Statistisk sentralbyrå
Statistics Norway

Search behavior, aggregate rationality and the discouraged worker effect

TALL SOM FORTELLER

John K. Dagsvik, Tom Kornstad and Terje Skjerpen

DISCUSSION PAPERS

906

Discussion Papers No. 906, April 2019
Statistics Norway, Research Department

John K. Dagsvik, Tom Kornstad and Terje Skjerpen

Search behavior, aggregate rationality and the discouraged worker effect

Abstract:

Discouraged workers are those who have given up search due to (perceived) low chances of obtaining work. In this paper we first develop a model for the probability of being in the labor force as a function of the probability of getting an acceptable job offer. This model is based on standard search theory and it is consistent with the notion of aggregate rationality. Second, we apply this model to analyze and compare the discouraged worker effect for two categories of Norwegian females: immigrants from non-Western countries and Norwegian-born. The empirical results show that estimated search cost (disutility) per unit of time is much higher for the Norwegian-born than for immigrants whereas total (expected) estimated search cost is – on average – higher for immigrants. We also propose an alternative measure of unemployment which includes the discourage worker effect.

Keywords: Discouraged workers; Modified unemployment rate; Aggregate rational expectations; Female immigrants; Labor force participation; Panel data; Random utility modeling

JEL classification: C33; C35; J21; J22; J61; J64

Acknowledgements: This research was made possible thanks to a grant from the Norwegian Research Council, under the project “Treff: Trygd i kontekst: Rettferdighet, effektivitet, fordeling”, project number 199836/H20. We are grateful for comments received at workshops at the Institute of Social Research, Oslo, the Research Department of Statistics Norway, and for comments by Ådne Cappelen, Arvid Raknerud, Steinar Strøm and Thor Olav Thoresen.

Address: John K. Dagsvik, Statistics Norway, Research Department. E-mail: john.dagsvik@ssb.no

Tom Kornstad, Statistics Norway, Research Department. E-mail: tom.kornstad@ssb.no

Terje Skjerpen, Statistics Norway, Research Department. E-mail: terje.skjerpen@ssb.no

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

© Statistics Norway
Abstracts with downloadable Discussion Papers
in PDF are available on the Internet:
http://www.ssb.no/en/forskning/discussion-papers
http://ideas.repec.org/s/ssb/dispap.html

ISSN 1892-753X (electronic)

Sammendrag

Et vanlig fenomen som observeres på mange arbeidsmarkeder er at arbeidstilbudet ser ut til å avhenge av konjunktursvingninger. Individer som søker arbeid synes å bli motløse under ugunstige konjunktursituasjoner fordi de tror at deres sjanser til å finne en akseptabel jobb er så små at kostnader og stress knyttet til jobbsøking oppveier de forventede fordelene med å søke. De potensielle arbeiderne trekker seg dermed ut av arbeidsmarkedet. Innen vårt rammeverk er en kvinne som ikke jobber, definert som motløs hvis hun ønsker å søke etter arbeid under veldig gode konjunkturforhold, men har trukket seg fra arbeidsstyrken under de aktuelle forholdene.

Vi studerer motløs arbeidereffekten separat for gifte/samboende kvinner født i ikke-vestlige land og i Norge med utgangspunkt i en modell basert på teorien for jobbsøking. Modellen spesifiserer hvordan sannsynligheten for å være i arbeidsstyrken avhenger av en parameter som representerer søkekostnad per tidsenhet samt sannsynligheten for å bli sysselsatt, gitt jobb søking. Modellen er konsistent med aktører (kvinner) som hver for seg kan avvike fra perfekt rasjonalitet når de vurderer verdien av jobbsøking, men som i gjennomsnitt tilfredsstiller restriksjoner som følger fra søketeori. Modellen benyttes til å analysere motløs arbeidereffekten separat for kvinnelige innvandrere og kvinner født i Norge ved hjelp av paneldata fra Arbeidskraftsundersøkelsene (AKU) for hvert kvartal fra andre kvartal 1988 til fjerde kvartal 2010.

Vi finner at estimert søkekostnad per tidsenhet er betydelig høyere for kvinner født i Norge enn for innvandrerkvinner fra ikke-vestlige land. En innvandrerkvinne med samme sannsynlighet for å skaffe seg arbeid som en kvinne født i Norge vil dermed ha større sjanse for å søke arbeid enn en kvinne født i Norge. Andelen motløse arbeidere er imidlertid for de fleste grupper mye høyere for innvandrerkvinner enn for kvinner født i Norge. Årsaken er at estimert total (forventet) søkekostnad (søkekostnad ganger forventet søketid) i gjennomsnitt er høyere for innvandrerkvinner enn for kvinner født i Norge.

Vi foreslår også et mål for ledighet som vi kaller modifisert arbeidsledighetsrate. Denne omfatter både registrerte arbeidsledige og motløse arbeidere. Eksempelvis er arbeidsledighetsraten for gifte eller samboende kvinner fra ikke-vestlige land om lag 17 prosent i 2005, mens den modifiserte arbeidsledighetsraten er om lag 25 prosent.

1. Introduction

A common phenomenon observed in many labor markets is that the supply of labor appears to depend on business cycles fluctuations. Workers who are searching for work seem to become "discouraged" under unfavorable business cycle conditions because they believe that their chances of finding an acceptable job are so small that the cost of searching for work outweighs the expected benefits from searching. The size of this effect is seen as depending on the expected search cost, which itself depends on the instantaneous search cost (search cost per unit of time) as well as on the chances of finding an acceptable job within a reasonable period of time. Search cost includes monetary as well as psychological "cost". In our setting a woman who does not work is defined as discouraged if she would like to search for work under "peak conditions" but has withdrawn from the labor force under the actual conditions. This notion is consistent with the definition used by many statistical agencies.[1] The discouraged worker effect is measured as the fraction of women who are discouraged. In this paper we analyze the discouraged worker effect and barriers to employment separately for women born in non-Western countries and women born in Norway.[2]

For economies in a boom, discouraged workers provide a hidden source of manpower since they participate to a larger extent in the labor market when chances of getting an acceptable job increase. In contrast, if the economy is in a recession, potential workers withdraw from the labor market and by doing so reduce observed unemployment. These workers are not captured by the standard measures of unemployment, as they are hard to identify. So, they can be considered as hidden unemployed people.

Structural analysis of the discouraged worker effect is of interest for several reasons. First, in several countries survey questionnaires include questions intended to measure the extent of the discouraged worker phenomenon. Typical survey data are, however, imprecise because they do not fully capture the conditions under which persons participating in the survey wish to search for work. As a result, direct measurement of the discouraged worker phenomenon might produce estimates that are difficult to interpret.[3] Second, in addition to measuring the actual (observed) discouraged worker effect, it might also be of interest to assess the level of this effect in hypothetical settings: that is, under conditions different from those that give rise to the observed business cycles. Third, it is also of considerable interest to analyze how the discouraged worker effect varies by key determinants such as

[1] For example, the US Bureau of Labor Statistics defines discouraged workers as "persons not in the labor force who want and are available for a job and who have looked for work sometime in the past 12 months, but who are not currently looking because they believe there are no jobs available or there are none for which they would qualify."

[2] Occasionally we use the term "immigrant women" as a synonym for "women born in non-Western countries".

[3] The Norwegian Labor Force Surveys also collect direct information on the discouraged worker effect, but the figures from the survey are not presented in the official statistics. One reason might be that the number of individuals providing this information is rather small as only individuals participating in the survey for the first or eighth time are asked the question.

wage rates and socioeconomic covariates. In order to address these three issues a structural approach is called for.

In this paper we propose a new measure related to barriers in the labor market that captures both the observed and hidden (discouraged) unemployed. This measure – which we refer to as the modified unemployment rate – thus captures the total effect of barriers and search costs in the labor market on actual employment. The modified unemployment rate is the (potential) number of women in the labor force under peak conditions minus the number of employed women under the actual conditions divided by the number of women in the labor force under peak conditions.

The motivation for comparing immigrant women and women born in Norway is that the level of employment is observed to be significantly lower among immigrant women than among women born in Norway and it is of interest to examine why. Typically, two explanations have been offered for this phenomenon. The first is that women might decide to search for work but fail to find an acceptable job because of barriers to employment. In Norway many immigrant groups seem to experience particular difficulties when searching for work and the unemployment rate is significantly higher among them than among women born in Norway, in particular when the economy is in a recession. The second explanation is related to differences in cultural background. Many immigrant women come from societies where women often do not participate in paid work. In addition, there is a third explanation, namely the discouraged worker effect. The discouraged worker phenomenon has not, in our view, received the attention it deserves in the public debate. Women may decide voluntarily to stay outside the labor market in economic downturns because the psychological as well as the monetary costs of searching for work are higher than the expected pay-off from continuing their search. According to our findings, the discouraged worker effect is particularly important among immigrant women, but, to a minor extent, also among women born in Norway with low educational qualifications.

In order to formulate a model for labor force participation we apply a particular search theoretic approach. Our approach is motivated by the fact that application of the standard search theory in empirical contexts is problematic to apply due to identification problems and unobserved heterogeneity in preferences and job opportunities. Flinn and Heckman (1982) and Heckman and Singer (1984) have shown that standard search theoretic models are fundamentally underidentified. Our model allows for unobserved heterogeneity in preferences, search costs and distribution of the value of job offers. Moreover, it is consistent with agents that may each deviate from perfect rationality when assessing their value of search, but satisfy the restrictions that follow from the standard search model *on* *average*.[4] From our theoretical approach we obtain an empirical (identified) model for the probability of participating in the labor force as a function of the probability of receiving an acceptable job offer. Based on this model, we use micro data to analyze labor force participation and barriers in the labor market separately for married/cohabiting non-Western female immigrants and similar women born in Norway.

Several studies of the discouraged worker phenomenon are based on macro data (Ehrenberg and Smith, 1988).[5] Empirical studies based on micro data include Ham (1986), Blundell et al. (1987, 1998), Connolly (1997), Başlevent and Onaran (2003), Bloemen (2005), Hotchkiss and Robertson (2006), and Dagsvik et al. (2013). Dagsvik et al. (2013) analyzed the discouraged worker effect among women living in Norway without controlling for their immigration status. The present study adds to the evidence from that study by analyzing the discouraged worker effect for female immigrants and females born in Norway, separately, using micro panel data from the Norwegian Labor Force Surveys (LFS) over a fairly long period of time: that is, for each quarter from the second quarter of 1988 to the fourth quarter of 2010. As opposed to most other studies in this field, including Dagsvik et al. (2013), we apply panel data with two observations for each individual. This type of data enables us to obtain more reliable estimates compared to studies that use only repeated cross-section data, because panel data provide observations on actual individual transitions in the labor market and therefore represent information on how individuals actually adjust behavior as a result of changes in observed and unobserved incentives and variation in preferences. However, the data do not contain information about search durations.

An interesting finding is that the estimates of the parameter representing search cost per unit of time is much higher for women born in Norway than for immigrant women. An immigrant woman facing the same probability of obtaining work as a woman born in Norway is then less likely to be discouraged from searching for work than a woman born in Norway. However, the fraction of discouraged workers is, for most groups, much higher for immigrant women than for women born in Norway. The reason is that the estimated total (expected) cost of search is, on average, found to be higher for immigrant women than for women born in Norway. The total expected cost is the cost per unit of time times the expected duration of search until an acceptable job offer arrives. Since the expected duration of search is typically found to be substantially longer for immigrant women than for women born in

[4] Thaler (2015), among others, has demonstrated in a number of studies that individuals only to a limited extent seem to behave rationally according to theory (see also Conlisk, 1996). In particular, the laboratory experiments analyzed in Brown et al. (2011) cast serious doubt about the ability of agents to assess a constant reservation wage in a stationary search environment.

[5] Some recent studies that have analyzed the discouraged worker issue using macro time series data are Benati (2001), Darby et al. (2001), Vendrik and Cörvers (2009), Österholm (2010), Emerson et al. (2011), Fuchs and Weber (2017) and Provenzano (2017). All these studies find significant discouraged workers effects, at least for subgroups in their samples.

Norway, the expected cost of search among immigrant women is higher than for women born in Norway.

The model is estimated on a sample that is not fully representative for the whole population. In order to use the model to simulate representative levels of labor force participation, employment and discouraged worker effect for the target population (married or cohabiting women), we have established a comprehensive micro population for 2005 with information about all the explanatory variables in the model.

Whereas the unemployment rate (across time and population groups) for married or cohabiting women from non-Western countries is about 17 per cent we find that the corresponding modified unemployment rate is about 25 per cent. For married or cohabiting women born in Norway the unemployment rate is about 3 per cent and the modified unemployment rate is about 5 per cent. For married or cohabiting women from non-Western countries who migrated to Norway less than 5 years ago, have length of schooling less than 13 years and have age less than 35 years the unemployment rate is about 33 per cent and the modified unemployment rate is about 53 per cent. These figures show that the discouraged worker effect is substantial. They also clearly indicate that it is important to account for the discouraged worker effect when discussing barriers in the labor market.

The paper is organized as follows. In Section 2 we define formally the notion of discouraged worker and the modified unemployment rate. In Section 3 the search-theoretic framework is developed. In Section 4 the empirical model is derived. Data issues are the topic of Section 5. In Section 6 we provide estimation results and assessment of fit. Finally, in Section 7 we formally define the concepts discouraged worker effect and modified unemployment rate and then quantify them for immigrant women and for women born in Norway, respectively.

2. Discouraged workers and modified unemployment

The purpose of this section is to formalize and make precise the concepts of discouraged workers and modified unemployment. For simplicity, the indexation of time and individual is suppressed in this section. To this end, let q be the conditional probability that the agent shall be employed given that the agent belongs to the labor force. The empirical counterpart of q is one minus the unemployment rate. Let $P(q)$ be the probability of being in the labor force. That is, $P(q)$ is the supply of labor (at the extensive margin). As mentioned in the introduction, it is commonly observed that the fraction of agents in the labor force seems to vary with the business cycle. That is, $P(q)$ seems to be an increasing function of q. In the subsequent sections we shall discuss a theoretical approach that will lead to a particular model for $P(q)$. For now, we simply assume that such a model has been established.

We define the peak condition in the labor market as a situation in which $q = q^0$ where q^0 is a reference conditional probability of being employed, given labor force participation, under the most favorable business cycle conditions for the workers. The probability of being a discouraged worker is given by $P(q^0) - P(q)$. That is, a woman is discouraged if she does not belong to the labor force under the current conditions but would prefer to enter the labor force under peak conditions. This notion of discouraged worker is consistent with the informal definition given in the introduction. The modified unemployment rate is defined by $(P(q^0) - P(q)q) / P(q^0)$. The empirical counterpart of the modified unemployment rate is the (potential) number of women in the labor force under peak conditions minus the number of employed women under the actual conditions divided by the number of women in the labor force under peak conditions. This measure then captures both ordinary observed unemployment and the discouraged worker effect. In empirical applications, one may define the reference level q^0 as the highest value of all the estimated acceptable job offer arrival rates (across population groups and across time).

Thus, in order to calculate modified unemployment rates, it is necessary to establish a model for the probability of being in the labor force, as a function of the conditional probability of being employed given participation in the labor force. This is the goal of the next section.

3. Modeling labor force participation

This section discusses our approach to characterizing the probability of being in the labor force as a function of the conditional probability of being in the labor force given participation. To this end, we first address the problem of characterizing the decision rule of whether to be in the labor force or not. Our approach departs from the standard one state job search model. In order to clarify the difference between our approach and the standard search-theoretic approach, we start by reviewing the standard search model.

3.1. The standard search model

In one version of the standard search model, the agent is assumed to operate in a stationary environment where job offers arrive according to a Bernoulli process in discrete time. Let U_1 be the present value of search, *K* an index that equals 1 if a job offer is arriving in the period and zero otherwise, U_2 the utility of the arriving job offer, κ the discount factor and *C* the real cost of search per unit of time, respectively. The agent is uncertain about which job offers arrive and when they arrive. However, she is supposed to know *C*, the job offer arrival rate and the distribution function of the utilities of the arriving job offers. By applying Bellman's optimality principle (Lippman and McCall, 1976), it follows that

$$U_1 = \kappa E^s(\max(U_1, KU_2/(1-\kappa))) - C, \tag{1}$$

where E^s denotes the expectation operator conditional on the information of the agent. The first term on the right-hand side of the equation is the discounted expected value of search. In principle, one can solve (1) for U_1 (the reservation value) as a function of κ, *C*, the expected job offer arrival rate $E^s K$ and the distribution function of U_2. In the particular case where $\kappa = 1$ an optimal policy still exists where now U_2 is interpreted as the lump sum value of the job offer over the infinite horizon. The value of search in this case is determined by

$$U_1 = E^s \max(U_1, KU_2) - C \tag{2}$$

(Flinn and Heckman, 1982). This simple job search model has been generalized in a number of ways, see Rogerson et al. (2005).

The application of (1) or (2) in empirical analyses poses several problems. As discussed by Heckman and Singer (1984) the search model described above is not identified even if there are no regressors or unobservables in the model. Even if data on accepted wages (when U_2 equals the wage rate) were available (which is typically not the case) the model is not identified unless the c.d.f. satisfies a recoverability condition (Heckman and Singer, 1984). This is due to the fact that the econometrician does not know the discount rate, the search cost, the job offer arrival rate and the distribution function of U_2. Specifically, the discount rate, search cost and the distribution of U_2 may vary across agents both with respect to observed and unobserved individual characteristics. Furthermore, perfect rational behavior, as represented by (3.1) or (3.2), may not hold, because, as indicated, the agent may have difficulties assessing the precise value of search– as represented by the equations above.

3.2. Unobserved heterogeneity and aggregate rationality

The analysis above is relevant for a given agent and an environment with only two states, "employed" and "unemployed". Now we consider a setting with three states, namely "out of the labor force", "unemployed" and "employed". Moreover, in our setting we have to deal with a population of agents where both preferences and job opportunities are heterogeneous. In Section 2.1 we mentioned that the standard search model is not identified. However, for our purpose we are not interested in identifying all the components of the search model. Specifically, it suffices to obtain a characterization of the value of being unemployed as a function of the probability of being employed given labor force participation. As we shall show in this section, this allows us to work with weaker assumptions than those of the standard approach, which will lead to a structural model that is useful for empirical analysis and will eventually enable us to calculate modified unemployment rates. In this case it is necessary to introduce indexation of time periods. The individual agent may either be out of the labor force, searching for work, or employed. The utility of being out of the labor force is denoted U_0. The utility of a job offer, U_2, will depend on the corresponding offered wage rate, but also on non-pecuniary attributes of the job offer, such as location, tasks to be performed, etc. We maintain the assumption that the agent operates as if she were in a stationary environment. Thus, although the utilities and the arrival rate of job offers may change over time periods, the agent is supposed to assess the value of search as if she were in a stationary environment, conditional on the available information at the current period. We also assume that an unemployed woman is only capable of taking into account the expected value of future job offers and ignores the possibility of withdrawing from the labor force and the possibility of being laid off in the future when calculating the value of search.

Assumption 1

The variables U_0 *and* U_2 *have the following structure:*

$$U_0 = u_0 + Z_0 + \varepsilon_0 \quad \text{and} \quad U_2 = u_2 + Z_2 + \varepsilon_2,$$

where $\{u_j\}$, j = 0,2, *are deterministic functions that depend on selected covariates and* $\{\varepsilon_0, \varepsilon_2\}$ *are zero mean random unobserved variables that are independent across time. Furthermore,* Z_0 *and* Z_2 *are i.i.d. zero mean random effects that are known to the agent and independent of* $\{\varepsilon_0, \varepsilon_2\}$.

The random effects Z_0 and Z_2 are supposed to capture unobservables that do not change over time. Specifically, Z_2 may capture systematic (unobservable) aspects of the job offers associated with the qualifications of the agent that are perfectly known to her. The random error terms ε_0 and ε_2 are supposed to capture the effect of unobservables that vary randomly over time. The assumption that the error terms $\{\varepsilon_0, \varepsilon_2\}$ are serially independent does not seem overly restrictive since unobservables that are permanent are captured by the random effects.

Assumption 2

The agents operate with discount factor κ that is equal to 1.

Under Assumption 2 it follows that the optimal policy is determined by (2). The agent is supposed to assess the (subjective) distribution of ε_2 conditional on unobservable factors that are known to the agents. However, she is allowed to make mistakes when evaluating the value of search. This means that instead of (3.2) we now assume that the following equation holds

$$U_1 = E^s \max(U_1, KU_2) - C + \Lambda \tag{3}$$

where Λ is an error term which represents the deviation from perfect rationality in the agent's evaluation of the value of search. In this setting the search cost per unit of time, or disutility *C*, is supposed to capture both economic as well as psychological costs and stress associated with job search.

Assumption 3 (Aggregate rational expectations)

Aggregate rational expectation holds in the sense that $E^p(\Lambda \mid Z_2) = 0,$ where E^p is the population expectation operator. Furthermore, the search costs are independent of the random effects and the job offer arrivals.

Assumptions 2 and 3 are motivated by bounded rationality. By the law of iterated expectations, we obtain that

$$E^p E^s(\max(U_1, KU_2) \mid Z_2) = E^p(\max(U_1, KU_2) \mid Z_2).$$

As a result, it follows from Assumption 3 by taking expectations on both sides of the equation in (3) that

$$E^p(U_1 \mid Z_2) = E^p(\max(U_1, KU_2) \mid Z_2) - E^p(C \mid Z_2). \tag{4}$$

Let $\lambda = E^p K,$ that is, λ is the probability (objective) of a job arrival in a period. Since by assumption the job offers arrive according to a Bernoulli process we have that

$$E^p(\max(U_1, KU_2) \mid Z_2) = (1-\lambda)E^p(U_1 \mid Z_2) + \lambda E^p(\max(U_1, U_2) \mid Z_2).$$

The above equation implies that

$$E^p(U_1 \mid Z_2) = E^p(\max(U_1, U_2) \mid Z_2) - \frac{c}{\lambda} \tag{5}$$

where $c = E^p(C \mid Z_2) = E^p C.$ Recall that, although suppressed in the notation here, the systematic terms λ, u_0 and u_2 may change over time periods (year), as indicated above. The relationship in (5) means that although the optimal search equation might not hold at the individual level, it holds on average within population groups with the same value of the random effect Z_2 and conditional on selected observed covariates (suppressed in the notation here). In this sense our approach can be viewed as a version of bounded rationality that relaxes the strict rationality assumption represented in standard search theory.

Above, it is implicit that the population expectation operator E^p is understood to be a conditional operator given observed covariates to be introduced later. There are two sources of randomness that affect the random variation in U_1. First, U_1 depends on the agent's subjective distribution of U_2 that may vary in a seemingly random manner. The variations in this distribution are due to the agent's inability to assess the distribution of the utilities of future job offers precisely. Second, U_1 depends on Λ which also may fluctuate over time in a random way. Variations in Λ are due to the agent's inability to calculate precisely the value of search given the job arrival rate, the subjective distributions of U_2 and the individual search costs. The error term Λ may also capture possible error in the agent's assessment of the arrival rate of job offers. Without further assumptions one cannot say more about the properties of U_1. Even with distributional assumptions about U_2 one cannot determine the distribution of U_1.

Assumption 4 (probabilistic rationality)

The error terms $\theta\varepsilon_j, j = 0,1,2,$ *are independent with Gumbel distribution* $\exp(-e^{-x})$ *where* $\theta > 0$ *is a dispersion parameter.* [6]

Assumption 4 is consistent with the following version of Luce Choice Axiom, (Luce, 1959) (equivalent to the Independence from Irrelevant Alternatives assumption, IIA). In order to give a brief explanation of the Choice Axiom, let Ω be a set consisting of the alternatives "working", "searching for work" and "out of the labor force", let *B* be a set, $B \subseteq \Omega,$ where *B* contains at least two alternatives and let $J(B)$ denote the most preferred alternative in *B*. Consider the relation

$$P(J(\Omega) = j \mid J(\Omega) \in B, \{Z_0, Z_2\}) = P(J(B) = j \mid \{Z_0, Z_2\})$$

[6] Remember that the Gumbel c.d.f. with zero mean has the form $\exp(-\exp((0.5772 - x)/\theta))$, where 0.5772 is Euler's constant and θ is a positive scale parameter.

for $j \in B \subseteq \Omega$. This relationship states that the choice of an alternative in the choice set Ω given that the most preferred alternative belongs to a subset *B*, is, on average, the same as making the choice from the set *B*. In other words, given that the most preferred alternative belongs to *B* then the alternatives in $\Omega \setminus B$ become irrelevant (on average). Luce (1977) refers to the assumption expressed in Assumption 4 as a *probabilistic rationality* postulate.[7] In other words, IIA is an aggregate rationality postulate. The IIA assumption can be restrictive in cases with unobserved attributes that are common for some alternatives. In our case where IIA is only supposed to hold conditional on the random effects Z_0 and Z_2 where the latter variable is common to alternatives 1 and 2, IIA seems reasonable.

Theorem 1

If Assumptions 1 to 4 hold, then the mean utility of search is determined by

$$u_1 \equiv E^p(U_1 \mid Z_2) = u_2 + Z_2 - \theta^{-1}\log[\exp(\theta c/\lambda) - 1] \qquad (6)$$

where $\theta = \pi / \sqrt{6 Var \varepsilon_j}$.

The proof of Theorem 1 is given in Appendix A. The result in Theorem 1 shows that one can express the mean value of search explicitly in terms of the mean value of job offers and c/λ. Note that λ^{-1} has the interpretation as the inter-arrival mean time between job offers. Hence, c/λ is the inter-arrival mean cost of search. Theorem 1 thus states that the average value of searching depends on the inter-arrival mean search cost. We also note that the relation in (6) has the intuitively reasonable property that when $c/\lambda \to 0$, then $u_1 \to \infty$. Thus, Theorem 1 yields a complete characterization of the value of search. Moreover, our theoretical approach has led to an essential simplification in that the average value of search can be expressed as a rather simple function of u_2, Z_2 and c/λ.

Unfortunately, the result in (6) is not directly applicable empirically because the job arrival probability λ is not observed. What is observed is the flow from unemployment to employment, and vice versa, which is the empirical counterpart of the probability of receiving an acceptable job offer conditional on job search. Let q denote the probability of receiving an acceptable

---

[7] In his conclusion, Luce (1977) characterizes the choice axiom as follows: "Perhaps the greatest strength of the choice axiom, and one reason it continues to be used, is as a canon of probabilistic rationality. It is a natural probabilistic formulation of K. J. Arrow's famed principle of the independence of irrelevant alternatives, and as such it is a possible underpinning for rational, probabilistic theories of social behavior. Thus, in the development of economic theory based on the assumption of probabilistic individual choice behavior, it can play a role analogous to the algebraic rationality postulates of the traditional theory."

job offer, given search. For the sake of interpretation, consider the distribution of the duration of search. Let T denote the total duration of search. It is easy to prove that T is distributed according to a geometric distribution which has the property that $ET = q^{-1}.$ Hence, whereas c is the expected (real) cost of search per unit of time, cq^{-1} can be interpreted as the total expected cost of search. Under the assumptions of Theorem 1 and the fact that $\theta\varepsilon_2 - \theta\varepsilon_1$ has a logistic c.d.f. that it follows readily that

$$P(U_2 > U_1) = P(u_2 + Z_2 + \varepsilon_2 > u_1 + \varepsilon_1) = \frac{1}{1+\exp(\theta u_1 - \theta u_2 - \theta Z_2)} = 1 - \exp(-\theta c / \lambda).$$

Consequently, since $q = \lambda P(U_2 > U_1)$ we obtain that

$$\frac{\theta c}{q} = \frac{\theta c}{\lambda P(U_2 > U_1)} = \frac{\theta c / \lambda}{1 - \exp(-\theta c / \lambda)} \tag{7}$$

which shows that $\theta c / q$ is determined by $\theta c / \lambda.$ Moreover, we have the following result.

Corollary 1

Under Assumptions 1 to 4 the mean value of search can be expressed as

$$u_1 = u_2 + Z_2 - \theta^{-1} h(\theta c / q),$$

where h is a function defined on $(0,\infty)$ *that is strictly increasing, concave and uniquely determined by the equation*

$$(1+\exp(-h(x)))\log(1+\exp(h(x))) = x.$$

Furthermore, when x increases h *approaches the identity mapping.*

The proof of Corollary 1 is given in Appendix A. The result of Corollary 1 is particularly interesting since it shows that under probabilistic rationality (Assumption 4) in addition to Assumptions 1 to 3 the function *h* is strictly increasing, concave and uniquely determined.

3.3. Labor force participation

We shall now discuss the implications of our theory for the probability of labor force participation and the discouraged worker effect. First, we need to consider the distributional properties of the random effects. From Assumptions 1 to 3 and Corollary 1 we obtain that the conditional probability that the agent will be in the labor force, given the random effects Z_0 and $Z_2,$ is equal to

$$P(U_1 > U_0 \mid Z_0, Z_2) = \frac{1}{1+\exp(-\theta(u_1 - u_0) + \theta Z_0))} \tag{8}$$

$$= \frac{1}{1 + \exp(-\theta u_2 + h(\theta c / q) + \theta u_0 - \theta Z_2 + \theta Z_0))}.$$

To characterize the distributional properties of the random effects we make the following assumption:

Assumption 5

The distribution of the random effect $\theta Z_2 - \theta Z_0$ *satisfies*

$$E\left(\frac{1}{1 + \exp(-v - \theta Z_2 + \theta Z_0)}\right) = \frac{1}{1 + \exp(-\alpha v)}$$

for any real v where $0 < \alpha \le 1$.

Assumption 5 asserts that the main difference between the unobservables represented by the error terms $(\varepsilon_0, \varepsilon_1)$ and the unobservables represented by the random effects is that $Z_2 - Z_0 + \varepsilon_1 - \varepsilon_0$ only differs from the distribution of $\varepsilon_1 - \varepsilon_0$ by a scale transformation which accounts for the fact that the variance of $Z_2 - Z_0 + \varepsilon_1 - \varepsilon_0$ is greater than the variance of $\varepsilon_1 - \varepsilon_0$. As a result, aggregation of the conditional choice probability above with respect to the random effects produces an unconditional choice probability of the same form, apart from a scale transformation of the precision parameter θ.

The invariance under aggregation property expressed in Assumption 5 is, however, not essential for our empirical analysis. An alternative would be to assume normally distributed random effects. Dagsvik (2018) has proved that Assumption 5 holds if and only if the distribution of $\theta Z_2 - \theta Z_0$ has p.d.f. $f_\alpha(z)$ defined on $(-\infty, \infty)$ that is given by

$$f_\alpha(z) = \frac{1}{\alpha\pi} \cdot \frac{\sin(\alpha\pi)}{e^{\alpha z} + e^{-\alpha z} + 2\cos(\alpha\pi)} \tag{9}$$

where the parameter α is related to the variance of $\theta Z_2 - \theta Z_0$ by

$$Var(\theta Z_2 - \theta Z_0) = \left(\alpha^{-2} - 1\right)\frac{\pi^2}{3}. \tag{10}$$

To make this paper self-contained Dagsvik's proof of (9) is also given in Appendix B.[8] The p.d.f. $f_\alpha(z)$ is symmetric around zero and has variance that increases without bounds as $\alpha \to 0$. The distribution $f_\alpha(z)$ is similar to the normal distribution but has heavier tails than the normal

---

[8] The result in (11) is related to results obtained by Cardell (1997). He proves that if the random effect is distributed as in (9) then Assumption 4 holds.

distribution. Moreover, it follows that the autocorrelation function of the error term process $\{Z_2 - Z_0 + \varepsilon_1 - \varepsilon_0\}$ is given by $1-\alpha^2$. The main advantage with the distribution $f_\alpha(z)$ is that it implies that the participation probability can be expressed on closed form by

$$P(U_1 > U_0) = \frac{1}{1+\exp(-\alpha(\theta u_2 - h(\theta c / q) - \theta u_0))}. \qquad (11)$$

4. Empirical model

In this section it is convenient to introduce indexation of both individuals and time. Let q_{it} denote the probability that individual *i* will be employed given participation in the labor force in year *t*. From Corollary 1 it follows that the function $h(x) \cong x$ when $x > 5$. Preliminary estimation results indicate that θc is of order of magnitude between 5 and 6 for immigrant women and between 28 and 29 for women born in Norway. Since $q_{it} < 1$ this implies that $\theta c / q_{it} > 5$ for immigrant women and $\theta c / q_{it} > 28$ for women born in Norway. Thus we can safely write[9]

$$h(\theta c / q_{it}) \cong \theta c / q_{it}.$$

(12)

Hence, according to Corollary 1 we can express the utility of search as

$$\theta U_{i1t} = \theta u_{i2t} + \theta Z_{i2} - \theta c q_{it}^{-1} + \varepsilon_{i1t}. \qquad (13)$$

Assumption 6

The systematic term of utility of the job offers is given by

$$u_{i2t} = E \log W_{it}$$

where W_{it} *is the agent-specific wage rate. The wage rate is modeled by*

$$\log W_{it} = \beta_{0t} + X_{it}\beta + \mu_i + \delta_{it}$$

(14)

where the intercept β_{0t} *may depend on time,* X_{it} *is a vector of covariates (given in Table C1 in Appendix C),* μ_i *is a random effect with zero mean and* $\{\delta_{it}\}$ *are serially uncorrelated random terms with zero mean. The systematic part of the utility of being out of the labor force is given by*

$$u_{i0t} = V_{it}\gamma \qquad (15)$$

[9] The estimation results do not depend critically on the assumption that $h(x) \cong x$. It is easily realized that the estimation results are consistent with a linear approximation of *h*.

where V_{it} is a vector of covariates (specified in Table 2). Furthermore, (13) holds where q_{it} is represented by a logit function depending on a vector of covariates (which are specified in Table C2 in Appendix C).

The wage equation in (15) is used to predict wage rates for women who do not work when estimating the model for labor force participation. The wage equation is also used to predict wages for those who work.[10] Hence, if follows from (14) and (15) that

$$u_{i2t} = \beta_{0t} + X_{it}\beta. \tag{16}$$

Since the number of observations in the respective subgroups of individuals in the labor force surveys is rather small, a (reduced form) logit model for q_{it} is introduced in order to obtain more reliable predictions of the probabilities of obtaining an acceptable job conditional on particular covariates.

Let $\tilde{Z}_i = \theta Z_{2i} - \theta Z_{0i}$. From (8), (10), (12) to (15) it follows that

$$\tilde{P}_{it}(\tilde{Z}_i) \equiv P(U_{i1t} > U_{i0t} \mid \tilde{Z}_i) = \frac{1}{1+\exp(-\theta(\beta_{0t} + X_{it}\beta - V_{it}\gamma) + \theta c q_{it}^{-1} - \tilde{Z}_i)}.$$

(17)

Furthermore, it follows from (2.17) that[11]

$$P_{it} \equiv E\tilde{P}_{it}(\tilde{Z}_i) = P(U_{i1t} > U_{i0t}) = \frac{1}{1+\exp(-\alpha\theta(\beta_{0t} + X_{it}\beta - V_{it}\gamma) + \alpha\theta c q_{it}^{-1})}.$$

(18)

Our sample has a rotating panel structure. In order to express the likelihood function, let $Y_{it}=1$ if the woman is in the labor force in year *t* and zero otherwise. Consequently, we can write the log-likelihood function as

$$L = \prod_i E\left(\prod_t \tilde{P}_{it}(\tilde{Z}_i)^{Y_{it}} (1-\tilde{P}_{it}(\tilde{Z}_i))^{1-Y_{it}} \tilde{P}_{i,t-1}(\tilde{Z}_i)^{Y_{i,t-1}} (1-\tilde{P}_{i,t-1}(\tilde{Z}_i))^{1-Y_{i,t-1}} \right) \tag{19}$$

where the expectation operator is taken with respect to $\tilde{Z}_i$. To calculate (4.9) we apply a Monte Carlo simulation approach. That is, we approximate *L* by $\tilde{L}$ given by

[10] In the predictions of $\log W_{it}$, the random effects are set to zero.

[11] It is of interest to note that the relations in (17) and (18) are similar to macro time series analyses where relative changes in labor force participation (or the corresponding log-odds) is specified as a linear function of unemployment $(1 - q_t)$ and other variables.

$$\tilde{L} = \prod_i M^{-1} \sum_{r=1}^{M} \prod_t \left(\tilde{P}_{it}(\tilde{Z}_{ir})^{Y_{it}} (1-\tilde{P}_{it}(\tilde{Z}_{ir}))^{1-Y_{it}} \tilde{P}_{i,t-1}(\tilde{Z}_{ir})^{Y_{i,t-1}} (1-\tilde{P}_{i,t-1}(\tilde{Z}_{ir}))^{1-Y_{i,t-1}} \right)$$

(20)

where $\tilde{Z}_{ir}, r = 1, 2, ..., M,$ are independent simulated copies of $\tilde{Z}_i$. For a detailed description of the simulation procedure, see Appendix D.

As previously mentioned, since we do not have precise estimates of q_{it}, we estimate a logit model for q_{it} based on the subsample of women who are in the labor force, separately for the two groups of women. An alternative specification would be to use more aggregate versions of the probabilities $\{q_{it}\}$ to represent the women's information about their chances in the labor market. For example, one could use the overall unemployment rate separately for the two groups of women as an estimate of $1-q_{it}$. In principle, one could estimate different versions of the model and check which of them are better able to explain the data. However, in our case this is difficult because the data only cover a few business cycle fluctuations. This creates difficulties in providing reliable identification of the discouraged worker effect without using variations in q_{it} across individuals.

5. Data

The data are obtained by linking information from the Norwegian Labor Force Surveys (LFS) 1988–2010 with information from the Norwegian Educational Database, registries with income information from the tax authorities (1988–2010) and the population registries with information about family composition in different years, as well as country of birth for immigrants and their first year of residence in Norway. Information about whether the person lives in a densely populated area is also obtained from the population registries. All registers and survey data are linked using a personal identification key.

In the selection of the sample we include only married women aged 25–60 years. The lower age limit excludes most women enrolled in higher education, while the upper age limit excludes women that might withdraw from the labor force due to early retirement. In addition to the selection based on age and marital status, we also exclude women who are disabled or claim that they are unable to work. Self-employed women and women hired in firms run by family members are also excluded.

The Norwegian LFS follow the international recommendations for labor force surveys where persons are classified as being employed, unemployed or outside the labor force, etc. Working time is measured as contractual hours of work on an annual basis in both the main and any possible second jobs. If this information is missing and the respondent is active in the labor market, information about actual working time is used. Nominal hourly wages are measured as labor income divided by annual working time. To ensure time consistency, we have chosen to use a measure of non-labor income that includes salary of the husband as well as stipulated labor income for self-employed husbands. The nominal hourly wage and non-labor income variables are deflated by the official Norwegian consumer price index, with 2010 as the reference year. Based on the information in the registries, we also calculate the number of children in each household aged 0–3 years, 4–6 years and 7–18 years. Education is measured in years of achieved level of schooling and work experience is defined as age minus length of schooling minus 7. Duration of residence in Norway in a particular year is calculated as the number of years from the first year of residence. Urbanity is a dummy variable which is equal to one if the person lives in a densely populated area, and zero otherwise. According to Statistics Norway a collection of houses is registered as a densely populated area if there are at least 200 people living there and the distances between the houses do not exceed 50 meters.

The Norwegian LFS are quarterly and the samples are rotating. In the estimation of the model we make use of the fact that it is possible to observe a person in the same quarter in two consecutive years. Thus, each woman in the sample is observed twice, and by observing women in the same quarter in both years, we avoid problems related to seasonal fluctuations. Note, however, that the sample includes observations from all four quarters during a year. The reason we exclude women who are not observed twice is that the behavior of this group of women in the labor market seems to be fundamentally different from that of other women.[12]

The empirical analysis is done separately for women born in Norway and for female non-Western immigrants. Non-Western immigrants include immigrants born in Eastern Europe, Africa, Asia, South and Central America. We have excluded immigrants born in Western countries, Australia, New Zealand and North America because we want to focus on immigrants with a substantially different cultural background compared to those born in Norway. In total the sample consists of 52,101 women born in Norway and 1,724 immigrant women. Table 1 gives summary statistics for the women for 1999, which is in the middle of the period of analysis. Membership rates in trade unions are high among Norwegian women and many

[12] One of the most frequent reasons for non-response in the LFS is the difficulty of getting in contact with the interview objects.

Table 1. Summary statistics for women born in non-Western countries and women born in Norway (1999)

	Women born in non-Western countries				Women born in Norway			
Variable	Mean	Std. dev.	Min.	Max.	Mean	Std. dev.	Min.	Max.
Age	37.1	7.3	25	60	42.0	9.5	25	60
Education (years)	12.4	3.3	6	20	12.6	2.8	9	20
Experience (years)	18.6	7.5	5	41	23.4	10.4	2	41
# children 0-3 years	0.4	0.6	0	2	0.3	0.5	0	2
# children 4-6 years	0.2	0.5	0	2	0.2	0.5	0	2
# children 7-18 years	0.9	1.1	0	4	0.7	1.0	0	4
Non-labor income[a]	343,531	185,439	58,401	185,439	385,643	185,243	58,401	1,325,439
Wage rate[b]	120.0	16.5	91.2	181.7	135.3	15.2	103.6	194.8
Participation rate	0.75	0.44	0	1	0.91	0.29	0	1

[a] In constant 2010 NOK.
[b]Predicted wage rate in constant 2010 NOK.

women work in the public sector. Dagsvik et al. (2016) provide more detailed information about the data used in the estimations.

So far, we have discussed the data used in the estimations. It is, however, of interest to apply the model for prediction of participation, unemployment and discouraged workers for the whole Norwegian population. As the sample used in the estimations of the model is not representative for our target population, in particular with respect to immigrant women, we have prepared another data set for prediction purposes. This data set is based on the Norwegian Income Registry 2005, representing the total Norwegian population.[13] For all women (of interest) it contains information about all the individual explanatory variables of the model: that is, non-labor income, length of schooling, (potential) work experience, duration of residence, urbanity dummy, age and the number of children in the specific age groups. The selection rules used for the micro population are as follows: we have removed women with public and private pensions in excess of 125,000 NOK (in nominal terms). This income limit is consistent with the maximum pension income in the data used in the estimation of the participation model, and by imposing this restriction we omit women who are unable to participate in the labor market. To capture the fact that we are modeling the decision to participate in paid work and not self-employment, women with more income from self-employment than wage incomes are excluded. In addition, about 20 per cent of the immigrant women are excluded due to missing information about their educational attainment. As in the estimations, we have carried out the

[13] Data for the Income Registry cannot be used for estimation of the model as it does not include sufficient information about labor market participation and unemployment.

simulations separately for married/cohabitating women of age 25–60 born in non-Western countries (41,339 obs.) and Norway (555,209 obs.).

6. Estimation results

Estimation of the participation model requires predictions from the wage equation as well as from the model for the probability of getting an acceptable job offer for each woman. These estimations are done separately for immigrant and non-immigrant women, on samples of employed women and women in the labor force, respectively. Table C1 and Table C2 in Appendix C contain the estimation results for the wage equation and the job offer probability, respectively.[14] The components of the (preference) vector V_{it} (cf. Eq. (3.4) and Table 2 below) are 1, age, age squared, real non-labor income, the number of children aged 0–3 years, the number of children aged 4–6 years and the number of children aged 7–18 years.

In Table 2 we report parameter estimates of the probability of labor force participation for women born in non-Western countries and women born in Norway. From the table we notice that the estimate of α, which represents the variance of the random effect as well as the strength of the serial correlation in the utility functions, is somewhat higher for the women born in non-Western countries than it is for those born in Norway. It follows from (3.12) that the estimated serial correlation of the error terms in the utility functions is equal to about 0.80 for immigrant women and 0.88 for women born in Norway. The estimate of θ, which is inversely proportionate to the standard deviation of ε_{jt}, is higher for women born in Norway (=12.3) than for immigrant women (=6.7). However, the most striking result in Table 2 is how different the estimates of the parameter that represents the cost (disutility) per unit of time (c) are between women born in Norway and women born in non-Western countries. The estimate for women born in Norway is more than three times as large as the estimate for women born in non-Western countries. We interpret this finding as being due to the difference in psychological costs. Non-Western immigrant women often come from economies where unemployment is high and accordingly it may be hard to get job offers. Thus, immigrant women are typically more used to demanding labor market conditions than women born in Norway. Consequently, immigrant women who are interested in entering the labor force will not be as easily discouraged as women born in Norway. Another possible explanation might stem from a selection effect: women who migrate may be more motivated for entering the labor force and less concerned

---

[14] In a preliminary stage we used Heckman's two-stage procedure for controlling for self-selection, but we found no significant selection effect.

with the psychological stress and uncertainty associated with job search than women who do not migrate.

Table 2. Estimation results for the probability of labor force participation. Women born in non-Western countries and women born in Norway

Variable/parameter	Women born in non-Western countries		Women born in Norway	
	Estimate	*t*-value	Estimate	*t*-value
$\alpha\theta$	3.011	8.474	4.237	42.195
Search cost per unit of time c	0.753	5.103	2.307	14.434
Constant, γ_1	–4.735	–11.981	–4.194	–57.393
Age, γ_2	0.046	2.170	0.019	5.335
$(\text{Age}/10)^2$, γ_3	–0.073	–2.733	–0.040	–9.449
(Real non-labor income)$\times 10^{-5}$, γ_4	–0.018	–1.956	–0.025	–15.903
No. of children 0–3, γ_5	–0.305	–6.760	–0.199	–28.780
No. of children 4–6, γ_6	–0.072	–2.224	–0.147	–22.804
No. of children 7–18, γ_7	–0.106	–5.236	–0.075	–18.221
Serial correlation,* α	0.450	13.568	0.344	56.524
No. of observations	3,448		104,202	
No. of observation units	1,724		52,101	
Log-likelihood	–1,424.8		–25,562.9	
M	150		150	
McFadden's ρ^2	0.40		0.65	

*Panel data are necessary in order to identify and estimate α and θ separately. If only independent cross-section data are available, one can still estimate $\alpha\theta$ and all the other parameters of the model.

Also, the estimate of the parameter relating to real non-labor income is not significantly different between the two groups. This estimate is substantially larger for women born in Norway than it is for women born in non-Western countries. Besides, the estimate for the latter group is on the border of being insignificant at the 5 percent level. The estimates of the parameters determining the effects of children are fairly equal for the two groups. For women born in Norway the smallest effect is found for children in the oldest age group, whereas for women born in non-Western countries the smallest estimate is found for children in the middle age group.

As measures of goodness-of-fit we have calculated McFadden's ρ^2 and also empirical and predicted participation rates. The values of ρ^2 are 0.40 for immigrant women and 0.65 for women born in Norway (Table 2), which indicate a fairly good fit. Table 3 displays aggregate predicted participation rates based on the estimated model together with observed participation rates in the sample. The predicted figures are average predicted participation probabilities across all women in the actual group, and we use the same sample that was used in the estimations (in-sample comparison).

We notice that the estimated models fit the data quite well.[15] The predictions capture the increase in labor market participation over time for both groups of women.

Table 3. Observed and predicted labor force participation rates

	Women born in non-Western countries			Women born in Norway		
Period	Observed	Std. dev.	Predicted	Observed	Std. dev.	Predicted
1988–1990	0.714	0.032	0.696	0.829	0.003	0.834
1991–1993	0.665	0.024	0.624	0.856	0.002	0.862
1994–1996	0.648	0.020	0.650	0.880	0.002	0.876
1997–1999	0.725	0.023	0.741	0.907	0.003	0.888
2000–2002	0.749	0.022	0.781	0.927	0.003	0.920
2003–2005	0.776	0.018	0.772	0.937	0.002	0.933
2006–2008	0.845	0.014	0.845	0.953	0.002	0.956
2009–2010	0.865	0.019	0.880	0.967	0.003	0.969
1988–2010	0.752	0.007	0.753	0.891	0.001	0.889

7. The discouraged worker effect and barriers to employment

In this section we formalize what we mean by the discouraged worker effect and how it relates to our empirical model and how we can use the model to obtain corresponding quantitative measures.

Recall that we define the peak condition in the labor market as a situation in which $q = q^0$ where q^0 is a reference probability of being employed, given participation in the labor force). To emphasize that the utility of search is a function of the probability of receiving an acceptable job we write $U_{1t} = U_{1t}(q_t)$, where U_{1t} is given in (13). We say that a worker is discouraged at time *t* if

$$U_{1t}(q^0) > U_{0t} > U_{1t}(q_t). \qquad (21)$$

In other words, the inequalities in (21) assert that a worker is discouraged if the value of search under the peak condition is greater than the utility of being out of the labor force, whereas the value of search under the actual labor market condition is less than the utility of not working. Thus, our notion of discouraged worker depends crucially on the assumption that information about the women's chances of obtaining an acceptable job are (on average) captured reasonably well by the probabilities $\{q_{it}\}$. In addition, it depends on the reference value q^0 that corresponds to the peak condition of the labor market. Let

$$P_t(q_t) = P(U_{1t}(q_t) > U_{0t}).$$

It follows from (7.1) that the probability of being a discouraged worker equals

[15] One reason why the figures in Table 3 vary so much over time is that the sample survey is rotating.

$$D_t(\tilde{q}_t) \equiv P(U_{1t}(q^0) > U_{0t} > U_{1t}(q_t)) = P(U_{1t}(q^0) > U_{0t}) - P(U_{1t}(q_t) > U_{0t}) = P_t(q^0) - P_t(q_t). \quad (22)$$

In this paper $D_t(q_t)$ is our measure of the discouraged worker effect. From (22) one may define the modified unemployment rate as $(P_t(q^0) - P_t(q_t)q_t) / P_t(q^0)$. The empirical counterpart of the modified unemployment rate is the (potential) number of women in the labor force under peak conditions minus the number of employed women under the actual conditions divided by the number of women in the labor force under peak conditions. This measure then captures both ordinary observed unemployment and the discouraged worker effect. In empirical applications, one may define the reference level q^0 as the highest value of all the estimated acceptable job offer arrival rates (across population groups and across time). In our sample, the highest value of q_{it} is about 0.99 and we have therefore chosen to let $q^0 = 1$.

Table 4 displays measures of the discouraged worker effect and the effect of barriers (represented by $\tilde{q}_t$) to employment for selected population groups. Specifically, we have divided the immigrant women into 12 specific subgroups depending on their duration of residence in Norway, their actual education and their age. We have also provided results for women born in Norway (see the four last rows). The last column of the table shows the number of observations in each group for the entire population in 2005. As regard barriers to employment, we note that the unemployment rate for young immigrant women with a low level of schooling and a short time since arrival (group 1) is high, about 33 percent, decreasing to about 22 per cent for group 6, where time since arrival is 5–10 years and the women are aged 35+. The lowest unemployment rate among immigrant women is for group 12, which contains women with high education, aged 45+, who have been in Norway for more than 10 years. In contrast, the unemployment rate for women born in Norway is much lower for all levels of education and age. For this group the probability of not getting an acceptable job varies between 1 and 7 percent.

Table 4. Labor force participation, employment and discouraged worker effect by age, education and duration of residence. 2005

Group	Duration of res. (D)	Length of edu-cation (E)	Age (A)	Partici-pation rate[a]	Employ-ment rate[b]	Un-employ-ment rate[c]	Disc. worker effect[d]	Modified unemploy-ment rate[e]	No. of obs.
Women born in non-Western countries									
1	D≤5	E≤13	A<35	0.542	0.385	0.331	0.225	0.526	4,144
2	D≤5	E≤13	A≥35	0.589	0.423	0.317	0.203	0.493	3,253
3	D≤5	E>13	A<35	0.798	0.645	0.197	0.077	0.269	2,319
4	D≤5	E>13	A≥35	0.828	0.672	0.193	0.067	0.253	1,601
5	5<D≤10	E≤13	A<35	0.639	0.496	0.244	0.145	0.387	2,902
6	5<D≤10	E≤13	A≥35	0.702	0.561	0.218	0.114	0.331	3,088
7	5<D≤10	E>13	A<35	0.828	0.709	0.147	0.050	0.196	1,149
8	5<D≤10	E>13	A≥35	0.859	0.746	0.133	0.039	0.171	1,729
9	D>10	E≤13	A<40	0.773	0.680	0.126	0.054	0.187	6,127
10	D>10	E≤13	A≥40	0.794	0.709	0.113	0.046	0.164	8,479
11	D>10	E>13	A<40	0.883	0.823	0.068	0.016	0.086	2,491
12	D>10	E>13	A≥40	0.906	0.854	0.058	0.011	0.070	4,117
All				0.753	0.642	0.172	0.086	0.254	41,399
Women born in Norway									
13		E≤13	A<35	0.868	0.812	0.067	0.067	0.135	61,910
14		E≤13	A≥35	0.919	0.893	0.029	0.019	0.049	279,315
15		E>13	A<35	0.950	0.923	0.028	0.012	0.041	60,557
16		E>13	A≥35	0.964	0.952	0.012	0.004	0.017	153,427
All				0.929	0.904	0.028	0.020	0.049	555,209

[a] Participation rate: $P(q)$. [b] Employment rate: $P(q)q$. [c] Unemployment rate: $1-q$. [d] DW effect: $D(q)$. [e]Modified unemployment rate: $(P(1)-P(\tilde{q})\tilde{q})\,/\,P(1)$.

As mentioned above, we use the modified unemployment rate to measure the total effect of barriers to employment. This rate is highest among immigrant women with short duration of residence in Norway and low education (groups 1 and 2). For these two groups, which differ with respect to age, the modified unemployment rate is as high as about 50 percent. At the other end of the scale, we find well-educated immigrant women who have lived for a long period of time in Norway (groups 11 and 12). For these two groups, the modified unemployment rates are about 9 and 7 percent respectively, mirroring a low level of discouraged workers. Among immigrant women with short duration of residence in Norway and low education, barriers are thus substantial. However, as duration of residence increases, immigrant women seem to integrate into the Norwegian labor market. Barriers then decrease and employment among immigrant women increases.

For Norwegian-born women the barriers are typically much smaller, but not always. By comparing the figures for the four groups in Table 4 (groups 13–16), we note that the modified unemployment rate is highest for young women born in Norway with a low level of schooling (group 13). For this group the modified unemployment rate is about 14 percent. For the other three groups, barriers are much smaller, and the low rates are due to both a low unemployment rate and a low discouraged worker effect.

Above we have discussed the distribution of the discouraged worker effect in the labor market for the target population. By dividing the population into a number of groups, we found that there is considerable heterogeneity in the barriers due to the composition of the different groups. We now wish to take a closer look at the difference in behavior for given population groups facing hypothetical levels of real wage rates and unemployment rates $(1-q)$. As regard wage rates, the distribution of the stochastic error term in the wage equation is assumed to be the same as that estimated from the data. Specifically, we simulate the labor market behavior of the respective groups of women with given age, a given log (real) wage rate, real non-labor income, number of children in the three different age groups and hypothetical unemployment rate. The upper part of Table 5 contains simulations for 14 different types of immigrant women, while the lower part contains similar results for 19 groups of women born in Norway. For the latter we present simulations for women with characteristics that are not only representative for women born in Norway (cases 15–25), but also similar to the ones used for immigrant women (cases 26–33). In order to reduce the number of simulations, we consider only women with real non-labor income equal to 380,000 NOK (at 2010 prices) for immigrant women and 490 000 NOK for women born in Norway. These values correspond to the median values in our samples used in the estimation of the model. Since the two groups of women also vary systematically with respect to wage rate and the probability of obtaining an acceptable job, the assumptions being made about the level of these variables also differ across the two groups. The lowest values of the mean wage rates correspond (approximately) to the first decile in the distribution of that variable in our sample for 2010, whereas the highest value is the ninth decile, and the value in the middle is the median value.

Looking at the results in Table 5 for women born in non-Western countries, we see that the discouraged worker effect varies from 0.001 to 0.185. The lowest rate is found for a childless woman aged 30 years with a predicted wage equal to 350 NOK and a probability of obtaining an acceptable job equal to 0.95, which corresponds to an unemployment rate of 5 percent. The highest rate is for a woman aged 45 with a wage equal to 100 NOK (at 2010 prices), with five children, where four of the children are in the oldest age group and the last child is in the next oldest age group, and a probability of obtaining an acceptable job equal to 0.75. As expected, the discouraged worker effect increases when there is an increase in the unemployment rate. By comparing the simulation results for immigrant women with different wage rates, we also notice that the discouraged worker effect depends on the wage rate. For women with high wage rates, the effect is small (cases 8 and 9), even when the unemployment rate is moderate, and the woman has many children (case 9). For women with low wage rates, the effect is considerably higher, even when the unemployment rate is moderate (cases 6 and 14).

The relationship between the number of children and the discouraged worker effect is more complicated, since a change in the number of children yields shifts in preferences and participation rates in the labor market. For young women with medium wage rates in Table 5 (cases 1–4), we note that more children increase the discouraged worker effect. However, by comparing the discouraged worker effect for cases 12 and 13 we note that there is a slight decrease for women with several children. Both these groups face low mean wage rates and belong to the oldest age group. Thus, we would expect them to have low participation rates. The participation rates are 0.488 for women with only two children and 0.267 for women with five children, according to the results in Table 5.

Compared to immigrant women, there is much less variation in the discouraged worker effect among women born in Norway. For these women (cases 15–25), the discouraged worker effect lies between 0 and 0.113, and it is only for middle-aged women with a low wage rate, a low *q* and five children (case 23) that the discouraged worker effect is higher than 0.077.

In the final part of Table 5 we present some simulations for women born in Norway, but with mean wage rates and unemployment rates that are more representative for immigrant women. Cases 26–28 relate to young women born in Norway with a mean wage rate equal to the median wage rate for immigrant women and varying the hypothetical unemployment rates. While these women are childless, others have one young child (cases 29–31) or a child in each age group (cases 32 and 33). By comparing women with similar characteristics – cases 26, 27, 32 and 33 for women born in Norway and cases 1–4 for

immigrant women – we find that the discouraged worker effect is lower among immigrant women than

Table 5. Predicted fraction of labor force participation and discouraged worker effect by age, wage rate, number of children and unemployment rate[a]

Case	Real wage rate[b]	Age	No. of children aged 0–3	4–6	7-18	Unempl. rate	Participation probability	Discour. worker eff.[c]	Modified unempl. rate[d]
Immigrant women									
1	180	30	0	0	0	0.25	0.933	0.034	0.277
2	180	30	0	0	0	0.15	0.952	0.015	0.163
3	180	30	1	1	1	0.25	0.764	0.109	0.344
4	180	30	1	1	1	0.15	0.822	0.051	0.200
5	100	30	1	0	0	0.25	0.486	0.183	0.455
6	100	30	1	0	0	0.15	0.574	0.094	0.269
7	100	30	1	1	1	0.25	0.356	0.185	0.506
8	350	30	0	0	0	0.05	0.995	0.001	0.051
9	350	30	1	1	1	0.15	0.972	0.009	0.158
10	180	45	0	1	1	0.15	0.889	0.034	0.181
11	180	45	0	1	4	0.15	0.753	0.067	0.219
12	100	45	0	1	1	0.25	0.488	0.182	0.454
13	100	45	0	1	4	0.25	0.267	0.17	0.542
14	100	45	0	1	4	0.15	0.342	0.095	0.335
Women born in Norway									
15	205	30	0	0	0	0.025	0.993	0.002	0.027
16	205	30	1	1	1	0.025	0.958	0.009	0.034
17	140	30	0	0	0	0.05	0.954	0.018	0.068
18	140	30	1	1	1	0.05	0.777	0.077	0.135
19	305	30	0	0	0	0.025	0.999	0	0.025
20	305	30	1	0	0	0.025	0.997	0.001	0.026
21	305	30	1	1	1	0.025	0.992	0.002	0.027
22	140	45	0	1	1	0.05	0.802	0.069	0.126
23	140	45	0	1	4	0.05	0.611	0.113	0.199
24	205	45	0	1	1	0.05	0.953	0.018	0.068
25	205	45	0	1	4	0.05	0.888	0.042	0.093
Women born in Norway with immigrants' characteristics									
26	180	30	0	0	0	0.25	0.812	0.179	0.386
27	180	30	0	0	0	0.15	0.952	0.039	0.183
28	180	30	0	0	0	0.05	0.985	0.006	0.056
29	180	30	1	0	0	0.25	0.651	0.329	0.502
30	180	30	1	0	0	0.15	0.896	0.084	0.223
31	180	30	1	0	0	0.05	0.967	0.013	0.063
32	180	30	1	1	1	0.25	0.422	0.528	0.667
33	180	30	1	1	1	0.15	0.772	0.178	0.310

[a] We also assume that non-labor income, measured in 2010 prices, is 380,000 NOK for immigrant women (cases 1−14, 26−33) and 490,000 NOK for women born in Norway (cases 15−25). [b] In constant 2010 NOK prices. [c] DW effect: $D(q)$. [d] Modified unemployment rate: $(P(1) - P(q)q) / P(1)$.

among women born in Norway. Table 5 also shows that the discouraged worker effect among women born in Norway with immigrant characteristics is quite sensitive to the level of the unemployment rate (equivalent to the probability of getting an acceptable job given search), in particular when the unemployment rate increases from 15 to 25 percent and there are several children in the family.

8. Concluding remarks

In this paper we have analyzed labor force participation and the discouraged worker phenomenon for married and cohabitating women born in non-Western countries and Norway, respectively. We have demonstrated that our empirical model is consistent with a search-theoretic framework based on aggregate rational expectations. The model is estimated separately for immigrant women from non-Western countries and women born in Norway. According to our estimation results, the two groups differ with respect to the estimate of the search costs per unit of time. Women born in Norway have higher probabilities of getting acceptable job offers, but they also have considerably higher estimated search costs per unit of time compared to immigrant women. A likely explanation for the latter feature is that the environments immigrant women are used to are much more demanding than the Norwegian one, so they may be accustomed to using more effort in order to achieve results. In total, however, the proportion of discouraged workers is significantly higher for some groups of immigrant women than for women born in Norway. The reason is that the probabilities of getting an acceptable job are substantially lower for immigrant women compared to women born in Norway and immigrant women will therefore, on average, need to search for a longer time than women born in Norway in order to get an acceptable job.

Not surprisingly, we find that educational qualifications are a key variable for the size of the discouraged worker effect for both married/cohabiting women born in non-Western countries and women born in Norway. A higher level of education is associated with both an increase in the wage rate the woman might expect to get in the labor market and her probability of getting a job offer she considers acceptable. Both these effects tend to reduce barriers in the labor market as measured by the discouraged worker effect and the modified unemployment rate, and years of schooling then influence barriers in the labor market positively through two channels.

Among immigrant women, duration of residence has a similar effect on barriers to employment as educational qualifications, as it is positively correlated with the wage rate and the probability of getting an acceptable job offer.

Although the analysis of this paper has focused on data from Norway we believe that our results also are of interest to other countries. First, we have demonstrated that a simple empirical model for labor force participation as a function of the probability of obtaining an acceptable job (1-

unemployment rate), wage rate and other covariates is consistent with the notion of aggregate rational search behavior. Second, provided data on labor force participation, unemployment, wages, incomes and suitable covariates are available for several cross-sections this model can be estimated, and subsequently be applied to predict the level of modified unemployment in counterfactual settings. As mentioned in the introduction, several countries (such as the US) collect data on discouraged workers. Such data are evidently useful for descriptive purposes. Still, we believe it is of interest to conduct a structural analysis that intends to explain how labor force participation varies over the cycle and depends on key socio-economic variables, such as the expected cost of search, the individual wage rate, and which can be used to analyze counterfactuals.

Conflict of Interest: The authors have received research grants from the Norwegian Research Council, under the project "Treff: Trygd i kontekst: Rettferdighet, effektivitet, fordeling", project number 199836/H20 and from the Research Department, Statistics Norway, Oslo, Norway. The authors declare that they have no conflict of interest.

References

Başlevent, C., & Onaran, Ö. 2003. Are married women in Turkey more likely to become added or discouraged workers? *Labour*, 17, 439–458.

Benati, L. 2001. Some empirical evidence on the ‘discouraged worker’ effect. *Economics Letters*, 70, 387−395.

Bloemen, H. G. 2005. Job search, search intensity, and labor market transitions. An empirical analysis. *Journal of Human Resources*, 40, 231–269.

Blundell, R., & MaCurdy, T. 1999. Labor supply: a review of alternative approaches. In *Handbook of Econometrics* Vol. 3A, Ashenfelter, O., & Card, D. (eds.). North-Holland, Amsterdam, 1559–1695.

Blundell, R., Ham, J., & Meghir, C. 1987. Unemployment and female labour supply. *Economic Journal*, 97, 44–64.

Blundell, R., Ham, J., & Meghir, C. 1998. Unemployment, discouraged workers and female labour supply. *Research in Economics*, 52, 103−131.

Brown, M., Flinn, C. J., & Schotter, A. 2011. Real-time search in the laboratory and the market. *American Economic Review*, 101, 948–974.

Cardell, N. S. 1997. Variance component structures for the extreme-value and logistic distributions with applications to models of heterogeneity. *Econometric Theory*, 13, 185–213.

Conlisk, J. 1996. Why bounded rationality? *Journal of Economic Literature*, 34, 669–700.

Connolly, S. 1997. A model of female labour supply in which supply is dependent upon the chances of finding a job. *Applied Economics*, 29, 1379–1386.

Dagsvik, J. K. 2018. The generalized extreme value model as a stable mixture of conditional logit models. Unpublished manuscript, Research Department, Statistics Norway.

Dagsvik, J. K., Kornstad, T., & Skjerpen, T. 2013. Labor force participation and the discouraged worker effect. *Empirical Economics*, 45, 401–433.

Dagsvik, J. K., Kornstad, T.,& Skjerpen, T. 2016. Discouraged worker effects and barriers against employment for immigrant and non-immigrant women. Discussion Paper 845, Statistics Norway.

Darby, J., Hart, R. A., & Vecchi, M. 2001. Labour force participation and the business cycle: A comparative analysis of France, Japan, Sweden and the United States. *Japan World Economy*, 12, 113−133.

Ehrenberg, R., & Smith, R. 1988. Modern labor economics: Theory and public policy, 3rd edition. Scott, Foreman, and Co., London.

Emerson, J. 2011. Unemployment and labor force participation in the United States. *Economic Letters*, 111, 203–206.

Flinn, C., & Heckman, J. J. 1982. New methods for analyzing structural models of labor force dynamics. *Journal of Econometrics*, 18, 115−168.

Fuchs, J., & Weber, E. 2017. Long-term unemployment and labour force participation: a decomposition of unemployment to test for the discouragement and added worker hypotheses. *Applied Economics*, 49, 5971−5982.

Ham, J. 1986. On the interpretation of unemployment in empirical labor supply analysis. In *Unemployment, search and labour supply*, Blundell, R., & Walker, I. (eds.). Cambridge University Press, Cambridge; 121−142.

Heckman, J. J., & Singer, B. 1984. Econometric duration analysis. *Journal of Econometrics*, 24**,** 63−132.

Hotchkiss, J. L., & Robertson, J. C. 2006. Asymmetric labor force participation decision over the business cycle. Federal Reserve Bank of Atlanta, Working Paper 2006-8.

Lippman, S. A., & McCall, J. J. 1976. The economics of job search: a survey, Part I. *Economic Inquiry*, 14, 155–189.

Luce, R. D. 1977. The choice axiom after twenty years. *Journal of Mathematical Psychology*, 15, 215–233.
McFadden, D. 1984. Econometric analysis of qualitative response models. In *Handbook of Econometrics*, Vol. 2, Griliches, Z., & Intriligator, M. D. (eds.). Elsevier Science Publishers BV, New York, 1395–1457.
Österholm, P. 2010. Unemployment and labor force participation in Sweden. *Economic Letters*, 106, 205–208.
Prowenzano, S. 2017. The empirics of hidden labor force dynamics in Germany. *Journal of Economics and Statistics*, 237, 373–406.
Rogerson, R., Shimer, R., & Wright, R. 2005. Search-theoretic models of the labor market: a survey. *Journal of Economic Literature*, XLIII, 959–988.
Samorodnitsky, G., & Taqqu, M. S. 1994. *Stable non-Gaussian random processes.* Chapman & Hall, New York.
Thaler, R. H. 2015. *Misbehaving: the making of behavioral economics.* W.W. Norton, New York.
Venders, M., Cörvers, F. 2009. Male and female labor force participation: the role of dynamic adjustments to changes in labour demand, goverrnment policies and autonomous trend. Maastricht University, ROA Research Memorandum 2009/13.
Yellott, J. I. 1977. The relationship between Luce's choice axiom, Thurstone's theory of comparative judgment, and the double exponential distribution. *Journal of Mathematical Psychology*, 15, 109–144.
Zolotarev, V. M. 1986. *One-dimensional stable distributions.* Translations of mathematical monographs, American Mathematical Society, 65. Translation from the original 1983 Russian edition.

Appendix A

Proof of Theorem 1:

From well-known results (Yellott, 1977), it follows that Assumption 4 is satisfied provided the error terms $\varepsilon_0, \varepsilon_1, \varepsilon_2$ are independent and distributed according to the Gumbel distribution. Since ε_0 and ε_2 are independent it must also be the case that ε_1 is independent of ε_0 and ε_2. Note that $\theta\varepsilon_j$ are standard Gumbel distributed with c.d.f. exp (-exp(-*x*)). Also, from well-known properties of the Gumbel distribution, it follows that

$$E^p(U_1, U_2) \mid Z_2) = E^p(\max(\theta u_1 + \theta\varepsilon_1, \theta u_2 + \theta Z_2 + \theta\varepsilon_2) \mid Z_2) = \log(\exp(\theta u_1) + \exp(\theta u_2 + \theta Z_2))$$

(A.1)

which together with (3.5) yields

$$\theta u_1 = \log(\exp(\theta u_1) + \exp(\theta u_2 + \theta Z_2)) - \frac{\theta c}{\lambda}. \tag{A.2}$$

Furthermore, (A.2) implies that

$$\exp(\theta u_1)(\exp(\theta c / \lambda) - 1) = \exp(\theta u_2 + \theta Z_2)$$

which yields the result of the theorem.

Q.E.D.

Proof of Corollary 1:

Recall that

$$q = \lambda P(U_2 > U_1) = \lambda P(\theta u_2 + \theta Z_2 + \theta\varepsilon_2 > \theta u_1 + \theta\varepsilon_1)$$

$$= \frac{\lambda}{1 + \exp(\theta u_1 - \theta u_2 - \theta Z_2)} = \lambda(1 - \exp(-\theta c / \lambda))$$

which yields

$$\frac{\theta c}{q} = \frac{\theta c / \lambda}{1 - \exp(-\theta c / \lambda)}. \tag{A.3}$$

From (A.3) we note that there is a unique correspondence between $\theta c / \lambda$ and $\theta c / q$. We can thus write the equation in Corollary 1 as

$$\log(\exp(\theta c / \lambda) - 1) = h(\theta c / q) \tag{A.4}$$

for some suitable function *h*. The relation in (A.4) implies that

$$\theta c / \lambda = \log(1 + \exp(h(\theta c / q)). \tag{A.5}$$

When (A.5) is inserted into (3.6) we obtain that $g(h(\theta c / \lambda)) = \theta c / q$ where

$$g(h) = (1 + \exp(-h)) \log(1 + \exp(h)).$$

The function g is strictly increasing and strictly convex. To realize this, note that

$$g'(h) = [\exp(h) - \log(1 + \exp(h))] \exp(-h).$$

Since the derivative of $\exp(h) - \log(1 + \exp(h))$ is positive for all *h* it follows that $g'(h) > 0$ for all *h*. Therefore, the equation $g(h(x)) = x$ defines the function $h(x)$ uniquely. Moreover, since $g'(h(x))h'(x) = 1$ it follows that *h* is strictly increasing. From (6) and (A.5) we therefore obtain that $\theta u_{1t} = \theta u_{2t} + \theta Z_2 - h(\theta c / \mu_t)$. Furthermore, it is easy to show that $g''(x) > 0$ which together with $g''(h(x))h'(x) + g'(h(x))h''(x) = 0$ implies that $h''(x) < 0$.

Q.E.D.

Appendix B

Distributional properties of ratios of independent stable random variables

Lemma 1

Assume that V_j, $j=1,2,$ are independent stable random variables that are distributed as $S_\alpha(1,1,0)$ with $\alpha<1,$ and let $Z=\alpha\log\left(V_1/V_2\right)$.[16] *Then the p.d.f. of Z is given by*

$$f_\alpha(z)=\frac{\sin(\alpha\pi)}{\pi\left(e^z+2\cos(\alpha\pi)+e^{-z}\right)}.$$

Proof of Lemma 1:

Since V_j, $j=1,2,$ are independent with c.d.f. $S_\alpha(1,1,0)$ it follows that the event $\alpha^{-1}Z\le z$ is equivalent to

$$U(z)\equiv\frac{V_1-e^zV_2}{\sigma(z)}\le 0,$$

where $\sigma(z)$ is the scale parameter of the stable random variable $U(z)$. From Property 1.2.1 in Samorodnitsky and Taqqu (1994, p. 10), it follows that the scale and the skewness parameters of $U(z)$ are one and $\beta(z)$ respectively, where the latter is given by

$$\beta(z)=\frac{1-e^{\alpha z}}{1+e^{\alpha z}}.$$

From Zolotarev (1986, equation (2.2.30), p. 79), it follows that

$$P(\alpha^{-1}Z\le z)=P\left(U(z)\le 0\right)=\frac{1}{2}-\frac{1}{\alpha\pi}\operatorname{Arctan}\left(\frac{\left(1-e^{\alpha z}\right)\psi}{e^{\alpha z}+1}\right) \tag{B.1}$$

where $\psi=\tan(\alpha\pi/2)$. From (B.1) it follows that the probability density of $\alpha^{-1}Z$ is given by

$$\frac{e^{\alpha z}\psi}{\pi\left(\left(1+e^{\alpha z}\right)^2+\left(1-e^{\alpha z}\right)^2\psi^2\right)}=\frac{2\psi e^{\alpha z}}{\pi\left(1+\psi^2\right)\left(\left(e^{\alpha z}-\left(\frac{\psi^2-1}{\psi^2+1}\right)\right)^2+\left(\frac{2\psi}{1+\psi^2}\right)^2\right)}$$

$$=\frac{\sin(\alpha\pi)e^{\alpha z}}{\pi\left(\left(e^{\alpha z}+\cos(\alpha\pi)\right)^2+\sin^2(\alpha\pi)\right)}=\frac{\sin(\alpha\pi)}{\pi\left(e^{\alpha z}+2\cos(\alpha\pi)+e^{-\alpha z}\right)}.$$

(B.2)

But (B.2) means that the p.d.f. of *Z* is given by

---

[16] The notation $S_\alpha(\sigma,\beta,\mu)$ means a stable distribution with index α, scale parameter σ, skewness parameter β and location parameter μ.

$$f_\alpha(z) = \frac{\sin(\alpha\pi)}{\pi\left(e^{\alpha z} + 2\cos(\alpha\pi) + e^{-\alpha z}\right)}.$$

Q.E.D.

Lemma 2

Assume that Assumptions 1, 2 and 4 hold. Then $Z_2 - Z_0$ *has p.d.f.* $f_\alpha(z)$ *defined on* $(-\infty,\infty)$ *that is given by*

$$f_\alpha(z) = \frac{1}{\alpha\pi} \cdot \frac{\sin(\alpha\pi)}{e^{\alpha z} + e^{-\alpha z} + 2\cos(\alpha\pi)}$$

where α *is a parameter,* $0 < \alpha \le 1$, *that is related to the variance of* $Z_2 - Z_0$ *by*

$$Var(Z_2 - Z_0) = \left(\alpha^{-2} - 1\right)\frac{\pi^2}{3}.$$

Furthermore, the participation probability is given by

$$P(U_{1t} > U_{0t}) = \frac{1}{1 + \exp(-\alpha(\theta u_{2t} - h(c/\mu_t) - \theta u_{0t}))}.$$

Proof of Lemma 2:

Consider a setting of binary choice with utilities $V_1 = Z_0 + \varepsilon_1$ and $V_2 = Z_2 + v + \varepsilon_2$ where ε_1 and ε_2 are independent standard Gumbel-distributed random variables that are independent of the random variables Z_0 and Z_2. Furthermore, Z_0 and Z_2 are independent. It follows by known results that

$$P(V_2 > V_1 \mid Z_0, Z_2) = \frac{1}{1 + \exp(-v - Z_2 + Z_0)}.$$

By Assumption 4 it follows that

$$P(\alpha V_2 > \alpha V_1) = \frac{1}{1 + \exp(-\alpha v)}.$$

But for this to be true it follows from Yellott (1977) that $\alpha Z_0 + \alpha\varepsilon_1$ and $\alpha Z_2 + \alpha\varepsilon_2$ must also be standard Gumbel-distributed. Since $P(\varepsilon_j \le y) = \exp(-e^{0.5772-y})$, j = 1, 2, and

$$P(\alpha Z_2 + \alpha\varepsilon_2 \le y) = EP(\varepsilon_2 \le y\alpha^{-1} - Z_2 \mid Z_2) = E^p \exp(-e^{0.5772+Z_2-y/\alpha})$$

it follows that we must have

$$E\exp(-e^{0.5772+Z_2-y/\alpha}) = \exp(-e^{0.5772-y}). \qquad \text{(B.3)}$$

Let $w = e^{0.5772-y\alpha^{-1}}$, which yields that $e^{0.5772-y} = w^\alpha e^{0.5772(1-\alpha)}$. When inserting for Z_2 in (B.3) we get

$$E\exp(-we^{Z_2}) = \exp(-w^{\alpha} e^{0.5772(1-\alpha)}) \qquad \text{(B.4)}$$

for $w \geq 0.$ We recognize the left-hand side of (B.4) as the Laplace transform of the distribution of $\exp(Z_2).$ From Proposition 1.2.12 in Samorodnitsky and Taqqu (1994, p. 15), it then follows that $\exp(Z_2)$ must be an $\alpha-$stable random variable that is maximally skew to the right with location parameter equal to zero. It also follows that the scale parameter σ is given, implicitly, by $\sigma^{\alpha} = e^{0.5772(1-\alpha^{-1})}\cos(\alpha\pi/2).$ This last equation implies that $\cos(\alpha\pi/2)$ is non-negative, which can only be the case if $\alpha \leq 1.$ Thus, we have proved that there exist independent random variables such that Assumption 5 is possible. From Lemma 1 we therefore obtain that $Z_2 - Z_0$ must have p.d.f. given by

$$\frac{1}{\pi}\cdot\frac{\sin(\alpha\pi)}{e^{\alpha z}+e^{-\alpha z}+2\cos(\alpha\pi)}.$$

In order to prove the variance formula, we use the fact that $Var(\varepsilon_1-\varepsilon_0) = 2Var\varepsilon_1 = \pi^2/3.$ Since $\varepsilon_1 - \varepsilon_0 + Z_2 - Z_0$ has the same distribution as $\alpha\varepsilon_1 - \alpha\varepsilon_0$ it follows that

$$Var(\alpha\varepsilon_{i1} - \alpha\varepsilon_0 + \alpha Z_2 - \alpha Z_0) = \alpha^2 Var(\varepsilon_1 - \varepsilon_0) + \alpha^2 Var(Z_2 - Z_0) = Var(\varepsilon_1 - \varepsilon_0)$$

from which the variance formula follows.

Q.E.D.

Appendix C

Estimation results for the wage equations

Table C1 displays the estimation results for the wage equations. Most of the variables enter the real wage equations in a significant manner. The estimated return on education is somewhat larger for women born in Norway than for women born in non-Western countries. In both equations experience has a positive effect on the real wage, but it is not easy to compare the results for the two groups since for immigrant women experience and duration of residence to some extent pick up similar features. The dummy for urbanity enters, as expected, with a positive and significant value. For both groups the estimated time effects are all positive and increase over time, accounting for business cycle variations and general growth of real wages over time which is not due to changes in the other explanatory variables.

Table C1. Estimates of wage equations

Variables	Women born in non-Western countries		Women born in Norway	
	Estimate	*t*-value	Estimate	*t*-value
Constant	4.083	53.67	4.110	445.94
Education	0.036	11.06	0.042	86.15
Experience	0.013	2.74	0.018	36.68
Experience squared/100	–0.018	–1.68	–0.028	–27.61
Dummy for urbanity			0.031	12.45
Duration of residence/10	0.107	2.89		
Duration of residence squared/100	–0.013	–1.22		
D91T93[a]	0.051	1.16	0.063	18.71
D94T96	0.095	2.10	0.087	24.20
D97T99	0.137	2.90	0.116	28.94
D00T02	0.177	3.80	0.164	39.04
D03T05	0.201	4.51	0.218	50.70
D06T08	0.274	6.38	0.291	66.36
D09T10	0.341	7.68	0.373	69.52
Variance of random effect	0.118		0.222	
Variance of genuine error term	0.088		0.204	
No. of observatiobs	2,768		98,544	
Log-likelihood	–1,545.2		–14,439.1	

[a] The variable D9193 is a dummy for the years 1991–1993, with a similar notation for the other time dummies. The dummy D8890 is excluded since a constant term is present.

Estimation results for probability of obtaining a job conditional on search

Table C2 shows the results from the estimation of the *q*-relations (probability of getting an acceptable job offer given search). As is evident from the estimation results in Table C2, model specifications are somewhat different with respect to explanatory variables. Work experience is included only in the specification for women born in Norway, while this variable is replaced by duration of residence for immigrant women. For both groups the sets of explanatory variables are expanded by including the log of the real non-labor income and the number of children aged 0–3 years and 4–6 years, respectively.

The education variable is significant in both relations, but the estimate of the coefficient attached to education in the equations for women born in non-Western countries is only a third of the corresponding estimate for women born in Norway. Thus, education is more important for job probability for women born in Norway than for women born in non-Western countries. For women born in Norway only the estimate of the coefficient attached to the linear term of experience is significant. As expected, experience has a positive effect on the job probability. Both the linear and the quadratic terms of the duration of residence impact the job probability significantly. The estimate of the coefficient attached to the linear term is positive whereas the estimate of the coefficient of the quadratic term is negative. The log of real non-labor income enters both relations positively, but the size of the estimated coefficient attached to this variable is somewhat larger for immigrant women. Both the variables on the number of children in two age groups enter with a negative effect on the job probability, and they are mostly significant at the 5 percent level. For both groups the estimated effect of the variable indicating the number of children aged 4–6 years is somewhat larger than for the number of children aged 0–3 years. The time dummies enter significantly for both groups and show a variation over time that mirrors business cycle variation.

Table C2. Estimates of the job offer probability, q

Variables	Women born in non-Western countries		Women born in Norway	
	Estimate	*t*-value	Estimate	*t*-value
Education	0.081	5.494	0.243	26.175
Work experience			0.060	6.344
Work experience squared/100			–0.017	–0.891
Duration of residence/10	1.201	5.978		
Duration of residence squared/100	–0.179	–2.801		
Log (real non-labor income)	0.260	3.691	0.154	4.906
No. of children aged 0–3	–0.166	–1.829	–0.140	–3.599
No. of children aged 4–6	–0.388	–3.978	–0.211	–5.350
D88T90[a]	–2.349	–2.615	–2.257	–5.556
D91T93	–3.391	–3.960	–2.422	–5.955
D94T96	–3.452	–4.019	–2.412	–5.904
D97T99	–3.061	–3.481	–2.327	–5.627
D00T02	–3.097	–3.480	–2.409	–5.772
D03T05	–3.303	–3.741	–2.654	–6.316
D06T08	–2.837	–3.178	–2.179	–5.101
D09T10	–2.837	–3.157	–2.019	–5.566
No. of observations	3,459		107,444	
McFadden's ρ^2	0.48		0.82	

[a] The variable D9193 is a dummy for the years 1991–1993, with a similar notation for the other time dummies.

Appendix D

Generating independent draws from the p.d.f. given in Eq. (9)

Let X_{ir}, r = 1, 2, …, M, be normally distributed N(0,1) and let $\Phi(x)$ denote the standard normal c.d.f. Let $g_b(z)$ be the p.d.f.

$$g_b(z) \equiv f_b(z/b)/b = \frac{1}{\pi b} \cdot \frac{\sin(\pi b)}{e^z + e^{-z} + 2\cos(\pi b)} \tag{D.1}$$

where b is a positive scalar. Define the random variable

$$A_{ir}(b) = \log\left(\frac{\tan(\pi b/2) + \tan\left((2\Phi(X_{ir}) - 1)\pi b/2\right)}{\tan(\pi b/2) - \tan\left((2\Phi(X_{ir}) - 1)\pi b/2\right)} \right) \tag{D.2}$$

for r = 1, 2,…,M, for individual i. The variable $A_{ir}(b)$ will then be distributed according to (D.1). Thus, one can simulate random variables from $g(y)$ by first draw independent standard normally distributed random variables and subsequently use (D.2) to calculate these random variables. Note next that $A_{ir}(\alpha)/\alpha$ has p.d.f. $f_\alpha(z)$ given in (9). Hence, we can write

$$P_{it}(Z_i) = \frac{1}{1 + \exp(-(\theta u_{i2t} - \theta c / q_{it} - \theta u_{i0t} + A_{ir}(\alpha)/\alpha)} \ . \tag{D.3}$$

The simulation procedure goes as follows. Let, for example, $b = \alpha_0 = 0.5$ be the starting value. Given this value of b, one can generate $A_{ir}(\alpha_0)$ by using (D.2). Then, by plugging the formula in (D.3) into the likelihood function with $A_{ir} = A_{ir}(\alpha_0)$, one can obtain a new estimate of α, $\alpha = \alpha_1$. Use this new value of α to generate $A_{ir}(\alpha_1)$. Then the procedure is repeated until convergence is obtained.