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Abstract:
This paper discusses the problem of specifying probabilistic models for choices (strategies) with uncertain outcomes. The most general case we consider is choice settings where the uncertain outcomes are sets which may contain more than one alternative. This is of interest for the following type of choice processes that take place in two stages: In stage one the agent has the choice between uncertain sets of alternatives and only knows the probabilities of which alternative that belongs to each set. Conditional on the choice in the first stage the content of the chosen set is revealed and the agent chooses (under perfect certainty) the most preferred one from this set. The standard setting in which the outcomes are single alternatives, follows as a special case of the model.

The point of departure is a generalization of Luce IIA assumption to choice experiments with uncertain outcomes and we analyze the implications when IIA is combined with particular assumptions about invariance with respect to aggregations of strategies.

Keywords: Random tastes, choice among uncertain sets, random utility models, bounded rationality, probabilistic choice models, independence from irrelevant alternatives.

JEL classification:

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1. Introduction

This paper develops a class of probabilistic choice models for choice experiments in which the outcomes are uncertain to the agent. This means that we assume that the agent's response to the same choice situation (with uncertain outcomes) is governed by a probability mechanism, and so in general he exhibits inconsistencies. By now there is a huge literature on stochastic choice models with certain outcomes. (For a summary of these models, see Suppes et al., 1989, ch. 17.) In fact, it was empirical observations of inconsistencies, dating back to Thurstone (1927), that lead to the study of probabilistic theories in the first place. Thurstone argued that one reason for the observed inconsistencies is that the agent has difficulties with assessing the precise value (to him) of the choice objects. While probabilistic models for certain outcomes have been studied and applied extensively in psychology and economics it seems that there is little interest for developing corresponding models for choice with uncertain outcomes (cf. Machina, 1985). As far as we know, there are no contributions in this field since the sixties. This is rather curious since one would expect that if an agent has problems with rank ordering alternatives with certain outcomes he would certainly find it difficult to choose among gambles. The importance of developing theoretically justified stochastic choice models in this context has been accentuated in two recent papers, Harless and Camerer (1994, p. 1287) and Hey and Orme (1994). For example, Hey and Orme, p.p. 1321-1322, argue;

> "Our results suggest quite strongly that the truth is <u>not</u> going to be found along this deterministic choice route, unless some account is taken of the errors. There is clearly a problem of identifying the underlying "true" model because of these errors — indeed it could be argued that the lack of significance for some of the top-level functionals (deterministic non-expected utility functionals) for some of the subjects in our study could simply result from this noise, ...".

In the next paragraph they conclude:

> "..., we are tempted to conclude by saying that our study indicates that behavior can be reasonably well modelled (to what might be termed a 'reasonable approximation') as 'Expected utility plus noise'. Perhaps we should now spend some time on thinking about the noise, rather than about even more alternatives to expected utility?"

The point of departure in this paper is to utilize some of the ideas that have emerged in the literature on discrete choice models with certain outcomes to obtain a theoretical rationale for similar models with uncertain outcomes.

The most general choice setting we have studied can be described as follows: The choice process takes place in two stages. In stage one the agent has the choice between a finite number of strategies. To each strategy is associated a set of alternatives which is revealed to the agent after a strategy has been selected. Conditional on the choice of strategy the agent obtains information about which alternatives belongs to his choice set (which may contain more than one alternative). In the second stage the agent chooses the most preferred alternative from the revealed choice set.

A typical example is the following: Suppose the agent's decision problem is to choose among different types of jobs. The feasible jobs are distributed across different regions, but the agent is uncertain about the locations of the feasible jobs. He can only find out which jobs are feasible in a particular region by moving to this region. In the first stage the strategies consist of the set of feasible regions. After a region has been selected the set of feasible jobs in that particular region will be revealed to the agent and he chooses the most preferred job from this set.

A second example is the choice of education. Here the student faces a discrete set of schooling alternatives each of which yields a set of uncertain job opportunities which are revealed after the chosen type of schooling has been completed. Thus the set of strategies is the set of feasible schooling alternatives. When the chosen school has been completed the set of feasible jobs will be revealed from which the agent can choose the most preferred one (second stage).

A third example is related to tourism. In the first stage the tourist has the choice between a set of travel destinations and he has only limited information about which sites and activities that are feasible given that a particular destination has been selected. In the second stage, i.e., when arriving at the destination, he chooses the preferred activity among the feasible ones.

The reason why we consider choice settings where the uncertain outcomes are sets that may contain more than one alternative is that, beyond the obvious interest in itself, it turns out that this case is useful for generating theoretically justifiable structures.

The usual setting in which the uncertain outcomes contains at most one alternative is evidently a special case of the general framework developed in this paper. Specifically, in this case one of the models discussed here is equivalent to the *Strict expected utility model* proposed by Becker et al. (1963a) and Luce and Suppes (1965). However, these authors provide no theoretical justification for their model other than the fact that it contains Luce model (for certain outcomes) as a special case. Becker et al. (1963a) and Luce and Suppes also consider other types of stochastic choice models for uncertain outcomes.

The paper is organized as follows: In the next section we discuss why a standard approach to modeling the kind of phenomenons mentioned above are likely to be rather intractable. In Section 3 we introduce an alternative approach. The new approach introduced here is analogous to Luce's (1959) theory of probabilistic choice. Specifically, we assume that the agent's choice between sets of strategies is probabilistic and satisfies the assumption known as "Independence from Irrelevant Alternatives". In Section 4 we demonstrate that the choice model obtained in Section 3 is compatible with a random utility representation in which the structural part of the random utilities can be interpreted as an "expected utility" (relative to the agent). In Section 5 the random utility representation is extended and some non-parametrically testable properties are obtained. In the final section we demonstrate that in particular choice settings the choice model developed under assumptions made in Section 3 has the same formal structure as a model for choice under perfect certainty but with choice sets that are latent to the analyst.

Although models developed in this paper are stochastic versions of the expected utility model, it is easily realized how these models in some cases could be extended to corresponding stochastic non-expected utility models. This is the case for the Rank Dependent Expected Utility Model (cf. Quiggin, 1982, Yaari, 1987, Chew, Karni and Safra, 1987), Allais (1979), and the Subjective Expected Utility Model, (Edwards, 1962, and Kahneman and Tversky, 1979). What all of these non-expected utility models have in common is that the conditional probabilities for the respective outcomes given the choice are replaced by a function of these probabilities.

2. Discussion of a standard modeling approach

Above we emphasised the need for developing stochastic choice theories to accommodate behavioral inconsistencies. When the outcomes are uncertain sets of alternatives we shall demonstrate later in this paper that our stochastic modelling approach yields a framework that is convenient for empirical modeling in contrast to the deterministic expected utility approach. In this section we shall illustrate that in the context of empirical modelling, the deterministic expected utility model, when combined with utilities that contain unobservable taste-shifters, is likely to produce rather intractable functional forms.

To this end let us consider an agent that has the choice between two sets B_s and $B_{s'}$. The agent does not know — ex ante — which alternatives that belong to B_s and $B_{s'}$. Let U_k be the agent's utility of alternative k. We assume that the utility function has the structure

$$U_k = b_k \varepsilon_k \tag{2.1}$$

(or alternatively an additive structure), where $\{\varepsilon_k\}$ are positive random variables and b_k are positive structural terms. As mentioned above we assume in this section that $\{\varepsilon_k\}$ are random only to the analyst. Thus to the agent the taste-shifters are known. Moreover, the agent is assumed to know the probability distribution of the random choice sets $\{B_s, B_{s'}\}$. When the choice sets (which are revealed in the second stage) are uncertain and the agent behaves according to the expected utility hypothesis, the agent will prefer s to s′ if

$$E\left(\max_{k \in B_s}(\beta_k \varepsilon_k) \mid \{\varepsilon_k\}\right) > E\left(\max_{k \in B_{s'}}(\beta_k \varepsilon_k) \mid \{\varepsilon_k\}\right). \tag{2.2}$$

As is clear from the notation in (2.2), expectation is evaluated with respect to the probability distribution of B_s and $B_{s'}$, respectively, conditional on the taste-shifters $\{\varepsilon_k\}$. While (2.2) describes the agent's decision rule, it is not immediately useful for empirical analyses due to the fact that $\{\varepsilon_k\}$ are unobservable. Accordingly, from the analyst's point of view it is necessary to calculate the choice probability,

$$P\left(E\left(\max_{k \in B_s}(\beta_k \varepsilon_k) \mid \{\varepsilon_k\}\right) > E\left(\max_{k \in B_{s'}}(\beta_k \varepsilon_k) \mid \{\varepsilon_k\}\right)\right). \tag{2.3}$$

Let N be the total number of alternatives. When the probability distribution of $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N)$ has been specified, (2.3) can in principle be calculated. However, by looking at (2.3) we realize that this will lead to very complicated and intractable mathematical expressions. Another, and more theoretical problem, is related to the choice of probability distribution of the unobservables and whether or not the multiplicative specification (2.1) should be replaced by an additive one, or possibly by a more flexible functional form. To clarify this point, let

$$U_k^* \equiv \log U_k = \beta_k^* + \varepsilon_k^*$$

where $\beta_k^* = \log\beta_k$ and $\varepsilon_k^* = \log\varepsilon_k$. In the case with perfectly certain choice sets the utility functions U_k and U_k^* are of course equivalent. However, this is not the case with uncertain choice sets since the decision rule (2.2) clearly depends on the choice of transform of the original utility function.

In light of the problems mentioned above we shall therefore abandon the standard approach and present an alternative approach below.

3. A "constant utility" approach

We mentioned above that in microeconomic theory the tradition is to assume that the consumer has a utility function that allows him to rank the alternatives in a consistent and unambiguous manner when faced with identical choice experiments. This approach has been critisized by psychologists and others (cf. Thurstone, 1927a,b; Luce, 1959; Tversky, 1972) to mention just a few who argue that when faced with a choice among several alternatives people often experience uncertainty and inconsistency. That is, they have difficulties with assessing the precise (subjective) value of the alternatives and consequently the choice outcomes in identical choice experiments may vary across experiments. To account for this empirical evidence the psychologists have developed probabilistic choice models. In the psychological choice literature one has traditionally distinguished between two types of choice models: In the constant utility model the decision rule is viewed as stochastic while utility is deterministic (Luce, Tversky). Luce model (Luce, 1959) is the most famous example of a constant utility model. Luce derives this model from his choice axiom (IIA) and demonstrates that it implies the existence of a unique (except for a multiplicative constant) scale (constant — or deterministic utility) from which choice probabilities can be expressed by a simple formulae. In the random utility model, utility is viewed as stochastic (Thurstone) while the decision rule is deterministic. In light of recent work by economists it seems that the difference between these models is only superficial. Specifically, Holman and Marley (cited in Luce and Suppes, 1965) and McFadden (1981) have demonstrated that the most familiar constant utility models such as the Luce model and Tversky's "elimination by aspects" can both be represented by random utility formulations.

The most famous contribution to the literature of probabilistic choice models is the monograph by Luce (1959). In the present section we shall discuss how his approach can be adapted to apply in the particular choice setting that is the concern of this paper. First we need some additional notation.

Recall that we consider the following choice process that takes place in two stages: In the first stage the agent has the choice between s=1,2,...,M, strategies. Conditional on the choice of a particular strategy s a finite set of feasible alternatives B_s, is revealed to him. Before the choice of strategy s the agent does not know which of the alternatives that belong to B_s[1]. Let $\bar{S}$ denote the total index set of strategies, i.e., $\bar{S}=\{1,2,...,M\}$, N the total number of alternatives and $\mathfrak{B}$ the family of subsets $\{B_s, s\in\bar{S}\}$. For a given set $B_s\in\mathfrak{B}$ define the utility of B_s as the utility of the most attractive alternative in B_s.

In general, the alternatives may depend on characteristics that are specific to B_s. However, we realize that this case is a special case of the general setup, which is obtained by letting the probabilities of alternatives that do not belong to B_s (say) be equal to zero.

Let $P(S_1;S_2)$ be the probability that the agent shall choose a strategy from S_1 when the set of feasible strategies is S_2, where $S_1\subset S_2\subset\bar{S}$. We assume that

$$P(S_1;S_2) = \sum_{s\in S_1} P(s;S_2). \tag{3.1}$$

Assumption A1

The sets in $\mathfrak{B}$ *are almost surely disjoint.*

Assumption A2

For any $S_1\subset S_2\subset\bar{S}$, $P(S_1;S_2)\in(0,1)$ *and*

$$P(S_1;\bar{S}) = P(S_2;\bar{S})P(S_1;S_2). \tag{3.2}$$

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[1] Although the agent in stage two is no longer uncertain about whether an alternative is feasible or not he may still be uncertain about the consequences of choosing a particular alternative. In this case the relevant decision rule in the second stage is to make the choice from B_s that maximizes expected utility.

Assumption A2 is the wellknown "Independence from irrelevant alternatives" (IIA), assumption which was proposed by Luce (1959).

Theorem 1

Assumption A2 holds if and only if

$$P(S_1;S_2) = \frac{\sum_{s\in S_1} a(s)}{\sum_{s\in S_2} a(s)} \tag{3.3}$$

where {a(s)} are positive scalars that are unique apart from a multiplicative constant.

A proof of Theorem 1 is provided by Luce (1959).

Observe that by (3.3) the structure of the choice probabilities is invariant under aggregation of strategies in the following sense: Let

$$a(S) \equiv \sum_{s\in S} a(s). \tag{3.4}$$

Then (3.3) takes the form

$$P(S_1;S_2) = \frac{a(S_1)}{a(S_2)} = \frac{a(S_1)}{a(S_1)+a(S_2-S_1)}. \tag{3.5}$$

From (3.5) we realize that the functional form of the choice probabilities is independent of S_1 and S_2, which means that the structure of (3.5) is independent of the aggregation level identified by S_1 and S_2.

Define $I_i(B_s)=1$ if alternative i belongs to B_s and zero otherwise, let $\bar{I}(B_s)=(I_1(B_s),I_2(B_s),...,I_N(B_s))$ and let Ω denote the agent's information set. Clearly, the choice set B_s is completely identified by $\bar{I}(B_s)$ and when A1 holds the probability distribution of $\{B_s\}$ is equivalent to the probability distribution of $\{\bar{I}(B_s)\}$. Let

$$g_k(s) \equiv E(I_k(B_s)|\Omega) = P(I_k(B_s)=1|\Omega) \tag{3.6}$$

for $s\subset\bar{S}$. Thus $g_k(s)$ is the probability that alternative k shall be feasible in the second stage given Ω and given that strategy s is chosen in stage one.

Assumption A3

The agent knows the probability distribution $g(s)=(g_1(s),g_2(s),\ldots,g_N(s))$.

More general, we shall define $g_k(S)$ as the probability that k becomes feasible given that some strategy in S, $S\subset\bar{S}$, is chosen. Obviously $g_k(S)=\sum_{s\in S} g_k(s)$.

Assumption A4

For $S\subset\bar{S}$, *the scalar* $a(S)$ *defined by (3.4) has the structure* $a(S)=f(S,g(S))$ *where the functional*

$$g(S) \to f\big(S,g(S)\big)$$

is continuous (with the Euclidian metric).

Assumption A4 is a natural assumption since the agent's "information" about his opportunities in the first stage, is represented by $\{g(s)\}$. In case $I_i(B_s)$ and $I_j(B_s)$ are stochastically independent for $i\neq j$, for all $B_s\in\mathfrak{B}$, the joint distribution of $\bar{I}(B_s)$ can be expressed by $g(s)$. However, if $I_i(B_s)$ and $I_j(B_s)$ are dependent, this is not the case, so that a proper representation of the agent's beliefs about his opportunities should include the joint distribution of $\bar{I}(B_s)$. Consequently, Assumption A4 is not appropriate when some of the components of $\bar{I}(B_s)$ are dependent.

The crucial property of Assumption A4 is that it is formulated on an aggregate level, say S. If A4 were assumed to hold only when S contains a single alternative we would obtain very little from this assumption. This will become clear after going through the arguments in the proof of the next result.

Theorem 2

Suppose that A2, A3 and A4 hold. Then

$$P(S_1;S_2) = \frac{a(S_1)}{a(S_2)} = \frac{\sum_{j\in S_1}\sum_{r=1}^{N}\beta_r g_r(j)}{\sum_{j\in S_2}\sum_{r=1}^{N}\beta_r g_r(j)} \tag{3.7}$$

for $S_1 \subset S_2 \subset \bar{S}$, *where* β_r , $r=1,2,...,N$, *are scalars that are uniquely determined apart from multiplication by a constant.*

The proof of Theorem 2 is given in the appendix.

Let $\tilde{P}(i;B_s)$ be the probability of choosing alternative i from B_s in the second stage given that strategy s was chosen in stage one. Recall that in the second stage B_s is known and the outcomes are certain. From (3.7) it follows that the choice probabilities in the second stage have the form

$$\tilde{P}(i;B_s) = \frac{\beta_i}{\sum_{r\in B_s}\beta_r} \tag{3.8}$$

provided strategy s has been selected in the first stage.

Let Q(i;S) be the probability that alternative i shall be the final outcome. When (3.7) and (3.8) are combined the next result follows immediately.

Corollary 1

The probability that alternative i is the choice outcome given that S_2 *is the set of feasible strategies, equals*

$$Q(i;S_2) = \sum_{s\in S_2} P(s;S_2)\tilde{P}(i;B_s). \tag{3.9}$$

In the special case where the choice sets contain only a single element, the model (3.7) was proposed by Becker et al. (1963a) and Luce and Suppes (1965), p. 360, under the name Strict expected utility model.

The next result is due to Becker et al. (1963a).

Corollary 2

Let $\mathfrak{S}$ consist of m+1 strategies where

$$g_r(m+1) = \frac{1}{m}\sum_{s=1}^{m} g_r(j). \tag{3.10}$$

for r=1,2,...,N. Then

$$P(m+1,\bar{S}) = \frac{1}{1+m}. \tag{3.11}$$

Proof:

From (3.7) it follows that

$$P(m+1,\bar{S}) = \frac{\sum_{r=1}^{N} \beta_r g_r(m+1)}{\sum_{s\in\bar{S}} \sum_{r=1}^{N} \beta_r g_r(s)}$$

$$= \frac{\sum_{r=1}^{N} \beta_r g_r(m+1)}{\sum_{r=1}^{N} \beta_r g_r(m+1) + m\sum_{r=1}^{N} \beta_r g_r(m+1)} = \frac{1}{m+1}$$

and the proof is complete.

Q.E.D.

Becker et al. (1963b) used the result of Corollary 2 to perform an experiment to test the prediction (3.11). They found that approximately 18 per cent of the subjects that participated in the experiment failed to satisfy this model. However, more empirical evidence is needed to assess the performance of the strict utility model. It is, for example, unlikely that this model is appropriate for all types of choice experiments.

Recall that the results above do not require a random utility representation. Similarly to Luce choice model (Luce, 1959), it may be viewed as a revealed preference result. In the next section we shall show, however, that a particular random utility representation exists that is consistent with (3.7) and which can be interpreted as a version of expected utility under bounded rationality.

4. A random utility representation

The hypothesis of a random utility index as a representation of preferences dates back to Thurstone (1927). Thurstone conducted psychophysical experiments in which individuals were asked to compare the intensities of physical stimuli. The interpretation of Thurstone's theory of random utilities is that while the decision rule is deterministic and follows from maximizing utility at each moment, the agent's tastes may fluctuate from one moment to the next in a way that is unpredictable to him. Alternatively, the agent is viewed as being unable to fix a definite (subjective) value of the alternatives.

We shall now answer the question of whether there exists a utility representation which implies choice probabilities as in Theorem 2. In settings where the agent knows the choice sets, Holman and Marley (see Luce and Suppes, 1965, p. 338), McFadden (1974), Yellott (1977) and Strauss (1979) have analyzed the problem of necessary and sufficient conditions for random utility models to satisfy IIA.

The choice probabilities that follow from a random utility model are defined formally by

$$P(S_1;S_2) = P\left(\max_{s\in S_1} V_s = \max_{s\in S_2} V_s\right) \tag{4.1}$$

for $S_1 \subset S_2 \subset \bar{S}$, where $\{V_s\}$ are random variables. When the joint c.d.f. of $(V_1,V_2,...,V_N)$ is specified (4.1) can, at least in principle, be calculated.

<u>Theorem 3</u>

The random utility model $\{V_s\}$ with

$$V_s = \eta_s \sum_r \beta_r g_r(s) \tag{4.2}$$

where η_s, $s=1,2,...$, are i.i.d. with

$$P(\eta_s \leq y) = exp(-y^{-1}) \tag{4.3}$$

implies that the choice probabilities are given by (3.7).

Proof:

When (4.2) and (4.3) hold the structure (3.7) follows readily by straight forward calculus.

Q.E.D.

When we take the logarithm of both sides of (4.2) we get an equivalent additive formulation with random term $\eta_s^* \equiv \log\eta_s$ that has c.d.f. $\exp(-e^{-y})$. The additive formulation is common in discrete choice theory, see e.g. Ben-Akiva and Lerman (1985).

Let us now compare the utility function of Theorem 3 with what follows from an analogue to the expected utility hypothesis. Note first that by letting $B_s=\{k\}$, $g_k(s)=1$, it follows that the corresponding utility reduces to $\beta_k\tilde{\eta}_k$ where the disturbances $\{\tilde{\eta}_k\}$ are i.i.d. with c.d.f. as in (4.3). Let

$$\eta_s \equiv \frac{\max_{k\in B_s}\left(\beta_k\tilde{\eta}_k\right)}{\sum_{k\in B_s}\beta_k} = \frac{\max_{k\in B_s}\left(\beta_k\tilde{\eta}_k\right)}{\sum_k \beta_k I_k\left(B_s\right)}. \tag{4.4}$$

By straightforward calculus it follows that η_s is stochastically independent of B_s and has c.d.f. as in (4.3). From (4.4) we get that

$$E\left(\max_{k\in B_s}\left(\beta_k\tilde{\eta}_k\right)|\eta_s,\Omega\right) = \eta_s\sum_k \beta_k E\left(I_k\left(B_s\right)|\Omega\right) = \eta_s\sum_k \beta_k g_k(s), \tag{4.5}$$

which demonstrates that

$$E\left(\max_{k\in B_s}\left(\beta_k\tilde{\eta}_k\right)|\eta_s,\Omega\right)$$

has the same c.d.f. as V_s given by (4.2). Recall that η_s captures the effect of the unpredictable taste-shifters. Note that by (4.4) η_s depends also on B_s. Thus we may interpret (4.5) as follows: According to (4.4) the utility of a strategy s consists of two components, of which one, η_s, is unpredictable to the agent. The agent does not even know the c.d.f. of $\{\eta_s\}$.[2] The best the agent can do is to evaluate expected utility of the structural part,

$$\sum_k \beta_k I_k(B_s).$$

We may interpret η_s as a variable that captures the agent's inability to deal with uncertainty. Therefore, when identical experiments are repeated the agent may choose different strategies each time, not only because of randomly fluctuating tastes but also because he is unable to assess precisely the value of the uncertain sets $\{B_s\}$.

Another way of expressing this goes as follows: At the moment of choice the agent ignores the fact that η_s depends on $\bar{I}(B_s)$ because he is unable to account for this dependence. (Recall that while η_s and $\bar{I}(B_s)$ may be dependent they are, however, stochastically independent.) When the dependence between $\bar{I}(B_s)$ and $\{\eta_s\}$ is ignored the term η_s only varies across experiments. At a particular moment in time it is thus treated as a constant by the agent when he applies the expectation operator.

We conclude this section by showing that the result of Corollary 1 also is consistent with a random utility representation. This is not obvious due to the fact that the choice process takes place in two stages and, unless explicitly assumed otherwise, the taste-shifters will be correlated across stages. However, due to a particular property of the extreme value distribution the result goes through.

<u>Theorem 4</u>

Let $U_k = \beta_k \tilde{\eta}_k$ be a random utility model where $\{\beta_k\}$ are positive scalars and $\{\tilde{\eta}_k\}$ are i.i.d. random variables with c.d.f.

$$P(\tilde{\eta}_k \leq y) = exp(-y^{-1}). \tag{4.6}$$

Let

$$V_s = \eta_s \sum_k \beta_k g_k(s) \tag{4.7}$$

where

[2] Note that the information represented by the c.d.f. given by (4.3) is not relevant here. The c.d.f. (4.3) only concerns variation in $\{\eta_s\}$ across identical choice experiments.

$$\eta_s \equiv \frac{\max_{k \in B_s}(\beta_k \tilde{\eta}_k)}{\sum_k \beta_k I_k(B_s)}. \tag{4.8}$$

Then

$$P\left(U_i = \max_{k \in B_s} U_k \,|\, V_s = \max_{r \in S_2} V_r,\ B_s \text{ is revealed}\right) = P\left(U_i = \max_{k \in B_s} U_k\right) = \tilde{P}(i; B_s). \tag{4.9}$$

A proof of Theorem 4 is given in the appendix.

The interpretation of (4.9) is that, conditional on the choice B_s in the first stage, the choice in the second stage only depends on $\{U_i, i \in B_s\}$.

5. Generalization

We shall now discuss how the model above can be generalized. Recall first that the model above contains the Luce model for choice with certain outcomes as a special case.

Strauss (1979) and Robertson and Strauss (1981) have examined the relationship between the Luce model and the corresponding random utility model when the utilities are allowed to be dependent. As a point of departure we shall adopt their approach to obtain a characterization of the random utility representation for choice with sure outcomes.

Assumption A5

The distribution of $\max_k U_k$ is independent of which utility attains the maximum.

Assumption A5′

Apart from a scale shift, then $\max_k U_k$ has the same c.d.f. as U_i, for any i.

Theorem 5

Suppose $U_i = \beta_i \eta_i$, *i=1,2,...,N, where* $\beta_i > 0$ *are positive constants and the c.d.f. of* $(\eta_1, \eta_2, \ldots, \eta_N)$ *is independent of* $\beta_1, \beta_2, \ldots, \beta_N$. *Then A2 and A5 imply that*

$$P\left(\bigcap_{i=1}^{N}(U_i \le y_i)\right) = \varphi\left(\sum_{i=1}^{N}\left(\frac{\beta_i}{y_i}\right)^{\alpha}\right) \tag{5.1}$$

for $y_i > 0$, *where* $\alpha > 0$ *is a constant and* $\varphi(\cdot)$ *is a function such that (5.1) is a proper c.d.f.*

Theorem 5′

Under the assumptions of Theorem 5, A2 and A5′ imply (5.1).

The proofs of Theorem 5 and 5′ are given in Robertson and Strauss (1981).

In the case where

$$\tilde{U}_j = \psi(U_j) = \psi(\beta_j \eta_j) \tag{5.2}$$

where ψ is an increasing function from R_+ to R_+ the c.d.f. of $\tilde{U}$ follows from (5.1) by replacing y_i by $\psi^{-1}(y_i)$. In the case with sure outcomes $\tilde{U}$ is of course equivalent to U. This is, however, not so when the outcomes are uncertain. From (5.1) it follows that

$$P\left(\bigcap_s \left(\max_{r \in B_s} \tilde{U}_r \le y_s\right)\right) = \varphi\left(\sum_s \left(\frac{\sum_r \beta_r^{\alpha} I_r(B_s)}{\psi^{-1}(y_s)^{\alpha}}\right)\right). \tag{5.3}$$

Define

$$\eta_s \equiv \frac{\left(\psi^{-1}\left(\max_{r \in B_s} \tilde{U}_r\right)\right)^{\alpha}}{\left(\sum_r \beta_r^{\alpha} I_r(B_s)\right)} \tag{5.4}$$

which means that

$$\max_{r \in B_s} \tilde{U}_r = \psi\left(\left(\eta_s \sum_r \beta_r^\alpha I_r(B_s)\right)^{1/\alpha}\right). \tag{5.5}$$

From (5.3) and (5.4) it follows that the c.d.f. of $(\eta_1, \eta_2, \ldots)$ equals

$$P\left(\bigcap_s (\eta_s \leq y_s)\right) = \varphi\left(\sum_s y_s^{-1}\right). \tag{5.6}$$

Analogous to the treatment above we define the value function by

$$V_s(\psi, \alpha) = E\left(\max_{r \in B_s} \tilde{U}_r \mid \{\eta_s\}, \Omega\right) \tag{5.7}$$

which the agent is supposed to maximize to find the best strategy.

<u>Assumption A6</u>

The distribution of $max_s V_s(\psi,\alpha)$ is independent of which variable attains the maximum.

Evidently, A5 is a special case of A6.

Let θ be a non-negative constant and define

$$\psi^*(x) = \begin{cases} \dfrac{x^\theta - 1}{\theta} & \text{for} \quad \theta > 0, \\ \log x & \text{for} \quad \theta = 0. \end{cases} \tag{5.8}$$

The function ψ^* is increasing and continuous. It is strictly convex for $\theta > 1$ and strictly concave when $\theta < 1$.

<u>Theorem 6</u>

Assume A1 and A3. Assume furthermore that $\{V_s(\psi,\alpha)\}$ satisfies A6 and that the corresponding choice probabilities (for choice of strategy) satisfy A2. Then $\psi = \psi^*$.

The proof of Theorem 6 is given in the appendix.[3]

Remark

From the proof of Theorem 6 it will be realized that θ also can be negative. However, there is no loss of generality in restricting θ to be non-negative since it can easily be veryfied that the corresponding choice probabilities depend on θ solely through |θ|.

When $\psi=\psi^*$ the value function takes the form

$$V_s(\psi^*,\alpha) = \begin{cases} \dfrac{\eta_s^{\theta/\alpha} E\left(\left(\sum_r \beta_r^\alpha I_r(B_s)\right)^{\theta/\alpha} | \Omega\right) - 1}{\theta} & \text{for } \theta \neq 0, \\ \dfrac{1}{\alpha}\log\eta_s + \dfrac{1}{\alpha} E\left(\log\left(\sum_r \beta_r^\alpha I_r(B_s)\right) | \Omega\right) & \text{for } \theta = 0. \end{cases} \tag{5.9}$$

Theorem 7

Under the assumptions of Theorem 6 and with $S_1 \subset S_2 \subset \bar{S}$ the choice probabilities are given by

$$P\left(\max_{s \in S_1} V_s(\psi^*,\alpha) = \max_{s \in S_2} V_s(\psi^*,\alpha)\right) = \frac{\sum_{s \in S_1} \left[E\left(\left(\sum_r \beta_r^\alpha I_r(B_s)\right)^{\theta/\alpha} | \Omega\right)\right]^{\alpha/\theta}}{\sum_{s \in S_2} \left[E\left(\left(\sum_r \beta_r^\alpha I_r(B_s)\right)^{\theta/\alpha} | \Omega\right)\right]^{\alpha/\theta}}, \tag{5.10}$$

when θ>0, and

[3] Similarly to Theorem 5′ it is readily realized that the result of Theorem 6 holds when A6 is replaced by a suitably modified version of A5′.

$$P\left(\max_{s\in S_1} V_s(\psi^*,\alpha) = \max_{s\in S_2} V_s(\psi^*,\alpha)\right) = \frac{\sum_{s\in S_1} exp\left(E\left(log\left(\sum_r \beta_r^\alpha I_r(B_s)\right)\right)|\Omega\right)}{\sum_{s\in S_2} exp\left(E\left(log\left(\sum_r \beta_r^\alpha I_r(B_s)\right)\right)|\Omega\right)}, \tag{5.11}$$

when $\theta=0$.

Proof:

Consider (5.9) with $\theta>0$ and let

$$m_s = \frac{\alpha}{\theta}\log\left[E\left(\left(\sum_r \beta_r^\alpha I_r(B_s)\right)^{\theta/\alpha}|\Omega\right)\right], \tag{5.12}$$

and observe that the joint c.d.f. of $(\log\eta_1, \log\eta_2,...)$ equals

$$\varphi\left(\sum_s e^{-y_s}\right).$$

From Strauss (1979), pp. 42-43 (eqs. (3.9) and (3.14)) we get that

$$P\left(\max_{s\in S_1} V_s(\psi^*,\alpha) = \max_{s\in S_2} V_s(\psi^*,\alpha)\right) = P\left(\max_{s\in S_1}(\log\eta_s+m_s) = \max_{s\in S_2}(\log\eta_s+m_s)\right) = \sum_{s\in S_1}\left(\frac{e^{m_s}}{\sum_{s\in S_2} e^{m_s}}\right) \tag{5.13}$$

which equals (5.10). The proof of (5.11) is completely analogous.

Q.E.D.

Note that while (5.10) and (5.11) satisfy A2 they do not satisfy A4.

Next we shall demonstrate that the choice model above contains the deterministic utility model as a special case.

Corollary 3

When $\alpha \rightarrow \infty$ the model given by (5.10) and (5.11) reduces to a deterministic model where the choice of strategy is determined by maximizing the deterministic utility function

$$V_s(\psi^*,\infty) \equiv E\left(max_r\left(I_r(B_s)\,\psi^*(\beta_r)\right)|\Omega\right). \tag{5.14}$$

The proof of Corollary 3 is given in the appendix.

When the sets $\{B_s\}$ contain at most one alternative we immediately get the next result.

Corollary 4

When every set in $\mathcal{B}$ contains at most one alternative (5.10) reduces to

$$P\left(\max_{s\in S_1} V_s(\psi^*,\alpha) = \max_{s\in S_2} V_s(\psi^*,\alpha)\right) = \frac{\sum_{s\in S_1}\left(\sum_r \beta_r^\theta g_r(s)\right)^{\alpha/\theta}}{\sum_{s\in S_2}\left(\sum_r \beta_r^\theta g_r(s)\right)^{\alpha/\theta}}, \tag{5.15}$$

for $\theta>0$, and (5.11) reduces to

$$P\left(\max_{s\in S_1} V_s(\psi^*,\alpha) = \max_{s\in S_2} V_s(\psi^*,\alpha)\right) = \frac{\sum_{s\in S_1} exp\left(\alpha\sum_r g_r(s)\log\beta_r\right)}{\sum_{s\in S_2} exp\left(\alpha\sum_r g_r(s)\log\beta_r\right)}, \tag{5.16}$$

wen $\theta=0$, where $S_1 \subset S_2 \subset \bar{S}$.

Observe that for $\alpha=\theta=1$, (5.15) coincides with the result of Theorem 3.
The next result extends the result of Corollary 2.

Theorem 8

Let $\mathfrak{B}$ consist of m+1 strategies where

$$P\left(\bar{I}(B_{m+1})=\bar{i}\right)=\frac{1}{m}\sum_{s=1}^{m}P\left(\bar{I}(B_s)=\bar{i}\right) \tag{5.17}$$

and $\bar{i}\in\{0,1\}^N$. Under the assumptions of Theorem 7

$$\left(\prod_{s=1}^{m}P_s\right)^{1/m}\leq P_{m+1}\leq\max_{s\leq m}P_s \tag{5.18}$$

when $\theta>0$, and

$$\left(\prod_{s=1}^{m}P_s\right)^{1/m}=P_{m+1} \tag{5.19}$$

when $\theta=0$, where

$$P_s\equiv P\left(V_s(\psi^*,\alpha)=\max_{j\leq m+1}V_j(\psi^*,\alpha)\right).$$

The proof of Theorem 8 is given in the appendix.

The observable properties (5.18) and (5.19) enable us to carry out non-parametric tests of the model given in Theorem 7. Unfortunately, empirical testing of (5.18) and (5.19) requires observations from a homogeneous sample (or alternatively many replications for each agent). In contrast, when $\theta=\alpha=1$ the result of Corollary 2 implies that the prediction of the fraction of time strategy m+1 is chosen is independent of the preference parameters and the probabilities, $\{g_i(s)\}$.

Corollary 5

Under the assumptions of Theorem 8 it follows that

$$P_{m+1}<\frac{1}{m+1} \tag{5.20}$$

when $\alpha>\theta$, and

$$P_{m+1} > \frac{1}{m+1} \tag{5.21}$$

when $\alpha<\theta$.

Proof:

From (5.10) it follows that for $\theta>0$

$$\sum_{s=1}^{m+1} \left(\frac{P_s}{P_{m+1}} \right)^{\theta/\alpha} = m+1, \tag{5.22}$$

or equivalently

$$\left(\frac{1}{m} \sum_{s=1}^{m} P_s^{\theta/\alpha} \right)^{\alpha/\theta} = P_{m+1}. \tag{5.23}$$

Since the left hand side of (5.23) is increasing as a function of α/θ, it follows that $\alpha/\theta>1$ implies

$$P_{m+1} > \frac{1}{m} \sum_{s=1}^{m} P_s = \frac{1}{m}\left(1-P_{m+1}\right) \tag{5.24}$$

which proves (5.20) when $\theta>0$. Consider next the case with $\theta=0$. By Jensen's inequality and (5.19) we get

$$\log P_{m+1} = \frac{1}{m} \sum_{s=1}^{m} \log P_s > \log\left(\sum_{s=1}^{m} P_s/m \right) = \log\left(\frac{1-P_{m+1}}{m} \right)$$

and therefore (5.20) holds when $\theta=0$. The second statement, (5.21), follows similarly.

Q.E.D.

Becker et al. (1963a) state a similar result as in Corollary 5.

6. Uncertainty versus aggregation of latent alternatives

Consider now an alternative choice setting. The agent now is perfectly certain about the choice sets. Each choice set B_s consists of disjoint subsets C_{rs}, i.e., $B_s = \cup_r C_{rs}$. The number of alternatives in C_{rs} is $m_r(s)$ and to the observing analyst it is assumed to be an unobservable that may vary across experiments. Thus we assume that $m_r(s)$ is a random variable with

$$Em_r(s) = mq_r(s) \tag{6.1}$$

where

$$m = \sum_j \sum_r m_r(j) \tag{6.2}$$

is assumed to be a constant integer. The analyst knows $\{q_r(s)\}$. Recall that the agent knows $\{m_r(s)\}$. The utility of alternative $i \in C_{rs}$ is assumed to be $\beta_r \varepsilon_{ri}$ where β_r is a positive constant that is independent of i and $\{\varepsilon_{ri}\}$ are i.i.d. random variables with c.d.f. as in (4.3). The alternatives in $\cup_j C_{rj}$ can therefore be interpreted as almost "similar" in the sense that their utilities have the same distribution. The corresponding choice probabilities,

$$\tilde{P}(s;S) \equiv P\left(\max_r\left(\max_{i \in C_{rs}}(\beta_r \varepsilon_{ri})\right) = \max_{j \in S, r}\left(\max_{i \in C_{rj}}(\beta_r \varepsilon_{ri})\right)\right) \tag{6.3}$$

are given by

$$\tilde{P}(s;S) = \frac{\sum_{r=1}^{N} \beta_r m_r(s)}{\sum_{j \in S} \sum_{r=1}^{N} \beta_r m_r(j)}. \tag{6.4}$$

When m is large we have that

$$\tilde{P}(s;S) \approx \frac{\sum_{r=1}^{N} \beta_r q_r(s)}{\sum_{j \in S} \sum_{r=1}^{N} \beta_r q_r(j)}. \tag{6.5}$$

Recall that from the viewpoint of the analyst, $q_r(s)$ is the probability that C_{rs} is non-empty, i.e., that an alternative of "type" r is feasible. Suppose next that the agent also does not know $\{m_r(s)\}$ but knows $\{q_r(s)\}$. Then the choice setting is completely analogous to the one treated previously with $g_r(s)=q_r(s)$, and by Theorem 2 we get that the first stage choice probabilities have the same structure as in (6.5). We can therefore conclude that in the choice setting discussed above and under the hypothesis of rational expectations the choice probabilities do not depend on whether the agent is uncertain or prefectly certain about his choice set.

7. Conclusion

In this paper we have examined the implications from IIA combined with different types of invariance assumptions in the context of a probabilistic formulation of discrete choice models with uncertain outcomes. These invariance assumptions (A4 to A6) concern properties of the choice models under aggregation of alternatives, or strategies. We have demonstrated that the functional forms of the corresponding choice models are simple, and consequently are convenient for empirical analyses. Moreover, we have derived non-parametrically observable properties. The invariance assumptions imply that the stochastic versions of expected utility must have distributions that are equivalent to type I or type III extreme value distributions. In the special case where the choice outcomes are single alternatives, the choice probabilities are expressed as a power — or exponential transform — of the expected utility given the favorable strategy to the sum over the possible strategies of the power — or exponential transformed — expected utilities given the respective strategies.

Appendix

Proof of Theorem 2:

From (3.4) and A4 we get:

$$f\left(S,\sum_{s\in S}\mathbf{x}(s)\right) = \sum_{s\in S} f(\{s\},\mathbf{x}(s)) \tag{A.1}$$

for $\mathbf{x}(s) \in [0,1]^N$, $S\subset\bar{S}$. Also it follows that for any s, $f(\{s\},\mathbf{0})=0$, because the probability of choosing a strategy which almost surely implies an empty choice set is zero. Hence with $\mathbf{x}(s')=0$, $\mathbf{x}(s)=\mathbf{x}$, for all $s'\neq s$, (A.1) reduces to

$$f(S,\mathbf{x}) = f(\{s\},\mathbf{x}) \tag{A.2}$$

for any $s\in S\subset\bar{S}$, and $\mathbf{x}\in[0,1]^N$. But then $f(\{s\},\cdot)$ must be independent of s which by (A.1) implies

$$f\left(\sum_{s\in S}\mathbf{x}(s)\right) = \sum_{s\in S} f(\mathbf{x}(s)). \tag{A.3}$$

In Aczél (1966) functional equations of the type (A.3) are treated. There it is demonstrated that (A.3) implies that $f(\cdot)$ is linear, say

$$f(\mathbf{x}) = \beta'\mathbf{x} \equiv \sum_{k=1}^{N} \beta_k x_k \tag{A.4}$$

where $\beta_k>0$, $k=1,2,\ldots,N$, are scalars. Now (3.7) follows from (3.3) and (A.4).

Q.E.D.

Proof of Theorem 4:

Note first that since the sets $\{B_s\}$ by A1 are disjoint it follows that U_i, $i\in B_s$ are independent of V_r for $r\neq s$. It therefore remains to prove that U_i, $i\in B_s$ are independent of V_s. Since

$$V_s d_s = \max_{k\in B_s} U_k, \tag{A.5}$$

where

$$d_s = \frac{\sum_k \beta_k I_k(B_s)}{\sum_k \beta_k g_k(s)} \tag{A.6}$$

we have

$$P\left(U_i = \max_{k\in B_s} U_k, V_s \le y\right) = P\left(U_i = \max_{k\in B_s} U_k, \max_{k\in B_s} U_k \le y\, d_s\right) \tag{A.7}$$

$$= P\left(U_i > \max_{k\in B_s - \{i\}} U_k, U_i \le y\, d_s\right) = \int_0^{yd_s} P\left(x > \max_{k\in B_s - \{i\}} U_k\right) P(U_i \in dx) = \int_0^{yd_s} \exp\left(-x^{-1} \sum_{k\in B_s} \beta_k\right) \beta_i x^{-2} dx$$

$$= \frac{\beta_i}{\sum_{k\in B_s} \beta_k} \cdot \exp\left(-(yd_s)^{-1} \sum_{k\in B_s} \beta_k\right) = \tilde{P}(i;B_s) P\left(\max_{k\in B_s} U_k \le yd_s\right) = \tilde{P}(i;B_s) P(V_s \le y).$$

This completes the proof.

Q.E.D.

The following lemma is useful for proving Theorem 6.

<u>Lemma 1</u>

Let h,v,w, and f be real-valued functions defined on an open interval $K \subset R_+$ having one as a limit point, with h and w strictly monotonic (or nonconstant, continuous); suppose that

$$h(xy) = v(x) + f(x)w(y) \tag{A.8}$$

whenever $x, y, xy \in K$.

Then either f is a constant function and there are constants α_0, α_1, α_2 and α_3, with α_0, α_3,>0, such that for all $x \in K$

$$\begin{aligned} h(x) &= \alpha_0 \log x + \alpha_1 + \alpha_2, \\ v(x) &= \alpha_0 \log x + \alpha_2, \\ w(x) &= (\alpha_0/\alpha_3)\log x + \alpha_1/\alpha_3, \\ f(x) &= \alpha_3, \end{aligned} \tag{A.9}$$

or f takes at least two distinct values and there are constants α_0, α_1, α_2, α_3, and λ with $\alpha_3>0$, and $\lambda\alpha_0<0$ such that for all $x \in K$

$$
\begin{aligned}
h(x) &= \alpha_0(1-x^{\lambda}) + \alpha_1 + \alpha_2, \\
v(x) &= (\alpha_0+\alpha_1)(1-x^{\lambda}) + \alpha_2, \\
w(x) &= (\alpha_0/\alpha_3)(1-x^{\lambda}) + \beta_1/\beta_3, \\
f(x) &= \alpha_3 x^{\lambda}.
\end{aligned} \tag{A.10}
$$

For a proof of Lemma 1 we refer to Falmagne (1985), p.p. 85-89.

Proof of Theorem 6

Consider the special case with one sure outcome, (1), and one (2) uncertain strategy. Assume furthermore that B_2 contains either alternative 2 or alternative 3. From (5.7) and (5.5) we obtain

$$V_2 = V_2(\psi,\alpha) = \psi\left(\eta_2^{1/\alpha}\beta_2\right)g_2(2) + \psi\left(\eta_2^{1/\alpha}\beta_3\right)\left(1-g_2(2)\right), \tag{A.11}$$

By A6 and Theorem 5 the c.d.f. of (V_1,V_2) must be of the form

$$P\left(V_1\le y_1, V_2\le y_2\right) = \varphi\left(\frac{\beta_1^{\alpha}}{\psi^{-1}(y_1)^{\alpha}} + \frac{b^{\alpha}}{\psi^{-1}(y_2)^{\alpha}}\right) \tag{A.12}$$

where b depends on $g_2(2)$. When $y_2=\infty$ we get

$$P\left(V_1\le y_1\right) = \varphi\left(\frac{\beta_1^{\alpha}}{\psi^{-1}(y_1)^{\alpha}}\right). \tag{A.13}$$

Since the distribution of V_1 does not depend on $g_2(2)$ we realize that the function φ cannot depend on $g_2(2)$. Therefore, it follows from (A.11) and (A.12) that

$$\psi\left(\eta_2^{1/\alpha}\beta_2\right)g_2(2) + \psi\left(\eta_2^{1/\alpha}\beta_3\right)\left(1-g_2(2)\right)$$

must have the same distribution as

$$\psi\left(\eta_2^{1/\alpha} b\right)$$

for all $g_2(2) \in [0,1]$. But then for almost all x>0 and $z \in [0,1]$

$$\psi(x\beta_2)z + \psi(x\beta_3)(1-z) = \psi(xb(z)). \tag{A.14}$$

Without loss of generality assume that $\beta_2 > \beta_3$. By assumption ψ is increasing and (A.14) therefore implies that b is increasing. Let c be the inverse of b, whence

$$\psi(xy) = \psi(x\beta_3) + \left(\psi(x\beta_2) - \psi(x\beta_3)\right)c(y). \tag{A.15}$$

We are now ready to apply the result of Lemma 1. Let $h(x)=\psi(x)$, $v(x)=\psi(\beta_3 x)$, $w(x)=c(x)$, and $f(x)=\psi(x\beta_2)-\psi(x\beta_3)$. Suppose f is not a constant. From (A.10) we get

$$h(x) = \psi(x) = \alpha_0(1-x^\lambda) + \alpha_1 + \alpha_2, \tag{A.16}$$

$$v(x) = \psi(\beta_4 x) = (\alpha_0+\alpha_1)(1-x^\lambda) + \alpha_2, \tag{A.17}$$

$$w(x) = c(x) = (\alpha_0/\alpha_3)(1-x^\lambda) + \alpha_1/\alpha_3, \tag{A.18}$$

and

$$f(x) = \psi(x\beta_2) - \psi(x\beta_3) = \alpha_3 x^\lambda. \tag{A.19}$$

When we combine (A.16), (A.17) and (A.19) we get the additional restrictions

$$\alpha_1 = \alpha_0(\beta_3^\lambda - 1)$$

and

$$\alpha_3 = \alpha_0(\beta_3^\lambda - \beta_2^\lambda) > 0.$$

Hence

$$\psi(x) = \alpha_0 + \alpha_1 + \alpha_2 - \alpha_0 x^\lambda = \alpha_0\beta_3^\lambda + \alpha_2 - \alpha_0 x^\lambda. \tag{A.20}$$

Without loss of generality we may fix the parameters such that $\alpha_0=-1$ and $\alpha_2=-\alpha_0\beta_3^\lambda$, and therefore ψ must be a power function. Conversely, when $\theta \neq 0$ it is easily veryfied that $\{V_s(\psi^*,\alpha)\}$ satisfies A6 for any set $\{g_k(s)\}$. Thus the conclusion of the theorem follows provided f is not a constant. When f is a constant we obtain similarly from (A.9) that

$\psi(x)=\log x$, and the proof is complete.

Q.E.D.

<u>Proof of Corollary 3</u>

Consider first w_s defined by

$$w_s = E\left(\left(\sum_r \beta_r^{\alpha} I_r(B_s)\right)^{\theta/\alpha} \mid \Omega\right) \tag{A.21}$$

as $\alpha\to\infty$. For simplicity and without loss of generality assume that $\beta_r>1$ for all r. By applying l'Hôpital's rule we obtain

$$\lim_{\alpha\to\infty} \frac{\log\left(\sum_r \beta_r^{\alpha} I_r(B_s)\right)}{\alpha} = \lim_{\alpha\to\infty} \frac{\sum_r \beta_r^{\alpha}(\log\beta_r) I_r(B_s)}{\sum_r \beta_r^{\alpha} I_r(B_s)} = \lim_{\alpha\to\infty} \frac{\sum_r (\beta_r/\beta)^{\alpha}(\log\beta_r) I_r(B_s)}{\sum_r (\beta_r/\beta)^{\alpha} I_r(B_s)} \tag{A.22}$$

where $\beta \equiv \max_r \beta_r$. Since $(\beta_r/\beta)^{\alpha}\to 0$ when $\alpha\to\infty$ for all β_r that are less than β it follows that the last expression above equals

$$\max_r\left((\log\beta_r) I_r(B_s)\right)$$

from which follows that

$$\lim_{\alpha\to\infty}\left(\sum_r \beta_r^{\alpha} I_r(B_s)\right)^{\theta/\alpha} = \max_r \beta_r^{\theta} I_r(B_s), \tag{A.23}$$

when $\theta>0$. But then, by Lebesgue's Dominated Convergence Theorem we also have

$$\lim_{\alpha\to\infty} w_s = E\left(\left(\max_r \beta_r^{\theta} I_r(B_s)\right) \mid \Omega\right). \tag{A.24}$$

Let $w \equiv \max_{s\in S_2} w_s$. Then we can express the choice probability of strategy j, given the choice set S_2, as

$$\frac{w_j^{\alpha}}{\sum_{s\in S_2} w_s^{\alpha}} = \frac{(w_j/w)^{\alpha}}{\sum_{s\in S_2} (w_s/w)^{\alpha}}.$$

When $\alpha \to \infty$ then the choice probabilities for those j for which $w_j<w$ tend towards zero while when $w_j=w$, the corresponding choice probability tends towards one. But this means that in the limit the agent will select the strategy that maximizes (5.14) with probability one. The case when $\psi^*(x)=\log x$ is proved similarly.

Q.E.D.

<u>Proof of Theorem 8</u>

Let

$$\mu(x) = \sum_{s=1}^{m+1} \left(\frac{P_s}{P_{m+1}}\right)^x. \tag{A.25}$$

From (5.10) it follows that

$$\mu\left(\frac{\theta}{\alpha}\right) = m + 1. \tag{A.26}$$

Provided $P_s \neq P_{m+1}$ for at least one s the function μ is strictly convex. Note also that $\mu(0)=m+1$. Thus when $\mu'(0)<0$ the equation $\mu(x)=m+1$ has one and only one positive solution while when $\mu'(0)\geq 0$ it has no positive solution. Hence for (A.26) to have a positive solution for θ/α we must have

$$\mu'(0) = \sum_{s=1}^{m} \log P_s - mP_{m+1} < 0. \tag{A.27}$$

When $P_s=P_{m+1}$ for all $s\leq m$ then $\mu'(0)=0$ and (A.26) holds for all positive θ/α. Thus the left hand side of (5.18) has been proved.

Note that (A.26) is equivalent to

$$\left(\frac{1}{m}\sum_{s=1}^{m} P_s^{\theta/\alpha}\right)^{\alpha/\theta} = P_{m+1}. \tag{A.28}$$

Obviously

$$\left(\frac{1}{m}\sum_{s=1}^{m} P_s^{\theta/\alpha}\right)^{\alpha/\theta} \leq \left(\frac{1}{m}\sum_{s=1}^{m} \max_{s\leq m} P_s^{\theta/\alpha}\right)^{\alpha/\theta} = \left(\max_{s\leq m} P_s^{\theta/\alpha}\right)^{\alpha/\theta} = \max_{s\leq m} P_s, \tag{A.29}$$

which proves the right hand side of (5.18).

Q.E.D.

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