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An Exploratory Study Based on Historical Data

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Abstract:
Based on the methodology developed by Hodrick & Prescott (1980), it is shown that monetary activity in Norway by no means obeys the cyclical patterns described by Lucas (1983). By constructing annual time series covering monetary data from 1900 to 1992, combined with the use of varying filtering parameter values, it is demonstrated that only credit volume has followed a procyclical pattern. Furthermore, prices are found to be countercyclical during the post war period. Tests of relative volatility and cyclical skewness are presented as well as prospects for future studies of business cycles in Norway based on historical data.

Keywords: Business cycles, history of monetary activity.

JEL classification: N13, N14, E32

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1. Introduction

In their study of business cycles, Kydland and Prescott (1990) claimed that in lack of fundamental empirical regularities, a significant part of modern business cycle theory had developed in an unfavourable direction. It was argued that what researchers believed to be unambiguous and historically stable regularities of economic covariation (Lucas (1983)), was easily contradictable. Consequently, during the relatively short period since the publishing of this seminal work, a large number of empirical researchers have shown increasing interest in reporting business cycle regularities based on historical data collected from a wide range of countries.

In line with the outlined trend, this paper explores some empirical regularities for monetary business cycles in Norway. To my knowledge, the reporting of Norwegian business cycle regularities has not been as well organised as in the United States. In an attempt to limit the scope of the study, I have chosen to focus on monetary aggregates leaving out variables like consumption, investment, government spending and employment. The method developed by Hodrick and Prescott (1980) is applied to extract a cyclical component from the trend component of a time series (the filtering properties are discussed in section 2). For every variable, cyclical patterns around the historical trend have been compared statistically to the cyclical behaviour of a reference indicator (real GDP).

Lucas (1983) tried to establish a definition of business cycles which seemed to replicate that of Burns and Mitchell (1946), stating that business cycles are to be viewed as movements about the trend in gross national product, and the comovements among different aggregate time series, see Lucas (1983, pg. 217). The definition is based on his interpretation of the methods used in Friedman (1963) and Burns and Mitchell (1946). In addition, Lucas emphasises that the comovements among aggregate variables represent the core properties of business cycles, basically because there are no regularities to find when analysing the univariate variation of GNP over time. One of the most discussed elements of his seminal paper is Lucas' attempt to establish laws of cyclical covariations between aggregate variables. Referring to the author: "*(i) Output across broadly defined sectors move together. (ii) Production of producer and consumer durables exhibit much greater amplitude than does the production of non durables.(iii) Production and prices of agricultural goods and natural resources have lower than average conformity. (iv) Business profits show high conformity and much grater amplitude than other series. (v) Prices generally are procyclical. (vi) Short term interest rates are procyclical, Long-term rates slightly so. (vii) Monetary aggregates and velocity measures are procyclical.*" (pp. 217-218). Lucas claims that these regularities are valid across countries and historical periods, and thus, declares them as "general rules". In response to this, a large group of researchers have tried to verify as well as falsify these regularities. Section 3 contains a review of some of these studies. In this paper, I particularly focus on regularity (v), (vi) and (vii), as they refer to the monetary part of the economy[1]. Due to the fact that most of the previous studies are more or less uniform in the choice of a reference indicator, I have chosen to work with the gross domestic product (GDP). GDP is not to be regarded as the same as GNP (the variable that Lucas picks), however, as a result of the long time span, it is necessary to pick a variable that is available and consistent over time. GDP is the only reference indicator that satisfies these demands.

To my knowledge, there has previously been few attempts to analyse Norwegian monetary variables within a business cycle concept, although there exists some international literature where such methods have been applied on data from Norway (Backus and Kehoe (1992)). Recent studies on stylised facts related to business cycles, have tended to emphasise the importance of choosing a sufficiently long time range from which one can generate more reliable results. Englund et.al. (1992) operate with data all the way back to 1861 and Backus and Kehoe mapped data from around 1870 up until today for ten different countries. To obtain reasonably long series, I have chosen to apply data for the period 1900 to 1992. Including monetary data from the late nineteenth century was found to not be satisfactory due to the inconsistent and poorly developed time series. In section 4, the historical data is presented, and the sources are to be found in appendix 3. A considerable amount of this part is focused on the problem of data inconsistency and whether the chosen series represent optimal indicators of monetary activity. The sources in use are extensively presented to enhance the data accessibility. In section 5, correlations between 7 different monetary variables and the reference indicator (real GDP) are presented and analysed on an annual level to establish some form of empirical regularities for the monetary part of the Norwegian economy. I emphasise the consequences for cyclical covariation between the monetary aggregates and the reference indicator when the filtering parameter value is changed. The covariations will be expressed as coefficients of correlation, and tested for stability when dividing the sample period in two subperiods. A small test of cyclical asymmetry is also applied. By analysing the skewness of the series, I explore whether the cyclical down-turns are more severe than the up-turns. Conclusions on how the variables covary are drawn and compared with the results from other studies listed in section 3.

Similarly to Kydland and Prescott, I do not intend to draw predictive conclusions. Neither do I claim any explanatory power. The purpose of the study is simply to obtain a set of cyclical regularities to which one may relate future as well as already existing results generated through theoretical simulations (e.g. business cycle modelling). With a well prepared data set, it is possible to test forecasting models retrospectively and hence obtain a more rigorous model building procedure.

[1] In section 5, results directly comparable to the listed regularities are presented. This way, Lucas' statements can be tested on the basis of Norwegian data.

2. Detrending and decomposition of the time series

To filter out a trend component, I have chosen to utilise the Whittaker-Henderson type A filter, first applied in economics by Hodrick and Prescott (1980). The filter is more commonly known as the Hodrick- Prescott filter and has later been applied in a large number of empirical studies primarily in the industrialised countries[2]. The filter is designed to produce a non-linear trend based on the variability of the series through time and can be presented in the following way:

(1) $$\min_{\{d_t\}}\left\{\sum_{t=1}^{N}(y_t-d_t)^2+\lambda\sum_{t=3}^{N}\left[(d_t-d_{t-1})-(d_{t-1}-d_{t-2})\right]^2\right\} \qquad t=1,\dots,N$$

where y_t is the aggregate time series, d_t is the trend component. One may define the cyclical component (z_t) is the deviation between these two elements (i.e. y_t - d_t).The filter consists of two parts. The first aims at minimising the squared deviation between the contemporaneous trend values and original values. Where as the second part penalises variation in the growth rate of the trend component. A weighting parameter is multiplied to the second term and determines what weight one assigns to the two presented properties. An interesting property of this filter is captured in the results generated by varying the λ value. If λ is set to infinity (or a sufficiently large number), the first term is ignored, so that the filter by virtue becomes a linear best fit. On the other hand, if λ is set to zero, the minimisation operator will minimise the first term unconditionally and we obtain a trend that is exactly equivalent to the original time series. In Figure 1. under appendix 2, I have depicted three alternative trends, the most variable one has λ value = 0 and is to be interpreted as the original series. The non-linear but smoother trend has λ-value = 200 whereas the almost dotted linear trend has λ-value = 800000.

Largely due to the simplicity of the algorithm, it is a rather easy task to control for variations in z_t when adjusting the λ-value. Due to this pivotal aspect of the analysis, I have chosen to map the salient data using multiple λ-values and hence, control for possible sensitivity in the results (see section 5). It is however, also necessary to approach the selection of a λ-value from a more theoretical perspective. In this respect, I present two arguments observed in the referred studies. Hodrick and Prescott (1980) chose to interpret the parameter λ as a measure of the relative variance in the cyclical component z_t and the second difference of the trend component [i.e. $\Delta^2 d_t=(d_t-d_{t-1})-(d_{t-1}-d_{t-2})$]. Assuming that $\Delta^2 d_t$ and z_t are - identically and independently normal distributed - $IIN(0,\sigma_1^2)$ and $IIN(0,\sigma_2^2)$ respectively, the authors claimed that equation (1) became a measure of the conditional expectation of d_t given y_t where λ is determined by the mentioned variances and can be expressed as[3]:

2 King and Rebelo (1993) present a rigorous analysis of the filter and its properties both from a time and frequency domain perspective.

3 For a more technical description of their interpretation, see Hodrick and Prescott (1980). My presentation is based on the somewhat more verbal analysis in Dantine & Girardin (1989).

(2) $$\lambda = \frac{\sigma_1^2}{\sigma_2^2}$$

Intuitively, this interpretation is appealing with respect to simplicity. Given the original series {y_t} and its variance, the expectation of the trend value {d_t} will only depend on a restriction on its own variance σ_1^2, which again determines the variance σ_2^2 of the cyclical component $z_t = y_t - d_t$. The λ parameter represents exactly such a restriction in equation (1) and may by this property be interpreted as how much one allows the trend component to vary compared to the variance of the original series. Hence, the minimisation problem in (1) becomes a measure of the expected value of the trend component conditional on the original series and the more subjectively determined λ value. As the variance of the second difference of the trend component goes to zero (as when one operates with linear trends), the λ-value goes to infinity. If $z_t = 0\ \forall t$, we have a case where the variance of z_t equals zero and the λ-value becomes zero (stating equivalence between the trend and the original series). To determine an operational λ-value one must apply an a priori restriction on the variance of the cyclical component as well as the trend component (i.e. an *ex ante* restriction on the relative sices of the two above mentioned variances). Hodrick and Prescott claimed that a 5% deviation from trend per quarter is a maximum of what is reasonable to expect. In addition, they believed that a maximum restriction on the quarterly variance of the second difference of the trend component could be set to 1/8 % (remember that their analysis was based on quarterly data). The values set into equation (2) yield

(2') $$\frac{(0,05)^2}{(0,00125)^2} = \frac{0,025}{0,000015625} \approx 1600$$

which is the value that Hodrick and Prescott have chosen to apply in their quarterly study. If one allows the second difference of the trend to vary four times as much in an annual setting without changing the restrictions on the cyclical component, one will generate a λ-value of 100, a value that is close to what usually has been applied in annual studies, see e.g. Englund et.al. (1992), Backus and Kehoe (1992), Correia et.al. (1992).

Obviously, this procedure confronts some important weaknesses both with respect to the correctness of the a priori beliefs and the assumption of identical and independent normal distribution[4]. The dependency on predetermined norms or beliefs of what business cycles look like seem to be the critical part of applying the filtering method. The only way to avoid this problem is probably to adopt a statistical model with a cycle generating mechanism, but even such a model could be criticised in a similar way as long as it depends on an a priori determined structural framework. As will be mentioned later, Englund et.al (1992) argued that there is no alternative to taking an a priori stand, and that agnosticism leads nowhere. They stated the following: "*To study business cycle fluctuations, it is necessary to take an a priori stand on exactly what one means with business cycles. Further, it may be the unwillingness to*

[4]Empirical evidence - see e.g. Danthine and Girardin (1989, pp. 37-39) - show that neither of the two measures tend to be normal in distribution, but even so, the interpretation of Hodrick and Prescott still serve an explanatory purpose on the intuitive level, which helps the user to develop a critical insight to the fundamental problems of the method.

take such an explicit stand that has led some researchers to the view that business cycle facts regarding the comovements between different variables are typically not robust."(pg. 355). Applying a filter with λ-value = 400, the authors argued - as opposed to Hodrick and Prescott - in a somewhat different way when determining this value. By applying spectral analysis, transforming the filtered data into the frequency domain with a Fourier transform[5], they were able to filter out all cycles that follow other frequencies than the one they had defined as the salient one. Englund et.al argued that frequencies shorter than three years and longer than eight, do not evolve from a business cycle mechanism, and that the average cycle frequency is 5 years. The higher frequencies are explained as consequences of wars, crop failures, strikes or other noise components. However, they did not explain from where the lower frequency results could possibly originate. After filtering out all other frequencies, they found 25 cycles over the period of 128 years, a result that corresponded perfectly with the a priori beliefs ($128/25 \approx 5$). On this basis, the mentioned λ-value was chosen and applied in the study. Instead of restricting the variance of the components, the authors imposed a frequencial limit of acceptance, and demonstrated simultaneously that there exists more than one way to argue for a specific value.

In the analysis in section 5, I have chosen not to argue for a specified λ-value, but rather to test the commonly used parameter value 200 against the higher value of 1600 which has been presented as a more reasonable value when operating with quarterly data. For reasons of curiosity, I have also chosen to present cyclical results when applying a near linear trend (i.e. λ=800000).

3. A short presentation of previous studies on business cycle regularities using the Hodrick-Prescott filter

After the paper by Hodrick and Prescott (1980), there has been published a large number of surveys mapping business cycle regularities for different countries over multiple spans of time. Seen from an international perspective, the paper by Backus and Kehoe (1992) is an important one. In addition to reporting stylised facts for Norway - including the analysis of M2 and prices - they map regularities for 9 other countries. The sample period is set to a minimum of one hundred years, implying that most of the series start before 1890. The authors report regularities for three separate periods to emphasise the aspect of stability over time. The three periods are: Pre-war (up till 1914), Inter-war (1920-1939) and Post-war (1950-1988). Using a λ-value equal to 100, Backus and Kehoe analyse contemporaneous correlations between real GNP and its main components (consumption, investment, government spending and net exports), as well as prices and the money stock. The respective definitions assigned to each country vary significantly, mainly due to the long time horizon and diverging methods of

[5]I have chosen not to present a theoretical analysis of the filter properties from a frequency domain perspective. For more on analysis in the frequency domain and the application of the Fourier transform, see Judge et.al. (1985, pp.314-315).

accounting[6]. I have found it most interesting to compare the cyclical patterns in Norway, Sweden, UK, and the United States. Real output for Norway, Sweden and UK is defined as the log of real GDP. In the United States it is defined as the log of real GNP. The price data for Norway is obviously consumer prices, the only price index in Norway that goes back to the mid nineteenth century. When comparing the results of Norway with the other three countries, there seems to be few qualitative differences (i.e. having a correlation carrying the opposite value of the others). It is a general tendency that the volatility (i.e. the measure of standard deviation of the series) of real output has been dampened after the World War 2. However, the short time span related to the interwar period might provoke a higher standard deviation than what is reasonable.

In most countries, one seems to observe a large negative contemporaneous correlation between real output and prices (usually defined as the consumer prices or the GDP deflator) in the post-war period. In contrast, the correlation was just as significant in the pre-war period in Norway, yet with the opposite sign (procyclical). It is interesting to observe that this phenomena has been most significant in Norway. Whereas the money stock is highly procyclical (positively correlated with real output) for the whole period in the United States, the money stock is probably contemporaneously acyclical (no cyclical pattern) in both Norway and Sweden.

If these results are good imitations of the "real" cycles, the conclusions will partly support Lucas' regularities (as stated in the introduction) with respect to cross country similarities, but contradict the belief that prices are pro cyclical and that cycles are equal over time.

There has been only a limited amount of analysis devoted to autoregressive properties of the series under study in this paper. In the paper by Kydland and Prescott (1990), quarterly data were used to report post-war regularities with a λ-value equal to 1600. The authors looked for a possibility of higher significant correlations when allowing the variables to be lagged or leaded. Although the survey only covers regularities in the United States, it is interesting to compare the results with the previous study, both because the λ-value is significantly different and because the authors apply annual and quarterly data, respectively. There are some striking similarities between the two studies. First, the contemporaneous correlation between real output and both prices and the money stock seem to be insensitive to the choice of annual or quarterly analysis, basically because the results reported in the two studies are relatively similar. These results may also support the theoretical considerations on whether to choose a larger λ-value when applying data with a higher frequency. However, the introduction of lagging and leading variables may indicate that the methods used by Backus and Kehoe could be insufficient with respect to quantitative accuracy. It is shown that both M2 and the price indexes achieve highest correlation with GNP lagged two periods. In this respect we say that such variables lead the cycle. If this is a general tendency, Backus and Kehoe might

[6]For an extensive presentation of the problems of data comparability, see Backus and Kehoe (1992) section 1 and appendix A. These problems will also be discussed under chapter 5.

have underestimated these correlation coefficients, not only for the United States, but potentially for the other countries as well.

Danthine and Girardin (1989) and Danthine and Donaldson (1993) are apparently the only students of the cyclical behaviour of interest rates and real output. In Danthine and Girardin (1989), we find a report on such regularities for Switzerland based on quarterly data from a rather limited period (1967:1 to 1984:3). They do however, compare these results to estimates from the United States (1950:1 to 1979:2), but are generally more concerned with testing different filter parameter values. Series filtered in four different ways (λ=0, linear trend, quadratic trend and λ=1600) are compared with American results obtained by applying a λ-value = 1600. An interesting result appears when comparing the results of the two first detrending methods with the regular Hodrick-Prescott filter parameter value. Looking at the price variables, it is striking to observe that the contemporaneous cross correlations with GNP is highly sensitive to the choice of detrending method. Consequently, Danthine and Donaldson (1993) claim that Kydland and Prescott (1990) failed to test their stylised facts in a sufficiently rigorous way. And that their contradiction of Lucas' cyclical price regularities may not be as convincing after all. However, the two first detrending methods presented by Danthine and Girardin (1989) are not regarded as appropriate tools, as they generate far too few cycles (In my data, a linear trend normally generate between 2 and 4 cycles over the century, a number that is unreasonably low). Danthine and Donaldson (1993) present stylised facts for monetary data from 11 different countries (not Norway).The sample period rarely extends 30 years, and by using quarterly data, they ignore to test their results against parameter sensitivity as was done in the 1989 paper. The authors find a general absence of strong regularities. They only find a tendency of more variable short term interest rates than long term interest rates. Furthermore, they say: *"Although short rates are generally positively correlated with output, no systematic pattern is observed vis à vis long rates....As to the correlation with output of both monetary measures, these are generally positive, with two exceptions in the case of M1, four in the case of M2."* (Danthine and Donaldson (1993) pg.10).

Although some surveys have been concerned with the cyclical properties of money, prices and interest rates - as presented in the last section - there is a surprising absence of focus on the covariation of interest rates and credit volume with GDP/GNP using series with observations back to the beginning of this century[7].

One may get the impression from the outlined surveys that empirical evidence support the hypothesis emphasising the non-existence of monetary links to the real economy. In the next two sections, I will focus on whether this is true in the case of Norway.

[7] To find a more extensive analysis of these mechanisms, one has to seek outside the literature where the Hodrick-Prescott filter has been applied. Benjamin M. Friedman (1986) presents a rigorous search for empirical covariations between the variables mentioned, but chooses to approach the problem using a vector autoregressive (VAR) model.

4. The sample data[8]

In this study, annual data form the basis for the analysis. This choice is based on the fact that very few economic variables for Norway were listed on a quarterly basis before 1950. Because large economic fluctuations can occur within a year, the annual perspective may ignore important fluctuations of a higher frequency. It is, on the other hand, convenient to avoid the problem of seasonal variations. Keeping this in mind, it is necessary to mention that the annual data are based on records not always gathered from the same season. Thus, even though the filtering procedure smoothes the data, it is not correct to claim that the seasonal problem is completely avoided. For instance, if half of the observations in a series is registered in June and the other half in December, one may expect seasonal disturbances whenever there exist a tendency of activity to peak around one of the dates mentioned. The more present and systematic these variations are, the less will the filter be able to adjust for seasonal disturbances.

Due to the long time horizon, I have been forced to employ a wide variety of reference sources to obtain continuos series. Some of the variables were rather easy to obtain (i.e. GDP and Consumer price indices), yet for most of them it has been necessary to use more than one source. Even though Statistics Norway (1968, 1978) cover most of the data analysed in this study, these sources fail to provide annual observations for the whole period. Either, these statistics report on a biannual or lower frequency for the beginning of this century, or the data are not available at all and must be searched for in other publications. As a consequence of the monetary focus, the study by Skånland (1967) has become an indispensable reference. When analysing time series composed from different sources, the problem of data inconsistency becomes inevitable. It has been my primary priority to obtain as consistent series as possible, but such a strategy necessarily restricts the number of available variables. Thus, what could have been regarded as an optimal indicator of a specific monetary activity is lost in the search for a more time consistent variable. Below, there will be mentioned a number of examples where the chosen variable is consistent in time but inferior to other variables as an estimator of activity.

I have chosen to include both war periods (1914 - 1918 and 1940 - 1945). In other studies (Backus and Kehoe (1992), Englund et.al (1992), Correia et.al. (1993)) the authors have excluded these period based partly on the lack of relevant data and partly on the highly irregular volatility in the variables as a result of extreme economic conditions. For Norway, the only variable omitted during this period is GDP and its components. I will however, argue that it is possible to generate reasonably good estimates of GDP, and that including data for this period may render more consistent trend estimates when applying the HP-filter. Due to the complex task of including these periods, I have chosen to discuss this problem in a separate section below.

[8] All sources are listed in appendix 3.

For reasons of structure and reader accessibility, I present each variable subsequently as listed in Table 1 through 6 in appendix 3 [9]. Real monetary variables are equivalent to nominal variables adjusted with the consumer price index, and need consequently no further explanations (original data are listed in appendix 1.).

Gross Domestic Product

The first consistent procedures of national accounting in Norway were developed in the nineteen thirties and forties. Statistics Norway applied these procedures to estimate GDP for two periods (1900-1929 and 1930-1951) using the less systematic data collected through these respective periods. From 1900 to 1927, no annual production statistic seem to exist, but sporadic lists of production volumes in the largest sectors of the economy are used to construct the aggregates. Trade statistics for Norway are available on an annual basis far earlier than 1900, and these numbers were considered as important information in the estimation process. Backus and Kehoe (1992, pg. 869) claim that the GDP data for Norway before 1927 are somewhat unreliable compared to other major industrialised countries. In Statistics Norway (1953, pp. 85-87), it is stated that the data for the period up till 1929 is especially narrow with respect to information, and that the period from 1915 to 1921 is most problematic due to the war economy and the vast price fluctuations referred to as "Dyrtida". The authors claim however, that the estimates give a reasonably good picture of the economic activity during this period. From 1930 onward, the production statistics are considered good according to international standards, see Statistics Norway (1978, pp. 88-91). As a result of a general reform of the national accounting system developed by the United Nations, some revised definitions were implemented in 1978. However, these changes are not believed to significantly alter the consistency of GDP, but could impose larger relative effects on the size of its components.

Consumer price index

In 1916, the first cost of living index (Laspeyre's formula) was published including estimates dated back to 1901. The calculations were based on the consumption patterns of working class families in the 6 largest cities during 1912 and 1913, see Statistics Norway (1968). In 1960, this index was changed to a consumer price index representing the whole population. The consumer price index may not be the best indicator of the general changes in the price level. The GDP-deflator generated by the estimates of GDP measured at market prices will naturally cover the development of prices in a broader way. However, estimates of the GDP-deflator can not be found for the period before World War 2, leaving CPI as the only applicable measure.

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9 Estimates of population growth are presented in appendix 1, and utilised in pr. capita measures. The numbers are found in any Statistical Yearbook published within the last ten years. Figures are estimated population means adjusted for updating censuses.

Credit Volume

From 1900 to 1956, the figures are based on loans offered to the public (non financial institutions, municipals, private persons and enterprises). The figures are believed to be robust with a few exceptions (for the following comments, see Skånland (1967, pp. 32-33)). Government balance sheets for financial transactions before 1930 are obsolete. Neither are these figures reliable during the World War 2. There are also some weaknesses with respect to the figures covering foreign transactions, but it is not clear to what extent these weaknesses make the data less reliable. In the period 1957 - 1975, the statistics also include loans to state enterprises, a change that probably affects the data consistency, presenting to high estimates for this period. In addition, loans from private financial corporations are included from 1965, altering the norm even further. In the subsequent sources, the data are believed to be consistent with the 1965 norm and needs no further specification. The chosen indicator for credit activity is the widest available. One could imagine that some parts of this market were more volatile than the aggregate measure. However, since the credit market was strictly regulated up till 1983 (i.e. both with respect to interest rates and credit volume), and little excess supply of credits has been observed (see Eide (1977)), there is reason to believe that only a few markets have operated with higher interest rates to capture the excess demand for capital. Although it is widely known that such markets emerged at the end of the seventies, there is no evidence of their existence earlier in this period and hence, the idea of a more cyclically volatile sub-indicator may turn out to be improper. On this basis, the wide measure of credit applied in this study may not significantly differ from alternative measures when comparing their cyclical properties.

M1 / M3

The definitions of monetary aggregates are taken from Isachsen (1976, pg. 27). The only difference is related to unused "cash credits" or credit lines in banks (i.e. unused credit arrangements offered by banks to private persons or enterprises). These figures are not available for the first part of the period and are consequently excluded from the analysis. Furthermore, deposits on savings accounts are treated as a part of M3 (this measure is only defined in M4 in Isachsen (1976)). The two monetary measures include the following statistics. M1 is composed by currency in circulation and demand deposits in commercial banks, savings banks and Postgiro. M3 is defined as M1 plus deposits on savings accounts (time deposits) in commercial and savings banks[10]. The advantage of presenting two monetary measures is captured in the potential differences in cyclical behaviour. If M3 is found to be more highly correlated with real GDP than is M1, a reasonable explanation would be to define deposits on savings accounts as a highly cyclical measure. Of course, this argument could be reversed to reveal the cyclical properties of the M1 components. The Bank of Norway produced extensive monetary statistics during the whole period, based on reports from banks and other financial

[10]M2 equals M0 - which is an even more narrow money measure than M1, as it excludes wage accounts - plus credit lines which I could not include in the analysis.

institutions. This activity was not reduced under the two wars and thus, there is no reason to believe that the figures are less reliable during these periods. An interesting phenomenon can be observed during the World War 2, when the money supply was increased significantly to stimulate the economic activity. As I will further discuss in section 5, this action may have contributed to the partly insignificant correlation of the aggregate money measures and real GDP. In 1943, Postgiro (the Norwegian postal transactions bank) was introduced, offering financial services considered to be a new way of banking. As a result, numbers obtained from Postgiro were processed in a separate account when calculating the monetary aggregates. Acquiring data from Postgiro has become the only post-war problem of the analysis. Apparently, for the period 1974 to 1979 it is hard to find any sources that separate regular demand deposits from tax withholding accounts in Postgiro. To solve this problem of omitted data, I have been forced to construct numbers for these 6 years by linear extrapolation from the 1973 figures to the 1980 figures. Naturally, one could have applied some statistical tools (like a Kalman filter) to generate more plausible results. However, considering the limited number of omitted observations, one may question whether such a procedure will significantly improve the estimates. Both demand deposits in commercial and savings banks and currency in circulation followed a near linear trend with low volatility during this period. Given these regularities, it seems plausible to assume a similar pattern for the omitted observations.
For deposits in general, there were some small revisions of the figures for the first half of the century based on statistical norms stating that the relative size of demand deposits compared to time deposits followed a trend that was not observed in the actual data (for more on this, see Skånland (1967, pp. 216-217)). From 1988 to 1991, the only figures published present money growth and its components. Numbers for these years are consequently added to the 1987 figures to obtain data expressed in totals.

Although the quality of the monetary data presented can be considered as relatively good, one may question how suitable they are as indicators of monetary supply. During the sample period, the financial markets have undergone vast changes, maybe most significantly in the non-banking sector. Because of the rapid growth of such institutions, the time series applied in this study may underestimate the activity of the later years. However, once again I have faced the dilemma of choosing between consistency and rigor, giving consistency the prior emphasis. For instance, statistics on savings in insurance companies are partly unreliable and partly obsolete for the earlier years. Thus, including this component would unquestionably worsen the quality of the data.

Interest on Deposit Accounts

Before presenting the interest measures, it is necessary to emphasis that all interest rates as well as credits were strictly controlled by the government up until 1977. The average interest on bank deposits between 1900 and 1943 is calculated by dividing interest payments on savings accounts from the Norwegian savings banks with the total amount of savings. Skånland (1967, pg. 217) claims that interest on demand deposits is insignificant and needs not to be included.

Obviously, these numbers are not completely reliable and there is reason to ask why the estimates are based on figures from the savings banks and not the commercial banks which represented a much larger part of the banking sector. From 1944, the numbers are based on interest reports in the Bank and Credit Statistics published by Statistics Norway. The figures are in nominal values, account only for deposits in commercial and savings banks and are average estimates. From 1975 and on, observations are based on interest offered at the end of the fourth quarter, registered by the Bank of Norway. Interest on deposit accounts as a variable is included in the analysis to represent short term interest rates. The idea is to compare how this variable covary with real GDP compared to how long term interest rates behave.

Interest on Norwegian Bearer Bonds

There is not much information obtained in the listed sources regarding consistency and quality of the estimate. Figures are in nominal values and do not include government bonds. The numbers are estimated averages based on annual registered values at Oslo stock exchange. When mapping long term interest rates, one must be aware of the importance of the remaining running time on the issued bond. Klovland (1976) presents a study on the effective interest rate of Norwegian bearer bonds and government bonds. He claims that the results are affected significantly if the problem of evaluating interest on bonds with different remaining running time is ignored. By implementing a model to estimate the actual interest rate based on profit curves, Klovland finds the numbers used in Isachsen (1976) to be biased and suggests simultaneously a way to avoid it. Due to limitations of space and focus, I have chosen not to take Klovland's critique into consideration. Consequently, the data set is not a perfect replication of long term interest rates, but must be considered an approximation.

The Bank of Norway's discount rate

This interest rate has been changed unsystematically through the history. To solve this problem, I have chosen to register the value at the end of each year (For some periods, the discount rate stood unchanged for several years). Hence, whereas the other variables are estimates of annual averages, these numbers are only observed at the same period of the year. Combining time series generated through different methods of calculation might reduce the comparability and the empirical value of the obtained results.

The numbers are completely reliable and we face no problems of time consistency. The discount rate is regarded as the main cause for changes in other interest rates and must by this property be regarded as a solid indicator of the interest rate policy in general.

Estimates of GDP for the period 1940 to 1945

Because Norway was only partly affected by the World War 1, most of the statistical work include these years. The period does not seem to be more volatile than the previous and following periods and the data quality is not considered to be inferior. As earlier mentioned, no official production statistics have been published for the World War 2. However, there exists a survey presenting some estimates of GNP during these years, see Ministry of Finance (1946) and Statistics Norway (1946). Based on statistics covering 13 broad sectors of the Norwegian economy, the gross domestic product as well as the gross national product (GNP) is calculated by summation. Shipping and whaling - two relatively large sectors at that time- are excluded from the statistics. The figures for 1940-1943 can be regarded as relatively reliable, but estimates for 1944 and 45 are not of the same quality. It would be too large a task to present weaknesses of estimation within each of the 13 sectors. For readers with special interest in this period, I will refer to the sources mentioned above. Because we operate with GDP, there is no reason to study the problem of estimating real capital depreciation. One would assume an unusually high rate of depreciation during the war, which again would affect the early post-war production potential. By rebuilding the country and its capital stock, the GDP grew in 1946 to almost twice its own size in 1939 (see appendix 1). Consequently, if one is determined to exclude the war period, one easily gets into problems of discontinuity. The filtered series will unquestionably be affected by the removal of this period. There is a chance that some of the cyclical downturn from this period would be transformed into changed trend values in an earlier or later period, distorting the general cyclical pattern. The economic as well as the monetary regime has changed along the whole century. Hence, if one as a researcher chooses not to consider institutional changes, one should not exclude the period of war purely on the basis of extreme institutional conditions. In my opinion, the World War 2 had a large effect on the post-war economy, and must be treated as an element of the cycle generating force.

5. An analysis of Norwegian monetary business cycles

Based on the time series presented in the previous section and the Hodrick-Prescott trend filtering method, I present data on the covariation of different monetary aggregates with real GDP, applying three different lambda values (200, 1600 and 800000). The covariation will be expressed as coefficients of correlation to obtain a unified and comparable measure[11]. Real GDP is measured in pr. capita terms to adjust for possible large fluctuations in the population. All variables except for the interest rates are expressed in natural logarithms[12]. The series presented as percentage deviation from trend (lambda value equal to 200) are depicted for both real and nominal values in Figure 2 to 12 in appendix 2. (Interest measures are only in nominal values). In Figure 2, I have compared the deviations from trend in real GDP pr. capita, when

[11] The correlation coefficient ρ is estimated in the following way:

$$\rho = \frac{Cov(X_t, Y_t)}{\sqrt{VarX_t}\sqrt{VarY_t}}$$

Where X and Y are the cyclical components of the spesified time series. By adjusting for the variance of each time series, one obtains a measure ρ that can take values defined within the range +/- 1. $\rho = 0$ implies no statistical correlation, where as $\rho = 1$ is a result of perfect correlation (or one to one linear relationship between the variables).

[12] On the basis of the discussion in appendix 4, this log transformation is not strictly necessary as the filter renders linearity in itself. However, I have wished to follow the tradition presented in earlier studies of this kind where log transformation of time series seam to be commonly applied.

Time series of more detailed components have been stored on EXCEL sheets and are available upon request.

applying the more commonly used λ- value = 200 and the near linear filter (λ-value = 800000) presented as the dotted line. The linear filter generally renders larger fluctuations with less cyclical features. Whereas the non linear filter generated 8 peaks and 8 troughs, the linear filter only produced 2 of each[13]. It is interesting to observe that the period between 1957 and 1964 is estimated to be a period of weak recession of about the same amplitude as the recession of the early seventies. Traditionally, this period has been treated as a part of the post war stabilised growth period (Hodne and Grytten (1992, chapter 12)) and certainly not viewed as a period of slight recession. In fact, the average growth rate of this period was noticeably lower than in the previous and following periods. In this way, a period of positive growth can in fact be regarded as a recession when applying the HP-filtering method. This necessarily opens for a more flexible interpretation of what booms and recessions are all about. Another interesting feature observed in Figure 2, is the relatively weak recessionary tendencies during the nineteen thirties. This observation is, however, not that controversial, considering what historians would call a mild recessionary hit compared to other countries. The thirties were considered to be hit by a strong recession, not simply because of the reduced production, but more due to the huge fluctuations in unemployment rates[14]. It must be mentioned that this time specific property is not robust to changes in the λ-value (the dotted line indicates a most significant recession in the thirties).

Some striking regularities can be found by comparing the figures. First, M1 and M3 as well as credit volume and consumer prices, show significantly less volatility in the post war period (all observations are based on λ=200). Secondly, all three interest series show sharply increased volatility from 1970 and on. The regularities can be verified by looking at the volatility estimates in Table 6. These observations are consistent with both the assumption that economic policy has been more stabilising after the World War 2, and the notion that the liberalisation of capital markets during the last two decades, may have generated more flexible interest rates. In dynamic macroeconomic theory it is generally believed that variables may depend on lagged values of other variables and that excluding these causal relations may generate incorrect results. To meet these propositions, I have tested the correlation coefficients for lagged and leading values of GDP (i.e. +/- 4 years). If a variable is most closely correlated with lagged GDP, we say that it lags the cycle. If the opposite is true, the variable is leads the cycle. For instance, if there is a high correlation between M1 and real GDP lagged one year, this indicates that a change in M1 will tend to affect real GDP one year later. In Table 1 to

[13]The peaks and troughs are measured as periods of positive and negative deviation fro trend respectively. It is implicitly assumed that a peak must be lead and followed by a trough and vice versa. Although this is no satisfactory measure of cyclical patterns, it is still a convenient description of how the lambda value alters the measures of deviation. The cyclical pattern presented in this way can be read directly from Table 2 by regular counting.

[14]Even the unemployment estimates have been questioned with respect to validity. Hodne and Grytten (1990 pp. 149-152) show that the estimates most frequently used (the traditional unemployment data) are based on statistics covering the number of unemployment workers in ten labour unions. This sample represented only 8 percentage of the total labour force and is believed to have been much more sensitive to cyclical variations, and hence, yielding unreasonably high rates of unemployment. A census in 1930 showed that 9 percentage of working force was unemployed, whereas the traditional statistics presented an unemployment data of 16,6 percentage points. Based on these observations, the authors claim that the traditional presentation of the labour market problems in the interwar years must be significantly exaggerated.

Table 3, I have presented the estimates using three different parameter values to test the sensitivity of the filter.

Table 1:
Cross Correlation Coefficients with Real GDP (Lambda = 200)

Variable:	Lag length of Real GDP y(t-4)	y(t-3)	y(t-2)	y(t-1)	**y(t)**	y(t+1)	y(t+2)	y(t+3)	y(t+4)
Real GDP	-0,2241	0,0157	0,2739	0,6418	**1**	0,6418	0,2739	0,0157	-0,224
Consumer Price Index	-0,2036	-0,351	-0,3926	-0,3481	**-0,2295**	-0,012	0,1385	0,254	0,3472
Credit Volume	-0,4315	-0,308	-0,1038	0,1732	**0,4646**	*0,6193*	0,5762	0,4358	0,2433
Real Credit	-0,3091	-0,029	0,2452	0,5352	**0,7591**	*0,7334*	0,5362	0,2574	-0,569
M1	-0,1056	-0,249	-0,3538	-0,3639	**-0,2018**	-0,047	0,0203	0,1043	0,18
M3	-0,0003	-0,14	-0,2712	-0,3178	**-0,2107**	-0,081	0,0228	0,0873	0,143
Real M1	0,0014	-0,11	-0,2403	-0,2947	**-0,1641**	-0,089	-0,0814	-0,0306	0,0213
Real M3	0,2811	0,2374	0,0649	-0,0736	**-0,051**	-0,122	-0,1535	-0,2056	-0,245
Interest on Deposit Accounts	-0,1502	-0,108	-0,0331	0,051	**0,0409**	0,1456	0,129	0,0921	0,1459
Interest on Bearer Bonds	0,0203	0,0412	0,0796	0,1133	**0,1255**	0,0147	-0,0753	-0,1136	-0,181
Bank of Norway's Discount rate	0,0791	0,0932	0,0697	0,0432	**-0,0256**	-0,094	0,0077	-0,0231	-0,078

Keep in mind that I will refer to the correlations based on λ-values =200 as the base figures, and that the results in Table 2 and Table 3 are meant purely to be treated as tests of coefficient stability[15]. For contemporaneous correlations with real GDP the results in general yield few significant signs of covariation. But the results give reason to believe that credit volume, both for real and nominal values, is positively correlated with contemporaneous real GDP. This pattern also holds when tested for alternative parameter values. The data in Table 1 may indicate that nominal credit volume may lead the cycle with one year. As mentioned in section 3, prices tend to be counter cyclical for the whole period and may lead the cycle with a year, but not on a very significant level. In Table 6, it is shown that post-war correlation between prices and real GDP is somewhat higher than pre-war and the whole period seen as one, a tendency that seems to be apparent in most of the western world, see e.g. Danthine and Donaldson (1993), or Backus and Kehoe (1992). Backus and Kehoe (1992) find a higher negative post-war correlation for Norway (-0,63) than I have presented in Table 6, yet the pattern is the same. The differences in magnitude may arise from different λ-values (see section 3) and from the fact that their sample period is somewhat shorter. Just as in Danthine and Girardin (1989), the GDP-price correlation in Norway is by no means insensitive to changes in the filtering parameter value. Estimates in Table 3 show an even higher correlation yet with the opposite sign when applying a near linear filter. The most unambiguous results are obtained for the three series on interest rates. None of them show significant correlation with real GDP, not even with a near linear trend will the best contemporaneous result yield more than a correlation of about 0,3. The tendency of low correlation with respect to interest rates has been verified in

[15]All tables present correlation coefficients without measures of significance. For the interested reader, these results are available upon request.

industrialised countries. For none of them are the correlations registered to be larger than 0,38 in absolute value and they vary significantly around zero. Hence, in this respect interest rates in Norway show the same pattern. Figures for the monetary aggregates M1 and M3 seem to indicate a certain extent of counter-cyclicity. For both measures correlations with GDP are the highest when lagged one period, indicating that they lag the cycle. When expressed in real terms, the correlations tend to loose significance, and they are highly sensitive to changes in parameter values. However, the counter-cyclical properties remain noticeable with a λ-value of 1600, which may be interpreted as a confirmation of the results in Table 1. Post-war correlations between M1 and real GDP are close to zero (see Table 6). Backus and Kehoe find almost the same post-war correlation as I do, but operate with a different definition of the pre-war period, producing completely different results. When splitting the series into two sub-periods, the only variable carrying some significance is nominal M3 during the post-war period.

Table 2:
Cross Correlation Coefficients with Real GDP (Lambda = 1600)

Variable:	Lag length of Real GDP y(t-4)	y(t-3)	y(t-2)	y(t-1)	y(t)	y(t+1)	y(t+2)	y(t+3)	y(t+4)
Real GDP	-0,041	0,197	0,4352	0,7336	**1**	0,7336	0,4352	0,197	-0,041
Consumer Price Index	-0,119	-0,208	-0,221	-0,1783	**-0,0879**	0,0534	0,1575	0,238	0,2982
Credit Volume	-0,193	-0,072	0,0967	0,3051	**0,5064**	0,6202	0,607	0,5184	0,3813
Real Credit	-0,132	0,1249	0,3667	0,6003	**0,7687**	0,7624	0,6293	0,4234	0,1759
M1	-0,051	-0,161	-0,236	-0,2507	**-0,1616**	-0,072	-0,0259	0,031	0,0827
M3	0,0635	-0,029	-0,108	-0,1384	**-0,0869**	-0,0204	0,0349	0,07	0,0984
Real M1	0,0208	-0,91	-0,208	-0,2733	**-0,2183**	-0,1863	-0,1851	-0,151	-0,109
Real M3	0,3406	0,2804	0,1281	-0,0103	**-0,0484**	-0,1388	-0,1951	-0,256	-0,298
Interest on Deposit Accounts	-0,16	-0,903	0,0081	0,0815	**0,146**	0,2432	0,2543	0,2396	0,2715
Interest on Bearer Bonds	0,096	0,0652	0,0393	0,0186	**-0,0153**	-0,1192	-0,2039	-0,248	-0,304
Bank of Norway's Discount rate	0,0866	0,0756	0,0301	-0,0137	**-0,0899**	-0,1667	-0,1143	-0,136	-0,181

Table 3:
Cross Correlation Coefficients with Real GDP - Near Linear Trend - (Lambda = 800000)

Variable:	Lag length of Real GDP y(t-4)	y(t-3)	y(t-2)	y(t-1)	y(t)	y(t+1)	y(t+2)	y(t+3)	y(t+4)
Real GDP	0,4088	0,5519	0,693	0,8589	**1**	0,8589	0,693	0,5519	0,4088
Consumer Price Index	0,1393	0,1328	0,1623	0,2142	**0,2849**	0,3735	0,4401	0,4916	0,5288
Credit Volume	0,2445	0,3223	0,4133	0,5127	**0,6138**	0,6824	0,7112	0,7143	0,6979
Real Credit	0,2625	0,3967	0,5251	0,6515	**0,7508**	0,7849	0,7705	0,7256	0,6602
M1	0,173	0,1085	0,0624	0,0536	**0,1016**	0,1373	0,1472	0,1647	0,1795
M3	0,2242	0,2268	0,2301	0,2563	**0,3104**	0,3657	0,4117	0,4452	0,47
Real M1	0,1215	0,0249	-0,078	-0,1529	**-0,1629**	-0,1885	-0,228	-0,245	-0,251
Real M3	0,3352	0,3484	0,3042	0,2643	**0,2644**	0,2488	0,242	0,2295	0,2293
Interest on Deposit Accounts	0,778	0,1306	0,1931	0,2509	**0,3158**	0,3796	0,4219	0,4581	0,5003
Interest on Bearer Bonds	0,0154	0,0636	0,1139	0,1723	**0,2364**	0,3087	0,3752	0,4294	0,4775
Bank of Norway's Discount Rate	0,0797	0,1161	0,1655	0,2194	**0,2883**	0,3461	0,3706	0,4128	0,4558

In Danthine and Donaldson (1993), only one country can refer to a higher correlation, but across nations this measure follows no specific pattern. When summarising the additional results from testing the stability of correlations with different parameter values, it is reasonable to state that the test only affected the stability of prices and partly M1 and M3. Hence, in the case of Norway, it seems that the choice of λ- value play a rather insignificant role.

In Tables 4 and 5, I have presented some estimates of the volatility and asymmetry of the series. In Table 6 the volatility is presented for two separate periods to test the stability of variation. The volatility is measured by the percentage standard deviation of the log of the variable and asymmetry is calculated by finding the skewness of the log of the variable[16] (the measure of skewness was also applied in Bergman and Jonung (1990) to test for asymmetry). Table 4 exhibit large differences in standard deviations. Prices have the largest fluctuation, but this is not the case for the post-war period. At the lower λ-values, M1 seems to be more volatile than M3, indicating that deposits on savings accounts may serve as a stabilising component. This pattern is also apparent in Table 6. Interest rates are generally associated with low volatility, but higher for the discount rate compared to the other two measures. As mentioned earlier, the volatility of interest rates has increased significantly since the pre-war period.

Real GDP and real credit are the only variables observed with negative skewness, implying asymmetric cycles giving relatively more severe recessions than booms. On the other side of the index, I find prices and real monetary aggregates to be positively skewed, with stability across λ-values. Although data presented in Table 6 have been presented earlier, it is necessary to emphasise that these data are primarily designed to test the stability of the coefficients over time. Of the ten monetary aggregates, four of them change signs of correlation, and 6 of them tend to stay qualitatively stable. Real credit and nominal M1 carry few quantitative changes over the periods and must be regarded as more stable than the others. In general, on the basis of the results in Table 6, one may conclude that monetary patterns are not observed to be stable over time. The conclusion can be interpreted as a serious criticism of the method in use, emphasising the absurdity in investigating correlations through times with no stability (i.e. any sub-period will yield different correlations).

[16]Skewness is a measure of how the observations are distributed around the mean (in this case interpreted as the trend), normally used in relation to whether a population or sample distribution is symmetric around it's mean (i.e. a normal distribution has zero skewness).
A measure of skewness can be given by (Alpha three measure of skewness):

$$\alpha_3 = \frac{E(X_t - \mu)^3}{\sigma^3}$$

Where μ is the mean of the series Xt and σ is the standard deviation.
In words, if $\alpha = 0$ the distribution is symmetric around the mean. If $\alpha > 0$, the right tale is elongated, and the opposite is true if $\alpha < 0$.The term in the denominator serves as a normalising parameter, transforming any value of skewness into a comparative range defined by the interval (-1,1). Applied on this survey, negative skewness implies that there exist stronger recessions than booms. The opposit will be the case if the sign is reversed. For more on this topic, see Kmenta (1986, pp.67-68).

Table 4:
Volatility of Variables (% Standard Deviation)

Variable:	Lambda value 200	1600	800000
Real GDP	1,233	1,4595	2,0174
Consumer Price Index	3,859	6,3797	10,776
Credit Volume	1,439	2,2694	5,8694
Real Credit	1,044	1,4422	3,0265
M1	3,283	5,2469	6,5292
M3	1,549	2,6	4,6626
Real M1	1,934	2,832	3,739
Real M3	0,7	0,9381	1,4318
Interest on Deposit Accounts	0,436	0,5831	1,9391
Interest on Bearer Bonds	0,469	0,6557	1,9545
Bank of Norway's Discount Rate	0,735	0,8062	2,1284

Table 5:
Measures of Cyclical Asymmetry (Skewdness)

Variable:	Lambda Value 200	1600	800000
Real GDP	-1,55	-2,22	-2,22
Consumer Price Index	0,84	0,98	1,21
Credit Volume	0,05	0,32	-0,14
Real Credit	-1,45	-2,13	-1,75
M1	0,55	0,57	0,47
M3	0,53	0,85	1,02
Real M1	0,91	0,82	0,48
Real M3	0,26	0,43	0,43
Interest on Deposit Accounts	1,16	0,8	0,45
Interest on Bearer Bonds	-1,17	-0,83	0,45
Bank of Norway's Discount Rate	-1,08	-1,41	0,94

Table 6:
Test of Coefficient Stability and Volatility of Time Series (l=200)

	Volatility (% stand. dev.)		Coefficients of Correlation with Real GDP (Contemporaneuos)	
Variable:	**prewar**	**postwar**	**prewar**	**postwar**
Real GDP	1,01	0,64	1	1
Consumer Price Index	5,56	1	-0,1372	-0,3692
Credit Volume	1,77	0,57	0,241	0,2987
Real Credit	0,84	0,53	0,642	0,4811
M1	4,29	1,11	0,0998	0,0266
M3	2,15	0,45	-0,0939	0,401
Real M1	2,32	1,13	0,2034	0,1112
Real M3	0,83	0,51	0,0318	0,52
Interest on Deposit Accounts	0,36	0,51	0,0719	-0,0232
Interest on Bearer Bonds	0,14	0,66	-0,2048	0,1658
Bank of Norway's Discount Rate	0,6	0,89	-0,117	0,2987

Prewar period is set to 1900 - 1939. Postwar period is set to 1946 - 1992.
Due to the high deviations during the period of the second world-war and problems of placing it under the two main periods, I have chosen to omit it in this analysis of stability.

Finally, it would be interesting to summarise the correspondence between my observations and the laws of cyclical movements expressed by Lucas (1977) and outlined in the introduction. First, prices are once again shown to be counter-cyclical directly contradictory to what Lucas thought. Second, short-term interest rates represented by interest on deposit accounts show no cyclical covariation with real GDP, and neither do long term rates measured as interest on bearer bonds. Thus, also this statement stands unproved. Finally, monetary measures (both M3 and M1) tend to be counter-cyclical in Norway, that is, if they do correlate with real GDP at all. There are no indications of a pro-cyclical pattern with respect to these variables.

6. Concluding comments

This study has given some indications on how the monetary part of the economy has covaried with real GDP during the last century. Based on the method developed by Hodrick and Prescott (1980), I have tested cross correlations between 10 monetary aggregates and the reference indicator real GDP. Alternative definitions on business cycles have been explored and compared, focusing primarily on Lucas' cyclical "laws" and the complex task of defining an optimal reference indicator. I have presented a theoretical consideration of the properties of the Hodrick-Prescott filter and elucidated the problem of defining an appropriate weighting parameter. Although the filter is a generalisation of earlier filtering approaches, it is still completely dependent on a priori restrictions levied on the properties of cyclical volatility as i Hodrick and Prescott (1980) or cyclical frequencies as in Englund et.al. (1992).

A survey on previous studies of business cycle regularities was presented to illuminate what could be viewed as international patterns of cyclical activity. Although prices tend to be counter-cyclical after the World War 2, and short rates are believed to be pro-cyclical, the reviewed papers give few reasons to believe that there exists an internationally systematic and uniform pattern with respect to monetary covariation with real GDP in general. In this study, only real credit volume is found to significantly covary with real GDP, and by the same time remain stable over the sub-periods. There is some evidence showing that the monetary aggregates M1 and M3 are countercyclical, but the evidence is not very robust and may even be insignificant when applying different filtering parameter values. Interest rates are shown to be acyclical under any circumstances, and prices tend to follow the international trend staying counter cyclical in the post-war period. However, data on prices show high sensitivity to filtering parameter changes. Generally, it is shown that monetary aspects of the Norwegian business cycle correspond to international patterns with a few exceptions. These exceptions are partly observed in the monetary aggregates M1 and M3 and partly in the short interest rates. When comparing the results of Norway with the regularities presented by Lucas, there seems to be few correspondences.

A large amount of work remains to be done to establish a qualitatively presentable chart of business cycle patterns. First, data from alternative periods must be presented to test the question of coefficient stability in a more rigorous way (e.g. an analysis where periods of war are excluded). Secondly, to construct a wider picture of the business cycle, alternative variables capturing other parts of the economy must be explored, such as the GDP components, unemployment, productivity, trade, competitive advantages, measures of supply and demand, etc. These steps are crucial elements in the qualitative updating of business cycle regularities in Norway.

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Appendix 1

The original data 1900 -1992. Annual observations

year	GDP (mill. kr)	CPI 1979=100	Mean population	year	GDP (mill. kr)	CPI 1979=100	Mean population
1900	1098	6,1	2230483	**1946**	11030	19,5	3126883
1901	1087	6	2254911	**1947**	12995	19,6	3165011
1902	1079	5,9	2275485	**1948**	14092	19,4	3201013
1903	1071	6,4	2287768	**1949**	15015	19,5	3234228
1904	1073	5,6	2297494	**1950**	16647	20,5	3265126
1905	1100	5,8	2308572	**1951**	20456	23,8	3295871
1906	1182	5,9	2319191	**1952**	22564	25,9	3327728
1907	1255	6,1	2328962	**1953**	22884	26,4	3360888
1908	1290	6,2	2345564	**1954**	24806	27,6	3394246
1909	1312	6,2	2367494	**1955**	26376	27,8	3427409
1910	1431	6,3	2383677	**1956**	29747	28,9	3459992
1911	1527	6,5	2400796	**1957**	31775	29,7	3491938
1912	1677	6,9	2423184	**1958**	31919	31,1	3522993
1913	1854	7,2	2446874	**1959**	33946	31,8	3522854
1914	1916	7,2	2472419	**1960**	36101	31,92	3581239
1915	2593	8,2	2497766	**1961**	39245	32,74	3609800
1916	3871	9,9	2522178	**1962**	42295	34,46	3638919
1917	4489	12,3	2550543	**1963**	45661	35,36	3666540
1918	5048	17,2	2577729	**1964**	50334	37,35	3694339
1919	6195	18,4	2602869	**1965**	55828	38,94	3723153
1920	7500	21,5	2634664	**1966**	60390	40,23	3753628
1921	5448	19,8	2667868	**1967**	66902	41,99	3786019
1922	4980	16,5	2694840	**1968**	71932	43,48	3818983
1923	4997	15,6	2713116	**1969**	77837	44,81	3850977
1924	5576	17,1	2728764	**1970**	79835	49,56	3877386
1925	5633	17,5	2746815	**1971**	89112	52,67	3903039
1926	4646	14,7	2763106	**1972**	98397	56,45	3933044
1927	4218	13,3	2774864	**1973**	111773	60,64	3960613
1928	4221	12,4	2784675	**1974**	130159	66,35	3985258
1929	4345	11,9	2795105	**1975**	148237	74,1	4007313
1930	4411	11,5	2807438	**1976**	170709	80,9	4026152
1931	3874	10,9	2823882	**1977**	191534	88,23	4043205
1932	3890	10,7	2841529	**1978**	213079	95,43	4058671
1933	3892	10,6	2858343	**1979**	238668	99,96	4072517
1934	4093	10,6	2874206	**1980**	285045	110,85	4085620
1935	4386	10,8	2889211	**1981**	327674	126	4099702
1936	4875	11,1	2903519	**1982**	362270	140,29	4114787
1937	5609	11,9	2918742	**1983**	402197	152,1	4128432
1938	5857	12,3	2935803	**1984**	452512	161,6	4140099
1939	6285	12,4	2954415	**1985**	500200	170,8	4152516
1940	5757	14,5	2973067	**1986**	513718	183,1	4167179
1941	5983	17	2990234	**1987**	561480	199,1	4186905
1942	5756	18	3008883	**1988**	583278	212,4	4209488
1943	5593	18,4	3032430	**1989**	621383	222,1	4226901
1944	5304	18,7	3060216	**1990**	660550	231,22	4241473
1945	5040	19	3091181	**1991**	686686	239,1	4261732
				1992	701650	244,7	4286432

year	Money M1 (Mill. kr.)	Money M3 (Mill. kr.)	Credit vol. (Mill. kr.)	year	Money M1 (Mill. kr.)	Money M3 (Mill. kr.)	Credit vol. (Mill. kr.)
1900	90,6	646,6	974	**1946**	4412	9272	4804
1901	88,5	686,5	1008	**1947**	4968	10123	6377
1902	86,9	697,9	1038	**1948**	5051	10573	7718
1903	85,4	721,4	1085	**1949**	5045	10957	9133
1904	86,2	743,2	1100	**1950**	4949	11100	10524
1905	93,7	761,7	1125	**1951**	5760	12243	12313
1906	94,9	832,9	1170	**1952**	6132	13013	13507
1907	99,5	897,5	1251	**1953**	6375	13634	15321
1908	100,8	938,8	1328	**1954**	6625	14312	17465
1909	107,5	1006,5	1405	**1955**	6805	14807	19648
1910	116,3	1065,3	1486	**1956**	7038	15512	21419
1911	132,9	1143,9	1614	**1957**	6447	16104	23850
1912	145,3	1221,3	1735	**1958**	6695	16522	25919
1913	157,6	1341,6	1891	**1959**	7174	17476	28336
1914	209	1440	2039	**1960**	7418	18575	30774
1915	321	1810	2336	**1961**	7797	19643	34253
1916	556	2737	3325	**1962**	12857	25618	37675
1917	861	3911	4594	**1963**	13555	27179	40992
1918	789	4875	5749	**1964**	14427	29150	44523
1919	815	5419	6523	**1965**	15544	30482	49175
1920	875	5835	7285	**1966**	16798	33951	54944
1921	725	5834	7101	**1967**	17991	36891	60680
1922	646	5626	6407	**1968**	20048	40780	65033
1923	591	4711	5939	**1969**	21776	44375	71138
1924	595	4482	5685	**1970**	24197	50689	78912
1925	525	4242	5366	**1971**	27013	57621	88549
1926	541	4132	5030	**1972**	30572	64810	105134
1927	499	3789	4854	**1973**	34350	72903	122536
1928	452	3606	4838	**1974**	39104	81690	139510
1929	469	3578	4855	**1975**	44696	93598	168823
1930	464	3537	4839	**1976**	52342	110360	205568
1931	470	3408	4902	**1977**	41977	110308	248107
1932	433	3289	4772	**1978**	45323	123282	273523
1933	428	3183	4723	**1979**	48935	139499	300001
1934	450	3114	4693	**1980**	52001	155391	329877
1935	488	3189	4740	**1981**	56830	175000	366525
1936	575	3120	4831	**1982**	63425	194620	417416
1937	607	3268	4964	**1983**	69661	214433	457853
1938	644	3447	5118	**1984**	95690	266882	524157
1939	780	3475	5492	**1985**	114069	305649	590919
1940	1779	4261	4942	**1986**	118888	313932	701320
1941	2784	5492	4575	**1987**	159817	356717	827517
1942	3721	6790	4213	**1988**	194553	375083	906785
1943	4547	8067	3868	**1989**	230301	409146	976424
1944	5244	9202	3620	**1990**	256860	432462	981707
1945	4319	8667	3946	**1991**	277250	480865	954204
				1992			

Year	% Interest on deposit accounts	Bank of Norway discount rate	% Interest on bearer bonds	Year	% Interest on deposit accounts	Bank of Norway discount rate	% Interest on bearer bonds
1900	4,76	6,5	4	1946	1,07	2,5	2,5
1901	4,41	5	4	1947	1,05	2,5	2,3
1902	3,51	5	4,1	1948	1,04	2,5	2,3
1903	3,95	5	4,1	1949	1,09	2,5	2,3
1904	3,72	5	4,2	1950	1,14	2,5	2,5
1905	3,54	5,5	4,2	1951	1,13	2,5	2,9
1906	3,71	5,5	4,3	1952	1,15	2,5	3,1
1907	3,73	6	4,3	1953	1,17	2,5	3,3
1908	3,8	5	4,3	1954	1,21	2,5	3
1909	3,54	4,5	4,2	1955	1,91	3,5	3,5
1910	3,54	4,5	4,2	1956	2,08	3,5	3,9
1911	3,6	5	4,2	1957	2,17	3,5	4
1912	4,1	5,5	4,2	1958	2,15	3,5	4,6
1913	4,29	5,5	4,2	1959	2,15	3,5	4,7
1914	3,93	5,5	4,2	1960	2,19	3,5	5,1
1915	3,97	5,5	4,6	1961	2,27	3,5	5
1916	3,73	5,5	4,8	1962	2,39	3,5	5,1
1917	4,1	6	4,8	1963	2,44	3,5	5
1918	4,6	6	5	1964	2,49	3,5	5
1919	4,6	6	5,1	1965	2,58	3,5	5,1
1920	5,27	7	5,4	1966	2,63	3,5	5,1
1921	5,5	6,5	5,6	1967	2,74	3,5	5
1922	4,35	5	5,5	1968	2,81	3,5	5
1923	4,41	7	5,4	1969	3,48	4,5	5,2
1924	5,19	6,5	5,6	1970	3,69	4,5	5,7
1925	4,24	5	5,6	1971	3,8	4,5	5,9
1926	4,02	4,5	5,5	1972	3,79	4,5	5,9
1927	3,43	5	5,5	1973	3,89	4,5	6
1928	4,01	5,5	5,5	1974	4,32	5,5	6,9
1929	3,99	5	5,5	1975	4,67	5	6,9
1930	3,32	4	5,4	1976	4,86	6	7,1
1931	3,35	6	5,2	1977	5,09	6	7,2
1932	3,29	4	5,2	1978	6,33	7	7,8
1933	2,83	3,5	5,1	1979	6,87	9	8,5
1934	2,61	3,5	5,1	1980	7,33	9	10
1935	2,62	3,5	4,9	1981	7,88	9	11,6
1936	2,64	4	4,7	1982	8,38	9	13,2
1937	3,03	3,5	4,6	1983	8,52	8	12,8
1938	2,7	4,5	4,5	1984	8,87	8	12,2
1939	2,74	3	4,4	1985	9,14	10,57	12,2
1940	2,13	3	3,7	1986	10,36	14,27	13,3
1941	1,59	3	3,7	1987	11,32	13,8	12,5
1942	1,46	3	3,6	1988	10,7	12,27	12,8
1943	1,42	3	3,6	1989	8,83	11	12,3
1944	1,4	3	3,52	1990	8,94	10,7	11,9
1945	1,45	2,5	3,45	1991	8,68	10	
				1992	9,05	11	

Appendix 2

Figure 1.

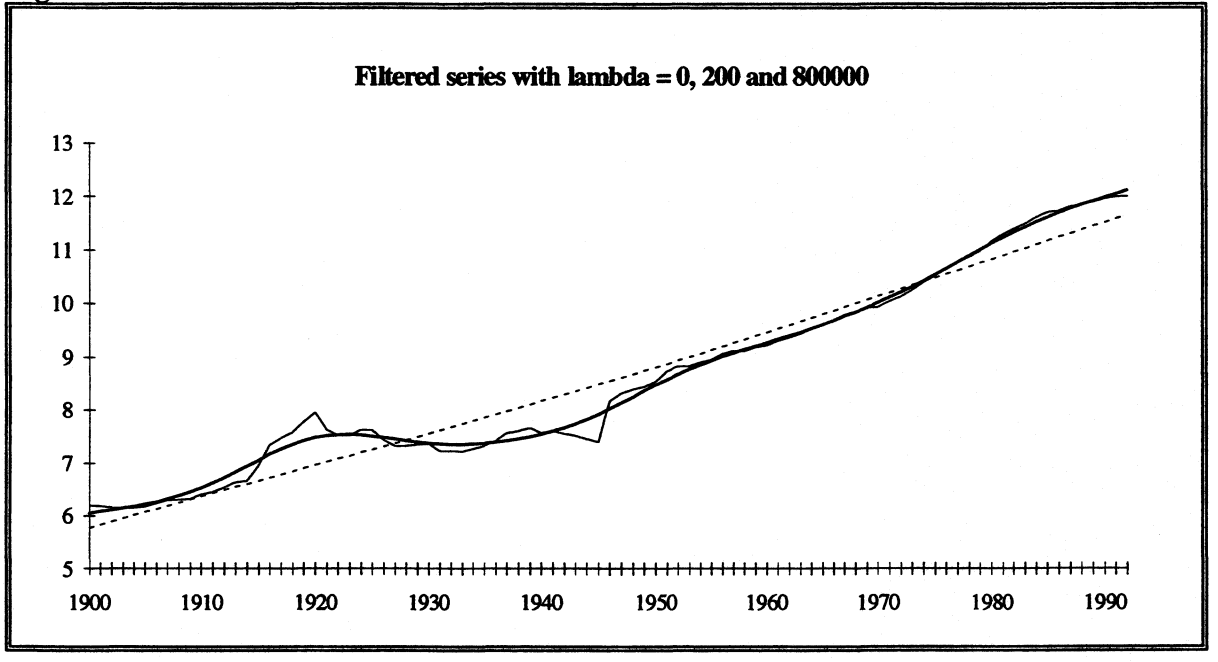


Figure 2.

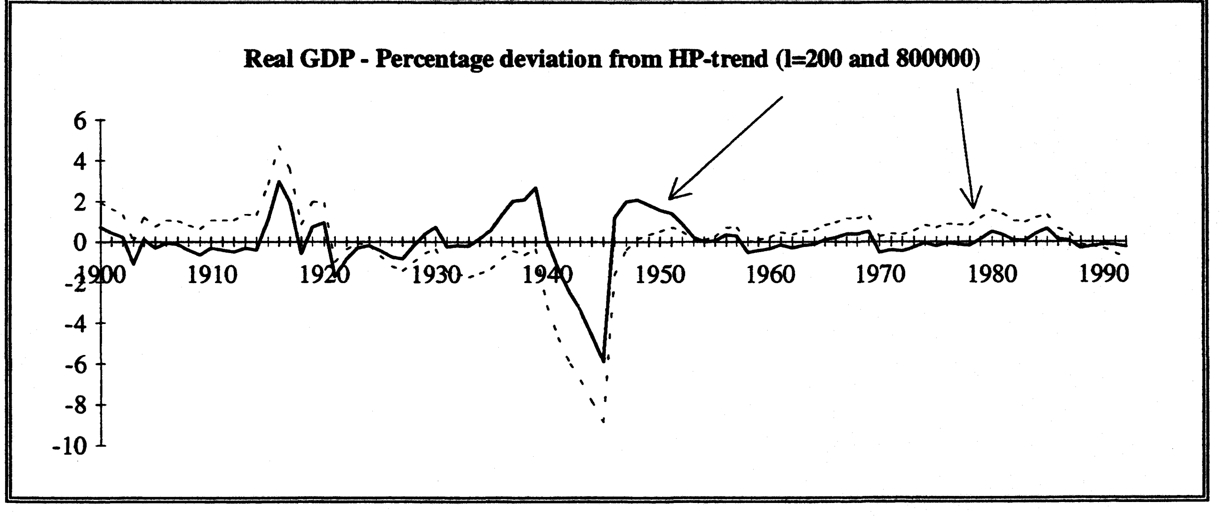


Figure3.

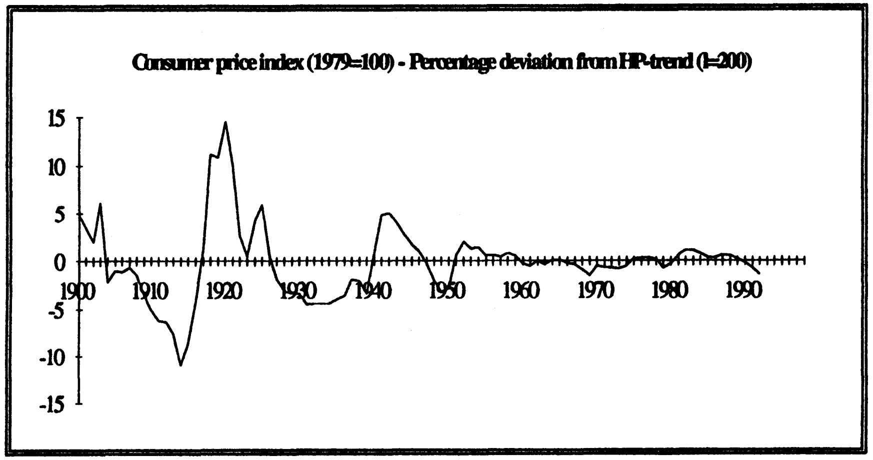


Figure 4.

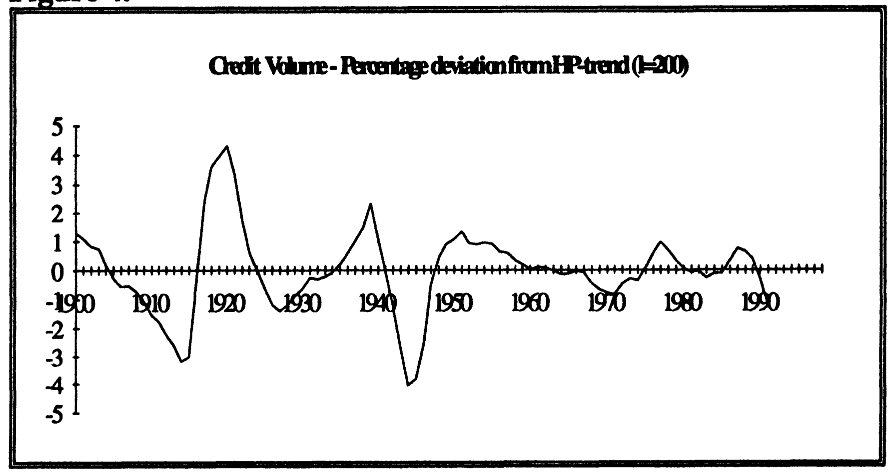


Figure 5.

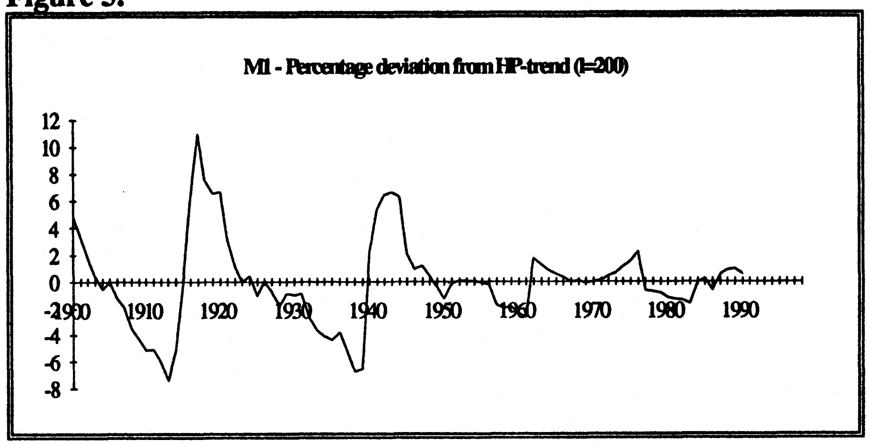


Figure 6.

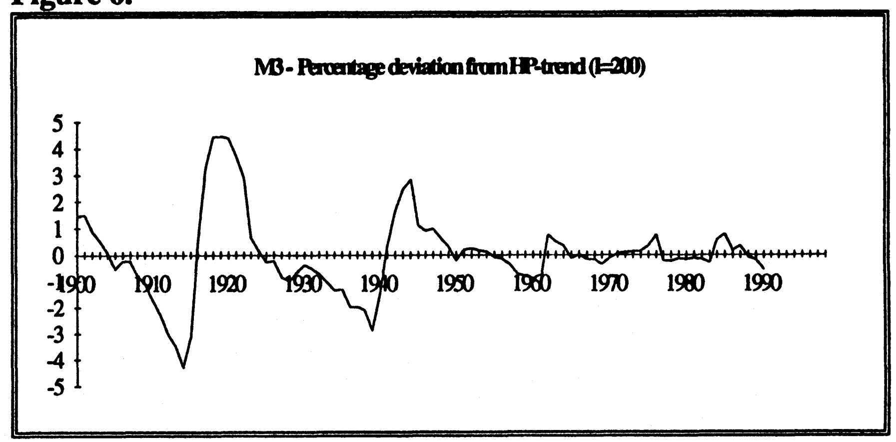


Figure 7.

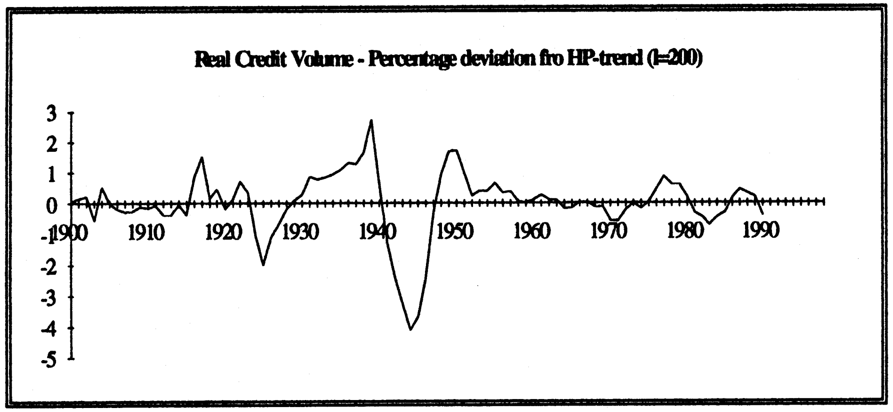


Figure 8.

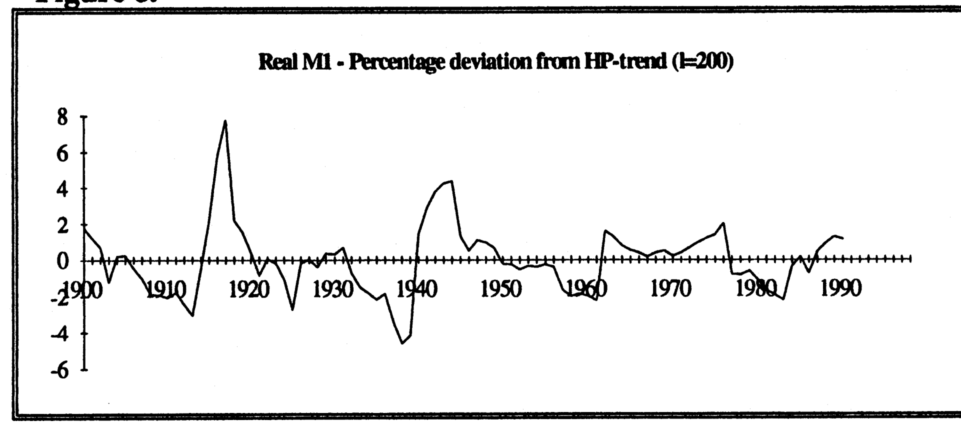


Figure 9.

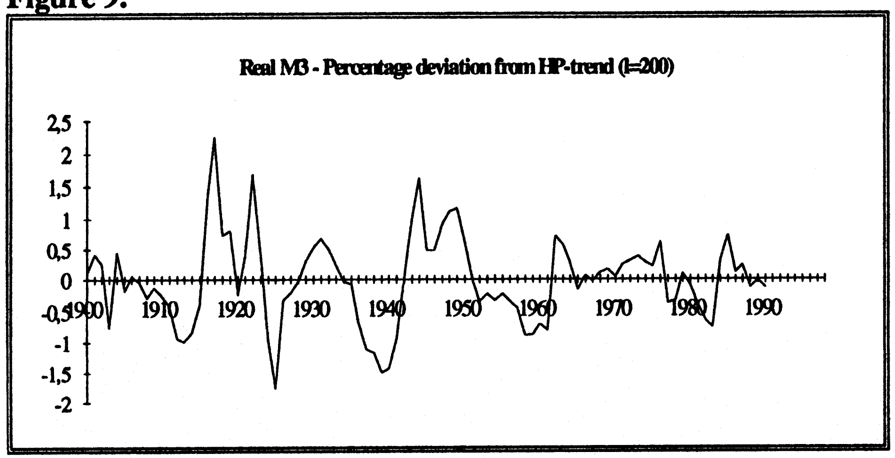


Figure 10.

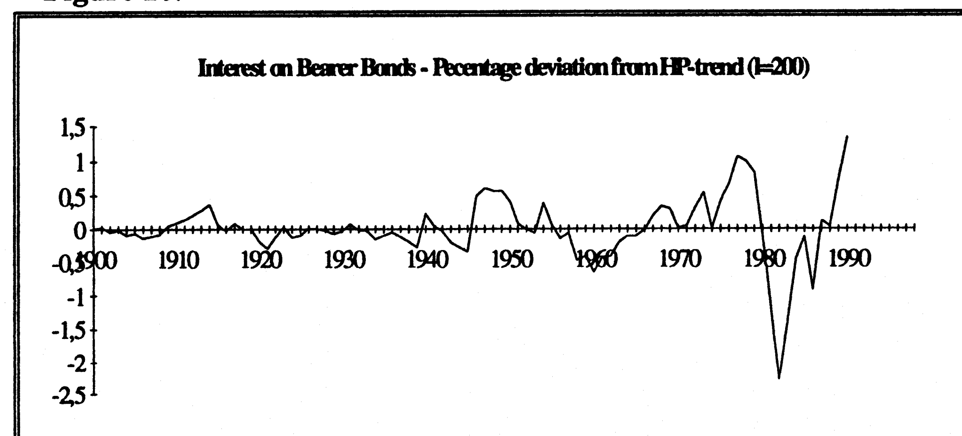


Figure 11.

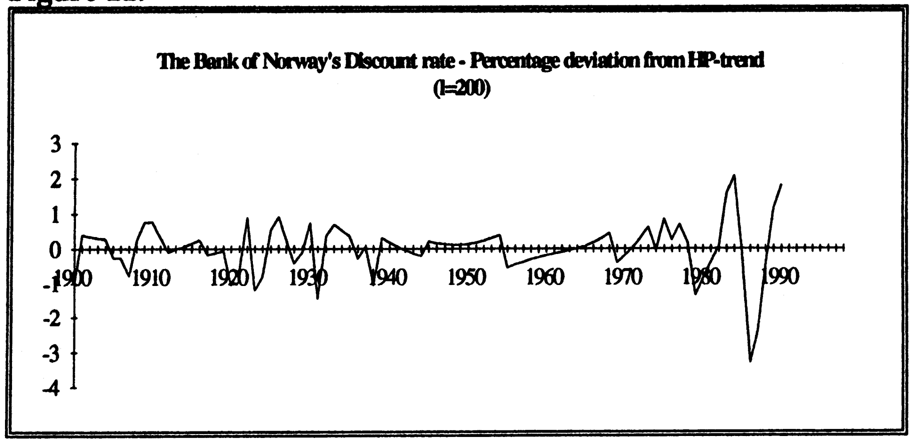
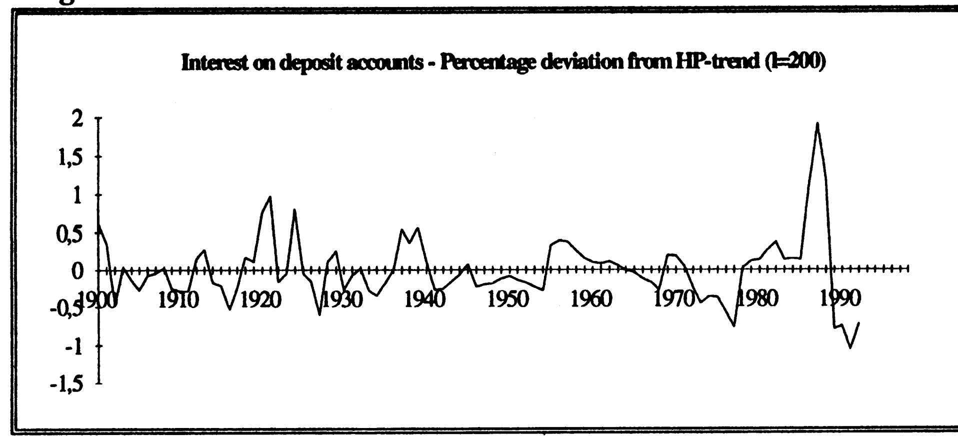


Figure 12.



Appendix 3

Gross Domestic Product:[17]

1900 - 1950: SN (1953) *"National Accounts 1900 - 1929"* NOS XI. 143. Table 1 pp 104-105

Ministry of Finans (1946) *"The National Budget for Norway, 1946"*, Occ. appendix 11.

SN (1946) *"The National Production in Norway 1935-1943"*, NOS X. 102.

1951 - 1969: SN (1968) *"Historical Statistics 1968"* NOS XII. 245. Table 61 pp 92-93.

SN (1978) *"Historical Statistics 1978"* NOS XII. 291. Table 51 pp 96-97.

1970 - 1992: SN (1971-93) *"Statistical Yearbook"* NOS. Tables for GDP and its components[18].

Consumer price index:

1900 - 1992: Bank of Norway "Data Base for Economic Statistics - Troll 8" - Variable A1600012.

Credit Volume:

1900 - 1956: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*, Table XI post A.VII and B.XII.

1957 - 1975: SN (1968) *"Historical Statistics 1968"* NOS XII. 245. Tab. 269 - Total loans.

SN (1978) *"Historical Statistics 1978"* NOS XII.291. Tab. 275 - Total loans.

1976 - 1991: SN (1981,84,86,90,93)*"Statistical Yearbook"* NOS. Tables for loans from financial inst.

M1 / M3:

Notes/ divisionary notes and coins in circulation.

1900 - 1975: SN (1968)*"Historical Statistics 1968"* NOS XII. 245. Table 247

SN (1978) *"Historical Statistics 1978"* NOS XII.291. Table 255.

1940 - 1945: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*. Tab.1 Post A2.

Demand deposits in Postgiro (tax-withholding accounts are excluded)

1943 - 1963: SN (1968) *"Historical Statistics 1968"* NOS XII. 245. Tab. 249.

1964 - 1973: Isachsen, Arne Jon (1976) "The demand for money in Norway" *Norges Banks Skriftserie*, no.3, Norges Bank. Oslo

1974 - 1979: Estimates computed by linear extrapolation.

1980 - 1992: Figures made available on request from Postgiro, Oslo - Norway.

Demand deposits and deposits subject to other terms - in commercial and savings banks.

1900 - 1956: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*. Tab.III

1957 - 1966: SN (1968) *"Historical Statistics 1968"* NOS XII. 245. Tables 252 and 253

1967 - 1975: SN (1978) *"Historical Statistics 1978"* NOS XII.291 SN 1978. Tables 261 and 262

1976 - 1987: SN (1978 - 89)*"Statistical Yearbook"* NOS. 1978-1989.Tables for private and public banks' balance.

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[17] Abbreviations:
NOS = Norges offisielle statistikk (Official Statistics of Norway)
SN = Statistics Norway, Oslo Norway.

[18] I have as far as possible tried to avoid preliminary estimates for the variables, using the oldest numbers in each statistical yearbook. However, for the latest years, this problem is harder to avoid, and for all variables, the observations from 1900-1992 must be considered as preliminary estimates.

1988 - 1991: SN (1993) *"Statistical Yearbook"* NOS. Table 454. post G.

Interest on Deposit Accounts:

1900 - 1963: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*. Table XIV

1964 - 1974: SN (1978) *"Historical Statistics 1978"* NOS XII.291. Table 270.

1975 - 1992: Bank of Norway "Data Base for Economic Statistics (Troll 8)" - Variable K834105C

Interest on Norwegian Bearer Bonds:

1900 - 1945: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*. Table XIV

1946 - 1991: SN (1993) *"Bank and Credit Statistics"* no.15 Table 12, pg. 16. June 20.

Interest on Norwegian Bearer Bonds:

1900 - 1945: Hermod Skånland (1967) *"The Norwegian credit market since 1900"*. Table XIV

1946 - 1991: SN (1993) *"Bank and Credit Statistics"* no.15, Table 12, pg. 16, June 20.

The Bank of Norway's discount rate:

1900 - 1976: SN (1978) *"Historical Statistics 1978"* NOS XII.291

1977 - 1992: IMF *"International Financial Statistics"* Post 60, under Norway.

Appendix 4

In matrix notation, see Danthine & Girardin (1989), the minimisation problem in (1) can be written as:

$$\text{(3)} \qquad \begin{aligned} &\underset{\{d\}}{min} \quad \left(z'z + \lambda(Wd)'Wd\right) \\ &st. \quad z = y - d \end{aligned}$$

To obtain equivalence between (1) and (3), it is necessary that :

$$\text{(4)} \qquad \mathbf{Wd} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_N \end{bmatrix}$$

Where **W** has the dimension $\left[(N+2) \times N\right]$.

Using vector differentiation, the solution to the minimisation problem in (3) is given by[19]

$$\text{(5)} \qquad \begin{gathered} \frac{\partial}{\partial \mathbf{d}}\left((\mathbf{y}-\mathbf{d})'(\mathbf{y}-\mathbf{d}) + \lambda(\mathbf{Wd})'\mathbf{Wd}\right) = 0 \\ \Updownarrow \\ \frac{\partial}{\partial \mathbf{d}}\left(\mathbf{y}'\mathbf{y} + \mathbf{d}'\mathbf{d} - 2\mathbf{y}'\mathbf{d} + \lambda \mathbf{d}'\mathbf{W}'\mathbf{Wd}\right) = 0 \\ \Updownarrow \\ 2\mathbf{d} - 2\mathbf{y} + \left[\mathbf{W}'\mathbf{W} + (\mathbf{W}'\mathbf{W})'\right]\lambda \mathbf{d} = 0 \end{gathered}$$

Because **W'W** is a symmetric matrix we can express (5) as;

$$\text{(6)} \qquad \begin{gathered} 2\mathbf{d} - 2\mathbf{y} + 2\lambda \mathbf{W}'\mathbf{Wd} = 0 \\ \Updownarrow \\ \mathbf{d} = \left(\mathbf{I} + \lambda \mathbf{W}'\mathbf{W}\right)^{-1}\mathbf{y} \end{gathered}$$

where **I** is the identity matrix.

For analytical purposes, it is important to give some attention to the Hodrick-Prescott filter as a means to transform non-stationary series to obtain stationarity. For instance, the short term as well as the long term interest rates have been considered as first order integrated variables. When analysing these series in the given setting (i.e. the reporting of business cycle regularities), it is often convenient to transform all the series to be stationary. Referring to King and Rebelo (1993),

[19] For a more theoretic presentation of vector and matrix differentiation, see Judge et.al (1988, pp.967-970).

the Hodrick-Prescott filter renders stationarity for any integrated series of order four I(4) or lower. To demonstrate this property, we find the first order condition of (1) to be:

(7)
$$\begin{aligned}&-2(y_t-d_t)+2\lambda\left[(d_t-d_{t-1})-(d_{t-1}-d_{t-2})\right]-4\lambda\left[(d_{t+1}-d_t)-(d_t-d_{t-1})\right]\\&+2\lambda\left[(d_{t+2}-d_{t+1})-(d_{t+1}-d_t)\right]=0\end{aligned}$$

expressing y_t as a function of d_t from (7) using the lag operator yields:

(8)
$$\begin{aligned}y_t&=(1+6\lambda)d_t-4\lambda d_{t-1}+\lambda d_{t-2}-4\lambda d_{t+1}+\lambda d_{t+2}\\&=\left[(1+6\lambda)-4\lambda L-4\lambda L^{-1}+\lambda L^2+\lambda L^{-2}\right]d_t=F(L)d_t\\&=\left[\lambda(1-L)^2\left(1-L^{-1}\right)^2+1\right]d_t\end{aligned}$$

where the lag operator is defined as:

(9)
$$L^i=d_{t-i}$$

To represent the cyclical component as a function of the expression in (8), we derive the following:

(10)
$$\begin{aligned}d_t&=\left[F(L)\right]^{-1}y_t=D(L)y_t\\&\Downarrow\\C(L)&=\left[1-D(L)\right]=\left[F(L)-1\right]\left[F(L)\right]^{-1}\end{aligned}$$

where C(L) is the lag polynomial for the cyclical component. The last expression in (8) set into (10) yields:

(11)
$$C(L)=\frac{\lambda(1-L)^2\left(1-L^{-1}\right)^2}{\lambda(1-L)^2\left(1-L^{-1}\right)^2+1}$$

In the numerator we have taken the difference from (t-2) to (t+2) (i.e. differencing four times), which implies that cyclical components derived using the Hodrick-Prescott filter will be stationary for up to fourth order integrated series.

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