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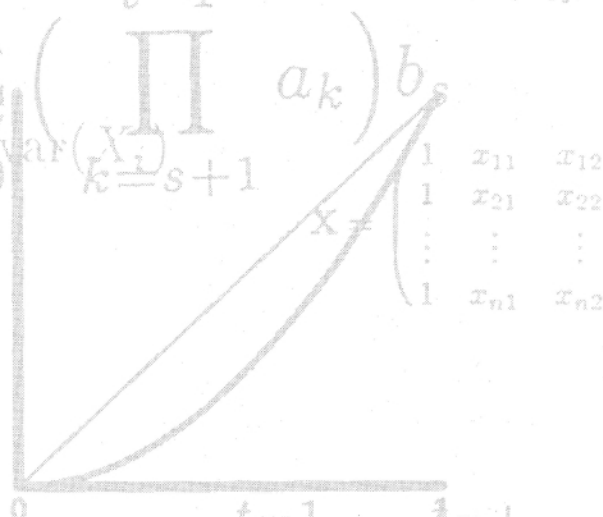
Brita Bye

# A Dynamic Equilibrium Analysis of a Carbon Tax

# Discussion Papers

$$+ 2 \sum_{i>j} \sum_{j=1} \text{COV}_a(X_i, X_j)$$

$$\text{var}\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i^2 \text{var}(X_i) + \sum_{s=0}^{t-1} \left(\prod_{k=s+1}^{t-1} a_k\right) b_s$$



$$\text{var}\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i^2 \text{var}(X_i) + \sum_{s=0}^{t-1} \left(\prod_{k=s+1}^{t-1} a_k\right) \sum_{i=1}^n (y_i - (\hat{a}x_i + \hat{b}))^2$$

*Brita Bye*

## **A Dynamic Equilibrium Analysis of a Carbon Tax**

**Abstract:**

This paper analyses the effects of a carbon tax on a small open petroleum producing economy, using an aggregate intertemporal general equilibrium model with differentiated products. The long run effects on welfare and capital accumulation of both a *unilateral* and an *international* carbon tax are emphasised. It is shown that the steady state welfare effect of a carbon tax can be positive or negative, depending on substitution effects which create efficiency losses, and income effects from changes in terms of trade. The presence of an initial tax wedge implies that there is an ambiguous relationship between the tax level and steady state welfare. With an *international* carbon tax the terms of trade gain is smaller and the petroleum revenue is reduced compared to a *unilateral* carbon tax, implying that for a petroleum producing economy an *international* carbon tax may be less beneficial than a *unilateral* carbon tax.

**Keywords:** Dynamic equilibrium analysis, Differentiated products, Carbon taxes.

**JEL classification:** D50, D60, D90, Q43.

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**Address:** Brita Bye, Statistics Norway, Research Department,  
P.O.Box 8131 Dep., N-0033 Oslo, Norway. E-mail: bby@ssb.no



# 1 Introduction

Utilisation of energy, especially combustion of fossil fuels, is an important source of environmental pollution. The increasing concern about global warming (the greenhouse effect), partly caused by increased emissions of carbon dioxide ( $CO_2$ ) which is the single most significant man-made greenhouse gas, has led to a large number of studies of how to curb further increase in these emissions and of the corresponding costs of imposing different means. One of the most common policy prescriptions for slowing greenhouse gas emissions is a carbon tax. This is a tax on fossil fuels in proportion to the amount of atmospheric carbon dioxide that is released when they are burned. Nordhaus (1993), Poterba (1993) and Jorgenson and Wilcoxon (1993) give an overview of the recent studies. Since the greenhouse effect is a global problem and the different countries' shares of greenhouse gas emissions are small, the most efficient way to reduce the emissions will be to make a multilateral agreement on a level of an *international* carbon tax. Nordhaus (1993) and Poterba (1993) discuss the optimal tax level.

Whether the carbon tax is *unilateral* or a result of a multilateral agreement on an *international* carbon tax to curb global emissions of  $CO_2$ , is important for a petroleum producing economy. According to Oliveira Martins *et al* (1992) an *international* carbon tax is likely to have the following effects on the world economy; a general recession will reduce the producers' oil price, the international price level will increase and international economic growth will be slower. Substitution from coal based to natural gas based electricity production due to lower carbon content in gas, can increase the demand for natural gas and the gas price may increase. But both ECON (1990) and Ingham *et al* (1993) argue that the demand for natural gas also will fall following a carbon tax, inducing a fall in the price of natural gas. Rosendahl (1994) shows that the reduction in the Norwegian petroleum wealth can be substantial following an international carbon tax.

Carbon taxes will in addition to short term effects on prices and costs, also have long term welfare effects through changing the rate of capital accumulation and economic growth. Hence, the suitable model framework is intertemporal general equilibrium models which generate optimal consumption-savings path. This paper presents an aggregate general equilibrium model to analyse the effects of a carbon tax on the stock of capital and welfare. The model pictures a small open, oil and gas producing economy with differentiated products, see Bye and Holmøy (1992). Intertemporally optimising producers and consumers have perfect foresight. A *unilateral* carbon tax will affect the price of fossil fuels and hence the price path of the optimal consumption bundle. Furthermore, the tax will increase the costs of producing capital goods, thereby changing the rate of capital accumulation and welfare. An *international* carbon tax will in addition affect the world markets for fossil fuels, having consequences for the petroleum revenue. In this paper we assume that a fall in the producers' oil price does not outweigh the tax such that the domestic purchaser price of fossil fuels increases, and the increase is relatively higher than the increase in the price on other competing imports due to lower carbon content in this commodity. As discussed in Oliveira

Martins *et al* (1992) the terms of trade gain is smaller for fossil fuels producing countries since the world market price of fossil fuels is reduced with an *international* tax. It turns out that a higher terms of trade gain may make the implementation of a *unilateral* carbon tax more beneficial for a petroleum producing economy. Any benefits for the country from lower global warming are not taken into consideration in the analysis.

As in Okuyama and Turnovsky (1994) the model is a dynamic equilibrium model, but the focus is on the long run solution, although the dynamic path of the economy is characterised. The dimensionality of the model makes it too complex for an analytical closed form description of the dynamics. The long run effects on the steady state equilibrium are analysed. In general the total effect on welfare of a tax change depends on the efficiency loss generated by the substitution effects of the tax rate change, and the income effects from changes in terms of trade. The analysis belongs to the literature of taxation in open economies surveyed by Dixit (1985) in a static framework.

Solving numerical disaggregated general equilibrium models with perfect foresight has been until recently quite difficult, such that most of the studies measuring the costs of reducing  $CO_2$ -emissions have used a model framework without intertemporal optimisation and hence no optimal consumption-savings path. Dean and Hoeller (1992) compare the costs of reducing  $CO_2$  -emissions in six global models where the only intertemporal optimisation model is the GLOBAL 2100 model, Manne (1992). Brendemoen and Vennemo (1994) analyse the effects on the Norwegian economy of an international carbon tax, using a numerical disaggregated general equilibrium model without any optimal consumption-savings choice. Håkonsen and Mathisen (1994) study how the costs of reducing  $CO_2$ -emissions depend on the model specification and the redistribution of the tax revenue, using a static general equilibrium model for Norway with constant stock of real capital. Jorgenson and Wilcoxon (1993) use a numerical disaggregated general equilibrium model with intertemporally optimising agents to analyse the costs of reducing  $CO_2$ -emissions for the US. A common result from these studies is that implementing a carbon tax has costs as lower GDP growth and reduced consumption.

Since global warming is a world issue and a small country's contribution to total global emissions of  $CO_2$  is negligible, most of the existing studies (Dean and Hoeller (1992), Manne (1992) and Brendemoen and Vennemo (1994)) consider an *international* carbon tax. But there are some arguments which can justify the implementation of a *unilateral* carbon tax. There are locally beneficial pollution effects and possibly lower traffic congestion following an introduction of a carbon tax, see Brendemoen and Vennemo (1994) and Håkonsen and Mathisen (1994). It is also possible to reduce existing distortionary taxes when implementing efficient environmental taxes, see e.g. Goulder (1994) and Håkonsen and Mathisen (1994).

The assumption of a positive terms of trade effect for a small open economy being initiated by product differentiation, deserves a comment. A vast literature on trade and imperfect competition identifies structures generating monopolistic competition and market power, see e.g. Helpman and Krugman (1985). The size of the country is not necessarily

relevant for this structure to be an equilibrium characteristics. Norman (1990) criticises the Armington approach as an approximation for oligopolistic structures and product differentiation at the firm level. On the other hand, econometric work at an aggregate industry level often end up with surprisingly small trade elasticities, see e.g. Lindquist (1993) for a recent econometric study of Norwegian industries export behaviour.

The paper is organised as follows: Section 2 gives a brief description of the structure of the model. In Section 3 long term effects of a tax on fossil fuels are discussed, and the differences between a *unilateral* and an *international* carbon tax are made precise. The transitional dynamics are also described. Section 4 summarises the main findings.

## 2 The model framework

### 2.1 Technology and preferences

Except for the presence of the carbon tax, the model is identical to the one analysed in Bye and Holmøy (1992). The economy is using two goods which are both used as capital input by the firms and consumed by the households. Good 1 is a composite good of a domestic and a foreign variety which are imperfect substitutes for each other. The domestic and the foreign varieties compete both on the domestic and on the world market. Good 2, fossil fuels, is supplied by imports only. The petroleum revenue is assumed to be exogenous which can be justified by the assumption that all domestic oil and gas production is exported at given world market prices and the production is too small to have any effects on these markets for fossil fuels. All firms have identical linearly homogeneous technology and all households have identical homothetic preferences. The structure of technology and preferences is assumed to be separable so that the two goods enter both the production function and the utility function through a macro commodity. The household preferences and the production technology are assumed to be similar with respect to the composition of the macro commodity. Due to constant returns to scale and competitive behaviour, the part of the demand structure that determines the composition of the macro commodity can be represented in dual terms by the following linearly homogeneous price functions:

$$(1) \quad P_1 = p_1(P_1^H, P_1^I)$$

$$(2) \quad P = p(P_1, P_2)$$

where  $P_2 = P_2^*(1 + \tau)$ .  $P_2^*$  is the world market price of fossil fuels and  $\tau$  denotes the carbon tax corresponding to the aggregate commodity fossil fuels. The domestic price  $P_1$  of the competing commodity is the unit cost of acquiring good 1. It is an ideal price index of the domestic producer price  $P_1^H$  and the price of the competing imports  $P_1^I$ . The price of the macro commodity  $P$  is formed in an analogous way and represents an ideal price index of the price of this competing commodity  $P_1$  and the price of fossil fuels  $P_2$ . The import prices

$P_1^I$  and  $P_2^*$  are both determined exogenously on the world market. The exchange rate is fixed to unity and all values are measured in domestic currency.

## 2.2 Producer behaviour

Due to competition among a sufficiently large number of domestic firms, the price on the domestic product is forced down to equal unit cost no matter where the commodity is sold. The individual firms are price takers in all markets and face given time paths of wages  $P^L$ , world market interest rate  $r$  and product prices. They rent labour  $L$ , but own the capital stock  $k$  which depreciates at a fixed rate  $\delta$ . Investment,  $J$ , is financed by retained profits. The supply of labour is assumed to be fixed, normalised to unity, and the labour market is assumed to be in equilibrium at all points of time. The production function is linearly homogeneous and the variables should be interpreted in per capita terms. The decision problem of the representative firm at time 0 is to choose the time paths of the control variable per capita gross investment so that the present value of the cash flow,  $V$ , is maximised:

$$(3) \quad V_0 = \int_{t=0}^{\infty} [P_1^H f(k) - PJ - P^L] e^{-rt} dt$$

subject to

$$(4) \quad \dot{k} = J - \delta k$$

$$(5) \quad k(0) = k_0$$

where  $k_0$  is predetermined and  $\dot{k}$  is backward looking. The transversality condition is given by

$$(6) \quad \lim_{t \rightarrow \infty} e^{-rt} P k = 0$$

The necessary f.o.c. are

$$(7) \quad P_1^H f'(k) = (r + \delta - \frac{\dot{P}}{P})P$$

$$(8) \quad P_1^H [f(k) - k f'(k)] = P^L$$

where  $\dot{P}$  is forward looking.

## 2.3 Consumer behaviour

The household sector is described by a representative consumer with an infinite horizon. The consumer's objective is to maximise total discounted utility with respect to total consumption  $c$  of the macro commodity. The consumption of leisure is fixed. Tax revenues are rebated to the consumer as lump-sum transfers, and no governmental sector is specified. Thus, all income in the economy is received by the representative consumer. The consumer's

income consists of wages, interest on financial wealth, net cash flow from the firms and lump sum transfers equal to the revenue from a carbon tax. The consumer's savings take the form of lending to the rest of the world at a given rate of interest. The time path of aggregate consumption is determined by solving the following maximisation problem:

$$(9) \quad \max_{(c)} U_0 = \int_0^{\infty} u(c_t) e^{-\rho t} dt$$

subject to the budget constraint:

$$(10) \quad \dot{b} = rb + P_1^H f(k) - Pc - PJ + \Omega + z$$

$$(11) \quad \lim_{t \rightarrow \infty} b e^{-rt} = 0$$

$$(12) \quad b(0) = b_0$$

$b$  is net financial wealth and  $b_0$  is predetermined.  $\Omega$  is lump sum transfer<sup>3</sup> and  $z$  is exogenous petroleum revenue.  $\rho$  is the subjective rate of time preference. Recall that  $P$  is determined by (1) and (2) because of the assumptions of separability in the preference structure. Equation (11) is the transversality condition.

From the utility maximisation we have the following first order condition:

$$(13) \quad \frac{u'_c}{P} = \mu$$

where  $\mu$  is the costate variable associated with the financial wealth accumulation equation (10). The costate variable evolves according to

$$(14) \quad \dot{\mu} = \frac{u'_c}{P} (r - \rho)$$

We assume the following well known specification of the utility function:

$$u(c_t) = \frac{\sigma^c}{\sigma^c - 1} c^{\left(\frac{\sigma^c - 1}{\sigma^c}\right)}, \quad \text{when } \sigma^c \neq 1$$

$$u(c_t) = \ln c_t, \quad \text{when } \sigma^c = 1$$

$\sigma^c$  is the intertemporal elasticity of substitution.

From the first order conditions of intertemporal utility maximisation we get the following simple relationship between consumption and the costate variable  $\mu$ :

$$(15) \quad c = (\mu P)^{-\sigma^c}$$

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<sup>3</sup> $\Omega = P_2^* \tau p'_2 (c + J)$  where  $p'_2 = \frac{\partial p(P_1, P_2)}{\partial P_2}$ . It is standard in the representative agent framework to assume that the agent being one of a large number of agents, is unable to infer his share of the total tax revenue. Hence (10) with  $\Omega$  treated as exogenous, is the budget constraint considered.



## 2.4 Equilibrium dynamics

The dynamic stability properties of an intertemporal model of a small open economy is discussed in numerous papers, see e.g. Sen and Turnovsky (1989). A necessary condition for obtaining a steady state solution is  $r = \rho$ . From the single country's perspective this is a "razor's edge" requirement as both the interest rate and the rate of time preference are exogenous in the model of a small open economy. For convenience it is assumed that the condition holds at all points in time.

The condition for equilibrium in the domestic product market is that domestic output equals domestic and foreign demand.

$$(16) \quad f(k) = p'_1(P_1, P_2)p'_{1H}(P_1^H, P_1^I)(c + \dot{k} + \delta k) + a\left(\frac{P_1^I}{P_1^H}\right)$$

The partial derivatives of the price function are defined as  $\frac{\partial p(P_1, P_2)}{\partial P_1} = p'_1(P_1, P_2)$  and  $\frac{\partial p_1(P_1^I, P_1^H)}{\partial P_1^H} = p'_{1H}(P_1^I, P_1^H)$  a.s.o. By Shephard's lemma the partial derivatives equal the commodity shares. Export demand  $a = a(P_1^I/P_1^H)$  is a decreasing function of the domestic product price relative to the world market price. The elasticity of substitution between domestic and foreign varieties of good 1 is assumed to be the same on the domestic market as on the export market<sup>4</sup>.

A reduced form of the dynamic equilibrium system is obtained by eliminating  $P$  from equations (1) and (2).  $P$  can be replaced by the function  $P = g(P_1^H, P_1^I, P_2) = p(p_1(P_1^H, P_1^I), P_2)$ , and we have  $g'_H(P_1^H, P_1^I, P_2) = p'_1(p_1(P_1^I, P_1^H), P_2)p'_{1H}(P_1^I, P_1^H)$  which is the home share of one unit of domestic absorption. We assume for simplicity that the exogenous variables  $P_1^I$ ,  $P_2^*$ ,  $\tau$  and  $\delta$  are constant through time. The dynamic system derived from (7), (16) and (10) with the tax term  $\Omega$  inserted, takes the following form<sup>5</sup>:

$$(17.a) \quad \dot{P}_1^H = h_1(k, P_1^H) = \frac{1}{g'_H} [(r + \delta)g - P_1^H f'(k)]$$

$$(17.b) \quad \dot{k} = h_2(k, P_1^H; \mu) = \frac{1}{g'_H} [f(k) - a] - (\mu g)^{-\sigma_c} - \delta k$$

$$(17.c) \quad \dot{b} = rb + P_1^H f(k) - G [(\mu g)^{-\sigma_c} + \dot{k} + \delta k] + z$$

where  $G \equiv g'_H P_1^H + g'_I P_1^I + g'_2 P_2^*$  and  $g'_I = p'_1 p'_{1I}$  and  $g'_2 = p'_2$ .  $G$  can be interpreted as the price of the domestic macro commodity exclusive of the tax on fossil fuels. Equation (17.c) gives the current account.

For any given  $\mu$ , the variables  $P_1^H$  and  $k$  can be solved from equations (17.a) and (17.b), and then  $b$  follows from equation (17.c). However the resulting accumulation of  $b$  will in general be inconsistent with the transversality condition imposed by (11) and will eventually explode. The complete equilibrium therefore requires a solution for  $\mu$  that satisfies (11), see

<sup>4</sup>This simplifies the analysis without losing any main points.

<sup>5</sup>The arguments of the export and price functions are suppressed such that  $g(P_1^H, P_1^I, P_2) = g$ ,  $g'_H(P_1^H, P_1^I, P_2) = g'_H$  a.s.o. in the rest of the paper.

e.g. Sen and Turnovsky (1989) and Brock and Turnovsky (1993) for the same kind dynamic structure. A special feature of this structure is the path dependency of the stationary solution. A closed form solution of the model can therefore only be found in approximate terms by linearising around the steady state  $(\bar{k}, \bar{P}_1^H)$  solution.

Such a linearisation of equations ( 17.a) and ( 17.b) yields

$$(18) \quad \begin{pmatrix} \dot{P}_1^H \\ \dot{k} \end{pmatrix} = \begin{pmatrix} h_{1P} & h_{1k} \\ h_{2P} & h_{2k} \end{pmatrix} \times \begin{pmatrix} P_1^H - \bar{P}_1^H \\ k - \bar{k} \end{pmatrix}$$

The elements in the Jacobian  $h_{1k} = \frac{\partial h_1(\bar{k}, \bar{P}_1^H)}{\partial k}$ ,  $h_{1P} = \frac{\partial h_1(\bar{k}, \bar{P}_1^H)}{\partial P_1^H}$  a.s.o. are evaluated at steady state and given in appendix A. Its determinant can be shown to be negative and the trace  $h_{1P} + h_{2k} = r > 0$ , so that there is a positive and a negative eigenvalue,  $\lambda_1 < 0$  and  $\lambda_2 > 0$ . Thus, the long-run equilibrium is a saddle-point. The stable solution for  $k$  is

$$(19) \quad k(t) = \bar{k} + (k_0 - \bar{k})e^{\lambda_1 t}$$

Inserting this into the linearised form of (18) we obtain the stable solution for  $P_1^H$ .

$$(20) \quad P_1^H(t) = \bar{P}_1^H + \frac{\lambda_1 - h_{2k}}{h_{2P}}(k_0 - \bar{k})e^{\lambda_1 t}$$

To determine the dynamics of the current account we linearise equation ( 17.c) around steady state.

$$(21) \quad \begin{aligned} \dot{b} &= r(b - \bar{b}) + (P_1^H f'(\bar{k}) - \delta G)(k_0 - \bar{k}) \\ &+ \left[ f(\bar{k}) - (\delta \bar{k} + (1 - \sigma_c)(\mu g)^{-\sigma_c})g'_H \right] (P_1^H - \bar{P}_1^H) \end{aligned}$$

Substituting the solution of  $P_1^H$  from ( 20) and  $k$  from ( 19) we obtain the following differential equation in  $b$ :

$$(22) \quad \dot{b} - rb = \alpha(k_0 - \bar{k})e^{\lambda_1 t} - r\bar{b}$$

where

$$\alpha \equiv P_1^H f'(k) - G\delta + \frac{\lambda_1 - h_{2k}}{h_{2P}} \left[ a + \tau g''_{2H}(c + \delta k) + G\sigma^c c \frac{g'_H}{P_1^H} \right]$$

and  $g''_{2H} = \frac{\partial^2 p'_2}{\partial P_1^H}$ . Assuming that the economy starts out with an initial stock of net financial wealth,  $b(0) = b_0$ , the solution to equation (22) is:

$$(23) \quad b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{\lambda_1 t}}{\lambda_1 - r} + \left[ (b_0 - \bar{b}) - \frac{\alpha(k_0 - \bar{k})}{\lambda_1 - r} \right] e^{rt}$$

In order for the transversality condition (11) to be satisfied, the last term must vanish, implying that:

$$(24) \quad b_0 - \bar{b} = \frac{\alpha(k_0 - \bar{k})}{\lambda_1 - r}$$

The solution which is consistent with long run solvency is then given by

$$(25) \quad b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{\lambda_1 t}}{\lambda_1 - r}$$

Equation (24) describes the equilibrium relationship between the change in the equilibrium stock of capital and the change in the equilibrium net wealth of the economy, depending crucially on the sign of  $\alpha$ . Consider the expression for  $\alpha$ . The first term  $P_1^H f'(k) - G\delta$  which is the increase in the net product in sector 1, is positive. In the second term  $\frac{\lambda_1 - h_2 k}{h_2 P} = \frac{\partial P_1^H}{\partial k}$  evaluated in steady state, is negative, see appendix A. The expression in the brackets  $a + \tau g_{2H}''(c + \delta k) + G\sigma^c c \frac{g_H'}{P_1^H}$  which summarises the increase in the export value and the intratemporal and intertemporal substitution away from the domestic product, is positive. Then it is possible that  $\alpha$  is negative and the relationship between  $k$  and  $b$  is positive. With a small price elasticity in demand, an increase in production caused by an increase in the capital stock induces a large fall in the price of the domestic product to clear the commodity market. The fall in the price outweighs the increase in the production such that the income is lower. Hence, it is necessary to have a higher level of net financial wealth to outweigh the fall in the production income. This is in contrast to a situation with prices given on the world market. When the domestic and foreign varieties are perfect substitutes, there is a negative relationship between  $k$  and  $b$ , see Sen and Turnovsky (1989). Although this assumption has been relaxed in the present study, the subsequent analysis will be confined to the case where the price elasticity is large enough to satisfy  $\alpha > 0$ .

The stationary solution of the linearised system is given by the following equations:

$$(26.a) \quad (r + \delta)\bar{g} = \bar{P}_1^H f'(\bar{k})$$

$$(26.b) \quad \frac{1}{g_H'}(f(\bar{k}) - \bar{a}) = (\bar{\mu}\bar{g})^{-\sigma^c} + \delta\bar{k}$$

$$(26.c) \quad r\bar{b} = \bar{G} [(\bar{\mu}\bar{g})^{-\sigma^c} + \delta\bar{k}] - \bar{P}_1^H f(\bar{k}) - z$$

$$(26.d) \quad \bar{b} - b_0 = \frac{\alpha}{r - \lambda_1}(k_0 - \bar{k})$$

which determine the steady state values  $\bar{P}_1^H$ ,  $\bar{k}$ ,  $\bar{b}$  and  $\bar{\mu}$ .

### 3 Analysis of a carbon tax

Since the analysis is based on the assumption of perfect foresight, the transitional adjustment is forward looking and is therefore determined in part by the expectations of the long run

steady state. As stated in the introduction, the dimensionality of the model makes it too complex for an analytical closed form description of the dynamics. Hence, the focus is on the long run solution, but the dynamic path is characterised in section 3.2. The effects of both a multilateral agreement on an *international* carbon tax to curb emissions of  $CO_2$  and a *unilateral* tax, are studied.

### 3.1 Long run response

To analyse the effects of a carbon tax on the steady states values of  $P_1^H$ ,  $k$ ,  $c$  and  $b$ , we first find the total logarithmic differentiates of the steady state solution (26.a) - (26.d) w.r.t.  $\tau$ ,  $P_1^I$ ,  $P_2^*$  and  $z$ . The three latter variables are only affected in the case of an *international* carbon tax. Since changes in the steady state level of consumption can be interpreted as changes in steady state welfare, we substitute the expression for consumption  $c = (\mu P)^{-\sigma^c}$  in the steady state solution (26.a) - (26.d).  $\check{P}$ ,  $\check{P}_1^H$ ,  $\check{k}$ , a.s.o. represent the logarithmic derivatives.

$$(27.a) \quad \check{P}_1^H = \frac{1}{\theta^I}(\theta_1 \theta_1^I \check{P}_1^I + \theta_2 \check{P}_2 + \frac{\theta^L}{\sigma^k} \check{k})$$

$$(27.b) \quad (\theta^K - \frac{\delta^H}{\bar{x}} \bar{k}) \check{k} = s_{IH}(\check{P}_1^H - \check{P}_1^I) + s_{12}(\check{P}_2 - \check{P}_1) + \frac{\bar{c}^H}{\bar{x}} \check{c}$$

$$(27.c) \quad \begin{aligned} \bar{G} \bar{c} \check{c} &= \bar{P}_1^H \bar{x} \check{P}_1^H + \bar{z} \check{z} + r \bar{b} \check{b} - \bar{G}(\bar{c} + \delta \bar{k}) \check{G} \\ &- (\bar{G} \delta \bar{k} - \bar{P}_1^H \bar{x} \theta^k) \check{k} \end{aligned}$$

$$(27.d) \quad \check{b} = -\frac{\alpha \bar{k}}{\bar{b}(r - \lambda_1)} \check{k}$$

We have the following definitions;

$$s_{12} \equiv (1 - \frac{\bar{a}}{\bar{x}}) \theta_2 \sigma_{12} > 0, \quad s_{IH} \equiv -(\theta_1^H \frac{\bar{a}}{\bar{x}} + \theta_1^I) \sigma_{IH} < 0$$

$$\theta^I \equiv (1 - \theta_1^H \theta_1), \quad c^H \equiv g'_H \bar{c}, \quad \delta^H \equiv g'_H \delta, \quad x \equiv f(k),$$

and the following expressions;

$$\check{G} = \frac{P}{G} [\theta_1 \check{P}_1 + \frac{\theta_2}{1 + \tau} \check{P}_2^* + \theta_1 \theta_2 \frac{\tau}{1 + \tau} \sigma_{12} (\check{P}_2 - \check{P}_1)]$$

$$\check{P}_1 = \theta_1^H \check{P}_1^H + \theta_1^I \check{P}_1^I, \quad \check{P}_2 = \check{P}_2^* + \frac{\tau}{1 + \tau} \check{\tau}, \quad \check{c} = -\sigma^c (\check{\mu} + \check{P})$$

$\sigma^k$  is the substitution elasticity between labour and capital in the production function.  $\theta^K$  and  $\theta^L$  are the cost shares of capital and labour respectively.  $\theta_i$ ,  $i = 1, 2$ , are the budget shares of good 1 and fossil fuels respectively.  $\theta_1^j$ ,  $j = H, I$ , are the budget shares of the domestic and imported variety of good 1 respectively.  $\theta^I$  can be interpreted as the economy's total import share.  $\sigma_{12}$  is the elasticity of substitution between good 1 and fossil fuels.  $\sigma_{12}$  incorporates the substitution possibilities between fossil fuels and other non-taxed energy commodities as hydro based electricity and between fossil fuels and other non-energy commodities. Other energy commodities is only a small fraction of the aggregate good 1, so the elasticity of substitution between fossil fuels and good 1 is likely to be smaller than  $\sigma_{IH}$  which is the elasticity of substitution between the domestic and foreign variety of good 1 on both the domestic market and the export market.  $s_{IH}$  summarises the substitution effects between the domestic and foreign variety of good 1.  $s_{12}$  can be interpreted as the effect on the demand for the domestic product induced by substitution between fossil fuels and good 1.  $s_{12}$  is positive because an increase in the price of fossil fuels redirects demand to good 1 which is partly produced domestically.

The reduced form equations for  $\check{k}$  and  $\check{c}$  can be derived from (27.a), (27.b), (27.c), (27.d) and the expressions for  $\check{G}$  and  $\check{P}_1$ . The derivation of the reduced form equations are given in appendix B, and the effects on  $\check{k}$  and  $\check{c}$  of changes in the exogenous variables are given by equations (B.12) and (B.14). The effects on steady state welfare of a carbon tax are easiest exposed by first considering a *unilateral* carbon tax. Then it is discussed whether an *international* carbon tax modifies the results.

## A unilateral carbon tax

With a *unilateral* carbon tax  $\check{P}_2 = \frac{\tau}{1+\tau}\check{\tau} > 0$  and  $\check{P}_2^* = \check{P}_1^I = \check{z} = 0$  since the economy is small and there are no repercussions on international goods markets. The reduced form equations (B.12) and (B.14) can now be written as

$$(28.a) \quad \check{k} = \frac{1}{\kappa} \left[ s_{IH} \frac{\theta_2}{\theta^I} \frac{\tau}{1+\tau} \check{\tau} + s_{12} \frac{\theta_1^I}{\theta^I} (1 - \theta_1 \theta^I \frac{\tau}{1+\tau\theta_1}) \frac{\tau}{1+\tau} \check{\tau} + \theta_1 \theta_1^H \frac{\bar{a}}{\bar{x}} \frac{\theta_2}{\theta^I} \frac{\tau}{1+\theta_1\tau} \check{\tau} \right]$$

$$(28.b) \quad \check{c} = \frac{1}{G\bar{c}} \left[ \gamma \check{k} + \bar{P}_1^H \bar{a} \frac{\theta_2}{\theta^I} \frac{\tau}{1+\tau} \check{\tau} - s_{12} \frac{\bar{P}_1^H \bar{x}}{\theta^I} \frac{\theta_1^I}{\theta_1^H} \frac{\tau}{1+\tau} \check{\tau} \right]$$

Consider first equation (28.b) which corresponds to (27.c) after substituting the equilibrium effects on  $P_1^H$ ,  $b$ ,  $k$  and  $G$ . The effects on consumption can be divided into a general equilibrium effect due to changes in the stock of real capital  $k$ , an income effect due to improved terms of trade, and finally a substitution effect. The two latter effects are also identified

in Dixit (1985) in a more general static treatment of tax policies in open economies. Since the tax income is redistributed as lump sum transfers, all other income effects disappears. Terms of trade is improved through higher export price, and the corresponding income effect is represented by the term  $\bar{P}_1^H \frac{\bar{a}}{\theta^I}$ . The income effect depends positively on the import share of fossil fuels relative to the total import share because the domestic product is used in production of the macro good which is used as real capital. Thus, an increase in  $P_1^H$  implies a rise in  $P$  and thereby higher production costs. This is a kind of input-output effect which is larger the smaller is the total import share in the macro good.

The other effect, which we denote the initial tax wedge effect, is the direct substitution effect on consumption from an initial tax represented by the term  $s_{12} \frac{\bar{P}_1^H \bar{x} \theta^I}{\theta^I} \frac{\tau}{1+\tau}$ . This effect is larger the larger the substitution between good 1 and fossil fuels which is well known from the normative theory of taxation. Furthermore, the initial tax wedge effect is increasing in the value of production corrected for the total import share, and the ratio of the import to the domestic budget share of good 1. The effect is modified by a large domestic budget share of good 1.

The general equilibrium effect following a change in  $k$  are multiplied with the term  $\gamma$  which summarises the impact of  $k$  on the net national income.  $\gamma$  is given by

$$\gamma \equiv \frac{\bar{P}_1^H \bar{x} \theta^L}{\theta^I \sigma^k} \left( \frac{\bar{a}}{\bar{x}} + s_{12} \frac{\tau}{1+\tau} \right) + \left( \bar{P}(r + \delta) - \bar{G}\delta - \frac{\alpha}{r - \lambda_1} \right) \bar{k}$$

Net national income is affected through; (i) the marginal change in net output in fixed prices measured by  $((r + \delta)\bar{P} - \bar{G}\delta)\bar{k}$ ; (ii) the change in the domestic price due to the change in the marginal product of capital, measured by the first term in the expression for  $\gamma$ ; (iii) the changes in interest payments on net foreign debt, measured by  $\frac{\alpha}{r - \lambda_1} \bar{k}$ .

From the reduced form equation of  $\check{k}$  in the unilateral case (28.a), it is easily seen that these substitution and income effects also affect the stock of real capital through their effects on aggregate demand. The positive income effect following an increase in terms of trade, represented by the term  $\theta_1 \theta_1^H \frac{\bar{a} \theta_2}{\bar{x} \theta^I} \frac{\tau}{1+\theta_1 \tau} \check{\tau}$ , implies higher demand for the macro commodity for consumption and investment purposes. The other effects are intratemporal substitution effects. The term  $s_{12} \frac{\theta^I}{\theta^I} (1 - \theta_1 \theta^I \frac{\tau}{1+\tau \theta_1}) \frac{\tau}{1+\tau} \check{\tau}$  summarises two effects. First, an increase in the tax rate induces substitution away from fossil fuels towards good 1. For a given import share of good 1, domestic production and capital input must increase proportionally in order to restore product market equilibrium. This positive effect is modified by the negative initial tax wedge effect in consumption and investment demand represented by the last term within the brackets. If  $\tau = 0$  initially,  $\bar{P} = \bar{G}$ ,  $\bar{P}_2 = d\tau$  and  $\bar{G} = \bar{P} - \theta_2 d\tau$ . Inserting for  $\bar{G}$  in (27.c) implies that the initial tax wedge effect disappears and the economy is in a first best situation before the tax is implemented.

The last effect present is the substitution effect between domestic and foreign varieties of good 1 represented by the term  $s_{IH} \frac{\theta_2}{\theta^I} \frac{\tau}{1+\tau} \check{\tau}$ , which is negative. An increase in the domestic product price following a carbon tax implies substitution away from the domestic variety both on the domestic and export market. Due to the assumptions made about the elastic-

ities of substitution, the term  $(s_{IH}\theta_2 + s_{12}\theta_1^I)$  is negative<sup>6</sup>, such that the negative effect on consumption and the stock of real capital is reinforced by the initial tax wedge effect.

A lower level of capital stock contributes to a fall in the domestic product price and consumption through the general equilibrium effects. With  $\tau > 0$ , it is easily seen from equation (28.b) that the level of consumption is decreasing in the initial tax rate. Hence, it is not possible to receive a welfare gain by multiple tax increases. If the initial tax rate is large enough, the negative initial tax wedge effect will outweigh the positive income effect from the change in terms of trade, and a further tax increase gives a welfare loss. If the initial tax rate is sufficiently small, the positive income effect from the terms of trade change may dominate the negative substitution effect, and welfare is higher in the new long run equilibrium.

To analyse the welfare effect along the dynamic adjustment path implies evaluating the welfare integral of the consumer given in equation (9), and as in Okuyama and Turnovsky (1994) this is too complex to be done analytically.

### An international carbon tax

With an *international* carbon tax  $\tilde{\tau} > 0$ ,  $\tilde{P}_2^* < 0$ ,  $\tilde{P}_2 > 0$ ,  $\tilde{P}_1^I > 0$ ,  $\tilde{z} < 0$  and  $\tilde{P}_2 > \tilde{P}_1^I$ . In section 1 possible effects on the world markets of an *international* carbon tax are reviewed. The increase in the domestic price of fossil fuels is smaller than in the *unilateral* case because the world market price falls, but it is assumed to be larger than the increase in the price of the foreign variety of good 1, i.e.  $(\tilde{P}_2 - \tilde{P}_1^I) > 0$ . From appendix B we have the reduced form equations.

$$(B.12) \quad \begin{aligned} \dot{k} = & \frac{1}{\kappa} \left[ \frac{1}{\theta^I} \left( s_{IH}\theta_2 + s_{12}\theta_1^I \left( 1 - \theta_1\theta^I \frac{\tau}{1 + \tau\theta_1} \right) \right) (\tilde{P}_2 - \tilde{P}_1^I) \right. \\ & \left. + \frac{\bar{P}}{\bar{G}} \frac{\theta_1\theta_1^H}{\bar{P}_1^H \bar{x}} \left( \bar{P}_1^H \bar{a} \frac{\theta_2}{\theta^I} \tilde{P}_2 - \frac{\bar{P}(\bar{c} + \delta\bar{k})}{1 + \tau} \theta_2 \tilde{P}_2^* + \left( \bar{P}_1^H \frac{\bar{a}}{\theta^I} - \bar{P}(\bar{c} + \delta\bar{k}) \right) \theta_1 \theta_1^I \tilde{P}_1^I + \bar{z}\tilde{z} \right) \right] \end{aligned}$$

$$(B.14) \quad \begin{aligned} \dot{c} = & \frac{1}{\bar{G}\bar{c}} \left[ \gamma\dot{k} - s_{12} \frac{\bar{P}_1^H \bar{x}}{\theta^I} \frac{\theta_1^I}{\theta_1^H} \frac{\tau}{1 + \tau} (\tilde{P}_2 - \tilde{P}_1^I) + \bar{P}_1^H \bar{a} \frac{\theta_2}{\theta^I} \tilde{P}_2 - \frac{\bar{P}(\bar{c} + \delta\bar{k})}{1 + \tau} \theta_2 \tilde{P}_2^* \right. \\ & \left. + \left( \bar{P}_1^H \frac{\bar{a}}{\theta^I} - \bar{P}(\bar{c} + \delta\bar{k}) \right) \theta_1 \theta_1^I \tilde{P}_1^I + \bar{z}\tilde{z} \right] \end{aligned}$$

The intratemporal substitution effects are modified with an *international* carbon tax due to the additional increase in the price of the foreign variety of good 1,  $P_1^I$ . The substitution

<sup>6</sup>We have

$$s_{IH}\theta_2 + s_{12}\theta_1^I = \theta_1^I \left( 1 - \frac{\bar{a}}{\bar{x}} \right) \theta_2 \sigma_{12} - \theta_2 \left( \theta_1^H \frac{\bar{a}}{\bar{x}} + \theta_1^I \right) \sigma_{IH}$$

By assuming  $\sigma_{12} = \sigma_{IH}$ , we have

$$s_{1H}\theta_2 + s_{12}\theta_1^I = -\theta_2 \frac{\bar{a}}{\bar{x}} \sigma_{12}$$

In the analysis  $\sigma_{12} > \sigma_{IH}$  by assumption, and the negative part of the intratemporal substitution effect will dominate, independently of  $\theta_2$ .

away from the domestic variety of good 1 is modified due to a smaller change in the relative price between the domestic and foreign varieties. In addition the change in the relative price between good 1 and 2 is smaller, hence the positive substitution effect will be reduced. The negative substitution effect of the initial tax wedge is also modified by the change in the world market prices  $P_2^*$  and  $P_1^I$ .

The increase in  $P_1^I$  has a similar positive income effect from the terms of trade increase as the increase in the price of fossil fuels,  $P_2$ . In addition, there is a direct income effect associated with the term  $\bar{P}(\bar{c} + \delta\bar{k})$ , of changes in the import prices  $P_1^I$  and  $P_2^*$ . Consider first the increase in  $P_1^I$ . The initial trade surplus  $\bar{z} + r\bar{b}$  consists of these two income effects. The term  $-\frac{(\bar{z} + r\bar{b})}{\theta I} = \bar{P}_1^H \frac{\bar{a}}{\theta I} - \bar{P}(\bar{c} + \delta\bar{k})$  can be interpreted as an intertemporal income effect. If the net financial wealth is positive, i.e.  $\bar{b} > 0$ , the positive interest income finances an import surplus in steady state. An import surplus implies that the negative income effect  $-\bar{P}(\bar{c} + \delta\bar{k})$  outweighs the positive income effect from the terms of trade gain of an import price increase. This contributes to a lower level of real capital and consumption. Since the tax revenue is redistributed lump sum, it is only the positive income effect from the reduction in the world market price of fossil fuels that matters, corrected for the initial tax level. Hence, there is an additional positive income effect from the fall in  $P_2^*$ . On the other hand, it is assumed that this positive effect is outweighed by the direct income effect from the petroleum revenue represented by the term  $\bar{z}\bar{z}$ , which is negative with an *international* carbon tax.

To summarise the effects of an *international* compared to a *unilateral* carbon tax, the substitution effects are smaller, due to the increase in  $P_1^I$  and fall in  $P_2^*$ . The change in world market prices contribute to modify the efficiency loss created by the tax rate increase. With an initial import (export) surplus the income effect of the increase in  $P_1^I$  is negative (positive). The reduction in petroleum revenue is assumed to dominate the positive income effect of lower world market price of fossil fuels, implying an additional negative effect. The positive effect on the terms of trade is modified by the increase in the import price of good 1 and the fall in the world market price of fossil fuels. Hence, a petroleum producing economy may be better off by implementing a *unilateral* compared to an *international* carbon tax, due to the existence of a positive terms of trade effect. For a non-petroleum producing economy an *international* tax is preferred.

Table 3.1. summarises the substitution and income effects, both direct and general equilibrium effects, of permanent changes in the exogenous variables  $P_1^I$ ,  $P_2^*$ ,  $\tau$  and  $z$ , on the steady state values of  $k$ ,  $P_1^H$ ,  $c$  and  $b$ . In the *unilateral* case only  $\tau$  is changing, and the effects present are the substitution effects and the income effect from the terms of trade gain of a corresponding change in  $P_2$ . With an *international* carbon tax we also have the substitution and income effects following the changes in  $P_1^I$ ,  $P_2^*$  and  $z$ .



**Table 3.1. Steady-state effects of increases in  $P_1^I$ ,  $P_2^*$ ,  $\tau$  and  $z$**

	Substitution effects		Income effect $\bar{P}_1^H \frac{\bar{a}}{\bar{x}}$	Income effect <sup>2</sup> $-\bar{P}(\bar{c} + \delta\bar{k})$	Direct petroleum revenue effect <sup>2</sup>
	$\dot{P}_2 > 0^1$	$\dot{P}_1^I > 0^2$	$\dot{P}_2 > 0^1, \dot{P}_1^I > 0^2$	$\dot{P}_2^* > 0, \dot{P}_1^I > 0$	$\dot{z} > 0$
Capital, $k$	-	+	+	-	+
Domestic product price, $P_1^H$	-	+	+	-	+
Consumption, $c$	-	+	+	-	+
Net foreign wealth, $b$	+	-	-	+	-

1.  $\dot{P}_2 = \dot{P}_2^* + \frac{\tau}{1+\tau}\dot{\tau}$

2. Only present with an international carbon tax.

### 3.2 Transitional dynamics

Consider the transitional dynamics following the implementation of a carbon tax. The dynamic adjustments of  $P_1^H$  and  $k$  can be described by a phase diagram of the dynamic system (17.a) and (17.b), see Figure 3.2.a and 3.2.b. In section 2 it was shown that the dynamic system (17.a) and (17.b) has the property of saddle-point stability, irrespective of the real value of the marginal utility  $\mu$ . We denote the loci generated by setting  $\dot{P}_1^H = 0$  in (17.a) and  $\dot{k} = 0$  in (17.b) as locus A and B respectively. The slope of these two loci are based on the steady state values of the derivatives which enter the dynamic system. This permits the use of (27.a)-(27.d) when deriving the analytical expressions for the slope of the loci. Equation (27.a) gives the slope of the relationship between  $P_1^H$  and  $k$  for locus A in the neighbourhood of steady state. It is easily seen that locus A is upward sloping in the  $P_1^H, k$ -diagram, and the elasticity of  $P_1^H$  w.r.t.  $k$  increases with the ratio between the cost share of labour and the elasticity of factor substitution, and decreases with the total import share.

Inserting equations (27.c) and (27.d) into equation (27.b) and using that  $\frac{\dot{P}}{G}(\frac{\tau}{1+\tau}) = \frac{\tau}{1+\tau\theta_1}$ , gives the slope of locus B.

$$(29) \quad \kappa^* \dot{k} = (s_{IH} - s_{12}\theta_1^H + \theta_1\theta_1^H \frac{\bar{a}}{\bar{x}})\dot{P}_1^H + s_{12}(\frac{\tau}{1+\tau\theta_1})\theta_1^I \dot{P}_1^H - s_{IH}\dot{P}_1^I + s_{12}(\frac{1}{1+\tau\theta_1})(\dot{P}_2 - \theta_1^I \dot{P}_1^I) + \theta_1\theta_1^H \frac{\bar{P}}{G} \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \dot{z} - (1 - \frac{\bar{a}}{\bar{x}}) \frac{\bar{P}}{G} (\theta_1\theta_1^I \dot{P}_1^I + \frac{\theta_2}{1+\tau} \dot{P}_2^*)$$

where

$$\kappa^* \equiv \theta^k (1 - \theta_1\theta_1^H \frac{\bar{P}}{G}) + \frac{\theta_1\theta_1^H r\alpha}{G(\delta+r)(r-\lambda_1)} > 0$$

Locus B is downward sloping in the  $P_1^H, k$ -diagram if the negative term  $s_{IH} - s_{12}\theta_1^H + \theta_1\theta_1^H \frac{\bar{a}}{\bar{x}}$  is not outweighed by the positive tax term  $s_{12}(\frac{\tau}{1+\tau\theta_1})\theta_1^I$ .<sup>7</sup> The absolute value of the elasticity of  $P_1^H$  w.r.t.  $k$  is larger the larger is the marginal elasticity of capital measured by the cost share of capital (in the neighbourhood of steady state), but the locus will be closer

<sup>7</sup>It can be shown that  $s_{IH} - s_{12}\theta_1^H + \theta_1\theta_1^H \frac{\bar{a}}{\bar{x}} < 0$  if  $\sigma_{12} > 1$  and  $\sigma_{IH} = \sigma_{12}$ . Hence, if  $\sigma_{IH} > \sigma_{12}$  as is assumed in the analysis,  $s_{IH} - s_{12}\theta_1^H + \theta_1\theta_1^H \frac{\bar{a}}{\bar{x}}$  is negative.

to horizontal the larger are the intratemporal substitution possibilities represented by the terms  $s_{IH}$  and  $s_{12}$ . The initial tax wedge contributes to a steeper slope (as long as the slope is negative), i.e. the larger the initial inefficiency, the larger is the necessary fall in the domestic product price to induce a given increase in the stock of capital. On the other hand, if the initial tax wedge is large enough, this effect together with the positive terms of trade effect represented by the term  $\theta_1 \theta_1^H \frac{\bar{a}}{\bar{x}}$ , may outweigh the negative intratemporal substitution terms, and the slope of locus B become positive. The stability properties though ensure that the slope will be less steep than the slope of locus A. If the slope is positive the locus will be closer to horizontal the larger the initial tax wedge.

From equation (27.a) it is easily seen that an increase in the import prices following a *unilateral* or an *international* carbon tax induces a positive shift in locus A for a given level of  $k$ . The shift in locus B is positive or negative depending partly on whether the carbon tax is *unilateral* or *international*. With a *unilateral* carbon tax  $\check{P}_2 = \frac{\tau}{1+\tau} \check{\tau}$  and  $\check{P}_1^I = \check{P}_2^* = \check{z} = 0$ . Inserting this into equation (29) gives

$$(30) \quad \kappa^* \check{k} = (s_{1H} - s_{12} \theta_1^H + \theta_1 \theta_1^H \frac{\bar{a}}{\bar{x}}) \check{P}_1^H + s_{12} (\frac{\tau}{1+\tau \theta_1}) \theta_1^I \check{P}_1^H + s_{12} (\frac{1}{1+\tau \theta_1}) \check{P}_2$$

For a given level of  $k$  locus B will have a positive shift with a *unilateral* carbon tax. However, the existence of an initial tax contributes to reduce the positive shift in locus B. The effects of a *unilateral* carbon tax are illustrated in figure 3.2.a.

With an *international* carbon tax  $\check{P}_2^* < 0$ ,  $\check{\tau} > 0$ ,  $\check{P}_2 > 0$ ,  $\check{P}_1^I > 0$  and  $\check{z} < 0$ . The shift in locus B is positive (negative) if the positive substitution and income effects represented by the terms

$$-s_{IH} \check{P}_1^I + s_{12} (\frac{1}{1+\tau \theta_1}) (\check{P}_2 - \theta_1^I \check{P}_1^I) - (1 - \frac{\bar{a}}{\bar{x}}) \frac{\bar{P}}{\bar{G}} \frac{\theta_2}{1+\tau} \check{P}_2^*$$

are larger (smaller) than the negative income effects represented by the terms

$$\theta_1 \theta_1^H \frac{\bar{P}}{\bar{G}} \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \check{z} - (1 - \frac{\bar{a}}{\bar{x}}) \frac{\bar{P}}{\bar{G}} \theta_1 \theta_1^I \check{P}_1^I$$

A negative shift in locus B is illustrated in figure 3.2.b. In both figure 3.2.a and 3.2.b,  $P_1^H$  is higher and  $k$  is lower in the new steady state situation.

Consider an *anticipated* implementation of a *unilateral* carbon tax, see figure 3.2.a. Assume that the implementation of the carbon tax is announced at time  $t_0$  to take place at time  $t_1$  in the future. At time  $t_0$  the economy is in the steady state solution  $S$ .  $k_0$  is given at  $t_0$  and  $P_1^H$  must jump upwards to offset the higher demand for capital due to expectations of future capital gains. The adjustment path is one that follows the dynamics dictated by the arrows around  $S$  until the import prices actually change. The initial jump in  $P_1^H$  is followed by a reduction in  $P_1^H$ , and the negative capital gains imply higher return to capital such that the demand for capital increases along the adjustment path from  $C_1$  to  $C_2$ . At the moment the tax rate changes, the adjustment path has to hit the saddle path leading

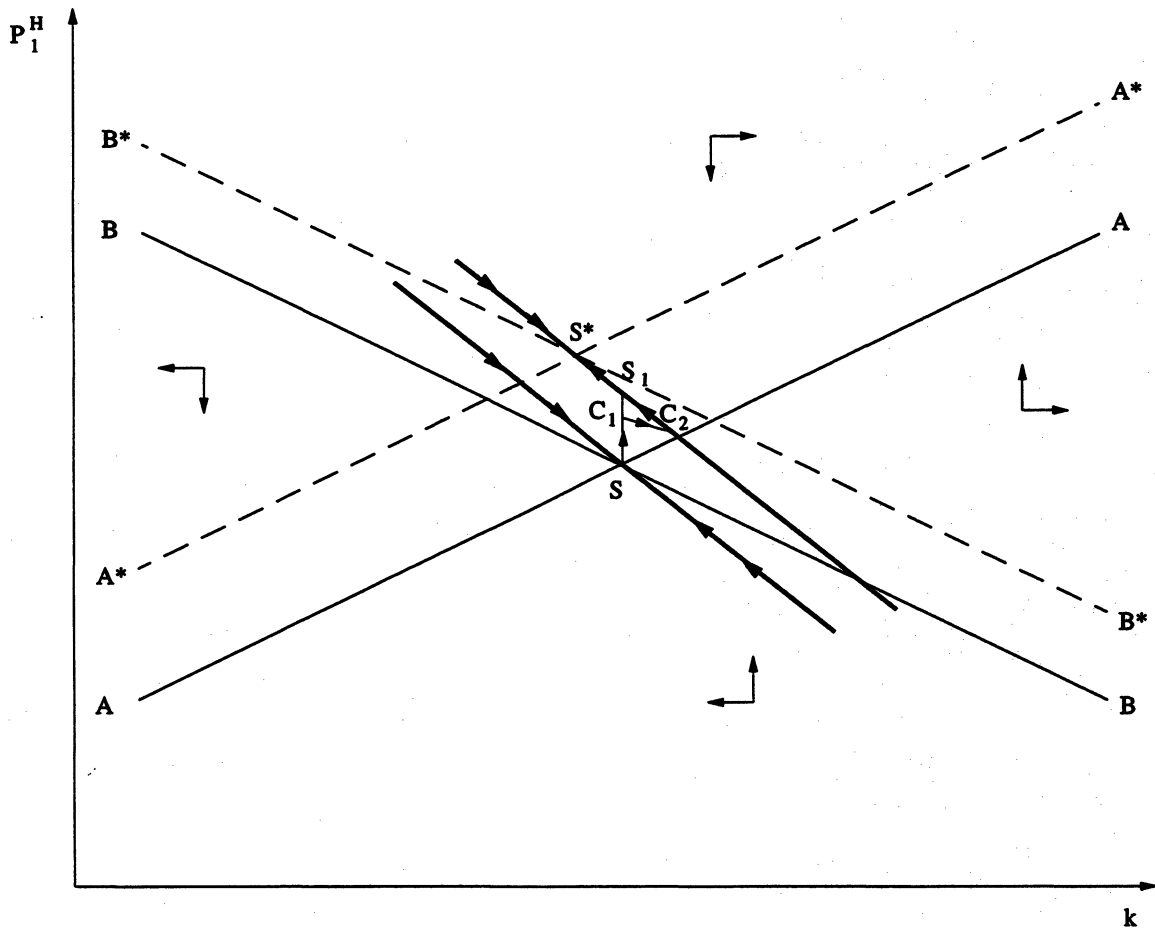
to the new steady state  $S^*$ . Such a path is shown by  $SC_1C_2S^*$  in figure 3.2.a. Along the saddle path positive capital gains imply lower return to capital such that the demand for  $k$  falls and  $P_1^H$  increases until the new steady state solution is obtained. Note that some kind of overshooting both in the stock of real capital and the domestic product price, may occur before the economy hits the new saddle path.

Figure 3.2.b illustrates the implementation of an *international* carbon tax when the negative income effects dominate. The tax is announced at  $t_0$  to take place at  $t_1$ . The fall in consumption demand is so large that it outweighs the higher investment demand for capital and induces a fall in the demand for  $k$  at  $t_0$ . Hence at  $t_0$ ,  $P_1^H$  must fall to offset the lower demand. The initial fall in  $P_1^H$  is followed by an increase in  $P_1^H$ . At time  $t_1$  the adjustment path hits the new saddle path at  $C_2$  leading to  $S^*$ . As in figure 3.2.a, positive capital gains imply lower return to capital such that the demand for  $k$  falls and  $P_1^H$  increases both along the adjustment path  $C_1C_2$ , and along the new saddle path. In contrast to the situation illustrated in figure 3.2.a, there is no overshooting in the stock of capital.

The consumer has perfect foresight and sees through the new path and steady state solution. Therefore the new level of  $\mu$  consistent with any announced changes at  $t_0$ , is implemented at  $t_0$ .

If the carbon tax is *unanticipated*, i.e. it is announced at time  $t_0$  to take place at  $t_0$ , the economy will immediately jump to  $S_1$  on the new stable path in figures 3.2.a and 3.2.b respectively. It will then move along this path until the new steady state solution is reached. The necessary price jump is larger and the change (reduction) in the capital stock is monotonic.

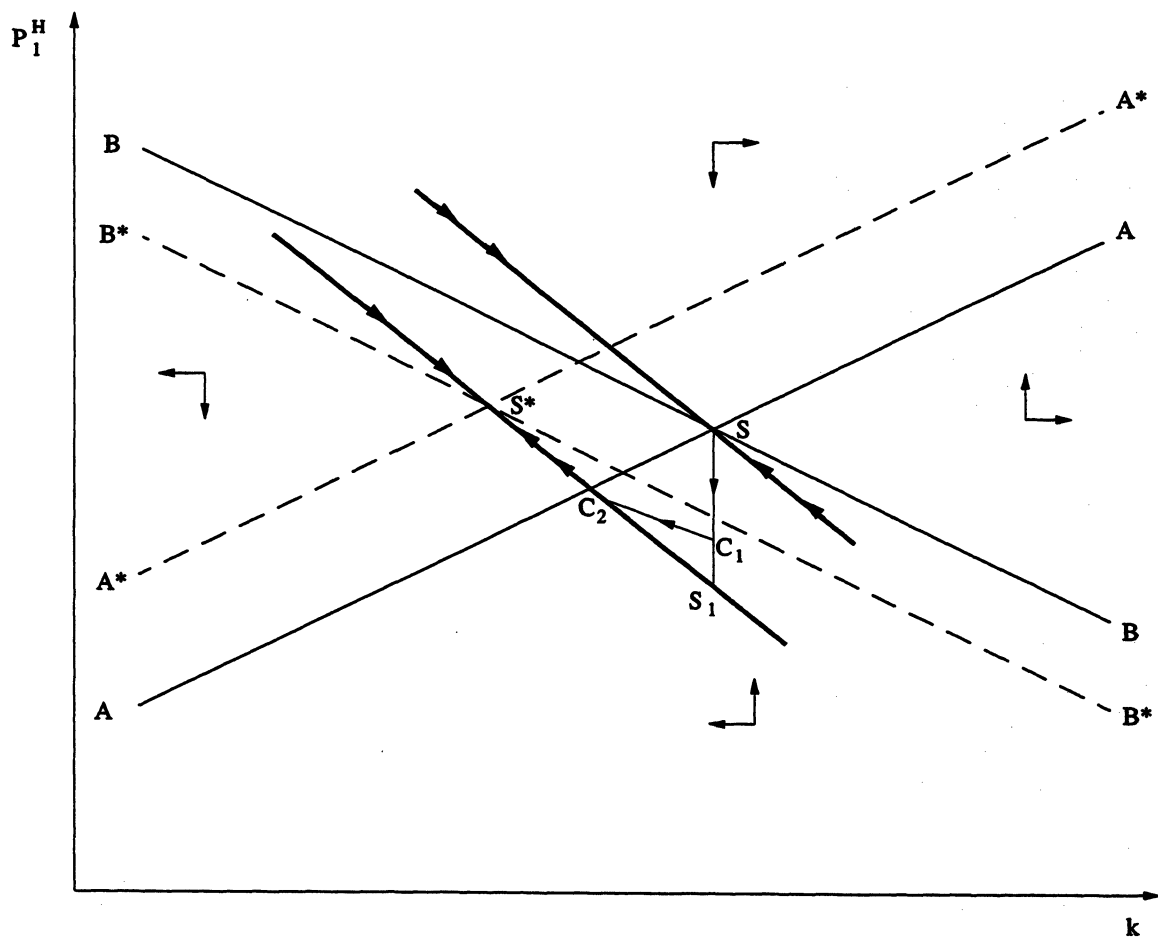
Figure 3.2.a. Dynamic effects of a unilateral<sup>8</sup> carbon tax.<sup>9</sup>



<sup>8</sup>The figure can also illustrate the implementation of an *international* carbon tax with the positive substitution and income effects dominating.

<sup>9</sup>Only a small area around the steady state is considered, such that all the steady state relationships are in effect in the figure.

**Figure 3.2.b. Dynamic effects of an international carbon tax.** <sup>10</sup>  
 Negative income effects dominate.



<sup>10</sup>Only a small area around the steady state is considered, such that all the steady state relationships are in effect in the figure.

## 4 Concluding remarks

This paper has analysed the effects of a carbon tax on a small open, petroleum producing economy. The model has differentiated products and is based on intertemporal optimisation. The long run effects on welfare and capital accumulation of both a *unilateral* and an *international* carbon tax are emphasised. It is shown that the steady state welfare effect of a carbon tax can be positive or negative, depending on substitution effects which create efficiency losses, and income effects from changes in terms of trade. The presence of an initial tax wedge implies that there is an ambiguous relationship between the tax level and steady state welfare. This is a common result in tax models with endogenous domestic prices; the terms of trade effect is of first order whereas the loss due to substitution is of second order.

In the *unilateral* case it is only the negative substitution effects from the increase in the price of fossil fuels and the positive income effect from the terms of trade improvement, which are present. Other income effects vanish since the tax revenue is redistributed lump sum. The positive terms of trade effect may give room for a welfare improvement even though the total effect on the stock of real capital should be negative. But, the welfare effect is negative if the initial tax rate is large enough, having created a large initial tax wedge. The presence of an initial tax wedge implies that it is not possible for the government to receive a welfare gain by multiple increases in the tax rate, even though it has positive terms of trade effects. This is independent of whether it is a *unilateral* or an *international* carbon tax. With an *international* carbon tax the effects on the terms of trade are smaller, or may be negative, for a petroleum producing economy. Hence, the income effects of the price changes may be negative. A lower level of petroleum revenue will furthermore reduce the stock of real capital and welfare, compared to the *unilateral* case. On the other hand, the substitution effects are smaller because the change in the relative price between the price of fossil fuels and the imported variety of good 1 is smaller. Both the reduction in the petroleum revenue and the smaller terms of trade gain imply that for a petroleum producing economy an *international* carbon tax may be less beneficial than a *unilateral* carbon tax.

The analysis is based on many restrictive assumptions. The effects of the petroleum sector on the economy is not explicitly modelled, but implemented through exogenous assumptions. Furthermore, the analysis has focused on the long run although the dynamic path of the economy is characterised. But the dimensionality of the model makes it too complex for an analytical closed form description of the dynamics.

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For the elements of the Jacobian we have in steady state:<sup>11</sup>

$$(A.1) \quad h_{1P} = -(r + \delta) \left( \frac{1}{\theta_1 \theta_1^H} - 1 \right) < 0$$

$$(A.2) \quad h_{1k} = -\frac{(P_1^H)^2}{P \theta_1 \theta_1^H} f''(k) > 0$$

$$(A.3) \quad h_{2P} = \frac{(\sigma_{IH}) \frac{a}{P}}{\theta_1 \theta_1^H} + \frac{\sigma^c c}{P_1^H} \theta_1 \theta_1^H + \theta_1^H \left( \frac{c + \delta k}{P_1^H} \right) (\sigma_{12} \theta_2 + \sigma_{IH} \frac{\theta_1}{\theta_1^H}) > 0$$

$$(A.4) \quad h_{2k} = \frac{r}{\theta_1 \theta_1^H} + \delta \left( \frac{1}{\theta_1 \theta_1^H} - 1 \right) > 0$$

The determinant of the linearised system can be written:

$$(A.5) \quad D = h_{1P} h_{2k} - h_{1k} h_{2P} < 0$$

The negative eigenvalue which corresponds to the stable solution is:

$$(A.6) \quad \lambda_1 = \frac{h_{1P} + h_{2k}}{2} - \left[ \frac{(-h_{1P} - h_{2k})^2}{4} - D \right]^{1/2}$$

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<sup>11</sup>See Bye and Holmøy (1992) for further discussion of the elements of the Jacobi matrix

In deriving the reduced form equations for  $\check{k}$  and  $\check{c}$ , the following expressions are extensively used.

$$(B.1) \quad g'_H = \theta_1 \theta_1^H \frac{\bar{P}}{\bar{P}_1^H}$$

$$(B.2) \quad \theta^k = \frac{\bar{P}(r + \delta)\bar{k}}{\bar{P}_1^H \bar{x}}$$

$$(B.3) \quad \left(1 - \frac{\bar{a}}{\bar{x}}\right) = \frac{\theta_1 \theta_1^H}{\bar{P}_1^H \bar{x}} \bar{P}(\bar{c} + \delta\bar{k})$$

$$(B.4) \quad r\bar{b} + \bar{z} = \bar{P}(\bar{c} + \delta\bar{k}) - \bar{P}_1^H \bar{x}$$

Combining (B.3) and (B.4) gives

$$(B.5) \quad \left(\theta^I - \frac{\bar{a}}{\bar{x}}\right) = \theta_1 \theta_1^H \frac{(r\bar{b} + \bar{z})}{\bar{P}_1^H \bar{x}}$$

The logarithmic differentiates of  $G$  and  $P$  are given by

$$(B.6) \quad \check{G} = \frac{\bar{P}}{\bar{G}} \left[ \theta_1 \check{P}_1 + \frac{\theta_2}{1 + \tau} \check{P}_2^* + \theta_1 \theta_2 \frac{\tau}{1 + \tau} \sigma_{12} (\check{P}_2 - \check{P}_1) \right]$$

$$(B.7) \quad \check{P}_1 = \theta_1^H \check{P}_1^H + \theta_1^I \check{P}_1^I$$

To find the reduced form equation of  $\check{k}$ , (27.d) substitutes  $\check{b}$  in (27.c) and (27.c) substitutes  $\check{c}$  in (27.b). Using (B.1) and (B.3), (27.b) can then be written as

$$(B.8) \quad \left( \theta^k + \theta_1 \theta_1^H \frac{\bar{P}\bar{k}}{\bar{G}\bar{P}_1^H \bar{x}} \left( \frac{r\alpha}{r - \lambda_1} - (r + \delta)\bar{P} \right) \right) \check{k} = s_{IH}(\check{P}_1^H - \check{P}_1^I) + s_{12}(\check{P}_2 - \check{P}_1) - \left(1 - \frac{\bar{a}}{\bar{x}}\right) \check{G} + \theta_1 \theta_1^H \frac{\bar{P}}{\bar{G}} \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \check{z}$$

Inserting for  $\check{G}$  in (B.8) and reorganizing gives

$$(B.9) \quad \left( \theta^k + \theta_1 \theta_1^H \frac{\bar{P}\bar{k}}{\bar{G}\bar{P}_1^H \bar{x}} \left( \frac{r\alpha}{r - \lambda_1} - (r + \delta)\bar{P} \right) \right) \check{k} = s_{IH}(\check{P}_1^H - \check{P}_1^I) + s_{12}(\check{P}_2 - \check{P}_1) - \theta_1 \frac{\tau}{1 + \tau} s_{12}(\check{P}_2 - \check{P}_1) + \theta_1 \theta_1^H \frac{\bar{P}}{\bar{G}} \check{P}_1^H - \left(1 - \frac{\bar{a}}{\bar{x}}\right) \frac{\bar{P}}{\bar{G}} \left( \theta_1 \check{P}_1 + \frac{\theta_2}{1 + \tau} \check{P}_2^* \right) + \theta_1 \theta_1^H \frac{\bar{P}}{\bar{G}} \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \check{z}$$

Inserting for  $\check{P}_1^H$  from (27.a) and  $\check{P}_1$  from (B.7), and moving all the expressions with  $\check{k}$  over to the left hand side, we get

$$\begin{aligned}
\kappa \check{k} &= s_{IH} \left( \frac{1}{\theta^I} (\theta_1 \theta_1^I \check{P}_1^I + \theta_2 \check{P}_2) - \check{P}_1^I \right) + s_{12} (\check{P}_2 - \left( \frac{\theta_1^I}{\theta^I} \check{P}_1^I + \frac{\theta_1^H \theta_2}{\theta^I} \check{P}_2 \right)) \\
&\quad - s_{12} \theta_1 \frac{\tau}{1 + \tau} \frac{\bar{P}}{G} (\check{P}_2 - \left( \frac{\theta_1^I}{\theta^I} \check{P}_1^I + \frac{\theta_1^H \theta_2}{\theta^I} \check{P}_2 \right)) + \theta_1 \theta_1^H \frac{\bar{P}}{G \theta^I} (\theta_1 \theta_1^I \check{P}_1^I + \theta_2 \check{P}_2) \\
&\quad - \left( 1 - \frac{\bar{a}}{\bar{x}} \right) \frac{\bar{P}}{G} \frac{\theta_1}{\theta^I} (\theta_1^I \check{P}_1^I + \theta_1^H \theta_2 \check{P}_2) - \left( 1 - \frac{\bar{a}}{\bar{x}} \right) \frac{P}{G} \frac{\theta_2}{1 + \tau} \check{P}_2^* \\
&\quad + \theta_1 \theta_1^H \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \frac{\bar{P}}{G} \check{z}
\end{aligned}
\tag{B.10}$$

where

$$\kappa \equiv \theta^k \left( 1 - \theta_1 \theta_1^H \frac{\bar{P}}{G} \right) + \frac{\theta_1 \theta_1^H r \alpha}{G(\delta + r)(r - \lambda_1)} + \frac{\theta^L}{\sigma^k \theta^I} \left( \theta_1^H (s_{12} - \theta_1 \frac{\bar{P}}{G} (s_{12} \frac{\tau}{1 + \tau} + \frac{\bar{a}}{\bar{x}})) - s_{1H} \right)$$

In deriving  $\kappa$  (B.2) is used.

Reorganizing (B.10), using (B.3), (B.4) and (B.5), and

$$\frac{\theta_1 \theta_1^I}{\theta^I} - 1 = -\frac{\theta_2}{\theta^I}, \quad 1 - \frac{\theta_1^H \theta_2}{\theta^I} = \frac{\theta_1^I}{\theta^I}, \quad \frac{\bar{P}}{G} \left( \frac{\tau}{1 + \tau} \right) = \frac{\tau}{1 + \tau \theta_1}$$

gives

$$\begin{aligned}
\check{k} &= \frac{1}{\kappa} \left[ \frac{1}{\theta^I} (s_{IH} \theta_2 + s_{12} \theta_1^I) (\check{P}_2 - \check{P}_1^I) + \frac{\bar{P}}{G} \frac{\theta_1 \theta_1^H}{\theta^I} \frac{\bar{a}}{\bar{x}} \theta_2 \check{P}_2 - \frac{\bar{P}}{G} \frac{\theta_1 \theta_1^H}{\theta^I} \frac{(\bar{z} + r \bar{b})}{\bar{P}_1^H \bar{x}} \theta_1 \theta_1^I \check{P}_1^I \right. \\
&\quad \left. + \frac{\bar{P}}{G} \theta_1 \theta_1^H \frac{\bar{z}}{\bar{P}_1^H \bar{x}} \check{z} - \frac{\bar{P}}{G} \left( 1 - \frac{\bar{a}}{\bar{x}} \right) \frac{\theta_2}{1 + \tau} \check{P}_2^* - s_{12} \theta_1 \theta_1^I \frac{\tau}{1 + \tau \theta_1} (\check{P}_2 - \check{P}_1^I) \right]
\end{aligned}
\tag{B.11}$$

Reorganizing (B.11) using (B.3), gives the reduced form equation for  $\check{k}$ .

$$\begin{aligned}
\check{k} &= \frac{1}{\kappa} \left[ \frac{1}{\theta^I} (s_{IH} \theta_2 + s_{12} \theta_1^I (1 - \theta_1 \theta^I \frac{\tau}{1 + \tau \theta_1})) (\check{P}_2 - \check{P}_1^I) \right. \\
&\quad \left. + \frac{\bar{P}}{G} \frac{\theta_1 \theta_1^H}{\bar{P}_1^H \bar{x}} (\bar{P}_1^H \frac{\theta_2}{\theta^I} \check{P}_2 - \frac{\bar{P}(\bar{c} + \delta \bar{k})}{1 + \tau} \theta_2 \check{P}_2^* + (\bar{P}_1^H \frac{\bar{a}}{\theta^I} - \bar{P}(\bar{c} + \delta \bar{k})) \theta_1 \theta_1^I \check{P}_1^I + \bar{z} \check{z}) \right]
\end{aligned}
\tag{B.12}$$

The reduced form equation for  $\check{c}$  is found by inserting (27.a) and (27.d) into (27.c), together with (B.6) and (B.7).

$$\begin{aligned}
\bar{G} \bar{c} \check{c} &= \gamma \check{k} + \frac{\bar{P}_1^H \bar{x}}{\theta^I} (\theta_1 \theta_1^I \check{P}_1^I + \theta_2 \check{P}_2) + \bar{z} \check{z} - \bar{P}(\bar{c} + \delta \bar{k}) \frac{\theta_1}{\theta^I} (\theta_1^I \check{P}_1^I + \theta_1^H \theta_2 \check{P}_2) \\
&\quad - \bar{P}(\bar{c} + \delta \bar{k}) \frac{\theta_2}{1 + \tau} \check{P}_2^* - s_{12} \frac{\bar{P}_1^H \bar{x}}{\theta^I} \frac{\theta_1^I}{\theta_1^H} \frac{\tau}{1 + \tau} (\check{P}_2 - \check{P}_1^I)
\end{aligned}
\tag{B.13}$$

where

$$\gamma \equiv \frac{\bar{P}_1^H \bar{x} \theta^L}{\theta^I \sigma^k} \left( \frac{\bar{a}}{\bar{x}} + s_{12} \frac{\tau}{1 + \tau} \right) + (\bar{P}(r + \delta) - \bar{G}\delta - \frac{\alpha}{r - \lambda_1}) \bar{k}$$

Reorganizing (B.13) using (B.3) and (B.4), gives the reduced form equation for  $\check{c}$ .

$$(B.14) \quad \check{c} = \frac{1}{\bar{G}\bar{c}} \left[ \gamma \check{k} - s_{12} \frac{\bar{P}_1^H \bar{x} \theta_1^I}{\theta^I \theta_1^H} \frac{\tau}{1 + \tau} (\check{P}_2 - \check{P}_1^I) + \bar{P}_1^H \bar{a} \frac{\theta_2}{\theta^I} \check{P}_2 - \frac{\bar{P}(\bar{c} + \delta \bar{k})}{1 + \tau} \theta_2 \check{P}_2^* \right. \\ \left. + (\bar{P}_1^H \frac{\bar{a}}{\theta^I} - \bar{P}(\bar{c} + \delta \bar{k})) \theta_1 \theta_1^I \check{P}_1^I + \bar{z}\check{z} \right]$$

Both  $\kappa$  and  $\gamma$  are assumed to be positive<sup>12</sup>.

The reduced form expression for  $\check{P}_1^H$  follows from (27.a) and (B.12), and the reduced form for  $\check{b}$  follows from (27.d) and (B.12).

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<sup>12</sup>If  $\tau = 0$ , it can be shown that the term  $\theta_1^H (s_{12} - \frac{\alpha}{\bar{x}} \theta_1) - s_{1H}$  in  $\kappa$  is positive if  $\sigma_{12} \theta_1 + \sigma_{1H} \theta_1^I > 1$ . The elasticities of substitution are assumed to be large enough to satisfy this requirement. This is consistent with the assumption that the demand elasticities should be large enough to ensure  $\alpha > 0$ , which is sufficient for  $\kappa$  to be positive.

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Statistics Norway  
Research Department  
P.O.B. 8131 Dep.  
N-0033 Oslo

Tel.: + 47 - 22 86 45 00  
Fax: + 47 - 22 11 12 38

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