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Inflation dynamics in a small open economy*

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Abstract

We evaluate the empirical performance of forward-looking models for inflation dynamics in a small open economy. Using likelihood-based testing procedures, we find that the *exact* formulation is at odds with Norwegian data. Moreover, some of the parameters in the model are not well identified. We also find that the *inexact* formulation is *not* rejected statistically using a test based on a minimum distance method. However, confidence regions reveal an identification problem with this model as well. Instead, we find a well-specified backward-looking model with imperfect competition underlying the price setting, a model that outperforms an alternative forward-looking model in-sample. The backward-looking model also forecasts somewhat better than the alternative forward-looking model during and after the recent financial crisis.

Keywords: Forward-looking, backward-looking, cointegrated vector autoregressive models, equilibrium correction models, likelihood-based methods and minimum distance method.

JEL classification: C51, C52, E31, F31.

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I. Introduction

Forward-looking models, based on rational expectations, optimising agents and imperfect competition in markets for goods, have long played a central role in understanding inflation dynamics in the economics profession and in central banks conducting inflation targeting. Since the influential papers by Roberts (1995), Fuhrer and Moore (1995), Galí and Gertler (1999), Galí *et al.* (2001) and Sbordone (2002), a large number of studies have been devoted to testing the role of expectations in inflation dynamics based on data from both closed and open economies, see Mavroeidis *et al.* (2014) and An and Schorfheide (2007) for reviews of the literature. Studies differ with respect to the data used, the sample period studied and the econometric methods applied. The supportive evidence on forward-looking behaviour in price formation is rather mixed.

In this paper, we follow Mavroeidis *et al.* (2014), among several others, and evaluate the empirical performance of forward-looking models based on Norwegian data and using a limited-information approach. Building on Sbordone (2002), our forward-looking models relate current inflation to expected future inflation and the difference between the actual price and the steady state value of levels as a theory-consistent forcing variable. The steady state value is specified as a mark-up over marginal costs, which, in turn, are determined by costs of both labour and imported intermediate goods along the lines of the open economy models in McCallum and Nelson (1999), Kara and Nelson (2003) and Batini *et al.* (2005). We contribute to the empirical literature by studying both the *exact* formulation in the sense of Hansen and Sargent (1991) and the *inexact* formulation in which a stochastic error term is included in the model. For the *exact* formulation, we employ the likelihood-based testing procedures suggested by Johansen and Swensen (1999, 2008). Because a similar treatment of the *inexact* formulation is more complicated to handle, we rely on a test based on a minimum distance approach along the lines of Sbordone (2002), and Magnusson and Mavroeidis (2010). Consequently, we are able to shed some light on the importance of introducing a stochastic error term to the empirical model, an econometric issue that is often neglected in the literature. Unlike most related studies, e.g. Galí and Gertler (1999), Galí *et al.* (2001) and Batini *et al.* (2005), we pay particular attention to time series properties of the variables involved, and the possible existence of unit roots, and we search for statistically well-specified underlying models as premises for valid statistical inference on the forward-looking models, cf. Mavroeidis *et al.* (2014). Another issue that has been little addressed in the literature is forecasting performance. Recently, Rumler and Valderrama (2010) and King and Watson (2012) have discussed this issue. The latter two authors find a large gap between inflation predicted by the Smets and Wouters (2007) forward-looking model for the US and actual inflation. The gap can only be closed by assuming large and exogenous mark-up shocks. They recommend devoting more attention to detailed analysis of the structural inflation equation in order to detect imperfect specifications. In our study, we also compare and contrast specifications of a reduced form forward-looking model with a backward-looking model counterpart as two competing models of inflation dynamics, both in-sample and out-of-sample in a forecasting competition.

Our empirical investigation, which is based on a data set not used in this setting earlier, produces several noteworthy findings. Firstly, we establish a well-specified empirical counterpart to the theory-consistent link between consumer prices and marginal costs. Secondly, we demonstrate that the *exact* formulation of the forward-looking model is at odds with the data. That is, the rational expectations hypothesis is *not* rejected statistically. However, when only economically meaningful parameters are allowed for, the model is *not* supported by the data. In addition, a plot of the likelihood surface reveals that some of the parameters may not be well-identified. Using alternative methods to those of Kurmann (2007), we also discuss the *inexact* formulation of the forward-looking model and find no indication that this model yields radically different results. In particular, the identification problem seems to also be present in this model. Thirdly, we establish a well-specified competing backward-looking model of inflation dynamics in a sample containing a major monetary policy regime shift. Finally, we find that the backward-looking model forecasts somewhat better than a reduced form forward-looking model during and after the recent financial crisis.

The rest of the paper is organised as follows: Section II outlines our forward-looking models, Section III describes the data, Section IV reports findings from the cointegration analysis, Section V reports the various tests of the forward-looking models and Section VI evaluates the empirical performance of the reduced form forward-looking model compared with the backward-looking model, and conducts a forecasting competition between these models. Section VII concludes.

II. Theoretical framework

Several approaches have been suggested in the literature to introducing open economy features into the price formation of firms. One approach treats imports as substitutable final consumer goods for domestically produced goods and assumes that the representative firm operates in imperfectly competitive markets facing regular downward sloping demand curves, see, e.g., Bårdsen *et al.* (2005, ch. 8.7) and Galí and Monacelli (2005). Profit maximisation then implies that prices are set as a mark-up over marginal costs, where the mark-up depends on the relative prices of domestic and imported consumer goods. A second approach treats imports as intermediate goods in production rather than as final consumer goods, see, e.g., McCallum and Nelson (1999), Kara and Nelson (2003) and Batini *et al.* (2005). Profit maximisation then implies that prices of imports become important determinants of marginal costs and not of the mark-up. A third approach treats imports as intermediate inputs as well as final consumer goods. Hence, prices of imports would be accounted for through both the mark-up and the marginal costs. A disaggregated model would then be necessary.

We rely on the second approach in this paper and treat imports as intermediate goods in production. Our main argument is that imports are rarely imported by consumers themselves, but rather by the wholesale and retail trade sector, and are used in combination with other inputs to supply domestic consumers with goods. These imported goods are usually both intermediate goods and final consumer goods. For instance, an imported agricultural product can be bought in a shop by the consumer, as part of a meal in a restaurant or as an input to the domestic food industry. Petrol is used both by consumers as a final good and by firms as input in production. In any case, the wholesale and retail trade sector adds trade margins and domestic cost components to the costs of imports to make up the consumer prices of imported goods.

Assuming a representative profit-maximising firm facing an isoelastic, downward-sloping demand curve and a Cobb-Douglas production function in labour and imports of intermediate inputs, we have

$$p_t^* = m_0 + \psi_1 ulc_t + \psi_2 uic_t, \tag{1}$$

where lower case letters indicate natural logarithms and p_t^*, m_0, ulc_t and uic_t are the steady-state value of the price level, the constant mark-up, the unit labour costs and the unit import costs of intermediates, respectively.[1] The price equation in (1) is homogeneous of degree one in the factor prices, i.e. $\psi_2 = (1 - \psi_1)$.

The various forward-looking models proposed in the literature on the new Keynesian Phillips curve share essentially the same semi-structural form, but differ with respect to the specific underlying pricing behaviour, including the forward-looking linear quadratic adjustment cost model of Rotemberg (1982), the models of staggered contracts developed by Taylor (1979, 1980) and the sticky price model of Calvo (1983). We build on equation (2.9) in Sbordone (2002) and specify our forward-looking models as

$$\Delta p_t = \delta E_t \Delta p_{t+1} - \lambda(p_t - p_t^*), \tag{2}$$

where $\Delta p_t = p_t - p_{t-1}$ is current inflation and $E_t \Delta p_{t+1}$ is expected inflation one period ahead, conditional on information available at time t. Now, inserting (1) in (2) yields a

[1]A full derivation of (1) is shown in Appendix 1. In the literature on the closed economy new Keynesian Phillips curve, it is common to assume that producers face isoelastic demand curves, so that the mark-up is a constant, see, e.g., Galí and Gertler (1999) and Galí *et al.* (2001).

forward-looking model of inflation dynamics

$$\Delta p_t = \delta E_t \Delta p_{t+1} - \lambda eqcm_t + \psi_0, \tag{3}$$

where

$$eqcm_t = p_t - \psi_1 ulc_t - \psi_2 uic_t \tag{4}$$

and $\psi_0 = \lambda m_0$. If our specification of price formation in a small open economy, as shown in (1), is supported by the data, then the equilibrium correction term in (4) with the homogeneity restriction imposed should be a stationary variable. This is a testable implication that we will return to in Section IV. Inspired by Galí and Gertler (1999) among others, we specify the so-called hybrid version of (3) as

$$\Delta p_t = \gamma_f E_t \Delta p_{t+1} + \gamma_b \Delta p_{t-1} - \lambda eqcm_t + \psi_0, \tag{5}$$

which allows for some firms to be backward-looking according to a rule of thumb hypothesis where past inflation drives current inflation. The semi-structural parameter spaces $0 \leq \gamma_f, \gamma_b \leq 1$ and $0 \leq \lambda$ are required in order to provide an admissible economic interpretation of (5). The hybrid forward-looking model reduces to its non-hybrid version when $\gamma_b = 0$. We note that (5) may be reparameterised as

$$\Delta p_t = \varphi_1 E_t \Delta p_{t+1} + \varphi_2 \Delta p_{t-1} + \varphi_3 \Delta ulc_t + \varphi_4 \Delta uic_t - \varphi_5 eqcm_{t-1} + \varphi_6, \tag{6}$$

where $\varphi_1 = \gamma_f/(1+\lambda)$, $\varphi_2 = \gamma_b/(1+\lambda)$, $\varphi_3 = \lambda\psi_1/(1+\lambda)$, $\varphi_4 = \lambda\psi_2/(1+\lambda)$, $\varphi_5 = \lambda/(1+\lambda)$ and $\varphi_6 = \psi_0/(1+\lambda)$. Hence, (6) becomes a backward-looking Phillips curve written in the usual equilibrium correction form when $\gamma_f = 0$.

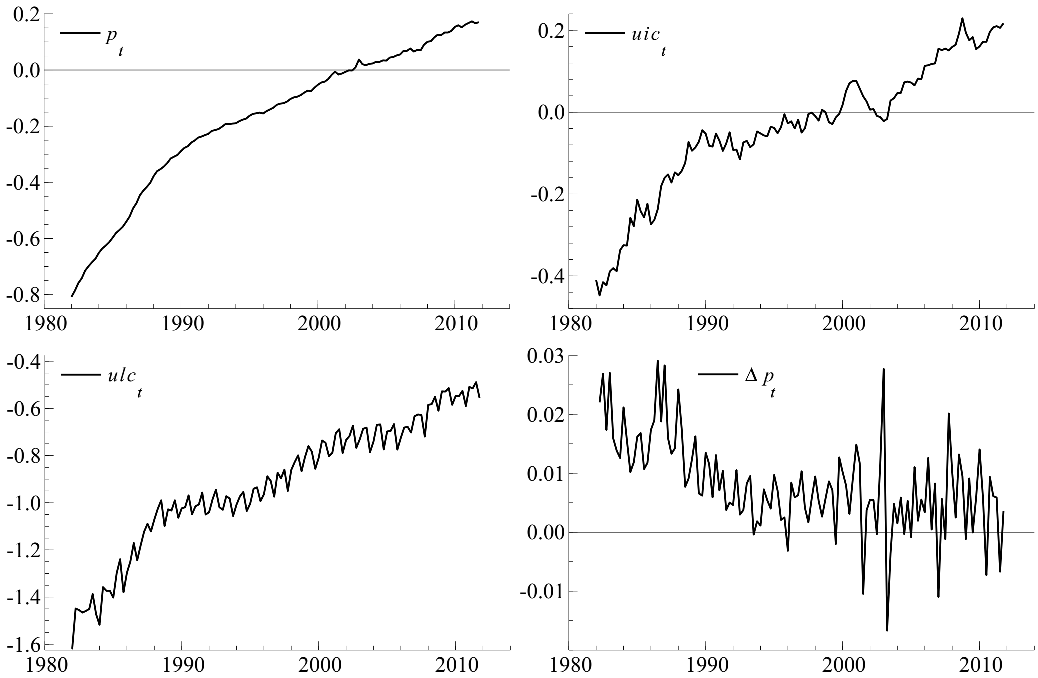
As pointed out by Mavroeidis *et al.* (2014), one may in principle allow for any number of lagged inflation terms in the model if the objective is to nest traditional Phillips curves. In addition to the backward-looking rule of thumb, lagged inflation terms could be motivated by staggered relative wage contracts and the indexation of prices to past inflation, see, e.g., Fuhrer and Moore (1995) and Christiano *et al.* (2005). We will return to these issues in the discussion of our empirical findings in Section VI.

III. Data

The empirical analysis is based on quarterly, seasonally unadjusted data that span the period $1982Q1 - 2011Q4$, from which data from the period $1982Q1 - 2005Q4$ and $2006Q1 - 2011Q4$ are used for estimation and *out-of-sample* forecasting, respectively. Mavroeidis (2004) concludes that the estimates of forward-looking models are less reliable when the sample covers periods in which inflation has been under effective policy control. The starting point of our estimation period is thus motivated by the fact that the 1970s and the early 1980s were characterised by massive governmental price controls in Norway, see Bowitz and Cappelen (2001). If the expectation term in the forward-looking model relationship is the most important factor determining the correlation between exchange rate movements and inflation, we would expect the relationship to depend closely on the monetary policy regime. We explored this hypothesis by ending the estimation period in $2001Q1$ rather than in $2005Q4$, since monetary policy in Norway changed fundamentally from exchange rate targeting to inflation targeting in late March 2001, see Boug *et al.* (2006) for details. It turns out, however, that the results from the cointegration analysis in the next section are virtually the same for the two ending points. By extending the estimation sample by twenty-four quarters for *out-of-sample* forecasting, we shed light on any change there may be in the link between exchange rate movements and domestic inflation following the financial crisis in 2008 and onwards. The *out-of-sample* forecasting ends in $2011Q4$ since available data on marginal costs for the years 2012 and 2013 are only preliminary figures from the national accounts.

We measure quarterly inflation by the official consumer price index (CPI) rather than by the GDP deflator often used in the new Keynesian Phillips curve literature. Prices set by agents in the economy are based on gross output and not on value added.

Figure 1: Time series for p_t, uic_t, ulc_t and Δp_t



Deflators based on value added are typically residuals in the national accounts, particularly those that follow the recommended principle of double-deflating, in which different deflators are used for gross output and material inputs. Hence, the GDP deflator is less related to micro price-setting behaviour than the consumer price index. There may be a problem, however, with using the consumer price index if indirect taxes change in a systematic way, although this also affects the GDP deflator. As noted below, we adjust for one episode of indirect tax changes in the sample period. In line with Batini *et al.* (2005), we employ the deflator for total imports as a proxy for unit import costs, whereas total labour costs relative to value added in the private mainland economy serve as a proxy for unit labour costs. The details can be found in Appendix 4. Figure 1 shows the log of the consumer price index (p_t), the log of unit import costs (uic_t) and the log of unit labour costs (ulc_t), together with the inflation rates (Δp_t) over the sample period.

During the estimation period, consumer price inflation shows rather large changes in the quarters $1986Q3$, $1996Q1$, $2001Q3$, $2003Q1$ and $2003Q2$. These changes are associated with a 12 per cent devaluation of the Norwegian currency in May 1986, a reduction in indirect tax rates during the first quarter 1996, a reduction in the VAT rate on food from 24 per cent to 12 per cent in July 2001 and a substantial increase and decrease in electricity prices during the first and second quarter 2003, respectively. That inflation increased considerably in the third and not in the second quarter of 1986 is due to delayed pass-through from exchange rate changes to import prices and consumer prices, see Boug *et al.* (2013). Fluctuations in electricity prices are to a large extent related to natural causes (e.g. temperature) and not much to immediate economic phenomena, since electricity is mainly based on hydroelectric power in Norway. We control for the above-mentioned episodes in the empirical analysis, using impulse dummies labelled $D86Q3$, $D96Q1$, $D01Q3$, $D03Q1$ and $D03Q2$. Consumer price inflation also shows some huge fluctuations during the forecasting period, especially in the quarter $2007Q1$ and in the years 2008 and 2009, which can likely be attributed to the substantial fall in electricity prices and the large movements in the exchange rate during the recent financial crisis, respectively. We further note that the time series exhibit a clear upward trend, but with no apparent mean-reverting property, suggesting that p_t, uic_t and ulc_t are nonstationary $I(1)$ series. Therefore, a reduced-rank vector autoregressive (VAR) model is a candidate as an empirical model.

IV. Cointegration analysis

We adopt the cointegration rank test suggested by Johansen (1995, p. 167) to find an empirical counterpart of (4). The point of departure of the $I(1)$ analysis and the tests that follow is a p-dimensional VAR of order k written as

$$X_t = A_1 X_{t-1} + \cdots + A_k X_{t-k} + \Phi_0 D_t + \Phi_1 + \Phi_2 t + \varepsilon_t, \tag{7}$$

where $X_t = (p_t, ulc_t, uic_t)'$, D_t includes seasonal dummies labelled SD_{it} $(i = 1, 2, 3)$ and the impulse dummies $D86Q3$, $D96Q1$, $D01Q3$, $D03Q1$ and $D03Q2$ as described above, t is a linear deterministic trend and $\varepsilon_{k+1}, \ldots, \varepsilon_T$ are independent Gaussian variables with expectation zero and (unrestricted) covariance matrix Ω. The initial observations of $X_1, \ldots, X_k$ are kept fixed. We follow common practice and restrict the linear trend to lie within the cointegrating space, whereas the deterministic components D_t and Φ_1 are kept unrestricted in (7). If X_t is $I(1)$, the presence of cointegration implies $0 < r < p$, where r denotes the rank or the number of cointegrating vectors of the impact matrix $\Pi = A_1 + \cdots + A_k - I$. The null hypothesis of r cointegrating vectors can be formulated as H_0: $\Pi = \alpha\beta'$, where α and β are $p \times r$ matrices, $\beta' X_t$ comprises r cointegrating $I(0)$ linear combinations and α contains the adjustment coefficients.

We find that $k = 3$ is the appropriate choice of lag length to arrive at a model with no serious misspecification in the residuals.[2] Table 1 shows the findings from applying the cointegration rank test to the data based on the VAR of order three.

Table 1: Tests for cointegration rank

r	λ_i	λ_{trace}	λ^a_{trace}
$r = 0$	0.262	47.22 [0.016]*	42.65 [0.052]
$r \leq 1$	0.136	18.95 [0.290]	17.12 [0.414]
$r \leq 2$	0.056	5.36 [0.555]	4.84 [0.626]

r denotes the cointegration rank and λ_i are the eigenvalues from the reduced-rank regression, see Johansen (1995). The λ_{trace} and λ^a_{trace} are the trace statistics without and with degrees of freedom adjustments, respectively. The p-values in square brackets, which are reported in OxMetrics, are based on the approximations to the asymptotic distributions derived by Doornik (1998). It should be noted that inclusion of impulse dummies in the VAR affects the asymptotic distribution of the reduced-rank test statistics. Thus, the critical values are only indicative. The asterisk * denotes rejection of the null hypothesis at the 5 per cent significance level.

We observe that the rank should be set to unity at the 5 per cent significance level (albeit the λ^a_{trace} statistics is a borderline case), indicating the existence of one cointegration relationship between consumer prices, unit labour costs and unit import costs. It may be worth pointing out that starting with a VAR with an unrestricted constant only yields the same result, $r = 1$. The p-values, not adjusted for degrees of freedom, corresponding to $r = 0$, $r \leq 1$ and $r \leq 2$ are 0.00, 0.12 and 0.09, respectively.

The null hypothesis that the linear trend, $\Phi_2 = 0$, can be eliminated from the VAR, assuming the rank to be unity, is not rejected by a likelihood ratio test. The p-value is 0.388 based on a χ^2 approximation with one degree of freedom. The corre-

[2]The preferred VAR includes an additional impulse dummy labelled $D84Q1$ to mop up a relatively large residual in 1984$Q1$ in the ulc_t-equation. Without $D84Q1$ the ulc_t-equation suffers from severe residual autoregressive heteroskedasticity. The cointegration analysis below is not significantly affected by any of the impulse dummies included in the preferred VAR.

sponding maximised value of the 2 log likelihood, which will be used in Section V, is 2536.72. Imposing a further restriction of homogeneity between p_t, ulc_t and uic_t entails a reduction in the value of the 2 log likelihood of 0.0097, which corresponds to a p-value of 0.921 using the same χ^2 approximation. We obtain the following empirical counterpart of (4) when the restrictions of homogeneity between p_t, ulc_t and uic_t and no linear trend in β are imposed:

$$\widehat{eqcm}_{1,t} = p_t - 0.641 ulc_t - 0.359 uic_t. \tag{8}$$

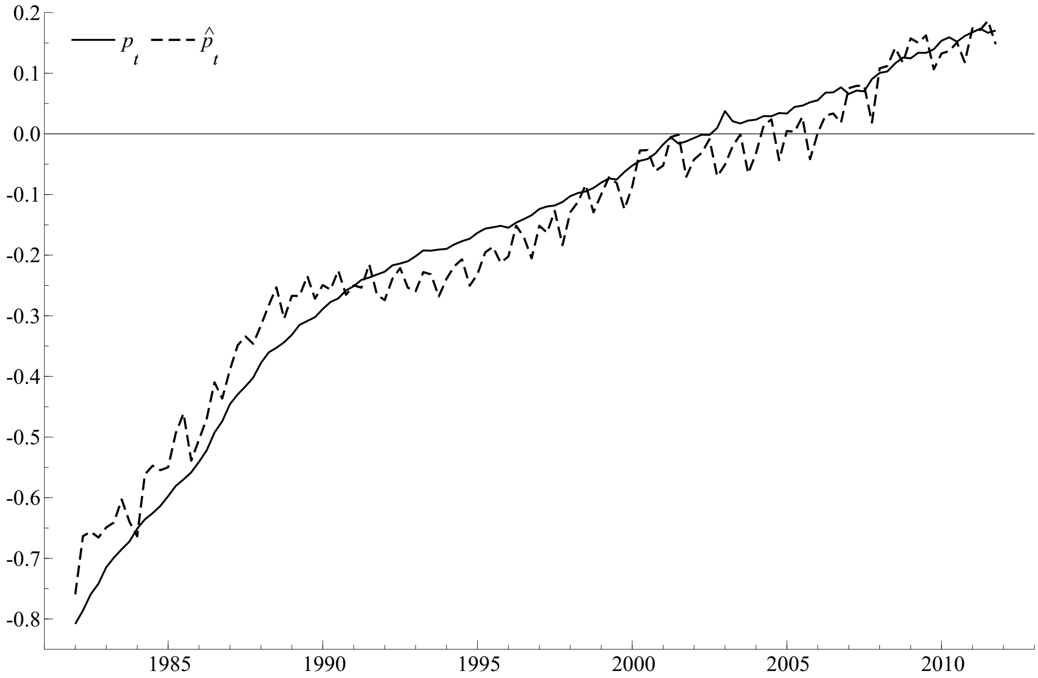
The issue of joint weak exogeneity of ulc_t and uic_t is more debatable. Here, the -2 log likelihood ratio value is 7.909. The p-value based on approximating the null distribution with a χ^2 distribution with two degrees of freedom is 0.02. However, investigating weak exogeneity more closely, using both parametric and non-parametric bootstrap methods, reveals that the asymptotic approximations are not accurate in our case. A bootstrap of the likelihood ratio test, as in Omtzigt and Fachin (2007), using the estimated values of the VAR coefficients and not imposing weak exogeneity and resampling the residuals, yields a p-value of 0.515. The outcome of a non-parametric bootstrap is similar. We conclude that the cointegration vector enters the Δp_t-equation only. Imposing, in addition, weak exogeneity of both ulc_t and uic_t yields

$$\widehat{eqcm}_{2,t} = p_t - 0.604 ulc_t - 0.396 uic_t. \tag{9}$$

We see that $\widehat{eqcm}_{1,t}$ and $\widehat{eqcm}_{2,t}$ are quite similar, which provides further evidence that the restriction of weak exogeneity of both ulc_t and uic_t can be justified. Figure 2 depicts time series for p_t and $\hat{p}_t$ based on (8) over the sample period.

It is evident that $\hat{p}_t$ matches p_t rather closely, both in-sample and out-of-sample. Hence, we interpret (8) as a long-run consumer price equation that corresponds well with the theory of mark-up pricing and takes into account that, for a small open economy like the Norwegian, features of an open economy such as import prices are expected to matter. The estimates in (8) are in line with previous findings based on Norwegian

Figure 2: Time series for p_t and $\hat{p}_t$ based on equation (8)



data, see, e.g., Bårdsen *et al.* (2005, p. 182). Nearly four decades ago, Aukrust (1977, p. 123) pointed out that the total direct effect on consumer prices that can be expected, under Norwegian conditions, from a proportionate increase in all import prices can be set at 0.33 per cent. Hence, (8) is also in line with the Scandinavian model of inflation, cf. Lindbeck (1979).

V. Tests of forward-looking models

An important econometric issue when testing the forward-looking model concerns whether the model is specified in its *exact* or *inexact* form by introducing a stochastic error term u_t. In general, the absence of an unobserved disturbance term ($u_t = 0$) may be a restrictive and nontrivial assumption since there are several justifications for why such a term could be included in the model, see, e.g., Sbordone (2005). To shed light on the importance of the disturbance term in our empirical case, we evaluate both versions of the model in this paper. However, as demonstrated by Boug *et al.* (2010) in the case of the new Keynesian Phillips curve within a bivariate VAR, the *exact* version is algebraically less involved and produces much simpler rational expectations restrictions than what follow from the *inexact* version under the assumption that u_t is a sequence of innovations, i.e. $E_t(u_{t+1}) = 0$. Hence, the numerical treatment of the *exact* model using likelihood-based methods is also much simpler than the *inexact* model. When a trivariate VAR is the underlying model, as is the case in the present study, the numerical treatment of the *inexact* model is even more complicated to handle using likelihood-based methods. As a consequence, we employ the likelihood-based testing procedures suggested by Johansen and Swensen (1999, 2008) for the *exact* model and a test based on a minimum distance approach, along the lines of Sbordone (2002) and Magnusson and Mavroeidis (2010), for the *inexact* model.

It is worth noting that, by considering the *exact* and *inexact* models as sub-models of a reduced-rank VAR, we avoid cases like those described by Beyer and Farmer (2007). In a single equation framework, they demonstrated that *exact* and *inexact* models exist that have the same likelihood and are hence empirically indistinguishable. As we shall see, the restrictions from an *exact* and *inexact* model on the coefficients of the VAR are quite different, and thus also the likelihoods of the two sub-models.

The exact version

The basic idea behind the procedure suggested by Johansen and Swensen (1999, 2008) is to start with a well-specified VAR model and, using the likelihood criterion, test the implications of the hybrid forward-looking model on the coefficients of the VAR. As explained in the previous section, a reduced-rank VAR model with three lags and a constant, but with no linear term, passed a test for homogeneity. We will therefore explore this model further.

Expressing (5) on level form, taking the homogeneity restriction into account, yields

$$\gamma_f E_t[p_{t+1}] - (1+\gamma_f)p_t + (\gamma_b + 1)p_{t-1} - \gamma_b p_{t-2} - \lambda(p_t - \psi_1 ulc_t - \psi_2 uic_t) + \psi_0 = 0,$$

or

$$\gamma_f E_t[p_{t+1}] - (1+\gamma_f)p_t + (\gamma_b + 1)p_{t-1} - \gamma_b p_{t-2} - \lambda eqcm_t + \psi_0 = 0,$$

where $\psi_1 + \psi_2 = 1$. With $c_1 = (\gamma_f, 0, 0)'$, $c_0 = (-1-\gamma_f-\lambda, \lambda\psi_1, \lambda\psi_2)'$, $c_{-1} = (\gamma_b+1, 0, 0)'$ and $c_{-2} = (-\gamma_b, 0, 0)'$, this can be written as

$$c_1' E_t[X_{t+1}] + c_0' X_t + c_{-1}' X_{t-1} + c_{-2}' X_{t-2} + \psi_0 = 0.$$

The fitted VAR model contains six impulse dummies and three seasonal dummies in addition to a constant. Denote the coefficients of the constant term as Φ_1 and the coefficients of the rest of the deterministic terms as Φ_0, so that the deterministic part of the VAR model can be written as $\Phi_0 D_t + \Phi_1$. The rational expectation hypothesis will imply restrictions also on these coefficients. Because we are not focusing on the properties that the dummies capture, we simply drop those restrictions, which, as explained in Boug *et al.* (2006), amounts to formulating the model as

$$c_1' E_t[X_{t+1} - \Phi_0 D_{t+1}] + c_0' X_t + c_{-1}' X_{t-1} + c_{-2}' X_{t-2} + \psi_0 = 0. \tag{10}$$

Using (7) with $\Phi_2 = 0$ to obtain an expression for $E_t[X_{t+1} - \Phi_0 D_{t+1}]$ and inserting it in (10) implies that the following restrictions must be satisfied

$$c_1'\Pi = -(c_1' + c_2' + c_3' + c_4'),\ c_1' A_2 = -c_{-1},\ c_1' A_3 = -c_{-2} \text{ and } c_1'\Phi_1 = -\psi_0, \tag{11}$$

where $\Pi = A_1 + A_2 + A_3 - I$. For fixed values of the semi-structural parameters $\gamma_f, \gamma_b, \lambda$ and $\psi = \psi_1 = (1 - \psi_2)$ the concentrated likelihood, $L_{c1}(\gamma_f, \gamma_b, \lambda, \psi)$, can be computed using the methods in Johansen and Swensen (1999).

However, in this particular case, the restrictions have a form that makes further simplifications possible. As explained in Appendix 2, we can use the methods of Johansen and Swensen (2008) to concentrate out the parameters γ_f, γ_b and λ, so that the concentrated or profile likelihood, $L_{c2}(\psi) = L_{c1}(\gamma_f(\psi),\ \gamma_b(\psi), \lambda(\psi), \psi)$, only depends on

$\psi = \psi_1 = (1-\psi_2)$ in the case of homogeneity. Hence, it is possible to compute for each value of ψ the value of the likelihood when the restrictions implied by (11) are satisfied. Thus, by computing the likelihood repeatedly for many values of ψ, it is possible to determine the maximal value of the likelihood.[3]

We can now test the four nested models, reduced-rank *without* homogeneity restriction, reduced-rank *with* homogeneity restriction, *exact* hybrid model and *exact* non-hybrid model using ordinary likelihood ratio tests. The results are summarised in Table 2.

Table 2: Likelihood ratio tests of the *exact* forward-looking model. Nested models

Model	$2\log L$	$-2\log LR$	df.	p-value
CVAR without homogeneity restriction	2536.72^a			
CVAR with homogeneity restriction	2536.71^a	0.01	1	0.92
Exact hybrid model	2535.15^b	1.56	4	0.82
Exact non-hybrid model	2529.64^b	5.60	1	0.02

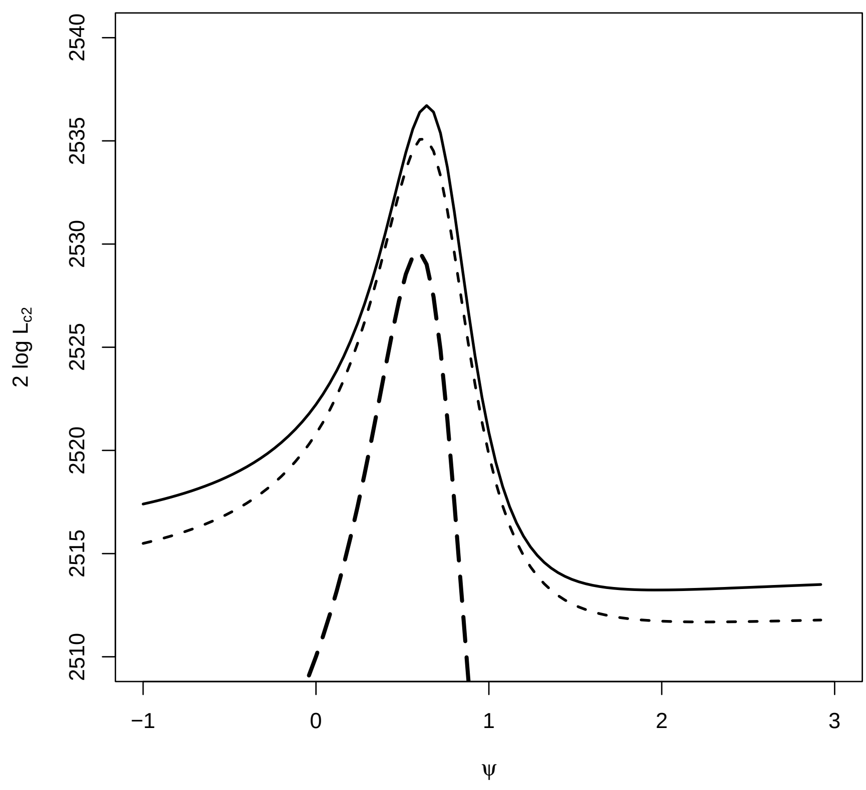
[a] Maximal values of the likelihood *without* the rational expectations restrictions imposed.
[b] Maximal values of the likelihood *with* the rational expectations restrictions imposed.

Using the usual top-down procedure, the test of the hybrid model is not rejected, whereas the non-hybrid model with $\gamma_b = 0$ is. Hence, it is a clear advantage to include the extra inflation lag in the expectation restrictions in the non-hybrid model. These impressions are also evident from the plots of the concentrated likelihoods shown in Figure 3.

The curve corresponding to the model imposing only the homogeneity restriction reaches a maximum at $\hat{\psi} = 0.641$. This is close to the maximum likelihood estimate of 0.621 in the hybrid model. To investigate this further, we have computed the restricted maximum likelihood estimates of the other semi-structural parameters γ_f, γ_b and λ for some reasonable fixed values of ψ, in addition to the maximum likelihood estimate $\hat{\psi} = 0.621$. The results are shown in Table 3. As one can see, all sums of γ_f and γ_b are far from 1. This is not surprising. Fitting a hybrid model with the additional restriction $\gamma_f + \gamma_b = 1$ yields a maximal value of $2\log L$ equal to 2489.10, corresponding to a value

[3]The procedure "optim" in the statistical package R [see http://www.r-project.org/ and R Development Core team (2006)] is used to carry out tests and estimation of the forward-looking models. The R-codes used for the procedures in this section and the bootstrap procedure in the previous section are available at http://folk.uio.no/~swensen/isoe/isoe.html.

Figure 3: Concentrated likelihood functions $2\log L_{c2}$ as functions of ψ for the CVAR with homogeneity restriction (solid line), the *exact* hybrid model (short dashed line) and the *exact* non-hybrid model (long dashed line)



of 46.05 of $-2\log LR$. The degree of freedom is 1, so the restriction is clearly rejected.

Table 3: Some parameter estimates of the *exact* hybrid forward-looking model

ψ	γ_f	γ_b	λ
0.80	3.56	−0.74	−0.14
0.62	5.16	−0.75	−0.27
0.40	5.84	−0.87	−0.23
0.10	4.83	−0.92	−0.11

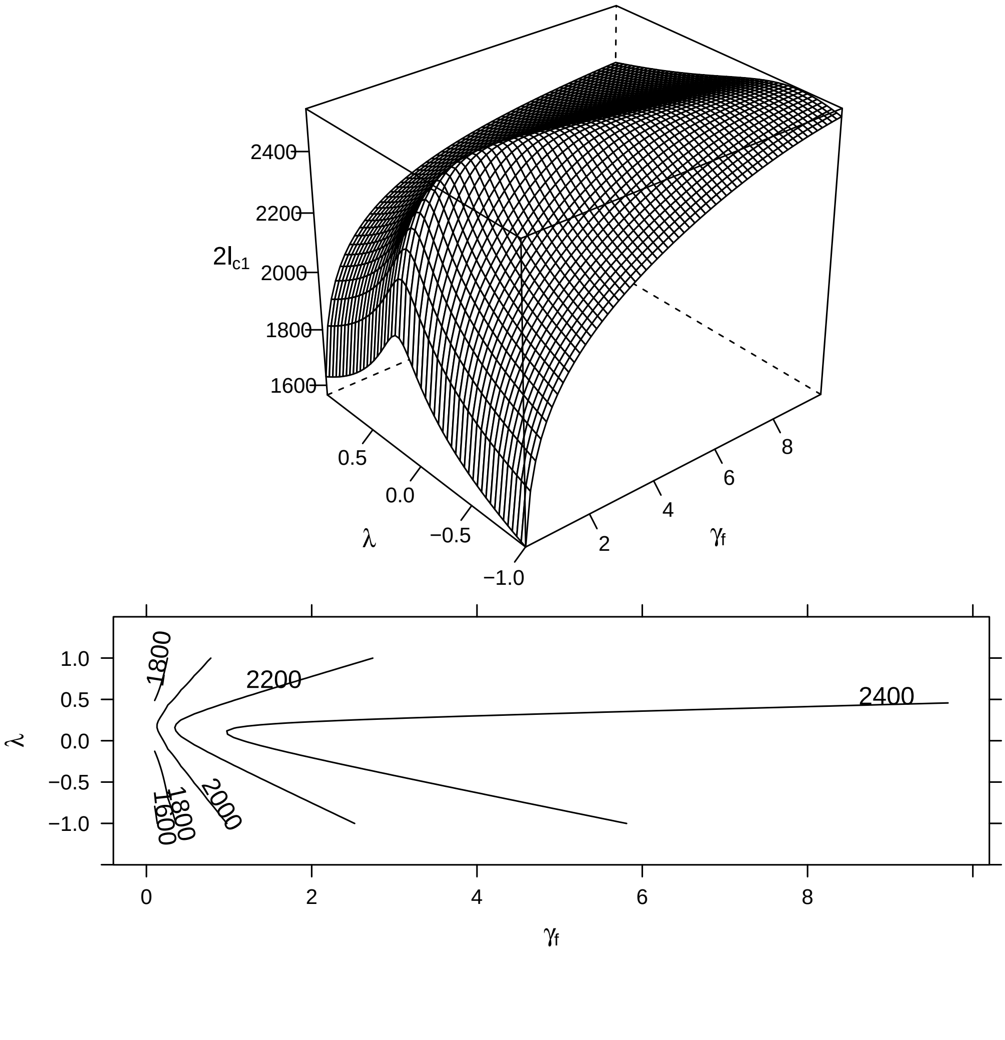
The estimates of γ_f, γ_b and λ are computed for reasonable values of ψ.

Additional evidence is provided by the plots in Figure 4 of the concentrated likelihood surface $2\log L_{c1}(\gamma_f, \gamma_b, \lambda, \psi)$ as a function of γ_f and λ with ψ and γ_b fixed at 0.621 and −0.745, respectively, which are the maximum likelihood estimates of these parameters. A rather striking feature is the smooth ridge in the γ_f direction, which indicates that this parameter is not well-identified.

The economically meaningful values of the semi-structural parameters are $0 \leq \gamma_f, \gamma_b, \psi \leq 1$ and $\lambda \geq 0$. It is evident from Figure 4 that the unrestricted maximum likelihood estimates will be outside this region. By defining $\gamma_f(\theta_f) = \exp(\theta_f)/(1 + \exp(\theta_f))$, $\gamma_b(\theta_b) = \exp(\theta_b)/(1 + \exp(\theta_b))$, $\psi(\theta) = \exp(\theta)/(1 + \exp(\theta))$ and $\lambda(\theta_l) = \exp(\theta_l)$ and maximising $L_{c1}(\gamma_f, \gamma_b, \lambda, \psi)$ with respect to $\theta_f, \theta_b, \theta$ and θ_l, the restriction that the parameters have a meaningful economic interpretation can be imposed. The maximal value of $2\log L_{c1}$ is then equal to 2437.66, corresponding to the estimates $\hat{\gamma}_f = 1.0, \hat{\gamma}_b = 3.3E-06, \hat{\lambda} = 0.016$ and $\hat{\psi} = 0.70$,, which are on the border of the permissible region. The likelihood ratio test for the null hypothesis that they belong to this region therefore has a non-standard asymptotic distribution, which is a convex combination of χ^2 distributions with different degrees of freedom. In this case, the critical values are smaller than the critical values computed from a χ^2_4 distribution. Since the difference between the maximal values of $2\log L$ is so large, $2535.15 - 2437.66 = 97.49$, and therefore exceeds all relevant critical values using a χ^2_4 distribution, the likelihood ratio test also rejects the null hypothesis that $0 \leq \gamma_f, \gamma_b, \psi \leq 1$ and $\lambda \geq 0$.

Thus, we conclude that the hybrid model with no restrictions imposed is not rejected by likelihood ratio tests. However, when only economically meaningful parameters are allowed for, the model is *not* supported by the data. In addition, γ_f seems to

Figure 4: Surface and contour plots of concentrated likelihood function $2l_{c1}$=$2\log L_{c1}$ as a function of γ_f and λ for the *exact* hybrid model with $\psi = 0.621$ and $\gamma_b = -0.745$. The maximal value is located at the point (5.16, -0.27)



be poorly identified, as indicated by the rather smooth ridge of the likelihood surface plot.

The inexact version

Another, perhaps more economically appealing, formulation of the restrictions (10) from the hybrid forward-looking model is

$$c_1' E_t[X_{t+1} - \Phi_0 D_{t+1}] + c_0' X_t + c_{-1}' X_{t-1} + c_{-2}' X_{t-2} + \psi_0 = u_t, \tag{12}$$

where the error term u_t is a sequence of innovations in the VAR model, i.e. $E_t[u_{t+1}] = 0$, and c_1, c_0, c_{-1} and c_{-2} are as defined earlier. Likelihood estimation of such a model must be handled using methods similar to those used by Boug *et al.* (2010) for bivariate systems, see also Kurmann (2007) and Fanelli (2008).

The reduced-rank VAR model can be written on level form as

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + A_3 X_{t-3} + \Phi_0 D_t + \Phi_1 + \epsilon_t. \tag{13}$$

Rewriting (12) at time $t+1$, using iterated expectations and inserting one-step ahead forecasts from the VAR, the restrictions on the coefficients implied by the hybrid model now take the form

$$\begin{aligned} c_1'(A_1^2 + A_2) + c_0' A_1 + c_{-1}' &= 0 && (14) \\ c_1'(A_1 A_2 + A_3) + c_0' A_2 + c_{-2}' &= 0 \\ c_1'(A_1 A_3) + c_0' A_3 &= 0 \\ c_1' \Phi_1 + (c_1' A_1 + c_0')(\Phi_0 D_{t+1} + \Phi_1) + \psi_0 &= 0. && (15) \end{aligned}$$

The model (13) with reduced rank equal to unity and homogeneity imposed contains $18 + 3 + 1 = 22$ autoregressive parameters in addition to the coefficients of the deterministic terms. There are not more than 9 restrictions on the coefficients of the VAR in (14), so using the reversed engineering approach of Kurmann (2007), expressing the likelihood in terms of the parameters of the inflation equation, and the semi-structural parameters γ_f, γ_b and λ, we end up with at least 16 freely varying parameters in addition to those from (15). Finding the maximum likelihood estimates in such situations represents a computational problem that is beyond the scope of the present paper. We therefore explore some alternative approaches.

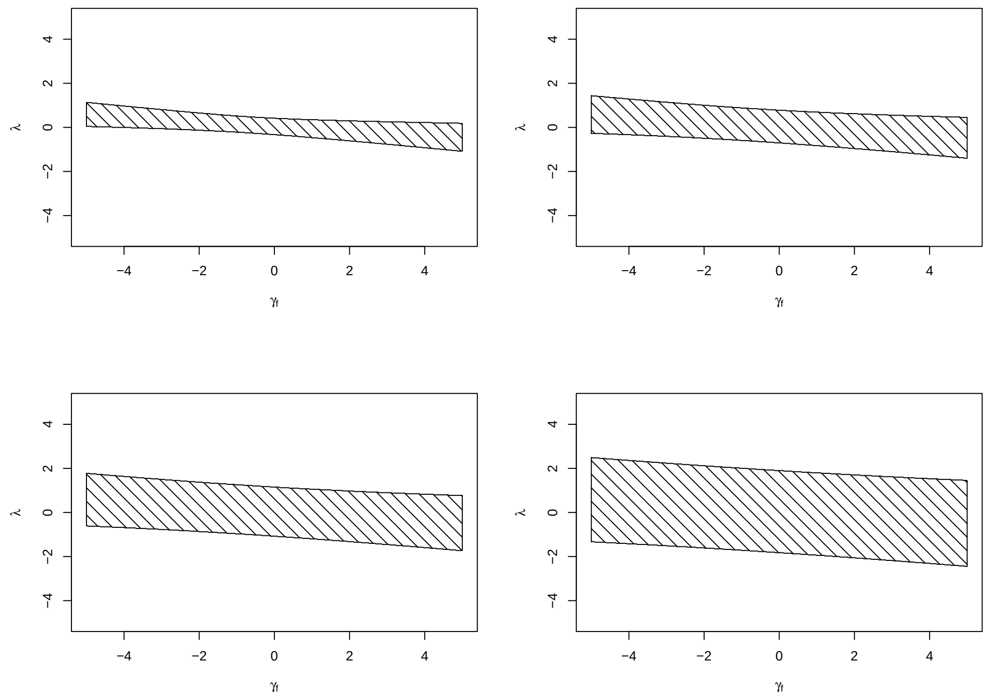
The equations (14) and (15) express necessary conditions for both the *exact* case, where $Var(u_t) = 0$, and the *inexact* case, where $Var(u_t) > 0$. By inspecting the equations, we can see that if (14) and (15) are fulfilled, the restriction that $c_1' A_1 + c_0' = 0$ is equivalent to the *exact* restrictions given in (11), see also Swensen (2014). We can then use a minimum distance method along the lines of Sbordone (2002) and Magnusson and Mavroeidis (2010) in a related setting, see also Newey and McFadden (1994) for a general exposition, to derive estimators for γ_f and λ occurring in c_1 and c_0 and test the restriction. The details are provided in Appendix 3. The test statistic equals 0.11, and the degree of freedom is 1, so the corresponding p-value is 0.74. This may not be surprising since the likelihood ratio test for the *exact* hybrid model, without imposing economic admissible parameters, was not rejected either.

To investigate further the strong indication of an identification problem, which is evident in the *exact* version, we computed joint confidence regions of the parameters γ_f, λ and γ_b using an approach reminiscent of how the Anderson-Rubin (1949) statistic is employed in similar situations. Consider the regression

$$\Delta p_t - \gamma_{f0}\Delta p_{t+1} + \lambda_0 \widehat{eqcm} - \gamma_{b0}\Delta p_{t-1} = \zeta_1' Z_{1,t} + \zeta_2' Z_{2,t} + error \tag{16}$$

where the coefficients of the endogenous variables, γ_{f0}, λ_0 and γ_{b0}, have specified values, $Z_{1,t} = (1, SD_{1t}, SD_{2t}, SD_{3t})'$ are exogenous variables and $Z_{2,t}$ are the instruments from the reduced-rank VAR model: $Z_{2,t} = (\Delta p_{t-2}, \Delta uic_{t-1}, \Delta uic_{t-2}, \Delta ulc_{t-1}, \Delta ulc_{t-2}, \widehat{eqcm}_{1,t-1})'$. The particular form of (16) arises by replacing the conditional expectation $E_t[\Delta p_{t+1}]$ in (5) with Δp_{t+1}, a common practice when estimating models of this form,

Figure 5: Estimated confidence region for γ_f, λ and γ_b with confidence level 0.999. Plots of (γ_f,λ) where γ_b is maximised over intervals (–2,2), (–4,4), (–6,6) and (–10,10)



see, e.g., Hansen *et al.* (1996) or Bårdsen *et al.* (2005). The errors will have a first order moving average structure, and, to take this dependency into account, we use the residuals from an ordinary least squares regression of (16) to estimate the correlation structure and then use a Wald statistic instead of the usual F-statistic. Projected regions consisting of the parameters $(\gamma_f, \lambda, \gamma_b)'$ for which a test of $\zeta_2 = 0$ with the level 0.001 is not rejected are shown in Figure 5, where the values γ_b that are maximised above belong to the intervals $(-2, 2)$, $(-4, 4)$, $(-6, 6)$ and $(-10, 10)$.

The increasing size of the projected regions indicates that the three-dimensional confidence region for $(\gamma_f, \lambda, \gamma_b)'$ is unbounded, which signifies that there is an identification problem, see, e.g., Dufour (2003). That the *exact* and *inexact* models may share several features is therefore confirmed by more than the non-rejection of the test.

VI. A competing backward-looking model

So far, the formal tests clearly indicate that the forward-looking model is at odds with the data. By themselves, however, these tests are not sufficient evidence that inflation expectations do not matter. Expectations may still matter for inflation dynamics if past information is relevant through its implications for the expectations of future inflation. Remember that, so far, we have only considered expectations as conditional mathematical expectations. In this section, we compare and contrast estimates of a reduced form of (5) with a backward-looking Phillips curve as a competing model of inflation dynamics. To this end, we first estimate a backward-looking Phillips curve based on (6) with $\gamma_f = 0$ and the same information set used in the testing of the forward-looking models. Based on the estimated backward-looking Phillips curve, we infer one-step ahead inflation expectations, $E_t \Delta p_{t+1}$, substitute into (5) and solve for inflation, Δp_t, to obtain a reduced form model. We then assess the fit of this reduced form model by asking whether there are any economically reasonable values of the semi-structural parameters, γ_f, γ_b and λ, that make the model consistent with the data in-sample. Finally, we evaluate the fit of the two models by means of a forecasting competition out-of-sample.

We rely on a general-to-specific modelling strategy in the estimation of the backward-looking model for Δp_t using the autometrics procedure available in OxMetrics, see Doornik and Hendry (2009). Our point of departure is a general *conditional* model for Δp_t with Δp_{t-1}, Δp_{t-2}, Δuic_t, Δuic_{t-1}, Δuic_{t-2}, Δulc_t, Δulc_{t-1}, Δulc_{t-2}, $\widehat{eqcm}_{2,t-1}$, 1, SD_{1t}, SD_{2t}, SD_{3t}, $D84Q1$, $D86Q3$, $D96Q1$, $D01Q3$, $D03Q1$ and $D03Q2$ as regressors. This general model is fully in accordance with the fitted reduced-rank VAR, both in terms of the number of lags and the weak exogeneity status of uic_t and ulc_t. Autometrics picks the following specific model in our case, together with diagnostic tests and standard errors in parenthesis[4]:

$$\begin{aligned}
\Delta p_t \;=\; & \underset{(0.061)}{0.150}\Delta p_{t-1} + \underset{(0.064)}{0.132}\Delta p_{t-2} - \underset{(0.011)}{0.059}\widehat{eqcm}_{2,t-1} + \underset{(0.005)}{0.028} + \underset{(0.0011)}{0.0058}SD_{1t} \\
& + \underset{(0.0012)}{0.0036}SD_{2t} - \underset{(0.0011)}{0.0036}SD_{3t} + \underset{(0.003)}{0.020}D86Q3 - \underset{(0.003)}{0.011}D96Q1 \\
& - \underset{(0.003)}{0.015}D01Q3 + \underset{(0.002)}{0.023}D03Q1Q2
\end{aligned} \quad (17)$$

$OLS, T = 93\ (1982Q4 - 2005Q4), \hat{\sigma} = 0.0033$

AR_{1-5}: $F(5, 77) = 1.76$ [0.13], $ARCH_{1-4}$: $F(4, 85) = 1.72$ [0.15],

$NORM$: $\chi^2(2) = 1.21$ [0.55], HET: $F(11, 78) = 1.58$ [0.12].

Several features of Norwegian inflation dynamics stand out from (17). Firstly, the economic variables entering the model are all highly significant. Consumer price inflation in Norway seems to be rather persistent, as represented by the significant autoregressive coefficients of Δp_{t-1} and Δp_{t-2}. The $\widehat{eqcm}_{2,t-1}$ appears with a t-value of -5.36, hence adding force to the results obtained from the cointegration analysis. Secondly, the sign of the impulse dummies corresponds well with the expected effects of the associated economic events described above. Thirdly, there are no significant contemporaneous short-run effects on inflation from unit import costs and unit labour costs in (17). No contemporaneous short-run effects combined with the small magnitude of the estimated loading coefficient (-0.059) imply very slow consumer price adjustment in the face of shocks in unit import costs and unit labour costs.

As stated by Fuhrer (2006) among others, lagged inflation is not simply a second-order add-on to the model, but is important when accounting for persistence in inflation. It is not commonplace to require an explicit economic interpretation of parameters in a VAR model, except those necessary for identification, but we think this is possible

[4] AR_{1-5} is a test for up to 5th order residual autocorrelation; $ARCH_{1-4}$ is a test for up to 4th order autoregressive conditional heteroskedasticity in the residuals; $NORM$ is a joint test for residual normality (no skewness and excess kurtosis) and HET is a test for residual heteroskedasticity, see Doornik and Hendry (2009). The numbers in square brackets are p-values. The dummy variable $D03Q1Q2$ combines the two dummy variables $D03Q1$ and $D03Q2$ and takes the value 1 in $2003Q1$ and -1 in $2003Q2$.

in our case. The Norwegian CPI includes housing rents with a weight of around 0.17. Most of the contracts in the housing market include an index clause that allows the owner to adjust the rent based on the observed increase in the CPI. This usually takes place in January based on the CPI in December. During the rest of the year, rents are not adjusted in line with inflation. In many cases, rents are not adjusted every year, but stay nominally constant as long as the contract lasts. However, the total index for rents will increase during the year as old contracts expire and new contracts are signed. In addition, the standard contract has a clause that states that the rent can be renegotiated every third year to bring it into line with current market prices. The CPI for imputed rents for households living in their own house or flat is based on the rent equivalence principle, such that imputed rent follows observed rent with some sampling modifications. The relevance to modelling aggregate CPI in Norway is the acknowledgement that the housing market is quite different from standard product markets and that nominal price rigidity and long lags are present and observable in the microdata on which the CPI is based.

Empirical evidence of constancy of (17) can be assessed from recursive test statistics, see Doornik and Hendry (2009). Neither one-step residuals with 2 estimated equation standard errors nor a sequence of break point Chow tests at the 1 per cent significance level indicate non-constancy. All recursive estimates vary little, especially relative to their estimated uncertainty. That no significant structural breaks are detected around the date of the shift in monetary policy regime from exchange rate targeting to inflation targeting (late March 2001) points to (17) *not* being subject to the Lucas critique.

To investigate whether incorporating forward-looking features into the model may lead to any improvement, we write the part of (17) not involving the impulse dummies as $\Delta p_t = a\Delta p_{t-1} + b\Delta p_{t-2} + c\widehat{eqcm}_{2,t-1} + d + seasonals$ and let that part be past information relevant to one-step ahead inflation expectations in (5) to obtain the following reduced form forward-looking model for Δp_t:

$$\Delta p_t = \beta_0 + \beta_1 \Delta p_{t-1} - \beta_2(\Delta ulc_t - \Delta uic_t) - \beta_3(\Delta uic_t - \widehat{eqcm}_{2,t-1}) + seasonals, \quad (18)$$

where

$$
\begin{aligned}
\beta_0 &= (\psi_0 + \gamma_f d)\varpi, \\
\beta_1 &= (\gamma_f b + \gamma_b)\varpi, \\
\beta_2 &= (\gamma_f c - \lambda)\psi_1\varpi, \\
\beta_3 &= (\gamma_f c - \lambda)\varpi, \\
\varpi &= (1 - a\gamma_f - c\gamma_f + \lambda)^{-1}.
\end{aligned} \tag{19}
$$

We see that (18) consists of the three composite parameters, β_1, β_2 and β_3, for determination of the three semi-structural parameters, γ_f, γ_b and λ, which are thereby exactly identified. Because both uic_t and ulc_t are weakly exogenous, we can generate the variables $(\Delta ulc_t - \Delta uic_t)$ and $(\Delta uic_t - \widehat{eqcm}_{2,t-1})$ and estimate (18) by means of OLS. By also including the impulse dummies from (17) as additional regressors, we obtain the following OLS estimate of (18):

$$
\begin{aligned}
\Delta p_t = & \underset{(0.067)}{0.126}\Delta p_{t-1} + \underset{(0.009)}{0.032}(\Delta ulc_t - \Delta uic_t) + \underset{(0.009)}{0.070}(\Delta uic_t - \widehat{eqcm}_{2,t-1}) \\
& + \underset{(0.004)}{0.035} + \underset{(0.0015)}{0.0029}SD_{1t} + \underset{(0.0015)}{0.0007}SD_{2t} - \underset{(0.0014)}{0.0061}SD_{3t} \\
& + \underset{(0.004)}{0.017}D86Q3 - \underset{(0.004)}{0.010}D96Q1 - \underset{(0.004)}{0.013}D01Q3 + \underset{(0.003)}{0.022}D03Q1Q2
\end{aligned} \tag{20}
$$

$OLS, T = 93\ (1982Q4 - 2005Q4), \hat{\sigma} = 0.0035$

AR_{1-5}: $F(5,77) = 1.77\ [0.13]$, $ARCH_{1-4}$: $F(4,85) = 1.19\ [0.32]$,

$NORM$: $\chi^2(2) = 4.14\ [0.13]$, HET: $F(11,78) = 1.68\ [0.09]$.

Like the estimated backward-looking Phillips curve, the estimated reduced form forward-looking model, is well specified, judging by the diagnostic tests. However, $\hat{\sigma}$ increases somewhat and calculations using (19) reveal that $\hat{\gamma}_f = 6.84, \hat{\gamma}_b = -0.908$ and $\hat{\lambda} = -0.408$, all of which are outside the regions containing sensible economic interpretations. These parameter estimates correspond to the earlier findings in Section

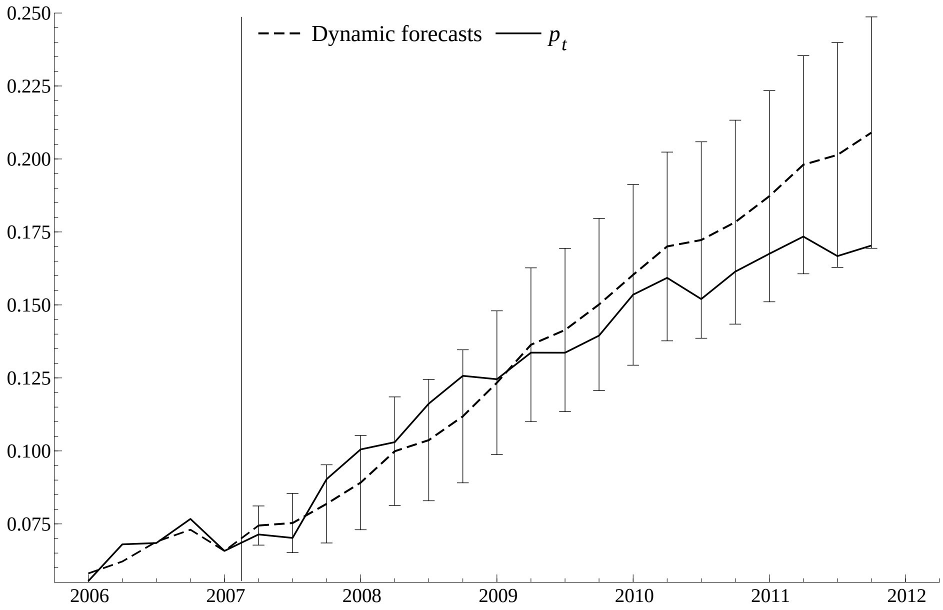
V (Table 3) and may not be surprising since both the hypotheses of weak exogeneity of uic_t and ulc_t and zero restrictions on the autoregressive parameters from the VAR are not rejected.

Although (20) has a slightly poorer fit than (17) and the parameter estimates are difficult to interpret economically, we compare the out-of-sample forecasting performance of the two competing models of inflation to shed light on their robustness with respect to relatively large movements in the exchange rate during the recent financial crisis. Taylor (2000) argues that the extent to which a firm matches exchange rate movements by changing its own price depends on how persistent the movements are expected to be. For a retail firm that adds services to its imports of goods, a depreciation of the exchange rate will raise the costs of the imports valued in domestic currency. According to Taylor (2000), if the depreciation is viewed as temporary, the retail firm will pass through less of the depreciation to its own price. In any case, if the price-setting behaviour changed significantly following the financial crisis, we should expect instabilities in the estimated Δp_t-equations, as indicated, for example, by poor out-of-sample forecasting ability.

To assess the forecasting performance of (20) and (17), we employ twenty-four quarters ($2006Q1-2011Q4$) of out-of-sample observations, including the period of the financial crisis. According to the theoretical model in Section II, both the forward-looking and the backward-looking models embody the price level. This feature is supported by the data, as is evident from the cointegration analysis and Figure 2. An inflation-targeting central bank is interested in forecasting inflation over a certain horizon (say two to four years), but may not be very concerned about forecasts of quarterly inflation a few quarters ahead. Thus, to evaluate the forecasting performance of the two competing models of inflation, we focus on the medium-term (ex ante) dynamic forecasts for the consumer price *level*.

A preliminary investigation of the two models reveals that a majority of the actual values of p_t stay within their corresponding confidence intervals over the forecasting period. However, the actual value of p_t is close to being outside the confidence intervals

Figure 6: The backward-looking model: Actual values and dynamic forecasts of $p_t \pm$ 2STD

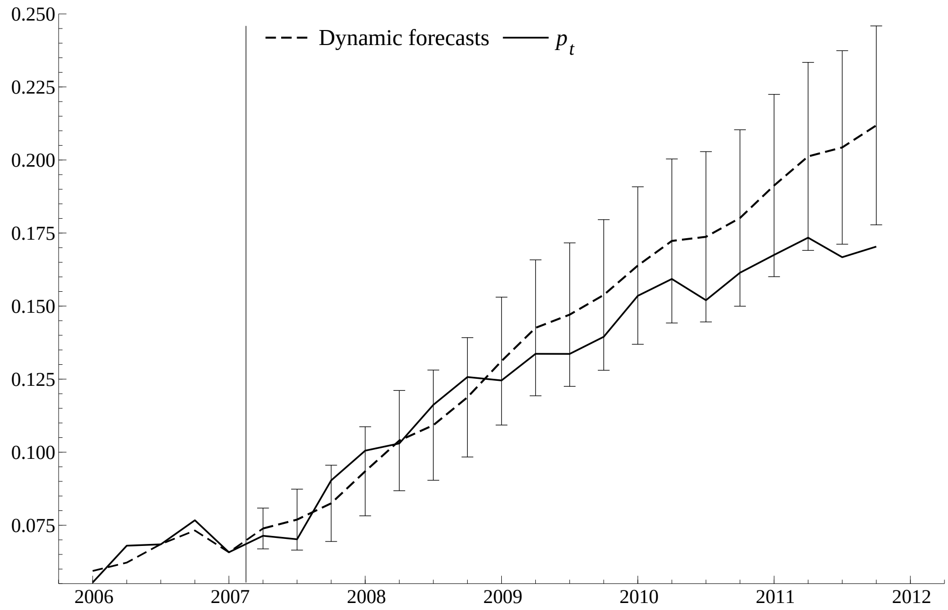


in the first quarter 2007. The actual value of p_t in $2007Q1$ and the values thereafter are likely to be influenced by the huge and transitory fall in electricity prices during the first quarter 2007. Consequently, the dynamic forecasts over-predict the actual values of p_t thereafter, irrespective of whether (20) or (17) is used as the underlying forecasting model.

To take a closer look at this hypothesis, we re-estimated the two models over the period $1982Q4 - 2007Q1$ with an impulse dummy in $2007Q1$ as a separate regressor, controlling for the substantial fall in electricity prices during that quarter. Hence, nineteen observations are now available for forecasting. The two re-estimated models are virtually unchanged from (20) and (17) with respect to both parameter estimates and diagnostics. Figures 6 and 7 depict actual values of p_t together with dynamic forecasts, adding bands of 95 per cent confidence intervals to each forecast, when the re-estimated models are used for forecasting.

We observe that the out-of-sample forecasting ability of each of the two competing models of inflation is reasonably good despite relatively large exchange rate movements in the wake of the financial crisis. That said, the forecasting performance of

Figure 7: The forward-looking model: Actual values and dynamic forecasts of $p_t \pm$ 2STD



the backward-looking model is somewhat better than the forward-looking model since the RMSE is 0.0167 and 0.0183, respectively. Moreover, the average yearly forecasting failure within the relevant three-year horizon for the Norwegian central bank is about 0.23 percentage points with the former model and about 0.35 percentage points with the latter model. Because no significant forecasting failures are evident with the backward-looking model, we may argue, in light of Taylor (2000), that the exchange rate movements during the financial crisis were perceived as transitory rather than permanent shocks, such that firms found it rational not to alter their pricing behaviour. We conclude that there may not be any value added of having forward-looking expectations once the model accounts for all relevant backward-looking terms, as is the case with (17).

Our results as regards forecasting can be compared to the results in King and Watson (2012). As mentioned in the introduction, they find that the Smets and Wouters (2007) forward-looking model of the US economy predicted inflation poorly after the financial crisis in 2008. Del Negro *et al.* (2015) argue that, when a standard DSGE model is extended by including financial frictions and the forecasts are conditional on the increased financial stress that occurred in autumn 2008, the forward-looking pricing equation forecasts inflation well after the financial crisis. Consequently, they accept that earlier inflation models did not perform well post sample and that previous DSGE models did not include parameters that were stable over time. That said, their modified model has one interesting feature in that lagged inflation becomes more important than is standard in the literature. This supports the argument by Fuhrer (2006) that lagged inflation is not a minor feature of the forward-looking model. Rather it *is* the model.

VII. Conclusions

In this paper, using Norwegian data, we have evaluated the empirical performance of forward-looking models for inflation dynamics in a small open economy. Our forward-looking models relate current inflation to expected future inflation and the difference between the actual price and the steady state value of levels as a theory-consistent forcing variable. The steady state value is specified as a mark-up over marginal costs, which, in turn, are determined by costs of both labour and imported intermediate goods in line with the open economy literature. The models thus include variables, both levels and differences, that require caution with respect to both the time series properties and possible cointegrated nature of the variables involved. Such econometric issues have typically been ignored in related studies on data from open economies and they raise questions about the validity of the existing statistical inference.

Using reduced-rank regressions, we first established a cointegrating vector between consumer prices and marginal costs. Thus, the theory-driven price equation underlying the models of inflation dynamics is well supported by the data. Secondly, using likelihood-based testing procedures, we found that the *exact* version of the forward-looking model is at odds with the data. That is, the rational expectations hypothesis is *not* rejected statistically, but when only economically meaningful parameters are allowed for, the model is *not* supported by the data. In addition, some of the parameters in the *exact* version of the model are not well identified. We also found that the *in-*

exact version is *not* rejected statistically using a test based on a minimum distance method. However, confidence regions obtained by inverting a test reminiscent of the Anderson-Rubin statistic reveal an identification problem in this model as well.

Finally, we established a well-specified dynamic backward-looking model, which, in addition to the theory-consistent forcing variable, includes backward-looking terms only. The backward-looking model of inflation dynamics outperforms a reduced form forward-looking model, it is reasonably stable in a sample containing a major monetary policy regime shift from exchange rate targeting to inflation targeting, and it forecasts well post-sample and during and after the recent financial crisis. All these findings provide strong evidence in favour of the backward-looking model and the Lucas critique does not seem to be important in our empirical context. Our findings are in line with Fuhrer (2006), among others, who points out that lagged inflation is not a second-order add-on to the optimising model, it *is* the model.

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Appendix 1

We show here how the steady state value of the price level in (1) is derived. As mentioned in the text, a representative firm is assumed to face a Cobb-Douglas production function of the form

$$Y_t = AL_t^{\alpha} I_t^{\beta}, \tag{A1}$$

where L_t and I_t denote labour and imports of intermediate inputs, respectively. Variable costs in production are given by the sum of labour costs and import costs

$$C_t = W_t L_t + PI_t I_t, \tag{A2}$$

where W_t is wage per hour and PI_t is the price of imports. Minimising variable costs given the production function leads to

$$W_t L_t = (\alpha/\beta) PI_t I_t. \tag{A3}$$

Solving for I_t in (A3) and inserting it into (A1) gives

$$L_t = L_0 (W_t/PI_t)^{-\beta/(\alpha+\beta)} Y_t^{1/(\alpha+\beta)}. \tag{A4}$$

Inserting (A4) into (A3) and solving for I_t gives a similar expression for I_t

$$I_t = I_0 (W_t/PI_t)^{\alpha/(\alpha+\beta)} Y_t^{1/(\alpha+\beta)}. \tag{A5}$$

Inserting (A4) and (A5) into (A2) gives the cost function, and it is straightforward to show that marginal costs, MC_t, become

$$MC_t = C_0 W_t^{\alpha/(\alpha+\beta)} PI_t^{\beta/(\alpha+\beta)} Y_t^{1/(\alpha+\beta)-1}, \tag{A6}$$

which is homogeneous of degree one in factor prices. Using the production function

(A1), we can express marginal costs as a function of two terms

$$MC_t = C_0(W_t L_t/Y_t)^{\alpha/(\alpha+\beta)}(PI_t I_t/Y_t)^{\beta/(\alpha+\beta)}. \tag{A7}$$

Using lower case letters to indicate logs, we have

$$mc_t = c_0 + \psi_1 ulc_t + \psi_2 uic_t, \tag{A8}$$

where ulc_t is labour unit costs, uic_t is import unit costs, $\psi_1 = \alpha/(\alpha+\beta)$ and $\psi_2 = \beta/(\alpha+\beta)$. As explained in the text, a representative profit-maximising firm facing an isoelastic demand function will set prices as a mark-up on marginal costs. In log form, we thus have $p_t^* = m_0 + mc_t$, where m_0 is the constant mark-up. Using the expression for marginal costs in (A8), we have

$$p_t^* = m_0 + \psi_1 ulc_t + \psi_2 uic_t, \tag{A9}$$

which is the steady state value of the price level given in (1).

Appendix 2

Equation (10) describing the estimated hybrid forward-looking model can be reformulated as

$$\begin{aligned} & c_1' E_t[\Delta X_{t+1} - \Phi_0 D_{t+1}] + (c_1' + c_0' + c_{-1}' + c_{-2}')X_t \\ & -(c_{-1}' + c_2')\Delta X_t - c_2' \Delta X_{t-1} + \psi_0 = 0. \end{aligned} \tag{A10}$$

If $e_1 = (1, 0, 0)'$ denotes the vector having the first element equal to 1 and zero otherwise, $c_1 = \gamma_f e_1,\ c_{-1} + c_{-2} = e_1$ and $c_{-2} = -\gamma_b e_1$. Furthermore, if we define $d = -(c_1' + c_0' + c_{-1}' + c_{-2}')/\lambda = (1, -\psi_1, -(1 - \psi_1))$ the model (A10) may be written

$$\begin{aligned} e_1' E_t[\Delta X_{t+1} - \Phi_0 D_{t+1}] &= \\ (\lambda/\gamma_f) d' X_{t-1} + (1/\gamma_f) e_1' \Delta X_t - (\gamma_b/\gamma_f) e_1' \Delta X_{t-1} - \psi_0 &= \\ \tau d' X_{t-1} + \tau_1 e_1' \Delta X_t + \tau_2 e_1' \Delta X_{t-1} + \mu_0 &= 0, \end{aligned}$$

where the parameters $\tau = \lambda/\gamma_f,\ \tau_1 = 1/\gamma_f,\ \tau_2 = -\gamma_b/\gamma_f$ and $\mu_0 = -\psi_0/\gamma_f$ vary freely. Thus, for fixed $\psi = \psi_1$ this has exactly the form treated in Johansen and Swensen (2008).

The marginal part of the model now takes the form

$$\Delta p_t = \tau(p_{t-1} - \psi ulc_{t-1} - (1 - \psi) uic_{t-1}) + \tau_1 \Delta p_{t-1} + \tau_2 \Delta p_{t-2} + \phi' D_t + \epsilon_{1t},$$

where $\phi = (\phi_1, \ldots, \phi_{10})$ are the coefficients of the seasonal dummies, the other six dummies and the constant. Note that there are four restrictions involved as the coefficients of $\Delta ulc_{t-1}, \Delta uic_{t-1}, \Delta ulc_{t-2}$ and Δuic_{t-2} are constrained to zero. The conditional part involves an unrestricted regression of $(\Delta ulc_t, \Delta uic_t)'$ on Δp_t, $(p_{t-1} - \psi ulc_{t-1} - (1 - \psi) uic_{t-1})$, Δp_{t-1}, Δulc_{t-1}, Δuic_{t-1}, Δp_{t-2}, Δulc_{t-2}, Δuic_{t-2}, 1 and the seasonal dummies and the impulse dummies.

For fixed values of ψ the numerical value of the likelihood can therefore be evaluated. Hence the maximum likelihood estimate of ψ can be determined. Once this is given, the estimates for τ, τ_1 and τ_2 can be found by ordinary least squares (OLS) from the marginal part with the estimate of ψ, and the semi-structural parameters γ_f, γ_b and λ computed.

Appendix 3

In the following, we present some details about the minimum distance approach used in the test of the restrictions $c_1'A_1 + c_0' = 0$. It is convenient to rewrite the restrictions as $B(\nu)\theta + h = 0$ where $B(\nu)$ is a 3×2 matrix depending on constants and the parameters ν of the autoregressive model, $\theta = (\gamma_f, \lambda)'$ and h is a vector where the elements are known constants.

In the alternative equilibrium correction parameterisation, the VAR model may be written as

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Phi_0 D_t + \Phi_1 + \epsilon_t,$$

where $\Pi = A_1 + A_2 + A_3 - I = \alpha\beta'$, $\Gamma_1 = -A_2 - A_3$ and $\Gamma_2 = -A_3$. Thus, $A_1 = \Pi + I - A_1 - A_2 = \Pi + I + \Gamma_1$. In particular, due to the super-consistency, the parameters in the cointegration vector, i.e. $\beta = (1, -\psi_1, -\psi_2)'$, where $\psi_1 + \psi_2 = 1$, can be considered as known. In this case ν corresponds to $\alpha = \{\alpha_i\}_{i=1}^3, \Gamma_1 = \{\gamma_{1,ij}\}_{i,j=1}^3, \Gamma_2 = \{\gamma_{2,ij}\}_{i,j=1}^3$, and

$$B(\nu)\theta + h = \begin{pmatrix} \alpha_1 + \gamma_{1,11} & -1 \\ -\alpha_1\psi_1 + \gamma_{1,12} & \psi_1 \\ -\alpha_2\psi_2 + \gamma_{1,13} & \psi_2 \end{pmatrix} \begin{pmatrix} \gamma_f \\ \lambda \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}.$$

If $\hat{\nu}$ is the usual estimator for ν, as in Johansen (1995), the minimum distance estimator for $\theta(\nu)$ can be expressed as $\hat{\theta}(\hat{\nu}) = [B(\hat{\nu})'B(\hat{\nu})]^{-1}B(\hat{\nu})'h$.

From theorem 13.5 in Johansen (1995), it follows that $\sqrt{T}(\hat{\nu} - \nu)$ is approximately multivariate Gaussian. Under the restriction that $B(\nu)\theta + h = 0$, one may write $\sqrt{T}(B(\hat{\nu})\hat{\theta}(\hat{\nu}) + h) = \sqrt{T}(B(\hat{\nu})\hat{\theta}(\hat{\nu}) + h - B(\nu)\theta - h) = \sqrt{T}[(B(\hat{\nu}) - B(\nu))\hat{\theta}(\hat{\nu}) + B(\nu)(\hat{\theta}(\hat{\nu}) - \theta)]$. From the δ-method, it follows that $\sqrt{T}(\hat{\theta}(\hat{\nu}) - \theta)$ has the same asymptotic distribution as $\frac{\partial B(\nu)}{\partial \nu}|_{\nu=\hat{\nu}}(\hat{\nu} - \nu)$. Thus, both terms in $\sqrt{T}(B(\hat{\nu})\hat{\theta}(\hat{\nu}) + h)$ may be expressed by $\sqrt{T}(\hat{\nu} - \nu)$, and the sum is therefore also asymptotically Gaussian. If the covariance matrix is estimated by plugging in the estimates for the unknown parameters, a test statistic for the null hypothesis $c_1'A_1 + c_0' = 0$ can be found. The asymptotic distribution is χ^2 with one degree of freedom.

Appendix 4

P: The official consumer price index (2002 = 1). Source: Statistics Norway.

UIC: Unit import costs proxied by the implicit deflator of total imports (2002 = 1). Source: Statistics Norway, the Quarterly National Accounts.

ULC: Unit labour costs proxied by YWP/QP, where YWP and QP are total labour costs and value added in the private mainland economy, respectively. Source: Statistics Norway, the Quarterly National Accounts.

$D84Q1$: Impulse dummy used to account for a large residual in the ulc_t-equation of the VAR. Equals unity in the first quarter of 1984, zero otherwise.

$D86Q3$: Impulse dummy used to control for the 12 per cent devaluation of the Norwegian currency in May 1986. Equals unity in the third quarter of 1986, zero otherwise.

$D96Q1$: Impulse dummy used to control for the reduction in indirect tax rates during the first quarter of 1996. Equals unity in the first quarter of 1996, zero otherwise.

$D01Q3$: Impulse dummy used to control for the drop in the VAT rate on food from 24 per cent to 12 per cent in July 2001. Equals unity in the third quarter of 2001, zero otherwise.

$D03Q1$: Impulse dummy used to control for the large increase in electricity prices during the first quarter of 2003. Equals unity in the first quarter of 2003, zero otherwise.

$D03Q2$: Impulse dummy used to control for the large decrease in electricity prices during the second quarter of 2003. Equals unity in the second quarter of 2003, zero otherwise.

$D07Q1$: Impulse dummy used to control for the large decrease in electricity prices during the first quarter of 2007. Equals unity in the first quarter of 2007, zero otherwise.