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**Simple examples on smoothing
macroeconomic time series**



1. Introduction*

1.1. Background

Underlying much of the recent research on business cycles is a picture that the observed variation in macroeconomic series consists of several components. The cyclic and seasonal variations are considered as superimposed on a secular trend. Isolation of the various components is thus vital before further analysis can be undertaken.

A fundamental problem in this context seems to be that the method of decomposition influences the possible answers one can obtain when the analysis is pushed one step further; that is when the isolated components are used as a basis for studying how different sectors of the economy interact. Searching macroeconomic series for "stylized facts" is therefore problematic in the sense that what one discovers may depend on the method which is chosen. It is essential to have an idea of what the purpose of the investigation is, and also of the properties and limitations of the methods that are employed.

The purpose of the present exposition is modest. We consider a couple of decomposition methods and look at the result to discover what conclusion one can expect. At the same time we try to keep in mind the problems outlined above and try to assess their importance.

The classical approach in business cycle research, due to Burns and Mitchell (1946), consists of fitting moving averages of different lengths to the data. The smoothed time series is used to classify the movements of the original series as a boom, recession, depression or recovery. The localization of the turning points in the different series of interest can then be used to infer about the propagation of cycles in the economy.

Another approach consists of viewing the cyclic behaviour of the series as a deviation from a trend. The main issue is how other series of interest relate to the GDP, so that the focus is on the relationship between the deviations of these series and the deviations of the GDP. This has been carried out by fitting a smooth curve to the respective series and subjecting the residuals from these fits to a closer scrutiny. In Kydland and Prescott (1990) and Blackburn and Ravn (1992), US and UK series respectively, are treated, and the focus is on the covariances between the residuals from the smoothed GDP and the other residuals. We shall carry out a similar investigation on Norwegian data, but in addition to looking at the covariances, we shall also consider the spectra and crossspectra of the residuals of the series.

We shall now give a few more details on how this is done. To smooth the series we use the so-called Hodrick-Prescott (HP) filter, which can be defined as the solution to the minimization problem

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$$(1) \quad \min_{g_{-1}, g_0, \dots, g_T} \sum_{t=1}^T \{ (y_t - g_t)^2 + \lambda [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \}$$

where y_1, \dots, y_T are the observations suitably transformed and λ is a smoothness parameter. The filter g_1, \dots, g_T can be seen as a compromise between the first term controlling the fit to the series and the second term which takes account of the smoothness of the filtered series. The extreme cases are $\lambda=0$, which reproduces the original series and $\lambda \rightarrow \infty$ which is the same as fitting a linear trend. Usually $\lambda=1600$. For some explanation of how one can compute $g_1, g_0, g_1, \dots, g_T$ see Appendix A.

The procedure has been designed for seasonally adjusted data, so some sort of seasonal adjustment may have to be applied first.

Once a smooth curve has been fitted the residuals

$$X_t = y_t - g_t \quad t = 1, \dots, T$$

can be formed, and these can now be analyzed further. Any findings based on the residuals will of course depend on how these residuals are formed, and it is therefore essential to investigate the robustness of the choices that have been made.

The HP procedure is controversial and has been subjected to further investigation in a number of recent studies. Harvey and Jaeger (1993) showed that the HP filter arises in a special case of a more general structural time series model, where certain parameters are set equal to fixed values. They estimated the general model without the imposed restrictions and found that for several of the series they considered, the two sets of parameters were rather different. Furthermore, the cyclic behaviour inferred from the estimated models and those where the HP filter were used for detrending, differed substantially in some cases. They concluded that spurious cycles may be created by mechanically applying the HP filter. This point has also been made by Cogley and Nason (1992). King and Robelo (1993) also studied the HP filtering procedure and compared it with the more traditional exponential smoothing filter. Their general conclusion was a warning against relying on the HP filter as a unique method of trend elimination. Canova (1993) applied a number of detrending techniques, among them two versions of the HP filter corresponding to $\lambda=1600$ and $\lambda=4$ to some major US macroeconomic time series. He found that the "stylized facts" varied across the detrending methods.

In addition to the approach of "fitting a smooth curve and looking at the residuals", we have also tried another approach. While the procedure sketched above may be considered as a variation upon the model of a linear deterministic trend, plus a stationary component, there have also been considerable attempts to use a model where the first differences are taken as starting point. One way to introduce this is to start with the fact that the first differences can be assumed to be a stationary process, and hence under some regularity conditions have an infinite moving average representation

$$(2) \quad \Delta y_t = y_t - y_{t-1} = \mu + \sum_{k=0}^{\infty} a_k \varepsilon_{t-k}$$

where $\varepsilon_t, \varepsilon_{t-1}, \dots$ are uncorrelated random variables with mean zero and variance σ_ε^2 and

$\sum_{k=0}^{\infty} |a_k| < \infty$. If quarterly or monthly data are considered, seasonal effects can be taken into

account by introducing appropriate dummies.

Now, if the process has a decomposition into a random walk component, Z_t , and a stationary part, c_t , so that

$$(3) \quad \begin{aligned} y_t &= Z_t + c_t \\ \Delta Z_t &= \mu + v_t \\ c_t &= \sum_{i=0}^{\infty} b_i \delta_{t-i} \end{aligned}$$

where the correlation between v_t and δ_t are arbitrary, Cochrane (1989) has emphasized that the value of the spectral density of Δy_t at zero equals the variance of the random walk component

multiplied by 2π . In terms of the quantities above it means that $\sigma_v^2 = \left(\sum_0^{\infty} a_j \right)^2 \sigma_\varepsilon^2 = 2\pi f_{\Delta y \Delta y}(0)$,

where $f_{\Delta y \Delta y}$ denotes the spectral density. The importance of this fact is that if $\sigma_v^2 = 0$, the model reduces to

$$(4) \quad y_t = t\mu + c_t$$

i.e. a linear trend plus a stationary component.

We mention in passing that if we require that shocks v_t and δ_t are identical, we get the famous Beveridge-Nelson decomposition introduced by Beveridge and Nelson (1981). It can be computed explicitly and thus the two components Z_t and c_t in (3) can be estimated and compared. A recent application to business cycle analysis can be found in Nicoletti and Reichlin (1993).

The two formulations (2) and (4) are two major competitors as models of macroeconomic time series. In a certain sense the approach based upon smoothing the series and analyzing the residuals can be seen as a variation of (4). Hence looking at estimates of $f_{\Delta y \Delta y}$ in the vicinity of zero must be of importance.

In addition spectral analysis is important for analyzing cyclic behaviour, so investigating the spectral properties of the residuals after having smoothed the series may also provide valuable information. Essentially the spectral density is a transformation of the variance/covariance structure

of a (second order) stationary time series and vice versa. Hence information found in one representation are in principle available in the other. However, they represent alternative views, and hence must be regarded as complimentary.

We shall for the sake of completeness provide some of the central ideas behind spectral analysis. These are well known and can be found in a lot of textbooks. We concentrate on the main points and on the probabilistic aspects. The estimation is done by a standard procedure and a short description can be found in the appendix.

Assuming that X_1, \dots, X_T stem from a stationary process with mean μ , their variance/covariance structure is given by $c_{XX}(k) = E(X_t - \mu)(X_{t+k} - \mu)$, $k=0,1,2, \dots$. The spectral density is defined as the Fourier transform of the covariances.

$$f_{XX}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-i\omega k} c_{XX}(k).$$

It always exists and is continuous provided $\sum_k |c_{XX}(k)| < \infty$, and can be inverted as

$$c_{XX}(k) = \int_0^{2\pi} \exp(i\omega k) f_{XX}(\omega) d\omega.$$

In particular $k=0$ gives

$$\text{Var } X_t = \int_0^{2\pi} f_{XX}(\omega) d\omega$$

which highlights the idea of the spectral representation as a decomposition of the variance of a stationary series.

In particular, values of $f_{XX}(\omega)$ for small values of ω will represent the lower frequencies which correspond to the longer cycles and higher frequencies will represent the shorter cycles.

If X_1, \dots, X_T are uncorrelated with constant variance σ^2 , $f_{XX}(\omega) = \frac{\sigma^2}{2\pi}$, so that in this case all

frequencies contribute the same amount to the variance.

There is an important relation between the spectra of two stationary time series which are related by a linear time invariant filter, i.e.

$$X_t = \sum_{k=-\infty}^{\infty} a_k Y_{t-k}$$

where $\sum_{k=-\infty}^{\infty} |a_k| < \infty$. Then

$$(5) \quad f_{XX}(\omega) = |A(\omega)|^2 f_{YY}(\omega)$$

where $A(\omega)$ is the so-called transfer function of the filter defined by $A(\omega) = \sum_{k=-\infty}^{\infty} a_k \exp(ik\omega)$.

We have already seen an example of the use of this relation. In equation (2) Δy is a filter of ..., $\varepsilon_1, \varepsilon_0, \varepsilon_1, \dots$. Hence $f_{\Delta y \Delta y}(\omega) = |A(\omega)|^2 f_{\varepsilon \varepsilon}(\omega)$. But the latter equals the constant $\sigma_\varepsilon^2/2\pi$, since the ε 's are uncorrelated with zero mean and constant variance. Furthermore, from the definition

$$|A(0)|^2 = \left(\sum_{k=0}^{\infty} a_k \right)^2. \text{ Hence } f_{\Delta y \Delta y}(0) = 2\pi \left(\sum_{i=0}^{\infty} a_i \right)^2 \sigma_\varepsilon^2. \text{ A further important illustration is provided}$$

by the difference operator, which corresponds to a filter with weights $a_0=1, a_1=-1$ and $a_i=0$ otherwise. Hence the transfer function is $A(\omega) = 1 - \exp(i\omega)$, which is 0 at $\omega=0$.

From equation (5) it is evident that the cyclic behaviour after using a filter can be due to both an effect in the original series and to the filter. This is an important problem. Often a preliminary filtering of the data is performed before the analysis is undertaken. For example, the seasonal pattern may not be of primary interest so a seasonal adjustment is carried out first. The spectral density of the seasonally adjusted data will then be a product of the spectral density of the original data and the squared modulus of the transfer function.

There are reasons to believe that problems of this kind are relevant for business cycle analysis. What one is interested in is the low frequencies. Some prior transformation is inevitable to get rid of the high frequency variation in the data, e.g. the seasonal variation. The question is then of course whether any peculiarities that one discovers is a genuine feature of the data or due to the particular filter that is employed.

The spectral density is a decomposition of the variance/covariance structure of a series. The Cramer representation provides a similar decomposition of the actual process. $\{X_t\}$ which may be written

$$X_t = \int_0^{2\pi} e^{it\omega} dZ(\omega)$$

where $\{Z(\omega): 0 \leq \omega \leq 2\pi\}$ is a right continuous process with orthogonal increments such that

$$E|Z(\omega) - Z(0)|^2 = \int_0^\omega f_{XX}(\lambda) d\lambda.$$

Thus in a sense X_t is decomposed into periodic functions with stochastic weights and the spectral density at a particular frequency is the variance of the weight assigned to this frequency.

Thus spectral decompositions should be well suited for discovering features which are periodic and repeated regularly. For the cyclic behaviour this means that it should be a valuable tool for discovering periodicities. If the emphasis is on waves of irregular length the usefulness may be more questionable, and a direct representation of the covariance structure may be preferable.

The same ideas carry over to the description of the relations between two stationary series X_t and Y_t . If $c_{XY}(k)$, $k=-1,0,1,\dots$ are the cross covariances i.e. $E(X_{t+k}-\mu_X)(Y_t-\mu_Y)$ the cospectrum is defined as

$$f_{XY}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-ik\omega} c_{XY}(k).$$

Also under appropriate conditions

$$c_{XY}(k) = \int_0^{2\pi} \exp(i\omega k) f_{XY}(\omega) d\omega.$$

While the spectral density is always a real function, the cospectrum is generally complex valued.

To describe it, it is usual to introduce a frequency dependent correlation, the so-called squared coherency

$$\gamma_{Y \cdot X}(\omega) = \frac{|f_{XY}(\omega)|^2}{f_{XX}(\omega)f_{YY}(\omega)}.$$

A large value of $\gamma_{Y \cdot X}$ at the frequency ω denotes a measure of the strength between X and Y at frequency ω .

Also relevant is the angle of $f_{XY}(\omega)$ at frequency ω , whose slope describes whether there is a lag or lead between the series X and Y at frequency ω . A positive slope denotes that the component of Y lags the corresponding component of X.

A feature that may have some importance in this connection is that if X and Y are stationary series, the effect of a filter cancels in the expression for $\gamma_{Y \cdot X}$ if $A(\omega) \neq 0$.

There are some alternative ways to estimate the spectral density and the cospectra. We shall give some indications in Appendix B. Here we shall confine us to some general remark. Since $f_{XX}(\omega)$ is completely unspecified beyond satisfying some smoothness conditions such as continuity, the procedure based upon estimating f_{XX} is non-parametric in nature. Hence, few assumption are imposed and more is demanded from the data. An alternative is to let f belong to some parametric class. A natural class is the ARMA class, where

$$f_{XX}(\omega) = \frac{\sigma_\varepsilon^2 |\theta(e^{i\omega})|^2}{2\pi |\varphi(e^{i\omega})|^2}$$

where θ is the moving average polynomial, φ is the autoregressive polynomial and σ_ε^2 is the variance of the errors. This turns out to be a versatile and useful class. However, using it one is faced with the traditional dilemma. If the unknown f_{XX} belongs to this class, more efficient use can be made of the data. On the other hand if f_{XX} does not belong to this class, but is nevertheless approximated by one, a bias is introduced.

1.2. The data

In the following we shall look at the twelve series. The data are taken from the KVDATA87 data base of Statistics Norway, except the M2 series which are taken from the financial data base TROLL8. All series are quarterly except the M2 series which is monthly. The range of definition are given for each series, together with the technical denomination.

1. Total gross domestic product (qff, 1966:1–1993:4)
2. Gross domestic product, mainland (qf6, 66:1–93:4)
3. Total private consumption (c, 66:1–93:4)
4. Investments mainland (jk6, 66:1–93:4)
5. Traditional export of goods (a4, 70:1–93:4)
6. Traditional imports of goods (i4, 66:1–93:4)
7. Labour hours (lw, 66:1–93:4)
8. Nominal wages pr. hour (yww/lw, 66:1–93:4)
9. Consumption price (pc, 66:1–93:4)
10. Productivity (qf6/lw, 66:1–93:4)
11. Real wages pr. hour (yww/(pc×lw), 66:1–93:4)
12. M2 (m5000132, 60:1–92:12)

2. Use of the HP-filter

2.1. The smoothed series

In figures 2.1.1–2.1.12 the logarithm of the data series are plotted. The smoothing is done as follows. At the end of each series two additional observations are estimated by linear regression including seasonal dummies. On the original series prolonged with the two estimated values at each end, a five term centered moving average is run. This constitutes a simple seasonal adjustment and the HP-filter provides an additional smoothing. The default value of $\lambda=1600$ of the HP-filter is used for all the series.

The results are displayed in the upper part of the figures. In the lower part the residuals from applying the HP-filter and the difference between the trend estimated by the HP-filter and the five term centered moving average is displayed.

Since $\lambda=\infty$ corresponds to fitting a linear trend, it is evident that the value of λ will have a crucial impact on the size of the residuals.

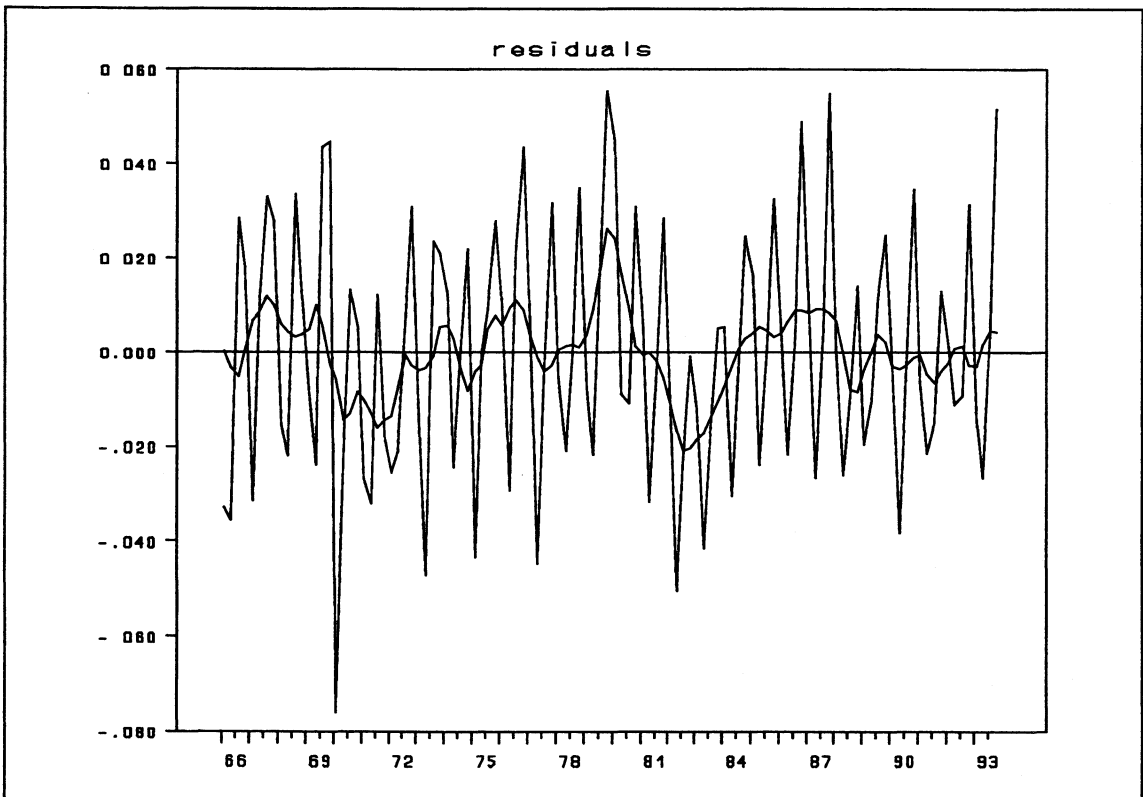
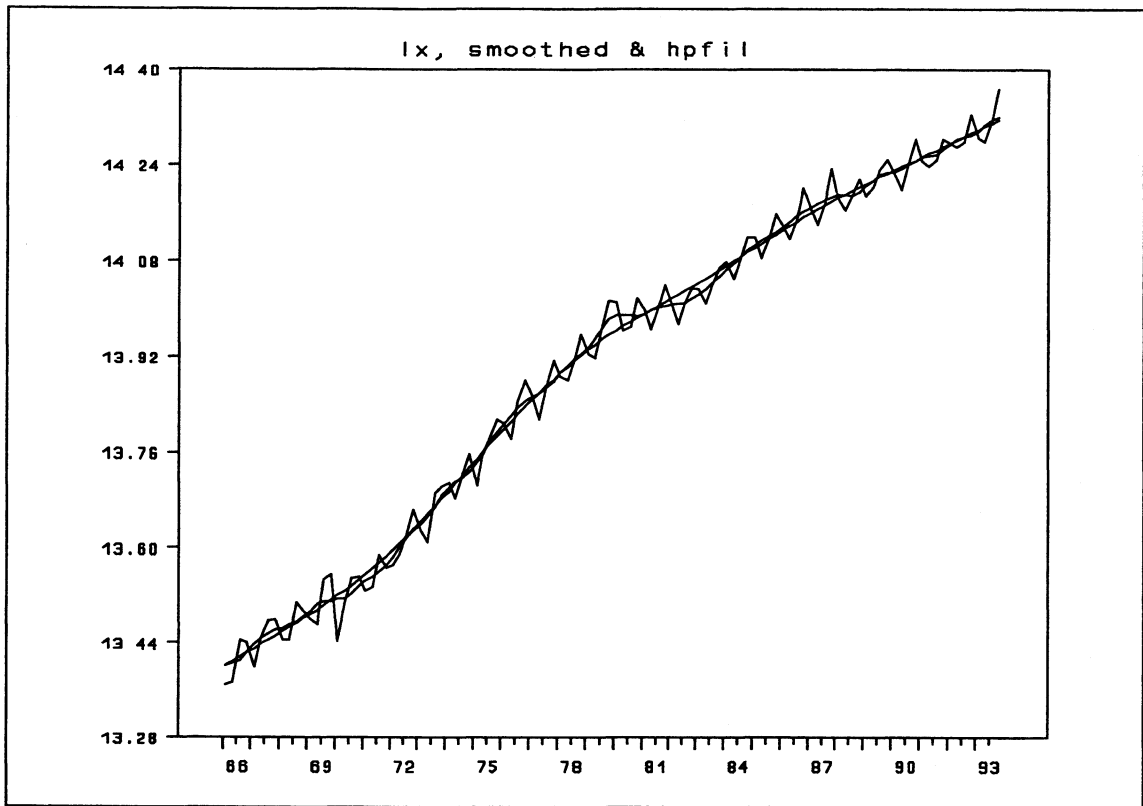


Figure 2.1.1. The logarithm of total GDP. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

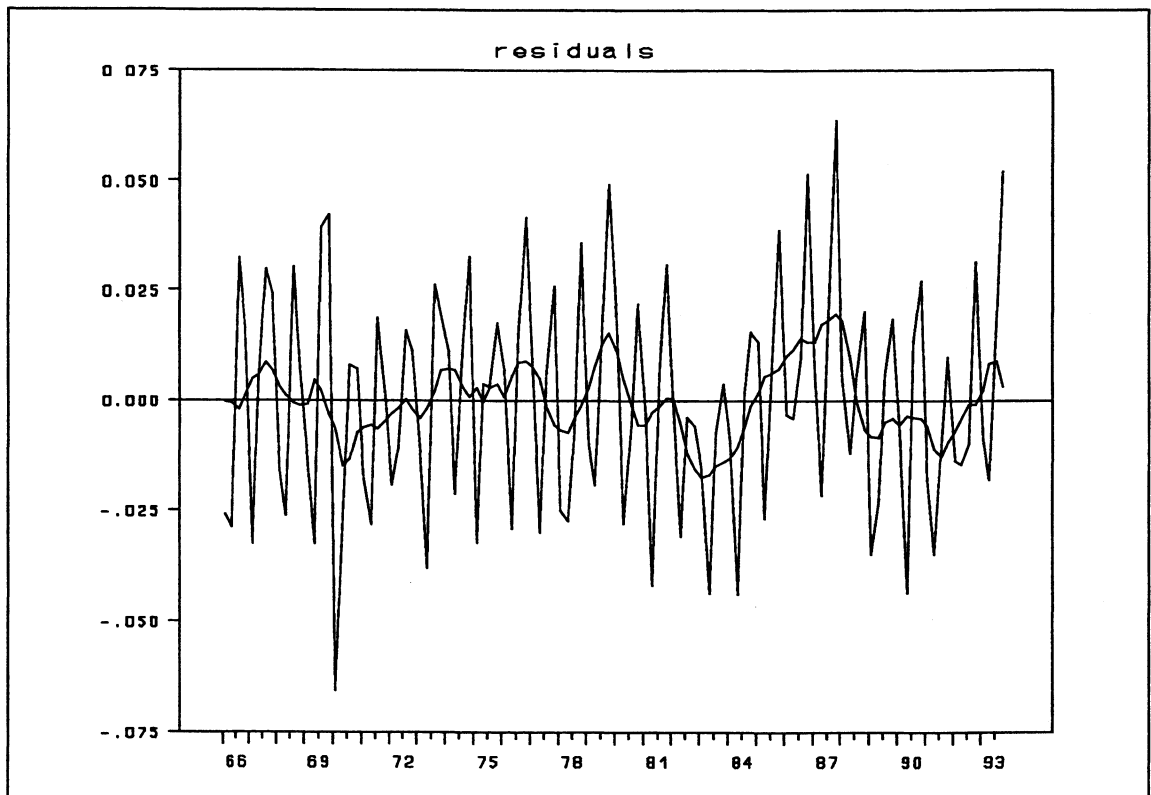
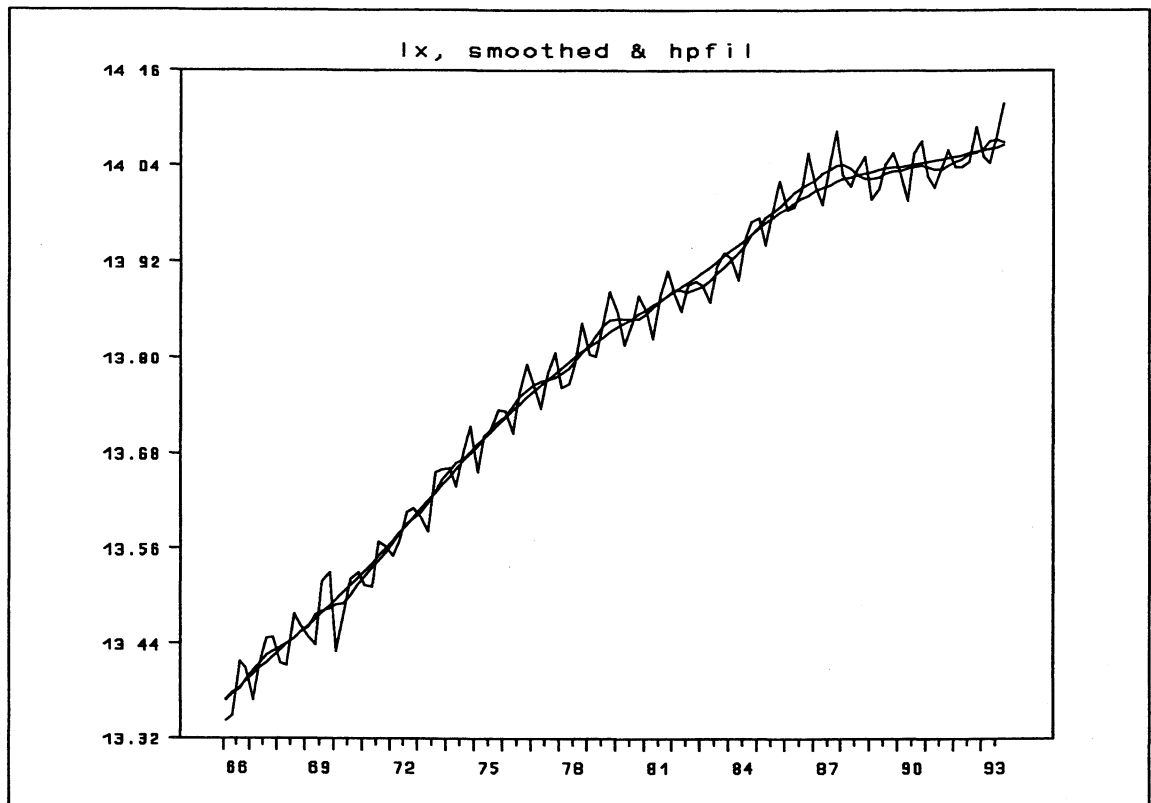


Figure 2.1.2. The logarithm of GDP mainland. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

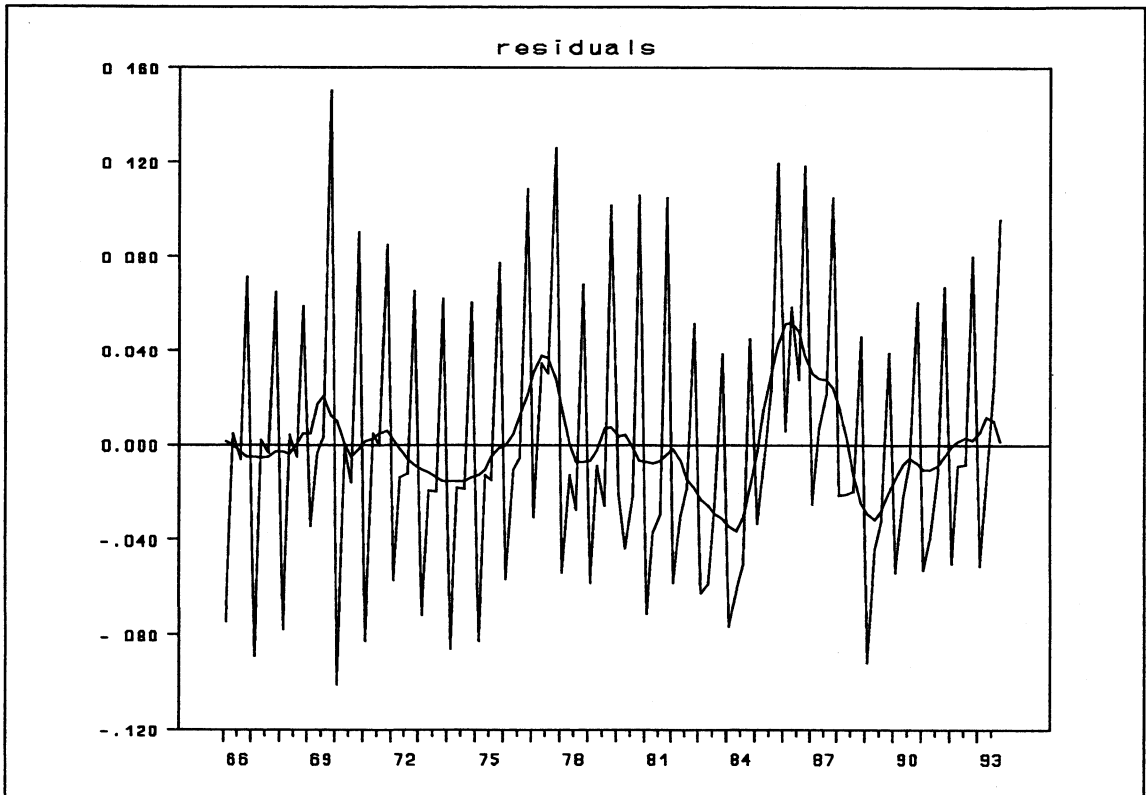
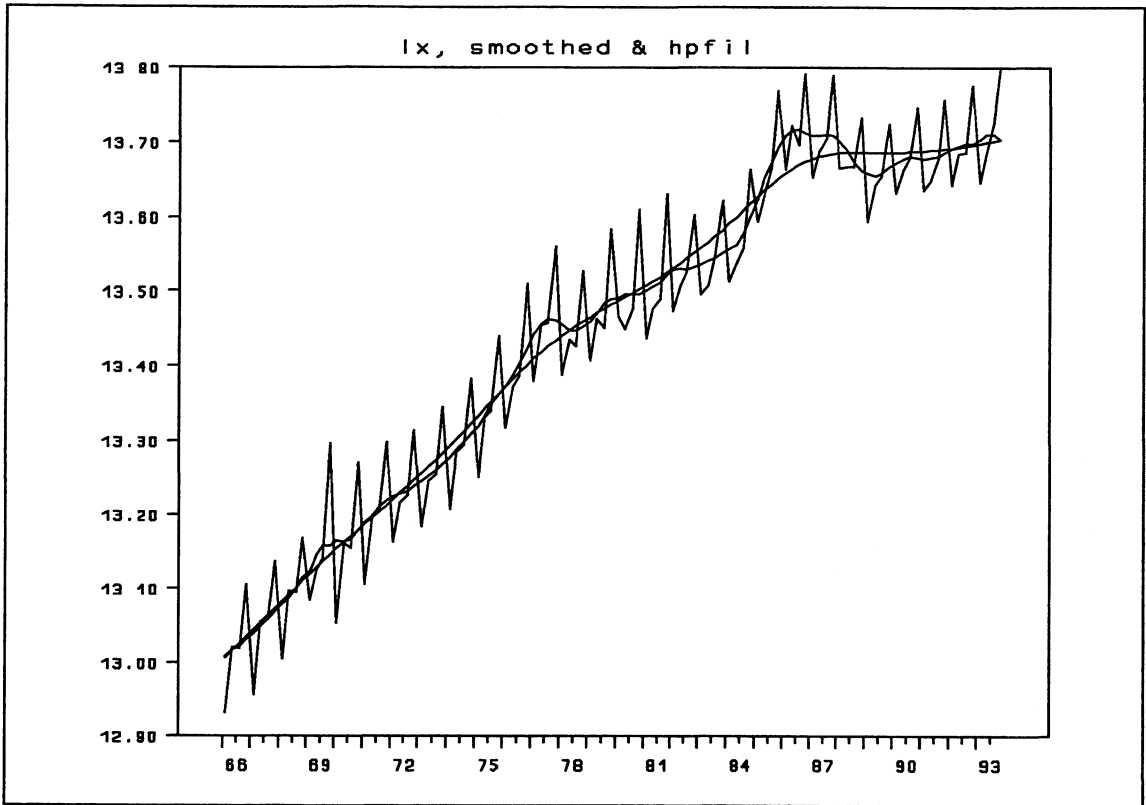


Figure 2.1.3. The logarithm of total private consumption.. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

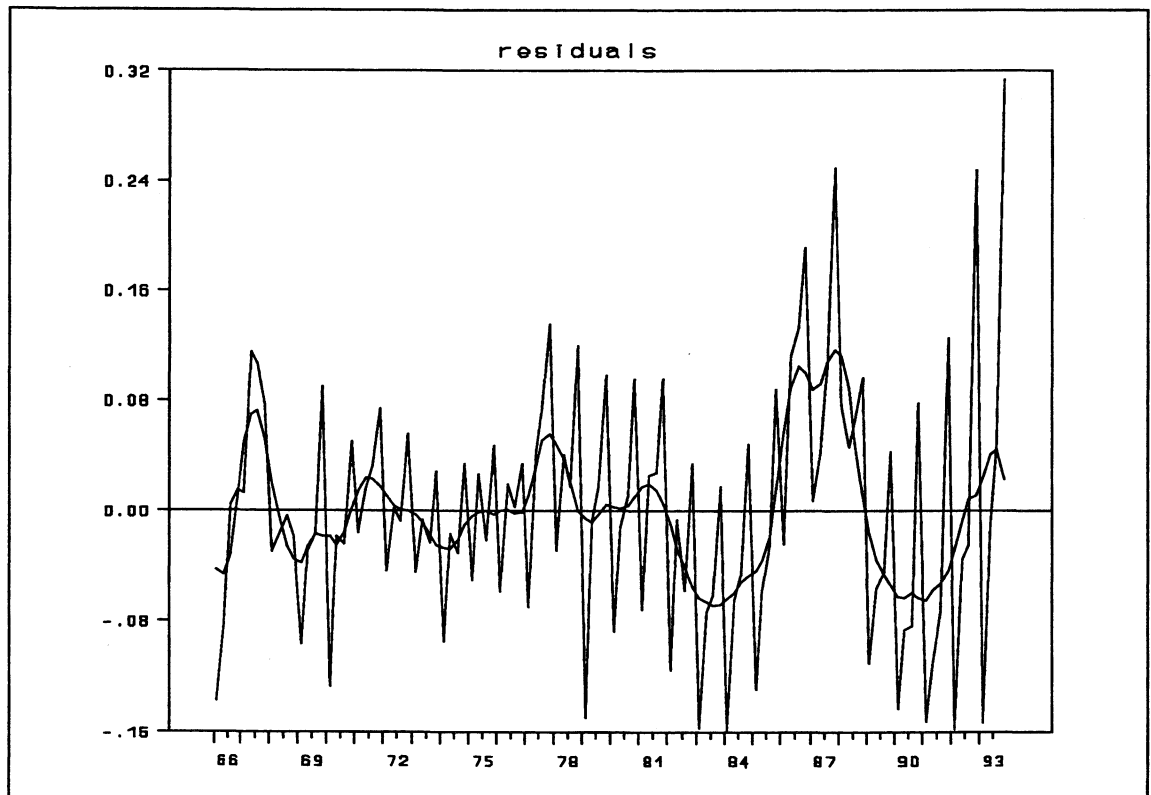
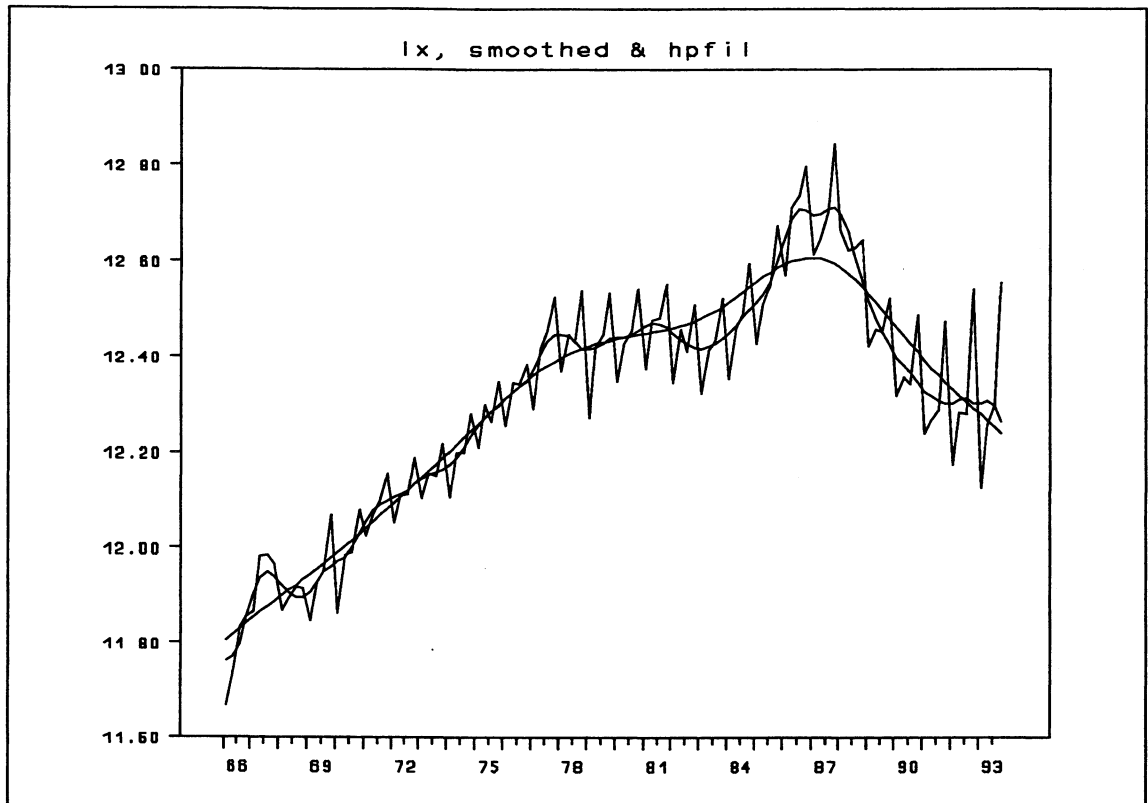


Figure 2.1.4. The logarithm of investments mainland.. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

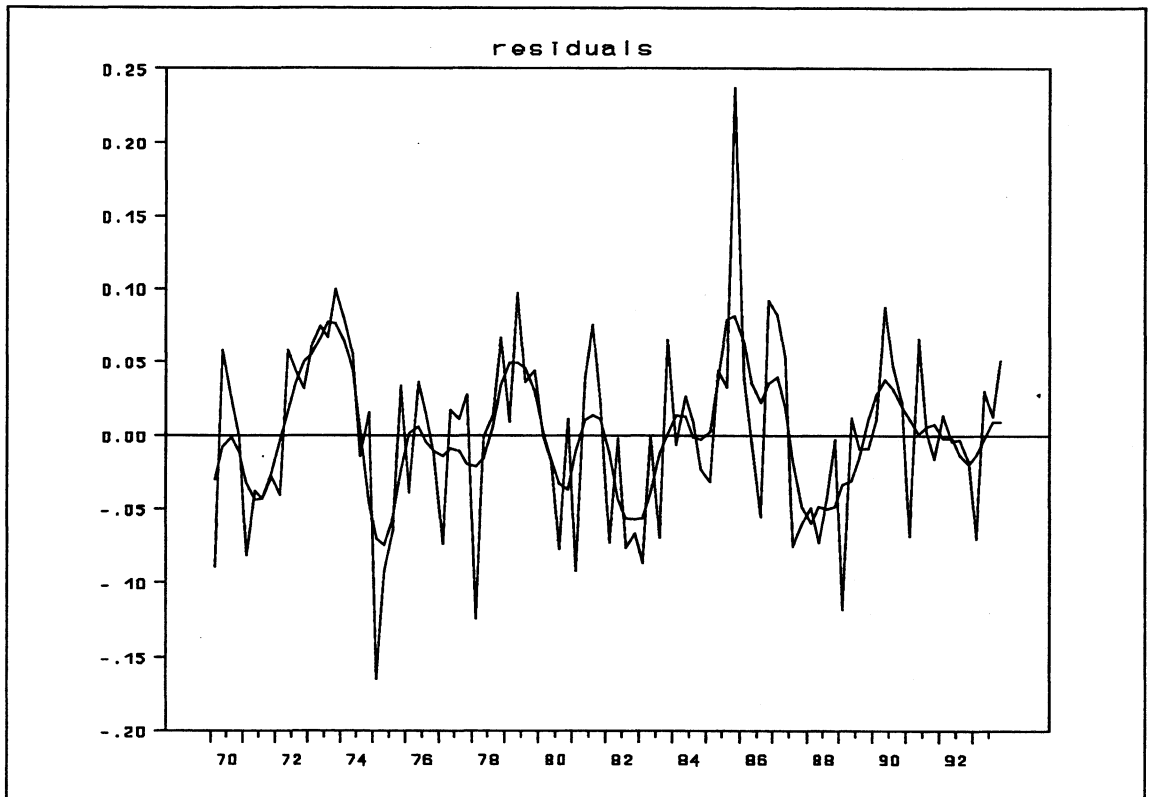
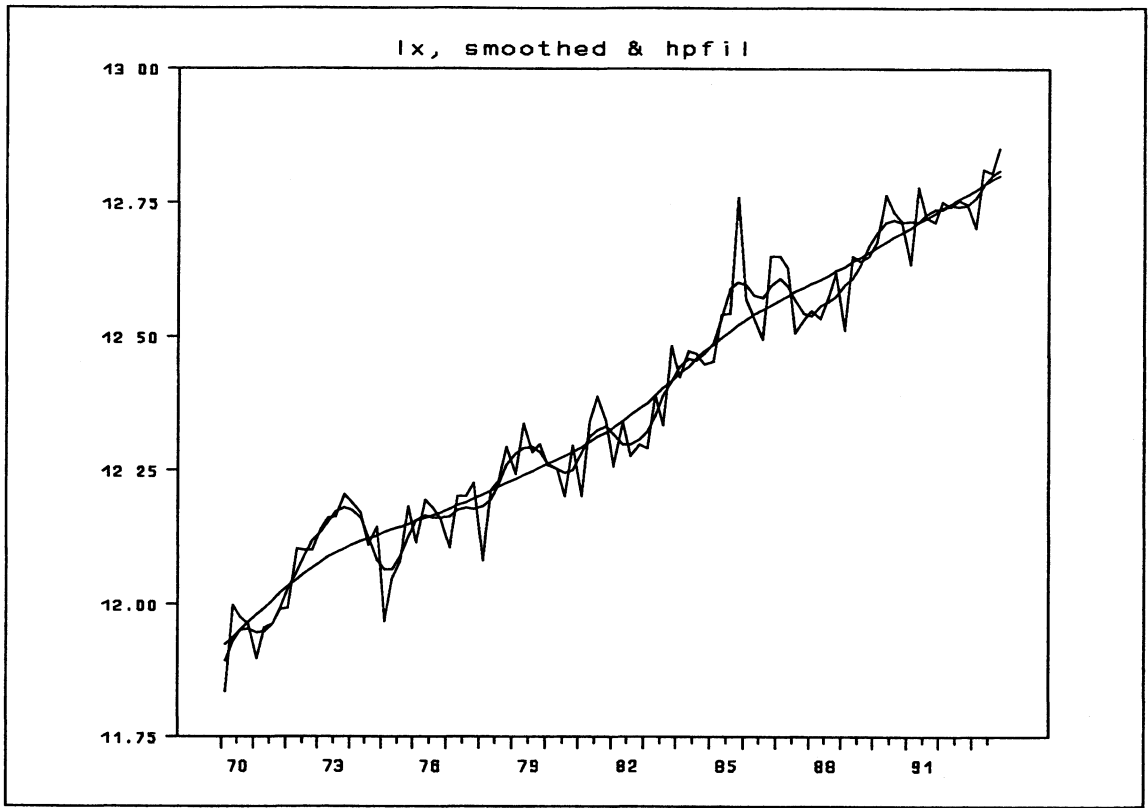


Figure 2.1.5. The logarithm of traditional export. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

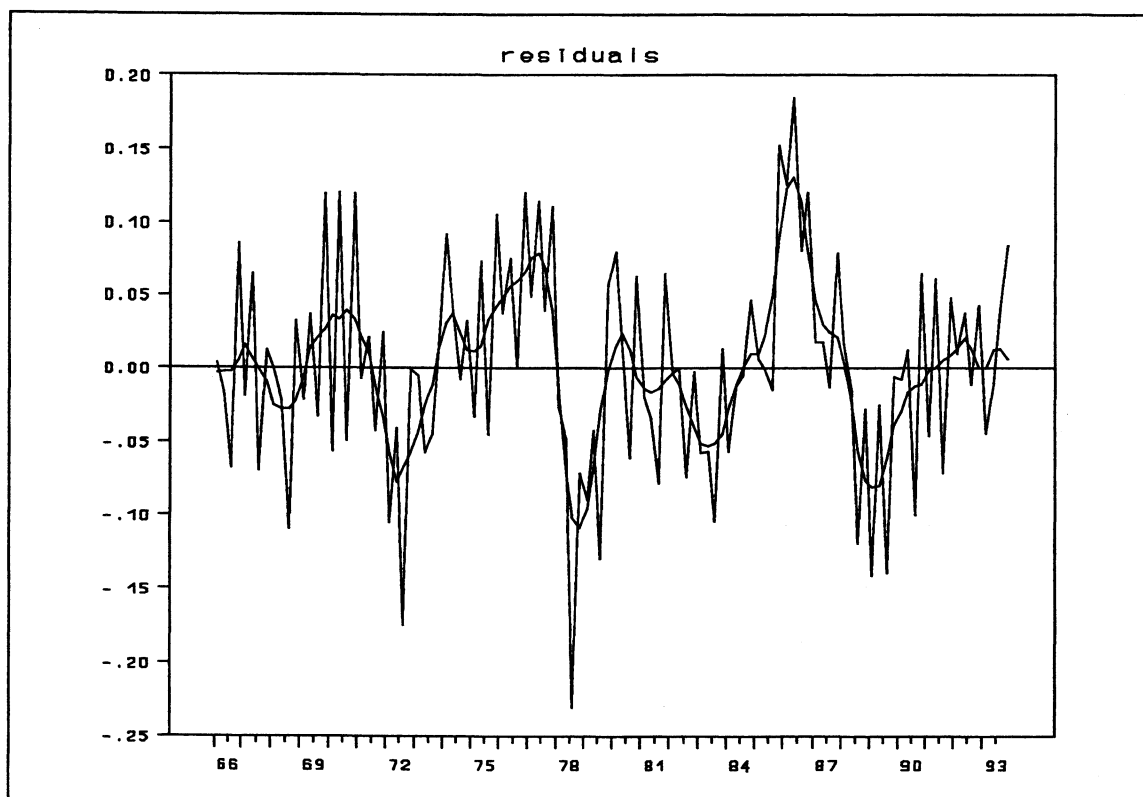
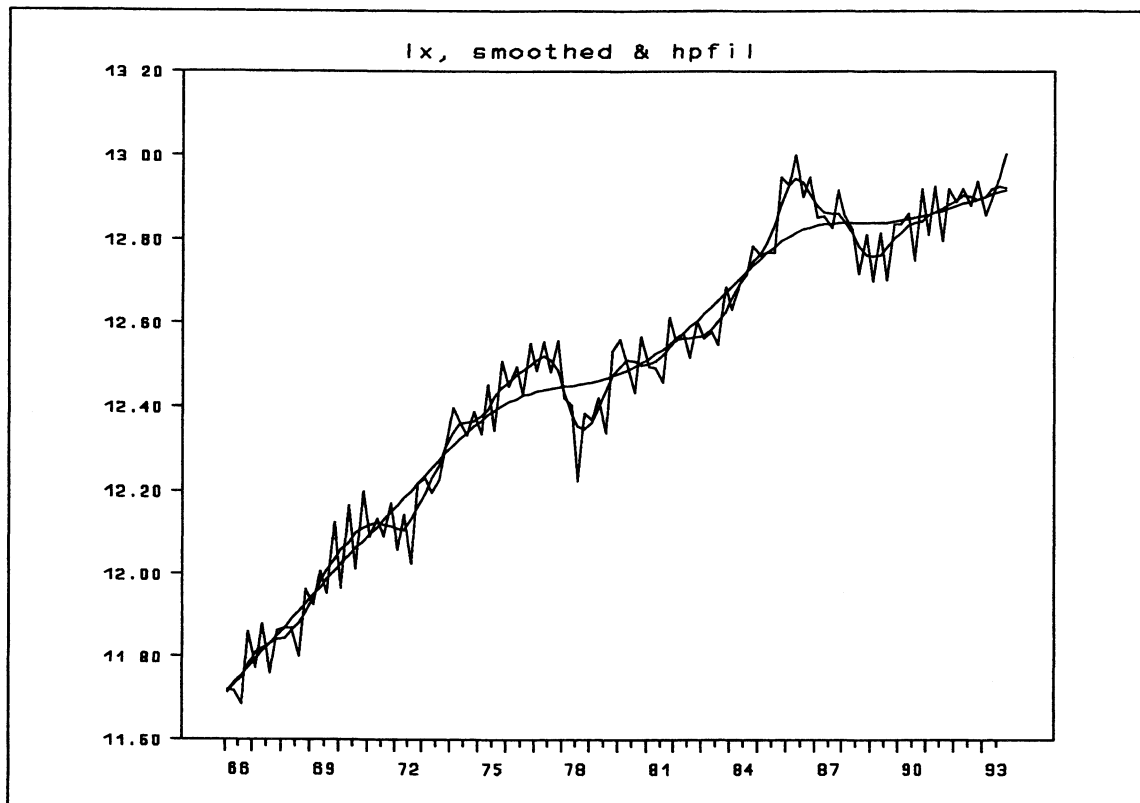


Figure 2.1.6. The logarithm of traditional import. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

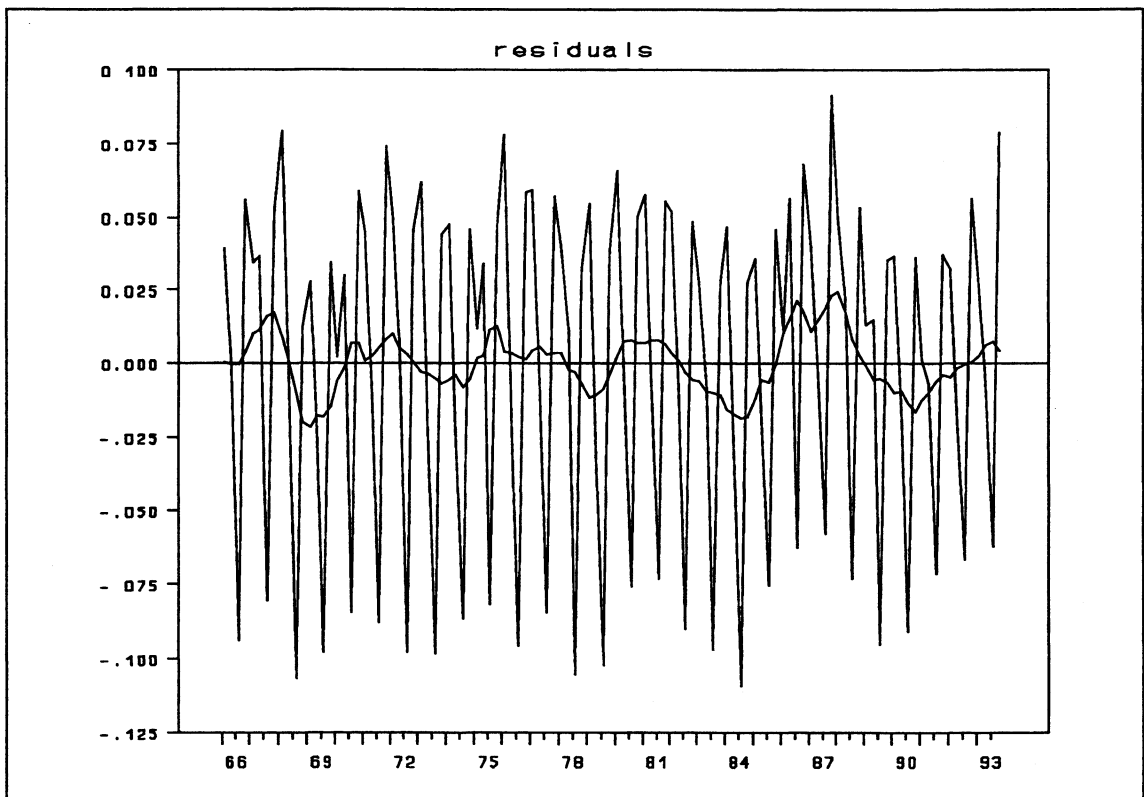
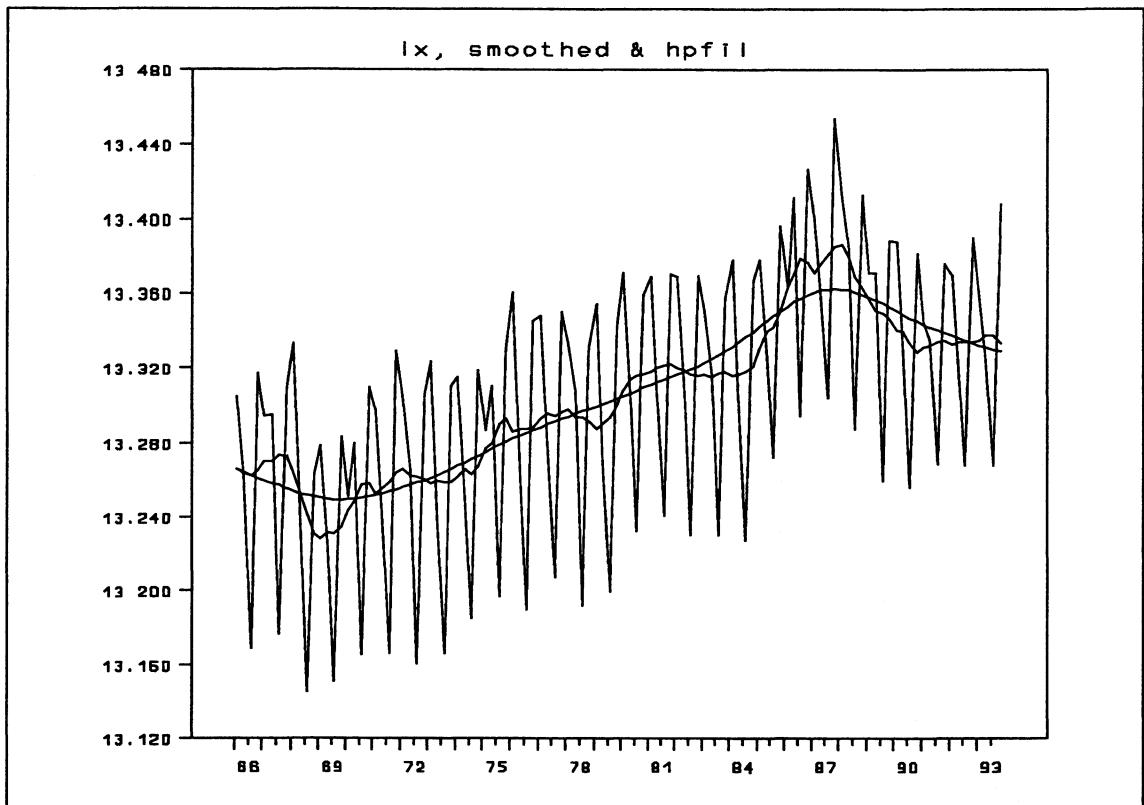


Figure 2.1.7. The logarithm of labor hours. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

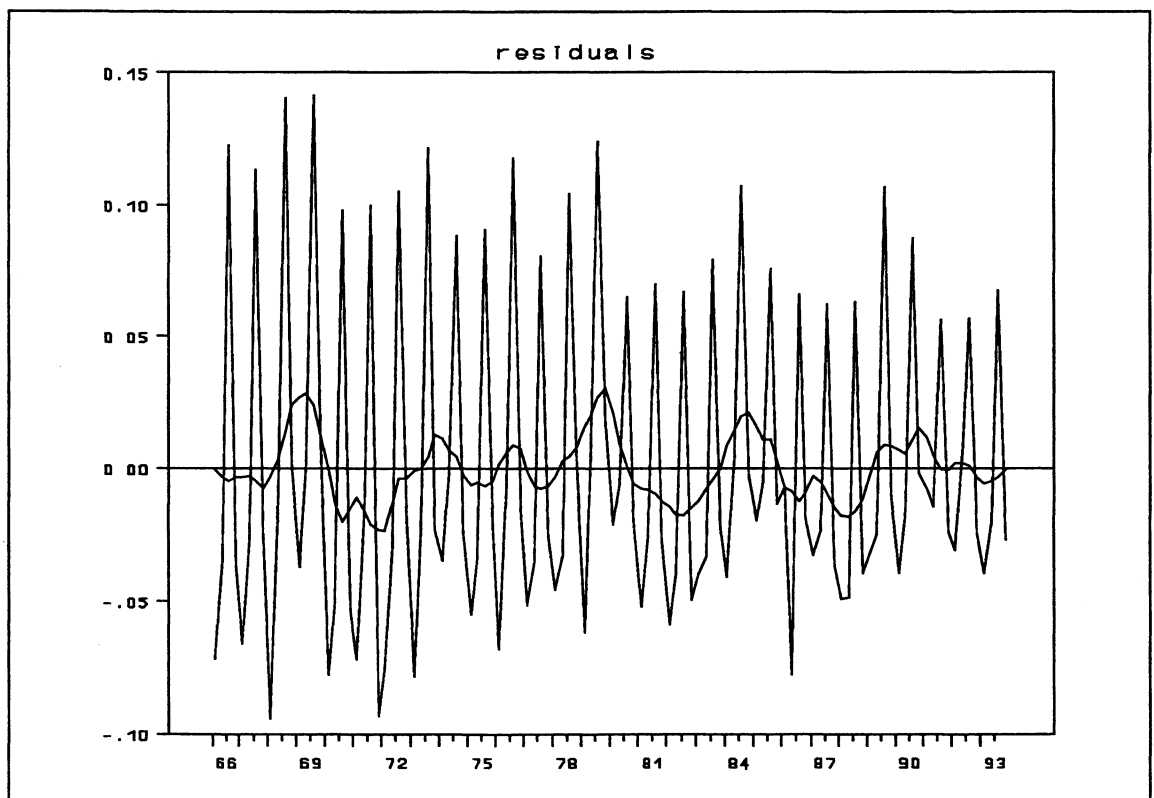
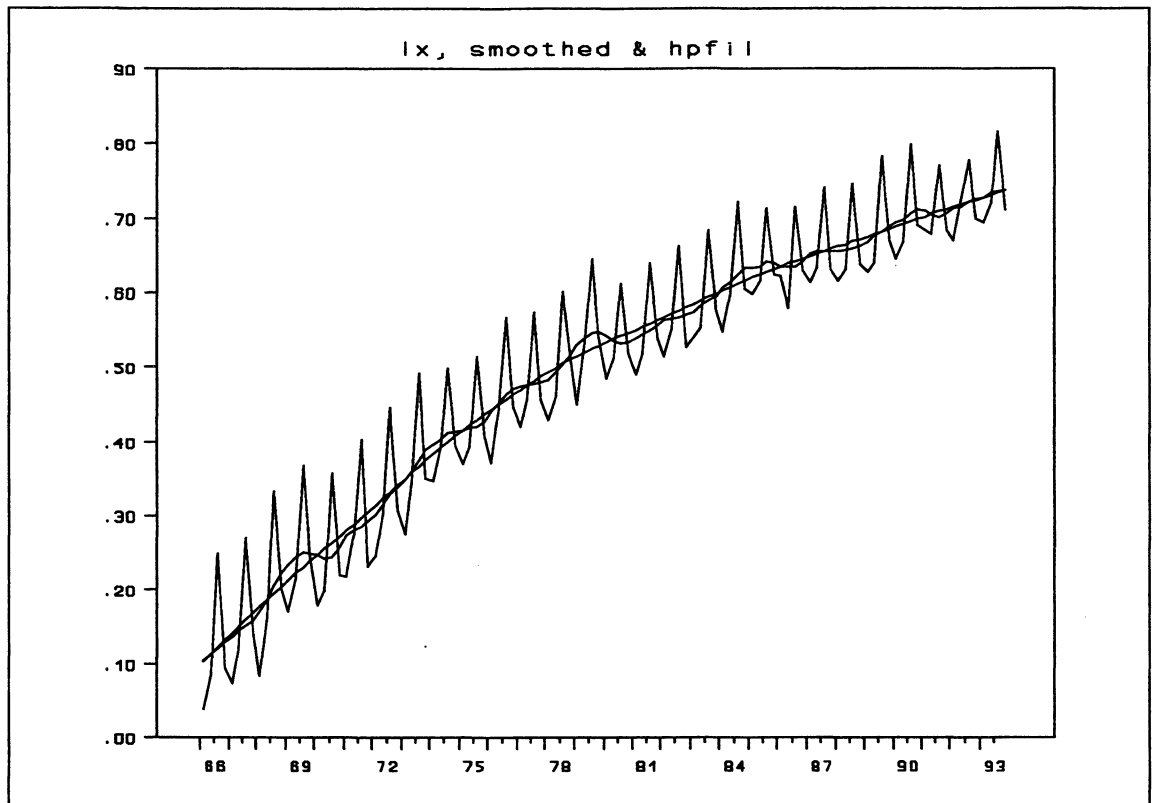


Figure 2.1.8. The logarithm of productivity. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

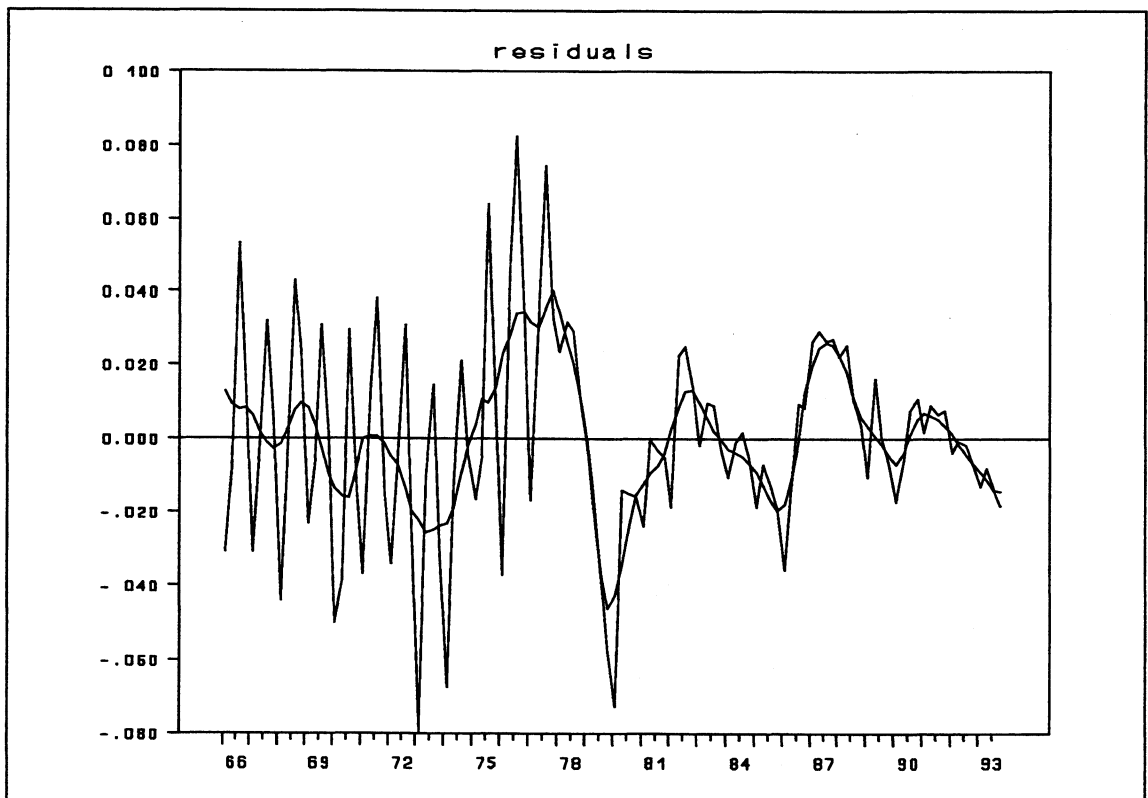
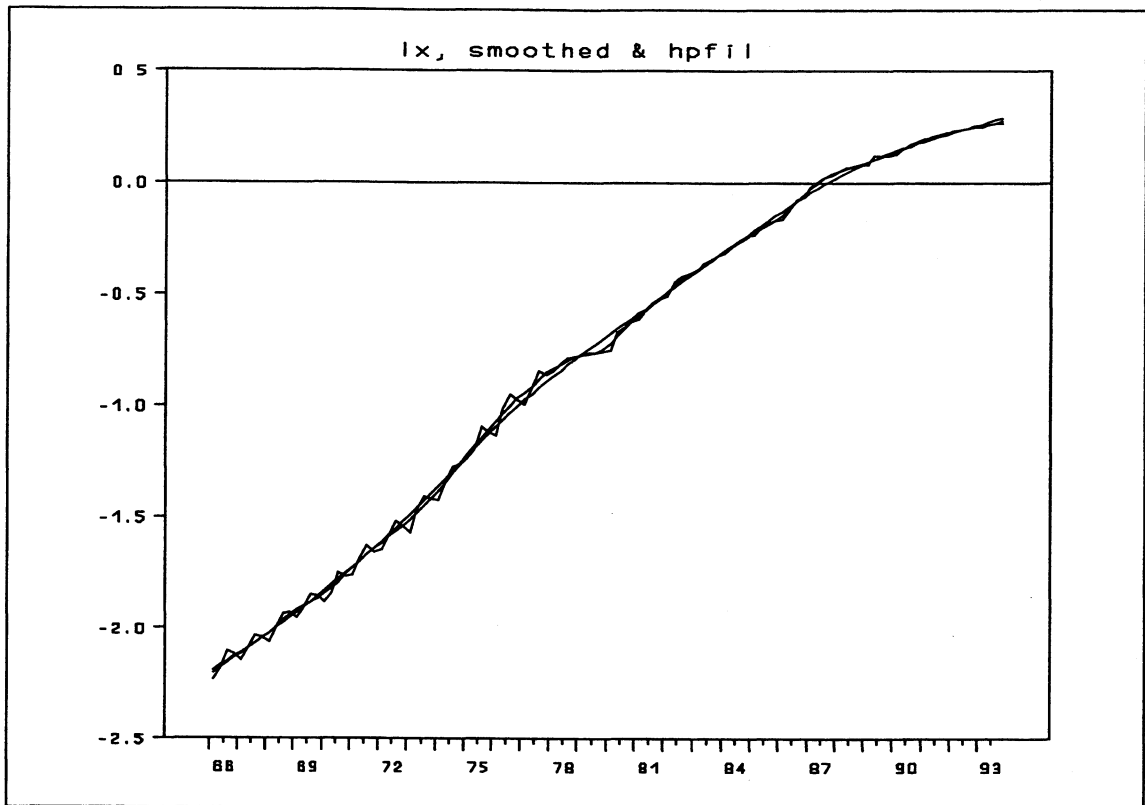


Figure 2.1.9. The logarithm of nominal wages. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

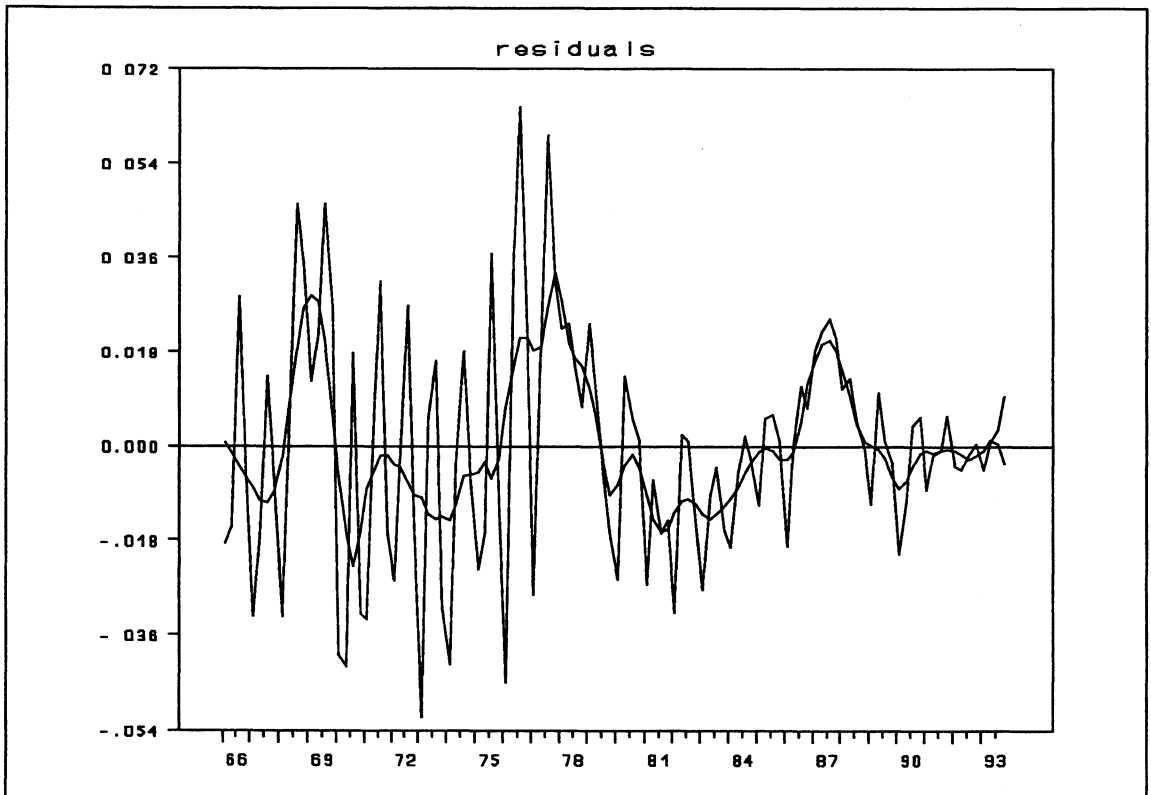
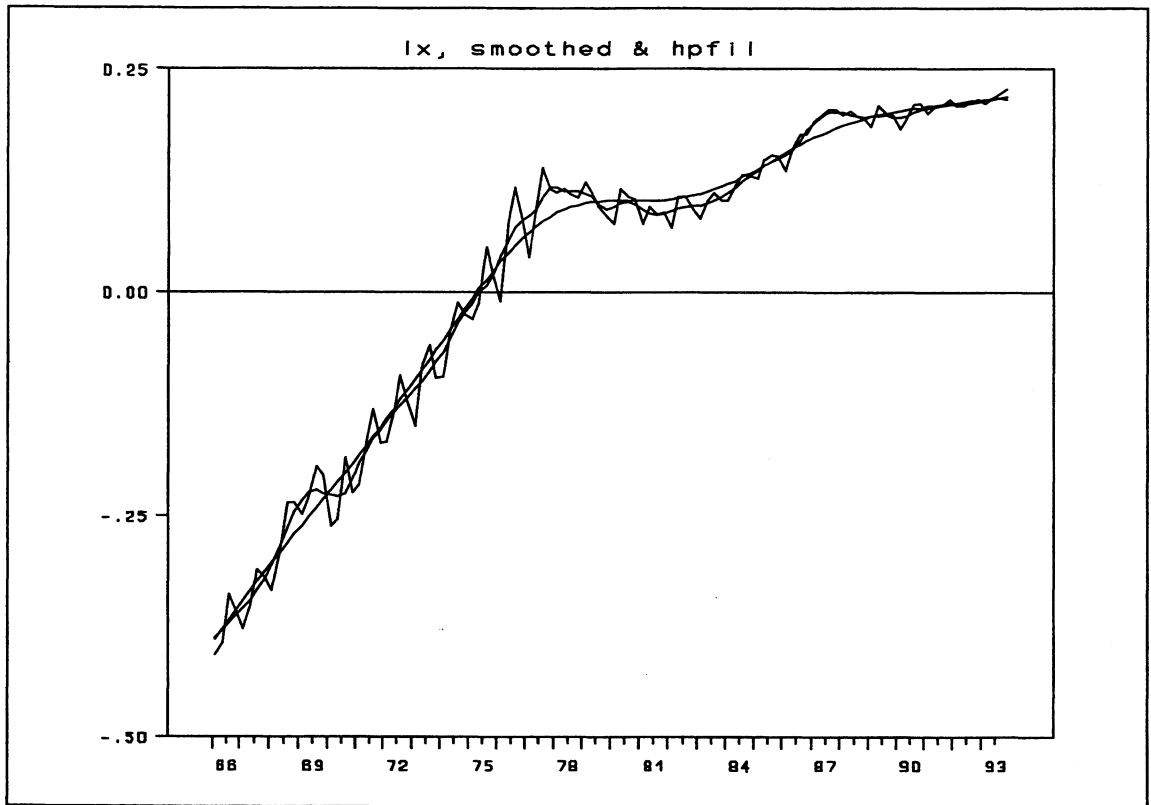


Figure 2.1.10. The logarithm of real wages. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

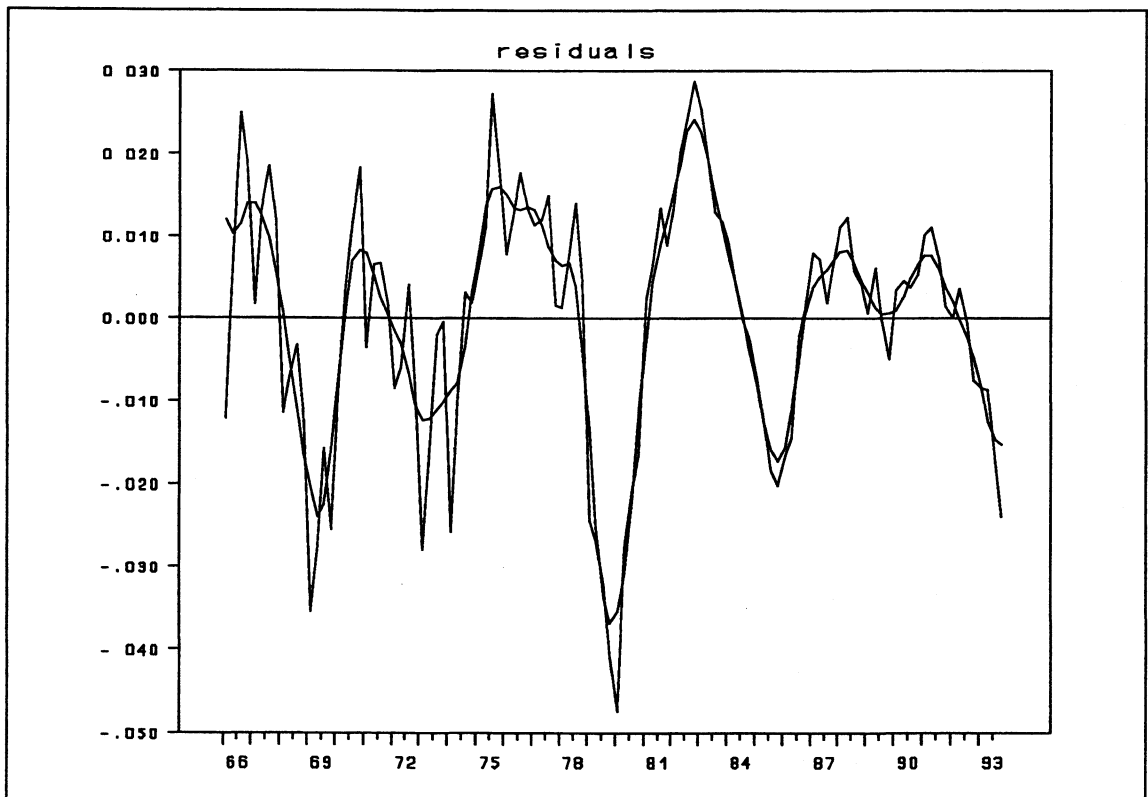
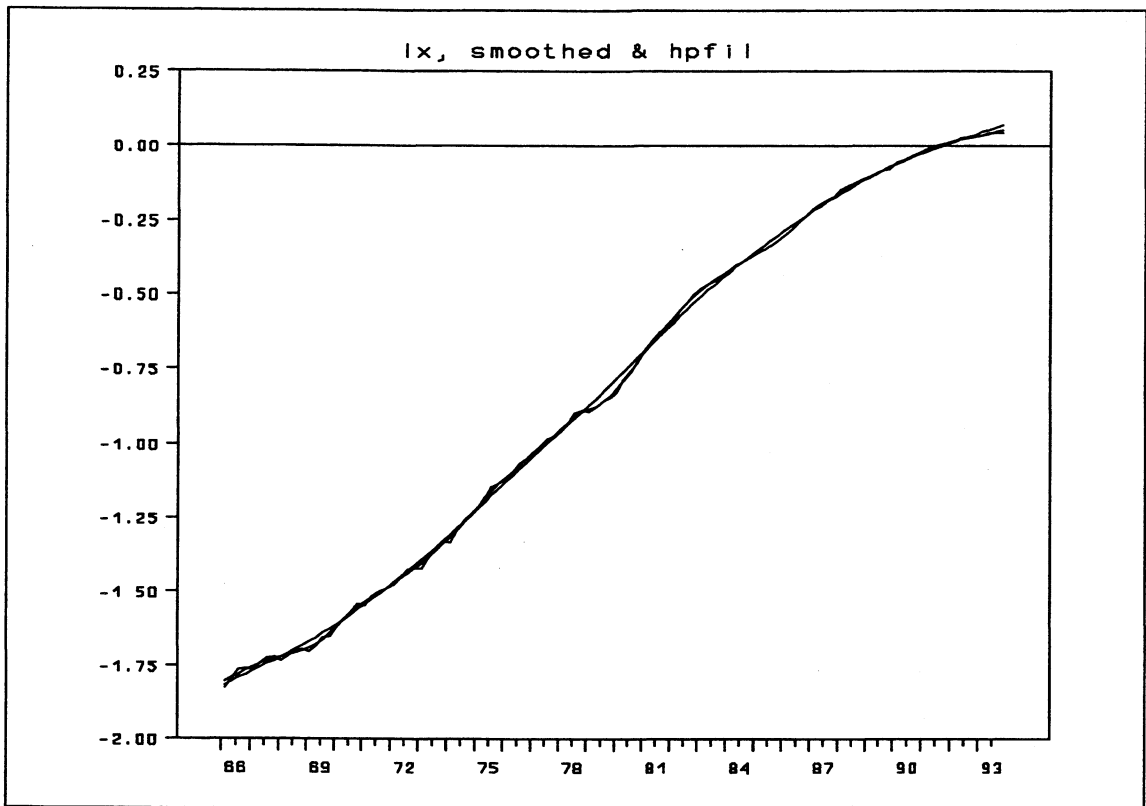


Figure 2.1.11. The logarithm of consumer price. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

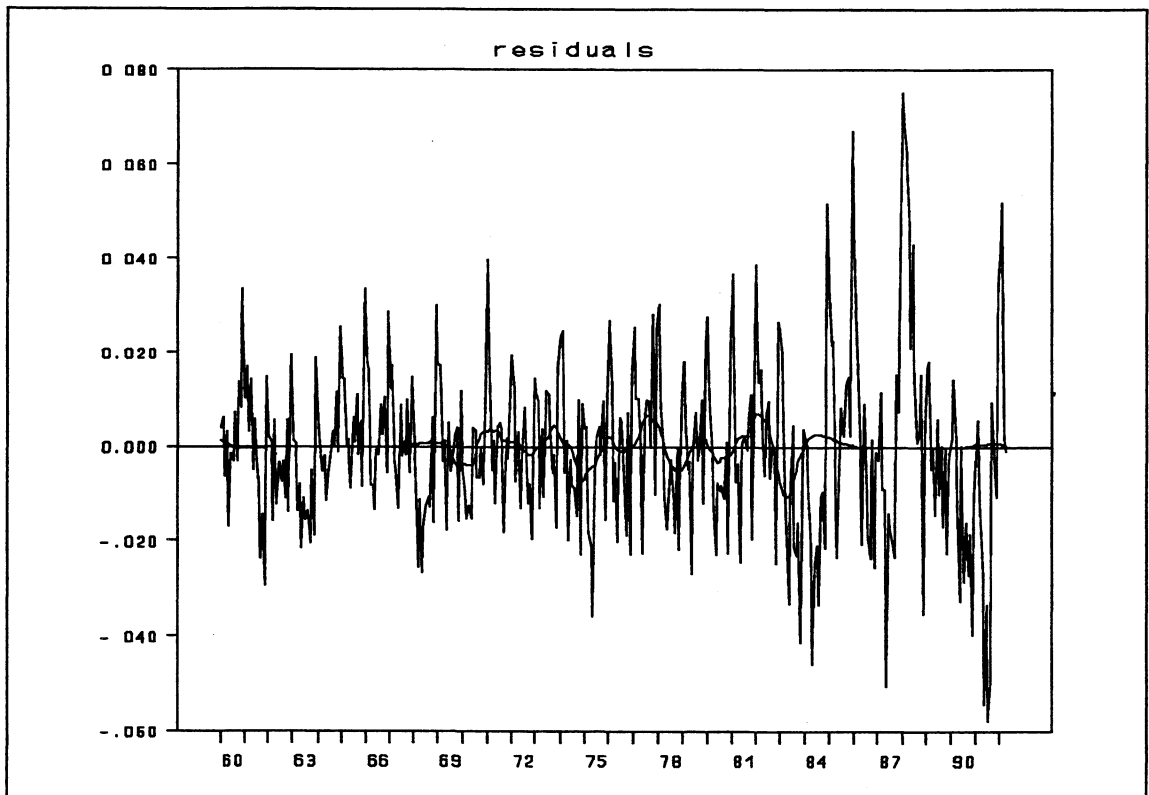
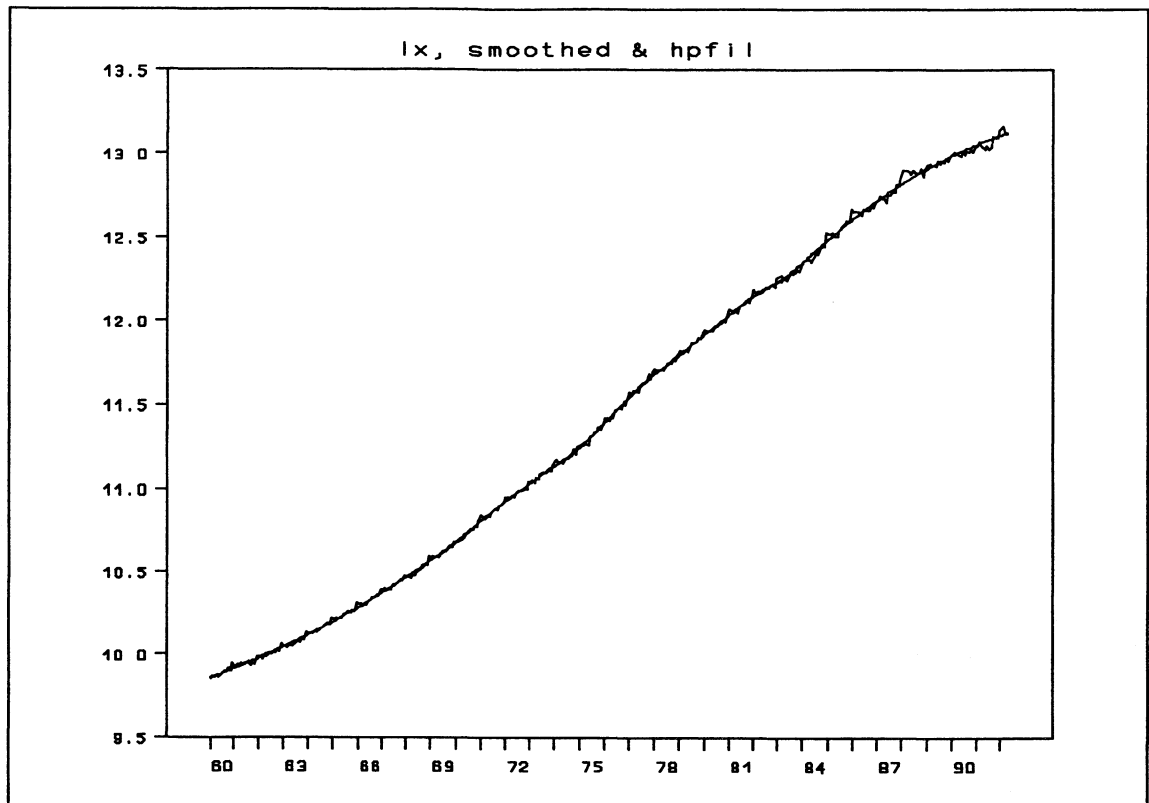


Figure 2.1.12. The logarithm of M2. In the upper panel the logarithm of the series, the five term centered moving average and the HP-filter with $\lambda=1600$. In the lower panel the residual from the HP-filter, and the difference of the five term centered moving average and the HP-filter.

2.2. Correlations based on the residuals

We first present a table of the cross-correlations between total GDP and the other series. Table 2.1.1 is based on the residuals between the observations and the series smoothed by the HP-filter.

Table 2.2.1. Correlations of the residuals of the GDP and the residuals of the other series after detrending with HP-filter

Var x	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
C	0.01	0.16	0.02	-0.08	0.00	0.45	0.08	0.07	0.18	0.33	0.08
JKS	-0.01	0.11	0.04	0.04	0.05	0.31	0.13	0.10	0.10	0.16	0.16
A	0.02	0.21	0.13	0.06	0.09	0.32	0.03	-0.06	0.01	0.00	-0.15
I	-0.02	0.02	-0.05	-0.09	-0.01	0.37	0.08	0.13	0.13	0.20	0.11
LW	0.04	0.02	-0.02	-0.07	-0.07	0.33	-0.09	0.05	0.15	0.05	0.14
PRD	-0.06	0.22	0.06	-0.04	0.05	0.46	0.13	-0.12	-0.07	0.23	-0.12
WW	-0.07	0.38	-0.07	-0.27	-0.14	0.30	-0.09	-0.27	-0.08	0.44	-0.05
RWW	-0.02	0.22	0.09	0.03	0.02	0.26	0.16	0.00	0.10	0.30	0.06
PC	-0.09	0.35	-0.16	-0.38	-0.21	0.22	-0.24	-0.37	-0.18	0.38	-0.10
M2	-0.10	0.43	-0.09	-0.28	-0.07	0.50	-0.03	-0.20	-0.02	0.51	-0.01

Since the comparable studies are usually done on seasonally adjusted data, we present in table 2.2.2 the correlations similar to those of table 2.2.1, but now based on the deviations between the seasonally adjusted data and the series filtered by the HP-filter.

Table 2.2.2. Correlations between the difference of the seasonally adjusted GDP and the trend estimated by the HP-filter, and similar differences in the other series

Var x	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
C	0.06	0.09	0.14	0.21	0.30	0.38	0.44	0.47	0.47	0.43	0.36
JKS	0.11	0.13	0.17	0.24	0.31	0.37	0.39	0.39	0.39	0.39	0.38
A	0.13	0.23	0.34	0.39	0.39	0.33	0.22	0.08	-0.06	-0.17	-0.23
I	-0.09	-0.08	-0.03	0.07	0.19	0.30	0.37	0.38	0.36	0.30	0.22
LW	0.00	-0.01	0.00	0.03	0.10	0.18	0.23	0.27	0.31	0.32	0.33
PRD	0.15	0.22	0.31	0.41	0.48	0.48	0.40	0.25	0.10	-0.04	-0.15
WW	0.14	0.05	-0.03	-0.10	-0.14	-0.16	-0.12	-0.06	0.03	0.12	0.19
RWW	0.22	0.25	0.27	0.31	0.33	0.35	0.37	0.38	0.38	0.36	0.32
PC	-0.03	-0.16	-0.29	-0.42	-0.50	-0.53	-0.51	-0.43	-0.32	-0.18	-0.04
M2	0.00	0.00	0.02	0.07	0.14	0.21	0.27	0.32	0.36	0.37	0.36

The correlations are between the current value of GDP and the value of the other series as indicated in the tables. Depending on whether the contemporaneous correlation is positive or negative the series are considered as pro- or countercyclical. We see that except for the seasonally adjusted values for the wages and prices all correlations indicate that the variables are procyclical. This

countercyclical feature of prices for seasonally adjusted data is also found by Kydland and Prescott (1990) in data from the US and by Blackburn and Ravn (1992) for data from the UK.

We also remark that for some variables there is a fairly large discrepancy between the impression from the correlations of the two tables. The productivity seems thus to be fairly symmetrically correlated with GDP, while based on the seasonal adjusted data there seems to be a tendency for it to lead the cycle since the highest correlations are between the current value of productivity and future values of GDP.

Also there may be some doubt about the significance. According to the figures computed by Harvey and Jaeger (1993) it seems that a significant value at the 5% level is around 0.4 when there are around 100 observations. This is based on a null hypothesis specifying two independent random walks. However, the calculations are based on a normal approximation where the variance involve the covariances in the series. Since these die slower in quarterly series, it is reasonable to believe that they should be somewhat higher in the case we consider. Hence according to traditional standards, most of the figures of table 2.2.1 and 2.2.2 are not significant taken individually.

Table 2.2.3 and 2.2.4 below contain similar correlations but now the GDP of mainland Norway is used.

Table 2.2.3. Correlations of the mainland GDP and the residuals of the other series after detrending with HP-filter

Var x	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
C	0.09	0.19	0.09	0.06	0.13	0.60	0.18	0.13	0.17	0.26	-0.02
JKS	0.04	0.18	0.13	0.16	0.18	0.45	0.27	0.25	0.21	0.23	0.20
A	0.10	0.26	0.21	0.17	0.09	0.27	-0.04	-0.09	-0.06	-0.13	-0.25
I	0.10	0.11	0.08	0.03	0.09	0.44	0.11	0.17	0.11	0.11	0.00
LW	0.07	0.02	0.07	0.04	-0.01	0.46	-0.04	0.17	0.27	0.09	0.17
PRD	-0.08	0.22	0.02	-0.04	0.06	0.44	0.09	-0.17	-0.18	0.15	-0.18
WW	-0.11	0.31	-0.06	-0.21	-0.10	0.29	-0.02	-0.21	-0.04	0.44	0.02
RWW	-0.05	0.19	0.09	0.04	0.03	0.26	0.20	0.00	0.09	0.30	0.09
PC	-0.11	0.29	-0.14	-0.32	-0.16	0.20	-0.18	-0.29	-0.12	0.38	-0.03
M2	-0.09	0.36	-0.05	-0.18	0.00	0.50	0.02	-0.13	0.00	0.45	-0.03

Table 2.2.4. Correlations between the difference of seasonally adjusted mainland GDP and the trend estimated by the HP-filter, and similar differences in the other series

Var x	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
C	0.24	0.28	0.35	0.45	0.55	0.63	0.62	0.55	0.43	0.28	0.14
JKS	0.21	0.29	0.39	0.50	0.60	0.67	0.69	0.65	0.59	0.51	0.42
A	0.32	0.41	0.47	0.46	0.38	0.25	0.08	-0.10	-0.25	-0.34	-0.36
I	0.23	0.23	0.26	0.32	0.40	0.45	0.44	0.38	0.26	0.12	-0.01
LW	0.12	0.16	0.23	0.30	0.40	0.50	0.54	0.56	0.55	0.49	0.42
PRD	0.03	0.10	0.21	0.32	0.37	0.35	0.22	0.04	-0.14	-0.25	-0.29
WW	-0.07	-0.06	-0.05	-0.05	-0.05	-0.03	0.03	0.11	0.21	0.30	0.36
RWW	0.08	0.14	0.20	0.26	0.31	0.35	0.37	0.39	0.39	0.39	0.36
PC	-0.16	0.20	-0.26	-0.31	-0.35	-0.36	-0.31	-0.22	-0.10	0.03	0.14
M2	0.01	0.05	0.13	0.23	0.33	0.40	0.42	0.41	0.37	0.32	0.27

2.3. Spectral analysis of the residuals from the HP filter

As explained in the introduction, spectral analysis is a representation of the correlation structure that is particularly valuable for detecting fixed cycles in a stationary time series. We shall in this section present the results from a spectral analysis from fitting a standard HP filter to the twelve chosen series. A fixed seasonal pattern is removed by regressing the residuals on a set of seasonal dummies. Since there is a break in the seasonal pattern in 1978:1, separate sets of dummies are used before and after this period. The logarithm of the spectral densities and the coherence and phase with respect to the series for GDP are then estimated.

The results are presented in figures 2.3.1–11. The grid in the figures showing the spectral densities are at the frequencies $0, \pi/8, 2\pi/8, \dots, \pi$. The cycle corresponding to $\pi/8$ is 16 quarters or four years, to $2\pi/8$ two years, and to $\pi/2$ one year. The only exception is the M2 series, which is monthly. Here the grid indicates the values $\pi/12, 2\pi/12, \dots, \pi$, so that the first grid corresponds to a 2 year cycle and the second to a yearly cycle. In these figures a 95% confidence interval is also indicated based on the assumption that the time series consisting of the residuals are stationary.

In most of the series there is a fairly strong seasonal component despite the fact that the fixed pattern has been removed. Also in most of the series there seems to be a pike in the specter corresponding to around five or six years. The exception is traditional export where movements are more frequent, a fact that is fairly evident by looking at the residuals plotted in figure 2.1.5. As mentioned in the introduction, there is a problem with interpreting these frequencies since these are exactly those induced by the HP filter when it is applied to data having a root at unity, cfr. Cogley and Nason (1993), Harvey and Jaeger (1993) and King and Rebelo (1993). The possibility that these cycles are spurious must therefore be kept in mind.

The lower panel of figures 2.3.2–11 are the coherence of the phase with the mainland GDP series. We remark that the cyclic behavior at the frequencies corresponding to the long run behavior is

much less pronounced than in the univariate specters. The most pronounced exception is productivity, which is not so surprising taking the definition of the series into account. This phenomenon should be compared with the fact mentioned in the introduction, that the coherence of a stationary time series is invariant to the effect of a filter. In this case the HP filter is applied to series which are non-stationary beyond any doubt, so the situations are not quite analogous. However, the lack of marked frequencies corresponding to the usual business cycles in the plot for the coherence does not exactly corroborate an assertion of strong comovements between the series.

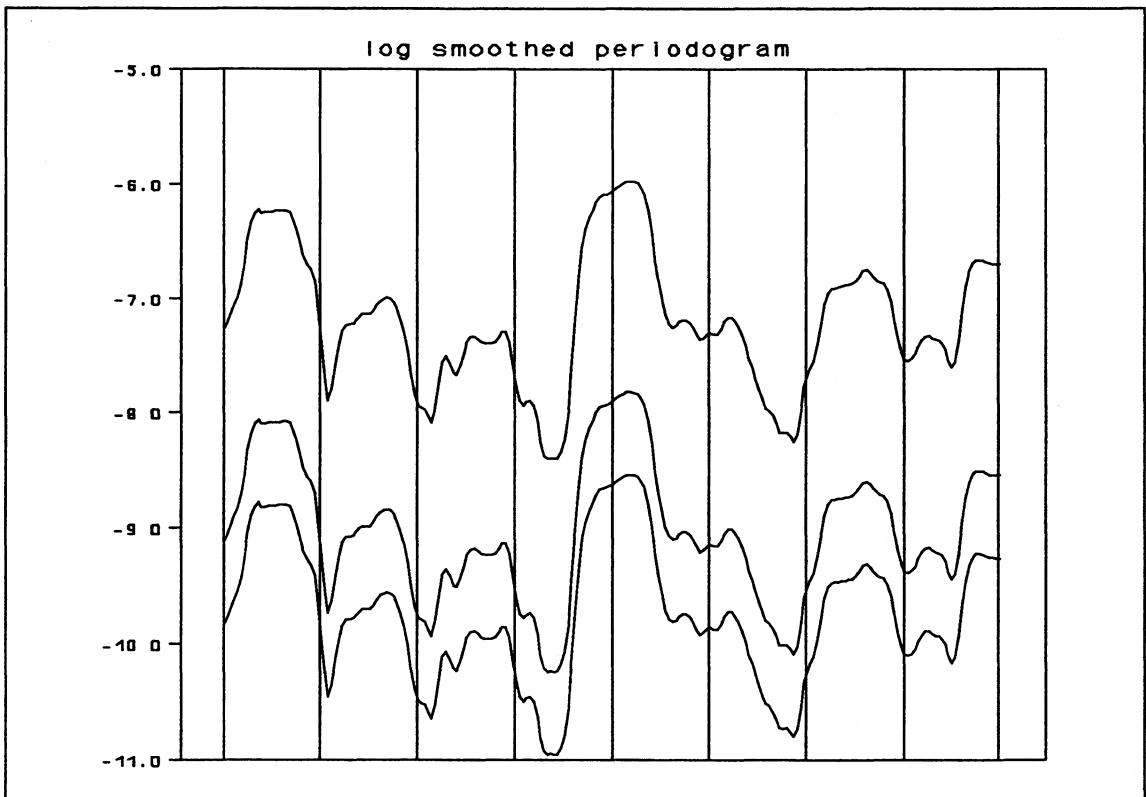
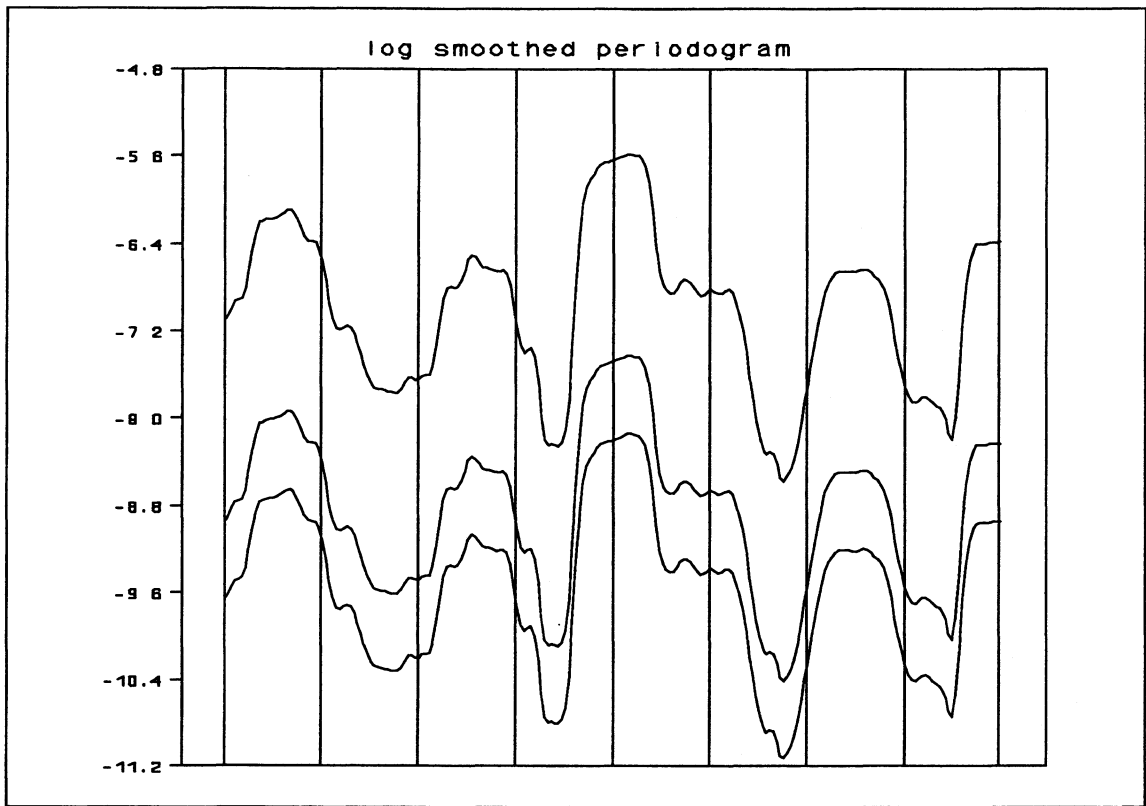


Figure 2.3.1. Estimated log-specter based on the residuals from smoothing with a HP filter. Total GDP in the upper panel and mainland GDP in the lower panel. A 95% confidence band is indicated.

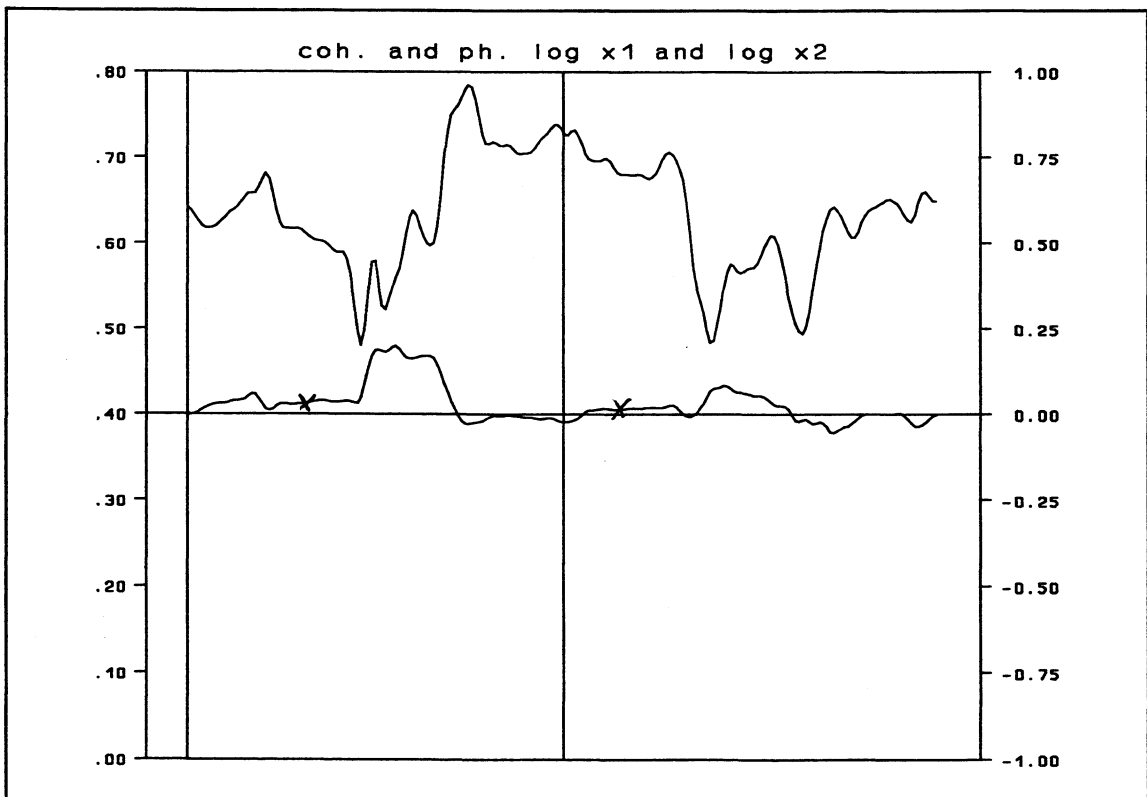
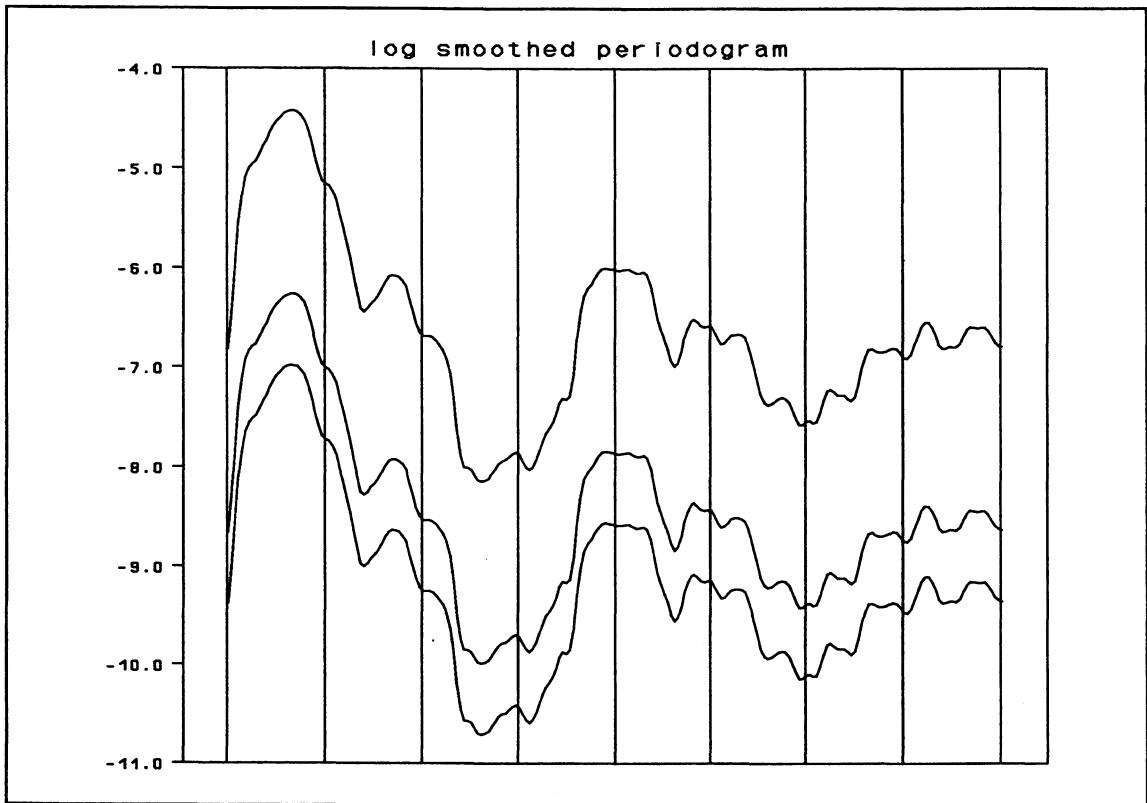


Figure 2.3.2. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing total private consumption with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

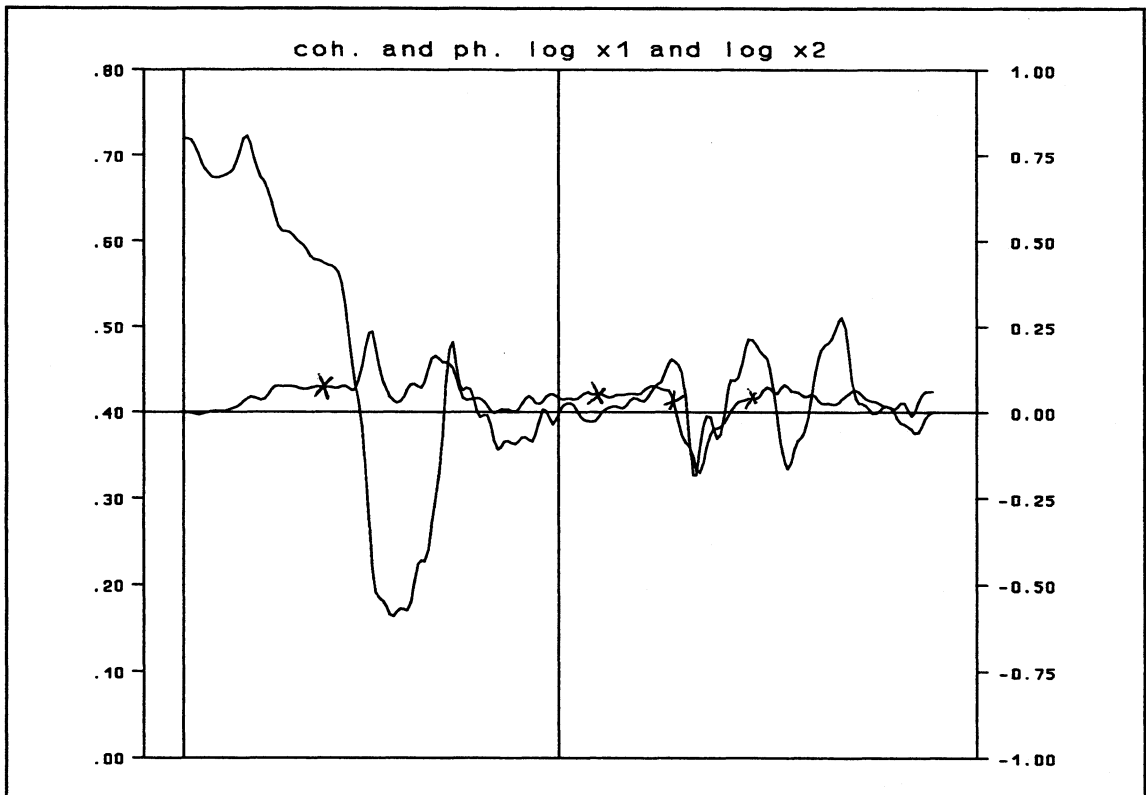
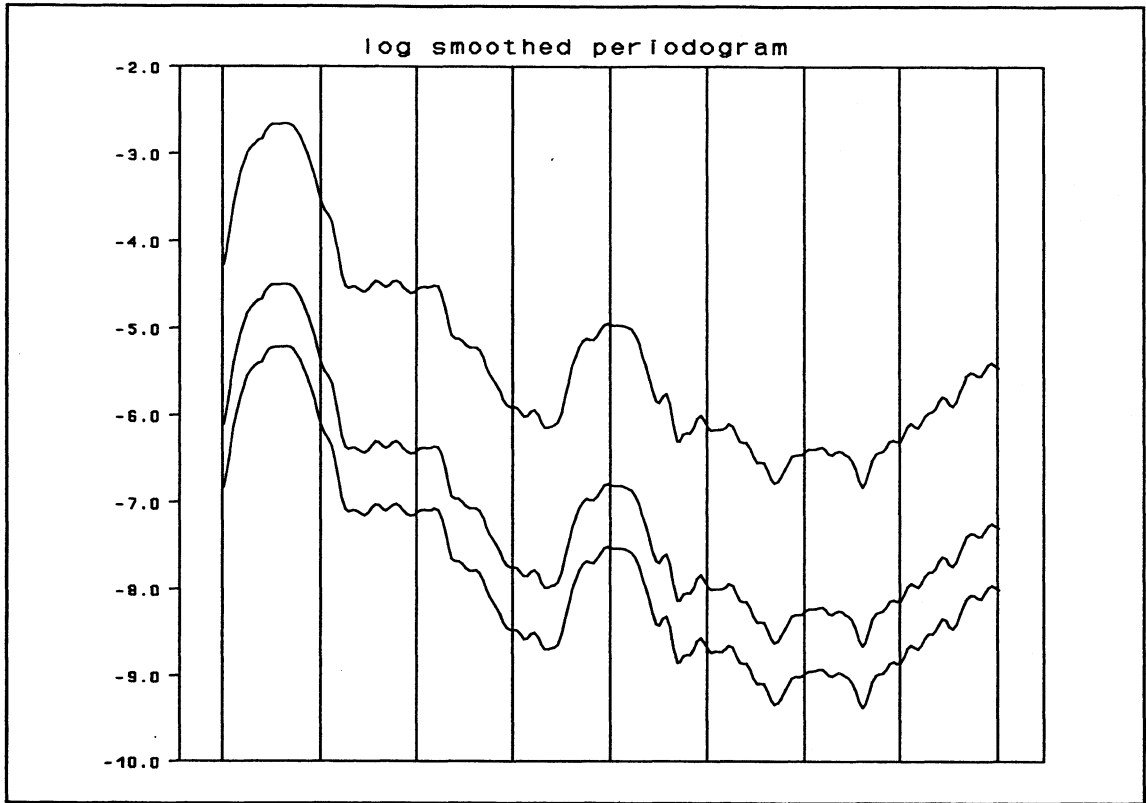


Figure 2.3.3. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing total investments in mainland with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

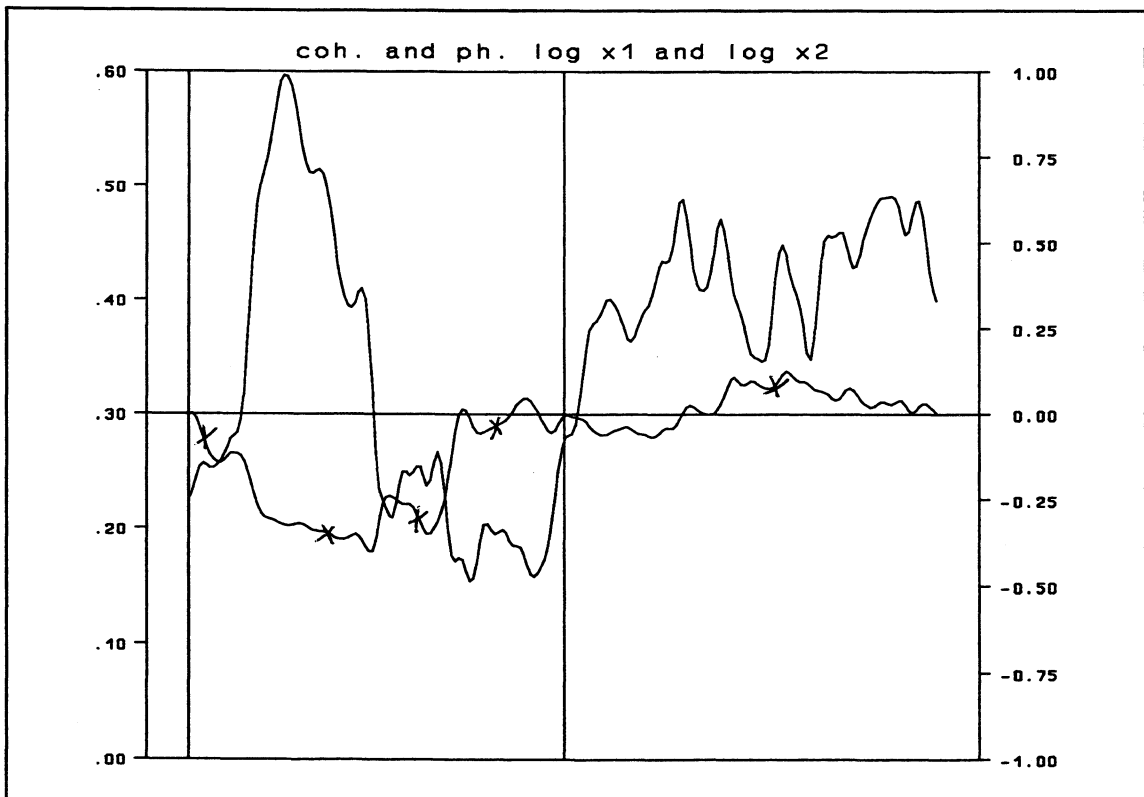
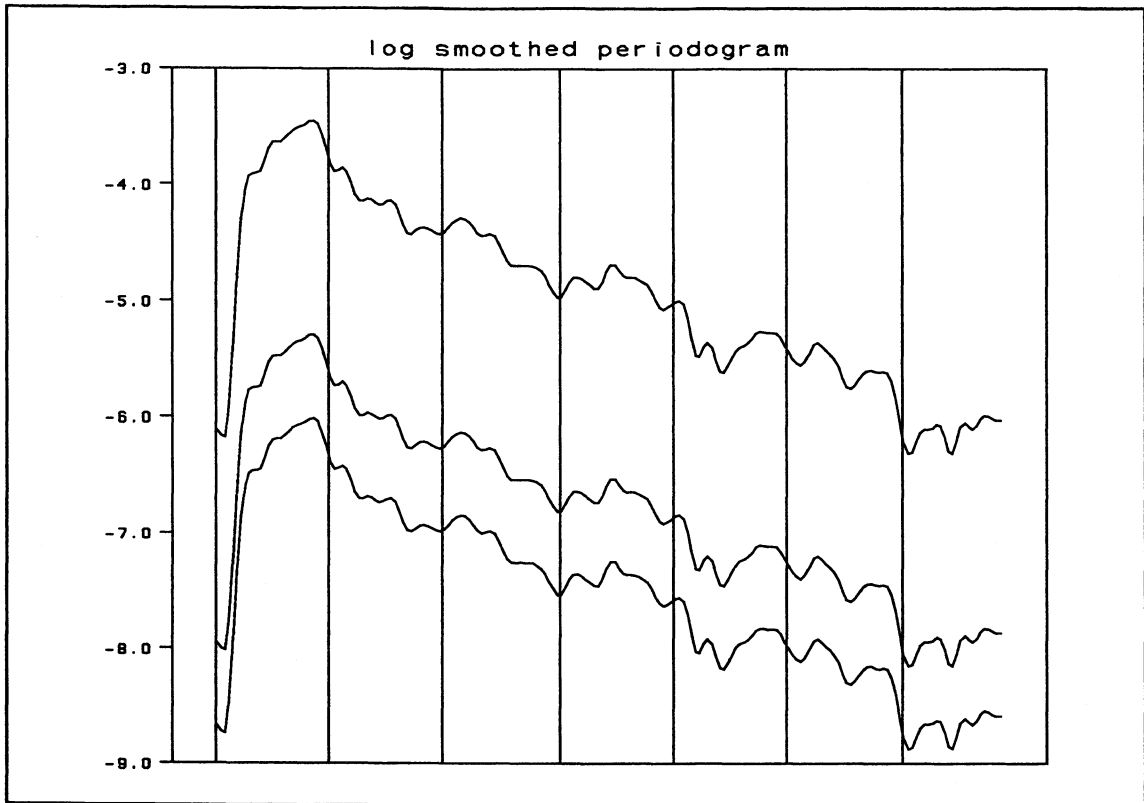


Figure 2.3.4. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing traditional export with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

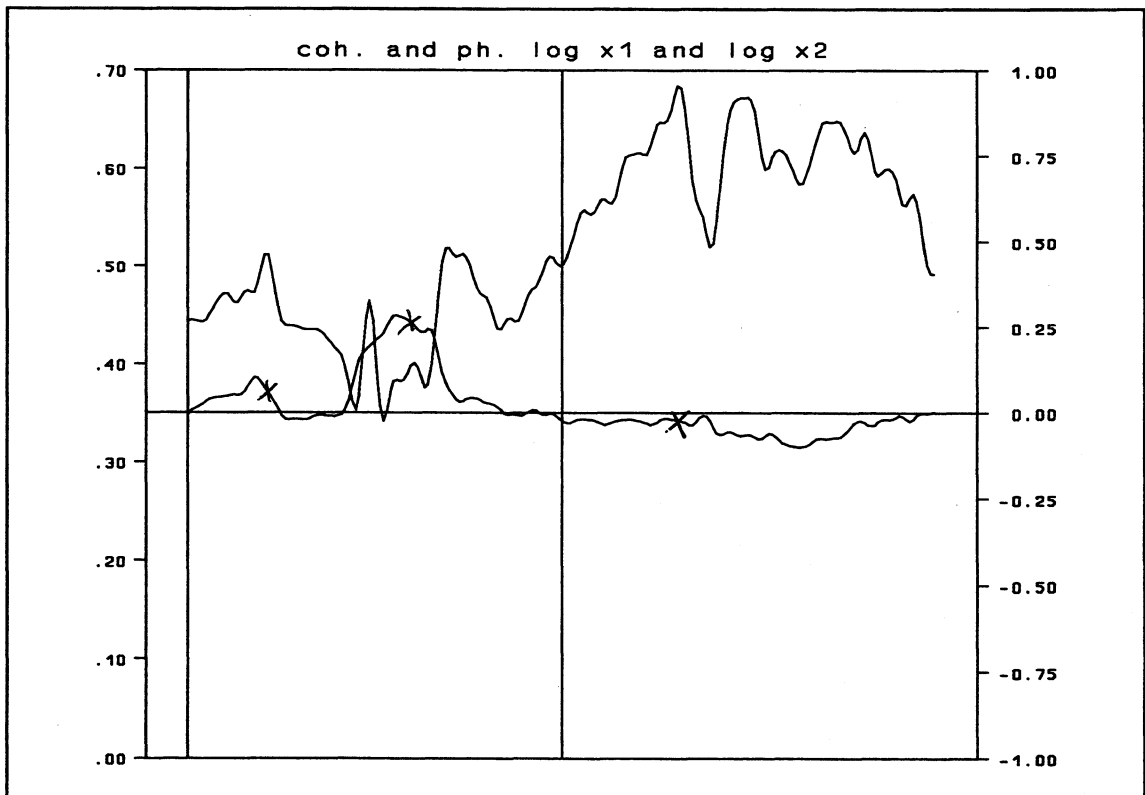
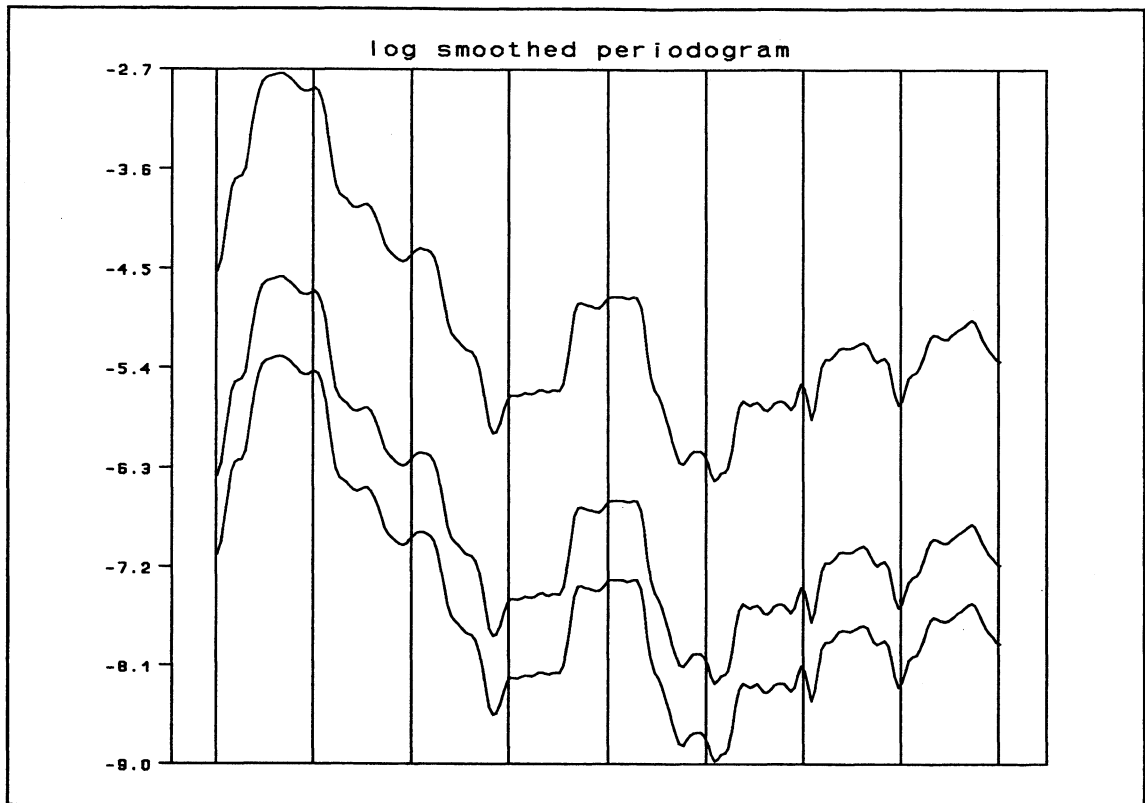


Figure 2.3.5. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing traditional import with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

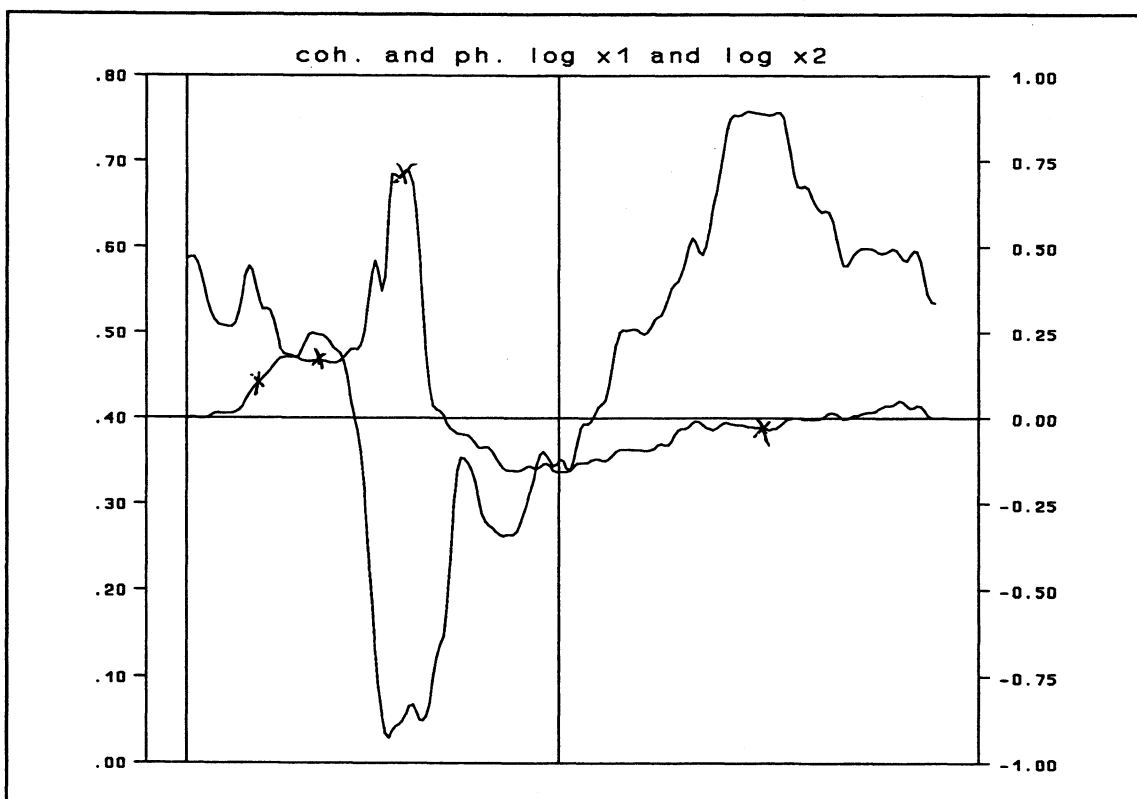
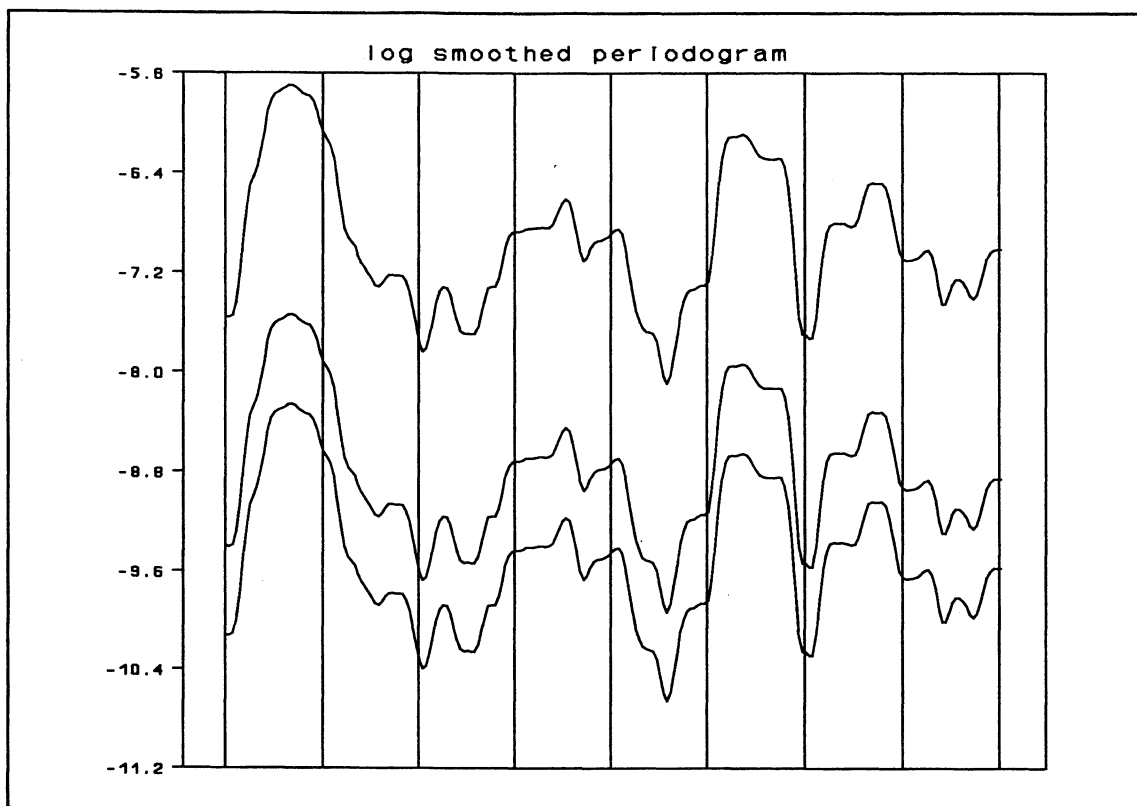


Figure 2.3.6. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing total hours with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

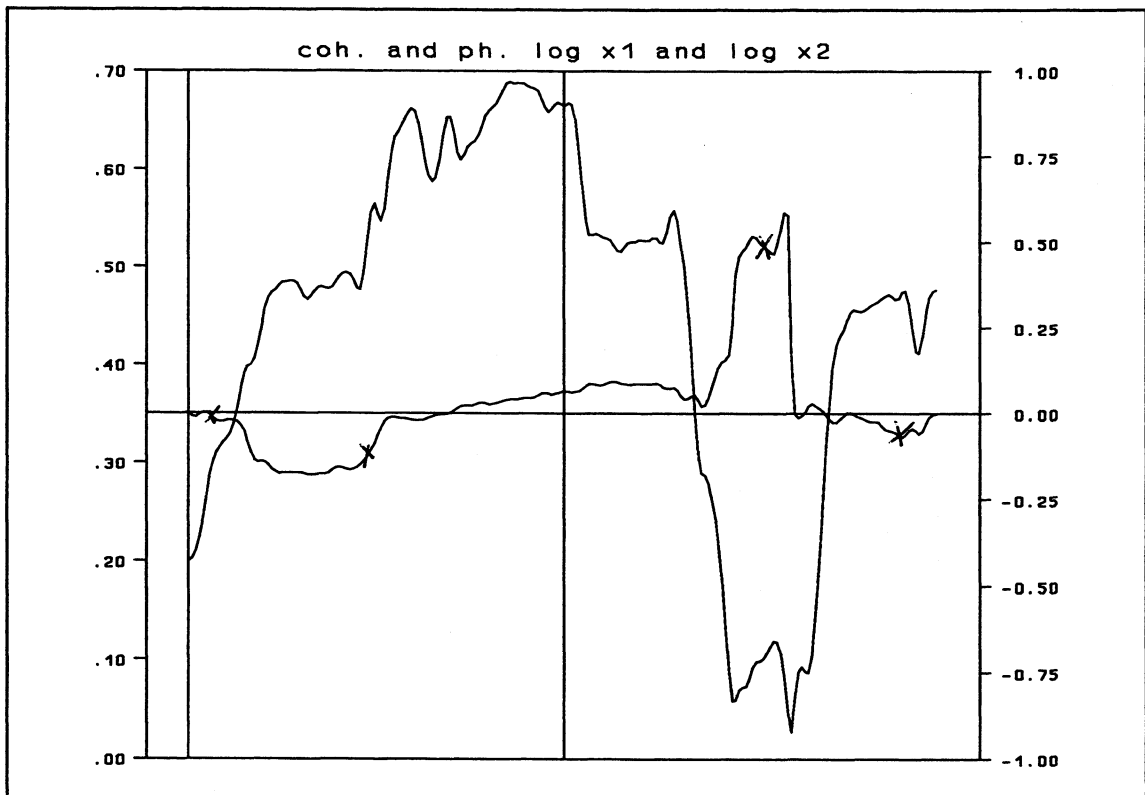
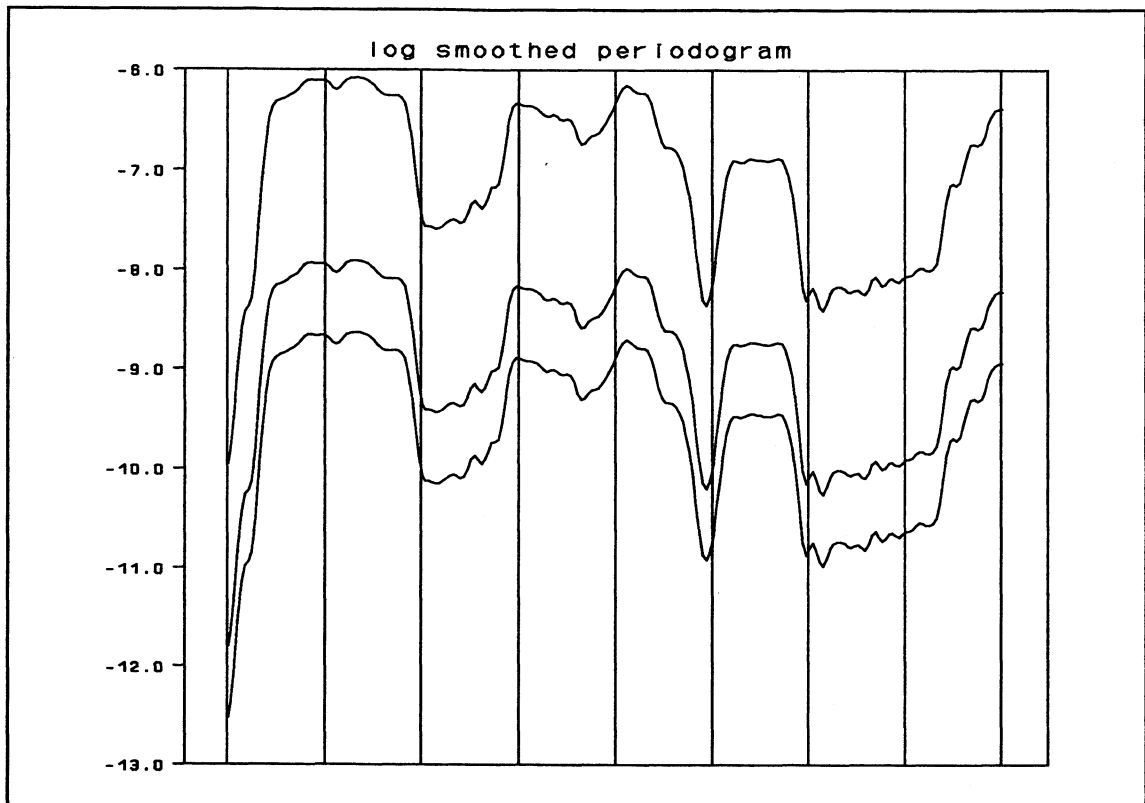


Figure 2.3.7. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing productivity with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

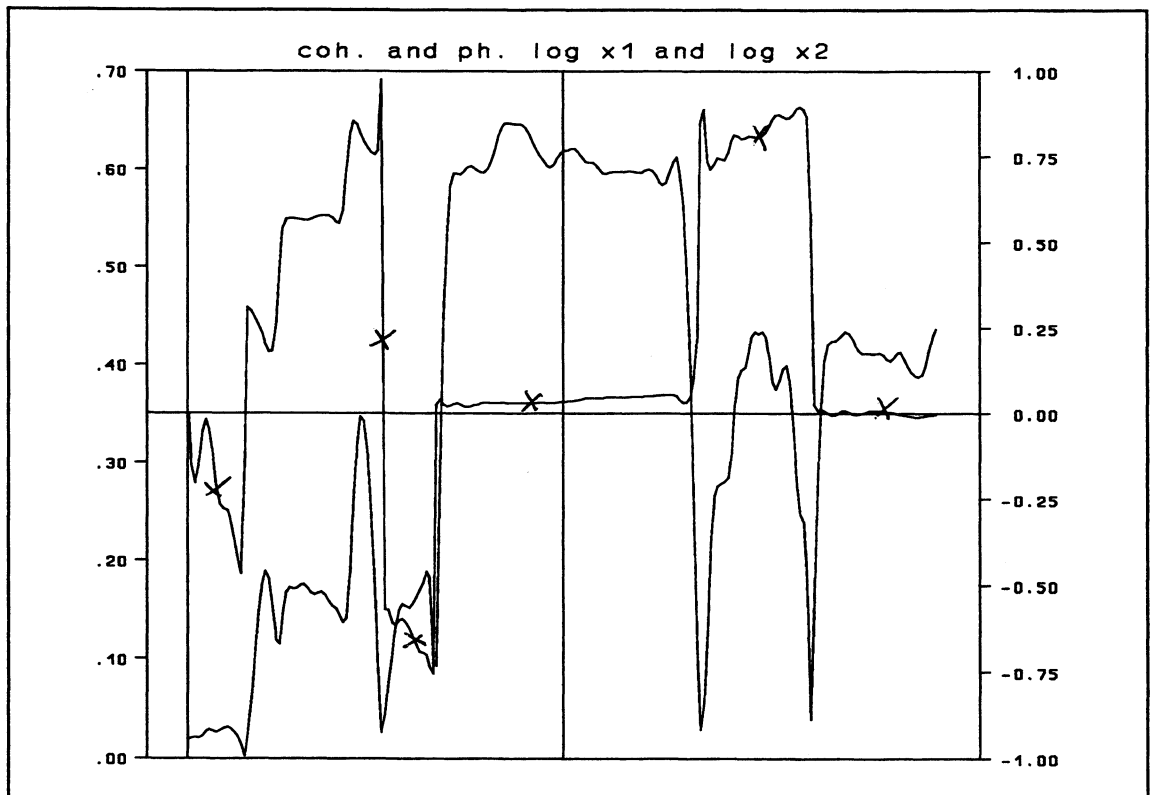
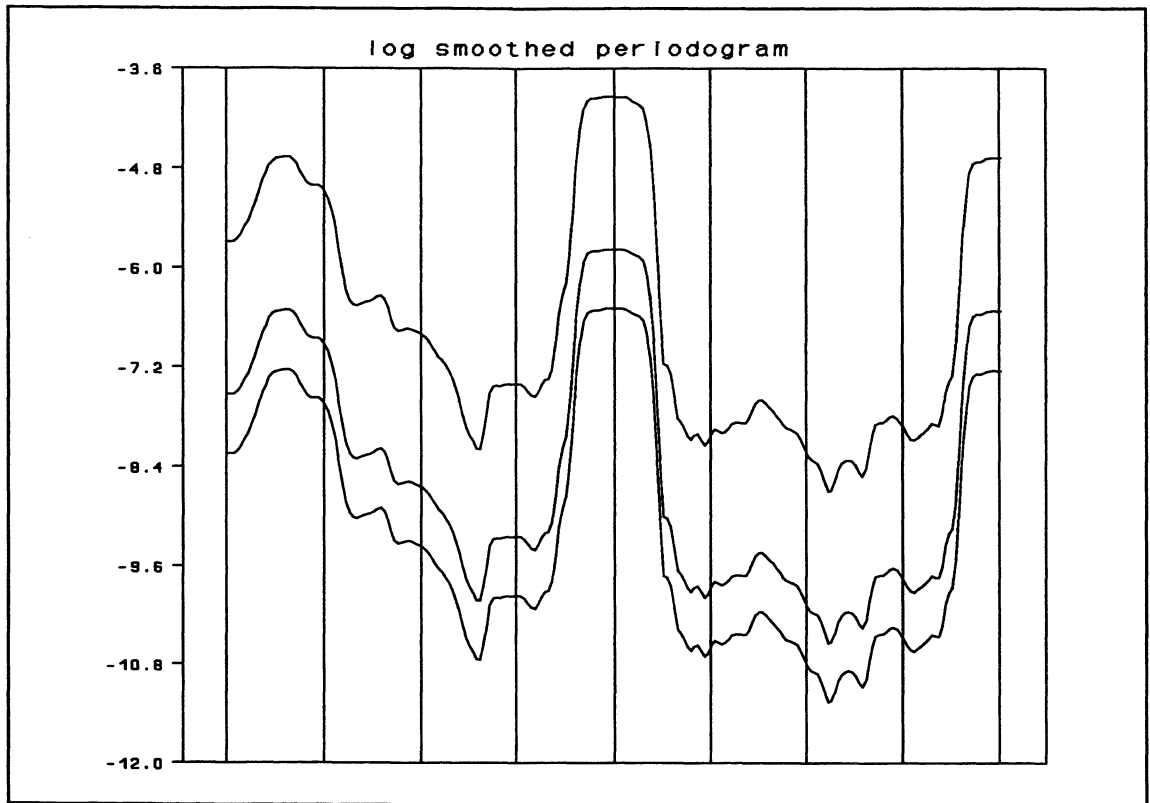


Figure 2.3.8. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing nominal wages with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

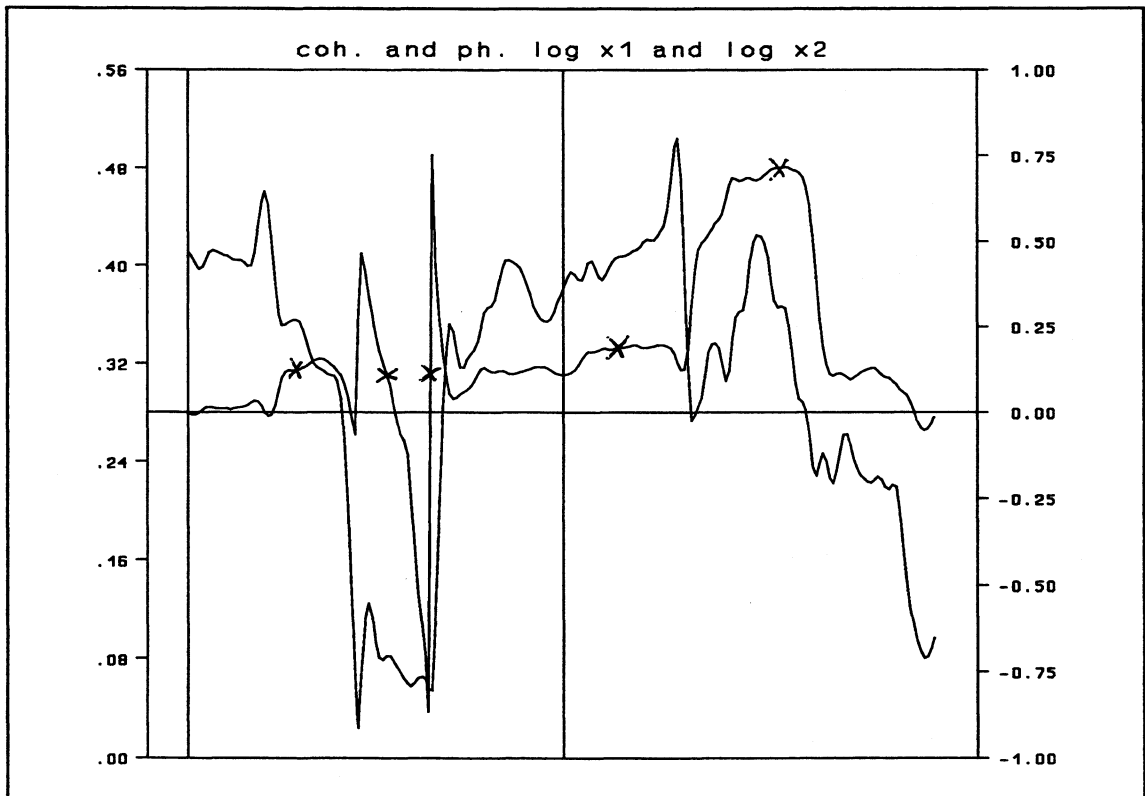
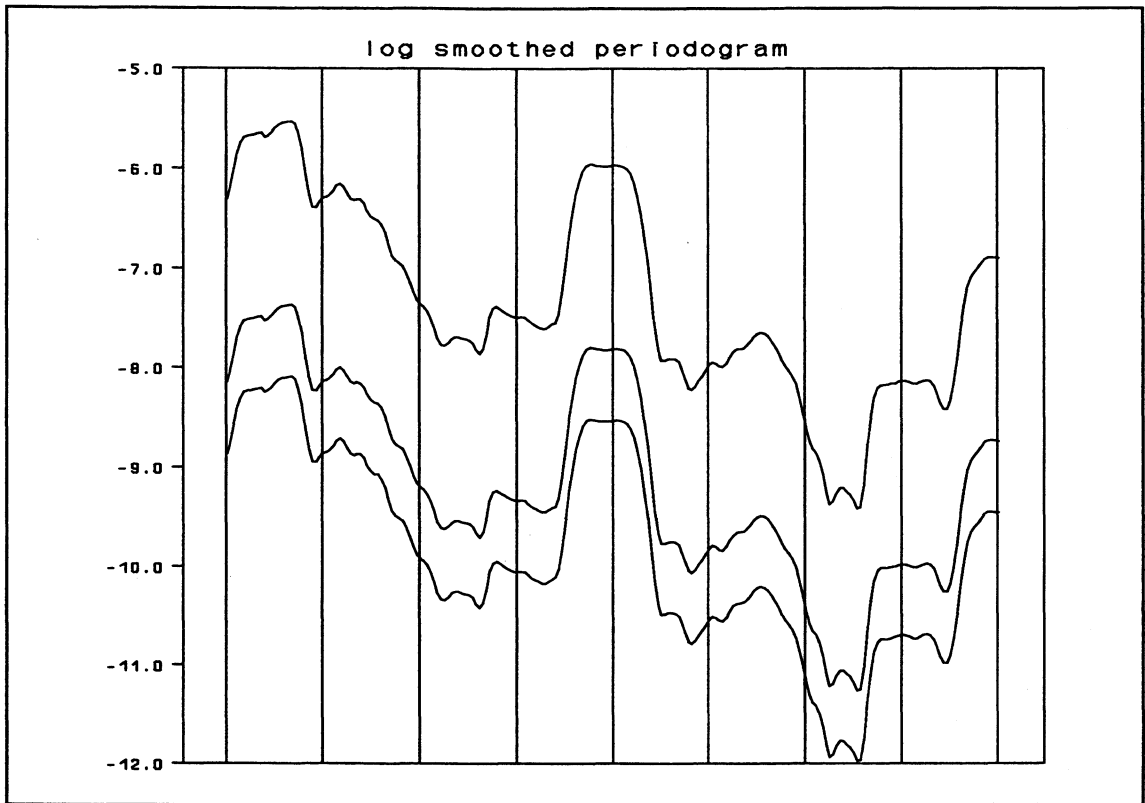


Figure 2.3.9. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing real wages with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

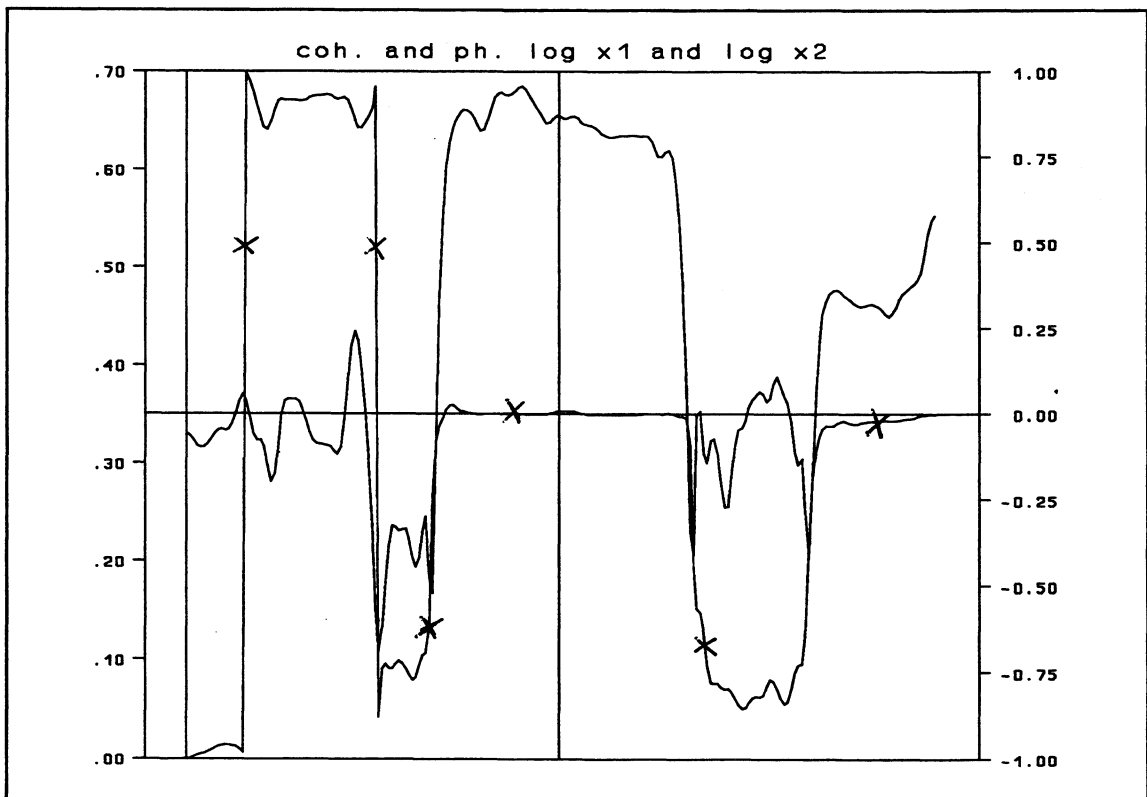
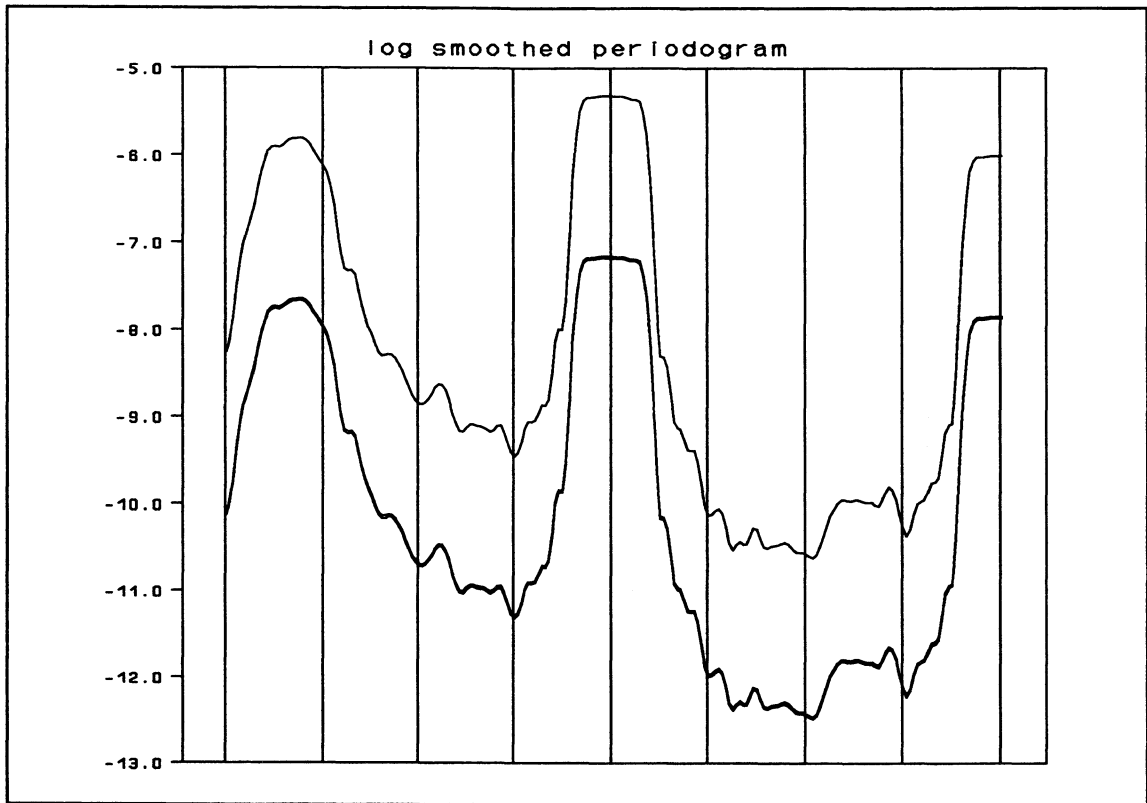


Figure 2.3.10. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing prices with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

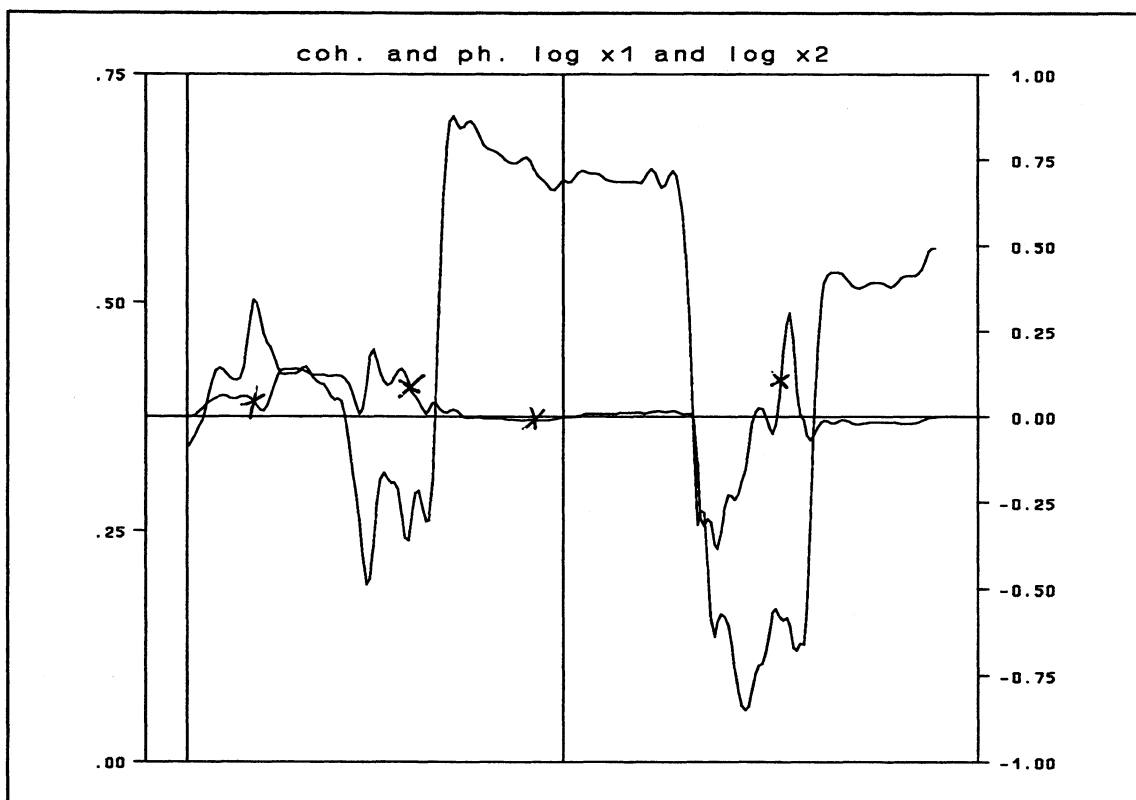
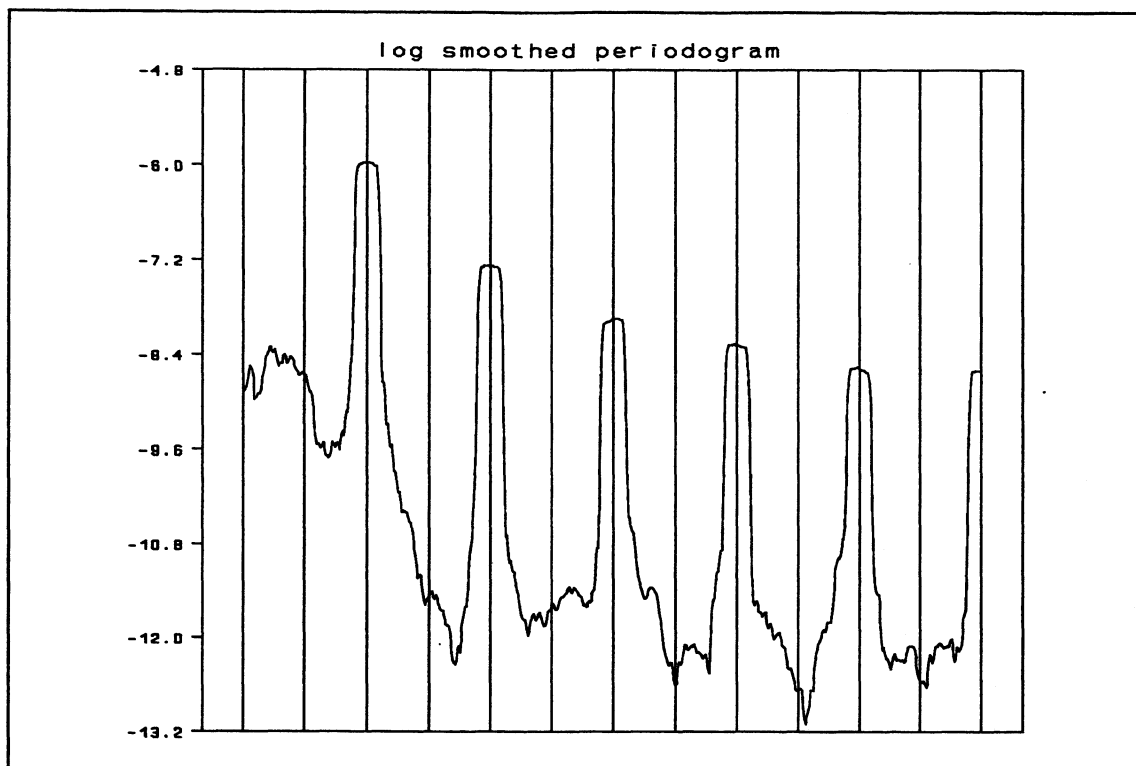


Figure 2.3.11. In the upper panel the logarithm of the estimated spectral density based on the residuals from smoothing M2 with a HP filter. A 95% confidence interval is indicated. In the lower panel the coherence and phase with the residuals from applying the HP filter on the mainland GDP. The scale on the left hand side refers to the coherence (the solid line), on the right hand side to the phase (the solid line with stars).

3. Analysing the first differences for a cyclical pattern

As mentioned in the introduction we have also considered the first differences of the twelve series selected. As mentioned there, it is of particular interest to consider the value of the spectral density at the value 0.

Figures 3.1.1–3.1.12 displays the logarithm of the estimated spectral densities after a fixed seasonal pattern has been removed by regressing the first differences on a set of seasonal dummies. As explained in the previous section we allow for a break in the seasonal pattern at 78:1. The grid are on the frequencies $\pi/8, 2\pi/8, \dots, \pi$. Hence the first grid corresponds to a four year cycle and the fourth to a yearly cycle. The M2 series is monthly and the grid are on the frequencies $\pi/12, 2\pi/12, \dots, \pi$. The first grid corresponds to a 24 month or 2 year cycle and the second to a yearly cycle. Also a 95% confidence interval is indicated.

A rather strong seasonal component remains in many of the series indicating a shift in the seasonal pattern over the period. Also we see that in most of the series the bulk of the variation can be attributed to variation in the high frequencies. The exception is the investments in the mainland and traditional import which contain some variation in the low frequencies. The same feature is present in the series for the wages, especially the nominal, and in prices and M2.

Concerning the value of the spectral density at zero, in the rest of the series it is not especially large. However, to neglect it does not seem to be warranted. Thus, in terms of Cochrane (1989), models containing unit roots are not ruled out by the impression from figures 3.1.1–3.1.12.

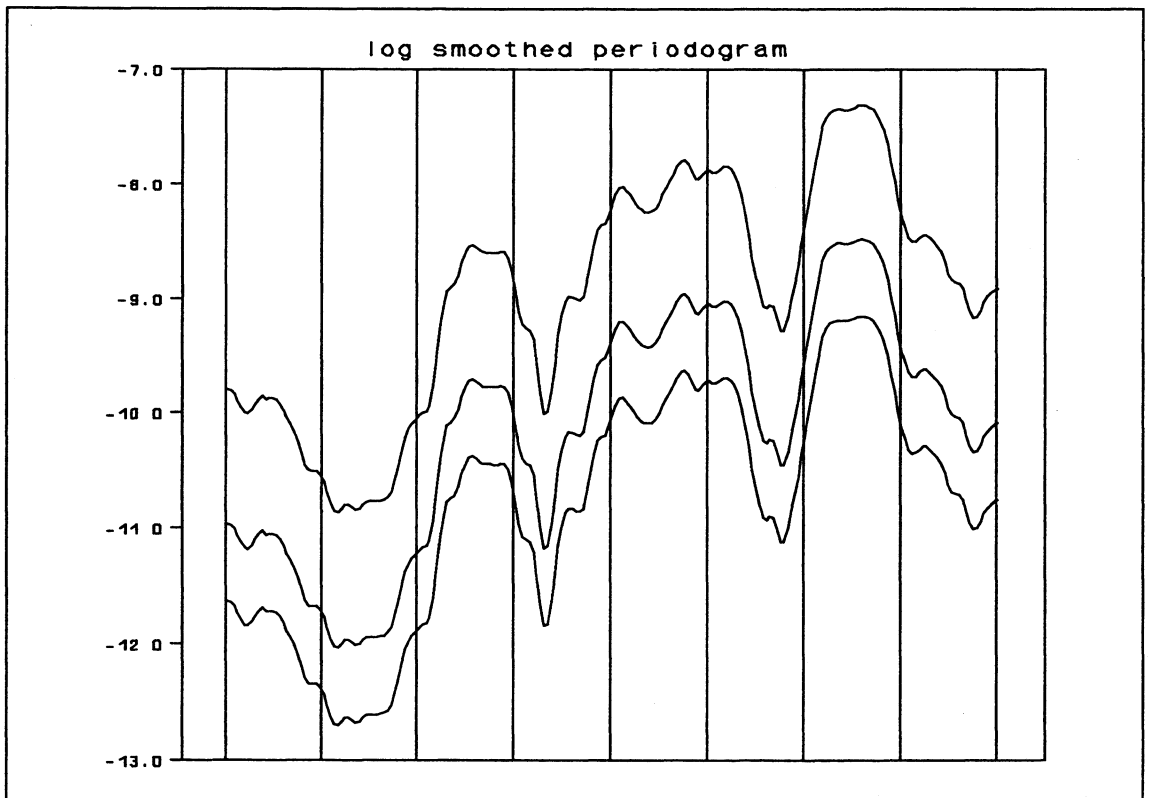
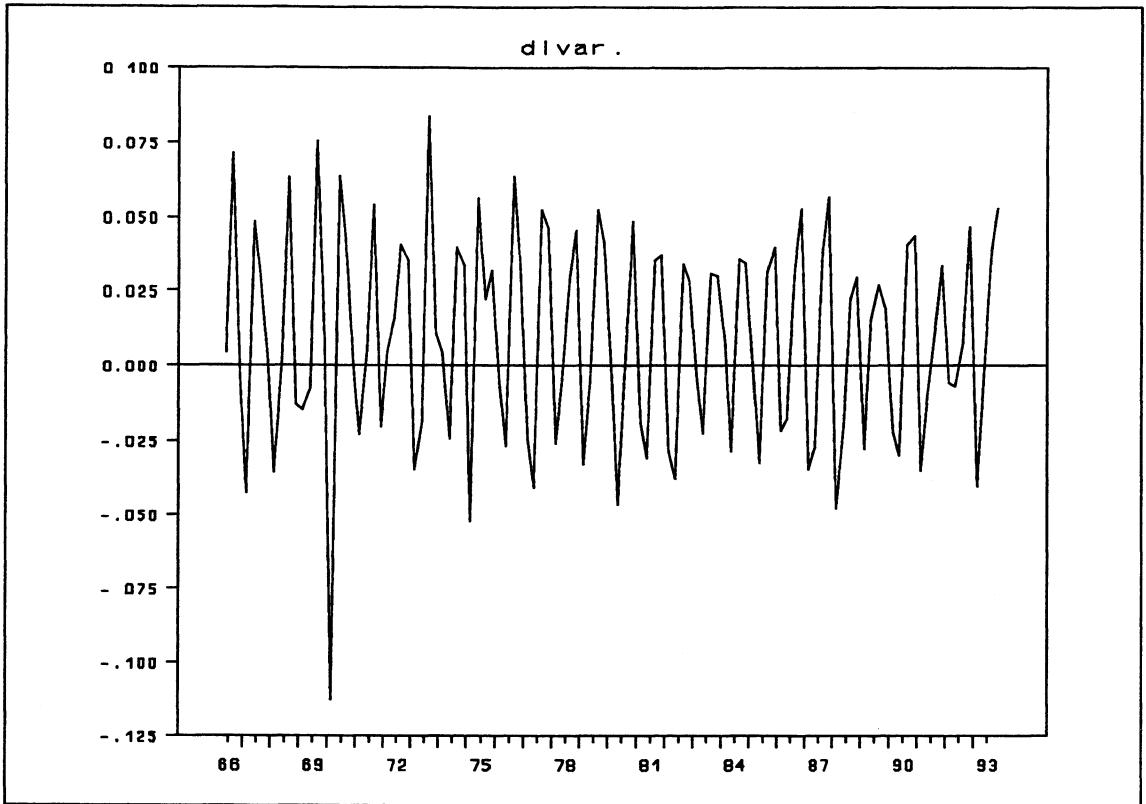


Figure 3.1.1. Total GDP. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

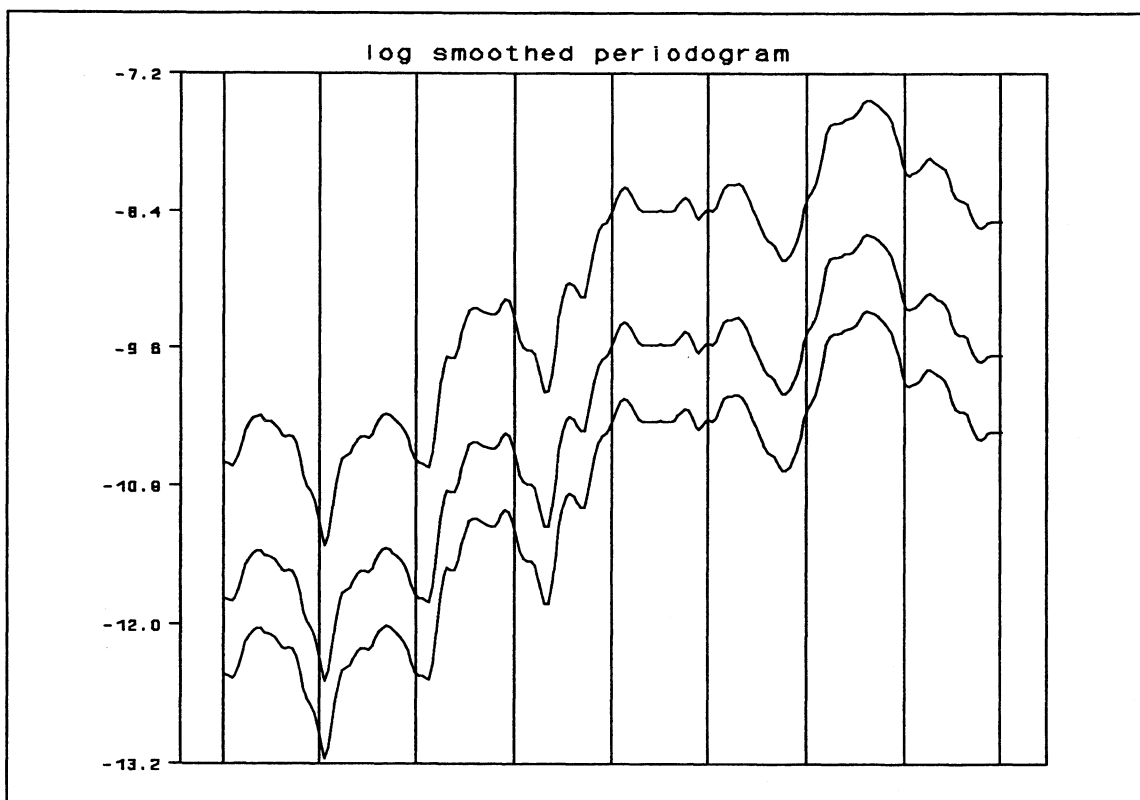
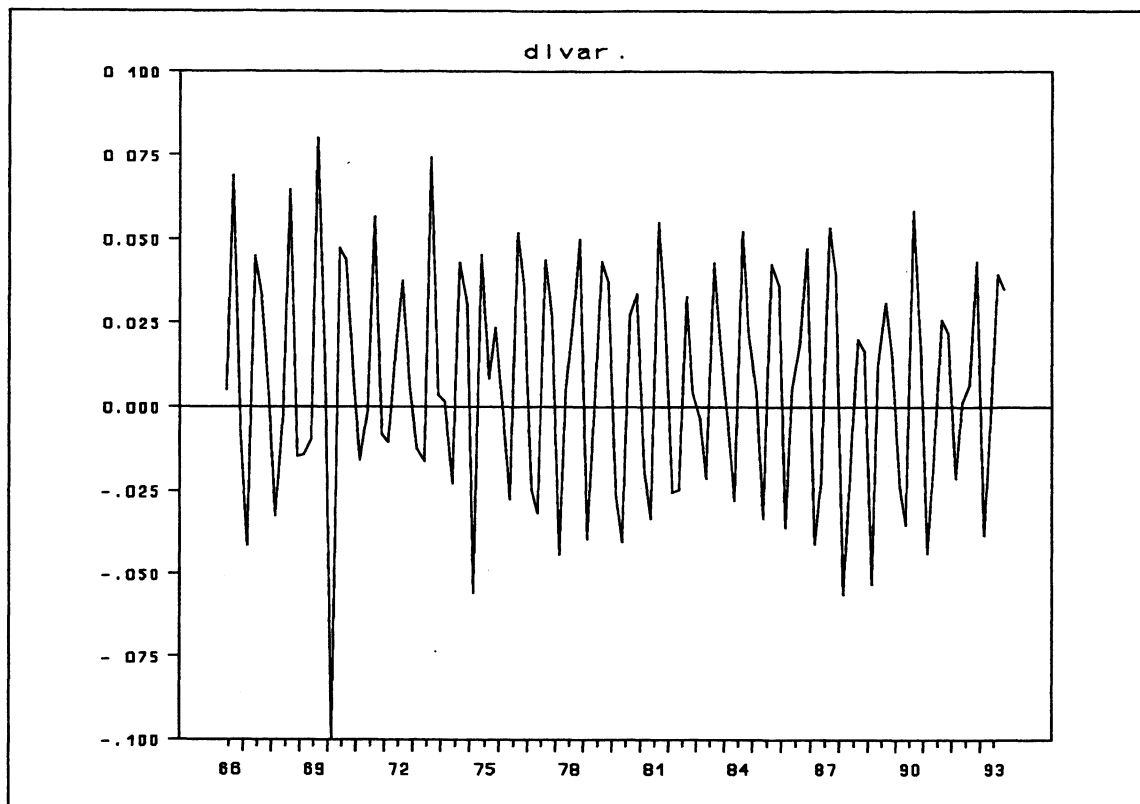


Figure 3.1.2. GDP mainland. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

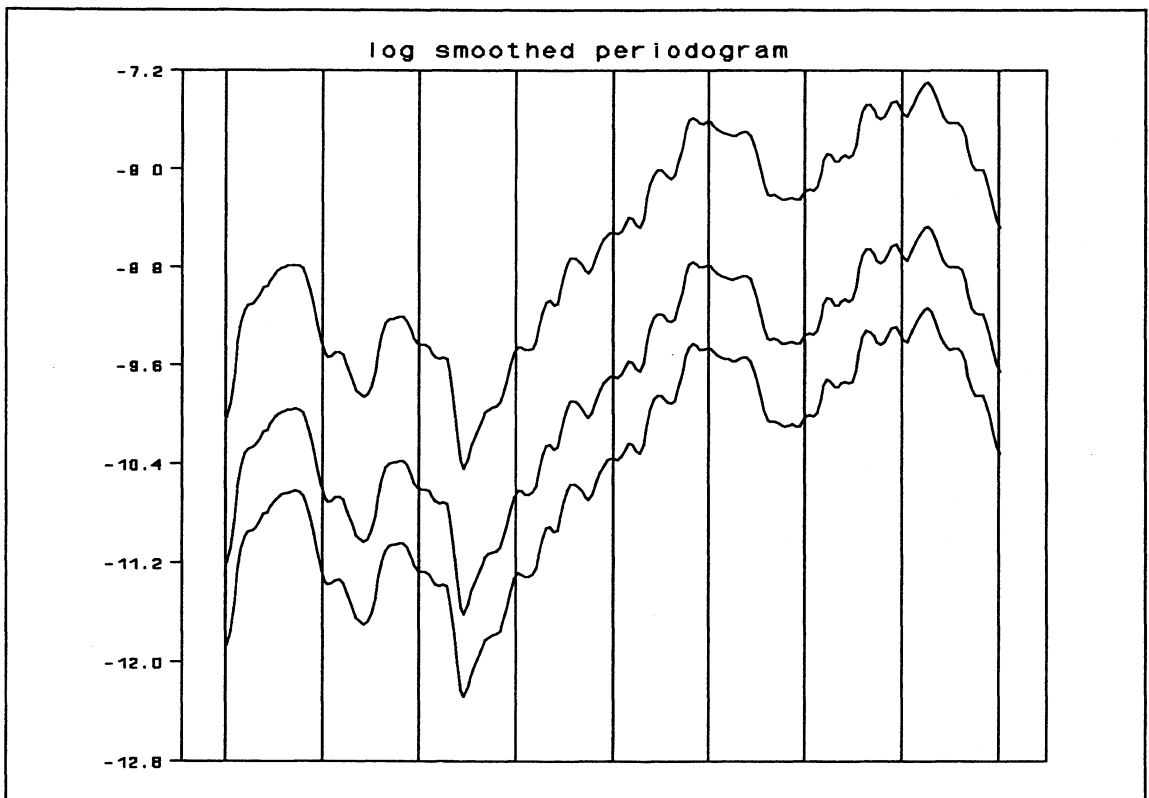
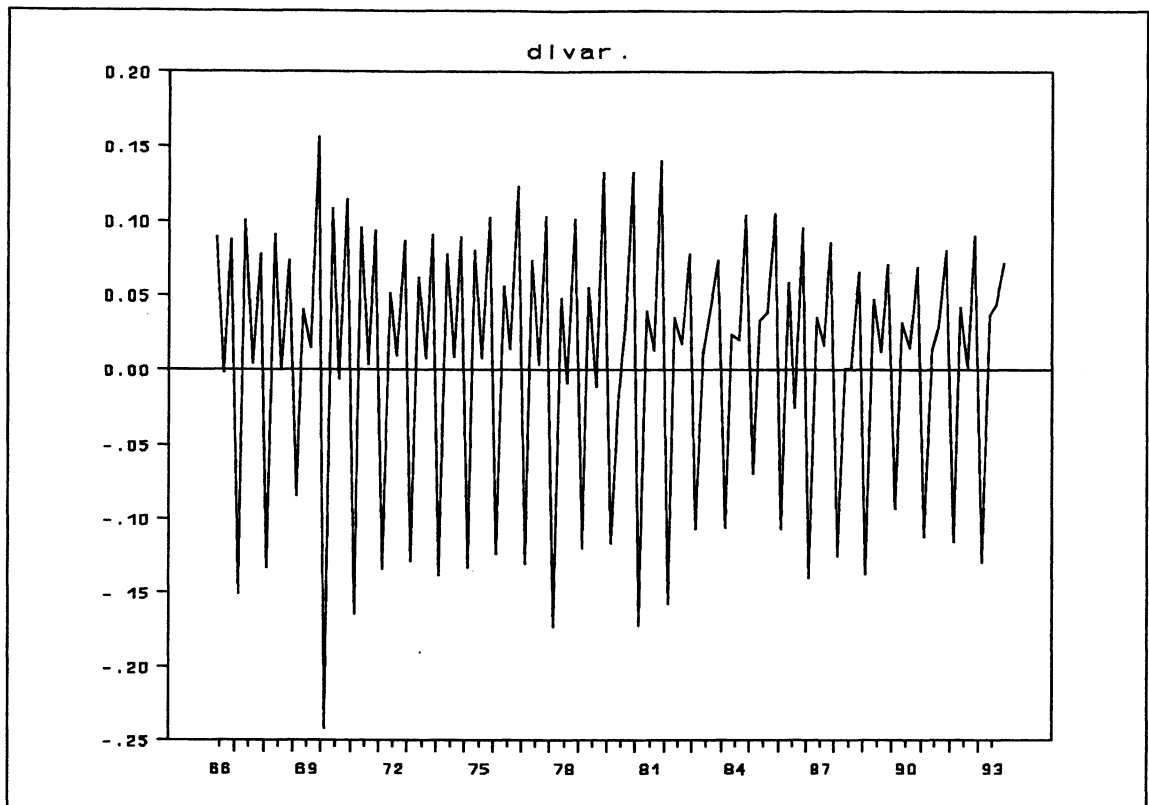


Figure 3.1.3. Total private consumption. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

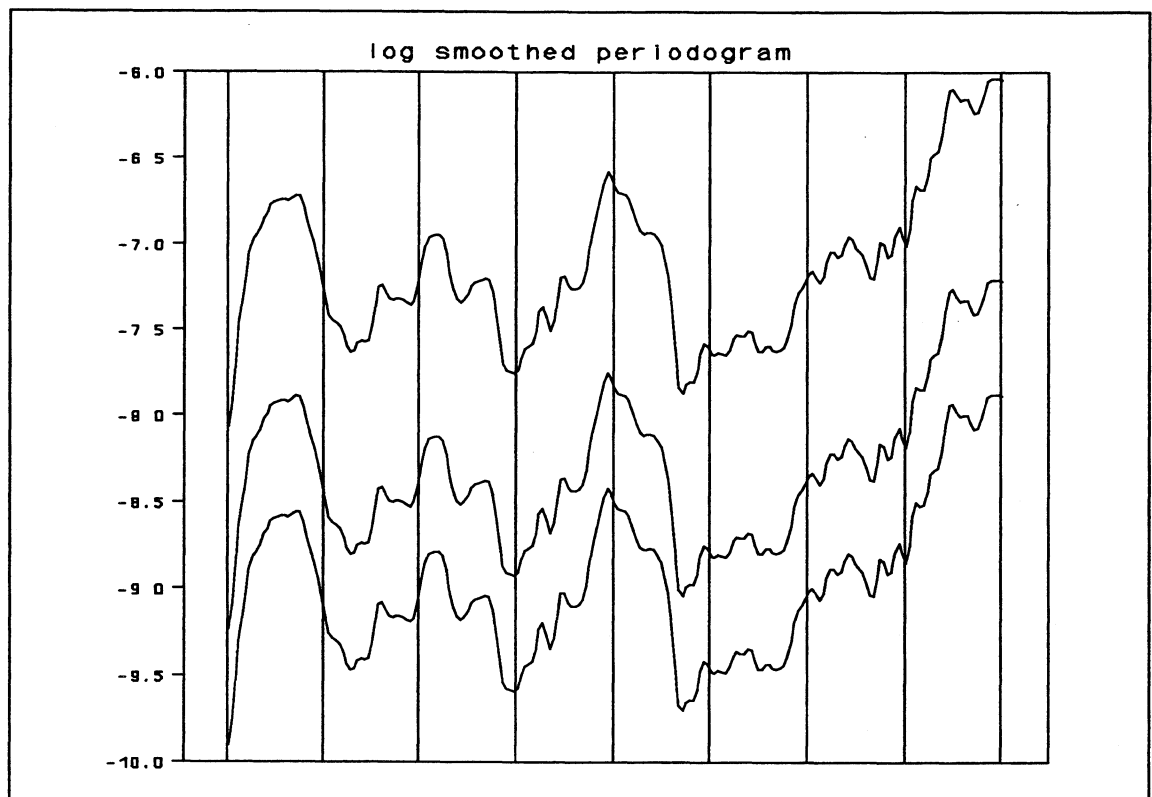
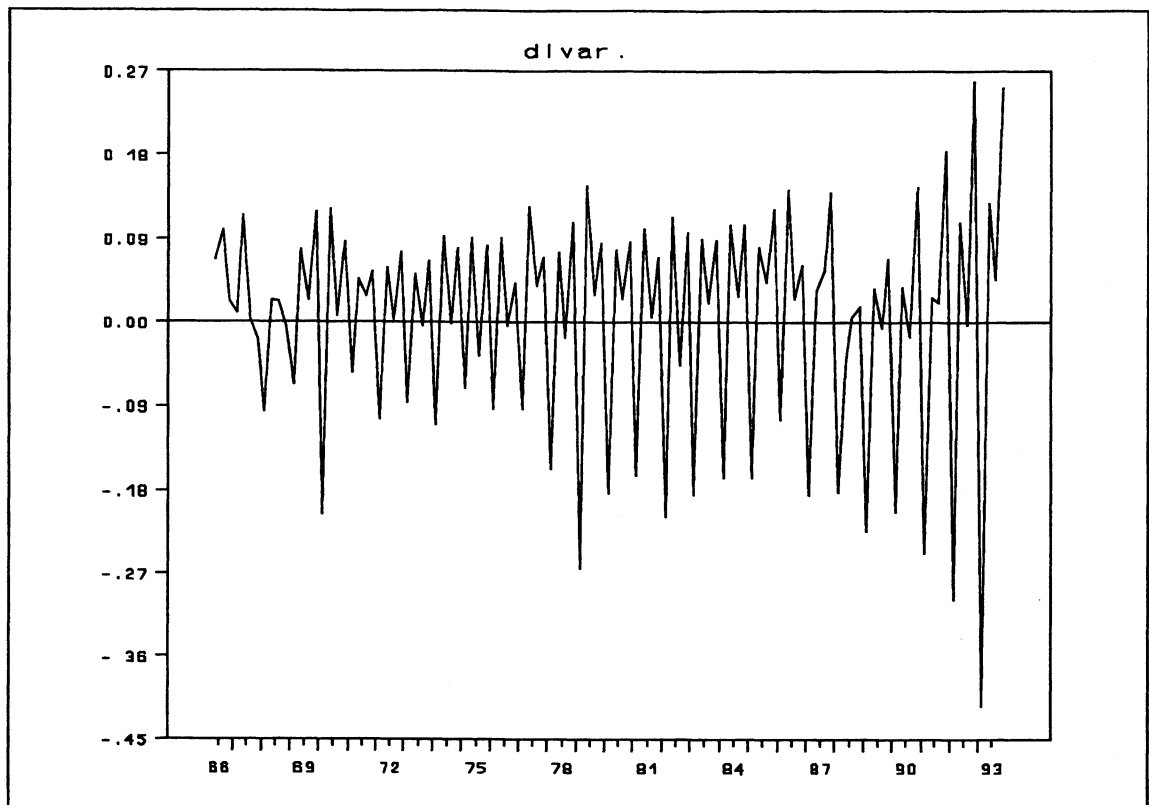


Figure 3.1.4. Investment mainland. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

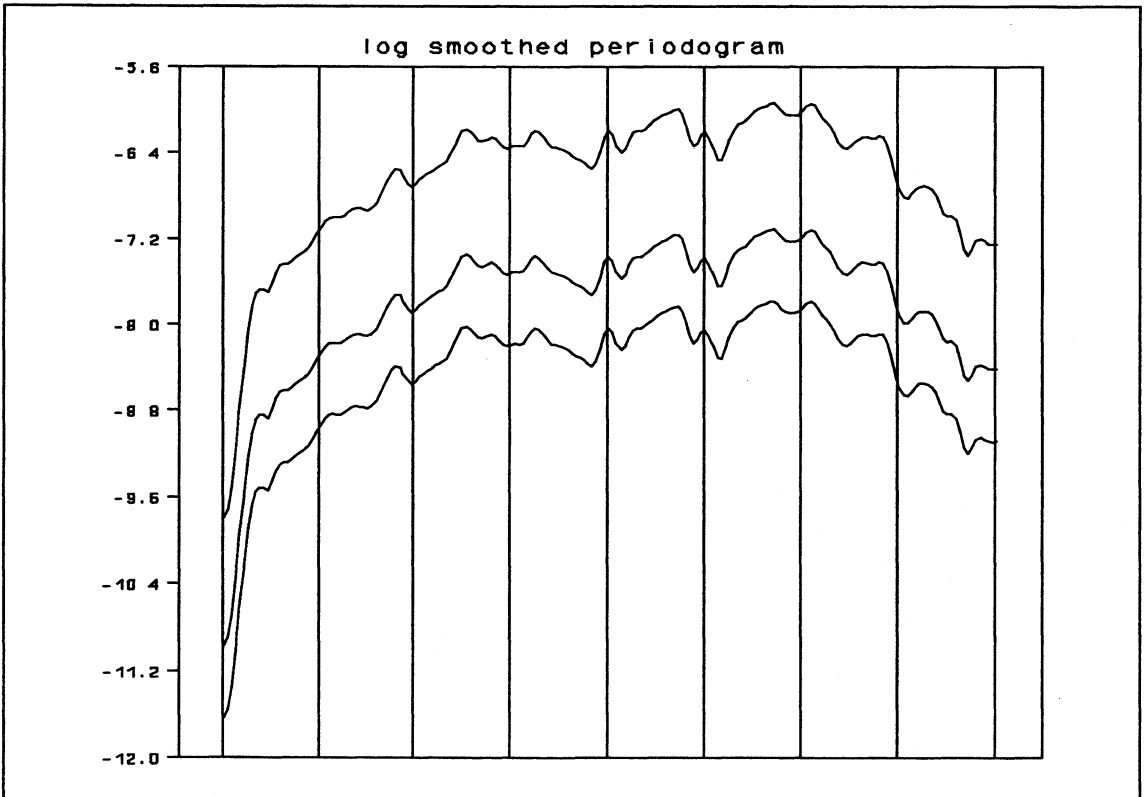
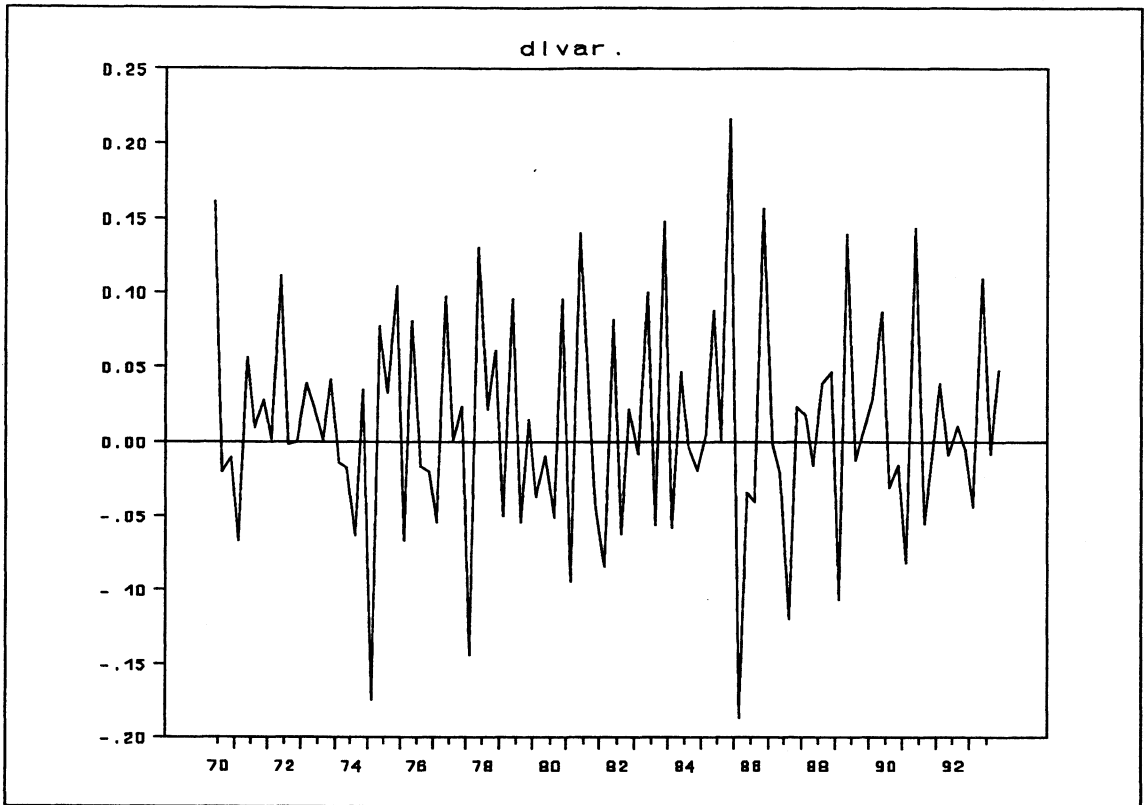


Figure 3.1.5. Traditional export. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

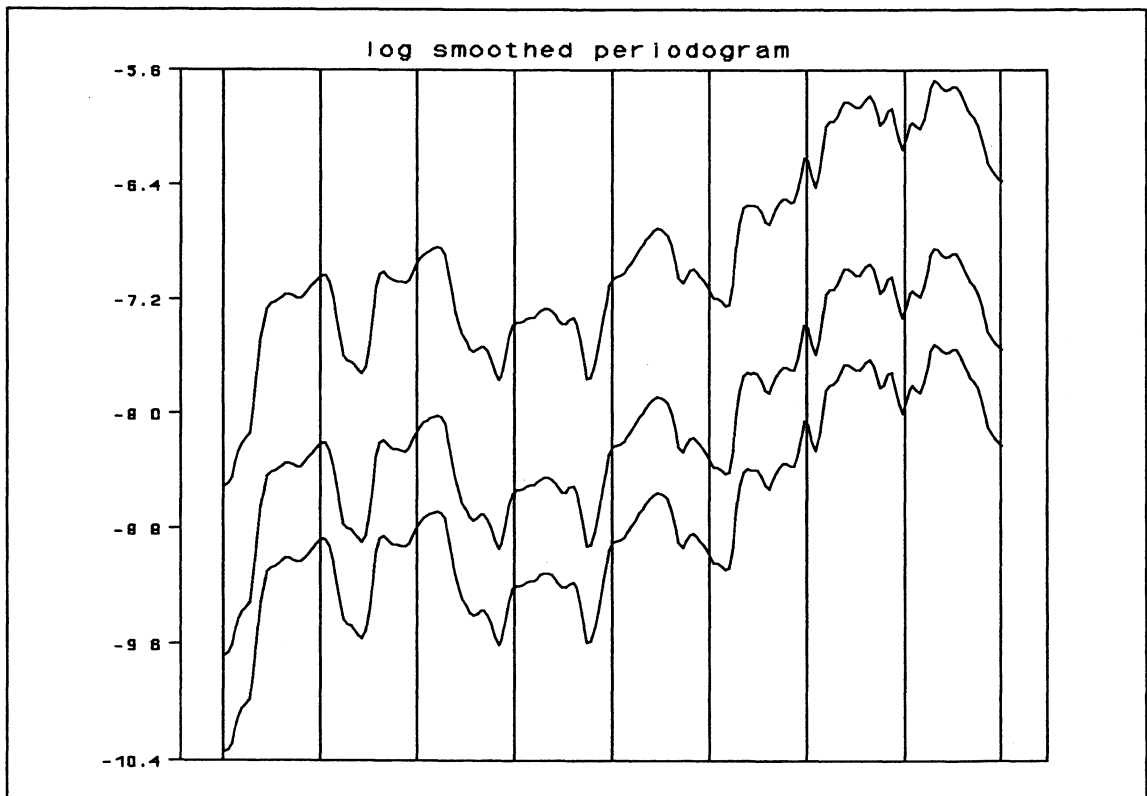
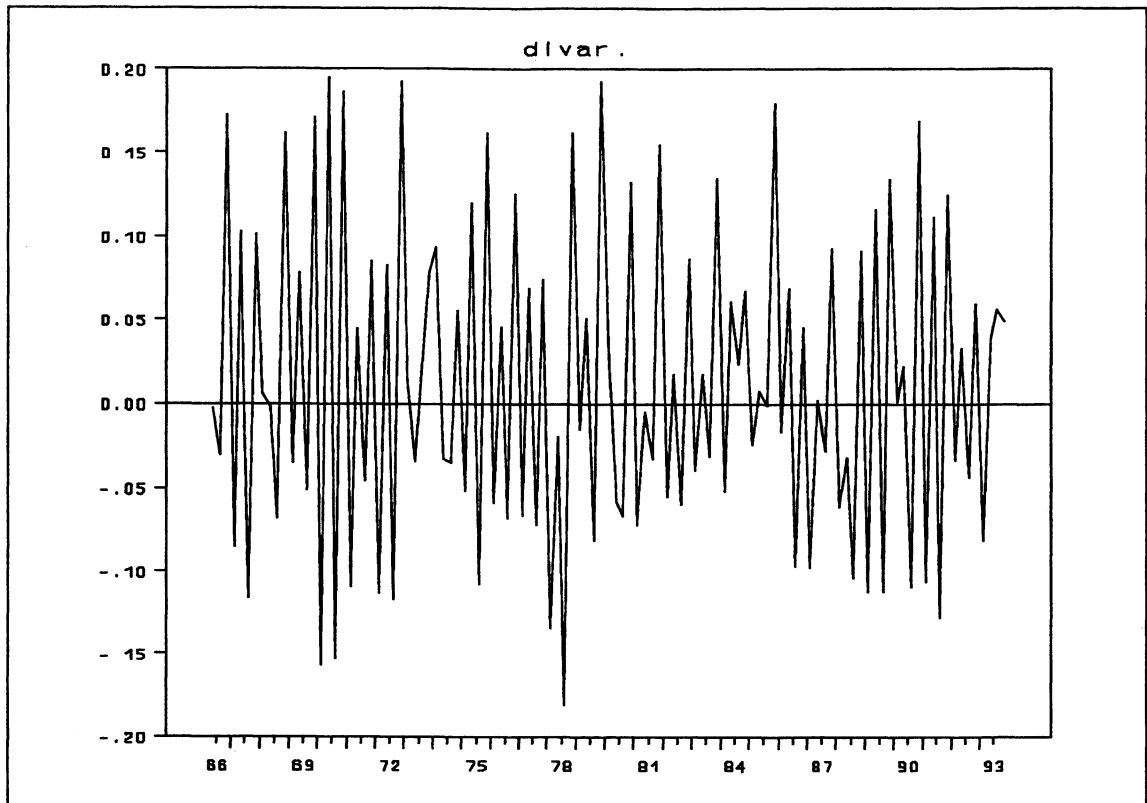


Figure 3.1.6. Traditional import. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

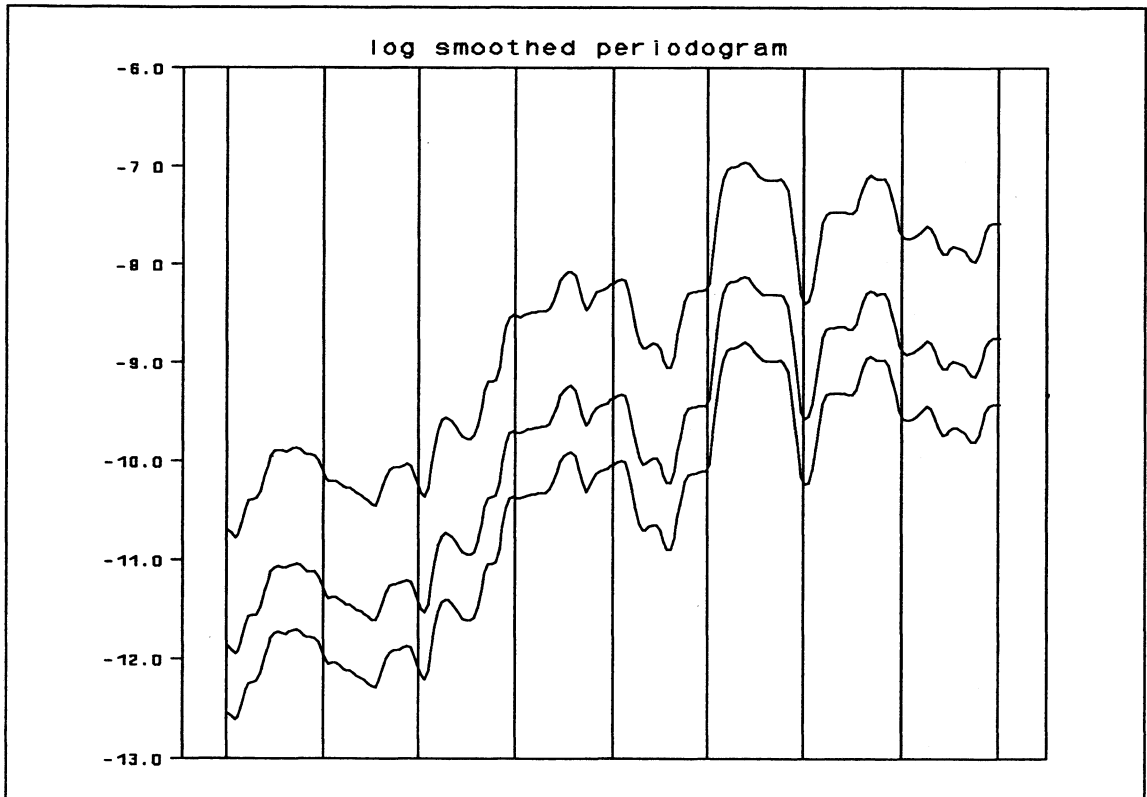
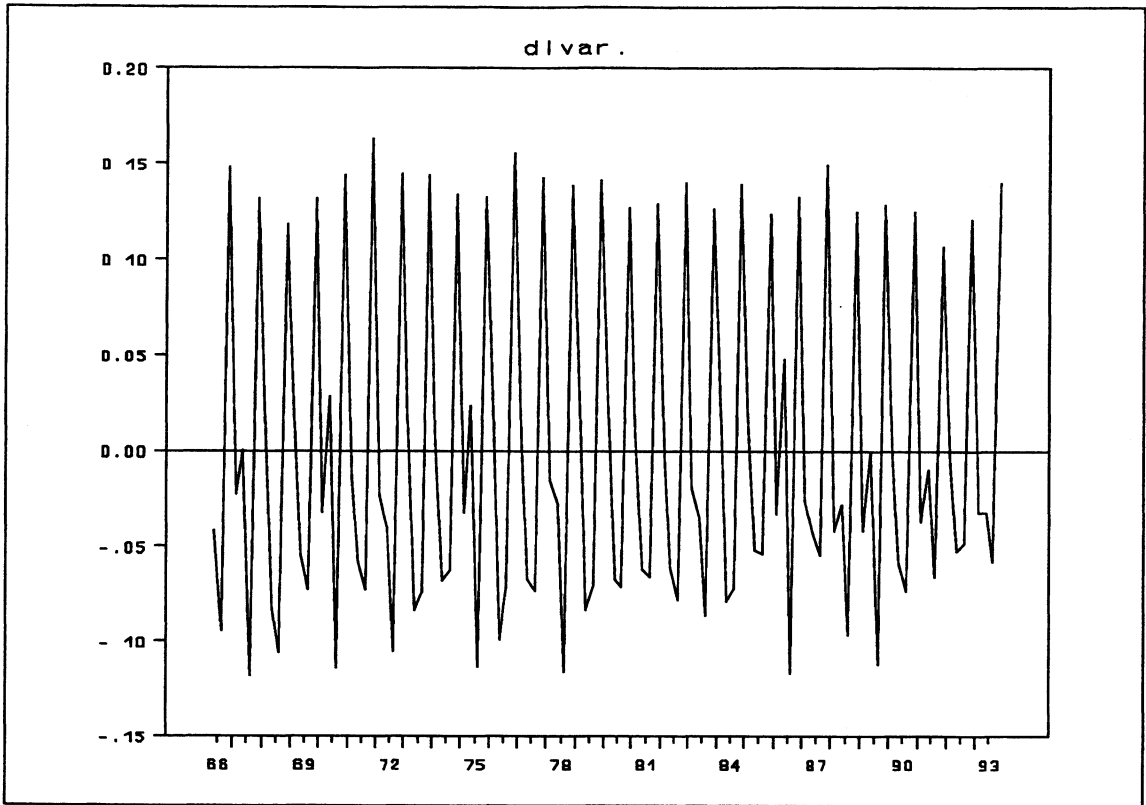


Figure 3.1.7. Total hours. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

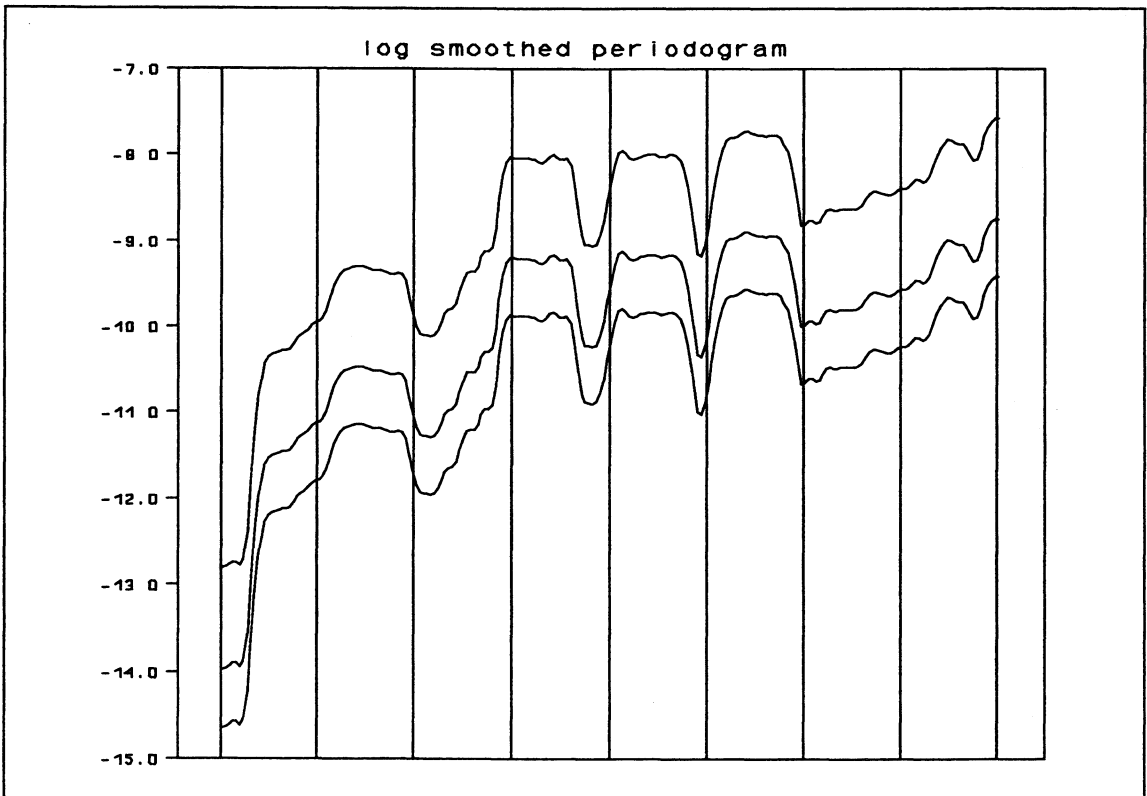
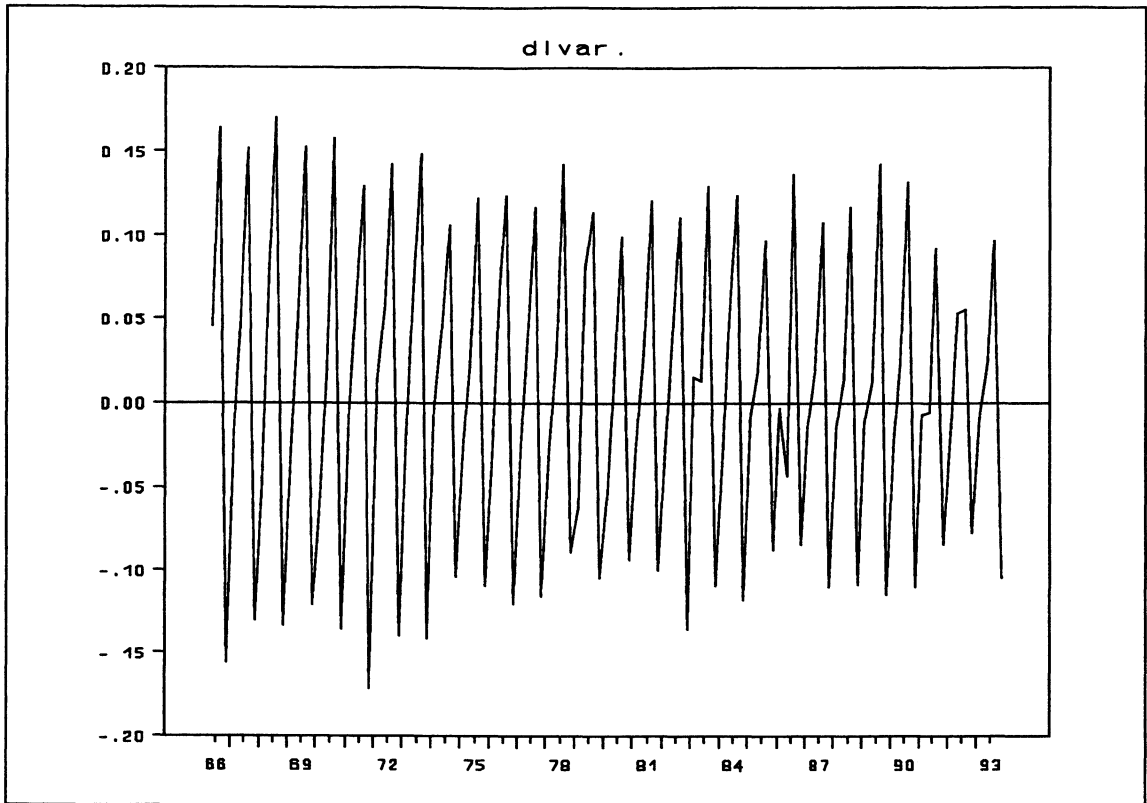


Figure 3.1.8. Productivity. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

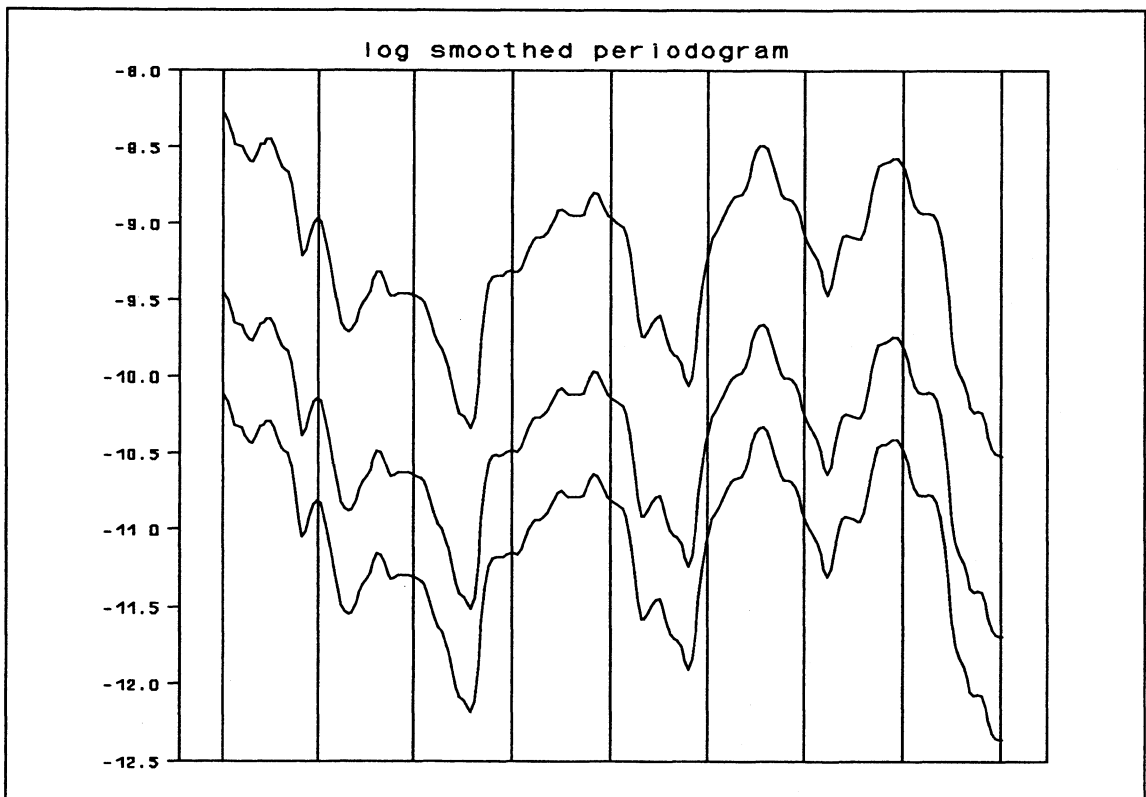
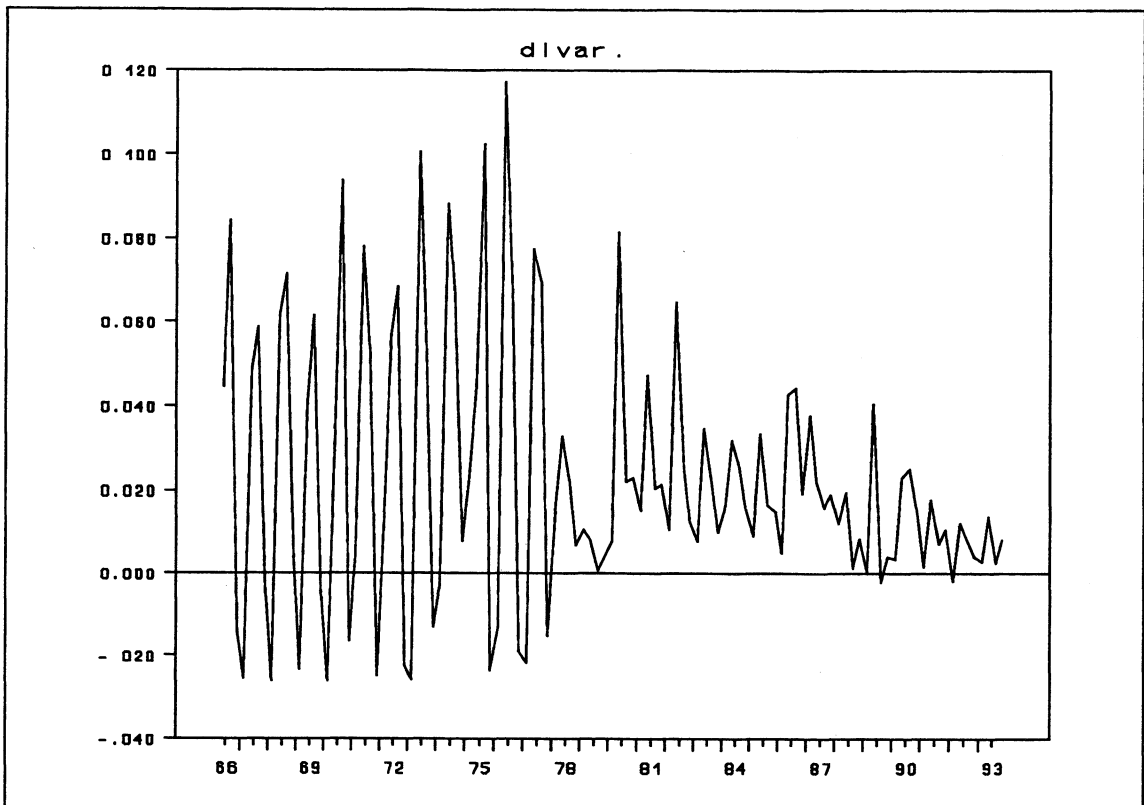


Figure 3.1.9. Nominal wages. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

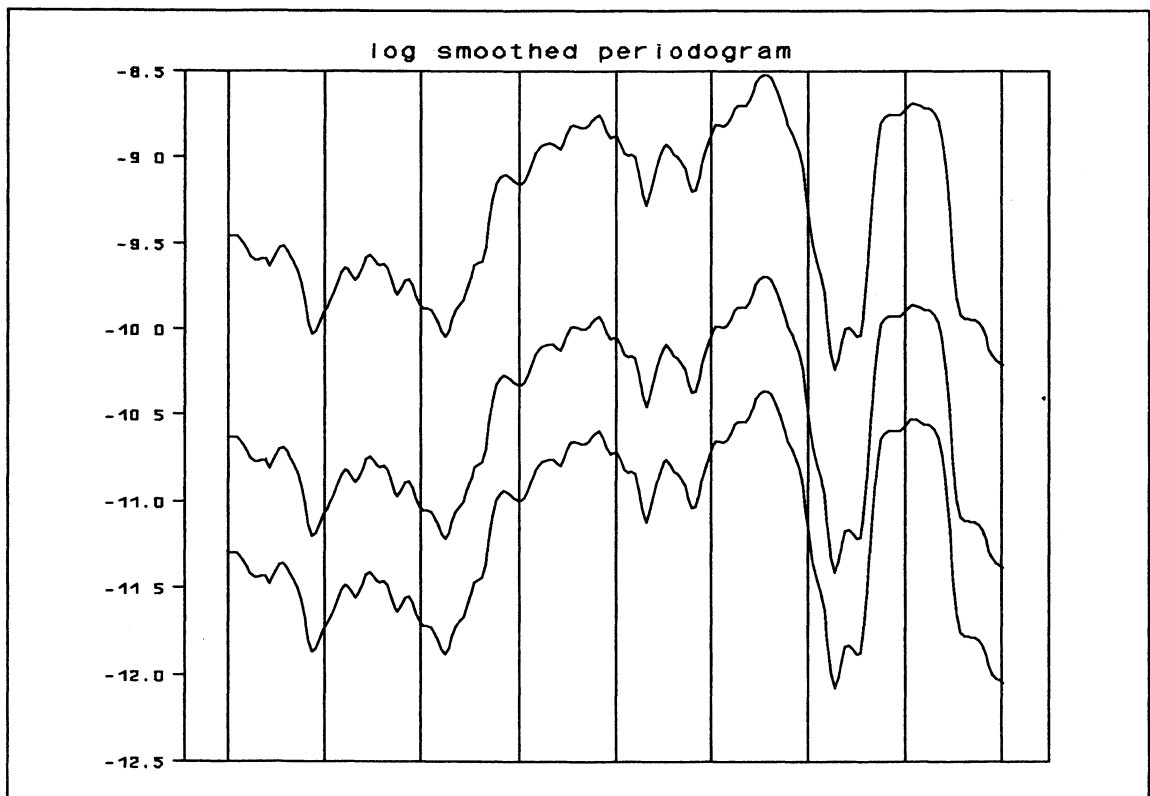
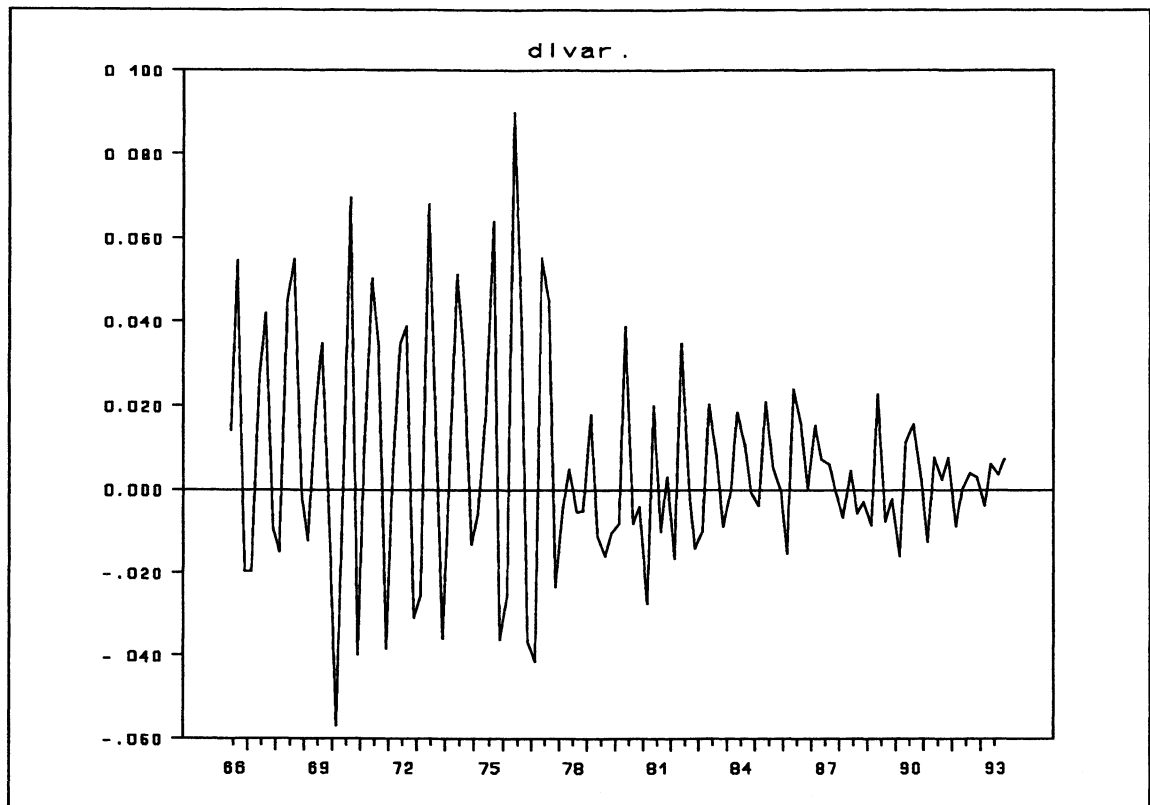


Figure 3.1.10. Real wages. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

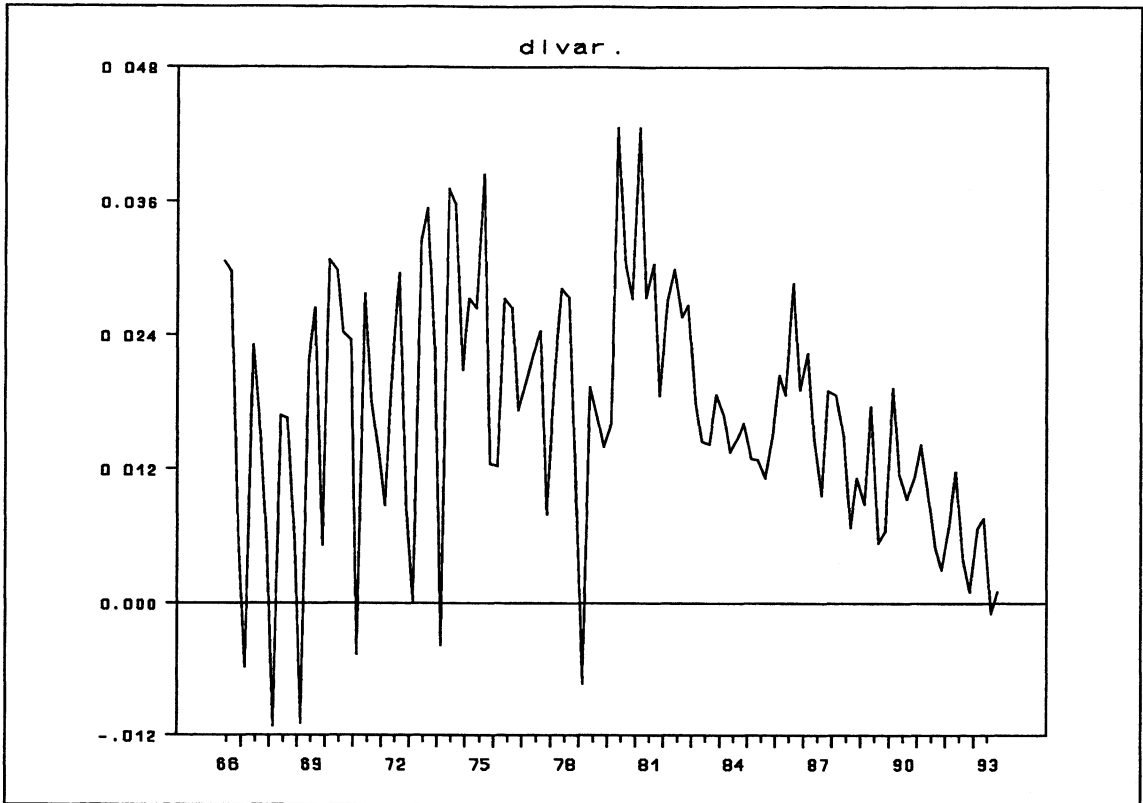


Figure 3.1.11. Prices. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

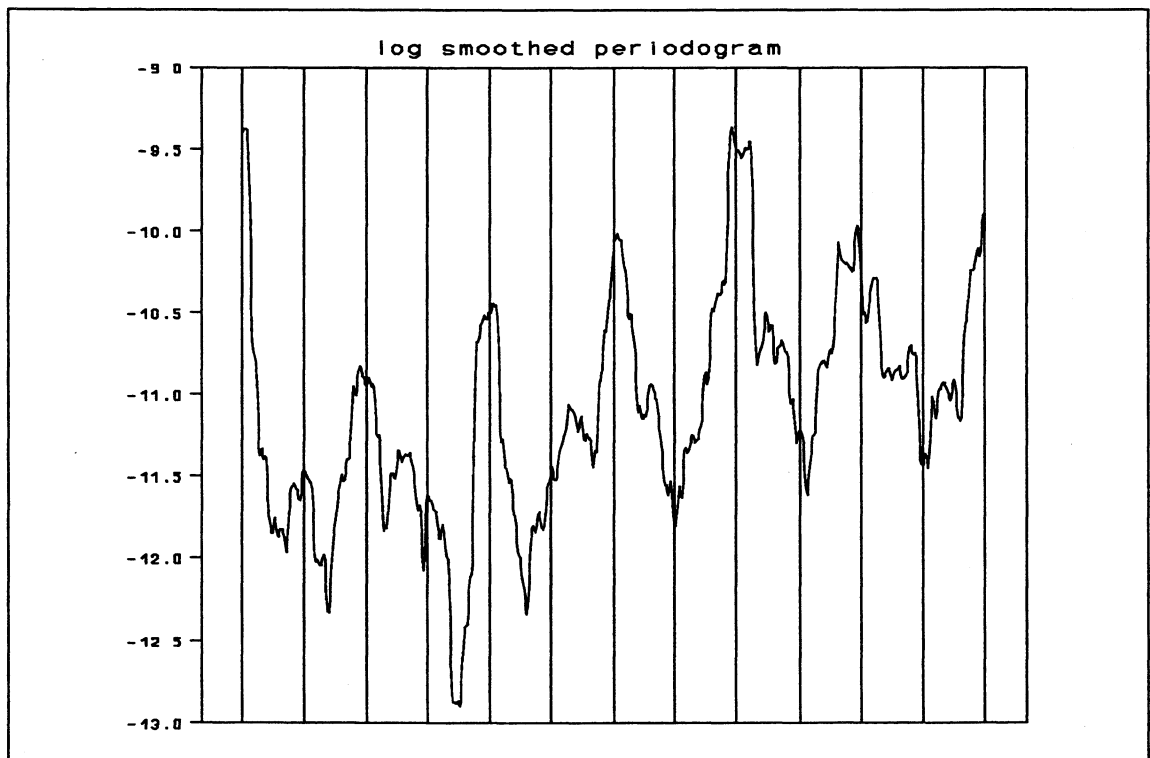
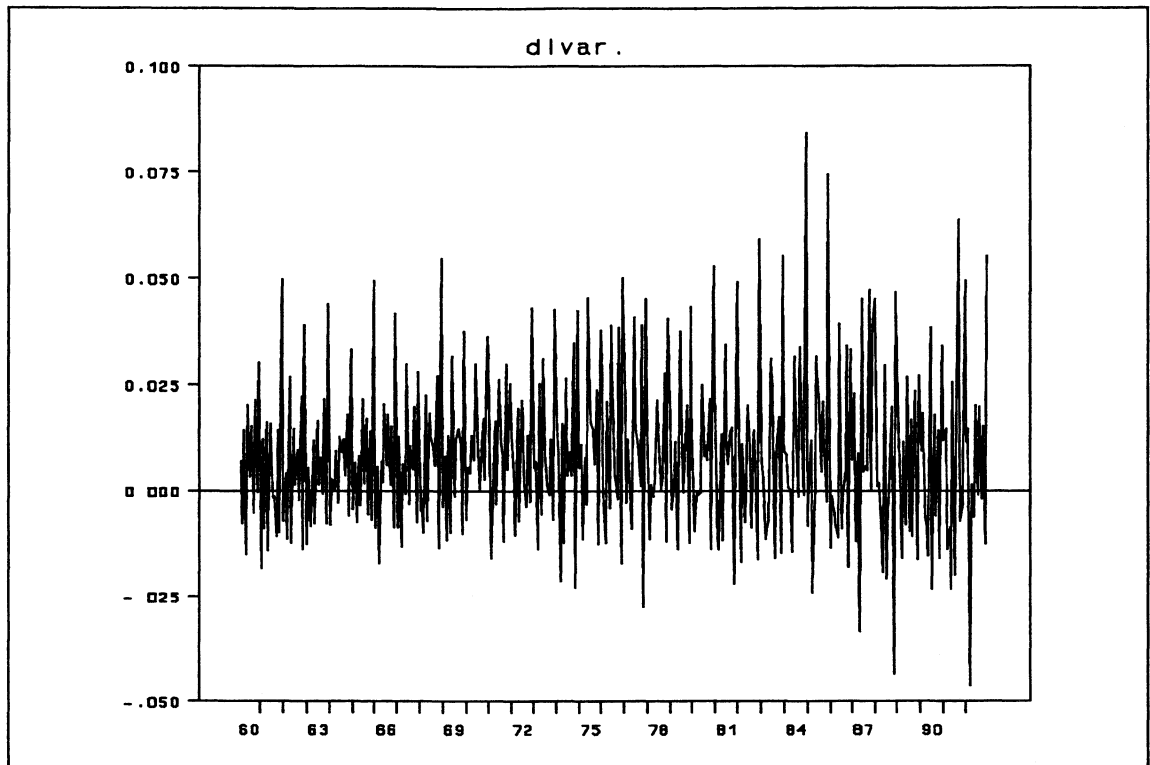


Figure 3.1.12. M2. The upper panel shows the differences of the logarithm of the original data. The lower is an estimate with a 95% confidence interval of the logarithm of the spectral density.

4. Conclusion

We have in this report considered some major Norwegian macroeconomic series from two points of view.

First we have applied the Hodrick-Prescott filter on the levels. It is evident that this cannot be used mechanically. The default value leads in some cases to an oversmoothing of the series, while the result for others seems to be more reasonable. Thus choosing a value depending on the series in question seems necessary.

This can be problematic when a further analysis is undertaken, since the results may depend on the chosen value to smooth the series.

When analyzing the residuals from using the HP filter, there are some indications of cyclic behavior in the individual series. However, this is not confirmed from the estimates of the cross correlation structure with the smoothed GDP. Most of the crosscorrelations are insignificant, and the coherencies show no particular pattern. The possibility that the apparent cycles are spurious can therefore not be ruled out.

Secondly we considered the spectra of the first differences. In the majority of the series, the high frequency variation is most pronounced. It may therefore be difficult to extract the variation associated with the low frequencies from the differenced series.

Appendix A

Computing the Hoderick-Prescott filter

The filter series is determined as the values g_{-1}, g_0, \dots, g_T minimizing

$$\sum_{t=1}^T [(g_T - g_t)^2 + \lambda (g_t - 2g_{t-1} + g_{t-2})^2].$$

The first order conditions consist of solving a system of linear equations. Carrying out the necessary differantiation and defining the $(T+2) \times (T+2)$ matrices

$$A_{-2} = \lambda \begin{pmatrix} 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & \dots & \dots & 0 \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 1 & -2 & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & 0 \end{pmatrix}$$

i.e. the i 'th row of A_{-2}/λ consists of the values 1, -2, 1 in the i , $i+1$ and $i+2$ 'th column, respectively $i = 1, \dots, T$ and otherwise zeros, the two last rows only consists of zeros;

$$A_{-1} = -2\lambda \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & \dots & \dots & 0 \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & 1 & -2 & 1 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 \end{pmatrix}$$

i.e. the first row of $A_{-1}/(-2\lambda)$ consists of only zeros, the i 'th row consist of 1, -2, 1 in the $i-1$, i and $i+1$ 'th column and otherwise zeros, $i = 2, \dots, T+1$, the last row consists of zeros only;

$$A_0 = \begin{pmatrix} 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & 0 \\ \lambda & -2\lambda & 1+\lambda & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \lambda & -2\lambda & 1+\lambda \end{pmatrix}$$

i.e. the two first rows consists of zeros only while the i 'th row consists of λ , -2λ , -2 in the $i-2$, $i-1$ and i 'th coloumns, respectively, $i = 3, \dots, T+2$. Also define the $T \times 2$ vectors

$$g = (g_{-1}, g_0, \dots, g_T)'$$

$$y = (0, 0, y_1, \dots, y_T)'$$

the first order conditions may be written

$$(A_{-2} + A_{-1} + A_0)g = y.$$

For large values of T , this can be a huge system. It may therefore be easier to exploit a recursive method for determining g_{-1}, g_0, \dots, g_T which is essentially the dynamic programming algorithm taking into account the simplification due to the fact that the minimization problem is quadratic in this case. We shall present the details. The algorithm is implemented in the routine of the econometric estimation package RATS computing the HP filter. The following definitions will be used

$$x_t = (g_t, g_{t-1}, 1)' \quad t = 1, \dots, T$$

$$u_t = g_t - 2g_{t-1} + g_{t-2} \quad t = 1, \dots, T$$

Then

$$(A1) \quad x_t = Ax_{t-1} + Bu_t$$

where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$B = (1, 0, 0)'$$

With these notations the minimization problem consists of finding the minimum of

$$(A2) \quad \sum_{t=1}^T \{ [c_t' x_t]^2 + \lambda u_t^2 \}$$

where

$$c_t = (1, 0, -y_t)'$$

The idea is now to express the value of g_T corresponding to a minimum of (A2) as a function of g_{T-1} , g_{T-2} , ..., g_0 , g_{-1} and $\{y_t\}_{t=1, \dots, T}$. Substituting in (A2) we can find the value of g_{T-1} corresponding to a minimum as a function of g_{T-2} , ..., g_{-1} and $\{y_t\}$. Continuing in this way lead to a minimization problem in two variables where the minimizing value can be computed explicitly in terms of $\{y_t\}_{t=1, \dots, T}$. Then we can use the expression (A1) to compute the other $\{g_t\}_{t=1, T}$ in terms of $\{y_t\}_{t=1, T}$. Here are the details. Since g_T is only appearing in the last term of the sum in (A2) we minimize

$$x_T' c_T' c_T x_T + \lambda u_T^2$$

with respect to g_T . The first order conditions express the optimal g_T in terms of g_{T-1} and g_{T-2} as

$$(1 + \lambda)g_T - 2g_{T-1} + g_{T-2} = y_T$$

or

$$u_T = \frac{y_T}{1 + \lambda} - \frac{2\lambda}{1 + \lambda}g_{T-1} + \frac{\lambda}{1 + \lambda}g_{T-2}$$

which can be written

$$u_T = -B' c_T c_T' A x_{T-1} / (1 + \lambda) = z_T' \cdot x_{T-1}$$

Now consider the minimization w.r.t. g_{T-1} and g_T . Then only the two last terms in the sum (A2) will be involved. They can be expressed as

$$(A3) \quad (x_{T-1}' c_{T-1}' c_{T-1} x_{T-1} + \lambda u_{T-1}^2) + (x_T' c_T' c_T x_T + \lambda u_T^2)$$

Now we use the expression of the optimal value of g_T in terms of g_{T-1} , ..., g_{-1} , to get

$$x_T = A x_{T-1} + B u_T = (A + B z_T') x_{T-1}$$

which means that the relevant part of (A2) can be written as

$$x'_{T-1} [c'_{T-1} c_{T-1} + (A' + z_T B') c'_T c_T (A + B z_T) + \lambda z_T z_T'] x_{T-1} + \lambda u_{T-1}^2 = x'_{T-1} D'_{T-1} D_{T-1} x_{T-1} + \lambda u_{T-1}^2.$$

where D_{T-1} can be expressed in terms of A, B and c_T . Minimizing this expression in terms of g_{T-1} parallels the minimization of (A2) in terms of g_T and leads to

$$u_{T-1} = z'_{T-1} x_{T-2}$$

for z_{T-1} expressed as an updating of z_T .

One can continue in this way until the optimal value of g_3 is expressed in terms of g_2, g_1, g_0, g_{-1} . Then it is necessary to take into account that g_1 and g_0 can be chosen so that u_1 and u_2 are equal to 0. That means that one has to minimize

$$(A4) \quad (y_1 - g_1)^2 + (y_2 - g_2)^2 + \sum_{i=3}^T [(y_i - g_i)^2 + \lambda (g_i - 2g_{i-1} + g_{i-2})^2]$$

By using the recursion explained above one can express the optimal value of the sum in (A2) as a function of g_1 and g_2 in the form $x'_2 D'_2 D_2 x_2$, which is easily minimized with respect to g_1 and g_2 . This determines the optimal values at g_1 and g_2 as functions of $\{y_t\}_{t=1, T}$. The optimal values for g_3, \dots, g_T can now be determined by using $u_3 = z'_3 x_2, x_3 = Ax_2 + Bu_3$ etc.

Appendix B

Estimating the spectral density

The estimation method for the spectral density and the crossspectral density is based on smoothing the periodogram, which is the standard method. The periodogram is defined as

$$I_{XX}^{(T)}(\omega) = \frac{1}{2\pi T} \left| \sum_{t=0}^{T-1} X_t e^{-i\omega t} \right|^2$$

where X_1, \dots, X_T are the observed value of a time series with spectral density $f_{XX}(\omega)$. Consider the values $\omega_j = \frac{j2\pi}{T}$, $j=0, \dots, T$. Then we have the following results under some mild regularity conditions.

$$EI_{XX}^T(\omega_j) = f_{XX}(\omega_j) + o\left(\frac{1}{T}\right) \quad j = 1, \dots, T-1$$

$$\text{var} I_{XX}^T(\omega_j) = f_{XX}(\omega_j) + o(1/T) \quad j = 1, \dots, T-1$$

$$\text{cov}(I_{XX}^T(\omega_j), I_{XX}^T(\omega_k)) = o(1/T) \quad j, k = 1, \dots, T-1 \quad j \neq k$$

The errors $o\left(\frac{1}{T}\right)$ denote terms which tends to zero no slower than $1/T$. This means that I_{XX} is an asymptotically unbiased estimator of f_{XX} . It is not consistent since the variance is not tending to zero. Also the estimates are asymptotically uncorrelated. This means that computing the estimates $I_{XX}^T(\omega_j)$ $j=1, \dots, T$ the estimates will fluctuate heavily. To obtain consistent estimates the usual method is to smooth adjacent values of I_{XX} . There are several ways to do this. What is important is that a bias is introduced if so many adjacent values are included that f_{XX} varies over the interval used. On the other hand the variance is less when the smoothing includes many values. Hence there is a tradeoff between unbiasedness and stable estimators and some compromise must be found. We refer to textbooks on spectral analysis for more details.

In practice the periodogram is computed at more values than ω_j , $j=1, \dots, T$. The figures in this report are based on $T'=432$, except for the monthly series where $T'=1536$, and $\omega'_j = \frac{j \cdot 2\pi}{T'}$ $j=1, \dots, T'-1$.

These values are smoothed using a moving average of 21 for the spectral densities except for the M2 series where there is no smoothing. When using these values for the crossspectra, the estimates of the coherence showed large variations. Hence we used a moving average of length 71 width.

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The Business Cycle History Project

The Business Cycle History Project consists of a number of subprojects aiming at analysing (the forces behind) business cycles in Norway in the period 1973-93, based on different, albeit coordinated methodologies. The project includes:

- Time series analyses of macroeconomic variables (identification of cycles, turning points, plus correlations between the variables).
- Analyses of effects of "exogenous shocks" by means of contrafactual simulations on the Statistics Norway's quarterly macroeconomic model (KVARTS), focusing on shocks from world markets, from the oil sector and from domestic economic policy, plus supply side shocks in the labour and commodity markets.
- Analyses of internal Norwegian business cycle dynamics as described by the dynamic properties of the quarterly model.

Reports from the project published so far:

Kjell Wettergreen: "Bestemmelse av konjunktuelle vendepunkter" (Identifying business cycle turning points). Notater 93/16

Torbjørn Eika: "Hvorfor steg arbeidsledigheten så mye" (What caused the huge increase in unemployment). Rapport 93/23. A summary published in "Faktorer bak økningen i arbeidsledigheten 1988-1991", Økonomiske analyser 2/93

Leo Andreas Grünfeld: Monetary Aspects of Business Cycles in Norway. An Exploratory Study Based on Historical Data. Discussion Papers No 131. Oktober 1994

Anders Rygh Swensen: Simple examples on smoothing macroeconomic time series. Documents 95/1

Most of the remaining reports will be published during 1995.

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