking individual differences over time, (ii) that the potential nuisance created by *fixed (additive) time specific* heterogeneity in OLS estimation can be eliminated by measuring all variables from their time specific means or taking time specific differences over individuals, and (iii) that efficient estimation in the presence of suitably structured *random individual or time specific heterogeneity*, can be performed by (Feasible) Generalized Least Squares.

It is, however, possible to construct such aggregate estimators from disaggregate building-blocks. Approaching estimation in this way, is far from an algebraic exercise. It is illuminating primarily because we can utilize the fact that regression coefficients can be estimated consistently from parts of a panel data set in a large number of ways and that some disaggregate estimators are more robust to bias than others. We can, for instance apply all observations from one or two individuals or from one or two periods only. By combining an increasing number of individual specific or period specific estimators, we can include an increasing part of the observations until, at the limit, we utilize the full panel data set. Such an investigation is interesting on the one hand because several familiar estimators (within, between, generalized least squares etc.) for coefficients in panel data models can be interpreted as known linear combinations of elementary estimators, on the other hand because we get suggestions of other estimators along the way.

The paper is structured as follows. After describing the model and some ways of transforming it (Section 2), we first, in Section 3, define *'disaggregate'* 'within individual' and 'within period' estimators, each of which can be given either an OLS or an Instrumental Variables (IV) interpretation. In Section 4, a more general moment estimator, obtained by an arbitrary weighting of these elementary estimators as well as its variance-covariance matrix, is constructed. We next reconsider nine familiar estimators of a slope coefficient vector in a linear regression equation for panel data with two-way random or fixed effects, three of which are 'within (group)', two are 'between (group)' estimators related to individual or time variation, one is the standard OLS (Ordinary Least Squares) and three are Generalized Least Squares (GLS) estimators. We show that these *'aggregate'* estimators can all belong to this class and demonstrate that our general estimator contains not only the nine estimators mentioned above, but also several others which are more robust to violation of the standard assumptions in random coefficient models. In this process, some textbook results [Maddala (1977, section 14–2) and Hsiao (2003, section 2.2)] and some results in Biørn (1994, 1996) are generalized. Both a standard regression framework and situations with simultaneity (correlation between individual effects, period effects, and/or disturbances on the one hand and the regressor vector on the other) and situations with random measurement errors in the regressor vector are considered. Among the latter estimators we select estimators which are more robust to simultaneity and measurement errors and more efficient than the 'disaggregate' estimators. Finally, an empirical illustration of robustness and efficiency loss, relating to manufacturing productivity, is given.

2 Model, notation, and transformations

Consider a linear regression equation relating y to a vector of K (stochastic) regressors $\boldsymbol{x}$, with data set from a panel of N (≥ 2) individuals observed in T (≥ 2) periods:

$$(2.1) \qquad y_{it} = k + \boldsymbol{x}_{it}\boldsymbol{\beta} + \epsilon_{it}, \quad \epsilon_{it} = \alpha_i + \gamma_t + u_{it}, \quad i = 1,\dots,N;\ t = 1,\dots,T,$$

where y_{it} and $\boldsymbol{x}_{it} = (x_{1it},\dots,x_{Kit})$ are the values of y and $\boldsymbol{x}$ for individual i in period t, $\boldsymbol{\beta} = (\beta_1,\dots,\beta_K)'$ is the coefficient vector, α_i and γ_t are random effects specific to individual i and period t, respectively, u_{it} is a genuine disturbance, and k is an intercept term. It is, however, possible to interpret α_i and γ_t as fixed effects, see Section 5. At the moment, we make the standard assumptions for two-way random effects models,

$$(2.2) \qquad u_{it} \sim \mathsf{IID}(0,\sigma^2), \quad \alpha_i \sim \mathsf{IID}(0,\sigma_\alpha^2), \quad \gamma_t \sim \mathsf{IID}(0,\sigma_\gamma^2), \quad i = 1,\dots,N;\ t = 1,\dots,T,$$

$$(2.3) \qquad u_{it},\ \alpha_i,\ \gamma_t,\ \boldsymbol{x}_{it} \quad \text{are independently distributed for all } i \text{ and } t,$$

which imply

$$(2.4) \qquad \mathsf{E}(\epsilon_{it}|\boldsymbol{X}) = 0, \qquad \mathsf{E}(\epsilon_{it}\epsilon_{js}|\boldsymbol{X}) = \delta_{ij}\sigma_\alpha^2 + \delta_{ts}\sigma_\gamma^2 + \delta_{ij}\delta_{ts}\sigma^2, \qquad \begin{array}{l} i,j = 1,\dots,N, \\ t,s = 1,\dots,T, \end{array}$$

where $\delta_{ij} = 1$ for $i = j$ and $= 0$ for $i \neq j$, and $\delta_{ts} = 1$ for $t = s$ and $= 0$ for $t \neq s$, and $\boldsymbol{X}$ is the $(NT \times K)$ matrix containing all the $\boldsymbol{x}_{it}$'s. Some of the assumptions in (2.2) and (2.3) will be relaxed later on.

The individual specific vectors and matrices, of dimension $(T \times 1)$ and $(T \times K)$, respectively, and the period specific vectors and matrices, of dimension $(N \times 1)$ and $(N \times K)$, respectively, are

$$\boldsymbol{y}_{i\cdot} = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{iT} \end{bmatrix}, \quad \boldsymbol{X}_{i\cdot} = \begin{bmatrix} \boldsymbol{x}_{i1} \\ \vdots \\ \boldsymbol{x}_{iT} \end{bmatrix}, \quad \boldsymbol{y}_{\cdot t} = \begin{bmatrix} y_{1t} \\ \vdots \\ y_{Nt} \end{bmatrix}, \quad \boldsymbol{X}_{\cdot t} = \begin{bmatrix} \boldsymbol{x}_{1t} \\ \vdots \\ \boldsymbol{x}_{Nt} \end{bmatrix},$$

which are stacked into

$$\boldsymbol{y}=\begin{bmatrix}\boldsymbol{y}_{1\cdot}\\ \vdots\\ \boldsymbol{y}_{N\cdot}\end{bmatrix},\quad \boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}_{1\cdot}\\ \vdots\\ \boldsymbol{X}_{N\cdot}\end{bmatrix},\quad \boldsymbol{y}_{*}=\begin{bmatrix}\boldsymbol{y}_{\cdot 1}\\ \vdots\\ \boldsymbol{y}_{\cdot T}\end{bmatrix},\quad \boldsymbol{X}_{*}=\begin{bmatrix}\boldsymbol{X}_{\cdot 1}\\ \vdots\\ \boldsymbol{X}_{\cdot T}\end{bmatrix}.$$

Further, let $\boldsymbol{e}_H$ be the $(H\times 1)$ vector of ones, $\boldsymbol{I}_H$ the H-dimensional identity matrix, $\boldsymbol{A}_H=\boldsymbol{e}_H\boldsymbol{e}_H'/H$, $\boldsymbol{B}_H=\boldsymbol{I}_H-\boldsymbol{A}_H$ and let $\boldsymbol{\alpha}=(\alpha_1,\dots,\alpha_N)'$ and $\boldsymbol{\gamma}=(\gamma_1,\dots,\gamma_T)'$. Alternative ways of writing (2.1) are then

$$\text{(2.5)}\qquad \boldsymbol{y}_{i\cdot}=\boldsymbol{e}_Tk+\boldsymbol{X}_{i\cdot}\boldsymbol{\beta}+\boldsymbol{\epsilon}_{i\cdot},\quad \boldsymbol{\epsilon}_{i\cdot}=\boldsymbol{e}_T\alpha_i+\boldsymbol{\gamma}+\boldsymbol{u}_{i\cdot},\quad i=1,\dots,N,$$

$$\text{(2.6)}\qquad \boldsymbol{y}_{\cdot t}=\boldsymbol{e}_Nk+\boldsymbol{X}_{\cdot t}\boldsymbol{\beta}+\boldsymbol{\epsilon}_{\cdot t},\quad \boldsymbol{\epsilon}_{\cdot t}=\boldsymbol{\alpha}+\boldsymbol{e}_N\gamma_t+\boldsymbol{u}_{\cdot t},\quad t=1,\dots,T,$$

where $\boldsymbol{\epsilon}_{i\cdot}$, $\boldsymbol{u}_{i\cdot}$, $\boldsymbol{\epsilon}_{\cdot t}$, $\boldsymbol{u}_{\cdot t}$ are defined in similar way as $\boldsymbol{y}_{i\cdot}$ and $\boldsymbol{y}_{\cdot t}$, and after deducting global means we obtain

$$\text{(2.7)}\qquad \boldsymbol{y}_{i\cdot}-\bar{\boldsymbol{y}}=(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})\boldsymbol{\beta}+\boldsymbol{\epsilon}_{i\cdot}-\bar{\boldsymbol{\epsilon}},\quad \boldsymbol{\epsilon}_{i\cdot}-\bar{\boldsymbol{\epsilon}}=\boldsymbol{e}_T(\alpha_i-\bar{\alpha})+\boldsymbol{B}_T\boldsymbol{\gamma}+\boldsymbol{u}_{i\cdot}-\bar{\boldsymbol{u}},$$

$$\text{(2.8)}\qquad \boldsymbol{y}_{\cdot t}-\bar{\boldsymbol{y}}_*=(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}}_*)\boldsymbol{\beta}+\boldsymbol{\epsilon}_{\cdot t}-\bar{\boldsymbol{\epsilon}}_*,\quad \boldsymbol{\epsilon}_{\cdot t}-\bar{\boldsymbol{\epsilon}}_*=\boldsymbol{B}_N\boldsymbol{\alpha}+\boldsymbol{e}_N(\gamma_t-\bar{\gamma})+\boldsymbol{u}_{\cdot t}-\bar{\boldsymbol{u}}_*,$$

where $\bar{\alpha}=(1/N)\sum_i\alpha_i$, $\bar{\gamma}=(1/T)\sum_t\gamma_t$, $\bar{\boldsymbol{X}}=(1/N)\sum_i\boldsymbol{X}_{i\cdot}$, $\bar{\boldsymbol{X}}_*=(1/T)\sum_t\boldsymbol{X}_{\cdot t}$, $\bar{\boldsymbol{y}}=(1/N)\sum_i\boldsymbol{y}_{i\cdot}$, $\bar{\boldsymbol{y}}_*=(1/T)\sum_t\boldsymbol{y}_{\cdot t}$, , etc. Premultiplying (2.5) by $\boldsymbol{B}_T$, (2.7) by $\boldsymbol{A}_T$, (2.6) by $\boldsymbol{B}_N$ and (2.8) by $\boldsymbol{A}_N$, give, respectively,

$$\text{(2.9)}\qquad \begin{aligned}\boldsymbol{B}_T\boldsymbol{y}_{i\cdot}&=\boldsymbol{B}_T\boldsymbol{X}_{i\cdot}\boldsymbol{\beta}+\boldsymbol{B}_T\boldsymbol{\epsilon}_{i\cdot},\\ \boldsymbol{A}_T(\boldsymbol{y}_{i\cdot}-\bar{\boldsymbol{y}})&=\boldsymbol{A}_T(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})\boldsymbol{\beta}+\boldsymbol{A}_T(\boldsymbol{\epsilon}_{i\cdot}-\bar{\boldsymbol{\epsilon}}),\end{aligned}$$

$$\text{(2.10)}\qquad \begin{aligned}\boldsymbol{B}_N\boldsymbol{y}_{\cdot t}&=\boldsymbol{B}_N\boldsymbol{X}_{\cdot t}\boldsymbol{\beta}+\boldsymbol{B}_N\boldsymbol{\epsilon}_{\cdot t},\\ \boldsymbol{A}_N(\boldsymbol{y}_{\cdot t}-\bar{\boldsymbol{y}}_*)&=\boldsymbol{A}_N(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}}_*)\boldsymbol{\beta}+\boldsymbol{A}_N(\boldsymbol{\epsilon}_{\cdot t}-\bar{\boldsymbol{\epsilon}}_*).\end{aligned}$$

We let $\boldsymbol{W}$, $\boldsymbol{V}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, with appropriate subscripts, symbolize matrices containing *within individual*, *within period*, *between individual*, and *between period* (co)variation, respectively. Define individual specific and period specific cross-product matrices as follows:

$$\text{(2.11)}\qquad \begin{aligned}\boldsymbol{W}_{XXij}&=\boldsymbol{X}_{i\cdot}'\boldsymbol{B}_T\boldsymbol{X}_{j\cdot}=\textstyle\sum_{t=1}^T(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jt}-\bar{\boldsymbol{x}}_{j\cdot}),\\ \boldsymbol{W}_{X\gamma i}&=\boldsymbol{X}_{i\cdot}'\boldsymbol{B}_T\boldsymbol{\gamma}=\textstyle\sum_{t=1}^T(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})'(\gamma_t-\bar{\gamma}),\end{aligned}\qquad i,j=1,\dots,N,$$

$$\text{(2.12)}\qquad \begin{aligned}\boldsymbol{V}_{XXts}&=\boldsymbol{X}_{\cdot t}'\boldsymbol{B}_N\boldsymbol{X}_{\cdot s}=\textstyle\sum_{i=1}^N(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})'(\boldsymbol{x}_{is}-\bar{\boldsymbol{x}}_{\cdot s}),\\ \boldsymbol{V}_{X\alpha t}&=\boldsymbol{X}_{\cdot t}'\boldsymbol{B}_N\boldsymbol{\alpha}=\textstyle\sum_{i=1}^N(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})'(\alpha_i-\bar{\alpha}),\end{aligned}\qquad t,s=1,\dots,T,$$

$$\text{(2.13)}\qquad \begin{aligned}\boldsymbol{B}_{XXii}&=(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})'\boldsymbol{A}_T(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})=T(\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}}),\\ \boldsymbol{B}_{X\alpha ii}&=(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})'\boldsymbol{e}_T(\alpha_i-\bar{\alpha})=T(\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}})'(\alpha_i-\bar{\alpha}),\end{aligned}\qquad i=1,\dots,N,$$

$$\text{(2.14)}\qquad \begin{aligned}\boldsymbol{C}_{XXtt}&=(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}}_*)'\boldsymbol{A}_N(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}}_*)=N(\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}}),\\ \boldsymbol{C}_{X\gamma tt}&=(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}}_*)'\boldsymbol{e}_N(\gamma_t-\bar{\gamma})=N(\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}})'(\gamma_t-\bar{\gamma}),\end{aligned}\qquad t=1,\dots,T,$$

etc., where $\bar{\boldsymbol{x}}_{i\cdot}=(\boldsymbol{e}_T'/T)\boldsymbol{X}_{i\cdot}$, $\bar{\boldsymbol{x}}_{\cdot t}=(\boldsymbol{e}_N'/N)\boldsymbol{X}_{\cdot t}$, $\bar{\boldsymbol{x}}=(\boldsymbol{e}_{NT}'/(NT))\boldsymbol{X}=(\boldsymbol{e}_{TN}'/(TN))\boldsymbol{X}_*$. These matrices have the following properties:

- $\boldsymbol{W}_{XXij}$, which has full rank K if $\boldsymbol{x}_{it}$ contains no individual specific variables, is the $(K \times K)$ matrix of *within individual covariation* in the x's of individuals i and j, and $\boldsymbol{V}_{XXts}$, which has full rank K if $\boldsymbol{x}_{it}$ contains no period specific variables, is the $(K \times K)$ matrix of *within period covariation* in the x's of periods t and s. If individual specific regressors occur, $\boldsymbol{W}_{XXij}$ has one zero column (and row) for each such variable, and if period specific regressors occur, $\boldsymbol{V}_{XXts}$ has one zero column (and row) for each such variable.
- $\boldsymbol{B}_{XXii}$ and $\boldsymbol{C}_{XXtt}$, which have rank 1, are the $(K \times K)$ matrices of *between individual cross-products* and *between period cross-products of* the x's of individual i and period t, respectively.
- $\boldsymbol{W}_{X\gamma i}$ is the $(K \times 1)$ vector of *within covariation* of the x's of individual i and the period specific effects, $\boldsymbol{V}_{X\alpha t}$ is the $(K \times 1)$ vector of *within covariation* of the x's of period t and the individual specific effects, $\boldsymbol{B}_{X\alpha i}$ is the $(K \times 1)$ vector of *between cross-products* of the x's of individual i and its individual specific effects, and $\boldsymbol{C}_{X\gamma t}$ is the $(K \times 1)$ vector of *between cross-products* of the x's of period t and its period specific effects.

Premultiplying the two equations in (2.9) by $\boldsymbol{X}'_{i\cdot}\boldsymbol{B}_T$ and $(\boldsymbol{X}_{i\cdot} - \bar{\boldsymbol{X}})'\boldsymbol{A}_T$, respectively, and premultiplying the two equations in (2.10) by $\boldsymbol{X}'_{\cdot t}\boldsymbol{B}_N$ and $(\boldsymbol{X}_{\cdot t} - \bar{\boldsymbol{X}}_*)'\boldsymbol{A}_N$, respectively, while using (2.11)–(2.14), we get

$$\boldsymbol{W}_{XYij} = \boldsymbol{W}_{XXij}\boldsymbol{\beta} + \boldsymbol{W}_{X\epsilon ij}, \qquad \boldsymbol{W}_{X\epsilon ij} = \boldsymbol{W}_{X\gamma i} + \boldsymbol{W}_{XUij}, \qquad i,j = 1,\dots,N, \tag{2.15}$$

$$\boldsymbol{B}_{XYii} = \boldsymbol{B}_{XXii}\boldsymbol{\beta} + \boldsymbol{B}_{X\epsilon ii}, \qquad \boldsymbol{B}_{X\epsilon ii} = \boldsymbol{B}_{X\alpha ii} + \boldsymbol{B}_{XUii}, \quad i = 1,\dots,N, \tag{2.16}$$

$$\boldsymbol{V}_{XYts} = \boldsymbol{V}_{XXts}\boldsymbol{\beta} + \boldsymbol{V}_{X\epsilon ts}, \qquad \boldsymbol{V}_{X\epsilon ts} = \boldsymbol{V}_{X\alpha t} + \boldsymbol{V}_{XUts}, \qquad t,s = 1,\dots,T, \tag{2.17}$$

$$\boldsymbol{C}_{XYtt} = \boldsymbol{C}_{XXtt}\boldsymbol{\beta} + \boldsymbol{C}_{X\epsilon tt}, \qquad \boldsymbol{C}_{X\epsilon tt} = \boldsymbol{C}_{X\gamma tt} + \boldsymbol{C}_{XUtt}, \qquad t = 1,\dots,T. \tag{2.18}$$

These can be considered 'moment versions' of Eq. (2.1),

3 Base estimators and their properties

The fact that $\boldsymbol{W}_{X\epsilon ij}$ and have zero expectations when (2.2) and (2.3) are satisfied, in combination with (2.15) and (2.17) motivate the following N^2 individual specific and T^2 period specific estimators of $\boldsymbol{\beta}$:

$$\widehat{\boldsymbol{\beta}}_{Wij} = \boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{XYij} = (\boldsymbol{X}'_{i\cdot}\boldsymbol{B}_T\boldsymbol{X}_{j\cdot})^{-1}(\boldsymbol{X}'_{i\cdot}\boldsymbol{B}_T\boldsymbol{y}_{j\cdot}), \qquad i,j = 1,\dots,N, \tag{3.1}$$

$$\widehat{\boldsymbol{\beta}}_{Vts} = \boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{XYts} = (\boldsymbol{X}'_{\cdot t}\boldsymbol{B}_N\boldsymbol{X}_{\cdot s})^{-1}(\boldsymbol{X}'_{\cdot t}\boldsymbol{B}_N\boldsymbol{y}_{\cdot s}), \qquad t,s = 1,\dots,T. \tag{3.2}$$

We denote them as *base estimators*, or *disaggregate estimators*, of $\boldsymbol{\beta}$. They can be given the following interpretations:

(i) $\widehat{\boldsymbol{\beta}}_{Wii}$ *is the OLS estimator based on observations from individual* i*, and* $\widehat{\boldsymbol{\beta}}_{Wij}$*, for* $j \neq i$*, is the IV estimator based on the 'within variation' of individual* j*,* $\boldsymbol{B}_T\boldsymbol{X}_{j\cdot}$*, using the 'within variation' of individual* i *in* $\boldsymbol{X}_{i\cdot}$*,* $\boldsymbol{B}_T\boldsymbol{X}_{i\cdot}$*, as IV matrix.*

(ii) $\widehat{\boldsymbol{\beta}}_{Vtt}$ *is the OLS estimator based on observations from period* t*, and* $\widehat{\boldsymbol{\beta}}_{Vts}$*, for* $s \neq t$*, is the IV estimator based on the 'within variation' of period* s*,* $\boldsymbol{B}_N\boldsymbol{X}_{\cdot s}$*, using the 'within variation' of period* t *in* $\boldsymbol{X}_{\cdot t}$*,* $\boldsymbol{B}_N\boldsymbol{X}_{\cdot t}$*, as IV matrix.*

All these N^2+T^2 estimators exist if all elements of $\boldsymbol{x}_{it}$ vary across individuals and periods, since this usually ensures that $\boldsymbol{W}_{XXij}$ and $\boldsymbol{V}_{XXts}$ have rank K.

If individual specific variables occur, so that $\boldsymbol{W}_{XXij}$ contains one or more zero rows and columns, their coefficients cannot be estimated from (3.1), but estimators for the coefficients of the other, *i.e.*, the two-dimensional or period specific variables, can be solved from $\boldsymbol{W}_{XXij}\widehat{\boldsymbol{\beta}}_{Wij} = \boldsymbol{W}_{XYij}$. Likewise, *if period specific variables occur*, so that $\boldsymbol{V}_{XXts}$ contains one or more zero rows and columns, their coefficients cannot be estimated from (3.2), but estimators for the coefficients of the other, *i.e.*, the two-dimensional or individual specific variables, can be solved from $\boldsymbol{V}_{XXts}\widehat{\boldsymbol{\beta}}_{Vts} = \boldsymbol{V}_{XYts}$.

Since inserting for $\boldsymbol{W}_{XYij}$ from (2.15) and for $\boldsymbol{V}_{XYts}$ from (2.17) in (3.1) and (3.2) gives, respectively,

$$\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta} = \boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{X\epsilon ij} = \boldsymbol{W}_{XXij}^{-1}(\boldsymbol{W}_{X\gamma i} + \boldsymbol{W}_{XUij}), \quad i,j = 1,\dots,N, \tag{3.3}$$

$$\widehat{\boldsymbol{\beta}}_{Vts} - \boldsymbol{\beta} = \boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{X\epsilon ts} = \boldsymbol{V}_{XXts}^{-1}(\boldsymbol{V}_{X\alpha t} + \boldsymbol{V}_{XUts}), \quad t,s = 1,\dots,T, \tag{3.4}$$

and (2.2) and (2.3) imply

$$\mathsf{E}(\boldsymbol{W}_{XUij}|\boldsymbol{X}) = \mathsf{E}(\boldsymbol{W}_{X\gamma i}|\boldsymbol{X}) = \mathbf{0}_{K1}, \quad i,j = 1,\dots,N, \tag{3.5}$$

$$\mathsf{E}(\boldsymbol{V}_{XUts}|\boldsymbol{X}) = \mathsf{E}(\boldsymbol{V}_{X\alpha t}|\boldsymbol{X}) = \mathbf{0}_{K1}, \quad t,s = 1,\dots,T, \tag{3.6}$$

we know that $\widehat{\boldsymbol{\beta}}_{Wij}$ and $\widehat{\boldsymbol{\beta}}_{Vts}$ are unbiased estimators for $\boldsymbol{\beta}$. Furthermore, $\widehat{\boldsymbol{\beta}}_{Wij}$ is consistent when $T \to \infty$ (*T-consistent*, for short), since then $\text{plim}(\boldsymbol{W}_{X\epsilon ij}/T) = \mathbf{0}_{K1}$, provided that $\text{plim}(\boldsymbol{W}_{XXij}/T)$ is non-singular, and $\widehat{\boldsymbol{\beta}}_{Vts}$ is consistent when $N \to \infty$ (*N-consistent*, for short), since then $\text{plim}(\boldsymbol{V}_{X\epsilon ts}/N) = \mathbf{0}_{K1}$, provided that $\text{plim}(\boldsymbol{V}_{XXts}/N)$ is non-singular.

However, some of the base estimators may be consistent even if conditions (2.2)–(2.3) are weakened. The following *robustness results* hold:

[1]*Since (3.3) does not contain* $\boldsymbol{\alpha}$*, all* $\widehat{\boldsymbol{\beta}}_{Wij}$ *are* T*-consistent even if* α_i *is treated as fixed or allowed to be correlated with* $\bar{\boldsymbol{x}}_{i\cdot}$*, but if* γ_t *is correlated with* $\bar{\boldsymbol{x}}_{\cdot t}$*, all* $\widehat{\boldsymbol{\beta}}_{Wij}$ *are inconsistent. Symmetrically, since (3.4) does not contain* $\boldsymbol{\gamma}$*, all* $\widehat{\boldsymbol{\beta}}_{Vts}$ *are* N*-consistent even if* γ_t *is treated as fixed or allowed to be correlated with* $\bar{\boldsymbol{x}}_{\cdot t}$*, but if* α_i *is correlated with* $\bar{\boldsymbol{x}}_{i\cdot}$*, all* $\widehat{\boldsymbol{\beta}}_{Vts}$ *are inconsistent.*

[2]*Endogeneity of or random measurement error in (some components of)* $\boldsymbol{x}_{it}$ *may cause* $\mathsf{E}(\boldsymbol{x}_{it}'u_{it}) \neq \mathbf{0}_{K1}$*. Then* $\operatorname{plim}(\boldsymbol{W}_{XUii}/T) \neq \mathbf{0}_{K1}$*, so that the OLS estimators* $\widehat{\boldsymbol{\beta}}_{Wii}$ *are inconsistent, but the IV estimators* $\widehat{\boldsymbol{\beta}}_{Wij}$ $(j \neq i)$ *remain* T*-consistent. Symmetrically, we then also have* $\operatorname{plim}(\boldsymbol{V}_{XUtt}/N) \neq \mathbf{0}_{K1}$*, so that the OLS estimators* $\widehat{\boldsymbol{\beta}}_{Vtt}$ *are inconsistent, but the IV estimators* $\widehat{\boldsymbol{\beta}}_{Vts}$ $(s \neq t)$ *remain* N*-consistent.*

In Appendix A it is shown that when (2.2)–(2.3) hold, the matrices of covariances between the individual specific and the period specific base estimators, respectively, can be expressed as

$$\begin{aligned}
(3.7)\qquad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Wij}, \widehat{\boldsymbol{\beta}}_{Wkl}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Wkl} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\gamma^2 + \delta_{jl}\sigma^2)\boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{XXik}\boldsymbol{W}_{XXlk}^{-1},
\end{aligned}$$

$$\begin{aligned}
(3.8)\qquad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Vts}, \widehat{\boldsymbol{\beta}}_{Vpq}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Vts} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Vpq} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\alpha^2 + \delta_{sq}\sigma^2)\boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{XXtp}\boldsymbol{V}_{XXqp}^{-1},
\end{aligned}$$

$$\begin{aligned}
(3.9)\qquad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Wij}, \widehat{\boldsymbol{\beta}}_{Vpq}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Vpq} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= \sigma^2\boldsymbol{W}_{XXij}^{-1}(\boldsymbol{x}_{iq} - \bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jp} - \bar{\boldsymbol{x}}_{\cdot p})\boldsymbol{V}_{XXqp}^{-1}, \quad i,j,k,l = 1,\dots,N, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t,s,p,q = 1,\dots,T.
\end{aligned}$$

Eq. (3.7) for $(k,l) = (i,j)$ and (3.8) for $(p,q) = (t,s)$ give in particular the variance-covariance matrices

$$\begin{aligned}
(3.10)\qquad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Wij}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\gamma^2 + \sigma^2)\boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{XXii}\boldsymbol{W}_{XXji}^{-1}, \quad i,j = 1,\dots,N,
\end{aligned}$$

$$\begin{aligned}
(3.11)\qquad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Vts}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Vts} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Vts} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\alpha^2 + \sigma^2)\boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{XXtt}\boldsymbol{V}_{XXst}^{-1}, \quad t,s = 1,\dots,T.
\end{aligned}$$

When (2.2)–(2.3) hold, $\widehat{\boldsymbol{\beta}}_{Wjj}$ and $\widehat{\boldsymbol{\beta}}_{Vss}$ are always more efficient than $\widehat{\boldsymbol{\beta}}_{Wij}$ $(j \neq i)$ and $\widehat{\boldsymbol{\beta}}_{Vts}$ $(s \neq t)$, respectively, *i.e.*, $\mathsf{V}(\widehat{\boldsymbol{\beta}}_{Wij}|\boldsymbol{X}) - \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Wjj}|\boldsymbol{X})$ for $i \neq j$ and $\mathsf{V}(\widehat{\boldsymbol{\beta}}_{Vts}|\boldsymbol{X}) - \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Vss}|\boldsymbol{X})$ for $t \neq s$ are positive (semi)definite matrices. The formal proof of this is

$$\begin{aligned}
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{Wij}|\boldsymbol{X}) - \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Wjj}|\boldsymbol{X}) &= (\sigma_\gamma^2 + \sigma^2)(\boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{XXii}\boldsymbol{W}_{XXji}^{-1} - \boldsymbol{W}_{XXjj}^{-1}) \\
&= (\sigma_\gamma^2 + \sigma^2)(\boldsymbol{A}_{WXij}^{-1}\boldsymbol{A}_{WXji}^{-1} - \boldsymbol{I}_K)\boldsymbol{W}_{XXjj}^{-1}, \\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{Vts}|\boldsymbol{X}) - \mathsf{V}(\widehat{\boldsymbol{\beta}}_{Vss}|\boldsymbol{X}) &= (\sigma_\alpha^2 + \sigma^2)(\boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{XXtt}\boldsymbol{V}_{XXst}^{-1} - \boldsymbol{V}_{XXss}^{-1}) \\
&= (\sigma_\alpha^2 + \sigma^2)(\boldsymbol{A}_{VXts}^{-1}\boldsymbol{A}_{VXst}^{-1} - \boldsymbol{I}_K)\boldsymbol{V}_{XXss}^{-1},
\end{aligned}$$

where

$$\boldsymbol{A}_{WXij} = \boldsymbol{W}_{XXii}^{-1}\boldsymbol{W}_{XXij}, \qquad \boldsymbol{A}_{VXts} = \boldsymbol{V}_{XXtt}^{-1}\boldsymbol{V}_{XXts}.$$

The latter are the $(K \times K)$ matrix of (sample) regression coefficients when regressing the block of $\boldsymbol{X}$ relating to j, *i.e.*, $\boldsymbol{X}_{j\cdot}$, on the block of $\boldsymbol{X}$ relating to individual i, *i.e.*, $\boldsymbol{X}_{i\cdot}$, and when regressing the block of $\boldsymbol{X}$ relating to period s, *i.e.*, $\boldsymbol{X}_{\cdot s}$, on the block of $\boldsymbol{X}$ relating to period t, *i.e.*, $\boldsymbol{X}_{\cdot t}$, respectively. Here all $(\boldsymbol{A}_{WXij}^{-1}\boldsymbol{A}_{WXji}^{-1} - \boldsymbol{I}_K)$, $j \neq i$, and all $(\boldsymbol{A}_{VXts}^{-1}\boldsymbol{A}_{VXst}^{-1} - \boldsymbol{I}_K)$, $s \neq t$, are positive (semi)definite matrices, provided that all variables in $\boldsymbol{x}_{it}$ are two-dimensional.

The structure of the variance-covariance matrices of the estimators is transparent in the *one-regressor* case, $K = 1$. Then (3.7) and (3.10) read

$$(3.12)\quad \mathsf{C}(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkl}|\boldsymbol{X}) = (\sigma_\gamma^2 + \delta_{jl}\sigma^2)\frac{W_{XXik}}{W_{XXij}W_{XXkl}}, \quad \mathsf{V}(\widehat{\beta}_{Wij}|\boldsymbol{X}) = (\sigma_\gamma^2 + \sigma^2)\frac{W_{XXii}}{W_{XXij}^2},$$

where W_{XXik}, $\widehat{\beta}_{Wij}$, etc. denote the scalar counterparts to $\boldsymbol{W}_{XXik}$, $\widehat{\boldsymbol{\beta}}_{Wij}$, etc. The coefficient of correlation between two arbitrary individual specific base estimators for the slope coefficient can therefore be written as

$$(3.13)\qquad \begin{aligned} \rho(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkl}|\boldsymbol{X}) &= \frac{\mathsf{C}(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkl}|\boldsymbol{X})}{[\mathsf{V}(\widehat{\beta}_{Wij}|\boldsymbol{X})\mathsf{V}(\widehat{\beta}_{Wkl}|\boldsymbol{X})]^{1/2}} \\ &= \frac{\sigma_\gamma^2 + \delta_{jl}\sigma^2}{\sigma_\gamma^2 + \sigma^2}\frac{W_{XXik}}{(W_{XXii}W_{XXkk})^{1/2}} \quad = \quad \rho(\epsilon_{jt}, \epsilon_{lt})R_{WXik}, \end{aligned}$$

where $R_{WXik} = W_{XXik}/(W_{XXii}W_{XXkk})^{1/2}$ is the sample coefficient of correlation between the x's of individuals i and k and $\rho(\epsilon_{jt}, \epsilon_{lt}) = (\sigma_\gamma^2 + \delta_{jl}\sigma^2)/(\sigma_\gamma^2 + \sigma^2)$ is the coefficient of correlation between ϵ_{jt} and ϵ_{lt}. If we therefore consider (2.5) *as an N-equation model* with one equation for each individual and with common slope coefficient, $\rho(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkl}|\boldsymbol{X})$ is simply the product of the coefficient of correlation between two ϵ disturbances from individuals (equations) j and l in the same period, and the coefficient of correlation between the values of the regressor (instrument) for individuals (equations) i and k. This means that $\rho(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkl}|\boldsymbol{X})$ has one equation specific component (j vs. l) and one instrument specific component (i vs. k). For $j = l$ and for $i = k$ (3.13) gives, respectively,

$$\begin{aligned} &\rho(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wkj}|\boldsymbol{X}) = R_{WXik}, && \text{for all } j;\ i \neq k && \text{[same equation (individual), different IV]}, \\ &\rho(\widehat{\beta}_{Wij}, \widehat{\beta}_{Wil}|\boldsymbol{X}) = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma^2}, && \text{for all } i;\ j \neq l && \text{[different equations (individuals), same IV]}. \end{aligned}$$

Symmetrically, (3.8) and (3.11) for $K = 1$ give

$$(3.14)\quad \mathsf{C}(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vpq}|\boldsymbol{X}) = (\sigma_\alpha^2 + \delta_{sq}\sigma^2)\frac{V_{XXtp}}{V_{XXts}V_{XXpq}}, \quad \mathsf{V}(\widehat{\beta}_{Vts}|\boldsymbol{X}) = (\sigma_\alpha^2 + \sigma^2)\frac{V_{XXtt}}{V_{XXts}^2}.$$

The coefficients of correlation can therefore be written as

$$(3.15)\qquad \begin{aligned} \rho(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vpq}|\boldsymbol{X}) &= \frac{\mathsf{C}(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vpq}|\boldsymbol{X})}{[\mathsf{V}(\widehat{\beta}_{Vts}|\boldsymbol{X})\mathsf{V}(\widehat{\beta}_{Vpq}|\boldsymbol{X})]^{1/2}} \\ &= \frac{\sigma_\alpha^2 + \delta_{sq}\sigma^2}{\sigma_\alpha^2 + \sigma^2}\frac{V_{XXtp}}{(V_{XXtt}V_{XXpp})^{1/2}} \quad = \quad \rho(\epsilon_{is}, \epsilon_{iq})R_{VXtp}, \end{aligned}$$

where $R_{VXtp} = V_{XXtp}/(V_{XXtt}V_{XXpp})^{1/2}$ is the coefficient of correlation between the x's in periods t and p and $\rho(\epsilon_{is}, \epsilon_{iq}) = (\sigma_\alpha^2 + \delta_{sq}\sigma^2)/(\sigma_\alpha^2 + \sigma^2)$ is the coefficient of correlation between ϵ_{is} and ϵ_{iq}. If we therefore consider (2.6) as a T-*equation model* with one equation for each period and with common slope coefficient, $\rho(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vpq}|\boldsymbol{X})$ is simply the product of the coefficient of correlation between two ϵ disturbances from periods (equations) s and q for the same individual, and the coefficient of correlation between the values of the regressor (instrument) in periods t and p. This means that $\rho(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vpq}|\boldsymbol{X})$ has one equation specific component (s vs. q) and one instrument specific component (t vs. p). For $s = q$ and $t = p$ (3.15) gives, respectively,

$$\begin{aligned}
&\rho(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vps}|\boldsymbol{X}) = R_{VXtp}, && \text{for all } s;\ t \neq p \quad \text{[same equation (period), different IV]},\\
&\rho(\widehat{\beta}_{Vts}, \widehat{\beta}_{Vtq}|\boldsymbol{X}) = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2}, && \text{for all } t;\ s \neq q \quad \text{[different equations (periods), same IV]}.
\end{aligned}$$

From (3.12) and (3.14) we find that the *inefficiency* when using the (within) variation of individual i as IV for the (within) variation of individual j relative to performing OLS on the observations from individual j and when using the (within) variation of period t as IV for the (within) variation of period s relative to performing OLS on the observations from period s, can be expressed simply as, respectively,

$$\frac{\mathsf{V}(\widehat{\beta}_{Wij}|\boldsymbol{X})}{\mathsf{V}(\widehat{\beta}_{Wjj}|\boldsymbol{X})} = \frac{1}{A_{WXij}A_{WXji}} = \frac{1}{R^2_{WXij}}, \tag{3.16}$$

$$\frac{\mathsf{V}(\widehat{\beta}_{Vts}|\boldsymbol{X})}{\mathsf{V}(\widehat{\beta}_{Vss}|\boldsymbol{X})} = \frac{1}{A_{VXts}A_{VXst}} = \frac{1}{R^2_{VXts}}. \tag{3.17}$$

Hence, R^{-2}_{WXij} (≥ 1) and R^{-2}_{VXts} (≥ 1) measure, respectively, the loss of efficiency when using estimators which are robust to inconsistency caused by simultaneity or random measurement error in the regressor, (i) by estimating a relationship for individual j by using as IV observations from another individual, i, rather than using OLS, and (ii) by estimating a relationship for period s by using as IV observations from another period, t, rather than using OLS.

4 A class of moment estimators

Since each of the $N^2 + T^2$ base estimators of $\boldsymbol{\beta}$, $\widehat{\boldsymbol{\beta}}_{Wij}$ and $\widehat{\boldsymbol{\beta}}_{Vts}$, only uses a minor part of the panel data set, they may not be considered real competitors to estimators constructed from the complete data set, when (2.2)–(2.3) are valid. And even if these assumptions are violated by correlation between $\boldsymbol{x}_{it}$ and u_{it}, between $\bar{\boldsymbol{x}}_{i\cdot}$ and α_i, and/or between $\bar{\boldsymbol{x}}_{\cdot t}$ and γ_t, aggregate estimators which are more efficient than any of the IV estimators $\widehat{\boldsymbol{\beta}}_{Wij}$ $(j \neq i)$ and $\widehat{\boldsymbol{\beta}}_{Vts}$ $(s \neq t)$ may exist. Yet, the insight provided by examining these base estimators as we have done in th previous section is useful when constructing composite estimators of $\boldsymbol{\beta}$, of which they can serve as building-blocks.

This motivates the construction of *a class of estimators of* $\boldsymbol{\beta}$ by weighting the individual specific or period specific (co)variation in $\boldsymbol{X}$ and $\boldsymbol{y}$, as defined in (2.11)–(2.12), as follows: Let $\boldsymbol{\theta} = (\theta_{ts})$ be a $(T \times T)$ matrix and $\boldsymbol{\tau} = (\tau_{ij})$ an $(N \times N)$ matrix of (positive, zero or negative) weights and define a *general moment estimator* as

$$\begin{aligned}\boldsymbol{b} = \boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\tau}) = &\left(\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \theta_{ts} \boldsymbol{V}_{XXts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \boldsymbol{W}_{XXij}\right)^{-1} \\ &\times \left(\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \theta_{ts} \boldsymbol{V}_{XYts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \boldsymbol{W}_{XYij}\right).\end{aligned} \tag{4.1}$$

Using (3.1)–(3.2) it can be written as a weighted average of the base estimators:

$$\begin{aligned}\boldsymbol{b} = &\left(\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \theta_{ts} \boldsymbol{V}_{XXts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \boldsymbol{W}_{XXij}\right)^{-1} \\ &\times \left(\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \theta_{ts} \boldsymbol{V}_{XXts} \widehat{\boldsymbol{\beta}}_{Vts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \boldsymbol{W}_{XXij} \widehat{\boldsymbol{\beta}}_{Wij}\right),\end{aligned} \tag{4.2}$$

or, in simplified notation,

$$\boldsymbol{b} = \textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{G}_{Vts} \widehat{\boldsymbol{\beta}}_{Vts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{G}_{Wij} \widehat{\boldsymbol{\beta}}_{Wij}, \tag{4.3}$$

where $\boldsymbol{G}_{Vts}$ and $\boldsymbol{G}_{Wij}$ are $(K \times K)$ weighting matrices, $\sum_t \sum_s \boldsymbol{G}_{Vts} + \sum_i \sum_j \boldsymbol{G}_{Wij} = \boldsymbol{I}_K$, given by

$$\begin{aligned}&\boldsymbol{G}_{Vts} = \boldsymbol{Q}^{-1} \theta_{ts} \boldsymbol{V}_{XXts}, && t, s = 1, \dots, T, \\ &\boldsymbol{G}_{Wij} = \boldsymbol{Q}^{-1} \tau_{ij} \boldsymbol{W}_{XXij}, && i, j = 1, \dots, N, \\ &\boldsymbol{Q} = \boldsymbol{Q}(\boldsymbol{\theta}, \boldsymbol{\tau}) = \textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \theta_{ts} \boldsymbol{V}_{XXts} + \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \boldsymbol{W}_{XXij}. &&\end{aligned} \tag{4.4}$$

None of the latter matrices are symmetric in general. If, however, $\theta_{ts} = \theta_{st}$ for all t, s and $\tau_{ij} = \tau_{ji}$ for all i, j, then $\boldsymbol{Q}' = \boldsymbol{Q}$.

The estimator $\boldsymbol{b}$ is unbiased for any $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ when (2.2) and (2.3) hold, and in Appendix B it is shown that its variance-covariance matrix is (This formula in the special case where $K = 1$ and $\sigma_\gamma^2 = 0$ is derived in Biørn (1994, Appendix A).)

$$\mathsf{V}(\boldsymbol{b}|\boldsymbol{X}) = \boldsymbol{Q}^{-1} \boldsymbol{P} (\boldsymbol{Q}^{-1})' = \boldsymbol{Q}(\boldsymbol{\theta}, \boldsymbol{\tau})^{-1} \boldsymbol{P}(\boldsymbol{\theta}, \boldsymbol{\tau}, \sigma^2, \sigma_\alpha^2, \sigma_\gamma^2) (\boldsymbol{Q}(\boldsymbol{\theta}, \boldsymbol{\tau})^{-1})', \tag{4.5}$$

where

$$\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{\theta}, \boldsymbol{\tau}, \sigma^2, \sigma_\alpha^2, \sigma_\gamma^2) = \sigma^2 (\boldsymbol{S}_V + \boldsymbol{S}_W + \boldsymbol{S}_{VW}) + \sigma_\alpha^2 \boldsymbol{Z}_V + \sigma_\gamma^2 \boldsymbol{Z}_W, \tag{4.6}$$

$$
\begin{aligned}
&\boldsymbol{S}_V = \boldsymbol{S}_V(\boldsymbol{\theta}) = \sum_{t=1}^{T}\sum_{p=1}^{T} \boldsymbol{V}_{XXtp}\left(\sum_{s=1}^{T}\theta_{ts}\theta_{ps}\right),\\
&\boldsymbol{S}_W = \boldsymbol{S}_W(\boldsymbol{\tau}) = \sum_{i=1}^{N}\sum_{k=1}^{N} \boldsymbol{W}_{XXik}\left(\sum_{j=1}^{N}\tau_{ij}\tau_{kj}\right),\\
&\boldsymbol{S}_{VW} = \boldsymbol{S}_{VW}(\boldsymbol{\theta},\boldsymbol{\tau}) = \sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_{ts}\tau_{ij}(\boldsymbol{x}_{is}-\bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jt}-\bar{\boldsymbol{x}}_{\cdot t}),\\
&\boldsymbol{Z}_V = \boldsymbol{Z}_V(\boldsymbol{\theta}) = \sum_{t=1}^{T}\sum_{p=1}^{T} \boldsymbol{V}_{XXtp}\left(\sum_{s=1}^{T}\theta_{ts}\right)\left(\sum_{r=1}^{T}\theta_{pr}\right),\\
&\boldsymbol{Z}_W = \boldsymbol{Z}_W(\boldsymbol{\tau}) = \sum_{i=1}^{N}\sum_{k=1}^{N} \boldsymbol{W}_{XXik}\left(\sum_{j=1}^{N}\tau_{ij}\right)\left(\sum_{l=1}^{N}\tau_{kl}\right).
\end{aligned} \tag{4.7}
$$

It is easily seen that $\boldsymbol{S}_{WV} = \mathbf{0}$ if either $\theta_{ts} = \theta$ for all t, s or $\tau_{ij} = \tau$ for all i, j, that $\boldsymbol{Z}_V = \mathbf{0}$ if $\sum_{s=1}^{T}\theta_{ts} = 0$ for all t, and that $\boldsymbol{Z}_W = \mathbf{0}$ if $\sum_{j=1}^{T}\tau_{ij} = 0$ for all i. The standard estimators in fixed and random effects models have at least one of these properties, which will be shown in the next section.

By utilizing (4.5)–(4.7), we can estimate $\mathsf{V}(\boldsymbol{b}|\boldsymbol{X})$ consistently from a panel data set for any weighting matrices $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ we may choose when consistent estimators of the variances σ^2, σ_α^2, and σ_γ^2 have been obtained.

5 Specific aggregate estimators

In this section, we consider specific members of the class of estimators described by (4.1). Some of these are familiar, others less familiar.

Aggregate within and between estimators

The estimator $\boldsymbol{b}$ contains several familiar estimators for fixed effects models as particular members. We first establish the weighting system $(\boldsymbol{\theta}, \boldsymbol{\tau})$ for six such estimators and comment on other, less familiar estimators which are more robust to violation of the basic assumptions. The results below generalize those in Biørn (1994, section 3), where only one regressor is included ($K = 1$) and period specific effects are disregarded ($\gamma_t = 0$).

We define, in the usual way [see, *e.g.*, Greene (2003, section 13.3.2)], the $(K \times K)$-matrices of *overall* (aggregate) *within individual* and *within period*, (co)variation as

$$\boldsymbol{W}_{XX} = \textstyle\sum_{i=1}^{N}\boldsymbol{W}_{XXii} = \sum_{i=1}^{N}\sum_{t=1}^{T}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot}), \tag{5.1}$$

$$\boldsymbol{V}_{XX} = \textstyle\sum_{t=1}^{T}\boldsymbol{V}_{XXtt} = \sum_{t=1}^{T}\sum_{i=1}^{N}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})'(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t}), \tag{5.2}$$

etc. The corresponding *overall between individual*, and *between period* (co)variation are

$$\boldsymbol{B}_{XX} = \textstyle\sum_{i=1}^{N}\boldsymbol{B}_{XXii} = T\sum_{i=1}^{N}(\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}}) = (1/T)\sum_{t=1}^{T}\sum_{s=1}^{T}\boldsymbol{V}_{XXts}, \tag{5.3}$$

$$\boldsymbol{C}_{XX} = \textstyle\sum_{t=1}^{T}\boldsymbol{C}_{XXtt} = N\sum_{t=1}^{T}(\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}}) = (1/N)\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{W}_{XXij}, \tag{5.4}$$

etc., where the last equalities are shown in Appendix C. The matrix of *overall* (co)variation and its decomposition into within and between variation is

$$\boldsymbol{T}_{XX} = \textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} (\boldsymbol{x}_{it} - \bar{\boldsymbol{x}})' (\boldsymbol{x}_{it} - \bar{\boldsymbol{x}}) = \boldsymbol{W}_{XX} + \boldsymbol{B}_{XX} = \boldsymbol{V}_{XX} + \boldsymbol{C}_{XX}, \tag{5.5}$$

which after inserting from (5.1)–(5.4) becomes

$$\begin{aligned} \boldsymbol{T}_{XX} &= \textstyle\sum_{i=1}^{N} \boldsymbol{W}_{XXii} + (1/T) \sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XXts} \\ &= \textstyle\sum_{t=1}^{T} \boldsymbol{V}_{XXtt} + (1/N) \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XXij}. \end{aligned} \tag{5.6}$$

Finally, the matrix of *residual* (co)variation, *i.e.*, the (co)variation which remains when all (co)variation between individuals and between periods is eliminated (also denoted as the *combined within-individual-and-period* (co)variation) is

$$\boldsymbol{R}_{XX} = \textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} (\boldsymbol{x}_{it} - \bar{\boldsymbol{x}}_{i\cdot} - \bar{\boldsymbol{x}}_{\cdot t} + \bar{\boldsymbol{x}})' (\boldsymbol{x}_{it} - \bar{\boldsymbol{x}}_{i\cdot} - \bar{\boldsymbol{x}}_{\cdot t} + \bar{\boldsymbol{x}}) = \boldsymbol{T}_{XX} - \boldsymbol{B}_{XX} - \boldsymbol{C}_{XX}, \tag{5.7}$$

which after inserting from (5.3), (5.4), and (5.6) becomes

$$\begin{aligned} \boldsymbol{R}_{XX} &= \textstyle\sum_{i=1}^{N} \Big(\boldsymbol{W}_{XXii} - (1/N) \sum_{j=1}^{N} \boldsymbol{W}_{XXij} \Big) \\ &= \textstyle\sum_{t=1}^{T} \Big(\boldsymbol{V}_{XXtt} - (1/T) \sum_{s=1}^{T} \boldsymbol{V}_{XXts} \Big). \end{aligned} \tag{5.8}$$

We see from (5.6) and (5.8) that $\boldsymbol{T}_{XX}$ and $\boldsymbol{R}_{XX}$ can be expressed in terms of the $\boldsymbol{W}_{XXij}$'s and the $\boldsymbol{V}_{XXts}$ in two symmetric ways.

We can now, combining the decompositions exemplified in (5.1)–(5.4) with (3.1)–(3.2), express the familiar within individual, within period, between individual, and between period estimators of $\boldsymbol{\beta}$ as

$$\widehat{\boldsymbol{\beta}}_W = \boldsymbol{W}_{XX}^{-1} \boldsymbol{W}_{XY} = \Big(\textstyle\sum_{i=1}^{N} \boldsymbol{W}_{XXii} \Big)^{-1} \Big(\sum_{i=1}^{N} \boldsymbol{W}_{XXii} \widehat{\boldsymbol{\beta}}_{Wii} \Big), \tag{5.9}$$

$$\widehat{\boldsymbol{\beta}}_V = \boldsymbol{V}_{XX}^{-1} \boldsymbol{V}_{XY} = \Big(\textstyle\sum_{t=1}^{T} \boldsymbol{V}_{XXtt} \Big)^{-1} \Big(\sum_{t=1}^{T} \boldsymbol{V}_{XXtt} \widehat{\boldsymbol{\beta}}_{Vtt} \Big), \tag{5.10}$$

$$\widehat{\boldsymbol{\beta}}_B = \boldsymbol{B}_{XX}^{-1} \boldsymbol{B}_{XY} = \Big(\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XXts} \Big)^{-1} \Big(\sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XXts} \widehat{\boldsymbol{\beta}}_{Vts} \Big), \tag{5.11}$$

$$\widehat{\boldsymbol{\beta}}_C = \boldsymbol{C}_{XX}^{-1} \boldsymbol{C}_{XY} = \Big(\textstyle\sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XXij} \Big)^{-1} \Big(\sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XXij} \widehat{\boldsymbol{\beta}}_{Wij} \Big). \tag{5.12}$$

Familiar results from panel data textbooks are: (i) $\widehat{\boldsymbol{\beta}}_W$ and $\widehat{\boldsymbol{\beta}}_V$, *as expressed by the first equalities in (5.9) and (5.10)*, are the MVLUE (Minimum Variance Linear Unbiased Estimator) of $\boldsymbol{\beta}$ in the cases with only fixed individual specific and with only fixed period specific effects, respectively. (ii) $\widehat{\boldsymbol{\beta}}_B$ and $\widehat{\boldsymbol{\beta}}_C$, as expressed by the first equalities in (5.11) and (5.12), are obtained by running OLS on relations expressed in terms of individual specific and in terms of period specific means, respectively. On the other hand, the expressions after the last equality signs in (5.9)–(5.12), in particular the two last ones are non-standard and are more interesting from our point of view. We find that: (iii) $\widehat{\boldsymbol{\beta}}_W$ and $\widehat{\boldsymbol{\beta}}_C$ utilize the *(co)variation across periods* in the panel data set and disregard the (co)variation across individuals, while (iv) $\widehat{\boldsymbol{\beta}}_V$ and $\widehat{\boldsymbol{\beta}}_B$ utilize the *(co)variation across individuals* and disregard the (co)variation across periods. This demonstrates that among these four estimators, $\widehat{\boldsymbol{\beta}}_W$ and $\widehat{\boldsymbol{\beta}}_C$ are related to *time-series analysis* and $\widehat{\boldsymbol{\beta}}_V$ and $\widehat{\boldsymbol{\beta}}_B$ are related to *cross-section analysis.*

With this in mind, it is interesting to reconsider the two remaining familiar estimators in the panel data literature: the total (standard OLS) ($\boldsymbol{T}$) and the residual ($\boldsymbol{R}$) estimators. Both can be written in two symmetric ways, either as

$$
\begin{aligned}
(5.13)\qquad \widehat{\boldsymbol{\beta}}_T &= \boldsymbol{T}_{XX}^{-1}\boldsymbol{T}_{XY} = (\boldsymbol{B}_{XX}+\boldsymbol{C}_{XX}+\boldsymbol{R}_{XX})^{-1}(\boldsymbol{B}_{XY}+\boldsymbol{C}_{XY}+\boldsymbol{R}_{XY}) \\
&= \Big(\textstyle\sum_{i=1}^{N}\boldsymbol{W}_{XXii}+(1/T)\sum_{t=1}^{T}\sum_{s=1}^{T}\boldsymbol{V}_{XXts}\Big)^{-1} \\
&\qquad \times\ \Big(\textstyle\sum_{i=1}^{N}\boldsymbol{W}_{XXii}\widehat{\boldsymbol{\beta}}_{Wii}+(1/T)\sum_{t=1}^{T}\sum_{s=1}^{T}\boldsymbol{V}_{XXts}\widehat{\boldsymbol{\beta}}_{Vts}\Big), \\
(5.14)\qquad \widehat{\boldsymbol{\beta}}_R &= \boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY} \\
&= \Big[\textstyle\sum_{i=1}^{N}\Big(\boldsymbol{W}_{XXii}-(1/N)\sum_{j=1}^{N}\boldsymbol{W}_{XXij}\Big)\Big]^{-1} \\
&\qquad \times\ \Big[\textstyle\sum_{i=1}^{N}\Big(\boldsymbol{W}_{XXii}\widehat{\boldsymbol{\beta}}_{Wii}-(1/N)\sum_{j=1}^{N}\boldsymbol{W}_{XXij}\widehat{\boldsymbol{\beta}}_{Wij}\Big)\Big],
\end{aligned}
$$

or as

$$
\begin{aligned}
(5.15)\qquad \widehat{\boldsymbol{\beta}}_T &= \Big(\textstyle\sum_{t=1}^{T}\boldsymbol{V}_{XXtt}+(1/N)\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{W}_{XXij}\Big)^{-1} \\
&\qquad \times\ \Big(\textstyle\sum_{t=1}^{T}\boldsymbol{V}_{XXtt}\widehat{\boldsymbol{\beta}}_{Vtt}+(1/N)\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{W}_{XXij}\widehat{\boldsymbol{\beta}}_{Wij}\Big), \\
(5.16)\qquad \widehat{\boldsymbol{\beta}}_R &= \Big[\textstyle\sum_{t=1}^{T}\Big(\boldsymbol{V}_{XXtt}-(1/T)\sum_{s=1}^{T}\boldsymbol{V}_{XXts}\Big)\Big]^{-1} \\
&\qquad \times\ \Big[\textstyle\sum_{t=1}^{T}\Big(\boldsymbol{V}_{XXtt}\widehat{\boldsymbol{\beta}}_{Vtt}-(1/T)\sum_{s=1}^{T}\boldsymbol{V}_{XXts}\widehat{\boldsymbol{\beta}}_{Vts}\Big)\Big],
\end{aligned}
$$

which follow from (3.1)–(3.2) and the decompositions exemplified in (5.5)–(5.8). We know that $\widehat{\boldsymbol{\beta}}_T$ is the MVLUE of $\boldsymbol{\beta}$ in the absence of any individual or period specific heterogeneity, whereas $\widehat{\boldsymbol{\beta}}_R$ has the same property when all the α_i's and γ_t's are interpreted as unknown constants (both fixed individual and period specific effects). The last equalities in (5.13) and (5.15) show clearly that the standard OLS estimator utilizes (co)variation both across individuals and periods.

Briefly, (5.9)–(5.16) show that all the six familiar aggregate estimators for fixed effects models belong to the class (4.2) and can be interpreted as follows:

(i) The OVERALL WITHIN INDIVIDUAL *estimator* $\widehat{\boldsymbol{\beta}}_W$ *and the* OVERALL BETWEEN PERIOD *estimator* $\widehat{\boldsymbol{\beta}}_C$ *are both matrix weighted averages of the* BASE INDIVIDUAL SPECIFIC *estimators* $\widehat{\boldsymbol{\beta}}_{Wij}$*, the former utilizing only the* N *individual specific OLS estimators, the latter also the* $N(N-1)$ *individual specific IV estimators.*

(ii) *The* OVERALL WITHIN PERIOD *estimator* $\widehat{\boldsymbol{\beta}}_V$ *and the* OVERALL BETWEEN INDIVIDUAL *estimator* $\widehat{\boldsymbol{\beta}}_B$ *are both matrix weighted averages of the* BASE PERIOD SPECIFIC *estimators* $\widehat{\boldsymbol{\beta}}_{Vts}$*, the former utilizing only the* T *period specific OLS estimators, the latter also the* $T(T-1)$ *period specific IV estimators.*

(iii) *The* (OVERALL) RESIDUAL *estimator* $\widehat{\boldsymbol{\beta}}_R$ *can be interpreted as a matrix weighted average of either all the* N^2 *base individual specific estimators or all the* T^2 *base period specific estimators.*

(iv) *The* TOTAL OLS *estimator* $\widehat{\boldsymbol{\beta}}_T$ *can be interpreted as a matrix weighted average of either (a) all the* N *individual specific OLS estimators, all the* T *period specific OLS estimators, and all the* $T(T-1)$ *period specific within period IV estimators, or (b) all the* T *period specific OLS estimators, all the* N *individual specific OLS estimators, and all the* $N(N-1)$ *individual specific within individual IV estimators.*

The weights are summarized in Table 1, panel A. In compact notation, we have

$$
\begin{aligned}
\widehat{\boldsymbol{\beta}}_R &= \boldsymbol{b}(\boldsymbol{B}_T, \boldsymbol{0}_{NN}) = \boldsymbol{b}(\boldsymbol{0}_{TT}, \boldsymbol{B}_N),\\
\widehat{\boldsymbol{\beta}}_B &= \boldsymbol{b}(\boldsymbol{A}_T, \boldsymbol{0}_{NN}),\\
\widehat{\boldsymbol{\beta}}_C &= \boldsymbol{b}(\boldsymbol{0}_{TT}, \boldsymbol{A}_N),\\
\widehat{\boldsymbol{\beta}}_W &= \boldsymbol{b}(\boldsymbol{B}_T, \boldsymbol{A}_N) = \boldsymbol{b}(\boldsymbol{0}_{TT}, \boldsymbol{I}_N),\\
\widehat{\boldsymbol{\beta}}_V &= \boldsymbol{b}(\boldsymbol{A}_T, \boldsymbol{B}_N) = \boldsymbol{b}(\boldsymbol{I}_T, \boldsymbol{0}_{NN}),\\
\widehat{\boldsymbol{\beta}}_T &= \boldsymbol{b}(\boldsymbol{I}_T, \boldsymbol{A}_N) = \boldsymbol{b}(\boldsymbol{A}_T, \boldsymbol{I}_N).
\end{aligned}
$$

For the total, residual and both within estimators the weights are given in two versions. The weights corresponding to (5.9)–(5.16) are given in rows 6, 8, 1, 2, 10, 4, 9, and 3, respectively, the weights in row 5 follow from (5.3), (5.5), and (5.6), and the weights in row 7 follow from (5.4), (5.5), and (5.6). We can derive their variance-covariance matrices *when the random effects specification is valid* [cf. (2.2)] by inserting the value of the weights in Table 1, panel A, into (4.5)–(4.7), using (5.1)–(5.8). The results are summarized in panel B. Compactly,

$$
\begin{aligned}
\mathsf{V}(\widehat{\boldsymbol{\beta}}_R|\boldsymbol{X}) &= \sigma^2 \boldsymbol{R}_{XX}^{-1},\\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_B|\boldsymbol{X}) &= (\sigma^2 + T\sigma_\alpha^2)\boldsymbol{B}_{XX}^{-1},\\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_C|\boldsymbol{X}) &= (\sigma^2 + N\sigma_\gamma^2)\boldsymbol{C}_{XX}^{-1},\\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_W|\boldsymbol{X}) &= (\boldsymbol{R}_{XX} + \boldsymbol{C}_{XX})^{-1}[\sigma^2 \boldsymbol{R}_{XX} + (\sigma^2 + N\sigma_\gamma^2)\boldsymbol{C}_{XX}](\boldsymbol{R}_{XX} + \boldsymbol{C}_{XX})^{-1},\\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_V|\boldsymbol{X}) &= (\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX})^{-1}[\sigma^2 \boldsymbol{R}_{XX} + (\sigma^2 + T\sigma_\alpha^2)\boldsymbol{B}_{XX}](\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX})^{-1},\\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_T|\boldsymbol{X}) &= (\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX})^{-1}\\
&\quad \times [\sigma^2 \boldsymbol{R}_{XX} + (\sigma^2 + T\sigma_\alpha^2)\boldsymbol{B}_{XX} + (\sigma^2 + N\sigma_\gamma^2)\boldsymbol{C}_{XX}]\\
&\qquad \times (\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX})^{-1}.
\end{aligned}
$$

Table 1: The General Moment Estimator (4.1)

A: Weights θ_{ts} and τ_{ij} for standard between and within estimators

	θ_{tt}	$\theta_{ts},\ s\neq t$	τ_{ii}	$\tau_{ij},\ j\neq i$	$\boldsymbol{\theta}$	$\boldsymbol{\tau}$
$\widehat{\boldsymbol{\beta}}_B$	$\frac{1}{T}$	$\frac{1}{T}$	0	0	$\boldsymbol{A}_T$	$\boldsymbol{0}_{NN}$
$\widehat{\boldsymbol{\beta}}_C$	0	0	$\frac{1}{N}$	$\frac{1}{N}$	$\boldsymbol{0}_{TT}$	$\boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_R$	$1-\frac{1}{T}$	$-\frac{1}{T}$	0	0	$\boldsymbol{B}_T$	$\boldsymbol{0}_{NN}$
$\widehat{\boldsymbol{\beta}}_R$	0	0	$1-\frac{1}{N}$	$-\frac{1}{N}$	$\boldsymbol{0}_{TT}$	$\boldsymbol{B}_N$
$\widehat{\boldsymbol{\beta}}_W$	$1-\frac{1}{T}$	$-\frac{1}{T}$	$\frac{1}{N}$	$\frac{1}{N}$	$\boldsymbol{B}_T$	$\boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_W$	0	0	1	0	$\boldsymbol{0}_{TT}$	$\boldsymbol{I}_N$
$\widehat{\boldsymbol{\beta}}_V$	$\frac{1}{T}$	$\frac{1}{T}$	$1-\frac{1}{N}$	$-\frac{1}{N}$	$\boldsymbol{A}_T$	$\boldsymbol{B}_N$
$\widehat{\boldsymbol{\beta}}_V$	1	0	0	0	$\boldsymbol{I}_T$	$\boldsymbol{0}_{NN}$
$\widehat{\boldsymbol{\beta}}_T$	1	0	$\frac{1}{N}$	$\frac{1}{N}$	$\boldsymbol{I}_T$	$\boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_T$	$\frac{1}{T}$	$\frac{1}{T}$	1	0	$\boldsymbol{A}_T$	$\boldsymbol{I}_N$

B: Values of $\boldsymbol{S}_V+\boldsymbol{S}_W$, $\boldsymbol{Z}_V$, $\boldsymbol{Z}_W$, $\boldsymbol{Q}$, and $\boldsymbol{Z}_{VW}$ defining Covariance Matrices

	$\boldsymbol{S}_V+\boldsymbol{S}_W$	$\boldsymbol{Z}_V$	$\boldsymbol{Z}_W$	$\boldsymbol{Q}$	$\boldsymbol{Z}_{VW}$
$\widehat{\boldsymbol{\beta}}_B$	$\boldsymbol{B}_{XX}$	$T\boldsymbol{B}_{XX}$	$\boldsymbol{0}$	$\boldsymbol{B}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_C$	$\boldsymbol{C}_{XX}$	$\boldsymbol{0}$	$N\boldsymbol{C}_{XX}$	$\boldsymbol{C}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_R$	$\boldsymbol{R}_{XX}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{R}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_W$	$\boldsymbol{C}_{XX}+\boldsymbol{R}_{XX}$	$\boldsymbol{0}$	$N\boldsymbol{C}_{XX}$	$\boldsymbol{C}_{XX}+\boldsymbol{R}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_V$	$\boldsymbol{B}_{XX}+\boldsymbol{R}_{XX}$	$T\boldsymbol{B}_{XX}$	$\boldsymbol{0}$	$\boldsymbol{B}_{XX}+\boldsymbol{R}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_T$	$\boldsymbol{B}_{XX}+\boldsymbol{C}_{XX}+\boldsymbol{R}_{XX}$	$T\boldsymbol{B}_{XX}$	$N\boldsymbol{C}_{XX}$	$\boldsymbol{B}_{XX}+\boldsymbol{C}_{XX}+\boldsymbol{R}_{XX}$	$\boldsymbol{0}$

GLS estimators for random effects models

We next reconsider the GLS estimator of $\boldsymbol{\beta}$, which is the MVLUE in the two-way random effects model (2.1)–(2.3). Consider first the following subclass of the estimator $\boldsymbol{b}$:

$$
\begin{aligned}
(5.17)\qquad \widehat{\boldsymbol{\beta}} &= \widehat{\boldsymbol{\beta}}(\mu_B,\mu_C,\mu_R) \\
&= (\mu_B\boldsymbol{B}_{XX}+\mu_C\boldsymbol{C}_{XX}+\mu_R\boldsymbol{R}_{XX})^{-1}(\mu_B\boldsymbol{B}_{XY}+\mu_C\boldsymbol{C}_{XY}+\mu_R\boldsymbol{R}_{XY}),
\end{aligned}
$$

where (μ_B,μ_C,μ_R) are scalar constants (one of which may be normalized to unity without loss of generality). Using the decompositions exemplified by (5.3), (5.4), and (5.8), it can be expressed in the (4.1) format either as

$$\widehat{\boldsymbol{\beta}} = \left(\frac{\mu_B}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XXts} + \mu_R \sum_{i=1}^{N} \boldsymbol{W}_{XXii} + \frac{\mu_C - \mu_R}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XXij} \right)^{-1}$$
$$\times \left(\frac{\mu_B}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XYts} + \mu_R \sum_{i=1}^{N} \boldsymbol{W}_{XYii} + \frac{\mu_C - \mu_R}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XYij} \right),$$

or as

$$\widehat{\boldsymbol{\beta}} = \left(\frac{\mu_C}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XXij} + \mu_R \sum_{t=1}^{T} \boldsymbol{V}_{XXtt} + \frac{\mu_B - \mu_R}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XXts} \right)^{-1}$$
$$\times \left(\frac{\mu_C}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{W}_{XYij} + \mu_R \sum_{t=1}^{T} \boldsymbol{V}_{XYtt} + \frac{\mu_B - \mu_R}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} \boldsymbol{V}_{XYts} \right),$$

compactly

$$\widehat{\boldsymbol{\beta}} = \boldsymbol{b}(\mu_B \boldsymbol{A}_T, \mu_C \boldsymbol{A}_N + \mu_R \boldsymbol{B}_N) = \boldsymbol{b}(\mu_B \boldsymbol{A}_T + \mu_R \boldsymbol{B}_T, \mu_C \boldsymbol{A}_N). \tag{5.18}$$

As shown thirty years ago by Fuller and Battese (1973, 1974), the two-way random effects GLS estimator of $\boldsymbol{\beta}$ utilizing (2.2)–(2.3), for known $(\sigma^2, \sigma_\alpha^2, \sigma_\gamma^2)$, can be written as

$$\begin{aligned} \widehat{\boldsymbol{\beta}}_{GLS} &= \widehat{\boldsymbol{\beta}}(\lambda_B, \lambda_C, 1) = (\lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX})^{-1} (\lambda_B \boldsymbol{B}_{XY} + \lambda_C \boldsymbol{C}_{XY} + \boldsymbol{R}_{XY}) \\ &= \left[\frac{\boldsymbol{R}_{XX}}{\sigma^2} + \frac{\boldsymbol{B}_{XX}}{\sigma^2 + T\sigma_\alpha^2} + \frac{\boldsymbol{C}_{XX}}{\sigma^2 + N\sigma_\gamma^2} \right]^{-1} \left[\frac{\boldsymbol{R}_{XY}}{\sigma^2} + \frac{\boldsymbol{B}_{XY}}{\sigma^2 + T\sigma_\alpha^2} + \frac{\boldsymbol{C}_{XY}}{\sigma^2 + N\sigma_\gamma^2} \right], \end{aligned} \tag{5.19}$$

where

$$\lambda_B = \frac{\sigma^2}{\sigma^2 + T\sigma_\alpha^2}, \qquad \lambda_C = \frac{\sigma^2}{\sigma^2 + N\sigma_\gamma^2}.$$

This is the MVLUE in the random effects model. Under normality of u_{it}, α_i and γ_t it is the Maximum Likelihood estimator. The corresponding estimators when only random individual effects occur ($\gamma_t = \sigma_\gamma^2 = 0$) and when only random period effects occur ($\alpha_i = \sigma_\alpha^2 = 0$) are, respectively,

$$\begin{aligned} \widehat{\boldsymbol{\beta}}_{GLS(\alpha)} &= \widehat{\boldsymbol{\beta}}(\lambda_B, 1, 1) = (\lambda_B \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX})^{-1} (\lambda_B \boldsymbol{B}_{XY} + \boldsymbol{C}_{XY} + \boldsymbol{R}_{XY}), \\ \widehat{\boldsymbol{\beta}}_{GLS(\gamma)} &= \widehat{\boldsymbol{\beta}}(1, \lambda_C, 1) = (\boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX})^{-1} (\boldsymbol{B}_{XY} + \lambda_C \boldsymbol{C}_{XY} + \boldsymbol{R}_{XY}). \end{aligned}$$

The weights θ_{ts} and τ_{ij} for these three GLS estimators are functions of λ_B and/or λ_C, as given, in two versions, in Table 2, panel A:

$$\begin{aligned} \widehat{\boldsymbol{\beta}}_{GLS} &= \boldsymbol{b}(\boldsymbol{B}_T + \lambda_B \boldsymbol{A}_T, \lambda_C \boldsymbol{A}_N) = \boldsymbol{b}(\lambda_B \boldsymbol{A}_T, \boldsymbol{B}_N + \lambda_C \boldsymbol{A}_N), \\ \widehat{\boldsymbol{\beta}}_{GLS(\alpha)} &= \boldsymbol{b}(\boldsymbol{B}_T + \lambda_B \boldsymbol{A}_T, \boldsymbol{A}_N) = \boldsymbol{b}(\lambda_B \boldsymbol{A}_T, \boldsymbol{I}_N), \\ \widehat{\boldsymbol{\beta}}_{GLS(\gamma)} &= \boldsymbol{b}(\boldsymbol{I}_T, \lambda_C \boldsymbol{A}_N) = \boldsymbol{b}(\boldsymbol{A}_T, \boldsymbol{B}_N + \lambda_C \boldsymbol{A}_N). \end{aligned}$$

In Appendix D it is shown that their variance-covariance matrices, when three variance components occur, can be written as

$$
\begin{aligned}
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS}|\boldsymbol{X}) &= \sigma^2[\boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX}]^{-1} = \left[\frac{\boldsymbol{R}_{XX}}{\sigma^2} + \frac{\boldsymbol{B}_{XX}}{\sigma^2 + T\sigma_\alpha^2} + \frac{\boldsymbol{C}_{XX}}{\sigma^2 + N\sigma_\gamma^2}\right]^{-1}, \\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}|\boldsymbol{X}) &= [\boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX}]^{-1} \\
&\quad \times [\sigma^2 \boldsymbol{R}_{XX} + \lambda_B^2(\sigma^2 + T\sigma_\alpha^2)\boldsymbol{B}_{XX} + (\sigma^2 + N\sigma_\gamma^2)\boldsymbol{C}_{XX}][\boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX}]^{-1}, \\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}|\boldsymbol{X}) &= [\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX}]^{-1} \\
&\quad \times [\sigma^2 \boldsymbol{R}_{XX} + (\sigma^2 + T\sigma_\alpha^2)\boldsymbol{B}_{XX} + \lambda_C^2(\sigma^2 + N\sigma_\gamma^2)\boldsymbol{C}_{XX}][\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX} + \lambda_B \boldsymbol{C}_{XX}]^{-1}.
\end{aligned}
$$

Then, of course, $\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}$ and $\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}$ are not MVLUE. If the one-way random effects model is valid, *i.e.*, if $\sigma_\gamma^2 = 0$ and if $\sigma_\alpha^2 = 0$, respectively, the latter two are simplified to

$$
\begin{aligned}
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}|\boldsymbol{X}) &= \left[\frac{\boldsymbol{R}_{XX} + \boldsymbol{C}_{XX}}{\sigma^2} + \frac{\boldsymbol{B}_{XX}}{\sigma^2 + T\sigma_\alpha^2}\right]^{-1}, \\
\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}|\boldsymbol{X}) &= \left[\frac{\boldsymbol{R}_{XX} + \boldsymbol{B}_{XX}}{\sigma^2} + \frac{\boldsymbol{C}_{XX}}{\sigma^2 + N\sigma_\gamma^2}\right]^{-1}.
\end{aligned}
$$

Table 2: The General Moment Estimator (4.1)
For Random Effects Models
$\lambda_B = \sigma^2/(\sigma^2 + T\sigma_\alpha^2), \quad \lambda_C = \sigma^2/(\sigma^2 + N\sigma_\gamma^2)$

A: Weights θ_{ts} and τ_{ij}

	θ_{tt}	$\theta_{ts},\ s\neq t$	τ_{ii}	$\tau_{ij},\ j\neq i$	$\boldsymbol{\theta}$	$\boldsymbol{\tau}$
$\widehat{\boldsymbol{\beta}}_{GLS}$	$1-\frac{1-\lambda_B}{T}$	$-\frac{1-\lambda_B}{T}$	$\frac{\lambda_C}{N}$	$\frac{\lambda_C}{N}$	$\boldsymbol{B}_T + \lambda_B \boldsymbol{A}_T$	$\lambda_C \boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_{GLS}$	$\frac{\lambda_B}{T}$	$\frac{\lambda_B}{T}$	$1-\frac{1-\lambda_C}{N}$	$-\frac{1-\lambda_C}{N}$	$\lambda_B \boldsymbol{A}_T$	$\boldsymbol{B}_N + \lambda_C \boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}$	$1-\frac{1-\lambda_B}{T}$	$-\frac{1-\lambda_B}{T}$	$\frac{1}{N}$	$\frac{1}{N}$	$\boldsymbol{B}_T + \lambda_B \boldsymbol{A}_T$	$\boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}$	$\frac{\lambda_B}{T}$	$\frac{\lambda_B}{T}$	1	0	$\lambda_B \boldsymbol{A}_T$	$\boldsymbol{I}_N$
$\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}$	1	0	$\frac{\lambda_C}{N}$	$\frac{\lambda_C}{N}$	$\boldsymbol{I}_T$	$\lambda_C \boldsymbol{A}_N$
$\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}$	$\frac{1}{T}$	$\frac{1}{T}$	$1-\frac{1-\lambda_C}{N}$	$-\frac{1-\lambda_C}{N}$	$\boldsymbol{A}_T$	$\boldsymbol{B}_N + \lambda_C \boldsymbol{A}_N$

B: Values of $\boldsymbol{S}_V + \boldsymbol{S}_W$, $\boldsymbol{Z}_V$, $\boldsymbol{Z}_W$, $\boldsymbol{Q}$, and $\boldsymbol{Z}_{VW}$ defining Covariance Matrices

	$\boldsymbol{S}_V + \boldsymbol{S}_W$	$\boldsymbol{Z}_V$	$\boldsymbol{Z}_W$	$\boldsymbol{Q}$	$\boldsymbol{Z}_{VW}$
$\widehat{\boldsymbol{\beta}}_{GLS}$	$\lambda_B^2 \boldsymbol{B}_{XX} + \lambda_C^2 \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$\lambda_B^2 T \boldsymbol{B}_{XX}$	$\lambda_C^2 N \boldsymbol{C}_{XX}$	$\lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}$	$\lambda_B^2 \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$\lambda_B^2 T \boldsymbol{B}_{XX}$	$N \boldsymbol{C}_{XX}$	$\lambda_B \boldsymbol{B}_{XX} + \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$\boldsymbol{0}$
$\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}$	$\boldsymbol{B}_{XX} + \lambda_C^2 \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$T \boldsymbol{B}_{XX}$	$\lambda_C^2 N \boldsymbol{C}_{XX}$	$\boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX} + \boldsymbol{R}_{XX}$	$\boldsymbol{0}$

Robustness

An interesting question is *robustness* of the members of the class $\boldsymbol{b}(\boldsymbol{\theta},\boldsymbol{\tau})$ to violation of (2.2)–(2.3). From (4.2) and conclusions [1] and [2] in Section 3 it follows that

- If $\boldsymbol{x}_{it}$ contains an IID *measurement error* vector, which becomes part of u_{it}, then (i) all estimators such that $\theta_{tt} = 0$, $\theta_{ts} \neq 0$ for some $s \neq t$, and all $\tau_{ij} = 0$, are N-consistent, and (ii) all estimators such that $\tau_{ii} = 0$, $\tau_{ij} \neq 0$ for some $j \neq i$, and all $\theta_{ts} = 0$, are T-consistent.
- *If endogeneity* of some variables in $\boldsymbol{x}_{it}$, gives rise to $\mathsf{E}(\boldsymbol{x}_{it}'u_{it}) \neq \boldsymbol{0}_{K1}$, while $\mathsf{E}(\boldsymbol{x}_{it}'u_{js}) = \boldsymbol{0}_{K1}$ for $j \neq i$ and/or $s \neq t$, then similar consistency results hold.

6 Empirical illustration: Factor productivity

In this section, we present an empirical application of some of the above results for a model with a single regressor ($K = 1$), relating to factor productivity. The data are from successive annual Norwegian manufacturing censuses, collected by Statistics Norway, for the sector *Manufacture of textiles* (ISIC 32), with $N = 215$ firms observed in the years 1983–1990, *i.e.*, $T = 8$. The y_{it}'s and x_{it}'s in this example are, respectively, the log of the material input and the log of gross production, both measured as values at constant prices, so that the (scalar) coefficient β can be interpreted as the *input elasticity of materials with respect to output*, assumed to be one in simple input-output analysis. The OLS estimate of β obtained from the complete data set ($NT = 1720$ observations) is $\widehat{\beta}_T = 1.1450$. From the residuals, $\widehat{\epsilon}_{it}$, which are consistent, and the resulting between individual, between period, and residual sum of squares,

$$B_{\widehat{\epsilon}\widehat{\epsilon}} = T\sum_{i=1}^{N}(\bar{\widehat{\epsilon}}_{i\cdot} - \bar{\widehat{\epsilon}})^2, \quad C_{\widehat{\epsilon}\widehat{\epsilon}} = N\sum_{t=1}^{T}(\bar{\widehat{\epsilon}}_{\cdot t} - \bar{\widehat{\epsilon}})^2, \quad R_{\widehat{\epsilon}\widehat{\epsilon}} = \sum_{i=1}^{N}\sum_{t=1}^{T}(\widehat{\epsilon}_{it} - \bar{\widehat{\epsilon}}_{i\cdot} - \bar{\widehat{\epsilon}}_{\cdot t} + \bar{\widehat{\epsilon}})^2,$$

we compute the ANOVA type estimates:

$$\widehat{\sigma}_\alpha^2 + \frac{\widehat{\sigma}^2}{T} = \frac{B_{\widehat{\epsilon}\widehat{\epsilon}}}{T(N-1)}, \qquad \widehat{\sigma}_\gamma^2 + \frac{\widehat{\sigma}^2}{N} = \frac{C_{\widehat{\epsilon}\widehat{\epsilon}}}{N(T-1)}, \qquad \widehat{\sigma}^2 = \frac{R_{\widehat{\epsilon}\widehat{\epsilon}}}{(N-1)(T-1)},$$

cf. Searle, Casella, and McCulloch (1992, section 4.7.iii), which give the estimated variance components

$$\begin{aligned}
\widehat{\sigma}_\alpha^2 &= \frac{1}{T(N-1)}\left[B_{\widehat{\epsilon}\widehat{\epsilon}} - \frac{R_{\widehat{\epsilon}\widehat{\epsilon}}}{T-1}\right] = 0.14394,\\
\widehat{\sigma}_\gamma^2 &= \frac{1}{N(T-1)}\left[C_{\widehat{\epsilon}\widehat{\epsilon}} - \frac{R_{\widehat{\epsilon}\widehat{\epsilon}}}{N-1}\right] = 0.00066,\\
\widehat{\sigma}^2 &= 0.03449.
\end{aligned}$$

Hence, the estimate of the total disturbance variance becomes $\widehat{\sigma}^2_\epsilon = \widehat{\sigma}^2_\alpha + \widehat{\sigma}^2_\gamma + \widehat{\sigma}^2 = 0.17909$. The corresponding shares representing individual heterogeneity, period heterogeneity, and residual variation are, respectively, $\widehat{\sigma}^2_\alpha/\widehat{\sigma}^2_\epsilon = 0.80372$, $\widehat{\sigma}^2_\gamma/\widehat{\sigma}^2_\epsilon = 0.00370$, and $\widehat{\sigma}^2/\widehat{\sigma}^2_\epsilon = 0.19259$. The corresponding (marginal) shares are $B_{YY}/T_{YY} = 0.93992$, $C_{YY}/T_{YY} = 0.00829$, $R_{YY}/T_{YY} = 0.05179$ for log-input and $B_{XX}/T_{XX} = 0.83525$, $C_{XX}/T_{XX} = 0.04216$, and $R_{XX}/T_{XX} = 0.12259$ for log-output. Not surprisingly, in both cases, the between firm variation dominates.

We have selected $N = 10$ firms randomly from the 215 in the full sample and included the $T = 8$ observations from each of them. All results below refer to this subsample of $NT = 80$ observations, except that the variance components have been estimated from the complete sample, as explained above.

The *firm specific IV/OLS estimates* of the input elasticity of materials $\widehat{\beta}_{Wij}$ for the $N = 10$ firms are given in the upper panel of Table 3, with the OLS estimates on the main diagonal, varying from -0.09 (firm 2) to 1.54 (firm 7), and the IV estimates in the off-diagonal positions. The standard errors, obtained from (3.10), are given in the lower panel. All standard errors are derived under the assumption that (2.2)–(2.3) are valid. Even for the OLS estimates, the precision is low. The corresponding within-firm coefficients of correlation of log-output, R_{WXij}, given in Table A3, panel A, show considerable variation, are often low, and exceed 0.9 in few cases only. This indicates that log-output for other firms tend to be weak instruments for 'own' log-output, cf. (3.10) and (3.12). Recall that the OLS estimates are inconsistent in the presence of endogeneity of or random measurement error in log-output; the IV estimates are T-consistent.

The weights which are given to the firm specific OLS estimates (Table 3) in the overall within-firm estimate of the materials input elasticity $\widehat{\beta}_W$, which is 0.9284 (standard error 0.0773), are reported in Table A1, panel A. The estimate for firm 1 by far dominates in this aggregate, with a weight of 38 per cent. The weights which are given to all the firm specific IV/OLS estimates (Table 3) in the overall between-year estimate $\widehat{\beta}_C$, which is 0.7269 (standard error 0.1628), are reported in Table A1, panel B. Again, the disaggregate estimate for $i = 1, j = 1$ by far dominates, with a weight of almost 15 per cent. Some off-diagonal weights are negative, which reflect negative correlation between the log-output of the relevant firms; cf. Tables A3, Panel A.

The *year specific IV/OLS estimates* $\widehat{\beta}_{Vts}$ for the $T = 8$ years are given in the upper panel of Table 4, with the OLS estimates on the main diagonal, varying between 1.21 (cross section from year 1989) and 1.64 (cross section from year 1985), and the IV estimates in the off-diagonal positions. All the $T^2 = 64$ estimates exceed one. Their standard errors, calculated from (3.11), are given in the lower panel. Overall, the precision is much higher than for the firm specific estimates. The corresponding across-year coefficients of correlation of log-output, R_{VXts}, given in Table A3, panel B, show far less variation than the corresponding across-firm correlation coefficients. This indicates that log-output for other years may be strong instruments for the year's 'own' log-output, cf. (3.11) and (3.14). Recall that the OLS estimates are inconsistent in the presence of endogeneity of or random measurement errors in log-output; the IV estimates are N-consistent.

The weights which are given to the period specific OLS estimates (Table 4) in the within-year estimate of the materials input elasticity $\widehat{\beta}_V$, which is 1.4528 (standard error 0.1717), are reported in Table A2, panel A. The weights vary much less than in Table A1, between 20 per cent (for 1984) and 8 per cent (for 1990). The weights which are given to all the period specific IV/OLS estimates in Table 4 in the overall between-firm estimate $\widehat{\beta}_B$, which is 1.5195 (standard error 0.1965), are reported in Table A2, panel B. Again, the weights vary less than in Table A1, and all off-diagonal weights are positive.

The residual estimate, the total (standard OLS) estimate, and the GLS estimate of the material input elasticity β (with standard error in parenthesis) are, respectively, $\widehat{\beta}_R = 0.9978$ (0.0875), $\widehat{\beta}_T = 1.4222$ (0.1646), and $\widehat{\beta}_{GLS} = 1.0147$ (0.0717). The latter two are both weighted averages of $\widehat{\beta}_B$, $\widehat{\beta}_C$, and $\widehat{\beta}_R$ (cf. Tables 1 and 2), which agrees with the numerical estimates $\widehat{\beta}_B = 1.5195$, $\widehat{\beta}_C = 0.7269$, and $\widehat{\beta}_R = 0.9978$.

Since all the aggregate estimators considered have either all $\theta_{tt} \neq 0$ or all $\tau_{ii} \neq 0$, they are inconsistent in cases of endogeneity of or measurement errors in the regressor, confer the discussion of robustness at the end of Section 5. If we *modify the between-firm estimator* $\widehat{\beta}_B$ by replacing $\theta_{ts} = 1/T$ for all (t, s) by $\theta_{ts} = 0$ for $s = t$, $\theta_{ts} = 1/T$ for $s \neq t$ (cf. Table 1), we get $\widehat{\beta}_{B*} = 1.5307$. This is N-consistent and is slightly above the (less robust) between-firm estimate $\widehat{\beta}_B = 1.5195$. Symmetrically, if we *modify the between-year estimator* $\widehat{\beta}_C$ by replacing $\tau_{ij} = 1/N$ for all (i, j) by $\tau_{ij} = 0$ for $j = i$, $\tau_{ij} = 1/N$ for $j \neq i$ (cf. Table 1), we get $\widehat{\beta}_{C*} = 0.5976$, which is T-consistent and is substantially below the (less robust) between-year estimate $\widehat{\beta}_C = 0.7279$. If, however, (2.2)–(2.3) hold, $\widehat{\beta}_{B*}$ is somewhat less efficient than $\widehat{\beta}_B$ (standard error 0.2007 against 0.1965), and $\widehat{\beta}_{C*}$ is markedly less efficient than $\widehat{\beta}_C$ (standard errors 0.2442 against 0.1628), *i.e.*, the efficiency loss when omitting the disaggregate OLS estimates (the diagonal elements of the lower panels of Tables 3 and 4) in aggregate estimator may be substantial.

Acknowledgements: I thank Terje Skjerpen for valuable comments after a careful reading of the paper.

Table 3: Firm Specific IV Estimates $\widehat{\beta}_{Wij}$. (OLS Along Main Diagonal) β = Materials - Output Elasticity. $N = 10, T = 8$.

Within deviation of firm i used as IV for within deviation of firm j

$i \downarrow \ j \rightarrow$	1	2	3	4	5	6	7	8	9	10
1	0.92	-0.03	1.29	3.41	0.99	0.92	1.74	1.23	0.12	-0.85
2	0.70	-0.09	1.92	1.80	1.15	4.94	3.20	1.42	0.69	4.67
3	0.95	-0.09	0.55	3.17	1.01	1.02	1.46	1.16	0.54	0.26
4	1.02	-0.43	14.42	1.22	0.78	-0.06	0.77	2.53	0.91	-2.77
5	0.94	-0.04	0.08	-3.46	0.99	0.94	1.62	1.16	0.36	-0.11
6	1.08	0.55	-0.64	0.67	1.05	0.90	1.21	1.13	0.92	0.74
7	1.11	-0.81	0.68	2.06	1.02	0.88	1.54	1.02	2.01	0.85
8	0.97	-0.02	0.32	-11.62	1.04	0.90	1.63	1.16	0.61	0.27
9	0.93	-0.05	2.91	1.39	1.14	1.14	0.91	1.30	0.53	-1.67
10	1.24	0.25	-2.19	0.38	1.07	0.79	1.58	0.91	-2.78	0.78

Standard errors

$i \downarrow \ j \rightarrow$	1	2	3	4	5	6	7	8	9	10
1	0.28	1.73	1.05	6.87	0.49	1.27	1.19	0.52	2.21	2.07
2	0.64	0.75	2.19	3.01	1.28	12.69	8.85	1.09	2.30	8.49
3	0.31	1.73	0.95	6.95	0.47	0.86	1.02	0.46	2.02	1.22
4	1.90	2.26	6.62	1.00	6.44	2.79	2.33	20.73	2.84	3.83
5	0.30	2.16	1.00	14.49	0.45	0.93	1.03	0.48	3.22	1.08
6	0.54	14.53	1.25	4.26	0.63	0.66	1.43	0.53	2.52	1.05
7	0.46	9.26	1.36	3.25	0.64	1.30	0.72	0.64	6.47	0.85
8	0.32	1.84	0.99	46.51	0.47	0.79	1.03	0.45	2.13	1.07
9	0.49	1.37	1.52	2.25	1.13	1.31	3.67	0.75	1.27	7.60
10	0.84	9.36	1.70	5.63	0.71	1.01	0.89	0.70	14.13	0.68

Table 4: Year Specific IV Estimates $\widehat{\beta}_{Vts}$. (OLS Along Main Diagonal) β = Materials - Output Elasticity. $N = 10, T = 8$.

Within deviation of year t used as IV for within deviation of year s

$t \downarrow \ s \rightarrow$	1983	1984	1985	1986	1987	1988	1989	1990
1983	1.267	1.433	1.572	1.383	1.514	1.567	1.407	1.613
1984	1.232	1.375	1.483	1.290	1.390	1.483	1.302	1.526
1985	1.374	1.508	1.642	1.465	1.589	1.576	1.468	1.663
1986	1.414	1.529	1.660	1.483	1.586	1.604	1.499	1.669
1987	1.441	1.595	1.751	1.588	1.606	1.652	1.435	1.618
1988	1.519	1.668	1.803	1.671	1.712	1.625	1.394	1.623
1989	1.454	1.589	1.676	1.584	1.570	1.477	1.212	1.487
1990	1.502	1.665	1.809	1.683	1.626	1.614	1.330	1.551

Standard errors

$t \downarrow \ s \rightarrow$	1983	1984	1985	1986	1987	1988	1989	1990
1983	0.080	0.073	0.099	0.105	0.113	0.118	0.142	0.158
1984	0.083	0.071	0.097	0.099	0.109	0.116	0.133	0.152
1985	0.086	0.074	0.093	0.093	0.103	0.105	0.123	0.140
1986	0.092	0.077	0.095	0.091	0.101	0.105	0.121	0.136
1987	0.094	0.080	0.100	0.096	0.096	0.097	0.106	0.118
1988	0.102	0.088	0.105	0.103	0.100	0.093	0.101	0.116
1989	0.117	0.097	0.117	0.113	0.105	0.097	0.097	0.115
1990	0.113	0.096	0.116	0.110	0.101	0.096	0.100	0.112

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Appendices

A: The covariance matrices of the base estimators

In order to derive the variance-covariance matrices of $\widehat{\boldsymbol{\beta}}_{Wij}$ and $\widehat{\boldsymbol{\beta}}_{Vts}$ when (2.2)–(2.3) is valid, we first need expressions for the variance-covariance matrices of $\boldsymbol{W}_{XUij}$, $\boldsymbol{V}_{XUts}$, $\boldsymbol{W}_{X\gamma i}$, and $\boldsymbol{V}_{X\alpha t}$. Since (2.2) implies

$$
\begin{aligned}
&\mathsf{E}(\boldsymbol{\alpha}\boldsymbol{\alpha}'|\boldsymbol{X}) = \sigma_\alpha^2 \boldsymbol{I}_N, \qquad \mathsf{E}(\boldsymbol{\gamma}\boldsymbol{\gamma}'|\boldsymbol{X}) = \sigma_\gamma^2 \boldsymbol{I}_T, \\
&\mathsf{E}(\boldsymbol{u}_{j\cdot}\boldsymbol{u}_{l\cdot}'|\boldsymbol{X}) = \delta_{jl}\sigma^2 \boldsymbol{I}_T, \quad \mathsf{E}(\boldsymbol{u}_{\cdot s}\boldsymbol{u}_{\cdot q}'|\boldsymbol{X}) = \delta_{sq}\sigma^2 \boldsymbol{I}_N, \qquad j,l = 1,\dots,N, \\
&\mathsf{E}(\boldsymbol{u}_{j\cdot}\boldsymbol{u}_{\cdot q}'|\boldsymbol{X}) = \sigma^2 \boldsymbol{i}_{Tq}\boldsymbol{i}_{Nj}', \qquad s,q = 1,\dots,T,
\end{aligned}
$$

where $\boldsymbol{i}_{Hj}$ denotes the j'th column of $\boldsymbol{I}_H$, we get, after some algebra,

$$
\text{(a.1)} \quad
\begin{aligned}
&\mathsf{E}(\boldsymbol{W}_{XUij}\, \boldsymbol{W}_{XUkl}'|\boldsymbol{X}) = \delta_{jl}\, \sigma^2\, \boldsymbol{W}_{XXik}, \\
&\mathsf{E}(\boldsymbol{W}_{X\gamma i}\, \boldsymbol{W}_{X\gamma k}'|\boldsymbol{X}) = \sigma_\gamma^2\, \boldsymbol{W}_{XXik}, \\
&\mathsf{E}(\boldsymbol{W}_{X\epsilon ij}\, \boldsymbol{W}_{X\epsilon kl}'|\boldsymbol{X}) = (\sigma_\gamma^2 + \delta_{jl}\sigma^2)\boldsymbol{W}_{XXik},
\end{aligned}
$$

$$
\text{(a.2)} \quad
\begin{aligned}
&\mathsf{E}(\boldsymbol{V}_{XUts}\, \boldsymbol{V}_{XUpq}'|\boldsymbol{X}) = \delta_{sq}\, \sigma^2\, \boldsymbol{V}_{XXtp}, \\
&\mathsf{E}(\boldsymbol{V}_{X\alpha t}\, \boldsymbol{V}_{X\alpha p}'|\boldsymbol{X}) = \sigma_\alpha^2\, \boldsymbol{V}_{XXtp}, \\
&\mathsf{E}(\boldsymbol{V}_{X\epsilon ts}\, \boldsymbol{V}_{X\epsilon pq}'|\boldsymbol{X}) = (\sigma_\alpha^2 + \delta_{sq}\sigma^2)\boldsymbol{V}_{XXtp},
\end{aligned}
$$

$$
\text{(a.3)} \quad
\left.
\begin{aligned}
&\mathsf{E}(\boldsymbol{W}_{XUij}\, \boldsymbol{V}_{XUpq}'|\boldsymbol{X}) \\
&\mathsf{E}(\boldsymbol{W}_{X\epsilon ij}\, \boldsymbol{V}_{X\epsilon pq}'|\boldsymbol{X})
\end{aligned}
\right\} = \sigma^2(\boldsymbol{x}_{iq} - \bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jp} - \bar{\boldsymbol{x}}_{\cdot p}), \qquad
\begin{aligned}
&i,j,k,l = 1,\dots,N, \\
&t,s,p,q = 1,\dots,T.
\end{aligned}
$$

Combining (a.1)–(a.3) with (3.3)–(3.4), it follows that the matrices of covariances between the individual specific and between the period specific base estimators, respectively, can be expressed as

$$
\begin{aligned}
\text{(a.4)} \quad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Wij}, \widehat{\boldsymbol{\beta}}_{Wkl}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Wkl} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\gamma^2 + \delta_{jl}\sigma^2)\boldsymbol{W}_{XXij}^{-1}\boldsymbol{W}_{XXik}\boldsymbol{W}_{XXlk}^{-1}, \quad i,j,k,l = 1,\dots,N,
\end{aligned}
$$

$$
\begin{aligned}
\text{(a.5)} \quad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Vts}, \widehat{\boldsymbol{\beta}}_{Vpq}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Vts} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Vpq} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= (\sigma_\alpha^2 + \delta_{sq}\sigma^2)\boldsymbol{V}_{XXts}^{-1}\boldsymbol{V}_{XXtp}\boldsymbol{V}_{XXqp}^{-1}, \quad t,s,p,q = 1,\dots,T,
\end{aligned}
$$

and that the matrices of 'cross covariances' between the two sets of estimators are

$$
\begin{aligned}
\text{(a.6)} \quad \mathsf{C}(\widehat{\boldsymbol{\beta}}_{Wij}, \widehat{\boldsymbol{\beta}}_{Vpq}|\boldsymbol{X}) &= \mathsf{E}[(\widehat{\boldsymbol{\beta}}_{Wij} - \boldsymbol{\beta})(\widehat{\boldsymbol{\beta}}_{Vpq} - \boldsymbol{\beta})'|\boldsymbol{X}] \\
&= \sigma^2\boldsymbol{W}_{XXij}^{-1}(\boldsymbol{x}_{iq} - \bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jp} - \bar{\boldsymbol{x}}_{\cdot p})\boldsymbol{V}_{XXqp}^{-1}, \\
&\qquad\qquad i,j = 1,\dots,N;\ p,q = 1,\dots,T.
\end{aligned}
$$

This completes the proof of (3.7)–(3.9).

B: The variance-covariance matrix of $\boldsymbol{b}$

Inserting for $\boldsymbol{W}_{XYij}$ and $\boldsymbol{V}_{XYts}$ from (2.15) and (2.17) in (4.1), using (4.4), we find

$$\begin{aligned}
\boldsymbol{b}-\boldsymbol{\beta} &= \boldsymbol{Q}^{-1}\left[\sum_{t=1}^{T}\sum_{s=1}^{T}\theta_{ts}\boldsymbol{V}_{X\epsilon ts}+\sum_{i=1}^{N}\sum_{j=1}^{N}\tau_{ij}\boldsymbol{W}_{X\epsilon ij}\right] \\
&= \boldsymbol{Q}^{-1}\left[\sum_{t=1}^{T}\sum_{s=1}^{T}\theta_{ts}\boldsymbol{V}_{XUts}+\sum_{t=1}^{T}\left(\sum_{s=1}^{T}\theta_{ts}\right)\boldsymbol{V}_{X\alpha t}\right. \\
&\qquad\left.+\ \sum_{i=1}^{N}\sum_{j=1}^{N}\tau_{ij}\boldsymbol{W}_{XUij}+\sum_{i=1}^{N}\left(\sum_{j=1}^{N}\tau_{ij}\right)\boldsymbol{W}_{X\gamma i}\right].
\end{aligned}$$

Combining this equation with (3.3), (3.4), and (a.1)–(a.3), we find that $\boldsymbol{b}$ is an unbiased estimator of $\boldsymbol{\beta}$ for any $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ and has variance-covariance matrix

$$\text{(b.1)}\qquad \mathsf{V}(\boldsymbol{b}|\boldsymbol{X})=\boldsymbol{Q}^{-1}\,\boldsymbol{P}\,(\boldsymbol{Q}^{-1})'=\boldsymbol{Q}(\boldsymbol{\theta},\boldsymbol{\tau})^{-1}\boldsymbol{P}(\boldsymbol{\theta},\boldsymbol{\tau},\sigma^2,\sigma_\alpha^2,\sigma_\gamma^2)(\boldsymbol{Q}(\boldsymbol{\theta},\boldsymbol{\tau})^{-1})',$$

where

$$\text{(b.2)}\qquad \boldsymbol{P}=\boldsymbol{P}(\boldsymbol{\theta},\boldsymbol{\tau},\sigma^2,\sigma_\alpha^2,\sigma_\gamma^2)=\sigma^2(\boldsymbol{S}_V+\boldsymbol{S}_W+\boldsymbol{S}_{VW})+\sigma_\alpha^2\,\boldsymbol{Z}_V+\sigma_\gamma^2\,\boldsymbol{Z}_W,$$

$$\text{(b.3)}\qquad \begin{aligned}
&\boldsymbol{S}_V=\boldsymbol{S}_V(\boldsymbol{\theta})=\sum_{t=1}^{T}\sum_{p=1}^{T}\boldsymbol{V}_{XXtp}\left(\sum_{s=1}^{T}\theta_{ts}\theta_{ps}\right), \\
&\boldsymbol{S}_W=\boldsymbol{S}_W(\boldsymbol{\tau})=\sum_{i=1}^{N}\sum_{k=1}^{N}\boldsymbol{W}_{XXik}\left(\sum_{j=1}^{N}\tau_{ij}\tau_{kj}\right), \\
&\boldsymbol{S}_{VW}=\boldsymbol{S}_{VW}(\boldsymbol{\theta},\boldsymbol{\tau})=\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_{ts}\tau_{ij}(\boldsymbol{x}_{is}-\bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jt}-\bar{\boldsymbol{x}}_{\cdot t}), \\
&\boldsymbol{Z}_V=\boldsymbol{Z}_V(\boldsymbol{\theta})=\sum_{t=1}^{T}\sum_{p=1}^{T}\boldsymbol{V}_{XXtp}\left(\sum_{s=1}^{T}\theta_{ts}\right)\left(\sum_{r=1}^{T}\theta_{pr}\right), \\
&\boldsymbol{Z}_W=\boldsymbol{Z}_W(\boldsymbol{\tau})=\sum_{i=1}^{N}\sum_{k=1}^{N}\boldsymbol{W}_{XXik}\left(\sum_{j=1}^{N}\tau_{ij}\right)\left(\sum_{l=1}^{N}\tau_{kl}\right).
\end{aligned}$$

This completes the proof of (4.5)–(4.7).

C: Proof of (5.3) and (5.4)

Since $\bar{\boldsymbol{x}}_{i\cdot}-\bar{\boldsymbol{x}}=\sum_{t=1}^{T}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})/T$, $\bar{\boldsymbol{x}}_{\cdot t}-\bar{\boldsymbol{x}}=\sum_{i=1}^{N}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})/N$, etc., and

$$\begin{aligned}
\sum_{i=1}^{N}(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})'\boldsymbol{A}_T(\boldsymbol{X}_{i\cdot}-\bar{\boldsymbol{X}})&=\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\boldsymbol{X}_{\cdot t}'\,\boldsymbol{B}_N\,\boldsymbol{X}_{\cdot s}, \\
\sum_{t=1}^{T}(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}})'\boldsymbol{A}_N(\boldsymbol{X}_{\cdot t}-\bar{\boldsymbol{X}})&=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{X}_{i\cdot}'\,\boldsymbol{B}_T\,\boldsymbol{X}_{j\cdot}
\end{aligned}$$

hold identically, (2.13) and (2.14) can be rewritten as

$$\text{(c.1)}\qquad \begin{aligned}
\boldsymbol{B}_{XXii}&=\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})'(\boldsymbol{x}_{is}-\bar{\boldsymbol{x}}_{\cdot s}), \\
\boldsymbol{B}_{X\alpha ii}&=\sum_{t=1}^{T}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{\cdot t})'(\alpha_i-\bar{\alpha}),
\end{aligned}\qquad i=1,\dots,N,$$

$$\text{(c.2)}\qquad \begin{aligned}
\boldsymbol{C}_{XXtt}&=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})'(\boldsymbol{x}_{jt}-\bar{\boldsymbol{x}}_{j\cdot}), \\
\boldsymbol{C}_{X\gamma tt}&=\sum_{i=1}^{N}(\boldsymbol{x}_{it}-\bar{\boldsymbol{x}}_{i\cdot})'(\gamma_t-\bar{\gamma}),
\end{aligned}\qquad t=1,\dots,T,$$

and the following identities hold

$$\text{(c.3)}\qquad \sum_{i=1}^{N}\boldsymbol{B}_{XXii}=\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\boldsymbol{V}_{XXts},\qquad \sum_{t=1}^{T}\boldsymbol{C}_{XXtt}=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{W}_{XXij}.$$

Similarly,

$$\sum_{i=1}^{N}\boldsymbol{B}_{X\alpha ii}=\sum_{t=1}^{T}\boldsymbol{V}_{X\alpha t},\qquad \sum_{t=1}^{T}\boldsymbol{C}_{X\gamma tt}=\sum_{i=1}^{N}\boldsymbol{W}_{X\gamma i}.$$

The overall between individual and overall between period (co)variation can then be written as

(c.4) $$\boldsymbol{B}_{XX} = \sum_{i=1}^{N} \boldsymbol{B}_{XXii} = T\sum_{i=1}^{N}(\bar{\boldsymbol{x}}_{i\cdot} - \bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{i\cdot} - \bar{\boldsymbol{x}}) = \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T} \boldsymbol{V}_{XXts},$$

(c.5) $$\boldsymbol{C}_{XX} = \sum_{t=1}^{T} \boldsymbol{C}_{XXtt} = N\sum_{t=1}^{T}(\bar{\boldsymbol{x}}_{\cdot t} - \bar{\boldsymbol{x}})'(\bar{\boldsymbol{x}}_{\cdot t} - \bar{\boldsymbol{x}}) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{W}_{XXij},$$

where the last equalities follow from (c.3), which completes the proof.

D: The variance-covariance matrix of $\widehat{\boldsymbol{\beta}}_{GLS}$

Recalling (5.11), (5.12), (5.14), and (5.19), the GLS weights in the variance-covariance matrix can be obtained from Table 2, panel A, by adding λ_B times the weights in row 1, λ_C times the weights in row 2, and 1 times the weights in row 3 (or 4). Expressions for the variance-covariance matrix of $\widehat{\boldsymbol{\beta}}_{GLS}$ can be derived by inserting the weights in Table 2, panel A, rows 1 or 2, into (4.5)–(4.7). The result is given in Table 2, panel B, row 1. In deriving the variance-covariance matrix of $\widehat{\boldsymbol{\beta}}_{GLS}$, we note that

$$\begin{array}{lll} \sum_{s=1}^{T} \theta_{ts} = \lambda_B, & \sum_{s=1}^{T} \theta_{ts}\theta_{ps} = \delta_{tp} - \dfrac{1-\lambda_B^2}{T} & t,p = 1,\dots,T, \\ \sum_{j=1}^{N} \tau_{ij} = \lambda_C, & \sum_{j=1}^{N} \tau_{ij}\tau_{kj} = \delta_{ik} - \dfrac{1-\lambda_C^2}{N} & i,k = 1,\dots,N, \end{array}$$

so that, using (4.7), we have

$$\boldsymbol{Z}_V = \lambda_B^2 \sum_{t=1}^{T}\sum_{p=1}^{T} \boldsymbol{V}_{XXtp} = \lambda_B^2 T \boldsymbol{B}_{XX}, \quad \boldsymbol{Z}_W = \lambda_C^2 \sum_{i=1}^{N}\sum_{k=1}^{N} \boldsymbol{W}_{XXik} = \lambda_C^2 N \boldsymbol{C}_{XX},$$

which are the expressions given in Table 2, panel B, columns 2 and 3. Obviously, $\boldsymbol{S}_{VW} = \boldsymbol{0}$. From (4.7), in combination with the weights in Table 2, rows 1 and 2, we get

$$\begin{array}{l} \boldsymbol{S}_V + \boldsymbol{S}_W = \boldsymbol{V}_{XX} - (1-\lambda_B^2)\boldsymbol{B}_{XX} + \lambda_C^2 \boldsymbol{C}_{XX} = \lambda_B^2 \boldsymbol{B}_{XX} + \boldsymbol{W}_{XX} - (1-\lambda_C^2)\boldsymbol{C}_{XX}, \\ \boldsymbol{Q} = \boldsymbol{V}_{XX} - (1-\lambda_B)\boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX} = \lambda_B \boldsymbol{B}_{XX} + \boldsymbol{W}_{XX} - (1-\lambda_C)\boldsymbol{C}_{XX}, \end{array}$$

both of which, since $\boldsymbol{V}_{XX} - \boldsymbol{B}_{XX} = \boldsymbol{W}_{XX} - \boldsymbol{C}_{XX} = \boldsymbol{R}_{XX}$, can be simplified to

$$\begin{array}{l} \boldsymbol{S}_V + \boldsymbol{S}_W = \boldsymbol{R}_{XX} + \lambda_B^2 \boldsymbol{B}_{XX} + \lambda_C^2 \boldsymbol{C}_{XX}, \\ \boldsymbol{Q} = \boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX}. \end{array}$$

These are the expressions given in Table 2, panel B, columns 1 and 4. Finally, noting that

$$\sigma^2(\boldsymbol{S}_V + \boldsymbol{S}_W) + \sigma_\alpha^2 \boldsymbol{Z}_V + \sigma_\gamma^2 \boldsymbol{Z}_W = \sigma^2[\boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX}],$$

the variance-covariance matrix of $\widehat{\boldsymbol{\beta}}_{GLS}$ can be written as

(d.1) $$\mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS}|\boldsymbol{X}) = \sigma^2[\boldsymbol{R}_{XX} + \lambda_B \boldsymbol{B}_{XX} + \lambda_C \boldsymbol{C}_{XX}]^{-1} = \left[\frac{\boldsymbol{R}_{XX}}{\sigma^2} + \frac{\boldsymbol{B}_{XX}}{\sigma^2 + T\sigma_\alpha^2} + \frac{\boldsymbol{C}_{XX}}{\sigma^2 + N\sigma_\gamma^2}\right]^{-1}.$$

The variance-covariance matrices of the *one-way* GLS estimators $\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}$ and $\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}$ when the *two-way* effects model is valid, obtained from Table 2, panel B, rows 2 and 3, are

$$\text{(d.2)}\quad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}|\boldsymbol{X}) = [\boldsymbol{R}_{XX}+\lambda_B\boldsymbol{B}_{XX}+\boldsymbol{C}_{XX}]^{-1} \times [\sigma^2\boldsymbol{R}_{XX}+\lambda_B^2(\sigma^2+T\sigma_\alpha^2)\boldsymbol{B}_{XX}+(\sigma^2+N\sigma_\gamma^2)\boldsymbol{C}_{XX}][\boldsymbol{R}_{XX}+\lambda_B\boldsymbol{B}_{XX}+\boldsymbol{C}_{XX}]^{-1},$$

$$\text{(d.3)}\quad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}|\boldsymbol{X}) = [\boldsymbol{R}_{XX}+\boldsymbol{B}_{XX}+\lambda_C\boldsymbol{C}_{XX}]^{-1} \times [\sigma^2\boldsymbol{R}_{XX}+(\sigma^2+T\sigma_\alpha^2)\boldsymbol{B}_{XX}+\lambda_C^2(\sigma^2+N\sigma_\gamma^2)\boldsymbol{C}_{XX}][\boldsymbol{R}_{XX}+\boldsymbol{B}_{XX}+\lambda_C\boldsymbol{C}_{XX}]^{-1}.$$

If the one-way random effects model is valid, *i.e.*, if $\sigma_\gamma^2 = 0$ and $\sigma_\alpha^2 = 0$, respectively, they can be simplified to

$$\text{(d.4)}\quad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\alpha)}|\boldsymbol{X}) = \left[\frac{\boldsymbol{R}_{XX}+\boldsymbol{C}_{XX}}{\sigma^2}+\frac{\boldsymbol{B}_{XX}}{\sigma^2+T\sigma_\alpha^2}\right]^{-1},$$

$$\text{(d.5)}\quad \mathsf{V}(\widehat{\boldsymbol{\beta}}_{GLS(\gamma)}|\boldsymbol{X}) = \left[\frac{\boldsymbol{R}_{XX}+\boldsymbol{B}_{XX}}{\sigma^2}+\frac{\boldsymbol{C}_{XX}}{\sigma^2+N\sigma_\gamma^2}\right]^{-1}.$$

Appendix tables

Table A1: Weights of $\widehat{\beta}_{Wij}$ in $\widehat{\beta}_W$ and $\widehat{\beta}_C$. $N = 10,\ T = 8$.

A. Weights of $\widehat{\beta}_{Wii}$ in $\widehat{\beta}_W$, per cent. Average weight = 10 per cent

$i \rightarrow$	1	2	3	4	5	6	7	8	9	10
	38.25	5.22	3.24	2.93	14.86	6.84	5.71	14.76	1.84	6.35

B. Weights of $\widehat{\beta}_{Wij}$ in $\widehat{\beta}_C$, per cent. Average weight = 1 per cent

$i \downarrow\ j \rightarrow$	1	2	3	4	5	6	7	8	9	10
1	14.95	-2.40	3.95	-0.60	8.50	3.28	-3.47	8.00	-1.87	2.00
2	-2.40	2.04	-0.70	0.51	-1.20	-0.12	0.17	-1.40	0.67	0.18
3	3.95	-0.70	1.27	-0.17	2.58	1.41	-1.18	2.61	-0.60	0.99
4	-0.60	0.51	-0.17	1.15	0.18	0.41	0.49	0.06	0.40	0.30
5	8.50	-1.20	2.58	0.18	5.81	2.78	-2.51	5.43	-0.80	2.38
6	3.28	-0.12	1.41	0.41	2.78	2.67	-1.23	3.28	-0.69	1.67
7	-3.47	0.17	-1.18	0.49	-2.51	-1.23	2.23	-2.50	0.25	-1.89
8	8.00	-1.40	2.61	0.06	5.43	3.28	-2.50	5.77	-1.21	2.41
9	-1.87	0.67	-0.60	0.40	-0.80	-0.69	0.25	-1.21	0.72	0.12
10	2.00	0.18	0.99	0.30	2.38	1.67	-1.89	2.41	0.12	2.48

Table A2: Weights of $\widehat{\beta}_{Vts}$ in $\widehat{\beta}_V$ and $\widehat{\beta}_B$. $N = 10,\ T = 8$.

A. Weights of $\widehat{\beta}_{Vtt}$ in $\widehat{\beta}_V$, per cent. Average weight = 12.5 per cent

$t \rightarrow$	1983	1984	1985	1986	1987	1988	1989	1990
	15.503	20.005	11.536	12.091	10.844	11.551	10.556	7.915

B. Weights of $\widehat{\beta}_{Vts}$ in $\widehat{\beta}_B$, per cent. Average weight = 1.56 per cent

$t \downarrow\ \ s \rightarrow$	1983	1984	1985	1986	1987	1988	1989	1990
1983	2.222	2.437	1.793	1.705	1.579	1.516	1.260	1.129
1984	2.437	2.868	2.084	2.043	1.857	1.749	1.517	1.330
1985	1.793	2.084	1.654	1.662	1.497	1.468	1.255	1.101
1986	1.705	2.043	1.662	1.733	1.555	1.499	1.303	1.154
1987	1.579	1.857	1.497	1.555	1.554	1.543	1.403	1.266
1988	1.516	1.749	1.468	1.499	1.543	1.656	1.524	1.328
1989	1.260	1.517	1.255	1.303	1.403	1.524	1.513	1.276
1990	1.129	1.330	1.101	1.154	1.266	1.328	1.276	1.135

Table A3: Coefficients of Correlation, Log-Output. $N = 10, T = 8$.

A. Within Firm, R_{WXij}

$i \downarrow\ \ j \rightarrow$	1	2	3	4	5	6	7	8	9	10
1	1.000	-0.435	0.909	-0.146	0.912	0.518	-0.601	0.861	-0.572	0.329
2	-0.435	1.000	-0.435	0.333	-0.347	-0.052	0.081	-0.408	0.550	0.080
3	0.909	-0.435	1.000	-0.144	0.952	0.765	-0.701	0.964	-0.626	0.559
4	-0.146	0.333	-0.144	1.000	0.069	0.235	0.309	0.022	0.446	0.178
5	0.912	-0.347	0.952	0.069	1.000	0.706	-0.696	0.938	-0.393	0.628
6	0.518	-0.052	0.765	0.235	0.706	1.000	-0.503	0.835	-0.501	0.647
7	-0.601	0.081	-0.701	0.309	-0.696	-0.503	1.000	-0.695	0.196	-0.803
8	0.861	-0.408	0.964	0.022	0.938	0.835	-0.695	1.000	-0.594	0.637
9	-0.572	0.550	-0.626	0.446	-0.393	-0.501	0.196	-0.594	1.000	0.090
10	0.329	0.080	0.559	0.178	0.628	0.647	-0.803	0.637	0.090	1.000

B. Within Year, R_{VXts}

$t \downarrow\ \ s \rightarrow$	1983	1984	1985	1986	1987	1988	1989	1990
1983	1.000	0.965	0.936	0.869	0.850	0.790	0.687	0.711
1984	0.965	1.000	0.957	0.916	0.879	0.803	0.728	0.737
1985	0.936	0.957	1.000	0.982	0.934	0.887	0.794	0.804
1986	0.869	0.916	0.982	1.000	0.947	0.885	0.805	0.823
1987	0.850	0.879	0.934	0.947	1.000	0.962	0.915	0.954
1988	0.790	0.803	0.887	0.885	0.962	1.000	0.963	0.969
1989	0.687	0.728	0.794	0.805	0.915	0.963	1.000	0.974
1990	0.711	0.737	0.804	0.823	0.954	0.969	0.974	1.000

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