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**Abstract:**
The present paper applies synthetic graphical modelling to the binary panel data of the Muscatine Coronary Risk Factor Study, where the observations are subject to non-response. The methodology combines the techniques of the graphical and generalized linear models. The purpose is to demonstrate the various aspects of the modelling approach than to obtain accurate empirical knowledge on the matter dealt with. We emphasize the flexibility, interpretation, and sensitivity (or robustness) of the models proposed.

**Keywords:** Panel design, non-response, graphical model, GLM

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# 1 Introduction

Obesity in school children was the subject in the Muscatine Coronary Risk Factor Study (Lauer 1975; Louisiana State University Medical Center 1978; Woolson and Clarke 1984). At each examination, a child is classified as either *obese* or *not obese* based on measurements of weight and height. The collected data are both longitudinal (i.e. each experimental unit is followed biennially over an eligible period of time) and cross-sectional (i.e. groups of both sexes and different initial ages are included). Table 1 lists the data from the last three examinations made in 1977, 1979 and 1981 which consist of 5 age groups, i.e. 5-7, 7-9, 9-11, 11-13 and 13-15, in both sexes. In this way each sex-age group constitute a *three-wave* binary panel subject to non-response, where e.g. B1 denotes the first age group (5-7) of boys, and G4 denotes the fourth age group (11-13) of girls, and so on. Meanwhile, denote by "N" not obese, by "O" obese and by "M" missing. For reasons which are not clear to us, no counts of the all-missing cell were available; and we shall treat them as if this has occurred out of randomness.

We shall not consider measurement error, i.e. misclassification of obesity, in the present study since it is believed to be of minimal effect given the fact that the classification does not depend on the wills of the participants and the examiners. Numerically, at each wave $i$, we denote by $x_i = 1$ that a child is obese and by $x_i = -1$ the opposite, and we denote by $r_i = 1$ that $x_i$ is available and by $r_i = 0$ the opposite. The observation $Y_i$ is therefore given as

$$Y_i = X_i R_i, \tag{1}$$

where $y_i = 0$ denotes non-response ("M"), $y_i = 1$ denotes obese ("O") and $y_i = -1$ denotes not obese ("N"). Several analyses of either the entire data set in Table 1 or some of its subsets can be found in the literature. Whereas Woolson and Clarke (1984), Lipsitz, Laird, and Harrington (1994), Fitzmaurice, Laird, and Lipsitz (1994) and Azzalini (1974) have proceeded under the assumption of ignorable non-response, Park and Davis (1993), Conaway (1994) and Baker (1995) have all found evidence of the opposite.

A common objective of these analyses has been the effect of age (denoted by $A$) and sex (denoted by $S$) on the marginal probabilities of obesity, in which respect Baker (1995) presented a model where these probabilities depend on the covariates via a logistic regression model. The methodology is related to that of Diggle and Kenward (1994), Molenberghs, Kenward, and Lesaffre (1997) and Fitzmaurice, Laird, and Zahner (1996) except that Baker (1995) allowed for more causal patterns of non-response than mere dropouts. More explicitly, for each $m$-wave panelist, the joint probability of $x = (x_1, ..., x_m)$

Table 1: Muscatine Coronary Risk Factor Study 1977 - 1981

| | Panels | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Data | B1 | B2 | B3 | B4 | B5 | G1 | G2 | G3 | G4 | G5 |
| NNN | 90 | 150 | 152 | 119 | 101 | 75 | 154 | 148 | 129 | 91 |
| NNO | 9 | 15 | 11 | 7 | 4 | 8 | 14 | 6 | 8 | 9 |
| NON | 3 | 8 | 8 | 8 | 2 | 2 | 13 | 10 | 7 | 5 |
| NOO | 7 | 8 | 10 | 3 | 7 | 4 | 19 | 8 | 9 | 3 |
| ONN | 0 | 8 | 7 | 13 | 8 | 2 | 2 | 12 | 6 | 6 |
| ONO | 1 | 9 | 7 | 4 | 0 | 2 | 6 | 0 | 2 | 0 |
| OON | 1 | 7 | 9 | 11 | 6 | 1 | 6 | 8 | 7 | 6 |
| OOO | 8 | 20 | 25 | 16 | 15 | 8 | 21 | 27 | 14 | 15 |
| NNM | 16 | 38 | 48 | 42 | 82 | 20 | 25 | 36 | 36 | 83 |
| NOM | 5 | 3 | 6 | 4 | 9 | 0 | 3 | 0 | 9 | 15 |
| ONM | 0 | 1 | 2 | 4 | 8 | 0 | 1 | 7 | 4 | 6 |
| OOM | 0 | 11 | 14 | 13 | 12 | 4 | 11 | 17 | 13 | 23 |
| NMN | 9 | 16 | 13 | 14 | 6 | 7 | 16 | 8 | 31 | 5 |
| NMO | 3 | 6 | 5 | 2 | 1 | 2 | 3 | 1 | 4 | 0 |
| OMN | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 2 | 0 |
| OMO | 0 | 3 | 3 | 4 | 1 | 1 | 4 | 4 | 6 | 1 |
| NMM | 32 | 45 | 59 | 82 | 95 | 23 | 47 | 53 | 58 | 89 |
| OMM | 5 | 7 | 17 | 24 | 23 | 5 | 7 | 16 | 37 | 32 |
| MNN | 129 | 42 | 36 | 18 | 13 | 109 | 47 | 39 | 19 | 11 |
| MNO | 18 | 2 | 5 | 3 | 1 | 22 | 4 | 6 | 1 | 1 |
| MON | 6 | 3 | 4 | 3 | 2 | 7 | 1 | 7 | 2 | 2 |
| MOO | 13 | 13 | 3 | 1 | 2 | 24 | 8 | 13 | 2 | 3 |
| MNM | 33 | 33 | 31 | 23 | 34 | 27 | 23 | 25 | 21 | 43 |
| MOM | 11 | 4 | 9 | 6 | 12 | 5 | 5 | 9 | 1 | 15 |
| MMN | 70 | 55 | 40 | 37 | 15 | 65 | 39 | 23 | 23 | 14 |
| MMO | 24 | 14 | 9 | 14 | 3 | 19 | 13 | 8 | 10 | 5 |

and $r = (r_1, ..., r_m)$ is factorized into

(2) $$p(x, r|b) = p(x|b)p(r|x, b)$$

where $b$ denotes some known covariates. A submodel on $p(r|x, b)$ specifies the non-response mechanism which is nonignorable unless $p(r|x, b) = p(r|b)$. An "outcome model", denoted by $M_O$, accounts for the marginal probabilities $p(x_i|b)$ for $1 \leq i \leq m$ — a logistic regression in this case. In addition, an "association model", denoted by $M_A$, is introduced to allow for "temporal associations" among the components of $x$. The probability $p(x|b)$ is in this way specified as a function of these two models, i.e. $p(x_1, ..., x_m|b) = g(M_O, M_A)$; and there are a number of parameterisations, i.e. $g(\cdot)$, through which the marginal outcome model can be combined with the association model — see e.g. the relevant references in Baker (1995).

Despite some inconvenience in presentation and possible difficulty in computation as pointed out by Baker (1995), this modelling approach seems natural for multivariate data typical of the analysis of contingency tables, or longitudinal studies using other designs than the panel. In contrast to such *symmetric* data, a noticable feature of the panel design is that the data and our interest are often *asymmetric* in the sense that, besides the marginal probabilities $p(x_i|b)$, we are primarily concerned with the transition probabilities $p(x_i|x_1, ..., x_{i-1})$ instead of, say, $Cov(x_i, x_{i-1})$. This is indeed one of the reasons why panel designs are indispensable for studies on inter-strata flows within the population, since the data collected are able to provide evidence of change on an individual basis, which stands in contrast to other designs for repeated surveys where different samples are taken on each occasion as e.g. in a trend survey.

Given non-response, it is natural from such an asymmetric point of view to model the panel data in terms of a more detailed factorized likelihood. For instance, the two factors of (2) can be further factorized into

(3) $$p(x|b) = p(x_1|b)p(x_2|x_1, b) \cdots p(x_m|x_1, ..., x_{m-1}, b)$$

and, similarly,

(4) $$p(r|x, b) = p(r_1|x, b)p(r_2|r_1, x, b) \cdots p(r_m|r_1, ..., r_{m-1}, x, b),$$

which is reasonable if the previous response (or non-response) pattern is helpful in "explaining" the response (or non-response) behaviour which follows. Notice that all the factors are conditional probabilities of a single variable here, since only one variable is interested at each wave both in (3) and (4). Moreover, (3) and (4) define a *complete* dependence structure in the sense that each variable is "explained" by *all* the variables which so far have been "explained". Reductions in the complete structure can be achieved if some of the conditioning variables are ruled out from the factors, which amounts to introducing certain conditional independence. For instance, simplifying $p(x_3|x_1, x_2, b)$ to $p(x_3|x_2, b)$ in (3) implies that $X_3$ is assumed to be independent of $X_1$ conditional to $x_2$.

We outline the basic approach of synthetic graphical modelling in Section 2. For the present binary panel data, we shall concentrate on the chained logistic regression models as a special case. In section 3 we study the various modelling aspects w.r.t. the effect of age and sex on obesity among school children. Section 4 contains a discussion on sensitivity analysis, followed by some concluding remarks.

# 2 Synthetic graphical modelling

## 2.1 Directed graphs for asymmetric data

The data from each panelist in Table 1 consist of $X = (x_1, x_2, x_3)$ and $R = (r_1, r_2, r_3)$, in addition to known konstants $A = (a_1, a_2, a_3)$ and $S = s$. The asymmetric dependence structure among these variables and constants implied by (3) and (4), either in their complete or reduced forms, can be summarized by a *directed graph*, acyclic to be sure. Indeed, Wermuth and Lauritzen (1983) named a factorization in the form of (3) and (4) "recursive" due to the *a priori* ordering among the variables induced by it. Synthetic graphical modelling thus employs the graphical techniques for the (conditional) dependence structure among the data developed in the theory of graphical models (Lauritzen 1996; Cox and Wermuth 1996).

We shall briefly go through the concepts relevant for the present analysis. Consider for instance the following two factoriztions of $p(x, r|a, s)$, i.e.

$$p(x, r|a, s) = p(x_1)p(x_2|x_1)p(x_3|x_1, x_2)p(r_1|x_1)p(r_2|x_2, r_1)p(r_3|x_3, r_2)$$

$$p(x, r|a, s) = p(x_1|a, s)p(x_2|x_1, a, s)p(x_3|x_1, x_2, a, s)p(r_1|x_1)p(r_2|x_2, r_1)p(r_3|x_3, r_1, r_2).$$

The dependence structure implied by the two differs in that the former assumes that (i) $(X, R)$ are independent of $(a, s)$, and (ii) $R_3$ is independent of $R_1$ conditional to $(r_2, x_3)$. The graphical representation of these two factorizations are, respectively, graph $\vec{G}_{1a}$ in Figure 1.a and $\vec{G}_{1b}$ in Figure 1.b.

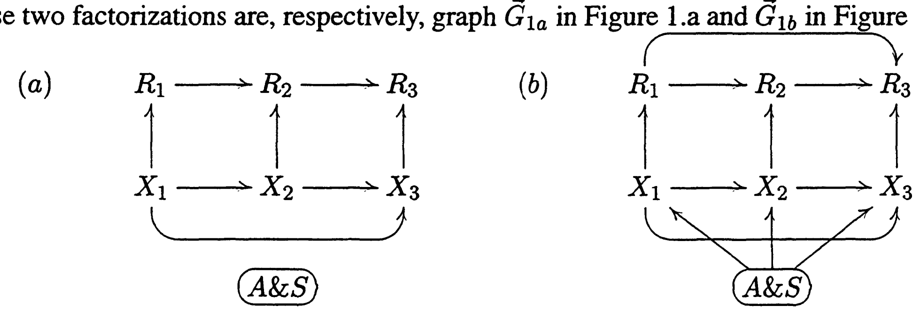



Figure 1. Two graphs for the three-wave non-response panel.

Formally, let $V$ contain all the variables and constants involved in a factorization of the joint probability of the data. There exists an arrow, in the corresponding directed graph, pointing from $V_i$ to $V_j$ if and only if $V_i$ is *explanatory* for $V_j$, i.e. $V_i$ is one of the conditioning variables/constants for the conditional probability of $V_j$. Together these define the corresponding directed graph, denoted by $\vec{G} = (V, E)$ where $E$ is the set of arrows. In standard graph theory, $V$ is the set of vertices. In particular, a vertex is framed here if it corresponds to some known constant of the model; and no arrow exists between any two framed vertices by stipulation. Hence $(A, S)$ is framed in Figure 1.

Graphical theory generalizes Markov chains to Markov fields. For instance, if two vertices $V_i$ and $V_j$ are *separated* by a subset $V_S \subset V$, then $V_j$ is independent of $V_i$ conditional to $V_S$, where $V_S$ separates $V_j$ from $V_i$ if to "travel" from $V_i$ to $V_j$ by following the arrows necessarily passes some vertices in $V_S$. Thus, while $(R_2, X_3)$ separates $R_3$ from $X_1$ in $\vec{G}_{1a}$, they do not in $\vec{G}_{1b}$ — $R_1$ is needed in addition.

One advantage of the graphical techniques which is particularly useful for us here, lies in their flexibility in isolating the various aspects of the dependence structure among the data by means of the induced subgraphs. Formally, a *subgraph* $\vec{G}_S = (V_S, E_S)$ of $\vec{G}$ induced by $V_S \subseteq V$ has $V_S$ as its vertex set and, for any $V_i$ and $V_j$ from $V_S$, $(V_i, V_j) \in E_S$ if and only if $(V_i, V_j) \in E$. It follows, among other things, that $V_i$ and $V_j$ are independent conditional to some subset of $V$ in $\vec{G}$ if and only if they exists some subset of $V_S$, possibly empty, conditioned on which $V_i$ and $V_j$ are independent in $\vec{G}_S$.

Inspection of (3) and (4) shows that $(R_1, R_2, R_3)$ conditional to $(x, b)$ have the same recursive order as that of $(X_1, X_2, X_3)$ conditional $b$. This means that the explanatory structure among the present $(R, X, A, S)$ can be elaborated in terms (i) that among $\{R_1, R_2, R_3\}$ and $\{X_1, X_2, X_3\}$ with suitable conditioning, and (ii) the way $X$ is joined to $(A, S)$ and $R$ to $(X, A, S)$. Figure 2 contains some possible subgraphs for $R$ or $X$. Under suitable conditioning, the relevant variables are independent of each other in $\vec{G}_{2a}$, they form a Markov chain in $\vec{G}_{2b}$, whereas $\vec{G}_{2c}$ depicts the complete structure.

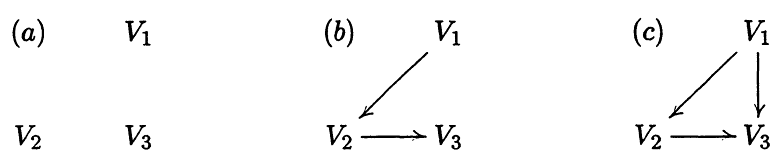
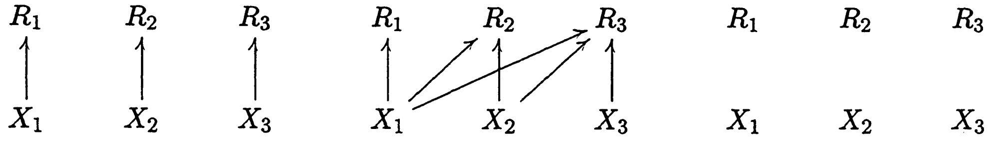



Figure 2. Some subgraphs induced by $R$ or $X$.

For a graphical representation of, say, the way in which $R$ is joined to $X$, we slightly extend notion of subgraphs so that they can also be induced by disjoint subsets of $V$. Formally, for any two disjoint subsets, say, $R$ and $X$ of $V$, an arrow $(V_i, V_j) \in E$ belongs to the subgraph induced by $(R, X)$ if and only if (i) $V_i \in R$ and $V_j \in X$, or (ii) $V_i \in X$ and $V_j \in R$. The asymmetry between $X$ and $R$ allows only case (ii) here; and Figure 3 contains some subgraphs induced by $R$ and $X$.




$(a)$ $(b)$ $(c)$

Figure 3. Some subgraphs induced by $R$ and $X$.

Directed graphs for $(R, X, A, S)$ can thus be generated by putting together the subgraphs induced by $R$, $X$, $X$ and $(A, S)$, and $R$ and $(X, A, S)$.

## 2.2 Chained logistic regression

Synthetic graphical modelling deviates from the graphical models once the corresponding graph has been constructed. While the graphical models are uniquely determined by their graphs, we allow for more detailed modelling of each factor. In particular, conditional GLMs (McCullagh and Nelder 1989) can be introduced recursively, whose covariates are generally given as a vector-valued function of the corresponding explanatory variables and constants. We call this function a *synthesizer*; hence the term "synthetic graphical modelling".

In (3) and (4), a conditional logistic regression can be introduced for each factor now that all the variables involved are binary. Thus, while the dependence structure among the data is represented by the corresponding directed graph, the explanatory details are spelled in terms of the synthesizers. Formally, let $V_1, ..., V_k$ be explanatory for binary $V_0$, the synthesizer for $V_0$ is then a vector-valued function $c_{v_0}(v_1, ..., v_k)$ such that $p = P[V_0 = 1 | v_1, ..., v_k]$ is given as

$$\text{(5)} \qquad \operatorname{logit} p = \log p - \log(1 - p) = \beta_{v_0} + c_{v_0}^T \beta.$$

In particular, the *identity* synthesizer is given as $(v_1, ..., v_k)^T$. Whereas a synthesizer is said to be *saturated* w.r.t. a set of binary variables $V_S \subseteq \{v_1, ..., v_k\}$ if for any nonempty subset $V_\alpha$ of $V_S$, $\prod_{v_i \in V_\alpha} v_i$ is an element of $\{c_{v_0}(v_1, ..., v_k)\}$. This follows once we notice that $V_S$ has $2^{|V_S|} - 1$ nonempty subsets and the same number of not-all-zero configurations. Thus, a synthesizer is saturated w.r.t. binary $v_1$ if $v_1 \in \{c_{v_0}(v_1, ..., v_k)\}$, it is saturated w.r.t. binary $\{v_1, v_2\}$ if $\{v_1, v_2, v_1 v_2\} \subseteq \{c_{v_0}(v_1, ..., v_k)\}$, and so on. Notice that, in the logistic regression context, the $(0, 1)$-specification of the variables is more convenient than the $(-1, 1)$-specification in (1) and will be adopted throughout the sequels.

Synthetic graphical modelling has thus led us to a family of chained logistic regression models, which are suitable for the present binary panel data with an asymmetric dependence structure. Suppose generic parameter $\beta$ and variables $(Z_1, ..., Z_m)$, where some of the components of $\beta$ may be common to several components of $Z$. Given non-response, not all the data are observed, and the m.l.e. $\hat{\beta}$ can be obtained using the EM algorithm. The E-step calculates the conditional expectations of the unobserved covariates of the GLMs. The M-step then solves the likelihood equation derived from the complete data, using e.g. the Fisher-scoring method which here coincides with the Newton-Raphson algorithm. For the logistic regression of $Z_i$, let $c_i$ be the coefficient-vector of $\beta$ conditional to $(z_1, ..., z_{i-1})$, where $c_{ij} \equiv 0$ unless $\beta_j$ is a parameter for $Z_i$. Denote by $\eta_i$ the corresponding conditional linear predictor, i.e. $\eta_i = c_i^T \beta$. The complete log-likelihood on $z$ is given as $l(\beta; z) = \sum_{i=1}^{m} [z_i \eta_i + \log(1 - p_i)]$, where $p_i = P[Z_i = 1 | z_1, ..., z_{i-1}]$. We have, for any $\beta_j$ and $\beta_k$,

$$\partial l(\beta; z)/\partial \beta_j = \sum_i c_{ij}(z_i - p_i) \qquad \partial^2 l(\beta; z)/\partial \beta_j \partial \beta_k = -\sum_i c_{ij} c_{ik} p_i (1 - p_i).$$

Given independent sample identically distributed as $Z$, and some starting value $\beta_{(0)}$, the Newton-Raphson algorithm updates, at the $r$-th step,

$$\beta_{(r+1)} = \beta_{(r)} - [(\frac{\partial^2 l}{\partial \beta^2})^{-1} (\frac{\partial l}{\partial \beta})]_{\beta = \beta_{(r)}}.$$

Convergence is rarely a problem with this type of model and data.

# 3 The effect of age and sex

## 3.1 Modelling the age effect

We investigate first the effect of age on the obesity of school children, separately for each sex. The vertex set of the model graphs is then $V_A = V \backslash \{S\} = \{X_1, X_2, X_3, R_1, R_2, R_3, a_1, a_2, a_3\}$. For the dependence structure among $V_A$, we assume that (i) $X_i$ does not depend on $a_j$ for $j \neq i$, and (ii) $R$ is independent of age conditional to $X$, so that the explanatory structure of a model on the effect of age is, possibly a reduction, of the form

$$p(x, r | a) = p(x_1 | a_1) p(x_2 | a_2, x_1) p(x_3 | a_3, x_1, x_2) p(r | x), \tag{6}$$

and $R$ is separated from $(a_1, a_2, a_3)$ by $X$ in the model graph. Figure 4 contains some of the explanatory structures allowed by (6). Notice how they are put together using the subgraphs from Figure 2 and 3 as explained before.

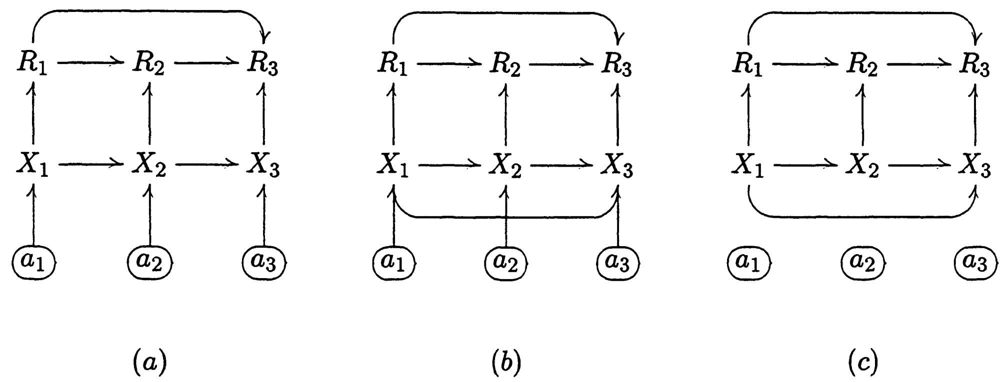


Figure 4. Some graphs involving the age effect for the three-wave non-response panel.

To set up the conditional logistic regressions, we set for simplicity the natural age of all children from a two-year age-group at the midpoint of that two-year interval, i.e. 6 for 5-7 age-group, 8 for 7-9 age-group, and so on. Meanwhile, denote by $c_i(a_i, x)$ the synthesizer for $X_i$ where $1 \leq i \leq 3$. To specify the marginal effect of age, we consider synthesizers of the form $c_i(a_i, x)^T = \{h_i(a_i), g_i(x)^T\}$. In other words, let $p_i = P[X_i = 1|a_i, x]$, the conditional logistic regression for $X_i$ under model (6) is of the form

$$\text{logit}\, p_i = \beta_0 + h_i(a_i)\beta_h + g_i(x)^T\beta_{g_i}, \tag{7}$$

where the parameters may vary from one panel to another, and $(\beta_0, \beta_h)$ are constants of $i$ within each panel. Notice that the contribution to the linear predictor for $X_i$ from Age is thus given as $\beta_0 + h_i(a_i)\beta_h$. In particular, within each panel, the identity $h_i(a_i) = a_i$ is equivalent to $h_i(a_i) = i - 1$, in which case the age effect simplifies to $\beta_0 + (i-1)\beta_h$. Meanwhile, we have $g_1(x) = \emptyset$ be default, and $g_2(x) = x_1$ unless $X_2$ is independent of $X_1$. Whereas $g_3(x) = x_2$ if $X_3$ is independent of $X_1$ given $x_2$; otherwise, $g_3(x) = (x_1, x_2)$ or $g_3(x) = (x_1, x_2, x_1x_2)$, of which the latter being the saturated case.

The synthesizers for $(R_1, R_2, R_3)$ are rather similar if we leave out the interaction between $X$ and $R$. That is, denote by $q_i$ the conditional probability of response at the $i$th wave, we have

$$\text{logit}\, q_i = \alpha_0 + k_i(x)^T\alpha_{k_i} + t_i(r)^T\alpha_{t_i}, \tag{8}$$

where $t_i(r)$ is similar to $g_i(x)$ in (7). The simplest choice for $k_i(x)$ is to let $k_i(x) = x_i$, in which case $R_i$ is independent $X_j$ conditional to $X_i$ and $R_j$ for $j \neq i$, as in Figure 4.

## 3.2 Age effects within each panel

Various chained logistic regression models have been fitted separately to all the 10 panels of Table 1, where the maximum attainable log-likelihood for each panel is given as $l(y; y) = \sum_{i=1}^{26} y_i \log y_i -$

$n \log n$, with $y_i$ being the counts of the 26 cells in each panel and $n = \sum_i y_i$ the size of panel. To summarize the results in words, we note:

1. For all the panels except G3, $\vec{G}_{2c}$ as the subgraph induced by $R$ was found to be necessary for a reasonable fit, together with saturated $t_2(r) = r_1$ and $t_3(r) = (r_1, r_2, r_1 r_2)$. As a matter of fact, other things being the same, the inclusion of $r_1 r_2$ as a covariate for $R_3$ typically resulted into tens of decrement in the deviance at the cost of one degree of freedom.

2. The subgraph induced by $X$ varied between $\vec{G}_{2b}$ and $\vec{G}_{2c}$. While $g_3(x) = (x_1, x_2, x_1 x_2)$ was sometimes necessary, some panels admitted even $(g_2, g_3) = (x_1, x_2)$ and $\beta_{g_2} = \beta_{g_3}$, in which case $(X_1, X_2, X_3)$ formed a Markov chain with identical transition probability conditional to $(a_1, a_2, a_3)$.

3. Age effect in the form of (7) with $h_i(a_i) = i - 1$ yielded reasonable fit in all the cases except for panel G1.

4. Subgraph $\vec{G}_{3a}$ induced by $(R, X)$ was found to yield reasonable fit in all cases except for panel G3, in which case $(R_1, R_2, R_3)$ depended on $(X_1, X_2, X_3)$ in the same way as $(X_1, X_2, X_3)$ on $(a_1, a_2, a_3)$.

For the numerical results, we denote by $M_{4a}$ the model with graph $\vec{G}_{4a}$ and by $M_{4b}$ that of $\vec{G}_{4b}$. For both models, we use $h_i(a_i) = i - 1$, and $k_i(x) = x_i$, and saturated $t_i(r)$. While $(g_2, g_3) = (x_1, x_2)$ and $\beta_{g_2} = \beta_{g_3}$ in $M_{4a}$, we set $g_2(x) = x_1$ and $g_3(x) = (x_1, x_2, x_1 x_2)$ in $M_{4b}$. To check on the age effect, we also fitted the model, denoted by $M_{4c}$, where $\beta_0 + h_i(a_i)\beta_h$ in (7) was substituted with $\eta_i$. Altogether, $p(x|a)$ has 7 parameters under $M_{4c}$ which is saturated for binary $(X_1, X_2, X_3)$; this is one more in number than $M_{4b}$ and four more in number than $M_{4a}$. The three models are the same w.r.t. $p(r|x)$.

The fitted log-likelihood, denoted by $\hat{l}(M; y)$, for all the 10 panels are given in Table 2, together with the respective $l(y; y)$. Compare $M_{4b}$ with $M_{4c}$, we find that only the deviance on G1 has decreased significantly from the former to the latter, i.e. 4.5 on one degree of freedom. This of course does not necessarily mean that there is no age effect here. Indeed, we obtain $\hat{\eta} = (-1.68, -2.75, -2.00)$, which might well be captured by a quadratic $\eta(i)$ requiring 3 parameters just like $(\eta_1, \eta_2, \eta_3)$. On the other hand, $M_{4a}$ appears poor-fitting for several panels compared $M_{4b}$. There are clearly a number of compromises between the two. However, we have omitted the details of such refinements now that they do not require theoretical considerations other than those exemplified above.

Table 2: Model fitting for the Muscatine Coronary Risk Factor Study data

| | Log-likelihood | | | | | | | | | | | Degree of |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B1 | B2 | B3 | B4 | B5 | G1 | G2 | G3 | G4 | G5 | Deviance | Freedom |
| $l(y;y)$ | -1165.6 | -1346.6 | -1381.5 | -1227.9 | -1107.1 | -1071.4 | -1243.5 | -1266.9 | -1189.1 | -1194.1 | | |
| $\hat{l}(M_{4a};y)$ | -1175.3 | -1367.4 | -1396.9 | -1238.8 | -1111.5 | -1080.8 | -1262.3 | -1283.4 | -1196.7 | -1199.2 | 237.4 | 120 |
| $\hat{l}(M_{4c};y)$ | -1171.5 | -1354.9 | -1387.0 | -1232.3 | -1109.6 | -1075.7 | -1249.8 | -1278.7 | -1193.0 | -1197.0 | 111.6 | 80 |
| $\hat{l}(M_{4b};y)$ | -1172.2 | -1355.8 | -1387.1 | -1233.2 | -1109.6 | -1078.0 | -1250.6 | -1279.7 | -1193.8 | -1197.4 | 127.2 | 90 |
| $\hat{l}(M_{4b,1};y)$ | -1176.8 | -1360.7 | -1391.3 | -1243.6 | -1119.9 | -1080.9 | -1253.2 | -1299.4 | -1198.7 | -1202.7 | 267.3 | 106 |
| $\hat{l}(M_{4b,2};y)$ | -1178.2 | -1355.8 | -1387.1 | -1234.2 | -1111.3 | -1079.3 | -1252.3 | -1283.0 | -1193.9 | -1197.4 | 157.6 | 98 |
| $\hat{l}(M_{4b,3};y)$ | -1180.3 | -1356.3 | -1387.9 | -1237.9 | -1115.9 | -1081.8 | -1256.2 | -1288.3 | -1194.0 | -1200.2 | 210.3 | 130 |
| $\hat{l}(M_{4b,4};y)$ | -1180.6 | -1356.6 | -1387.8 | -1238.3 | -1115.8 | -1081.4 | -1256.1 | -1288.9 | -1194.2 | -1200.7 | 213.5 | 134 |
| $\hat{l}(M_{4b,5};y)$ | -1180.8 | -1356.6 | -1387.8 | -1238.4 | -1116.1 | -1081.5 | -1256.2 | -1288.9 | -1194.2 | -1200.9 | 215.3 | 139 |

Parameter $\alpha_{k_i}$ in (8) where $k_i(x) = x_i$ accounts for the difference in log-odds of response due to the obesity state of a child at the $i$th wave, whose estimates under $M_{4b}$ are mostly negative — especially for $R_2$ and $R_3$. A negative value has the interpretation that a child who is not obese is more likely to respond than another who is obese. Similary, parameters $\beta_{t_i}$ account for the effects of the earlier non-response behaviour. However, we shall not go into the relevant details here due to the abnormal absence of the all-missing groups. From experiences as well as an inspection of the data in Table 1, we tend to believe that these have been censored after the data were collected, and the assumption that they have occurred out of randomness as in the previous analysis is likely not to be correct.

## 3.3 Age effects on boys and girls

The data of Table 1 overlap each other in the sense that independent samples of the same age are available from different times. For instance, the third wave of panel G2 (age 7-9 in 1977), the second wave of panel G3 (age 9-11 in 1977) and the first wave of panel G4 (age 11-13 in 1977) are all samples of age-12 school girls, and can be considered to be independent of each other. In extending the age-effect model from each independent panel to the consecutive panels of the same sex, we must take into consideration the cohort effect among the panels. In particular, we refer to the assumption that $(\beta_0, \beta_h)$ in (7) are identical for all the consecutive panels of the same sex as the assumption of *constant* age effect. This can be relaxed so that only $\beta_h$ remains the same, which we refer to as the assumption of *minimal common* age effect.

For the five consecutive panels of each sex, identity $h_i(a_i) = a_i = 6 + 2(i + j - 2)$ at the $i$th wave of the $j$th panel (or cohort) is equivalent to $i + j - 2$, in which case (7) is modified into

$$\text{(9)} \qquad \operatorname{logit} p_{ij} = \beta_{0,j} + (i + j - 2)\beta_{h,j} + g_i(x)^T \beta_{g_i,j} \qquad 1 \le i \le 3 \quad 1 \le j \le 5,$$

where $p_{ij}$ refers to the $j$th panel at its $i$th wave. Notice that the cohort effect here is only formulated for $p(x|a)$; we shall treat the cohort effect in $p(r|x)$ as a nuisance, and allow it to vary freely between the panels.

Allowing also $\beta_{g_i,j}$ of (9) to vary, we found that the assumption of constant age effect had to be rejected in favour of the minimal common age effect even under such relaxed settings. Indeed, the estimated $\beta_h$ under the former is 0.05 for girls and 0.00 for boys; none of which is significant. More explicitly, denote by $M_{4b,1}$ the model with the constant age effect, which differs from $M_{4b}$ only in that $(\beta_{0,j}, \beta_{h,j}) = (\beta_0, \beta_h)$ for all the panels of the same sex. Denote similarly by $M_{4b,2}$ the model with the minimal common age effect, where only $\beta_{h,j}$ is held constant. The fitted log-likelihoods for all the 10 panels under both models are given in Table 2. Notice that the independence among the panels implies that the log-likelihood on boys, or girls, is simply the sum of those on each panel. Such a splitting provides a clear view over the composition of the overall deviance. For instance, inspections of $\hat{l}(M_{4b,2}; y)$ show that the largest contribution towards the overall deviance for boys came from B1. Indeed, applying $M_{4b,2}$ to B2 - B4 alone yielded fitted log-likelihoods $(-1356.2, -1387.6, -1233.7, -1110.5)$, which are about the same as those in Table 2.

## 3.4 The effect of sex

Take first pairs of panels of the same age, i.e. (B1,G1) and (B2,G2) and so on. In general terms, the sex effect means that the joint probability $p(x, r|a, Boy)$ differs from $p(x, r|a, Girl)$. For the dependence structure, again we assume that $R$ is independent of $(A, S)$ conditional to $x$. The model admits the factorization, possibly in a reduced form,

$$p(x, r|a, s) = p(x_1|s, a_1)p(x_2|s, a_2, x_1)p(x_3|s, a_3, x_1, x_2)p(r|x). \tag{10}$$

Correspondingly, we modify the model graph by adding to it a framed vertex S which points to $X_1$, $X_2$ and $X_3$. Regarding the cohort effect in $p(r|x)$ and $\beta_{g_i}$ of (7) as nuisance, the effect of sex on a pair of panels of the same age group is trivial if we allow both $\beta_0$ and $\beta_h$ to differ from boys to girls. Whereas fixing $\beta_0$ and allowing $\beta_h$ to differ implies that $p_1 = P[X_1 = 1]$ is identical for boys and girls to begin with, which seems liable to *a priori* objections.

To incorporate the effect of sex into a model for all the data in Table 1, we modify (9) into

$$\operatorname{logit} p_{ijs} = \beta_{0,j,s} + s\beta_s + (i + j - 2)\beta_{h,j,s} + g_i(x)^T \beta_{g_i,j,s}, \tag{11}$$

where $(\beta_{0,j,s}, \beta_{h,j,s}, \beta_{g_i,j,s})$ are all sex-specific. We shall keep regarding the cohort effect in $p(r|x)$ as nuisance. Meanwhile, we recover $M_{4b,2}$ by setting $(\beta_{h,j,s}, \beta_s) = (\beta_{h,s}, 0)$. Denote therefore by $M_{4b,3}$ the model derived from $M_{4b,2}$ by setting $(\beta_{h,j,s}, \beta_{g_i,j,s}) = (\beta_h, \beta_{g_i,s})$, and by $M_{4b,4}$ that where $(\beta_{h,j,s}, \beta_{g_i,j,s}) = (\beta_h, \beta_{g_i})$ for all the 10 panels. Whereas to further restrict the base-line effect, i.e. $\beta_{0,j,s}$, we set $\beta_{0,j,s} = \beta_{0,j}$ in addition, denoted by $M_{4b,5}$. Notice that under $M_{4b,5}$, boys and girls are subjected to the same transitions from the first wave to the second, and from the second to the third, except from a constant difference in log-odds $\beta_s$ between any pair of panels from the same age-group. The cohort effect is entirely accounted for by the base-line effect $\beta_{0,j}$ for pairs of panels, and the non-response mechanism $p(r|x)$ of each panel. The fitted log-likelihoods are given in Table 2.

# 4 Discussion: sensitivity analysis

Sooner or later, every practitioner of statistical methods of analysis will have to face the trade-off between the goodness-of-fit of a model and its explanatory power; and no universally applicable criterion is available. The dilemma seems to be rooted in that neither of the concepts involved can be quantified unequivocally. Not only can the various statistics of goodness-of-fit contradict each other because they measure different aspects of a model, but there are also important areas, such as model selection for predictive inference, where the commonly used goodness-of-fit statistics simply fail. The matter is as elusive on the other end. For instance, a typical misconception here mixes the explanatory power of a model with the complexity of its structure which is in turn reduced to its number of free parameters. Now any set of data can be fitted perfectly, e.g. with zero deviance when this is the measure of "goodness" adopted, if a free parameter is assigned to each free observation. At the same time, this might also occur with some rather complicated parametric model which happens to use up all the degree of freedom in the data. However, while the first model possesses no explanatory power whatsoever, the same may not be said of the second one. It is therefore interesting if the family of models under the investigation, despite its richness, can display some degree of robustness w.r.t. the interest of inference. In the present case, this is the marginal obesity among school children and the effect of sex and age on it.

Among the models listed in Table 2, $M_{4b}$ and $M_{4c}$ treated each panel on its own. Whereas we obtained $\hat{\beta}_h = -0.50$ and $\hat{\beta}_s = 0.15$ under all the three models $M_{4b,3}$, $M_{4b,4}$ and $M_{4b,5}$, so that the synthetic graphical models considered here do display robustness w.r.t. the estimates of the effect of age and sex, and the deviances (or $p$-values) of the fitted models are misleading in this respect.

To examine the robustness of the models w.r.t. the estimates of the marginal obesity, we have listed the key estimates for school boys and girls in Table 3 — together with those reported in Baker (1995). To assure the validity of the mean values involved, we need to assume that the total numbers of school boys or girls did not changed significantly over the years between 1977 and 1981. It is hardly surprising that $M_{4b,3}$ and $M_{4b,5}$ should give smoother estimates of the marginal probabilities than the others, especially at the two ends of the age spectrum. In fact, the estimates under $M_{4b,3}$ or $M_{4b,5}$ agrees well with those reported by Baker (1995) except from the upper end, i.e age 17-19. (It is not clear how one should interpret the strict monotonicity, both in age and sex, in Baker's estimates, which seemed to counter our intuition on the matter. In addition, Baker's model had a deviance of 1141.9 on 231 degrees of freedom, which was not plausible compared to, say, $M_{4b,5}$ here, according to the Akaike Information criterion (Akaike 1974) used in Baker (1995) for model selection.)

Table 3: Estimates of the marginal obesity among school boys and girls

| | | School boys | | | | | | | School girls | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | Year | 5-7 | 7-9 | 9-11 | 11-13 | 13-15 | 15-17 | 17-19 | 5-7 | 7-9 | 9-11 | 11-13 | 13-15 | 15-17 | 17-19 |
| | 1977 | .06 | .18 | .18 | .25 | .20 | - | - | .16 | .14 | .27 | .21 | .23 | - | - |
| $M_{4c}$ | 1979 | - | .18 | .19 | .23 | .24 | .21 | - | - | .17 | .24 | .27 | .31 | .28 | - |
| | 1981 | - | - | .25 | .22 | .24 | .20 | .18 | - | - | .25 | .26 | .23 | .30 | .25 |
| | Mean | .06 | .18 | .21 | .23 | .23 | .20 | .18 | .16 | .16 | .25 | .25 | .26 | .29 | .25 |
| | 1977 | .08 | .16 | .18 | .24 | .20 | - | - | .12 | .15 | .26 | .22 | .22 | - | - |
| $M_{4b}$ | 1979 | - | .17 | .21 | .24 | .26 | .21 | - | - | .19 | .22 | .28 | .29 | .29 | - |
| | 1981 | - | - | .25 | .22 | .23 | .20 | .18 | - | - | .24 | .26 | .22 | .30 | .25 |
| | Mean | .08 | .16 | .21 | .23 | .23 | .20 | .18 | .12 | .17 | .24 | .25 | .24 | .30 | .25 |
| | 1977 | .17 | .17 | .19 | .19 | .17 | - | - | .18 | .19 | .20 | .22 | .21 | - | - |
| $M_{2f,3}$ | 1979 | - | .21 | .21 | .23 | .24 | .22 | - | - | .23 | .25 | .26 | .29 | .28 | - |
| | 1981 | - | - | .21 | .21 | .24 | .24 | .21 | - | - | .22 | .24 | .27 | .30 | .28 |
| | Mean | .17 | .19 | .20 | .21 | .22 | .23 | .21 | .18 | .21 | .22 | .24 | .26 | .29 | .28 |
| | 1977 | .16 | .17 | .18 | .19 | .18 | - | - | .18 | .19 | .21 | .22 | .21 | - | - |
| $M_{2f,5}$ | 1979 | - | .20 | .21 | .23 | .25 | .23 | - | - | .23 | .24 | .27 | .28 | .26 | - |
| | 1981 | - | - | .20 | .21 | .23 | .25 | .23 | - | - | .23 | .25 | .27 | .29 | .27 |
| | Mean | .16 | .18 | .20 | .21 | .22 | .24 | .23 | .18 | .21 | .23 | .25 | .25 | .28 | .27 |
| Baker (1995) | | .15 | .17 | .19 | .21 | .24 | .26 | .29 | .17 | .19 | .21 | .23 | .26 | .28 | .31 |

Table 4: Estimates of the marginal obesity among school children

| | Age group | | | | | | |
|---|---|---|---|---|---|---|---|
| Model | 5-7 | 7-9 | 9-11 | 11-13 | 13-15 | 15-17 | 17-19 |
| $M_{4c}$ | .11 | .17 | .23 | .24 | .24 | .25 | .22 |
| $M_{4b}$ | .10 | .17 | .23 | .24 | .24 | .25 | .22 |
| $M_{4b,3}$ | .18 | .20 | .21 | .22 | .24 | .26 | .24 |
| $M_{4b,5}$ | .17 | .20 | .21 | .23 | .24 | .26 | .25 |
| Woolson and Clarke (1984) | .11 | .17 | .21 | .23 | .23 | .21 | .17 |
| Conaway (1994) | .13 | .15 | .23 | .22 | .20 | .20 | .20 |

Meanwhile, assuming in addition that there are about as many boys and girls within the population, we have listed in Table 4 the estimated marginal obesity among school children — together with those reported by Woolson and Clarke (1984) and Conaway (1994), which rather resemble those under model $M_{4b}$ or $M_{4c}$ here except for a flatter upper end. Again, we would like to caution against too much emphasis in any of these estimates due to the absence of the all-missing cell, especially if these had been censored as we suspect.

# 5 Concluding remarks

The synthetic graphical modelling approach of this paper combines two powerful statistical methods of analysis, i.e. the graphical and generalized linear models. While the dependence structure among the data is depicted by the corresponding graph, its details are spelled in terms of the conditional GLMs. As a sepcial case, the chained logistic regression model is suitable here given the asymmtry among the variables of the study. The fact that several existing sets of estimates on the present data, derived under various alternative models, are close to those obtained under certain members of the family of models considered here demontrates the richness as well as the flexibility of our approach. The unified treatment allows non-response to depend on both the object variables and the non-response at some earlier points. This can be useful for analysis of incomplete-data in general, such as when measurement-error and non-response are present at the same time. Finally, applications of mixed graphs together with other types of GLMs should enable us to handle more complex data which contain both symmetric and asymmetric dependence structures, while preserving the conceptual clarity of the models.

## References


Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control* **19**, 716–23.

Azzalini, A. (1974). Logistic regression for autocorrelated data with application to repeated measures. *Biometrics* **50**, 767–75.

Baker, Ŝ. (1995). Marginal regression for repeated binary data with outcome subject to non-ignorable non-response. *Biometrics* **51**, 1042–52.

Conaway, M. (1994). Causal nonresponse models for repeated categorical measurements. *Biometrics* **50**, 1102–16.

Cox, D. and N. Wermuth (1996). *Multivariate Dependencies: Models, analysis and interpretation.* London: Chapman and Hall.

Diggle, P. and M. Kenward (1994). Informative drop-out in longitudinal data analysis (with discussion). *Applied Statistics* **43**, 49–93.

Fitzmaurice, G., N. Laird, and S. Lipsitz (1994). Analyzing incomplete longitudinal binary responses: A likelihood-based approach. *Biometrics* **50**, 601–12.

Fitzmaurice, G., N. Laird, and G. Zahner (1996). Multivariate logistic models for incomplete binary responses. *Journal of the American Statistical Association* **91**, 99–108.

Lauer, R. (1975). Coronary heart disease risk factors in school children: The muscatine study. *Journal of Pediatrics* **86**, 697–706.

Lauritzen, S. (1996). *Graphical Models*. Clarendon Press, Oxford.

Lipsitz, S., N. Laird, and D. Harrington (1994). Weighted least squares analysis of repeated categorical measurements with outcome subject to non-response. *Biometrics* **50**, 11–24.

Louisiana State University Medical Center, Mayo Clinic, U. o. I. M. C. (1978). Cardiovascular profile of 15,000 children of school age in three communities 1971-1975. In *DHEW publication no. (NIH) 78-1472. Washington, D.C.: U.S. Department of Health, Education and Welfare; Public Health Service; Bethesda; National Heart, Lung and Blood Institute.*

McCullagh, P. and J. Nelder (1989). *Generalized Linear Models* (2nd edn). London: Chapman and Hall.

Molenberghs, G., M. Kenward, and E. Lesaffre (1997). Informative dropouts in ordinal longitudinal data. *Biometrika* **84**, 33–44.

Park, T. and C. Davis (1993). A test of the missing data mechanisms for repeated categorical data. *Biometrics* **49**, 631–8.

Wermuth, N. and S. Lauritzen (1983). Graphical and recursive models for contingency tables. *Biometrika* **70**, 537–52.

Woolson, R. and W. Clarke (1984). Analysis of incomplete longitudinal categorical data. *Journal of the Royal Statistical Society A* **147**, 87–99.

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