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Is the marginal cost of public funds equal to one?

TALL SOM FORTELLER

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Is the marginal cost of public funds equal to one?

Abstract:
In a recent article Bas Jacobs found that the marginal cost of public funds (MCF) is one when taxation gives second best resource allocation. This conclusion is based on a claim that there are certain shortcomings with the standard definition of MCF, for example that the size and sign of the standard MCF measure is sensitive to the choice of the untaxed good. A less frequently used definition of MCF is therefore applied instead. If a lump-sum tax is a marginal source for public revenue and taxation is optimal, MCF is one with the proposed definition. The contribution of the present paper is two-fold. First, it finds the standard MCF-measure is not sensitive to the choice of the untaxed good. Second, it finds that the proposed alternative definition has undesirable properties, for example that it could give negative MCF-measures along the upward sloping part of the Laffer-curve and is sensitive to the choice of the untaxed good also in cases where this does not make sense. The present paper therefore concludes that there is a weak basis for the conclusion that MCF is one with optimal taxation.

Keywords: Marginal cost of public funds, taxation, lump-sum taxes, public goods.

JEL classification: H20, H40, H50.

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Sammendrag

I en artikkel tidsskriftet International Tax and Public Finance fant Bas Jacobs at marginalkostnaden for skattefinansiering av kollektive offentlige goder (MCF) er én. Denne konklusjonen bygger på en kritikk av standarddefinisjonen av MCF, for eksempel at størrelsen og fortegnet på standard MCF-mål er følsomt for valget av det ubeskattede godet. Jacobs anvender derfor en mindre vanlig definisjon av MCF. Hvis lump-sum-skatter er en marginal kilde for offentlig finansiering og beskatningen er optimal, er MCF én med den anvendte definisjonen. Det foreliggende arbeidet har to hovedfunn. For det første finner jeg at standard MCF-mål ikke er følsomt for valg av ubeskattet gode, slik Jacobs hevder. For det andre finner jeg at den foreslåtte alternative definisjonen av MCF anvendt av Jacobs har uønskede egenskaper, for eksempel at den kan gi negative MCF-verdier langs den stigende delen av Laffer-kurven og er følsom overfor valget av ubeskattet gode også i tilfeller der dette ikke gir mening. Jeg konkluderer med at det ikke er grunnlag for å hevde MCF er én.

1 Introduction

Jacobs (2018) challenges previous research on the question of possible costs of taxation and the question of optimal supply of tax-funded public goods. If the conclusions in Jacobs (2018) are confirmed by further research, they have important consequences. Accordingly, influenced by Jacobs' analyses the Dutch government recently decided to set the marginal cost of public funds to one in cost benefit analyses of public projects.

The present paper is mainly a follow up to Jacobs (2018) and questions his conclusions. Different definitions of the marginal cost of public funds (MCF) will be discussed. It is therefore reasonable to begin with a brief overview of some important contributions to the literature on this issue.

Due to the limited possibilities for use of lump-sum taxes and likely efficiency losses from other taxes, Pigou (1947, p. 34) concluded that public *expenditure ought not to be carried out so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen.* In other words, Pigou concluded that the traditional rule for optimal allocation of resources between private and public sector, later formalized by Samuelson (1954), would give too much public expenditure (Diamond & Mirrlees, 1971).

Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) made it clear that this is not necessarily correct. The reason is that a tax increase has both a substitution effect and an income effect. The substitution effect draws in the direction of reduced labour supply. This creates an unintended distortion and represents a cost of taxation. In addition, a tax increase has an income effect. Reduced after-tax-income makes individuals increase their labour supply if leisure is a normal good. Increased labour supply will represent an efficiency gain if there already is a tax on labour income which has caused an inefficiently low supply of labour. If the income effect exceeds the substitution effect, the tax increase gives an efficiency improvement which means that *the marginal cost of public funds* (MCF) is less than one.

Accordingly, Stiglitz and Dasgupta (1971) found that "whether the conventional rule represents an under or over supply depends simply on whether the supply of labour is backward bending or upward sloping."

Neither Stiglitz and Dasgupta (1971) or Atkinson and Stern (1974) used the term MCF. Their articles have nevertheless been considered as the starting point for what today is the standard definition of MCF. Hence, since the publication of these two contributions it has been clear that MCF could be smaller than one even when distortionary taxes are the marginal source of public funding. Possible confusion caused by this conclusion was discussed by Ballard and Fullerton (1992), who explained clearly why this conclusion, which perhaps might be considered a paradox, makes sense, see also Håkonsen (1998) who compared different defintions of MCF.

Because its effects on income distribution is an important aspect of the design of tax schemes, an important contribution by Sandmo (1998) was to accentuate the need to analyze the issue with models with a set of heterogenous individuals and to propose a transparent theoretical model for MCF in such a setup. Christiansen (1981) and Kaplow (1996) applied models with heterogenous individuals to the conditions for the Samuelson Rule to be satisfied.

Despite the significant number of studies on the issue, there does not seem to be convergence towards consensus with regard to the size of MCF. It seems likely that countries with different tax levels and systems have different MCFs (Dahlby, 2008). However, this is not the only reason for the lack of convergence in the literature. With more general models some contributions have indicated that MCF is relatively high, see for example Dahlby and Ferede (2012), Browning (1976), Browning (1987), Browning and Liu (1998), and Feldstein (1999). Also Kleven and Kreiner (2006) found relatively high MCF estimates with a model with entry and exit possibilities in the labour market. Others have argued that MCF is closer to one, see for example Ballard (1990) and Stuart (1984). Theoretical contributions from Christiansen (2007) and Kaplow (1998) draw in the same direction. Despite the ambiguous conclusions in the literature, cost-benefit analyses of public projects often assume that there are costs related to taxation and assume implicitly that the conventional Samuelson Rule has to be corrected for that costs. Many countries have as a general recommendation for cost-benefit analyses of tax-funded public projects that the costs should be multiplied with a certain factor, for example 1.2, to take MCF into account.

Jacobs (2018) argues against this practise. He found that the standard definition of MCF has some "undesirable" properties. Based on the critique of the traditional approach, he argues that another definition of MCF should be applied. With that approach, he found that MCF equals one in second best.[1] An important reason for this result is the assumption that the lump-sum tax is a marginal source for public revenue in addition to income taxes or consumption taxes.[2]

Jacobs' critique of the standard approach lists three undesirable properties of the standard definition of MCF:

1. The standard measure of MCF of lump-sum taxes are different from one even when taxes are optimized.

2. The standard measure of MCF is not directly related to the marginal excess burden of taxation (MEB).

3. The standard measure of MCF is found to be sensitive to the choice of the untaxed good with optimal taxation.

In addition, Jacob's conclusion that MCF is one with optimal taxation is based on the assumption that lump-sum-taxes could be a marginal source of public funding. This is in contrast to most earlier contributions, where the starting point for the discussion has been that the possibilities for lump-sum taxation is limited and therefore not should be considered as a marginal source of finance (Pigou, 1947; Stiglitz & Dasgupta, 1971; Diamond & Mirrlees, 1971; Atkinson & Stern, 1974).

The present paper has two main elements. First, it responds in detail to the third element of Jacobs' critique of the standard definition of MCF; the claim that the standard MCF-measure is sensitive to the choice of the untaxed good. My starting point is that MCF is defined as the shadow price of the public budget constraint divided by the average of the individuals' marginal utility of income net of taxes, see for example Sandmo (1998). With that definition the standard MCF-measure is not sensitive to the choice of the untaxed good with optimal taxation. I will show that the reason why Jaobs (2018) found that MCF is sensitive to the

[1] With a set of individual lump-sum taxes the optimal resource allocation (first best) could be achieved.

[2] The proposed alternative definition of MCF was analyzed and discussed in Håkonsen (1998) who also studied other alternative representations of MCF.

choice of the untaxed good was caused by a comparison of two different definitions of MCF. In the case of an income tax, and the consumption tax normalized to zero, MCF was defined as the shadow price of the public budget constraint divided by the average of the individuals' marginal utility of *net income* (after-tax income). This is in accordance with most previous literature. However, in the case of a consumption tax, and the income tax normalized to zero, MCF was implicitly defined as the shadow price of the public budget constraint divided by the average of the individuals' marginal utility of *gross income*.

Next, I will show that the proposed alternative definition of MCF has some undesirable properties which makes it less useful than the standard definition. Hence, there are reasons to stick to the standard definition of MCF, which is not one with optimal taxation and access to lump-sum taxes.

A numerical example illustrates the results. The concluding section includes brief discussions of the two first elements of the critique of the standard MCF measure.

2 Some properties of MCF with the standard definition

Let there be a total of n individuals. Henceforth, it is understood that i refers to one of the n individuals under consideration and the index i always runs from $i = 1, ..., n$. With regard to model formulation and notation, I will, with a few exceptions, follow Sandmo (1998).

The individuals' utility functions are:

$$u_i = u(c_i, l_i, G),\ u_c,\ u_l,\ u_G > 0, \tag{1}$$

where c_i is consumption of a private good, l_i is leisure and G is consumption of a pure public good. Individuals maximize u_i with respect to c_i and l_i, taking G as given, subject to the budget constraint:

$$(1 + \tau)p_c c_i = (1 - t)w_i h_i + a, \tag{2}$$

where h_i is labour supply, τ is the consumption tax, t is the wage tax, w_i is an individual wage rate which reflects the individuals' productivity, p_c is producer price of the consumption good, T is the endowment of time such that $h_i + l_i = T$, and a is a uniform lump-sum transfer (positive, zero or negative). When a is called a lump-sum *tax*, I have a negative value of a in mind. In the following the consumer good price is normalized to one ($p_c = 1$).

When formulating the maximization problem of the n individuals and the corresponding Lagrange-function, one should have in mind that how individuals' budget constraint are included in the Lagrange function determines the interpretation of the Lagrange multiplier and its size. The budget constraint is therefore normalized as follows:

$$c_i = \frac{1-t}{1+\tau} w_i h_i + \frac{a}{1+\tau}. \tag{3}$$

This is equivalent to

$$c_i + p_i l_i = M_i, \tag{4}$$

where p_i is the price of leisure and M_i could be labeled the virtual income:

$$p_i = \frac{1-t}{1+\tau} w_i, \tag{5}$$

$$M_i = \frac{1-t}{1+\tau} w_i T + \frac{a}{1+\tau}. \tag{6}$$

For later reference, define also the effective tax rate and effective lump-sum transfer as:

$$t^* = \frac{t+\tau}{1+\tau}, \tag{7}$$

$$a^* = \frac{a}{1+\tau}. \tag{8}$$

Obviously, any combined change of t, τ, and a that does not change either t^* or a^* will not change behaviour of the individuals nor public revenue.

The Lagrange function related to the individuals' utility maximization could be written:

$$L_i(c_i, h_i, G, w_i, t, \tau, a) = u(c_i, T - h_i, G) - \lambda_i \left(c_i - \frac{1-t}{1+\tau} w_i h_i - \frac{a}{1+\tau} \right). \tag{9}$$

First order conditions:

$$u_c(c_i, l_i, G) = \lambda_i, \tag{10}$$

$$u_l(c_i, l_i, G) = -\lambda_i \frac{1-t}{1+\tau} w_i. \tag{11}$$

With the normalization of the budget constraint before inclusion in the Lagrange-function, λ_i is equal to the marginal utility of consumption, or, in other words, *marginal utility of income net of taxes*. This is essential because MCF will be defined as the shadow cost of the public budget constraint divided by the average of the n Lagrange-multipliers λ_i.

The two first order conditions in (10) and (11) together with the budget constraint (4) give individual Marshallian demand for consumer goods and leisure together with labour supply as functions of p_i and M_i:

$$c_i = c(p_i, M_i; G), \tag{12}$$

$$l_i = l(p_i, M_i; G), \tag{13}$$

$$h_i = T - l(p_i, M_i; G) = h(p_i, M_i; G). \tag{14}$$

Let $v_i = v(t, \tau, a, w_i, G)$ be the indirect utility function. The envelope theorem gives that:

$$\frac{\partial v_i}{\partial t} = -\lambda_i \frac{1}{1+\tau} w_i h_i, \tag{15}$$

$$\frac{\partial v_i}{\partial \tau} = -\lambda_i \frac{1}{1+\tau} c_i, \tag{16}$$

$$\frac{\partial v_i}{\partial a} = \lambda_i \frac{1}{1+\tau}, \tag{17}$$

$$\frac{\partial v_i}{\partial G} = \frac{\partial u_i}{\partial G}. \tag{18}$$

The public budget constraint is:

$$\tau \sum_i c_i + t \sum_i w_i h_i - na = qG, \tag{19}$$

where q is the unit productions costs of the public good. The government maximizes the sum of the individuals' utilities subject to the public budget constraint. The corresponding Lagrange function is:

$$L_g = \sum_i v(t, \tau, a, G, w_i) + \mu \left(\tau \sum_i c_i + t \sum_i w_i h_i - qG - na \right). \tag{20}$$

The first order conditions for the government's maximization problem are:

$$\frac{1}{1+\tau} \sum_i \lambda_i = \mu \left(n - \tau \sum_i \frac{\partial c_i}{\partial a} - t \sum_i w_i \frac{\partial h_i}{\partial a} \right), \tag{21}$$

$$\frac{1}{1+\tau} \sum_i \lambda_i w_i h_i = \mu \sum_i \left(\tau \frac{\partial c_i}{\partial t} + w_i h_i + t w_i \frac{\partial h_i}{\partial t} \right), \tag{22}$$

$$\frac{1}{1+\tau} \sum_i \lambda_i c_i = \mu \sum_i \left(c_i + \tau \frac{\partial c_i}{\partial \tau} + t w_i \frac{\partial h_i}{\partial \tau} \right), \tag{23}$$

$$\sum_i \frac{\partial u_i}{\partial G} = \mu \left(q - \tau \sum_i \frac{\partial c_i}{\partial G} - t \sum_i w_i \frac{\partial h_i}{\partial G} \right). \tag{24}$$

The Lagrange multiplier μ is the shadow value of public consumption, which in optimum should be equal to marginal costs of government expenditure financed by taxes. Therefore μ is the starting point for definitions of MCF.

However, μ has to be normalized in some way or another. In models with a single individual or a set of identical individuals, the marginal utility of income (net of taxes) is an obvious choice for normalization, see for example Håkonsen (1998). With a set of heterogenous agents there is no such obvious way to normalize μ. I follow Sandmo (1998) and define what could be considered as the standard MCF-measure:

Definition 1 (The standard MCF) *The marginal cost of public funds is defined as*

$$MCF = \frac{\mu}{\bar{\lambda}}, \tag{25}$$

where $\bar{\lambda}$ is the average marginal utility of income net of taxes:

$$\bar{\lambda} = \frac{1}{n}\sum_i \lambda_i. \tag{26}$$

Throughout the paper bars indicate average values of variables. Define also $\lambda = \sum_i \lambda_i$, $y_i = w_i h_i$, $y = \sum_i y_i$, $c = \sum_i c_i$, and public revenue as $R = \tau \sum_i c_i + t\sum_i y_i - na$. Moreover, throughout let ε_{xz} represent the elasticity of a variable x with respect to a parameter z. Note also that it follows from the definition of a covariance that:

$$cov(\lambda_i, y_i) = \frac{1}{n}\sum_i \lambda_i y_i - \bar{\lambda}\bar{y}. \tag{27}$$

First consider the case where a lump-sum transfer is combined with an income tax. Using the f.o.c. in (21) and (22) and assuming that $\tau = 0$, we have:

$$MCF_a = \frac{1}{1 - \frac{t}{n}\sum_i w_i \frac{\partial h_i}{\partial a}}, \tag{28}$$

$$MCF_t = \frac{1 - \delta_{\lambda y}}{1 - \theta_t}, \tag{29}$$

where the subscript a and t indicate that MCF of a lump-sum tax and an income tax are considered, respectively, and where:

$$\delta_{\lambda y} = -\frac{cov(\lambda_i, y_i)}{\bar{\lambda}\bar{y}}, \tag{30}$$

$$\theta_t = -\sum_i \frac{y_i}{y}\varepsilon_{h_i t}. \tag{31}$$

where $\varepsilon_{h_i t}$ is the elasticity of individual i's labour supply with respect to the tax rate t.

Next, consider the case where a consumption tax is combined with a lump-sum tax. Using the f.o.c. in (21) and (23) and assuming that $t = 0$, we have:

$$MCF_a = \frac{1}{1+\tau}\frac{1}{1-\frac{\tau}{n}\sum_i \frac{\partial c_i}{\partial a}}, \tag{32}$$

$$MCF_\tau = \frac{1}{1+\tau}\frac{1-\delta_{\lambda c}}{1-\theta_\tau}, \tag{33}$$

where the subscript a and τ indicate that MCF of a lump-sum transfer and a consumption tax are considered, respectively, and where

$$\delta_{\lambda c} = -\frac{cov(\lambda_i, c_i)}{\bar{\lambda}\bar{c}}, \tag{34}$$

$$\theta_\tau = -\sum_i \frac{c_i}{c}\varepsilon_{c_i \tau}. \tag{35}$$

where $\varepsilon_{c_i t}$ is the elasticity of individual i's consumption of the private good with respect to the tax rate t.

Jacobs (2018) found that with his approach, standard MCF in second best was less than one if the consumption tax was set to zero, and greater than one if the income tax was set to zero. This is not the case with the setup in the present paper:

Proposition 1 (MCF with optimal taxation) *Assume that the government implements a lump-sum tax in combination with either an income tax or a consumption tax, in both cases such that second best is achieved. Then it follows from*

(28) and (32) that the size of MCF is less than one and not sensitive to whether the solution is a result of a combination of a lump-sum tax and an income tax, or a combination of a lump-sum tax and a consumption tax.

Proof. See appendix A. ■

The question is then why Jacobs (2018) found that with the standard definition MCF is greater than one with an optimal combination of a consumption tax and a lump-sum tax. This apparent shortcoming with the standard definition of MCF followed from his interpretation of what exactly should be the standard definition. In accordance with what is standard Jacobs defined MCF as the shadow price of the public budget constraint normalized with the average of the shadow prices of the individual budget constraints. This is apparently in accordance with Definition 1. However, there is an important difference. In construction of the Lagrange function related to individual maximization Jacobs (2018) looked to the budget constraint (2) and applied the Lagrange function:

$$L_i{}'(\cdot) = u(c_i, T - h_i, G) - \lambda_i' \left((1+\tau)c_i - (1-t)w_i h_i - a\right), \tag{36}$$

where λ_i' is Lagrange-multiplier. This means that the f.o.c. in (10) now is replaced with the following f.o.c.:

$$\frac{u_c(c_i, h_i, G, w_i)}{1+\tau} = \lambda_i'. \tag{37}$$

It follows that the Lagrange-multiplier λ_i' in the case with an income tax (and $\tau = 0$) is equal to the marginal utility of *net* income (after tax payment). In the case with a consumption tax (and $t = 0$), λ_i' is the marginal utility of *gross* income (before tax payments). Because MCF is defined as $\mu/\bar{\lambda}'$, this means that we are dealing with two different definitions of MCF, one in the case with an income tax and another in the case with a consumption tax. This explains why MCFs are found to be different in the two cases.

A key point here is that the size of λ_i' varies with the size of the consumption tax irrespective of whether the effective tax rate has changed. For example, consider a situation where the three tax parameters t, τ and a are given values such that the second best allocation is achieved. With this type of model and set of tax parameters, the second best solution could be achieved with different combinations of the three tax parameters (Fullerton, 1997). However, irrespective of the choice of tax parameters within the set of combinations, as long as second best is achieved, MCF should be unaltered. This will be the case with the definition of MCF as applied in this paper, where $MCF = \mu/\bar{\lambda}$. However, *any* change of τ will change the size of $\bar{\lambda}'$ also when it is combined with changes of t and a such that the effective tax rates t^* and a^* are unaltered. This explains why Jacobs (2018) found the standard MCF-measure to be sensitive to the choice of the untaxed good. This is not an undesirable property with the standard MCF definition, but a weakness with his definition of MCF. With the chosen formulation of the Lagrange-function in Jacobs (2018), see (36), a better choice would have been to define MCF as follows:

$$MCF = \frac{1}{1+\tau}\frac{\mu}{\bar{\lambda}'}. \tag{38}$$

Then MCF would not have been sensitive to the choice of the untaxed good.

The next section will analyze some properties of the MCF-definition preferred by Jacobs (2018). In preparation for this, some corresponding properties of the standard MCF-measure are analyzed in the following.

Because transfers received by individuals are not subject to income taxation, while a consumption tax indirectly apply to transfers also, there will always be an asymmetry with regard to the effects of increasing the income tax and the consumption tax if $a \neq 0$. Hence, when $a \neq 0$ one cannot expect MCF to be insensitive to the choice of the untaxed good.

With the assumption that $a = 0$ this is different. Then the effects of a consumption tax and a labour tax are identical with respect to resource allocation and income distribution, and therefore also welfare. It follows that costs related to public funding are the same with the two tax instruments in this case. Thus, if $a = 0$, MCF should be the same in these two cases also outside the second best solution.[3] Proposition 2 shows that this is the case with the standard definition of MCF.

In preparation for Proposition 2, note that if $a = 0$ it follows from (29) and

[3]The term 'second best' could here perhaps be confusing. Proposition 1 considers MCF in second best assuming that a lump-sum tax could be implemented at the preferred level. If a lump-sum tax for different reasons is not considered as an option, the second best solution will be different from the one considered in Proposition 1.

(33) that:

$$MCF_t = \frac{1-\delta_{\lambda y}}{\varepsilon_{Rt}}, \quad (39)$$

$$MCF_\tau = \frac{1}{1+\tau}\frac{1-\delta_{\lambda c}}{\varepsilon_{R\tau}}, \quad (40)$$

where ε_{Rt} and $\varepsilon_{R\tau}$ are the elasticities of public revenue R with respect to the income tax t and the consumption tax τ, respectively.

Proposition 2 (MCF without lump-sum taxes) *Assume that* $a = 0$. *Let* $MCF_{\tilde{t}}$ *represent the standard measure of the marginal cost of public funds when* $\tau = 0$ *and* $t = \tilde{t} \geq 0$. *Correspondingly, let* $MCF_{\hat{\tau}}$ *represent the case where* $\hat{\tau} = \tilde{t}/(1-\tilde{t})$ *and* $t = 0$. *Then we have that*

$$MCF_{\tilde{t}} = MCF_{\hat{\tau}} \quad (41)$$

irrespective of whether the solution is second best.

Proof. See appendix A. ■

Proposition 2 shows a desirable property with the standard MCF-definition. The expressions in (39) and (40) reveals other desirable properties. The Laffer-curve is usually found to be concave (Trabandt & Uhlig, 2011). When approaching the top of the Laffer-curve, increasing the tax rate further is by definition steadily less effective with respect to revenue collection. Hence, when approaching the top of the Laffer-curve it is desirable that an MCF-measure approaches infinity.

To check whether this is the case with the standard MCF-measure, consider first the numerators of (39) and (40). If individual income y_i is assumed to increase with ability w_i (agent monotonicity), then it is reasonable to assume that $cov(\lambda_i, y_i)$ and $cov(\lambda_i, c_i)$ are both negative. Hence, the numerators of (39) and (40) are both positive. With regard to the denominators, ε_{Rt} and $\varepsilon_{R\tau}$ are the elasticities of the revenue with respect to the tax rates. Consequently, with the standard definition, MCF is positive along the entire upward sloping part of the Laffer-curve. As the economy approaches the top of the Laffer-curve, ε_{Rt} and $\varepsilon_{R\tau}$ converge towards zero from above, which means that MCF approaches infinity. A numerical example which illustrates this will be presented in Section 4.

3 Some properties of MCF with the Diamond-based definition

Due to the alleged undesirable properties with the standard MCF-measure, Jacobs (2018) recommends another definition. In the following some properties of the proposed MCF-measure is analyzed. Because Håkonsen (1998) and Jacobs (2018) are starting points for this, their setup for individuals' utility maximization is followed. They applied a Lagrange function corresponding to the formulation in (36). The envelope theorem now gives that:

$$\frac{\partial v_i}{\partial a} = \lambda_i', \tag{42}$$

$$\frac{\partial v_i}{\partial t} = -\lambda_i' w_i h_i, \tag{43}$$

$$\frac{\partial v_i}{\partial \tau} = -\lambda_i' c_i. \tag{44}$$

The public budget constraint is given by (19) and the Lagrange-function in (20) is again applied for the government's maximization problem. Using (42)-(44), the first order conditions with respect to the three tax parameters then become:

$$\mu\left(n - \tau\sum_i \frac{\partial c_i}{\partial a} - t\sum_i w_i \frac{\partial h_i}{\partial a}\right) = \sum_i \lambda_i', \tag{45}$$

$$\mu\sum_i\left(\tau\frac{\partial c_i}{\partial t} + w_i h_i + t w_i \frac{\partial h_i}{\partial t}\right) = \sum_i \lambda_i' w_i h_i, \tag{46}$$

$$\mu\sum_i\left(c_i + \tau\frac{\partial c_i}{\partial \tau} + t w_i \frac{\partial h_i}{\partial \tau}\right) = \sum_i \lambda_i' c_i. \tag{47}$$

When the standard MCF-measure was defined in the previous section, I followed Sandmo (1998) and normalized by dividing with the *average* marginal utility of net income. Håkonsen (1998) and Jacobs (2018) suggested to divide by the average social marginal value of income as defined by Diamond (1975):

Definition 2 (Diamond-based MCF) *Based on Diamond (1975), define the* *social marginal value of income to individual i as:*

$$\alpha_i = \lambda_i' + \mu \left(tw_i \frac{\partial h_i}{\partial a} + \tau \frac{\partial c_i}{\partial a} \right). \quad (48)$$

The marginal cost of public funds in the following called the Diamond-based MCF-measure is defined as

$$MCF^D = \frac{\mu}{\bar{\alpha}}, \quad (49)$$

where μ is the shadow value of the public budget constraint.

Here it should be noted that it reasonable to label $\lambda_i' = (1+\tau)^{-1} u_c(c_i, h_i, G)$ as the private marginal value of income, which is included as the first term on the right hand side of (48). Note also that when MCF-measures in the remainder of the paper have the superscript D, it refers to Diamond-based measures.

By plugging α_i as defined in (48) into (46) and (47) we obtain the proposed MCF-measures:

$$MCF_t^D = \frac{1 - \delta_{\alpha y}}{1 + \varepsilon_{yt} + \rho_y}, \quad (50)$$

$$MCF_\tau^D = \frac{1 - \delta_{\alpha c}}{1 + \varepsilon_{c\tau} + \rho_c}, \quad (51)$$

where

$$\delta_{\alpha y} = -\frac{cov(\alpha_i, y_i)}{\bar{\alpha}\bar{y}}, \quad (52)$$

$$\delta_{\alpha c} = -\frac{cov(\alpha_i, y_i)}{\bar{\alpha}\bar{c}}, \quad (53)$$

$$\rho_y = t \sum_i \frac{y_i}{y} w_i \frac{\partial h_i}{\partial a}, \quad (54)$$

$$\rho_c = \tau \sum_i \frac{c_i}{c} \frac{\partial c_i}{\partial a}. \quad (55)$$

It should here be noted that the denominator in (50) could be written as in the following expression (see Appendix B):

$$MCF_t^D = \frac{1-\delta_{\alpha y}}{1-\theta_t^H} \tag{56}$$

where

$$\theta_t^H = -\sum_i \frac{y_i}{y}\varepsilon_{h_i t}^H \tag{57}$$

and where $\varepsilon_{h_i t}^H$ is the elasticity of the Hicksian (compensated) labour supply with respect to the tax rate. As pointed out by Håkonsen (1998), this means that the Diamond-based definition of MCF when $\tau = 0$ is exactly the same definition that Ballard and Fullerton (1992) labeled the Pigou-Harberger-Browning approach, which depends heavily on the compensated labour supply elasticity.

With the assumption that $a = 0$, we could simplify (50) and (51) to:

$$MCF_t^D = \frac{1-\delta_{\alpha y}}{\varepsilon_{Rt}+\rho_y}, \tag{58}$$

$$MCF_\tau^D = \frac{1-\delta_{\alpha c}}{\varepsilon_{R\tau}+\rho_c}. \tag{59}$$

First, consider (58). Because marginal utility of consumption is decreasing, it is reasonable to assume that the $cov(\alpha_i, y_i)$ is negative. Hence, the numerator is positive. The second term of the denominator, ρ_y, is negative because leisure is assumed to be a normal good. This means that MCF_t^D approaches infinity at a level of t which is to the left of the top of the Laffer-curve. Furthermore, for tax levels where $0 < \varepsilon_{Rt} < -\rho_y$, we have that MCF_t^D is negative although we are still along the upward sloping part of the Laffer-curve. Hence, for this set of tax levels it is difficult to see that the Diamond-based definition of MCF of an income tax provides any meaningful estimates of the costs of public funding.

Next consider (59), i.e. the case where $t = 0$ and a consumption tax is applied instead. We still assume that the numerator is positive. The second term of the denominator (ρ_c) is positive for obvious reasons. This means that MCF_τ^D is positive along the entire upward sloping part of the Laffer-curve. Moreover, MCF_τ^D is positive also for tax levels where $-\rho_y < \varepsilon_{R\tau} < 0$, which is along the downward sloping part of the Laffer-curve. This reveals that MCF_{τ}^{D} could be different from MCF_{t}^{D} for the same effective tax rate t^{*} also when $a = 0$. This is an undesirable property of the Diamond base MCF-measure, because tax-based funding of a public project in these two cases gives exactly the same values of p_i and M_i, and identical effects on all individuals' private consumption, labour supply, and utility. Hence, the MCF should have been identical in those two cases.

4 Numerical illustrations

To illustrate some of the results in the two preceding sections, a simulations of a numerical model with 10 individuals were carried out. For detailed description of the applied, model, see Appendix C. The utility functions were specified as CES-functions that include private consumption, leisure, and public consumption. The elasticity of substitution between leisure and private consumption was assumed to be 1.25, which means that labour supply is upward sloping.

Assuming that the lump-sum transfer a could be set at any preferred level, the second best solution is achieved with a lump-sum transfer $a = -3.3$ combined with an income tax $t = 0.25$. A lump-sum transfer $a = -4.4$ combined with a consumption tax $\tau = 0.33$ will give the same effective tax rate and transfer, cf. (7) and (8) and therefore also the same allocation of resources. With standard definition $MCF = 0.81$ and the Diamond-based definition, $MCF^{D} = 1$ in this second best solution.

Simulations were also carried out when the lump-sum transfer were assumed to be zero, see Figure 1. The horizontal axis measures public revenue as share of total income, i.e. the effective tax rate, see (7). The left vertical axis measures MCF while the right vertical axis measures total labour supply, welfare and public revenue.

The Laffer-curve is blue and peaks at the point where the share of income collected is equal to 0.85. The green curves represent the standard MCF-measure with respect to both an income tax and a consumption tax. In accordance with the results in section 2, the upper green curve increases towards infinity as we come close to the top of the Laffer-curve from the left. At the top of the Laffer-curve $\varepsilon_{Rt} = \varepsilon_{R\tau} = 0$. Thus, at this point the standard MCF-measure is not defined, see

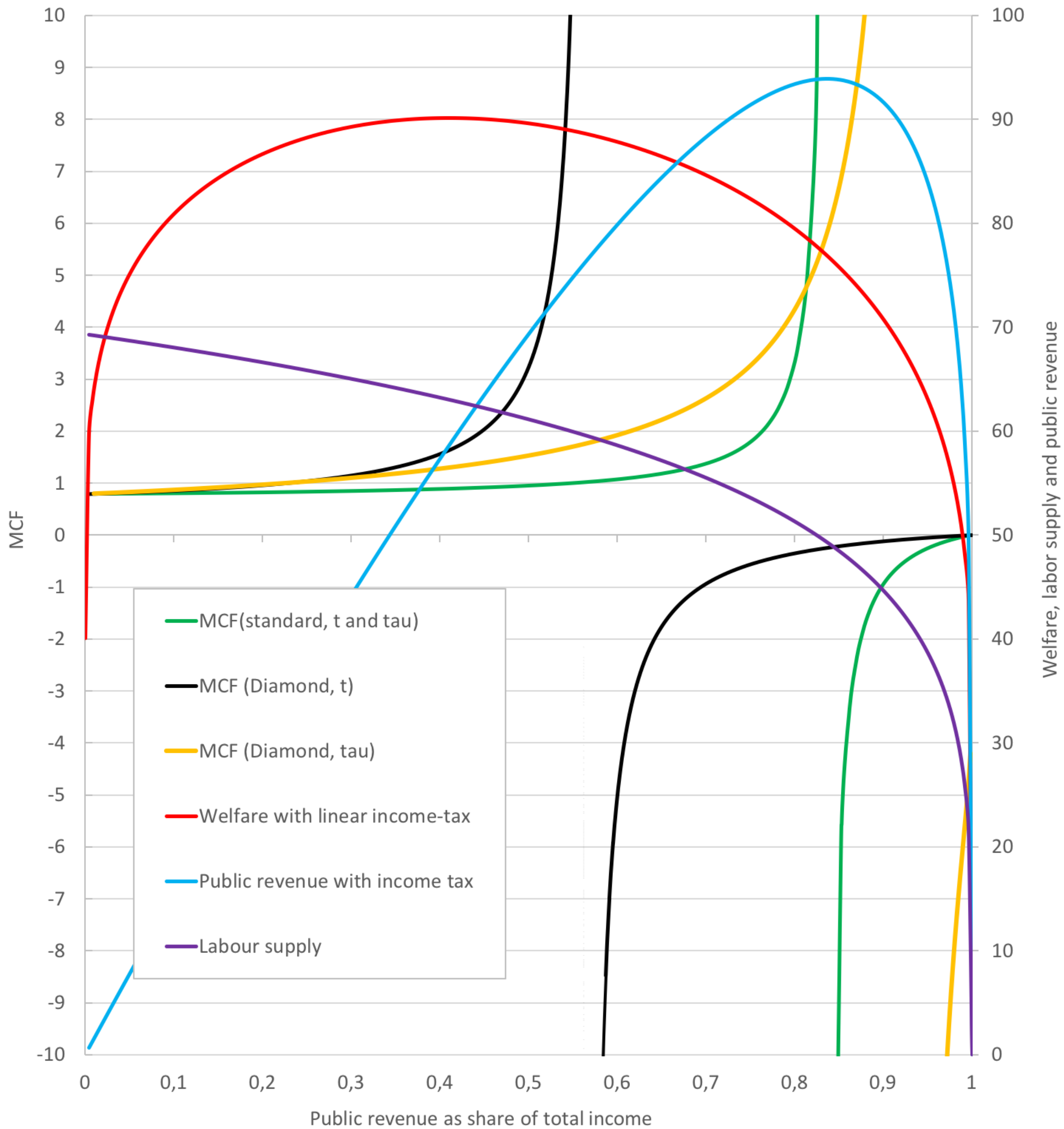

Figure 1: A numerical example with 10 individuals with different labour productivity factors. The horizontal axis measures the share of income collected by either an income tax or a consumption tax. The blue curve shows the collected revenue (the Laffer-curve). The red curve shows the aggregate utility of the consumers (welfare). The purple curve shows labour supply. The green curve shows the standard MCF-measure of both the consumption tax and the income tax. The black and the yellow curves show MCF with the Diamond-based definition and an income tax and a consumption tax, respectively. The dashed lines are asymptotes.

(39) and (40). To the right hand side of the Laffer-curve maximum, both ε_{Rt} and $\varepsilon_{R\tau}$ are negative. Hence, the standard MCF-measures become negative as well, see Figure 1.

The black curve represents the Diamond-based MCF-measure when an income tax is applied. In accordance with results in section 3, this MCF-measure approaches infinity at a point to the left of the Laffer-curve maximum (when t approaches 0.57 from below). Within the range where $0.57 \leqq t \leqq 0.85$, which belongs to the upward sloping part of the Laffer curve, the Diamond-based MCF with respect to the income tax is negative.

The yellow curve represents the Diamond-based MCF-measure when a consumption tax is applied. In accordance with the results in the previous section, the yellow curve does not coincide with the black curve, representing the Diamond-based MCF-measure with an income tax.

5 Discussion and conclusion

Jacobs (2018) found that the standard definition of MCF has some undesirable properties. First, it was considered as a shortcoming that MCF of lump-sum taxes are different from one even when they are optimized. Second, that MCF is not directly related to the marginal excess burden of taxation (MEB). Third, that the standard measure of MCF is found to be sensitive to the choice of the untaxed good.

In accordance with the first element of the critique of the standard MCF, the right hand side of (28) shows that if the income tax is positive, MCF of the lump-sum tax is less than one. The reason is that the lump-sum tax has no distortionary substitution effects, but has an income effect which draws in the direction of increased labour supply. If there is a positive income tax, which has caused an inefficiently low labour supply, increased labour supply represents an efficiency gain. This is an essential point of both Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974), and their critique of Pigou (1947). The same arguments apply to a lump-sum tax combined with a consumption tax, cf. (32) and Proposition 1. Hence, it might be argued that we are dealing with a strength with the standard definition of MCF, not a shortcoming.

With regard to the second element of the critique, that the standard MCF is not directly related to MEB, Jacobs (2018) argues that Pigou (1947), Harberger (1964), and Browning (1976) suggested such a relationship. It is outside the aspirations of this paper to discuss this question thoroughly. At this point I simply refer to the discussion in contributions such as Stiglitz and Dasgupta (1971), Atkinson and Stern (1974) and Ballard and Fullerton (1992).

The present paper analyzes in more detail the question of whether the standard MCF-measure is sensitive to the choice of the untaxed good. That result in Jacobs (2018) was not confirmed. On the contrary, it was found that MCF is the same in second best irrespective of the choice of the untaxed good. The reason was simply that MCF in the present paper was defined as the shadow cost of the public budget constraint divided by the average of the individuals' marginal utility of net-of-tax income. In the case with an income tax Jacobs (2018) applied the same definition of MCF. However, his MCF of a consumption tax was implicitly defined as the shadow price of the public budget constraint divided by the average of the individuals' marginal utility of gross income. This explains why he found that a standard MCF-measure is sensitive to the choice of the untaxed good.

Because Jacobs (2018) found that the standard MCF-measure was undesirable, another and less frequently used definition was suggested and applied. With this definition the shadow price of the public budget constraint was normalized with what Diamond (1975, p. 338) defined as the social marginal utility of private income.

The present paper studied some properties of this alternative definition of MCF in a case where the lump-sum transfer was set to zero. In that case public revenue collection with a consumption tax has exactly the same effects on distribution and resource allocation as revenue collection with an income tax. Hence, it would be reasonable to expect that an MCF-measure in this case is insensitive to the choice of the untaxed good also outside second-best optimum.

The present paper checked to what extent the two MCF-definitions passed this test. The standard definition passed the test as MCF with this definition is not influenced by the choice of the untaxed good also outside second best. In contrast, with the Diamond-based MCF-measure the size of MCF is sensitive to the choice of the untaxed good when we are no longer in second best.

The present paper found other reasons why the standard definition of MCF should be preferred to the Diamond-based definition. It was shown that with the Diamond-based definition less meaningful MCF-estimates could be the result also along the upward sloping part of the Laffer-curve.

Finally, Jacobs' conclusion that the marginal cost of public funds is one is based on an assumption that lump-sum-taxes could be a marginal source of public funding. This is in contrast to most earlier contributions, where the starting point for the discussion since the first contributions has been that the possibilities for lump-sum taxation is limited and therefore not should be considered as a marginal source of finance (Pigou, 1947; Stiglitz & Dasgupta, 1971; Diamond & Mirrlees, 1971; Atkinson & Stern, 1974). Jacobs agues that reducing for example tax credits could have the properties of lump-sum taxation. However, it is not specified what type of tax credits he has in mind. In practise it might be difficult to find good examples of tax credits with pure lump-sum properties. Hence, in practise the assumption that lump-sum taxation could be a marginal source for public funding might be too optimistic.

Appendices

A Proofs

Proof of Proposition 1. First, assume that the second best allocation of resources is achieved with a combination of a consumption tax $\hat{\tau}$ and a lump-sum transfer $\hat{a}$ (while $t = 0$). Then an income tax $\tilde{t} = \hat{\tau}/(1+\hat{\tau})$ combined with a lump-sum transfer $\tilde{a} = \hat{a}/(1+\hat{\tau})$ will also lead to be the second best allocation, cf. (7) and (8).

In the case where $t = 0$, while $\tau = \hat{\tau}$ and $a = \hat{a}$ we have from the budget constraint (3) that

$$\frac{\partial c_i}{\partial \hat{a}} = \frac{1}{1+\hat{\tau}}\left(1 + w_i \frac{\partial h_i}{\partial \hat{a}}\right). \tag{A.1}$$

Because $\tilde{a} = \hat{a}/(1+\hat{\tau})$, we have that

$$\frac{\partial h_i}{\partial \hat{a}} = \frac{1}{1+\hat{\tau}} \frac{\partial h_i}{\partial \tilde{a}}. \tag{A.2}$$

Plugging (A.2) into (A.1) gives that

$$\frac{\partial c_i}{\partial \hat{a}} = \frac{1}{1+\hat{\tau}}\left(1 + w_i \frac{1}{1+\hat{\tau}} \frac{\partial h_i}{\partial \tilde{a}}\right). \tag{A.3}$$

Inserting into equation (32) gives that when $t = 0$ we have that

$$MCF_a = \frac{1}{1 - \frac{\hat{\tau}}{1+\hat{\tau}} \frac{1}{n} \sum_i w_i \frac{\partial h_i}{\partial \tilde{a}}}, \tag{A.4}$$

which is equal to MCF_a as defined in (28) if $t = \hat{\tau}/(1+\hat{\tau})$.

Because leisure is assumed to be a normal good, $\partial h_i/\partial a < 0$. Then it follows from (A.4) that $MCF \leq 1$ irrespective of case considered. ■

Proof of Proposition 2. This proposition claims that if if $a = 0$ and $\tau =$

$t/(1-t)$, then $MCF_t = MCF_\tau$. This is true only if

$$\frac{1-\delta_{\lambda y}}{\varepsilon_{R\tilde{t}}} = \frac{1-\delta_{\lambda c}}{(1+\hat{\tau})\varepsilon_{R\hat{\tau}}}, \tag{A.5}$$

cf. (39) and (40).

First, define:

$$m = \frac{1-t}{1+\tau}. \tag{A.6}$$

When $a = 0$, $p_i = mw_i$ and $\mathrm{M}_i = mw_iT$. Then we have from (14) that $y = \sum_i w_i h_i(mw_i, mw_iT)$. We could then define the function

$$y = y(m, w_1, ..., w_n). \tag{A.7}$$

Starting with the denominator of the l.h.s. of equation (A.5) and the case where $a = \tau = 0$, then $R = ty$ and it follows that

$$\frac{\partial R}{\partial t} = y + \frac{\partial y}{\partial m}\frac{\partial m}{\partial t} = y - t\frac{\partial y}{\partial m}. \tag{A.8}$$

Thus, the denominator of the l.h.s. of equation (A.5) could be written as follows:

$$\varepsilon_{Rt} = 1 - \frac{t}{y}\frac{\partial y}{\partial m}. \tag{A.9}$$

Next, consider the denominator of the r.h.s. of equation (A.5) and the case where $a = t = 0$. Then we have that

$$R = \tau c = \frac{\tau}{1+\tau}y. \tag{A.10}$$

We have that

$$\frac{\partial y}{\partial \tau} = \frac{\partial y}{\partial m}\frac{\partial m}{\partial \tau} = -\frac{\partial y}{\partial m}\frac{1}{(1+\tau)^2}, \tag{A.11}$$

and it follows that

$$\frac{\partial R}{\partial \tau} = \frac{1}{(1+\tau)^2}y - \frac{\tau}{(1+\tau)^3}\frac{\partial y}{\partial m}. \tag{A.12}$$

Thus, we have that

$$(1+\tau)\varepsilon_{R\tau} = \left(1 - \frac{\tau}{1+\tau}\frac{1}{y}\frac{\partial y}{\partial m}\right). \tag{A.13}$$

Taking into account that $\tau = t/(1-t)$ and (A.9), it follows that the denominator of the l.h.s is equal to the denominator of the r.h.s. of equation (A.5).

Finally, consider the numerators of of equation (A.5). It follows that when $a = 0$, then $c_i = my_i$ and

$$cov(\lambda_i, c_i) = m \cdot cov(\lambda_i, y_i). \tag{A.14}$$

This means that $\delta_{\lambda y} = \delta_{\lambda c}$ if $a = 0$, cf. (30) and (34). Hence, the numerator of the l.h.s. is equal to the numerator of the r.h.s. of equation (A.5). ■

B The relationship between the Diamond-based MCF-measure and the PHB-tradition.

The Marshallian demand for leisure and supply of labour were defined in (13) and (14), respectively. Let $l_i^H = l_i^H(p_i, u_i)$ and $h_i^H = h_i^H(p_i, u_i)$ be the corresponding Hicksian demand for leisure and supply of labour, respectively. The Slutsky-equation gives that

$$\frac{\partial l_i}{\partial p_i} = \frac{\partial l_i^H}{\partial p_i} - l_i \frac{\partial l_i}{\partial M_i}, \tag{B.1}$$

which means that:

$$\frac{\partial h_i^H}{\partial p_i} = \frac{\partial h_i}{\partial p_i} + (T - h_i)\frac{\partial h_i}{\partial a}. \tag{B.2}$$

Assuming that $\tau = 0$, we have that

$$\frac{\partial h_i}{\partial t} = \frac{\partial h_i}{\partial p_i}\frac{\partial p_i}{\partial t} + \frac{\partial h_i}{\partial M_i}\frac{\partial M_i}{\partial t} \tag{B.3}$$

$$= -w_i \frac{\partial h_i}{\partial p_i} - \frac{\partial h_i}{\partial a} w_i T, \tag{B.4}$$

$$\frac{\partial h_i^H}{\partial t} = -w_i \frac{\partial h_i^H}{\partial p_i} \tag{B.5}$$

It follows that:

$$\frac{\partial h_i}{\partial p_i} = -\frac{1}{w_i}\frac{\partial h_i}{\partial t} - \frac{\partial h_i}{\partial a} T, \tag{B.6}$$

$$\frac{\partial h_i^H}{\partial p_i} = -\frac{1}{w_i}\frac{\partial h_i^H}{\partial t} \tag{B.7}$$

Plugging into (B.2) gives that:

$$\frac{\partial h_i^H}{\partial t} = \frac{\partial h_i}{\partial t} + w_i h_i \frac{\partial h_i}{\partial a}. \tag{B.8}$$

Equation (50) could be rewritten to:

$$MCF_t^D = \frac{1 - \delta_{\alpha y}}{1 + \frac{t}{y}\sum_i w_i \left(\frac{\partial h_i}{\partial t} + y_i \frac{\partial h_i}{\partial a}\right)}. \tag{B.9}$$

(56) then follows from (B.8) and (B.9).

C Description of the model used in the numerical example

In the numerical model used for illustrative purposes in section 4, there are $n = 10$ individuals with the following utility function:

$$u_i = x\left(\alpha^{1-\rho} c_i^\rho + \beta^{1-\rho} l_i^\rho + \gamma^{1-\rho} G^\rho\right)^{\frac{1}{\rho}} \tag{C.1}$$

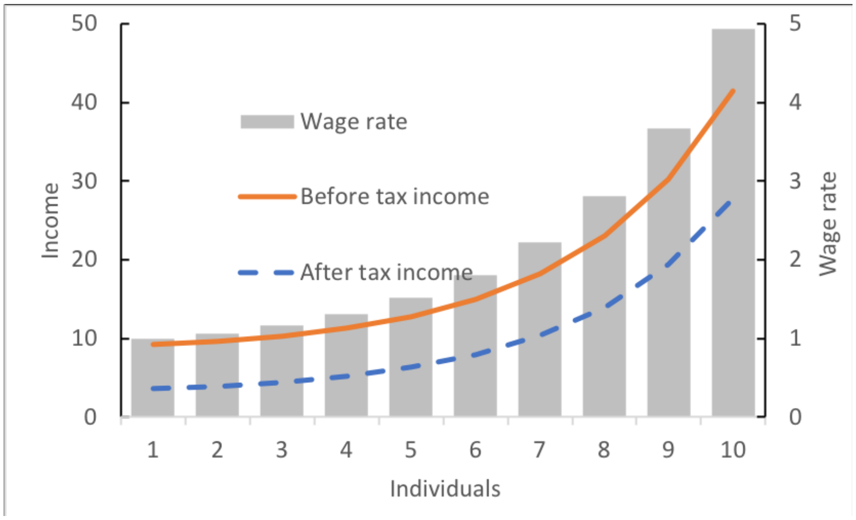

Figure 2: Wage rates w_i and gross and net incomes in second best, for $i = 1, ..., 10$.

where x, α, β, γ, and ρ are parameters. Define

$$s = \frac{1}{1-\rho}. \tag{C.2}$$

s is the elasticity of substitution between consumption and leisure and was set to 1.25. The parameters α and β where calibrated such that $\alpha^{1-\rho} = 0.3$ and $\beta^{1-\rho} = 0.7$.

The wage rates were determined such that:

$$w_1 = 1 \tag{C.3}$$

$$w_i = (w_{i-1})^g \quad \text{if} \quad i > 1, \tag{C.4}$$

were g is a parameter which was calibrated to a value that gave a Gini-index of gross income distribution of 27.5 in second best. Both the individual wage rates and individual before and after tax incomes in second best are shown in Figure 2.

The complete list of applied parameter values follows:

Parameter	Value	Parameter	Value
x	0.25	γ	0.10
α	0.22	β	0.64
ρ	0.20	s	1.25
T	24.00	g	1.03

Second best was achieved with an effective tax rate $\mathrm{t}^* = 0.25$ combined with an effective lump-sum transfer $a^* = -3.30$. Individuals spent between 8.4 and 9.2 time units working of total time endowments of $T = 24$ time units. An effective level $t^* = 0.41$ gave welfare maximum if the lump-sum transfer was set to zero.

References

Atkinson, A., & Stern, N. H. (1974). Pigou, taxation and public goods. *Review of Economic Studies*, *41*(1), 119-128.

Ballard, C. L. (1990). Marginal welfare cost calculations: Differential analysis vs. balanced-budget analysis. *Journal of Public Economics*, *41*(2), 263-276.

Ballard, C. L., & Fullerton, D. (1992). Distortionary taxes and the provision of public goods. *Journal of Economic Perspectives*, *6*(3), 117-131.

Browning, E. K. (1976). The marginal cost of public funds. *Journal of Political Economy*, *84*(2), 283-298.

Browning, E. K. (1987). On the marginal welfare cost of taxation. *The American Economic Review*, *77*(1), 11-23.

Browning, E. K., & Liu, L. (1998). The optimal supply of public goods and the distortionary cost of taxation: Comment. *National Tax Journal*, *51*(1), 103-116.

Christiansen, V. (1981). Evaluation of public projects under optimal taxation. *The Review of Economic Studies*, *48*(3), 447-457.

Christiansen, V. (2007). Two approaches to determine public good provision under distortionary taxation. *National Tax Journal*, *60*(1), 25-43.

Dahlby, B. (2008). *The marginal cost of public funds.* Cambridge, Mass.: MIT Press.

Dahlby, B., & Ferede, E. (2012). The effects of tax rate changes on tax bases and the marginal cost of public funds for canadian provincial governments. *International Tax and Public Finance*, *19*(6), 844-883.

Diamond, P. A. (1975). A many-person ramsey tax rule. *Journal of Public Economics*, *4*(4), 335-342.

Diamond, P. A., & Mirrlees, J. A. (1971). Optimal taxation and public production i: Production efficiency. *The American Economic Review*, *61*(1), 8-27.

Feldstein, M. (1999). Tax avoidance and the deadweight loss of the income tax. *The Review of Economics and Statistics*, *81*(4), 674-680.

Fullerton, D. (1997). Environmental levies and distortionary taxation: Comment. *The American Economic Review*, *87*(1), 245-251.

Håkonsen, L. (1998). An investigation into alternative representations of the marginal cost of public funds. *International Tax and Public Finance*, *5*(3), 329-343.

Harberger, A. C. (1964). The measurement of waste. *The American Economic Review*, *54*(3), 58-76.

Jacobs, B. (2018). The marginal cost of public funds is one at the optimal tax system. *International Tax and Public Finance*, *25*(4), 883-912.

Kaplow, L. (1996). The optimal supply of public goods and the distortionary cost of taxation. *National Tax Journal*, *49*(4), 513-533.

Kaplow, L. (1998). A note on the optimal supply of public goods and the distortionary cost of taxation:. *National Tax Journal*, *51*(1), 117-125.

Kleven, H. J., & Kreiner, C. T. (2006). The marginal cost of public funds: Hours of work versus labor force participation. *Journal of Public Economics*, *90*(10-11), 1955-1973.

Pigou, A. C. (1947). *A study in public finance* (Third Edition ed.). London: MacMillan.

Samuelson, P. A. (1954). The pure theory of public expenditure. *The Review of Economics and Statistics*, *36*(4), 387-389.

Sandmo, A. (1998). Redistribution and the marginal cost of public funds. *Journal of Public Economics*, *70*(3), 365-382.

Stiglitz, J. E., & Dasgupta, P. (1971). Differential taxation, public goods, and economic efficiency. *The Review of Economic Studies*, *38*(2), 151-174.

Stuart, C. (1984). Welfare costs per dollar of additional tax revenue in the united states. *The American Economic Review*, *74*(3), 352-362.

Trabandt, M., & Uhlig, H. (2011). The laffer curve revisited. *Journal of Monetary Economics*, *58*(4), 305-327.