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Child Care in the Welfare State

A critique of the Rosen model

Abstract:
A recent study of the welfare state in Sweden, Rosen (1995, 1996, 1997), concludes that child care subsidies may lead to substantial deadweight losses that may impede economic growth and the future of the welfare state. In this article we show that the deadweight losses are highly sensitive to some parameter restrictions implied by Rosen's theoretical model. We then critically review the relation between the parameter values in Rosen's model. Moreover, as a first approach to extend Rosen's model, we analyze the case of positive externalities associated with child quality. The positive externality provides a rationale for child care subsidies, as expected, and also influences the optimal income tax rate.

Keywords: Household Production, Externalities, Optimal Taxation, Subsidies

JEL classification: D13, D62, H21

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1. Introduction

The provision of child care is a crucial issue in the organization of the welfare state. As married women have increased their labor market participation, much of the unpaid care work has been replaced by subsidized public child care or market substitutes. In a recent study of the Swedish welfare state Sherwin Rosen (1995, 1996, 1997) concludes that subsidized child care leads to excessive consumption of child care services, so that "women are encouraged to work too much in the state-subsidized household sector, taking care of other families' household needs, and not enough in the material goods sector." (Rosen 1997) p. 83. Rosen points out that although child care subsidies encourage market work that income taxes discourage, they introduce other distortions because they require increased taxes on other goods.

However, the provision of high-quality child care is crucial for the well-being of children themselves as well as for the future resources of society in the form of human capital, human capabilities and social capital. High-quality investments in children have positive externalities to society, and high child care costs may be balanced against high gains from positive externalities. If there is underproduction of child care in the absence of subsidies, the true deadweight losses resulting from the subsidies may be smaller than assessed by Rosen (or perhaps non-existent), see Folbre (1994). Moreover, subsidized child care has a role to play in ensuring employment and tax bases (Bergstrom and Blomquist, 1996). Distributional concerns might also be an argument for subsidized child care. The cost of raising children has large implications for the distribution of income and welfare between men and women, between families at different income levels, between families with children and the childless, between single parents and two-parents families, and between children from different family types.

In Rosen's model the gains from bringing up children only appear via "child quality" as a consumption good in the parents' utility function, i.e. a view-point of "children as pets", the term borrowed from England and Folbre (1998). Alternative assumptions about child quality and equity concerns are difficult to analyze in an optimal tax model, and it would be unfair to criticize Rosen for not having introduced such issues into his analysis. However, as a first step towards expanding Rosen's model to include a wider range of the benefits of high-quality child care, we have in this article analyzed the effect on the optimal tax and subsidy rates of a positive externality generated by parents' consumption of child quality.

Rosen's critique of child care subsidies relies to some extent on a numerical illustration of deadweight loss in Sweden based on benchmark parameter values for tax and subsidy rates, labor supply elasticities, substitution elasticities, cost shares and budget shares. The theoretical model – before the optimal tax problem is introduced – implies some parameter restrictions that Rosen has not taken into account in the numerical example. When we take the parameter restrictions implied by the theoretical model into account, we find that the results are highly sensitive to the parameter restrictions, and in some examples the deadweight losses are substantially reduced as compared to Rosen's case.

The outline of the article is as follows. In Section 2 we give a summary of Rosen's model. In Section 3 we critically review the consistency of Rosen's parameter values. In Section 4 we extend the Rosen model with a stylized representation of positive externalities generated by the consumption of child quality. We show how Rosen's solution for optimal income tax and subsidy rates is modified when the externality is taken into account.

2. A summary of the Rosen model

In this section we summarize the Rosen model, with references to Rosen (1997). The *Journal of Economic Literature* article (Rosen 1996) is an nontechnical discussion based on a working paper (Rosen 1995) which has also appeared as Rosen (1997) in an edited volume by Freeman, Topel and Swedenborg (1997). The two main building blocks of the Rosen model are the "child quality" production function and the parents' utility function. Child quality z is modelled as a function of parental child care time h and purchased child care M,

$$z = f\left(h, M\right) \qquad (1)$$

where production is assumed to exhibit constant returns. The elasticity of substitution between h and M in child quality production is denoted by σ_p, and $\theta = wh/qz$ is the cost share of parental child care time in the production of child quality, where q is the price of child quality, endogenously determined by cost minimization, and w is the after tax wage rate. The subsidized (taxed) price of purchased child care is p. In equilibrium $qz = wh + pM$. The purchased child care is interpreted as the hired help of others, irrespective of whether the paid child care takes place in the home or in a day care center. The production function (1) for child quality implies imperfect substitution between h and M. In contrast, Sandmo (1990) assumes perfect substitution between parental child care and purchased child care. His model implies separability between consumption of material goods and parental child care.

The utility function of the parents depends on consumption of material goods *x* and child quality *z*,

(2) $$u = u(x, z).$$

The elasticity of substitution between *x* and *z* in consumption is denoted by σ_c. Moreover, η_{zq} is the uncompensated demand elasticity of child quality with respect to the implicit price of child quality, η_{zI} is the income elasticity of child quality, and $\varphi = qz/(x + qz)$ is the budget share of child quality in total consumption (full income). Full income *I* is defined by $x + qz = w(h + t) = I$, where *t* is labor market time. In Rosen's model leisure is not included, so that the time constraint is

(3) $$t + h = 1$$

when total time is normalized to one. Rosen assumes that production of *x* and *M* are linear in their time inputs *l* and *m* where $t = m + l$. Moreover, the marginal product of labor in material goods production is 1, so that $x = l$ and $M = \alpha m$, where α is a constant "reflecting the number of children per day-care mother ($\alpha = 4$ in Sweden)" (p. 94). The wage rate and producer price *W* and *P* are given, and the tax rates are defined by

(4) $$w = W - \tau$$

(5) $$p = P + \rho$$

where τ is the tax rate on labor income and ρ is the tax rate (if positive) or subsidy rate (if negative) on purchased child care. The consumer budget equation is given by

(6) $$x + pM = wt$$

where consumption of material goods *x* is numeraire. Combining (3) and (6) gives the full income budget equation

(7) $$x + wh + pM = x + qz = I = w = 1$$

where $w = 1$ in competitive equilibrium since the marginal product of labor in material goods production is 1 and *x* is the numeraire (see p. 94). Rosen applies a dual approach, where consumers combine *h* and *M* to minimize production costs for fixed *z*, and choose *x* and *z* to maximize *u(x,z)*, for given cost of *z*. Based on the indirect utility function Rosen derives the optimal tax and subsidy rates given by (12) on p. 97. In Appendix A we have summarized the elasticities of parental child care time, purchased child care and labor supply with respect to the wage rate and the price of purchased child care, as given in Rosen (1997).

3. A critique of Rosen's numerical illustrations

Rosen's analysis of deadweight loss in Sweden is based on a numerical example with benchmark parameters for tax and subsidy rates, labor supply elasticities, substitution elasticities, cost shares and budget shares. However, the theoretical model – before the optimal tax problem is introduced – implies some parameter restrictions that Rosen has not taken into account in the numerical example.

Let us now consider the parameter restrictions implied by Rosen's model. The definitions of the cost share θ and budget share φ imply that $h = \theta\varphi$. Recall the definitions of θ and φ that yield $\theta\varphi = \left(wh/qz\right)\left(qz/\left(x+qz\right)\right) = wh/I$ and the definition of full income that gives $I = w$ when total time is normalized to one. Hence, we have that $h = \theta\varphi$ irrespective of the wage rate.

To ensure consistency, the parameters in Rosen's numerical example should reflect the constraint that $h = \theta\varphi$. On p. 100, Rosen introduces his numerical example and assumes that $h = 0.5$, $\theta = 0.5$ and $\varphi = 0.25$. This implies that $\theta\varphi = 0.125$ which violates the condition $h = \theta\varphi$.

Table A.1 in Appendix A shows Rosen's numerical example for the deadweight loss and the corresponding values for the substitution elasticities and the labor supply elasticities from Rosen (1997), Table 2.3, p. 100. For given labor supply elasticities Rosen has computed the corresponding substitution elasticities and deadweight loss, see (A.12) and (A.14) in Appendix A. In order to illustrate the consequences of the parameter restriction $h = \theta\varphi$ for Rosen's results, we have computed the values for σ_c and the deadweight loss in two different cases, see Tables 1 and 2. In Table 1 we have retained Rosen's assumption of $h = 0.5$ and adjusted the values for θ and φ. As an example we consider a case where $pM = x = 0.5wt$, so that purchased child care and material consumption each amount to half of total consumption. This yields parameter values $\theta = 2/3$ and $\varphi = 3/4$, so that $h = \theta\varphi = 0.5$.

Table 1 shows that the parameter restriction $h = \theta\varphi$ and the case of $\theta = 2/3$ and $\varphi = 3/4$ imply that the deadweight loss is substantially reduced as compared to Rosen's example, for all parameter combinations except for one case with a high value of σ_p.

Table 1. Substitution elasticities in consumption and deadweight loss for alternative parameter values: Rosen's model with the restriction $h = \theta\varphi = 0.5$, $\theta = 2/3$ and $\varphi = 3/4$

	η_{tw} = 1/3		η_{tw} = 2/3		η_{tw} = 1	
σ_p	σ_c	DWL	σ_c	DWL	σ_c	DWL
0	4.94	0.36	6.91	0.51	8.94	0.66
1	2.97	0.22	4.94	0.37	6.97	0.52
2	1.00	0.08	2.97	0.23	5.00	0.38
3	NA	NA	1.00	0.09	3.03	0.24

In Table 2 we have retained Rosen's assumption of $\theta = 0.5$ and $\varphi = 0.25$ and adjusted h so that $h = \theta\varphi = 0.125$ and $1 - h = 0.875$. This implies that $pM = wh$ and $x = 6pM$.

Table 2. Substitution elasticities in consumption and deadweight loss for alternative parameter values: Rosen's model with the restriction $h = \theta\varphi = 0.125$, $\theta = 0.5$ and $\varphi = 0.25$

	η_{tw} = 1/3		η_{tw} = 2/3		η_{tw} = 1	
σ_p	σ_c	DWL	σ_c	DWL	σ_c	DWL
0	7.61	1.83	13.05	3.13	18.33	4.4
1	6.28	1.51	11.72	2.82	17	4.09
2	4.95	1.20	10.39	2.50	15.67	3.77
3	3.61	0.88	9.05	2.19	14.33	3.46

Table 2 shows that the restriction $h = \theta\varphi$ and the case of $\theta = 0.5$ and $\varphi = 0.25$ imply that the deadweight loss becomes substantially larger than in Rosen's example. By comparing Tables 1 and 2 with Table A.1 we obtain the following conclusion.

Conclusion 1: *The Rosen model implies the parameter restriction* $h = \theta\varphi$. *If we retain Rosen's assumption of* $h = 0.5$ *and adjust* θ *and* φ *so that e.g.* $\theta = 2/3$ *and* $\varphi = 3/4$, *the deadweight loss is almost halved as compared to Rosen's example, except for one case with a high value of* σ_p. *Alternatively, if we retain Rosen's assumptions of* $\theta = 0.5$ *and* $\varphi = 0.25$, *this implies that* $h = 0.125$, *and in this case the deadweight loss is substantially higher than in Rosen's example.*

In order to explain the large difference between Tables 1 and 2, note that the labor supply elasticity with respect to the child care price depends negatively on σ_p and positively on $(1-\varphi)\sigma_c$, see (A.13). In Table 1 where φ is large, $1-\varphi$ is small and the positive effect of σ_c on the labor supply elasticity is diminished so that the negative effect of σ_p may dominate. Hence, increased subsidies tend to increase the labor supply and reduce the deadweight loss. In Table 2 where φ is small, the positive effect of σ_c on the labor supply elasticity may dominate the negative effect of σ_p. Hence, increased subsidies tend to reduce the labor supply and increase the deadweight loss.

As indicated by this discussion, the labor supply elasticity with respect to the child care price must be taken into account in order to explain the relationship between the substitution elasticities and the deadweight loss. Since Rosen expresses his assumptions in terms of the labor supply elasticity with respect to the wage rate rather than the child care price, it must be noted that these two elasticities are different in Rosen's model, see the expressions for η_{tw} and η_{tp} in (A.12) and (A.13). This is illustrated in Table 3, that gives labor supply elasticities with respect to the wage rate and the price of child care for Rosen's parameters. The first column reproduces the assumptions of Table A.1, whereas the second column gives the corresponding labor supply elasticity with respect to the child care price.

Table 3. Labor supply elasticities w.r.t. wage rate and child care price. Parameters from Table A.1

σ_p	σ_c	η_{tw}	η_{tp}
0	3.20	0.33	1.33
1	1.88	0.33	0.33
2	0.56	0.33	-0.67
0	4.11	0.67	1.67
1	2.78	0.67	0.67
2	1.44	0.67	-0.33
0	5	1	2
1	3.67	1	1
2	2.33	1	0

The difference between the two columns in Table 3 is found directly from (A.12) and (A.13) as

$$(8) \quad \eta_{tp}-\eta_{tw}=-\frac{h}{1-h}\eta_{hp}+\frac{h}{1-h}\eta_{hw}=\frac{h}{1-h}\left\{-2(1-\theta)\sigma_p+(1-2\theta)(1-\varphi)\sigma_c+(1+\varphi-2\theta\varphi)\eta_{zI}\right\}.$$

For Rosen's benchmark parameter values of $h=0.5$, $\theta=0.5$ and $\eta_{zI}=1$, (8) is simplified to

(9) $$\eta_{tp} - \eta_{tw} = 1 - \sigma_p$$

independently of σ_c and φ. For discussions of the effect of child care subsidies it is useful to notice that $\eta_{tp} \neq \eta_{tw}$ except in the case described above.

Conclusion 2: *In Rosen's benchmark case with* $h = 0.5$*,* $\theta = 0.5$ *and* $\eta_{zI} = 1$*, the labor supply elasticities with respect to the wage rate and the price of child care coincide only in the case with substitution elasticity in household production equal to one, indenpendently of* σ_c *and* φ*.*

Next we will analyze the consequences of the additional parameter restrictions implied by Rosen's model for optimal taxation. We realize that Rosen's example is not meant to reflect the optimal tax model, but we find that these restrictions provide a better understanding of the relationships between the parameters. The parameter restrictions implied by the optimal tax model are of three types. First, in order to avoid a zero value denominator, we must have $\sigma_p \neq 0$ and $\sigma_c \neq \eta_{zI}$ in addition to $h \neq 0$, $M \neq 0$ and $\varphi \neq 1$. Secondly, in order to ensure a positive income tax rate, we must have $\sigma_c > \eta_{zI}$. Finally, in order to ensure a subsidy rather than a tax on purchased child care, we must have $\sigma_p > \sigma_c$.

Table 4 illustrates a case where we have chosen parameter values that satisfy these restrictions as well as $h = \theta\varphi$ with assumptions from Table 1 that $h = 0.5$, $\theta = 2/3$ and $\varphi = 3/4$. In Table 4 we have considered different values of the substitution elasticities so that $\sigma_p > \sigma_c > 1$, where we have retained the assumption of $\eta_{zI} = 1$. For each value of σ_p and σ_c we compute the corresponding labor supply elasticities with respect to w and p. Since we here consider given values of σ_c (in contrast to Rosen's example where σ_c is endogenized to match given values of η_{tw}) the labor supply elasticity η_{tw} is endogenous.

Conclusion 3: *Rosen's model for optimal taxation implies three types of parameter restrictions, (i)* $\sigma_p \neq 0$ *and* $\sigma_c \neq \eta_{zI}$ *in order to avoid zero denominator, (ii)* $\sigma_c > \eta_{zI}$ *in order to have a positive income tax rate and (iii)* $\sigma_p > \sigma_c$ *in order to have a subsidy rather than a tax on purchased child care. If these restrictions are included in the numerical example, the deadweight loss is considerably reduced as compared to Rosen's example.*

Table 4. Labor supply elasticities with respect to wage rate and child care price, and deadweight loss, with $h = 0.5$, $\theta = 0.67$ and $\varphi = 0.75$

Elasticity of substitution in		Labor supply elasticity		Deadweight loss
Production of child quality σ_p	Consumption σ_c	η_{tw}	η_{tp}	$\tau = 0.7$, $\rho = -0.9$
4.5	4	1.66	-0.91	0.147
4	3.5	1.41	-0.78	0.129
3.5	3	1.16	-0.66	0.110
3	2.5	0.91	-0.54	0.092
2.5	2	0.66	-0.41	0.074
2	1.5	0.41	-0.29	0.055

Conclusion 3 follows by comparing Table 4 with Rosen's example in Table A.1. The reason why the deadweight losses are so small in Table 4 compared to Rosen's example and Table 1 is that the elasticity of substitution in consumption is fairly low and less than the elasticity of substitution in child quality production. As explained above, when the negative effect of σ_p on η_{tp} dominates the positive effect of σ_c, labor supply tends to increase and the deadweight loss is reduced. Table 4 thus illustrates that the low deadweight losses in Table 1 are further reduced for low values of σ_c.

The literature on estimates for substitution elasticities may be consulted for choice of parameter values. A series of recent articles have analyzed the role of household production in business cycles, see e.g. Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991), McGrattan, Rogerson and Wright (1997), and Ingram, Kocherlakota and Savin (1997). The latter article has a discussion of the substitution elasticity between market and nonmarket goods that may be applied for the trade-off between material goods and child quality in Rosen's model.

They use the values $\sigma = -1.5$ (market and nonmarket goods are complements) and $\sigma = 0.5$ (market and nonmarket goods are substitutes) where σ is the elasticity of the CES-aggregate of market and nonmarket goods. McGrattan et al. (1997) estimate σ to be around 0.4. Rupert, Rogerson and Wright (1995) also estimate σ to be around 0.4. In Table 5 we illustrate the labor supply elasticities and deadweight loss consistent with $\sigma = 0.4$ which gives an elasticity of substitution in consumption of $1/(1-0.4) = 1.67$. We have used the other assumptions in Table 4. In this case we also find that the deadweight loss is substantially reduced as compared to Rosen's example.

Table 5. Labor supply elasticities with respect to wage rate and child care price, and deadweight loss, for $\sigma_c = 1.67$ and other parameter values and restrictions as in Table 4

Elasticity of substitution in		Labor supply elasticity		Deadweight loss
Production of child quality σ_p	Consumption σ_c	η_{tw}	η_{tp}	$\tau = 0.7$ $\rho = -0.9$
4.5	1.67	1.27	-1.10	0.064
4	1.67	1.10	-0.93	0.064
3.5	1.67	0.94	-0.77	0.063
3	1.67	0.77	-0.60	0.063
2.5	1.67	0.61	-0.44	0.062
2	1.67	0.44	-0.27	0.062

Table 5 illustrates that the deadweight loss is further reduced when σ_c declines as compared to the values in Table 4. Low substitution elasticity in consumption means that parents are less inclined to substitute child quality with more material consumption.

4. Optimal taxes and subsidies in the case of externalities

As a first step towards expanding the notion of social benefits of child quality, we now extend the Rosen model to the case with a positive externality created by consumption of child quality. Inspired by Sandmo (1975), we assume that the externality is a function of the total consumption of child quality *Z,* where $Z = nz$, hence, we have assumed that the economy consists of *n* consumers with identical preferences and productivities. The utility function of the representative parents given by (2) is thus replaced by

$$(10) \qquad u = u(x, z, Z), \text{ with } u_E = \frac{\partial u}{\partial Z} > 0$$

where subscript *E* here and in the following denotes the derivative with respect to the externality.

This representation of the externality is of course highly stylized, but it captures the feature that child quality is not only a private consumption good for parents. Devoting resources to child quality contributes to the atmosphere of society and creates a positive externality of the public good type, over and above the private utility. Alternatively, the externality could have been included in the production function since a high level of child quality in general favorably influences the production of child quality. In other words, it is easier to bring up children if other children are well-behaved. In our context, however, the point is to illustrate the effect of including the externality in the Rosen model, and (10) is an analytically convenient starting point.

With the utility function (10), the consumer demand functions and the indirect utility function in general contain the externality as an argument. In Appendix B we have derived the optimal tax and subsidy rates in the general case of non-separable externalities.

In the following we will consider the special case of separable externalities, with separability between the externality term Z and the private goods x and z. Then the marginal rates of substitution between x and z will be independent of the amount of externality. The same is true of the demand functions, so that the feedback effect vanishes. Under these assumptions the consumers benefit from the positive externality, but it does not change the way they behave, see Sandmo (1998) and Håkonsen (1999). In the case of separable externalities the demand functions do not depend on Z, but the externality appears as an argument in the indirect utility function. Hence, the indirect utility function is given by

(11) $$G(w,p,Z)=u\big(x(q,I),z(q,I),nz(q,I)\big)$$

where we have substituted $Z=nz(q,I)$. The government budget constraint for expenditure level g is given by

(12) $$g=n\tau(1-h)+n\rho M.$$

The optimal tax problem can be formulated as maximization of the welfare function, which is the sum of the indirect utility functions, with respect to consumer prices, subject to the budget constraint (12). The Lagrangian becomes

(13) $$L=nG(w,p,Z)+\nu\big[g-n(W-w)(1-h)-n(p-P)M\big]$$

where ν is the shadow price. In appendix B we have derived the optimal tax structure in the general case of non-separable externalities. We will now explain how the optimal tax structure in the general case in Appendix B simplifies under the assumption of separable externalities, where $x_E=z_E=h_E=M_E=0.$ In the general case of non-separable externalities the implicit price of child quality q depends on the externality so that $q_E<0$. This effect vanishes in the case of separable externalities. When $q_E=0,$ the optimal tax structure given in (B.37) and (B.38) simplifies to

(14) $$\frac{\tau}{w} = (1-\beta)\frac{\theta\sigma_p + (1-\theta)\sigma_c}{\theta\varphi\sigma_p(\sigma_c - \eta_{zI})} + \beta n \frac{u_E}{u_z}$$

(15) $$-\frac{\rho}{p} = (1-\beta)\frac{\sigma_p - \sigma_c}{\varphi\sigma_p(\sigma_c - \eta_{zI})} + \beta n \frac{u_E}{u_z}$$

where $\beta = -\mu / \nu$ and μ is the shadow price in the utility maximization problem for the consumer. We have that $\beta > 0$ since the marginal utility of money for the consumer μ is positive, and ν is negative since the government's tax requirement implies a withdrawal of resources from private sector.

We immediately note that our solution for the optimal tax and subsidy rates is given as weighted average of two terms, where the first term consists of efficiency terms, and the second term is the marginal social benefit of the externality generating commodity. The efficiency terms or Ramsey terms of (14) and (15) are precisely those that Rosen has found in his (12), if we insert the equilibrium solution $w = 1$ and $p = 1/\alpha$.

The structure of our solution is similar to the optimal subsidy rate in the presence of externalities as developed in Sandmo (1975), where he showed that the optimal subsidy (tax) rate on the externality generating commodity is a weighted average of the efficiency term and the externality term. In fact, in the case of separable externalities we have found an optimal tax structure where the efficiency terms are precisely those of the Rosen model and externality terms are precisely those of the Sandmo model.

In our model, the positive externality is generated from consumption of child quality *z* which depends on both parental child care *h* as well as purchased child care *M* via the child quality production function $z = f(h, M)$. Hence, both the tax rate τ and the subsidy rate ρ are affected by the externality terms. From (14) and (15) we see that the presence of positive externalities caused by consumption of *z* implies that the consumption of *z* and its input factors *h* and *M* should be encouraged through higher income tax, hence reducing the price of *h*, and higher subsidies, reducing the price of *M*. In Rosen's model, a subsidy is warranted only when $\sigma_p > \sigma_c$. If $\sigma_p = \sigma_c$, efficiency considerations call for no subsidies (or taxes) on purchased child care. But in our model with positive externalities subsidies are called for even with $\sigma_p = \sigma_c$.

As noted by Sandmo (1975) the externality term nu_E/u_z expresses the marginal rate of substitution between child quality as a public good and as a private good. With increasing β, the weight of the externality term in the optimal tax formula increases relative to the weight of the efficiency term. Recall that $\beta = -\mu/\nu,$ and as Sandmo (1975) notes, we might interpret β as the marginal rate of substitution between private and public income. The higher β is, the higher is the marginal value of private income compared to public income, and the lower the tax requirement. In the case of $\beta = 1,$ the tax requirement is exactly satisfied by the Pigovian tax, so that no additional distortionary taxes or subsidies are called for.

The difference between the income tax rate and subsidy rate is given by

$$\text{(16)} \qquad \frac{\tau}{w} - \left(-\frac{\rho}{p}\right) = (1-\beta) \; \frac{\sigma_c}{\theta\varphi\sigma_p\left(\sigma_c - \eta_{zI}\right)}$$

The difference between the tax and subsidy rates given by (16) is proportional to σ_c, which represents the "automatic" subsidization of *h* via the income tax. If $\sigma_c = 0$, the optimal tax and subsidy rates thus coincide. Note that in Rosen's model we cannot have $\sigma_c = 0$ as $\sigma_c > \eta_{zI}$ is required in order to have a positive tax rate. In the externality model, however, a positive tax rate is attained even if $\sigma_c = 0$ provided that the externality term is larger than the efficiency term.

5. Concluding remarks

Rosen's calculations of deadweight losses from child care subsidies are highly sensitive to parameter restrictions from his theoretical model. Taking these parameter restrictions into account we find some examples where the deadweight losses are substantially reduced as compared to Rosen's case, while in other situations the deadweight losses may increase. Our calculations illustrate that the deadweight losses are substantially reduced when the substitution elasticity is low, i.e. when parents are less inclined to substitute child quality with more material consumption as the child care subsidies increase.

We have also extended Rosen's model to the case of positive externalities from the consumption of child quality. We find that not only the optimal subsidy rate, as expected, but also the income tax rate should be higher in the case of a positive externality. Our stylized externality model is only meant as a first step towards expanding the notion of social benefits of child quality.

Subsidies blur the price signals that are supposed to reveal the preferences of parents for various child care arrangements, yet there might be other strong arguments for subsidized child care. If parents underestimate the quality of purchased child care, if mothers underestimate the cost of lost income from employment, and if society wants equality in the sense of similar employment rates for women and men, child care should be subsidized. Also, distributional concerns might be an argument for subsidized child care. Although it is difficult to quantify the positive externalities, the theoretical analysis gives new insight into the optimal tax problem. Constructing empirical measures for the externalities of child quality and distributional concerns is a challenge for further research.

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Appendix A

Elasticity expressions in the Rosen model

Rosen (1997) gives the following expressions for the elasticities η_{hw} and η_{hp} of parental child care time (h) and η_{Mp} and η_{Mw} of purchased child care (M) with respect to the wage rate (w) and the price of purchased child care (p), see p. 93, formula (3):

(A.1) $$\eta_{hw} = -(1-\theta)\sigma_p + \theta\eta_{zq} + \eta_{zI}$$

(A.2) $$\eta_{hp} = (1-\theta)(\sigma_p + \eta_{zq})$$

(A.3) $$\eta_{Mp} = -\theta\sigma_p + (1-\theta)\eta_{zq}$$

(A.4) $$\eta_{Mw} = \theta\sigma_p + \theta\,\eta_{zq} + \eta_{zI}$$

In the text on p. 106 Rosen (1997) gives the relationship

(A.5) $$\eta_{zq} \equiv -(1-\varphi)\sigma_c - \varphi\eta_{zI}$$

Inserting (A.5) in (A.1)-(A.4) gives

(A.6) $$\eta_{hw} = -(1-\theta)\sigma_p - \theta(1-\varphi)\sigma_c + (1-\theta\varphi)\eta_{zI}$$

(A.7) $$\eta_{hp} = (1-\theta)\left[\sigma_p - (1-\varphi)\sigma_c - \varphi\,\eta_{zI}\right]$$

(A.8) $$\eta_{Mp} = -\theta\sigma_p - (1-\theta)(1-\varphi)\sigma_c - (1-\theta)\varphi\,\eta_{zI}$$

(A.9) $$\eta_{Mw} = \theta\sigma_p - \theta(1-\varphi)\sigma_c + (1-\theta\varphi)\eta_{zI}$$

In the text on p. 99 Rosen (1997) notes that

(A.10) $$\eta_{tw} = \frac{\partial\, log(1-h)}{\partial\, log\, h} = -\frac{h}{1-h}\eta_{hw}$$

and we add the corresponding restriction

(A.11) $$\eta_{tp} = \frac{\partial\, log(1-h)}{\partial\, log\, h} = -\frac{h}{1-h}\eta_{hp}$$

Combining (A.10) with (A.6) gives

(A.12) $$\eta_{tw} = -\frac{h}{1-h}\eta_{hw} = \frac{h}{1-h}\left[(1-\theta)\sigma_p + \theta(1-\varphi)\sigma_c - (1-\theta\varphi)\eta_{zI}\right]$$

Combining (A.11) with (A.7) gives

(A.13) $$\eta_{tp} = -\frac{h}{1-h}\eta_{tp} = \frac{h}{1-h}(1-\theta)\left[-\sigma_p + (1-\varphi)\sigma_c + \varphi\eta_{zI}\right]$$

Finally, Rosen derives the deadweight loss DWL in terms of compensating variation, found by expanding the expenditure function, which gives, see (13) on p. 98 in Rosen (1997),

(A.14) $$DWL = \left(\theta(1-\theta)\sigma_p(\tau+\rho)^2 + (1-\varphi)\sigma_c(\theta\tau - (1-\theta)\rho)^2\right)qz/2$$

where τ and ρ are the tax and subsidy rates. In the footnote to his table 2.3, Rosen explains that the values for σ_c in Table A.1 are those implied by his equation (14), i.e. our (A.12), for the indicated values of σ_p and η_{tw}, when he assumes that $h = 0.5$, $\theta = 0.5$, $\varphi = 0.25$ and $\eta_{zI} = 1$ (explained in the text on p.99 and 100). Then he uses his equation (13), i.e. our (A.14), for corresponding pairs of σ_p and σ_c and his benchmarks values $\tau = 0.7$ and $\rho = -0.9$ to compute the deadweight loss as a fraction of qz.

Table A.1. Deadweight loss (DWL) for alternative parameters: Sherwin Rosen's model, $\tau = 0.7$ and $\rho = -0.9$

	$\eta_{tw} = 1/3$		$\eta_{tw} = 2/3$		$\eta_{tw} = 1$	
σ_p	σ_c	DWL	σ_c	DWL	σ_c	DWL
0	3.20	0.77	4.11	0.99	5.00	1.20
1	1.88	0.46	2.78	0.67	3.67	0.89
2	0.56	0.14	1.44	0.36	2.33	0.57
3	NA	NA	0.11	0.04	1.00	0.26

Source: Rosen (1997, Table 2.3, p.100)

Optimal tax structure with externality in the utility function and non-separable externalities

In the general case of non-separable externalities the indirect utility function is given by

(B.1) $$G(w,p,Z) = u(x(q,I,Z), z(q,I,Z), nz(q,I,Z))$$

where we have used the definition $Z = nz$ and applied the demand functions $x = x(q,I,Z)$ and $z = z(q,I,Z)$ where the externality appears as an argument. The partial derivatives of the indirect utility function (B.1) are given by

(B.2) $$\begin{aligned} G_w &= u_x\left(x_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + x_I + x_E\frac{\partial Z}{\partial w}\right) \\ &+ u_z\left(z_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + z_I + z_E\frac{\partial Z}{\partial w}\right) \\ &+ u_E n\left(z_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + z_I + z_E\frac{\partial Z}{\partial w}\right) \end{aligned}$$

and

(B.3) $$\begin{aligned} G_p &= u_x\left(x_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + x_E\frac{\partial Z}{\partial p}\right) \\ &+ u_z\left(z_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + z_E\frac{\partial Z}{\partial p}\right) \\ &+ u_E n\left(z_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + z_E\frac{\partial Z}{\partial p}\right). \end{aligned}$$

Here we have used the definition of $q = q(w,p,Z)$ as well as the condition $I = w$ given in the consumer budget equation

(B.4) $$x + q(w,p,Z)z = I = w.$$

From the utility maximization problem we have that

(B.5) $$u_x = \mu$$

and

(B.6) $$u_z = \mu q(w,p,Z).$$

Partially differentiating the budget constraint (B.4) with respect to *w* and *p* we obtain

(B.7) $$x_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + x_I + x_E\frac{\partial Z}{\partial w} + q\left(z_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + z_I + z_E\frac{\partial Z}{\partial w}\right) + z\left(q_w + q_E\frac{\partial Z}{\partial w}\right) = 1$$

and

(B.8) $$x_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + x_E\frac{\partial Z}{\partial p} + q\left(z_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + z_E\frac{\partial Z}{\partial p}\right) + z\left(q_p + q_E\frac{\partial Z}{\partial p}\right) = 0\,.$$

Multiplying (B.7) and (B.8) by μ, inserting in (B.2) and (B.3) and using (B.5) and (B.6) we find

(B.9) $$G_w = \mu\left(1 - z\left(q_w + q_E\frac{\partial Z}{\partial w}\right)\right) + nu_E\left(z_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + z_I + z_E\frac{\partial Z}{\partial w}\right)$$

and

(B.10) $$G_p = -\mu z\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + nu_E\left(z_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + z_E\frac{\partial Z}{\partial p}\right).$$

In order to make the notation more compact, we now follow Sandmo (1998) and consider the function

(B.11) $$Z = nz\big(q(w,p,Z),I,Z\big).$$

Taking the partial derivatives of (B.11) we obtain

(B.12) $$\frac{\partial Z}{\partial w} = n\left(z_q\left(q_w + q_E\frac{\partial Z}{\partial w}\right) + z_I + z_E\frac{\partial Z}{\partial w}\right)$$

and

(B.13) $$\frac{\partial Z}{\partial p} = n\left(z_q\left(q_p + q_E\frac{\partial Z}{\partial p}\right) + z_E\frac{\partial Z}{\partial p}\right).$$

We rewrite (B.12) and (B.13) to obtain

(B.14) $$\frac{\partial Z}{\partial w} = \frac{n(z_q\, q_w + z_I)}{1 - n\, z_q\, q_E - n\, z_E} = nk(z_q\, q_w + z_I)$$

and

(B.15) $$\frac{\partial Z}{\partial p} = \frac{n\, z_q\, q_p}{1 - n\, z_q\, q_E - n\, z_E} = nk\, z_q\, q_p$$

where

(B.16) $$k = \frac{1}{1 - n\, z_q\, q_E - n\, z_E}.$$

Using (B.12) and (B.13) and recalling that in the case of non-separable externalities we have that $h = zq_w(w, p, Z)$ and $M = zq_p(w, p, Z)$, we finally obtain

(B.17) $$G_w = \mu(1 - h) + (u_E - \mu\, z\, q_E)\frac{\partial Z}{\partial w}$$

and

(B.18) $$G_p = -\mu\, M + (u_E - \mu\, z\, q_E)\frac{\partial Z}{\partial p}.$$

Hence, the first-order conditions for (25) are

(B.19) $$\mu(1 - h) + (u_E - \mu\, z\, q_E)\frac{\partial Z}{\partial w} + \nu\left[\tau\left(\frac{\partial h}{\partial w} + h_E\frac{\partial Z}{\partial w}\right) + (1 - h) - \rho\left(\frac{\partial M}{\partial w} + M_E\frac{\partial Z}{\partial w}\right)\right] = 0$$

and

(B.20) $$-\mu\, M + (u_E - \mu\, z\, q_E)\frac{\partial Z}{\partial p} + \nu\left[\tau\left(\frac{\partial h}{\partial p} + h_E\frac{\partial Z}{\partial p}\right) - M - \rho\left(\frac{\partial M}{\partial p} + M_E\frac{\partial Z}{\partial p}\right)\right] = 0$$

which, in the case of no externalities, i.e. $u_E = q_E = h_E = M_E = 0,$ is equivalent to the conditions given in Rosen equation (11). From (B.11), (B.14) and (B.15) we immediately have

(B.21) $$\eta_{zw} = \frac{w}{z}\frac{\partial z}{\partial w} = k\frac{w}{z}(z_q\, q_w + z_I)$$

and

(B.22) $$\eta_{zp} = \frac{p}{z}\frac{\partial z}{\partial p} = k\frac{p}{z} z_q \, q_p \, .$$

Note that in (B.21) and (B.22) the factor *k* captures all the terms involving the externalities, hence, the other factors contain no externality terms, and we can directly apply Rosen's conversion to Slutsky elasticities and substitution elasticities. We thus obtain

(B.23) $$\begin{aligned}\eta_{zw} &= k\frac{w}{z}\left(z_q q_w + z_I\right) = k\frac{w}{z}\left(\left(\frac{z}{q}\varepsilon_{zq} - zz_I\right)\frac{h}{z} + z_I\right)\\ &= k\left(-\theta\left(1-\varphi\right)\sigma_c + \left(1-\theta\varphi\right)\eta_{zI}\right)\end{aligned}$$

where we have defined the Slutsky elasticity

(B.24) $$\varepsilon_{zq} = \frac{q}{z} z_q = -(1-\varphi)\sigma_c$$

and used the Slutsky equation and the result that in the two-commodity case the Slutsky elasticity is proportional to the elasticity of substitution. Similarly, we find

(B.25) $$\eta_{zp} = -k\,(1-\theta)\left((1-\varphi)\sigma_c + \varphi\,\eta_{zI}\right).$$

Converting (B.19) and (B.20) to elasticities we obtain

(B.26) $$\frac{\mu+\nu}{\nu} w(1-h) + \frac{1}{\nu}\left(u_E - \mu\, z\, q_E\right) n\, z\, \eta_{zw} = -\tau\, h\left(\eta_{hw} + \frac{h_E}{h} n\, z\, \eta_{zw}\right) + \rho\, M\left(\eta_{Mw} + \frac{M_E}{M} n\, z\, \eta_{zw}\right)$$

and

(B.27) $$\frac{\mu+\nu}{\nu} pM - \frac{1}{\nu}\left(u_E - \mu\, z\, q_E\right) n\, z\, \eta_{zp} = \tau\, h\left(\eta_{hp} + \frac{h_E}{h} n\, z\, \eta_{zp}\right) - \rho\, M\left(\eta_{Mp} + \frac{M_E}{M} n\, z\, \eta_{zp}\right)$$

where η_{zw} and η_{zp} denote the elasticities of $z(q(w,p),I,Z)$ with respect to *w* and *p*, obtained directly from (B.21) and (B.22). From Appendix A we have that η_{hw} and η_{hp} denote the elasticities of $h(w,p,z)$ with respect to *w* and *p*, and η_{Mw} and η_{Mp} denote the elasticities of $M(w,p,z)$ with respect to *w* and *p*.

Solve the two linear equations in τ and ρ to obtain

$$\text{(B.28)} \quad \begin{aligned}\tau = &-\frac{1}{D}\frac{\mu+v}{v}M\left(w(1-h)\left(\eta_{Mp}+\frac{M_E}{M}n\,z\,\eta_{zp}\right)+pM\left(\eta_{Mw}+\frac{M_E}{M}n\,z\,\eta_{zw}\right)\right)\\ &-\frac{1}{D}\frac{1}{v}\left(u_E-\mu\,z\,q_E\right)n\,zM\left(\eta_{zw}\left(\eta_{Mp}+\frac{M_E}{M}n\,z\,\eta_{zp}\right)-\eta_{zp}\left(\eta_{Mw}+\frac{M_E}{M}n\,z\,\eta_{zw}\right)\right)\end{aligned}$$

and

$$\text{(B.29)} \quad \begin{aligned}\rho = &-\frac{1}{D}\frac{\mu+v}{v}h\left(pM\left(\eta_{hw}+\frac{h_E}{h}n\,z\,\eta_{zw}\right)+w(1-h)\left(\eta_{hp}+\frac{h_E}{h}n\,z\,\eta_{zp}\right)\right)\\ &+\frac{1}{D}\frac{1}{v}\left(u_E-\mu\,z\,q_E\right)n\,zh\left(\eta_{zp}\left(\eta_{hw}+\frac{h_E}{h}n\,z\,\eta_{zw}\right)-\eta_{zw}\left(\eta_{hp}+\frac{h_E}{h}n\,z\,\eta_{zp}\right)\right)\end{aligned}$$

where D is the determinant of the coefficient matrix of the system (B.26) - (B.27),

$$\text{(B.30)} \quad D = hM\begin{vmatrix} -\eta_{hw}-\frac{h_E}{h}n\,z\eta_{zw} & \eta_{Mw}+\frac{M_E}{M}n\,z\eta_{zw} \\ \eta_{hp}+\frac{h_E}{h}n\,z\eta_{zp} & -\eta_{Mp}-\frac{M_E}{M}n\,z\eta_{zp}\end{vmatrix} = hM(1-\varphi)\sigma_p\left(\sigma_c-\eta_{zI}\right)k_1$$

where

$$\text{(B.31)} \quad k_1 = 1 + n\,k\theta\frac{h_E}{h}z + n\,k\,(1-\theta)\frac{M_E}{M}z\,.$$

Note that $k_1 = 1$ in the case of separable externalities. The solution for the optimal tax rates imposes the restrictions that $h \neq 0,\; M \neq 0,\; \sigma_p \neq 0$, $\sigma_c \neq \eta_{zI}$, $\varphi \neq 1$ and $k_1 \neq 0$. We will now show how the expressions in (B.28) and (B.29) can be simplified into expressions corresponding to Rosen's solution given by his equation (12) plus an externality correction term corresponding to Sandmo's solution given by his equation (23) in Sandmo (1975). Utilizing Rosen's elegant expressions for the elasticities, see Appendix A, we derive the following auxiliary results:

$$\text{(B.32)} \quad -w(1-h)\eta_{Mp} - pM\,\eta_{Mw} = w(1-\varphi)\left(\theta\sigma_p + (1-\theta)\sigma_c\right)$$

$$\text{(B.33)} \quad -w(1-h)\eta_{zp} - pM\,\eta_{zw} = kw(1-\theta)(1-\varphi)\sigma_c$$

$$\text{(B.34)} \quad \eta_{Mp}\,\eta_{zw} - \eta_{Mw}\,\eta_{zp} = k\theta(1-\varphi)\sigma_p\left(\sigma_c - \eta_{zI}\right)$$

(B.35) $$-w(1-h)\eta_{hp} - pM\,\eta_{hw} = -w(1-\theta)(1-\varphi)(\sigma_p - \sigma_c)$$

(B.36) $$\eta_{hw}\,\eta_{zp} - \eta_{hp}\,\eta_{zw} = k(1-\theta)(1-\varphi)\sigma_p(\sigma_c - \eta_{zI})$$

Using the definitions of the cost share θ and the budget share φ, we note e.g. that $wh(1-\theta) = pM\theta$, $pM = (1-\theta)\varphi w$, $w\theta - wh = x$ and $x = w(1-\varphi)$. We also recall that $h = \theta\varphi$ and $u_z = \mu q$. Moreover, we define $\beta = -\mu/v$, and we immediately see that both τ and ρ appear as weighted averages of efficiency terms and externality terms. With these substitutions it immediately follows that

(B.37) $$\frac{\tau}{w} = (1-\beta)\frac{\theta\sigma_p + (1-\theta)\sigma_c}{\theta\varphi\sigma_p(\sigma_c - \eta_{zI})k_1} + (1-\beta)\,\frac{(1-\theta)\sigma_c nkz\,M_E/M}{\theta\varphi\sigma_p(\sigma_c - \eta_{zI})k_1} + \beta\frac{nk}{k_1}\left(\frac{u_E}{u_z} - z\frac{q_E}{q}\right)$$

and

(B.38) $$-\frac{\rho}{p} = (1-\beta)\frac{\sigma_p - \sigma_c}{\varphi\sigma_p(\sigma_c - \eta_{zI})k_1} - (1-\beta)\,\frac{\sigma_c nkz\,h_E/h}{\varphi\sigma_p(\sigma_c - \eta_{zI})k_1} + \beta\frac{nk}{k_1}\left(\frac{u_E}{u_z} - z\frac{q_E}{q}\right).$$

The solution contains two efficiency terms and a composite externality term, depending on both u_E and q_E .With $q_E < 0$ the externality term is positive. The first efficiency term equals Rosen's solution. The second efficiency terms represent the additional need for tax revenue to finance the increased demand for child quality due to the positive externality. The additional term in the optimal income tax rate represents the tax requirement to finance the subsidized child care via M_E, whereas the corresponding term in the optimal subsidy rate represents the income tax loss from parental child care via h_E. We assume that these demand feedback effects are small compared to the pure externality term, so that the total externality effect on the subsidy rate is positive.

The demand feedback effects cancel out when we consider the difference between the income tax rate and the subsidy rate. Applying the definition of k_1 in (B.31) we find that the difference between the optimal income tax and subsidy rate also in the general case is given by

(B.39) $$\frac{\tau}{w} - \left(-\frac{\rho}{p}\right) = (1-\beta)\frac{\sigma_c}{\theta\varphi\sigma_p(\sigma_c - \eta_{zI})}$$

as we found in (16) for the case of separable externalities.