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Tom Kornstad

Are Predicted Lifetime Consumption Profiles Robust with respect to Model Specifications?

Abstract:
Structural models of lifetime labour supply and consumption require functional specifications of preferences as well as other assumptions that can be difficult to assess a priori. Misspecifications of the model might lead to biases in the estimates of preferences, that may influence the simulation results obtained from the model. In this paper we study to what extent predicted distributions of lifetime consumption are robust with respect to the specification of preferences. The simulation results show that simulated distributions of lifetime consumption strongly depend on the estimate of the intertemporal substitution rate as well as on the estimate of the time preference rate relative to the marginal interest rate net of taxes.

Keywords: Lifetime preferences, life cycle consumption, testing modelspesifications, empirical models.

JEL classification: D91, C52, D12.

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Address: Tom Kornstad, Statistics Norway, Research Department. E-mail: tom.kornstad@ssb.no

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Statistics Norway
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Telephone: +47 62 88 55 00
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1 Introduction

Life cycle models of labour supply and consumption have received much attention in the literature due to their micro- and macroeconomic policy implications. However, it is not straight forward to apply this type of models in evaluation of policy experiments. The difficulties are related to the issue of functional form in the empirical specification, and to the fact that important variables often are unobserved (such as fixed costs of working for example) or corrupted by measurement errors. Difficulties in determining the size of the rate of time preference relative to the marginal interest rate is also a problem that complicates applications of this type of models.

In the context of policy simulation experiments it is of course of great importance to assess to what extent the results are robust with respect to empirical misspecifications, typically due to unknown functional form and distributions of unobservables. The problem of robustness is more accentuated in empirical life cycle studies than in the simpler case of static settings since estimation of empirical life cycle models is very data demanding if preferences are non-separable both within- and across periods. Estimation thus requires simplifying assumptions, see the works of MaCurdy (1981), Heckman and MaCurdy (1980), Browning, Deaton and Irish (1985) and Blundell (1987) for a review and a discussion of the most used empirical specifications.[1]

Unfortunately, simulation experiments are difficult to carry out since it is impossible to find a closed form solution for the distribution of consumption over the life

[1]Empirical life cycle studies typically assume that preferences are intertemporally separable and many studies also assume intraperiod separability, cf. Hall (1978), Bover (1989), Heckman and MaCurdy (1980), MaCurdy (1981, 1983), Hall and Mishkin (1982), Zeldes (1989) and Runkle (1991) for examples of studies using micro data. The demand for durables is ignored or treated rather superficially in most analyses, and it is often assumed that the intertemporal rate of substitution is constant across households and over time, cf. Heckman and MaCurdy (1980), MaCurdy (1981) and Altonji (1986) for examples of labour supply analyses, and Wickens and Molana (1984), Hall (1988) and Browning (1989) for consumption analyses.

cycle for most specifications of preferences. Hence, there has not been a systematic testing of the robustness of the simulation results with respect to the assumptions underlying the estimated models. Recently, however, Attanasio et al. (1999) show that empirical life cycle models can be used for policy simulations by applying a simulation technique based on the method proposed by Deaton (1991).

Attanasio et al. explore whether empirical life cycle models can reproduce the humps and bumps one typically observes in the distribution of consumption over the life cycle. Specifically, they examine the total effect of a number of partial hypothesis including the specification of preferences. The present paper, in contrast, studies to what extent simulated distributions of consumption across the life cycle are robust with respect to changes in only one of these underlying assumptions, namely the specification of preferences. For the sake of simplicity we have chosen a particular class of utility functions (within-period preferences of the Box-Cox type[2]), and analyze the effects of changing the parameter values of this function. In particular, we examine the effects of assuming different magnitudes of the parameter determining the intertemporal substitution elasticity, and different magnitudes of the rate of time preference relative to the marginal interest rate net of taxes.

The model specifications are quite parsimonious. It is assumed that households live in an environment of perfect certainty, and that there are no binding constraints in the credit markets. Even though these simplifications may seem unreasonable from an empirical point of view, this simple model contains important characteristics of the more sophisticated models. Thus, we think that the findings from this model are of interest for more complex models as well. Notice also that whereas the present analysis studies consumption, the results are also relevant for lifetime labour supply analyses since these analyses often apply the same framework.

According to our simulation results, distributions of consumption across the life cycle are quite sensitive to the magnitude of the intertemporal substitution

[2]The Box-Cox function is a quite flexible specification of within-period preferences that includes the quadratic and the Stone Geary function as special cases.

elasticity and the rate of time preference. In the discussion of the effects of these parameters, it is appropriate to distinguish between the level and the profile of lifetime consumption. The profile of lifetime consumption is primarily determined by the intertemporal substitution elasticity and the distance between the rate of time preference and the marginal interest rate. For a given distance between these two rates, their levels have only a minor impact on the consumption profile. And the larger this distance is, and the larger the intertemporal substitution elasticity is (both measured in absolute values), the larger is the relative change in consumption from one year to another. An exception is the case where the rate of time preference equals the marginal interest rate. In this case consumption is evenly distributed across the life cycle independent of the magnitude of the time preference rate. Notice that this assumption is often applied in macro economic analyses since the no-Ponzi-game condition may otherwise be violated (see Blanchard and Fischer 1989).

The implication that the distribution of consumption across the life cycle depends on the distance between the rate of time preference and the marginal interest rate might be a reason for further reflection about the specification of life cycle models. In many countries the marginal interest rate net of taxes varies considerable over time and across families, and if the magnitude of the rate of time preference is independent of these variations, there can be considerable gaps between these two rates. The signs of the gaps might also vary from period to period. If the intertemporal substitution elasticity is not close to zero, this type of models would then predict considerable variations in the level of consumption across periods due to changes in the interest rate. Status today is that we don't know precisely to what extent these predictions are consistent with real household behavior.

The level of lifetime consumption is determined primarily by total household incomes and to a lesser extent by the specification of preferences. Interest rates also influence the consumption levels as increased interest rates increase interest expenditures paid on loans and capital incomes from holding assets. We don't examine further the determination of the level of lifetime consumption since our focus is the specification of preferences.

The paper is organized as follows. Section 2 presents the model specifications used in the simulations. Parameter values for the exogenous state variables as well as the simulation results are presented in Section 3. Finally, the paper is summarized in Section 4.

2 Model specification

The analysis focuses the behavior of one particular household, and ignores the possibility of overlapping generations. The household is assumed to live in an environment of perfect certainty with respect to future prices, variables determining future preferences and it's lifetime T.[3] Lifetime preferences over consumption are given by

$$U_0 = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \frac{(C_t + z_0)^\sigma - 1}{\sigma}, \qquad \sigma < 1, \sigma \neq 0, \tag{1}$$

where ρ is the rate of time preference, C_t is consumption of non-durables[4] in period t, and z_0 and σ are parameters that are constant over time. The parameter z_0 is often interpreted as the subsistence level of consumption, and in this case $z_0 \leq 0$, but there are other interpretations as well which imply that z_0 can be any real number, see Kornstad (1995). Taste modifier variables are ignored in the specification of preferences as we don't analyze the effects of shifts in preferences.

Within period preferences are assumed to be quasi-concave, and the parameter σ is then less than one. When $\sigma = 0$, our specification of preferences is defined by $\ln(C_t + z_0)$. Notice also that σ influences the intertemporal substitution elasticity[5]

[3]There are several works studying the effects of lifetime uncertainty on precautionary saving, see for instance Hubbard and Judd (1987) and Hurd (1989).

[4]In my Dr. polit thesis, Kornstad (1995), I present arguments for defining Z_t as a particular Hicks composite goods of durables and non-durables, but this will complicate the analysis and we thus focus on the case of non-durables.

[5]The intertemporal substitution elasticity measures the percentage change in consumption in any two periods in response to a percentage change in the relative price for those periods.

of consumption, which is equal to $1/(\sigma - 1)$ for $z_0 = 0$. The magnitude of this elasticity as well as the intertemporal substitution elasticity for labour supply have been important issues within empirical life cycle studies of consumption and labour supply. Blundell et al. (1989) and Attanasio and Browning (1993) point out that the estimates of these elasticities may depend crucially on model specifications, and status today is that we don't know precisely the magnitudes of these parameters, nor the robustness of the results with respect to model specifications. Most empirical studies, however, find that the intertemporal substitution elasticity for consumption is between -1 and 0, which corresponds to $0 < \sigma < 1$ (for $z_0 = 0$).

In the determination of consumption the household faces a set of wealth constraints that are assumed to hold. We ignore the possibility of being retired and receiving retirement pension, and the household receives labour incomes in all periods of life. If there is no income taxation and no binding constraints in the credit market, the wealth constraints are given by

$$w_t + r_t A_{t-1} = p_t C_t + (A_t - A_{t-1}), \qquad t = 0, 1, \cdots, T, \tag{2}$$

where w_t is the household's labour incomes at age t, r_t is the lending and borrowing rate, A_t is assets, and p_t is price of consumption. The initial (A_{-1}) and terminal (A_T) stock of assets are assumed to be given exogenously, and they are finite[6].

The first order conditions corresponding to maximizing the time preference discounted sum of lifetime preferences with respect to $C_0, C_1, \ldots, C_T$, subject to the wealth constraints (2), include the constraints and the equations

$$(C_t + z_0)^{\sigma-1} = \lambda_t p_t, \qquad t = 0, 1, \ldots, T, \tag{3}$$

[6]By ingoring the terminal condition (A_T) for assets, it follows that the optimal consumption path will yield higher and higher debt over time in order to enable the household to pay interest expenditures on the existing debt. To prevent unreasonable accumulation of debt, macro economic analyses thus often assume that the no-Ponzi-game condition is fulfilled. According to this assumption, total net debt cannot increase asymptotically faster than the interest rate.

and

$$\lambda_t = \frac{1 + r_{t+1}}{1 + \rho} \lambda_{t+1}, \qquad t = 0, 1, \ldots, T - 1, \tag{4}$$

where $\lambda_t = \lambda_t^*(1 + \rho)^t$ and λ_t^* is the marginal utility of wealth. Notice that r_{t+1} is the marginal interest rate net of taxes in the case of income taxation.

In order to obtain an equation for the consumption profile across the life cycle, we substitute equation (3) into equation (4). For constant consumption prices this Euler equation for consumption can be written as

$$\ln \frac{C_{t+1} + z_0}{C_t + z_0} = \frac{1}{\sigma - 1} \ln \frac{1 + \rho}{1 + r_{t+1}}, \quad \sigma < 1, \quad t = 0, 1, \ldots, T, \tag{5}$$

where the coefficient $1/(\sigma - 1)$ is the intertemporal rate of substitution when $z_0 = 0$.

This expression implies the following relations between the marginal interest rate r_{t+1} and the rate of time preference ρ:

$$C_{t+1} = C_t \quad \text{if} \quad r_{t+1} = \rho, \tag{6}$$

$$C_{t+1} > C_t \quad \text{if} \quad r_{t+1} > \rho, \tag{7}$$

$$C_{t+1} < C_t \quad \text{if} \quad r_{t+1} < \rho. \tag{8}$$

That is, if the marginal interest rate (net of taxes) equals the rate of time preference, consumption is constant across the life-cycle no matter the values of the rates of time preference and intertemporal substitution. If the marginal interest rate exceeds the rate of time preference, consumption increases over the life-cycle. In this case the incentive to wait overcomes impatience, and it pays to postpone consumption. And the larger the absolute value of the intertemporal substitution elasticity is, the larger is the increase in consumption. But consumption does not increase indefinitely since lifetime consumption is constrained by the wealth constraints and the initial and terminal condition on wealth.

In contrast, consumption decreases over the life cycle if the marginal interest rate is less than the rate of time preference. Notice, however, that equations (5) do not determine how changes in the interest rate influence the level of consumption. The consumption path rises more steeply (that is, falls less steeply if the marginal interest rate is less than the rate of time preference) by age if the interest rate increases, but this increase also leads to a shift in the consumption path.

3 Simulation results

The first order conditions constitute a simultaneous equation system of $2(T+1)+T$ equations that determine the distribution of C_t, A_t and λ_t across the life cycle. The exogenous variables include the distributions of wage incomes (w_t), consumption prices (p_t) and interest rates (r_t) across the life cycle, and the initial (A_{-1}) and terminal (A_T) stock of assets. Since the budget sets are convex, the first order conditions are both necessary and sufficient conditions for optimum, and one can apply a simulation algorithm method that uses these conditions in the determination of the optimal consumption path.

It is assumed that the terminal stock of assets converted into its period 0 equivalent equals the initial stock of assets, and the household's lifetime consumption expenditures must then equal its lifetime wage incomes. Consumption prices are fixed equal to one in all periods, and annual disposable labour income is NOK 263000[7]. The interest rate is assumed to be constant over time, and is specified in the figures that present the simulation results.

Figures 1-5 show how the predicted distribution of consumption over a period of $T = 20$ years depends on the parameters of the preference function (z_0, σ and ρ) and our assumptions about lifetime wage incomes, consumption prices and interest rates. The used values of σ are based on the observation that most empirical life cycle studies find that the intertemporal elasticity of substitution (I) lies between 0 and -1.[8] Thus, the distribution of consumption over the life cycle is predicted

[7]This figure corresponds to the average labour incomes net of taxes for married couples in the data applied in Kornstad (1995).

[8]Blundell, Browning and Meghir (1989) report that the average elasticity of intertemporal substitution in consumption (I) is about -1. Other studies often find that it is closer to zero, cf. for instance Altonji 1986 (I is close to -0.3) and Blundell, Meghir and Neves 1993 (I is close to

for $\sigma \in \{-19, -1, 0\}$, which corresponds to $I \in \{-.05, -.5, -1\}$, respectively, when $z_0 = 0$.

Figure 1: The distribution of consumption across life cycle for various values of z_0. The time preference rate is .01 and σ is equal to -1.

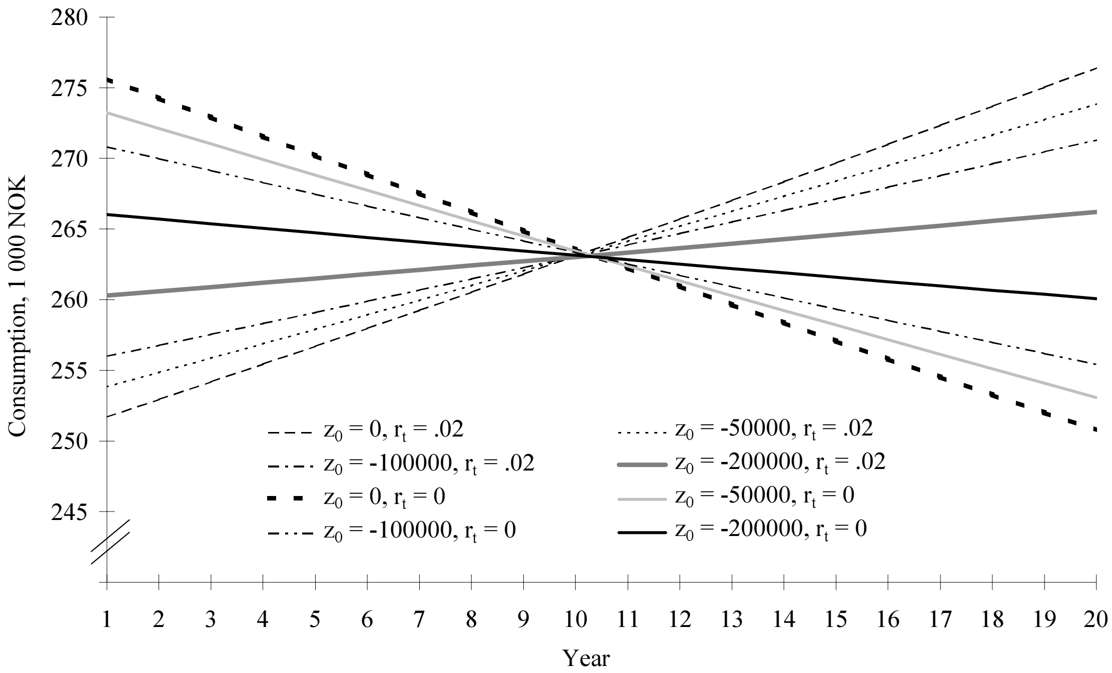


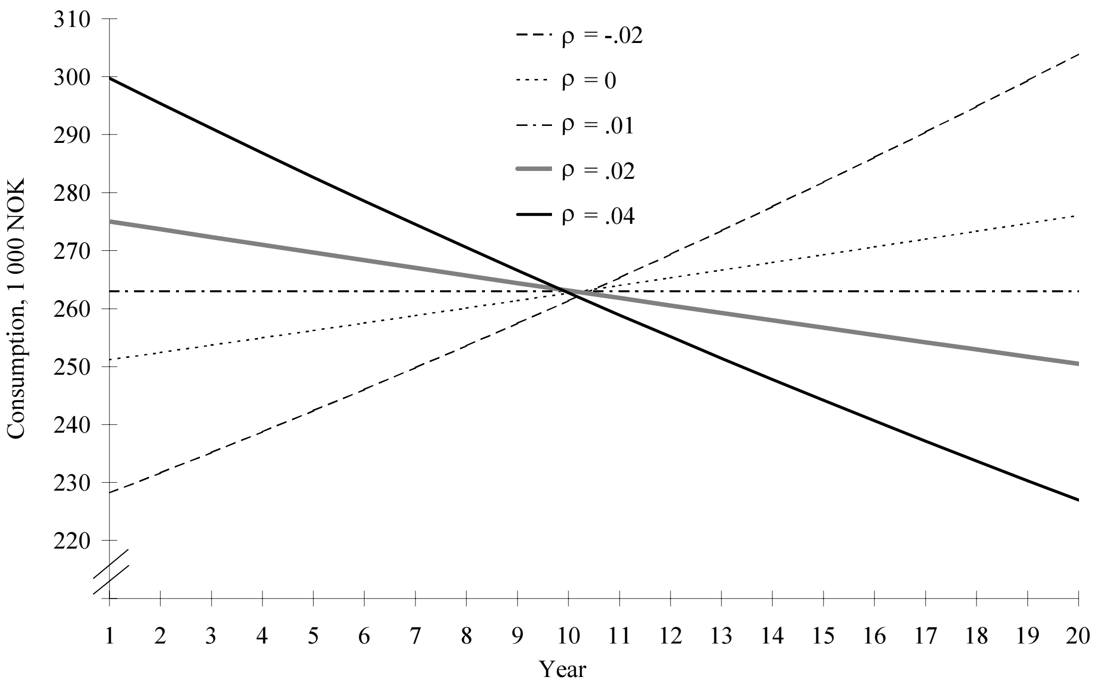
Figure 1 shows how the distribution of consumption across the life cycle depends on the value of z_0, for $z_0 \in \{-200000, -100000, -50000, 0\}$. Non-positive values for z_0 are chosen as z_0 is interpreted as subsistence level of consumption.[9] It is assumed that $\sigma = -1$ (the intertemporal substitution elasticity is $-.5$ for $z_0 = 0$), and that the rate of time preference is .01. The real marginal interest rate (net of taxes) takes two different values, 0 percent (falling graphs) and 2 percent (rising graphs). Notice that for reasonable values of the interest rate and the rate of time preference it is primarily the distance between these two rates that is relevant for

-0.5).

[9]Notice that equation (5) is not defined for $z_0 \leq -Z_t$.

the consumption *profile* across the life cycle, whereas the levels of these two rates are of less importance, see also equation (5).

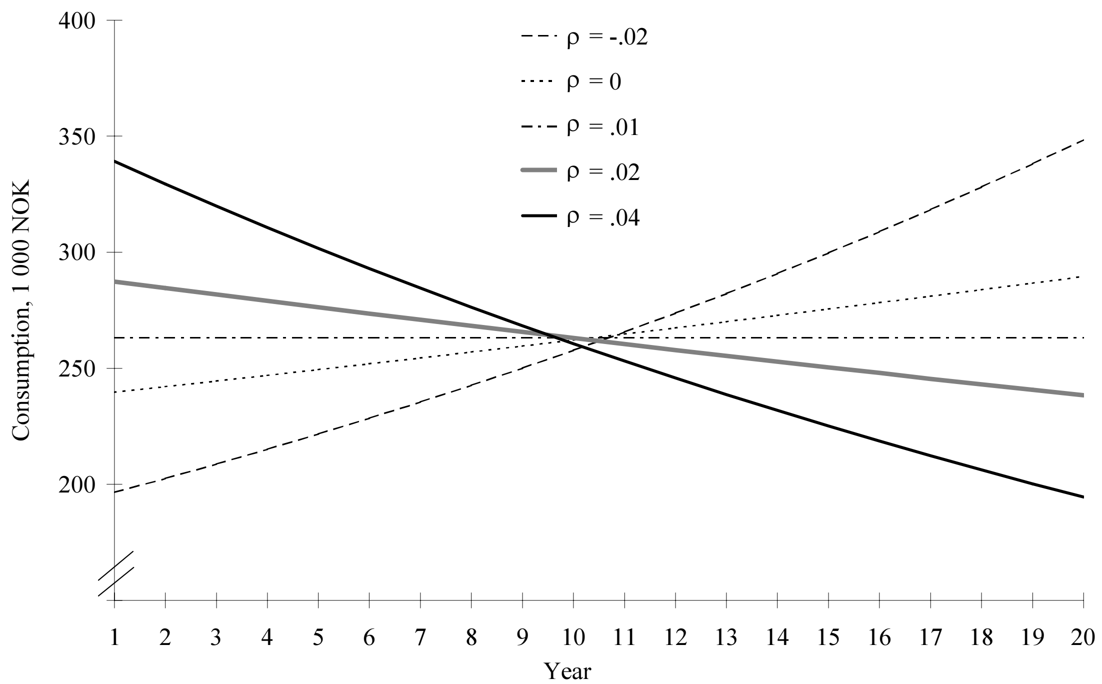
Figure 2: The distribution of consumption across life cycle for various values of the time preference rate. The marginal interest rate net of taxes is .01, the intertemporal substitution elasticity is -.5 ($\sigma = -1$) and z_0 is 0.



From the figure we see that an increase in z_0 from -200000 to 0 leads to a steeper consumption path across the life cycle, but annual consumption does not change very much. If z_0 is -200000 and the rate of time preference differs from the interest rate by only one percentage point as it does in this figure, consumption is close to permanent wage income (= 263000 NOK) for all years. An increase in z_0 from -200000 to 0 yields a maximum change in consumption of about 10000 NOK according to our results and assumptions, but related to annual consumption this change is modest. This conclusion remains valid for reasonable values of σ, ρ, r_{t+1} and T, and we conclude that the consumption profile across the life cycle is relatively insensitive to changes in z_0.

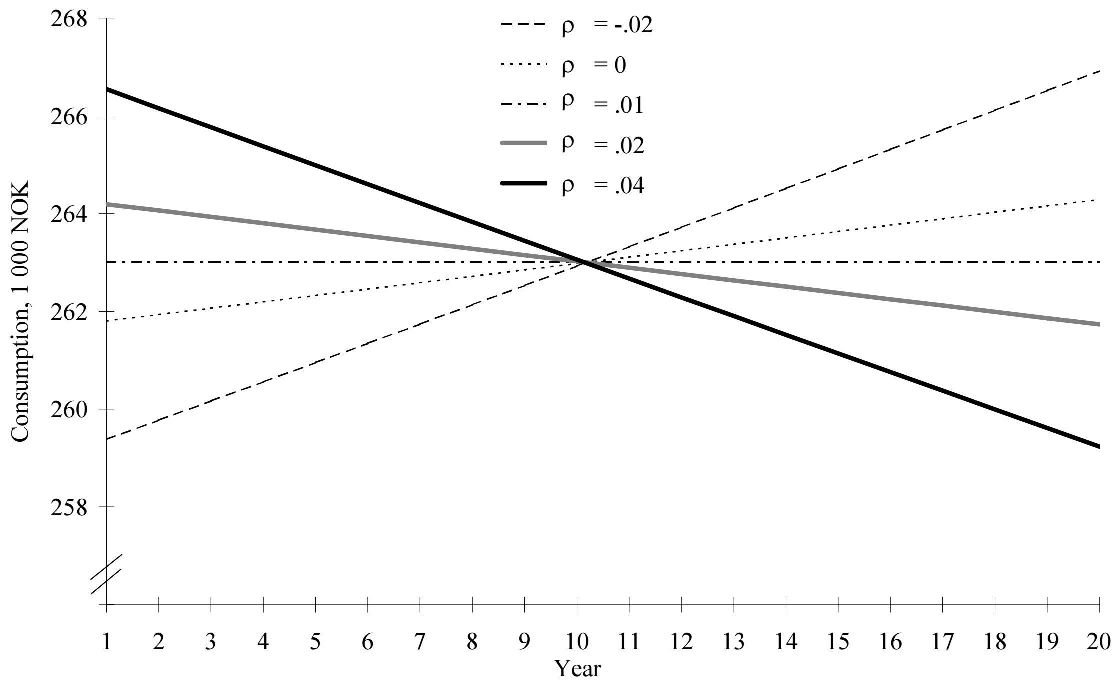
The rate of time preference or, to be more precisely, the magnitude of $\ln(1+\rho)$ relative to $\ln(1+r_{t+1})$ might of cause have larger effects on the distribution of consumption across the life cycle. Whereas ρ is typically assumed to be a fixed parameter in empirical analyses, the real marginal interest rate net of taxes varies considerably across both persons and time in real life. According to Statistics Norway (2000), the real average borrowing rate on bank loans in Norway during the period 1991-1999 has varied between about -5 percent to about 7 percent. If the rate of time preference is independent of the interest rates, this means that there can be a considerable difference between the values of these two rates.

Figure 3: The distribution of consumption across life cycle for various values of the time preference rate. The marginal interest rate net of taxes is .01, the intertemporal substitution elasticity is -1.0 ($\sigma = 0$) and z_0 is 0.



An illustration of how the ratio between these two rates influences the lifetime consumption profile can be found in Figure 2, where it is assumed that the intertemporal elasticity of substitution is -.5 and $z_0 = 0$. The real marginal interest rate net of taxes is one percent, and $(\rho \times 100) \in \{-2, 0, 1, 2, 4\}$. The horizontal line is consumption when the real marginal interest rate equals the rate of time preference, see equation (5). The graphs for $\rho = 0$ and $\rho = .02$ represent a one percentage point difference between the rate of time preference and the interest rate, whereas the graphs for $\rho = -.02$ and $\rho = .04$ represent a 3 percentage points difference.

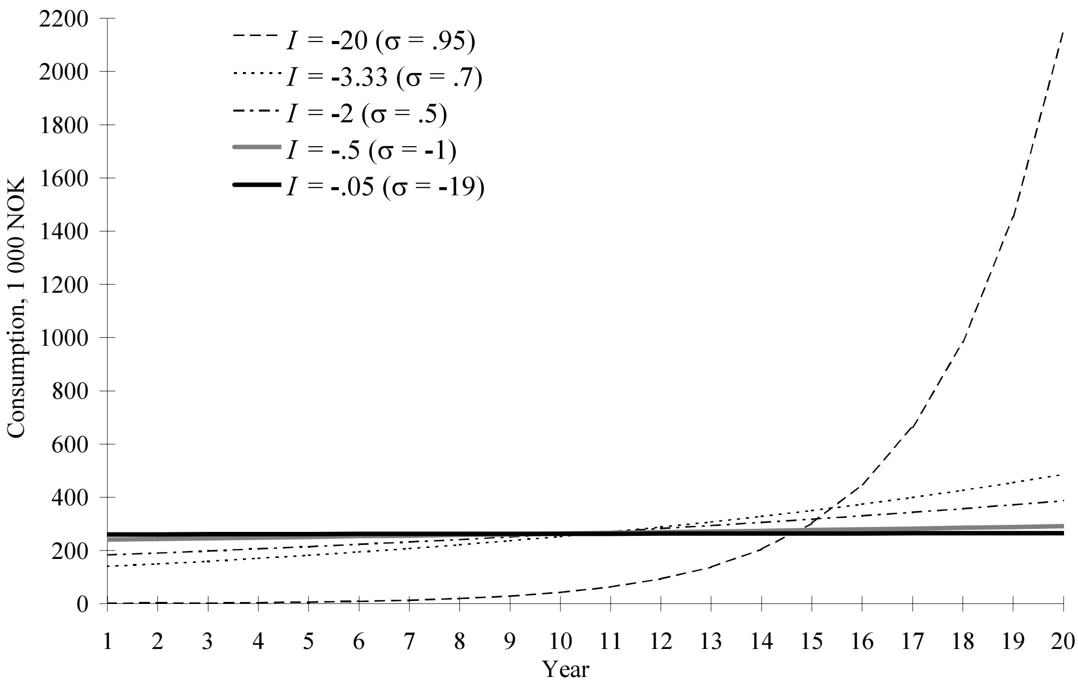
Figure 4: The distribution of consumption across life cycle for various values of the time preference rate. The marginal interest rate net of taxes is .01, the intertemporal substitution elasticity is -.05 ($\sigma = -19$) and z_0 is 0.



According to the figure, the difference between the real marginal interest rate and the rate of time preference has a significant effect on the lifetime consumption path even if the intertemporal substitution elasticity is as small as $-.5$. If the intertemporal substitution elasticity is greater (in absolute value), the impact of a partial change in the rate of time preference increases, see Figure 3. This figure indicates that the distribution of lifetime consumption is quite sensitive to the magnitude of the rate of time preference if the intertemporal substitution elasticity is equal to -1. In contrast, large as well as small changes in the difference between the rate of time preference and the marginal interest rate have only little effects on the distribution of consumption across the life cycle if the intertemporal substitution elasticity is (very) close to zero, see Figure 4.

It follows from figures 2-4 that small biases in the estimates of intertemporal substitution elasticities might have significant effects on the predicted distribution of consumption over the life cycle. For policy evaluations it is thus of great importance whether the elasticities are close to 0 or close to -1, even if the rate of time preference and the marginal interest rate net of taxes are not very different.

Figure 5: The distribution of consumption across life cycle for various values of the intertemporal substitution elasticity (I). The marginal interest rate is .03, the time preference rate is .01 and z_0 is 0.



For the utility function (1) to be quasi-concave, σ must be less than one. In Figure 5 we study the effects on lifetime consumption when σ tends to this limit. The distribution of lifetime consumption is shown for $\sigma \in [-19, -1, .5, .7, .95]$, which corresponds to $I \in \{-.05, -.5, -2, -3.33, -20\}$. It is assumed that the interest- and rate of time preference are 3 percent and 1 percent respectively. From the figure we see that also small differences in the interest- and time preference rates have very large effects on the distribution of lifetime consumption when σ tends to the limit. When the difference between these two rates increases, this picture is dramatized further. Reasonable estimates of σ should then be considerably less than one.

4 Summary

In this paper we have studied the effects on the distribution of consumption over the life cycle of varying the specification of preferences within a particular class of functional forms. Not surprisingly we find that the predicted distribution of consumption over the life cycle is sensitive to the specification of preferences. Although the model specifications are quite simple, we believe that this finding is relevant for more sophisticated model specifications as well. Our findings suggest that the specification of preferences should be an important subject for research in the future.

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