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Non-parametric Identification of the Mixed Hazards Model with Interval-Censored Durations

Abstract:

Econometric duration data are typically interval-censored, that is, not directly observed, but observed to fall within a known interval. Known non-parametric identification results for duration models with unobserved heterogeneity rely crucially on exact observation of durations at a continuous scale. Here, it is established that the mixed hazards model is non-parametrically identified through covariates that vary over time within durations as well as between observations when durations are interval-censored. The results hold for the mixed proportional hazards model as a special case.

Keywords: duration analysis, interval-censoring, non-parametric identification

JEL classification: C41

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1 Introduction

The topic of this paper is non-parametric identification in duration models specified as continuous time models, when the data considered are interval-censored.[1] There are two separate reasons why the issue is important. First, econometric duration data for some phenomena, such as unemployment durations, tend to be available at a coarser level of time units than what is used in economic theory. Hence it is important to investigate to what extent it is possible to recover the structure of a model at a finely-grained time scale from such imperfect observations. Secondly, core identification results for econometric duration models seem to be extremely dependent on the exact observation of durations. It is thus important to investigate to what extent such results generalize to finely-grained interval-censored observations. Data are after all never continuous, but at best finely-grained interval-censored.

The combination of continuous time hazard rate models and interval-censored duration data is common enough to have generated a voluminous literature. There are basically three approaches to estimation of models in this setting. The first is to simply assume away the interval-censoring in the sense that data are treated as if they were not censored. Not surprisingly, this may lead to problems, see e.g. Bergstrøm and Edin (1992) or Røed and Zhang (2002). The second approach is to derive the likelihood of the interval-censored observations from a continuous time model and use this likelihood as a basis for estimation. Flinn and Heckman (1982) give an early discussion of this. The third approach is to specify the model as a discrete duration model. A discrete duration model may or may not be consistent with a hazard rate model. For cases where the discrete time models are consistent with such underlying continuous time models, the second and third approaches are equivalent. Han and Hausman (1990) and Sueyoshi (1995) estimate discrete duration models that are consistent with hazard rate models, while e.g. van den Berg and van Ours (1994) estimate discrete duration models that are not consistent with hazard rate models, but on the other hand allow for simplification of some estimation procedures.

There is a large econometric literature on non-parametric identification of Mixed Hazards (MH) models, usually appearing under the guise of Mixed Proportional Hazards (MPH) models, see van den Berg (2001) for details. Elbers and Ridder (1982) proved identification of the MPH model under an assumption of finite mean for the heterogeneity distribution, while Heckman and Singer (1984a) proved a similar result with alternative assumptions on the fatness of the right hand tail of the same distribution. Ridder (1990) clarified these issues and showed how the differing assumptions on the tail or mean of the heterogeneity distribution serve to identify the model while taking the limit as spell lengths approach zero. Different assumptions regarding the fatness of the tail lead to qualitatively different types of duration dependence. Hence, the condition on the distribution of heterogeneity is a crucial identifying assumption. Heckman and Honoré (1989) and Abbring and van den Berg (2003) generalize these results

[1] Interval-censored duration data, also known as discrete or grouped duration data, are not observed directly, but have end points that are observed to lie within a known time window.

to competing risks models. Brinch (2007) proved that variation over time in covariates, in addition to variation over observations, is sufficient to identify the MH model without assuming a proportional hazards structure. Identification with interval-censored durations have also been studied, though with few positive results. Sueyoshi (1995) demonstrates the impossibility of recovering the hazard function behaviour within intervals. Ridder (1990) shows that the MPH model is not identified under assumptions corresponding to the classical results for uncensored data, but that the model is identified if covariates are assumed to enter the structural hazard function linearly. McCall (1994) showed that the model is still identified when the coefficients associated with the linear function of covariates are interval specific. Further semi-parametric identification results are discussed in Meyer (1995) and Bierens (2008). Heckman and Navarro (2007) presents results for sequential discrete choice models that generalize the problems studied in this paper. These results are discussed further below.

This paper provides two new identification results. In contrast to the main results from the literature, exactly observed durations are not required. In contrast to the other papers that consider identfication under interval-censoring, parametric restrictions on the effects of covariates are not required. In contrast to most of the literature, the proportional hazards assumption is not required. However, in constrast to most of the literature, time-varying covariates are required. The first contribution of the present paper is to show that it is, in contrast to other identification results for MH models, possible to generalize the result from Brinch (2007) to the case with interval-censored durations. The result is however not possible to generalize directly, as covariates that range over a continuous set are necessary with interval-censored durations, in constrast to the case without interval-censoring. Further, and more trivially, it is not possible to identify the within-interval structural hazard function, but only the integrated structural hazard function over the relevant intervals. The second contribution is a partial identification result that does not rely on analytic continuation of the Laplace transform of the distribution function for unobserved heterogeneity. The result shows that with time-varying continuous covariates, the integrated structural hazard function may be identified for all periods following the first change in covariates, even without identification of the distribution of unobserved heterogeneity. A similar result does not exist in the literature, even for data without interval-censoring.

2 The identification results

Define a covariate path as a function $x : \mathbb{R}_+ \to X$. Let $S \subset (\mathbb{R}_+ \times X)$ be a family of such paths.

The role of a structure of the MH model is to specify the distribution of a non-negative random variable T as a function on S. The survival function of T, $G : (\mathbb{R}_+ \times S) \to [0, 1]$ is given by

$$G(t; x) = \mathcal{L}(\Lambda(0, t, x)), \tag{1}$$

where $\mathcal{L}$ is the Laplace transform (see Feller, 1971) of another non-negative random variable V, and

$$\Lambda(s,t;x) = \int_s^t \lambda(r;x(r))\,dr, \tag{2}$$

where $\lambda : (\mathbb{R}_+ \times X) \to \mathbb{R}_+$. λ is denoted the *structural hazard function* and Λ the *integrated structural hazard function.*

A *structure of the MH model* can be defined as a pair $\{\mathcal{L}, \Lambda\}$ that satifies the above requirements. The *MH model* is defined as the set of all such admissible pairs.

A wider class of models will also be studied. This class is an extension of the GAFT (Generalized Accelerated Failure Time) class discussed in Ridder (1990), which does not impose the separability condition corresponding to the proportional hazards assumption. Let us call this class EGAFT, for Extended GAFT. A structure of the EGAFT model is defined as a pair $\{L, \Lambda\}$, where the survival function is given by

$$G(t;x) = L(\Lambda(0,t,x)), \tag{3}$$

where Λ is defined as for the MH model and L is any positive, strictly decreasing, continuously differentiable function with L(0)=1.

The MH model is a subset of the EGAFT class, as the Laplace transform of a random variable satisfies the requirements of L in the EGAFT definition. There are, however, EGAFT structures that are not MH structures.

With interval-censoring, whether or not durations have ended is only observed at a finite number of points in time. E.g. in the study of unemployment spells, whether individuals have left unemployment is only observed at the end of each month.

Definition 1 *A random variable T on R_+ is interval-censored with observation set $Q = \{t_1, \ldots, t_n\}$ if and only if (i) T is not observed, (ii) a random variable T' on $\{1, 2, \ldots, n\}$ is observed, (iii) $T' = i \iff T \in [t_i, t_{i+1})$, for $i = 1, \ldots, n-1$ and $T' = n \iff T \geq t_n$ and (iv) Q is a set of known positive real numbers with $t_1 < t_2 < \ldots < t_n$.*

We would say that a model is *identified* under interval-censoring if the structure of the model is uniquely determined by the survival function at the observation set Q. There is no hope of full identification of these continuous time models under interval-censoring, so the identification results will be stated in terms of the requirements of *empirically equivalent* structures, structures that imply the same survival function (as a function of covariate processes) at the points in time given by Q.

Identification depends crucially on variation over time in covariates. It is possible to specify a large number of different covariate processes relevant for time-varying covariates. The approach taken here is to characterize the properties that the process must satisfy rather than to specify results conditional on some specific covariate process. The requirements are specified in Assumption 1.

Assumption 1 *Let U be an open subset of R^2. There exists a function $z : U \to S$, denoted $z(r_1, r_2)$, such that when $x = z(r_1, r_2)$ $x(s)$ is independent of r_2 for $s \leq t_a$, $x(s)$ is independent of r_1 for $s > t_a$, and $\Lambda(0, t_b, z(r_1, r_2))$ is a continuously differentiable and non-constant function on U, with $t_a, t_b \in Q$, with $t_a < t_b$.*

Assumption 1 requires variation over time in covariates, in the sense that the integrated structural hazard rate over one time interval is not always a function of the integrated structural hazard rate over another time interval. Hence, Assumption 1 fails if all covariates are constant over time or if the value the covariates at one point of time is sufficient for deriving the full covariate path (such as duration, age or any constant covariate intersected with such a covariate). In addition, covariates take on values on an open set and the integrated hazard rates are continuously differentiable and not independent of the (relevant) covariates.

The archetypical time-varying covariate process for an interval-censored duration model is a jump covariate process with jump times corresponding to the points in time in the observation set, as in e.g. Sueyoshi (1992). Thus, $x(s) = x_1 \in X$ for $s < t_1$, $x(s) = x_2 \in X$ for $t_1 \leq s < t_2$, etc. In such a case, the identification results are straightforward to apply. We can just take r_1 and r_2 to be covariate values in different time intervals and define $x(s) = z(r_1, r_2) = r_1$ for $s < t_1$, and $x(s) = z(r_1, r_2) = r_2$ for $t_1 \leq s < t_2$. Note that the covariates need only to change at one point of time for the results in this paper to apply. Assumption 1 is thus usually satisfied in applications that include time-varying continuous covariates, such as in e.g. Arulampalam and Stewart (1995) or Carling et al. (1996). The only relevant reason why Assumption 1 should fail in presence of (proper) time-varying covariates is that the relevant covariates are discrete, as in Røed and Zhang (2003). Assumption 1 is however not limited to this sort of archetypical covariate process. It is quite conceivable with covariates that change at a finer time scale than observations and that such changes are observed or can be computed. An example of such a time-varying covariate that can change continuously over time is the present value of the future benefit flow as unemployed, for analyses of unemployment duration data. The fact that Assumption 1 reaches beyond the archetypical covariate process discussed above is important because the discrete nature of covariate processes in duration data can be a feature of ad hoc discretization rather than a real feature of the data generating process of the covariate. In this context it is nice to know that such discretization over time of covariates is not a necessary condition for identification.

Lemma 1 *Suppose durations are given by the EGAFT model and are interval-censored with observation set Q. Under Assumption 1, for given z, two empirically equivalent structures of the EGAFT model $\{L, \Lambda\}$ and $\{K, \Pi\}$, must satisfy*

$$\Pi(0, t_a, z(r_1, r_2)) = D_a + C\Lambda(0, t_a, z(r_1, r_2)) \tag{4}$$

and

$$\Pi(t_a, t_b, z(r_1, r_2)) = D_b + C\Lambda(t_a, t_b, z(r_1, r_2)) \tag{5}$$

for $(r_1, r_2) \in U$*, where* D_a*,* D_b *and* $C > 0$ *are constants.*

Proof. Given a function z, the survival probability to t_b can now be defined as a function of r_1 and r_2.

For two structures $\{L, \Lambda\}$ and $\{K, \Pi\}$ to be empirically equivalent

$$\frac{\partial G(t_b; x)/\partial r_2}{\partial G(t_b, x)/\partial r_1} = \frac{\partial \Lambda(t_a, t_b, z(r_1, r_2))/\partial r_2}{\partial \Lambda(0, t_a, z(r_1, r_2))/\partial r_1} = \frac{\partial \Pi(t_a, t_b, z(r_1, r_2))/\partial r_2}{\partial \Pi(0, t_a, z(r_1, r_2))/\partial r_1} \tag{6}$$

Equations (4) and (5) follow immediately by integration over r_1 and r_2, respectively. Different C in equations (4) and (5) would not be consistent with equation (6). ■

Lemma 1 follows almost immediately from the fact that G, under Assumption 1, can be defined as a continuously differentiable and nonconstant function of r_1 and r_2. This function may be used to define yet another function that specifies r_2 as a function of r_1, by requiring G to be constant. A constant G implies a constant integrated structural hazard function and thus, the ratio of the derivatives of the integrated structural hazard function w.r.t. r_1 and r_2 follows.

Theorem 1 *Suppose durations are interval-censored with observation set* Q *and that the family of covariate processes satisfies Assumption 1, for some* $t_a, t_b \in Q$*. Then, two structures* $\{\mathcal{L}, \Lambda\}$ *and* $\{\mathcal{K}, \Pi\}$ *of the MH model are empirically equivalent if and only if*

$$\Pi(0, t_i, x) = C\Lambda(0, t_i, x), \tag{7}$$

for all $t_i \in Q$*, with* $C > 0$ *a constant, and*

$$\mathcal{L}(Cw) = \mathcal{K}(w), \tag{8}$$

for all $w \in R_+$*.*

Thus, two empirically equivalent structures are identical up to a scale transformation of the distribution of unobserved heterogeneity and a corresponding (inversely proportional) scale transformation of the integrated structural hazard function evaluated at the points of time in the observation set.

Proof. Since the MH model is also an EGAFT model, Lemma 1 applies. Thus, for two structures $\{\mathcal{L}, \Lambda\}$ and $\{\mathcal{K}, \Pi\}$ of the MH model to be empirically equivalent,

$$\mathcal{L}\left(\Lambda(0, t_a, z(r_1, r_2))\right) = \mathcal{K}\left(D_a + C\Lambda(0, t_a, z(r_1, r_2))\right). \tag{9}$$

Equation (9) must hold for all r_1 on an open interval. Thus, the equation must also hold for all Λ on an open interval, that is

$$\mathcal{L}(w) = \mathcal{K}(D_a + Cw) \tag{10}$$

for all w on an open interval.

Since $\mathcal{L}$ and $\mathcal{K}$ are analytic, see e.g. Feller (1971), equation (10) must hold for all w such that the functions are defined. Thus

$$1 = \mathcal{L}(0) = \mathcal{K}(D_a), \tag{11}$$

which implies $D_a = 0$.

It follows that the integrated structural hazard function is identified up to a scale factor at the points in time of the observation set, as

$$\Pi(0, t_i, x) = \mathcal{K}^{-1}(G(t_i; x)) = C\mathcal{L}^{-1}(G(t_i; x)) = C\Lambda(0, t_i, x) \tag{12}$$

for all $x \in S$. ■

Note that the assumption of a finite mean for the distribution of unobserved heterogeneity is not required, as in e.g. Elbers and Ridder (1982).[2]

Another way to ensure (partial) non-parametric identification of the EGAFT model that does not rely on an analytic continuation, is the following:

Assumption 2 *For each $t_i \in Q$, with $t_i > t_a$, Assumption 1 holds with $t_b = t_i$, with the same z for all t_i. Further, there exists $(r_1^{i'}, r_2^{i'}) \in U$ and $(r_1^{i''}, r_2^{i''}) \in U$ such that $G(t_i, z(r_1^{i'}, r_2^{i'})) \geq G(t_{i-1}, z(r_1^{i''}, r_2^{i''}))$.*

The survival probability to time t is a function of the covariates. Assumption 2 specifies that the ranges of the survival probabilities to time t and $t+1$ overlap for all $t \in Q$ such that $t \geq t_a$. The relevance of Assumption 2 depends on the length of intervals and on the explanatory power of the covariates. Equal survival probabilities imply equal integrated structural hazard functions, so the unidentified additive term D_b from Lemma 1 in the interval leading up to $t+1$ is identified:

Theorem 2 *Suppose durations are interval-censored with observation set Q and that the family of covariate processes satisfies Assumption 2. Let $s_2 = \sup_{r_1, r_2} G(t_a, z(r_1, r_2))$ and $s_1 = \inf_{r_1, r_2} G(t_n, z(r_1, r_2))$. Two empirically equivalent structures $\{L, \Lambda\}$ and $\{K, \Pi\}$ of the EGAFT model must satisfy the equations*

$$\Pi(0, t_i, x) = D_a + C\Lambda(0, t_i, x) \tag{13}$$

[2]The technical reason for this difference is that the variation in the data serve to identify the Laplace transform up to a scale parameter. For the classical MPH identification results, the data only serve to identify the Laplace transform up to a scale parameter and an additional parameter affecting the shape of the distribution, corresponding to $\mathcal{L}(C_1 w^{C_2}) = \mathcal{K}(w)$ in the notation from Theorem 1, with C_1 and C_2 being constants. The role of the assumptions on the fatness of the tail or the finiteness of the mean in the literature is to identify the C_2-constant. Clearly, this problem does not appear in Theorem 1 and no corresponding assumptions are necessary.

for all x and t_i such that $G(t_i, x) \in (s_1, s_2)$ and

$$K(D_a + Cw) = L(w), \tag{14}$$

for all w such that $L(w) \in (s_1, s_2)$ where D_a and $C > 0$ are constants. Any two structures conforming to these equations will in fact be empirically equivalent for all $t_i \in Q$, $x \in S$ such that $G(t_i, x) \in (s_1, s_2)$.

Proof. Lemma 1 applies. In particular, for two structures to be empirically equivalent,

$$\Pi(t_a, t_i, z(r_1, r_2)) = D_i + C\Lambda(t_a, t_i, z(r_1, r_2)) \tag{15}$$

for each $t_i > t_a$.

From Assumption 2, there are, for $i = a+1$, $(r_1^{i'}, r_2^{i'}) \in U$ and $(r_1^{i''}, r_2^{i''}) \in U$ such that $G(t_i, z(r_1^{i'}, r_2^{i'})) \geq G(t_{i-1}, z(r_1^{i''}, r_2^{i''}))$. Clearly, $G(t_i, z(r_1^{i'}, r_2^{i'})) \leq G(t_{i-1}, z(r_1^{i'}, r_2^{i'}))$. Since G is continuous on U, by Assumption 1 and the definition of the EGAFT class, there are, by the Intermediate Value Theorem, also $(r_1^{i'''}, r_2^{i'''}) \in U$ such that $G(t_i, z(r_1^{i'}, r_2^{i'})) = G(t_{i-1}, z(r_1^{i'''}, r_2^{i'''}))$. Thus,

$$\begin{aligned} &\Pi(0, t_i, z(r_1^{i'}, r_2^{i'})) = \Pi(0, t_{i-1}, z(r_1^{i'''}, r_2^{i'''})) = \\ &\quad D_a + C\Lambda(0, t_{i-1}, z(r_1^{i'''}, r_2^{i'''})) = D_a + C\Lambda(0, t_i, z(r_1', r_2')), \end{aligned} \tag{16}$$

thus $D_i = 0$. The same argument can then be applied for $i = a + 2$, $i = a + 3$, etc.

Thus, equation (13) holds for all x such that $x = z(r_1, r_2)$, with $(r_1, r_2) \in U$. Thus

$$K(D_a + C\Lambda(t_i, z(r_1, r_2))) = L(\Lambda(t_i, z(r_1, r_2))) \tag{17}$$

for $(r_1, r_2) \in U$, $t_i \geq t_a$, and hence equation (14) holds for all w such that $L(w) \in (s_1, s_2)$.

Now,

$$L(\Lambda(0, t_i, x)) = K(D_a + C\Lambda(0, t_i, x)) \tag{18}$$

for all $x \in S$, $t_i \in Q$ such that $L(z) \in (s_1, s_2)$ and

$$L(\Lambda(0, t_i, x)) = K(\Pi(0, t_i, x)), \tag{19}$$

for all $x \in S$, $t_i \in Q$, imply equation (13) for all x and t_i such that $L(z) \in (s_1, s_2)$.

The converse is trivial (by substitution). Two models that satisfy equations (13) and (14) need however not be empirically equivalent for survival probabilities outside the interval (s_1, s_2). ■

Theorem 2 thus shows that identification of the MH model is only partially dependent on exact identification of the distribution function for unobserved heterogeneity. Most of the integrated structural hazard function is non-parametrically identified without relying on analytical continuation of the Laplace transform.

Theorem 2 has no continuous time counterpart in the literature. However, the result is easy to apply to continuous time data as a limit result: If two models are not empirically equivalent with interval-censored duration, they are not equivalent with uncensored durations either. Thus, with continuous cross-sectional variation in time-varying covariates, the structural hazard function is identified from the first point in time where "past" covariate values may differ from "current" covariate values, without relying on analytical continuation. Assumption 2 is easily satisfied with sufficiently short time between the elements in the observation set.

3 Discussion

Theorem 1 in this paper can be seen as an interval-censored version of the result in Brinch (2007). The main difference is that the covariates in Brinch (2007) were only required to take on two values, whereas the structural hazard function in this paper is required to be a differentiable function of covariates taking on values on an open set. Such an extension is necessary, as one can not identify general continuous functions from a finite number of cell probabilities, which is the empirical content of interval-censored duration models without continuous variation in covariates. Even though the proportional hazards assumption is not used in the current paper, the identification results provided here are also new for the widely used MPH model.

It is useful to contrast the EGAFT class studied above to the basic (non-parametric) models studied in Heckman and Navarro (2007), HN in the following. The basic discrete duration model in their paper is specified as

$$\left.\begin{array}{l} D(t) = 1(I(t) > 0) \\ I(t) = f_t(x(t)) - \eta(t) \end{array}\right\} \text{if} D(t-1) = 0, t = 1, \ldots, \tag{20}$$

where $\eta(t)$ are random terms that may be dependent over time and f_t are functions of the covariates $x(t)$. The EGAFT model may be described similarly as

$$\left.\begin{array}{l} D(t) = 1(I(t) > 0) \\ I(t) = \sum_{s=0}^{t} f_s(x(s)) - \eta \end{array}\right\} \text{if} D(t-1) = 0, t = 1, \ldots, \tag{21}$$

where η is a random term with distribution function corresponding to $1 - L$ from equation (3). The EGAFT class is a non-parametric ordered response model with covariate dependent thresholds. The EGAFT class fits into the framework of HN by specifying $\eta(t)$ as η for $t = 1$ and

$$\eta(t) = -f_{t-1}(x(t-1)) + \xi(t-1), \tag{22}$$

for $t > 1$, where $\xi(t)$, $t = 1, \ldots$ are random variables distributed as $\eta(t)$, conditional on $\eta(t) > f(x(t))$. The framework of HN is thus more general than the EGAFT class. However, the restrictions inherent in the EGAFT class relative to the framework of HN are useful for identification purposes, as they allow us to use Theorem 2 to uncover duration dependence, while similar results do not exist for the HN framework. The non-parametric results in HN rely on limiting attention to special classes of functions, following Matzkin (1994). The model is only identified if one limits f_t to classes of functions where no function can be expressed as a strictly increasing transformation of another. It seems unlikely that this can be interpreted as identification of structural duration dependence. It is more natural to interpret this as a meta-rule that can be used to decide whether semi-parametric models are identified. Since neither class of models is given any structural interpretation, and the necessary assumptions for identification based on HN and the current paper are not nested, their relative relevance must be considered context-dependent. The identification results for the EGAFT class are relevant primarily because they apply to the MH model.

Theorem 2 suggests the possibility of estimating EGAFT models with the motivation of providing a specification test of MH models. Sueyoshi (1994) considers such estimation in the context of the GAFT class. Different results based on EGAFT and MH specifications will *indicate* that the data generating process is not likely to be an MH model. However, MH models are usually semi-parametrically estimated following Heckman and Singer (1984b) and there exists no asymptotic distribution theory for their estimator, so formal testing is difficult. Within the context of semi-parametric estimation it should be possible to estimate EGAFT class models based on methods for ordered response models, following Klein and Spady (1993) and Klein and Sherman (2002), giving estimators with known asymptotic properties. Bearse, Canals-Cerdá and Rilstone (2007) provide similar semi-parametric estimators for GAFT-class duration models, without taking interval-censored durations into account. Theorem 2 suggests that when applying such estimators, caution is required when interpreting the duration dependence in the structural hazard functions at the start of spells, as the information one discards by moving from the MH model to the EGAFT class leaves the model only partially identified from a non-parametric point of view.

It is unfortunately not possible to generalize the results in this paper to dependent competing risks (DCR) models. In the DCR model, destination specific hazard functions are defined as $v_i\lambda_i(t)$, conditional on (dependent) random variables $V_i = v_i$ for $i = 1, \ldots, m$, where m is the number of destination states. The MH model is thus a DCR model with one destination state. Unfortunately, the DCR model does not lend itself to interval-censored data as easily as the MH model. The integrated structural transition intensities are not identified even if we know the heterogeneity distribution, because there is not a one-to-one relationship between the integrated structural transition intensities and the transition probabilities over an interval, as the transition probabilities depend on the within-interval behaviour of the destination specific structural hazard functions. The identification of the DCR model has not even been proved with uncensored durations, without additional assumptions of proportional hazards and restrictions on the heterogeneity distribution.

4 Conclusion

This paper has proved that the MH model is non-parametrically identified through cross-sectional and temporal variation in covariates, even in the case where durations are interval-censored. This result closes a gap between non-parametric identification results, relying on continuous observation of durations and applied econometric duration analyses, where it is usually appreciated that durations are interval-censored. The results provided here emphasize the importance of finding suitable time-varying covariates for the identification of unobserved heterogeneity and structural duration dependence.

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