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Abstract:
We outline a novel procedure to identify the role of measurement errors in explaining the empirical dispersion in productivity across establishments. The starting point of our framework is the typical errors-in-variable model consisting of a measurement equation and a structural equation for the true productivity. The key idea in our identification strategy is to estimate the variance of the measurement errors in order to deduce the variance of the companion true variable. Specifically, we estimate a dynamic panel model where establishment-specific productivity is modelled as a first order autoregressive process augmented with year dummies and establishment-specific unobserved heterogeneity, represented by random effects. Considering 3 Norwegian manufacturing industries, as an example, we find that about 4 per cent of the measured dispersion is caused by measurement errors.

Keywords: Labor productivity; Productivity dispersion; Establishment performance

JEL classification: C23; C26; J24

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Sammendrag

Spredningen i produktivitet blant virksomheter er stor. Standardavviket i arbeidsproduktiviteten, målt ved bruttoprodukt per timeverk, er typisk i størrelsesordenen 30 til 100 prosent (Bartelsman og Wolf, 2017). Også i Norge er spredningen i arbeidsproduktivitet i denne størrelsesordenen. Flere årsaker til det store standardavviket har vært analysert i litteraturen: forskjeller i kvaliteten på ledelsen i virksomhetene (Bloom og Van Reenen, 2010), ulik bruk av produksjonsfaktorer (Crepon mfl., 1998), hvor substituerbare produktene er (Syverson, 2004) og konkurranseforholdene i markedet (Bloom mfl., 2013). Selv om det er velkjent at målefeil potensielt kan være en viktig forklaring bak den store spredningen i arbeidsproduktivitet så har det vært veldig lite forskning for å identifisere hvor stort dette bidraget er.

I denne artikkelen utleder vi en ny metode for å beregne hvor mye av spredningen i produktivitet blant virksomheter som skyldes målefeil. Rammeverket vårt tar utgangspunkt i litteraturen om målefeil, se f.eks. Meijer mfl. (2017). Vi estimerer en modell basert på et dynamisk panel hvor produktivitets-nivået følger en første-ordens autoregressiv prosess med årlige og virksomhetsspesifikke dummy-variabler. For at resultatene våre skal kunne sammenlignes med det man ofte finner i litteraturen definerer vi relativ produktivitet som logaritmen av det virksomhetsspesifikke produktivitetsnivået i forhold til det gjennomsnittlige produktivitetsnivået. Vår analyse indikerer at om lag 4 prosent av den målte spredningen i produktivitet kan spores tilbake til målefeil.

1. Introduction

It is widely accepted that dispersion of productivity across establishments and industries is large. A common way to measure dispersion is by looking at the standard deviation across establishments where the productivity of each establishment is measured relative to a reference point, such as the mean productivity level at a given point in time. Using this procedure, it is typically found that the standard deviation across establishments is large and lies in the range of 30 to 100 per cent, see Bartelsman and Wolf (2017).

Several reasons have been put forward to explain this large dispersion in productivity: quality of management (Bloom and Van Reenen, 2010), different input usage, as the intensity of R&D or other intangible capital (Crepon et al., 1998), product substitutability (Syverson, 2004), product market rivalry (Bloom et al., 2013), or market distortions (Hsieh and Klenow, 2009), to name a few. Although it is acknowledged that a sizable portion of productivity dispersion may also be due to measurement errors, little research has been devoted to identify how much they actually contribute.

In this paper, we outline a novel procedure to identify the role of measurement errors in explaining the empirical dispersion in productivity across establishments. We define productivity as the log of the ratio between gross nominal output and the number of man-hours of employees. One reason for the presence of measurement errors in productivity is that our labor input variable relates to labor input according to the labor contract, which may deviate from the actual man-hours executed. Another source of measurement errors is misclassification which occurs when the main part of the establishment's production belongs to another industry than the one considered, cf. e.g. Bartelsman et al. (2009, p. 28). As emphasized by among others Jones (2016, Ch. 4.10), applying an output measure in constant prices using a deflator which is common for all units in an industry raises identification issues. Hence, we use gross output in nominal terms and thus consider a revenue productivity measure.

The starting point is the typical errors-in-variable framework consisting of a measurement equation and a structural equation for the true productivity. The key idea in our identification strategy is to estimate the variance of the measurement errors in a consistent way such that we can deduce the variance of the companion true variable. To this end we build on the econometric theory of measurement errors, see e.g. Meijer et al. (2017). Specifically, we estimate a dynamic panel model where the log of establishment-specific productivity is modelled as a first order autoregressive process augmented with year dummies and establishment-specific unobserved heterogeneity, represented by random effects. To get the results on a form that is comparable with earlier studies, we deduce the (model-based) standard deviation of the log of productivity scaled by a geometric mean of the productivity of the establishments that are present in a specific year. Such a calculation is done both for the variable contaminated by measurement errors and for the true variable. Our findings indicate that only about 4 per cent of measured dispersion in productivity is caused by measurement error.

The rest of this paper is organised as follows. Section 2 outlines the procedure and the model for establishment-specific productivity. Section 3 describes the data and presents the results. Section 4 concludes.

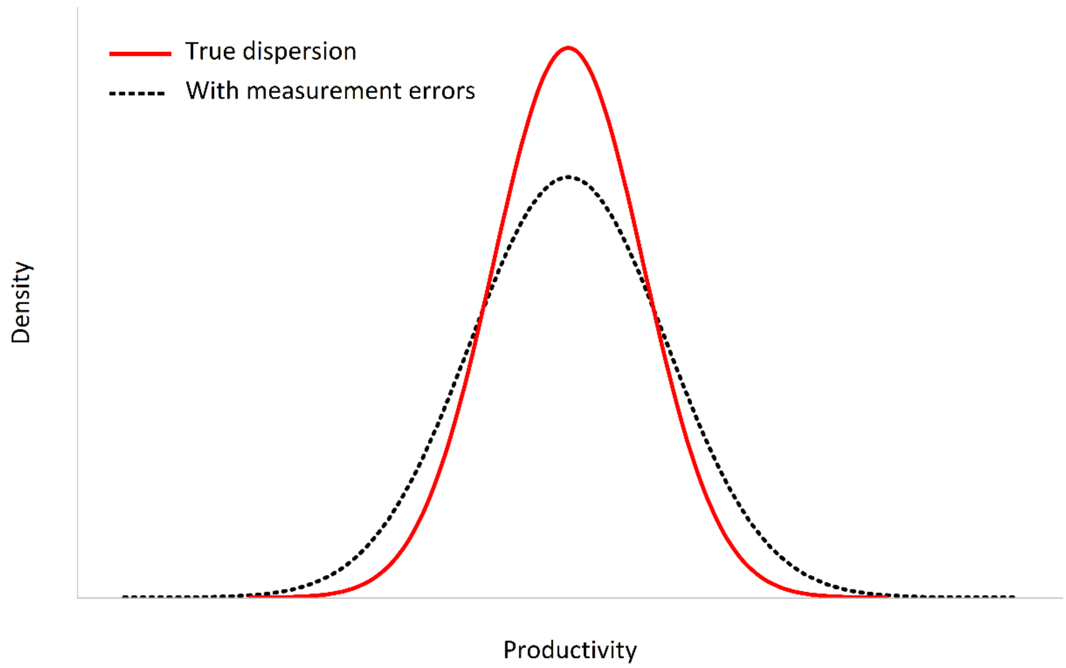

Figure 1. Productivity dispersion with and without measurement errors

2. Modelling framework

Figure 1 illustrates the conceptual challenge of measuring the true dispersion in productivity across establishments. Measurement errors make the tales in the empirically observed distribution fatter and increase dispersion compared with the true distribution of productivity across establishments. Analytically, this may be illustrated by considering the following econometric model

$$y_{it} = y_{it}^{*} + u_{it}, \tag{1}$$

where y_{it} denotes observed productivity and y_{it}^* true productivity of establishment *i* in year *t*. The last symbol in Eq. (1), u_{it}, denotes a random measurement error. Let the variances of the observed productivity and the true productivity variables be denoted by σ_{yy}^2 and $\sigma_{y^*y^*}^2$, respectively. It follows from taking the variance of Eq. (1) that the presence of measurement errors leads to a wider dispersion in productivity, i.e. $\sigma_{yy}^2 > \sigma_{y^*y^*}^2$.

To identify how much of the variance of observed productivity is due to measurement errors, we apply the following model for true productivity

$$y_{it}^* = \sum_{j=t_0}^{t_1} \alpha_j Dj_t + \beta y_{i,t-1}^* + \mu_i + \varepsilon_{it}. \tag{2}$$

The variables $Dt_{0t},...,Dt_{1t}$ represent dummy variables for the years. Establishment-specific unobserved heterogeneity is represented by random effects, μ_i. The last symbol on the right-hand side of Eq. (2), ε_{it}, is a genuine error. The systematic part of Eq. (2) involves the autoregressive slope coefficient β, which is between zero and unity, and the year effects $\alpha_{t_0},...,\alpha_{t_1}$. For the three uncorrelated unobserved variables we assume that they all have expectation 0 and that $Var(u_{it}) = \sigma_{uu}^2$, $Var(\mu_i) = \sigma_{\mu\mu}^2$ and $Var(\varepsilon_{it}) = \sigma_{\varepsilon\varepsilon}^2$ for all *i* and *t*.[1]

The key idea in our identification strategy is to obtain a consistent estimate of the variance of the measurement error, which then in turn, will enable us to estimate the variance of the true productivity consistently. Inserting for y_{it}^* from Eq. (1) into Eq. (2) yields

$$y_{it} = \sum_{j=t_0}^{t_1} \alpha_j Dj_t + \beta y_{i,t-1} + \mu_i + \varepsilon_{it} + u_{it} - \beta u_{i,t-1}, \tag{3}$$

where we let $\eta_{it} = \mu_i + \varepsilon_{it} + u_{it} - \beta u_{i,t-1}$ denote the gross error term. It follows from our assumptions that

$$Var(\eta_{it}) = \sigma_{\mu\mu}^2 + \sigma_{\varepsilon\varepsilon}^2 + (1+\beta^2)\sigma_{uu}^2, \tag{4}$$

$$Cov(\eta_{it}, \eta_{i,t-1}) = \sigma_{\mu\mu}^2 - \beta\sigma_{uu}^2 \tag{5}$$

and

---

[1] Our modelling framework also covers the situation when one has systematic measurement errors in the sense that Eq. (1) is augmented with an intercept. In that case the intercept in Eq. (3) will be the sum of the intercept in the measurement equation, i.e. the one in Eq. (1), and the intercept in Eq. (2). However, the two components cannot be identified, only their sum.

$$Cov(\eta_{it}, \eta_{i,t-s}) = \sigma^2_{\mu\mu}, s > 1. \tag{6}$$

Consistently with Eq. (4)- Eq. (6), we may write

$$\sigma^2_{\mu\mu} = Cov(\eta_{it}, \eta_{i,t-2}), \tag{7}$$

$$\sigma^2_{uu} = \frac{\sigma^2_{\mu\mu} - \mathrm{cov}(\eta_{it}, \eta_{i,t-1})}{\beta} \tag{8}$$

and

$$\sigma^2_{\varepsilon\varepsilon} = Var(\eta_{it}) - \sigma^2_{\mu\mu} - (1+\beta^2)\sigma^2_{\varepsilon\varepsilon}. \tag{9}$$

By utilizing that y_{it} follows a stationary process one might, as shown in the appendix, deduce the following expression for the variance of the observed productivity variable

$$Var(y_{i,t}) = \sigma^2_{yy} = \sigma^2_{uu} + \frac{\sigma^2_{\varepsilon\varepsilon}}{1-\beta^2} + \frac{\sigma^2_{\mu\mu}}{(1-\beta)^2}. \tag{10}$$

Correspondingly, one obtains the following formula for the true productivity variable

$$Var(y^*_{it}) = \sigma^2_{y^*y^*} = \frac{\sigma^2_{\varepsilon\varepsilon}}{1-\beta^2} + \frac{\sigma^2_{\mu\mu}}{(1-\beta)^2}. \tag{11}$$

In empirical work attention is often devoted to the standard deviation of productivity less the mean productivity for the establishments present in a specific year. It is shown in the appendix that in our framework this measure can be approximated by $\sqrt{\sigma^2_{yy}}$ in the case when the measurement errors are present and by $\sqrt{\sigma^2_{y^*y^*}}$ in the case when they have been eliminated.

Estimation of the variances in Eq. (10) and Eq. (11) requires estimates of both β and the variances of the unobserved components. These unknown parameters are estimated sequentially. We start with the β parameter. To get rid of the establishment-specific unobserved heterogeneity we take the first difference in Eq. (3) and obtain the following equation

$$\Delta y_{it} = \sum_{j=t_0+1}^{t_1} \alpha_j \Delta Dj_t + \beta \Delta y_{i,t-1} + \Delta\varepsilon_{it} + \Delta u_{it} - \beta \Delta u_{i,t-1}. \tag{12}$$

Note that $\Delta y_{i,t-1}$ is correlated with the 'gross error term', $\Delta\varepsilon_{it} + \Delta u_{it} - \beta\Delta u_{i,t-1}$. The same is true for $\Delta y_{i,t-2}$ since this lagged difference is correlated with $\beta\Delta u_{i,t-1}$. Thus, $\Delta y_{i,t-2}$ is not suitable as an instrument variable for $\Delta y_{i,t-1}$ in Eq. (12). Hence, we employ the variable $\Delta y_{i,t-3}$ as an identifying instrument. Since we are using a differenced variable lagged three years, we drop the dummy variables related to the initial years. Thus, instead of Eq. (12) the equation to be estimated is

$$\Delta y_{it} = \sum_{j=t_0+3}^{t_1} \alpha_j \Delta Dj_t + \beta\Delta y_{i,t-1} + \Delta\varepsilon_{it} + \Delta u_{it} - \beta\Delta u_{i,t-1}. \tag{13}$$

Let $\hat{\beta}$ denote the IV-estimate of β. To proceed, we plug in the estimate of β in Eq. (3) and obtain:[2]

$$qy_{it} = y_{it} - \hat{\beta} y_{i,t-1} = \sum_{j=t_0}^{t_1} \alpha_j Dj_t + \mu_i + \varepsilon_{it} + u_{it} - \beta u_{i,t-1}, \tag{14}$$

where the quasi-differenced variable, qy_{it}, is a generated regressand. We estimate the year effects consistently by OLS and denote the estimates by $\tilde{\alpha}_{t_0},...,\tilde{\alpha}_{t_1}$. Note that the standard errors in a two stage estimation procedure need to corrected, see Dumont et al. (2005). However, the standard errors are not needed for estimating the size of measurement errors: given consistent estimates of $\tilde{\alpha}_{t_0},...,\tilde{\alpha}_{t_1}$, the variance components are estimated by utilizing the gross residuals given by

$$\hat{\eta}_{it} = qy_{it} - \sum_{j=t_0}^{t_1} \hat{\alpha}_j Dj_t \tag{15}$$

and Eqs. (7)-(9).

3. Empirical application

The modelling framework outlined above can be utilized for any operationalization of labour productivity. In the empirical application below, we consider a revenue labour productivity measure, i.e., we define productivity as the log of the ratio between gross nominal output and the number of man-hours of employees. We apply our framework using data from the years 2000-2014 to three Norwegian industries at the 2-digit NACE code level. Thus, in the application t_0 = 2001 and t_1 = 2014. The three industries are (i) Manufacture of food products (Industry 10), (ii) Manufacture of wood and

[2] To avoid introducing new notation we retain the notation used for the unobserved variables on the right-hand side of the equation.

of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials (Industry 16) and (iii) Printing and reproduction of recorded media (Industry 18). Equation (3) is specified for a single industry. We also consider a pooled case in which we estimate the model using data for different industries. In this case the model is augmented with industry dummies for all industries, except the one which is the reference industry. The procedure is analog to the one used in the single industry case. Final estimates of industry and calendar effects are obtained from an equation analog to Eq. (14), where fixed industry effects have been added.

Table 1 shows the estimate of the autoregressive parameter, β, and the estimates of the different components involved in the decomposition of the variance of log revenue productivity. There are three sources of variation which, respectively, stem from the contribution of measurement errors, the contribution of establishment-specific unobserved heterogeneity represented by random effects and the contribution of genuine errors in the specification of the true log revenue productivity variable. In the three last rows of Table 1 we report the share of the variance of log revenue productivity coming from the three identified sources. Our main interest is in the contribution by the measurement errors. In the pooled case, the share of the variance of productivity coming from the measurement errors is 7.3 percent. Looking at the individual industries the largest contribution is found for Industry 16 and the lowest for Industry 10. From Table 1 one can also see that the estimated variance of productivity, i.e. the estimate of σ_{yy}^2, is not very different from the corresponding empirical measure reported in the fourth-last row in Table 1.

Table 1. Decomposing the variance of observed productivity[a]

Terms	Industry			
	10	16	18	Pooled
β	0.800	0.580	0.649	0.759
	(4.237)	(1.815)	(2.632)	(5.289)
σ_{uu}^2	0.032	0.048	0.018	0.037
$(1-\beta^2)^{-1}$	2.773	1.507	1.731	2.360
$\sigma_{\varepsilon\varepsilon}^2$	0.115	0.100	0.086	0.101
$(1-\beta^2)^{-1}\sigma_{\varepsilon\varepsilon}^2$	0.319	0.151	0.148	0.238
$(1-\beta)^{-2}$	24.908	5.673	8.154	17.238
$\sigma_{\mu\mu}^2$	0.013	0.035	0.016	0.014
$(1-\beta)^{-2}\sigma_{\mu\mu}^2$	0.336	0.196	0.130	0.234
σ_{yy}^2	0.687	0.395	0.296	0.509
Empirical variance of log productivity	0.691	0.382	0.270	0.525
$\sigma_{uu}^2(\sigma_{yy}^2)^{-1}$	0.047	0.122	0.061	0.073
$(1-\beta^2)^{-1}\sigma_{\varepsilon\varepsilon}^2(\sigma_{yy}^2)^{-1}$	0.464	0.382	0.500	0.468
$(1-\beta)^{-2}\sigma_{\mu\mu}^2(\sigma_{yy}^2)^{-1}$	0.489	0.496	0.439	0.460

[a]*t*-values in parentheses are based on standard errors calculated from a heteroscedastic-consistent matrix. The number of observations used for estimating β is 12,635; 8,305 and 7,063 in industries 10, 16 and 18, respectively. The number of observations used for estimating the year effects, which are not reported in the table, is 21,273; 15,654 and 13,046 in industries 10, 16 and 18, respectively. The number of observations used for estimating the variance components is 14,984; 10,205 and 8,622 in industries 10, 16 and 18, respectively.

In Table 2 we focus on the (model-based) standard deviation of productivity less the mean of productivity for all establishments in a specific year. We report results based on our model both for the variable contaminated by measurement errors and for the companion true variable. The results reported in Table 2 can be used to infer the positive bias caused by neglecting measurement errors when reporting figures on the spread in productivity. In the pooled case, the spread is decreasing from 0.71 to 0.69 when measurement errors are allowed for. The largest decrease is found for Industry 16 where the estimated spread when measurement errors are not corrected for is 0.63 and the estimated spread with correction for measurement errors is 0.59.

Table 2. Spread in observed and true productivity on an annual basis (in per cent)

	Industry			
Spread	10	16	18	Pooled
$\sqrt{\sigma^2_{yy}}$	82.9	62.9	54.4	71.4
$\sqrt{\sigma^2_{y^*y^*}}$	80.9	59.0	52.7	68.7
Difference	2.0	3.9	1.7	2.7

4. Conclusion

In this paper, we have outlined a novel procedure to identify the role of measurement errors in explaining the empirical dispersion in productivity across establishments. The starting point of our framework is the typical errors-in-variable model consisting of a measurement equation and a structural equation for the true productivity. The key idea in our identification strategy has been to estimate the variance of the measurement errors in order to deduce the variance of the companion true variable. Specifically, we have estimated a dynamic panel model where establishment-specific productivity is modelled as a first order autoregressive process augmented with year dummies and establishment-specific unobserved heterogeneity, represented by random effects. Using the case of 3 manufacturing industries in Norway as an illustrative example, we found that about 4 per cent of the measured dispersion is caused by measurement errors.

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APPENDIX A. Derivation of various formulae

Treating the calendar variables as deterministic and taking the variance of both sides of Eq. (3) yields

$$\begin{aligned} &Var(y_{it}) = Var(\varepsilon_{it}) + Var(u_{it}) + Var(\beta y_{i,t-1} + \mu_i - \beta u_{i,t-1}) = \\ &\sigma_{\varepsilon\varepsilon}^2 + \sigma_{uu}^2 + \beta^2 Var(y_{i,t-1}) + \sigma_{\mu\mu}^2 + \beta^2\sigma_{uu}^2 + 2\beta Cov(y_{i,t-1}, \mu_i) - 2\beta^2 Cov(y_{i,t-1}, u_{i,t-1}). \end{aligned} \tag{A1}$$

Consider first the term next to the last in Eq. (A1). Multiplying Eq. (3) with μ_i and taking expectations yields the following expression for $Cov(y_{i,t-1}, \mu_i)$ when we, consistent with our stationarity assumption, impose $Cov(y_{i,t-1}, \mu_i) = Cov(y_{i,t}, \mu_i)$:

$$Cov(y_{i,t-1}, \mu_i) = \frac{\sigma_{\mu\mu}^2}{1-\beta}. \tag{A2}$$

Furthermore, from stationarity it follows that $Cov(y_{i,t-1}, u_{i,t-1}) = Cov(y_{i,t}, u_{i,t})$. Multiplying Eq. (3) with u_{it} and taking expectations yields, in view of Eq. (A2)

$$Cov(y_{i,t-1}, u_{i,t-1}) = \sigma_{uu}^2. \tag{A3}$$

Inserting from Eqs. (A2) and (A3) in Eq. (A1) and, in view of stationarity, setting $Var(y_{i,t}) = Var(y_{i,t-1})$ yields

$$\begin{aligned} &Var(y_{it}) = \sigma_{\varepsilon\varepsilon}^2 + \sigma_{uu}^2 + \beta^2 Var(y_{i,t}) + \sigma_{\mu\mu}^2 + \beta^2\sigma_{uu}^2 + \frac{2\beta\sigma_{\mu\mu}^2}{1-\beta} - 2\beta^2\sigma_{uu}^2 \Leftrightarrow \\ &(1-\beta^2)Var(y_{i,t}) = (1-\beta^2)\sigma_{uu}^2 + \sigma_{\varepsilon\varepsilon}^2 + \frac{(1+\beta)\sigma_{\mu\mu}^2}{1-\beta} \Leftrightarrow \\ &Var(y_{i,t}) = \sigma_{uu}^2 + \frac{\sigma_{\varepsilon\varepsilon}^2}{1-\beta^2} + \frac{\sigma_{\mu\mu}^2}{(1-\beta)^2}, \end{aligned} \tag{A4}$$

which corresponds to Eq. (10).

We are interested in the population standard deviation of productivity for an establishment less the mean of productivity for all the establishments present in a specific year, which may be written as

$$Std(y_{it} - \frac{1}{N_t}\sum_{j=1}^{N_t} y_{jt}) = Std(\frac{N_t - 1}{N_t} y_{it} - \frac{1}{N_t}\sum_{j \neq i}^{N_t} y_{jt}), \tag{A5}$$

where N_t denotes the number of establishments present in year *t*. According to our model assumptions the observations from different establishments are independent. Thus, we simply obtain

$$Std\left(\frac{N_t - 1}{N_t} y_{it} - \frac{1}{N_t} \sum_{j \neq i}^{N_t} y_{jt}\right) = \sqrt{\frac{N_t^2 - 2N_t + 1}{N_t^2} \sigma_{yy}^2 + \frac{N_t - 1}{N_t^2} \sigma_{yy}^2} = \sqrt{\frac{N_t^2 - N_t}{N_t^2} \sigma_{yy}^2} = \sqrt{\frac{N_t - 1}{N_t} \sigma_{yy}^2} \approx \sqrt{\sigma_{yy}^2}. \quad \text{(A6)}$$

Correspondingly for the true values we have

$$Std\left(y_{it}^* - \frac{1}{N_t} \sum_{j=1}^{N_t} y_{jt}^*\right) = Std\left(\frac{N_t - 1}{N_t} y_{it}^* - \frac{1}{N_t} \sum_{j \neq i}^{N_t} y_{jt}^*\right) \approx \sqrt{\sigma_{y^* y^*}^{*2}}. \quad \text{(A7)}$$

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