Erik Biørn and Tor Jakob Klette

Errors in Variables and Panel Data: The Labour Demand Response to Permanent Changes in Output

Discussion Papers

Erik Biørn and Tor Jakob Klette

Errors in Variables and Panel Data: The Labour Demand Response to Permanent changes in Output*

Abstract:
This paper examines panel data modelling with latent variables in analyzing log-linear relations between inputs and output of firms. Our particular focus is on (i) the "increasing returns to scale puzzle" for labour input and (ii) the GMM estimation in the context of errors-in-variables and panel data. The IV's used for the observed log-differenced output are log output (in level form) for other years than those to which the difference(s) refer. Flexible assumptions are made about the second order moments of the errors, the random coefficients, and other latent variables, allowing, inter alia, for arbitrary heteroskedasticity and autocorrelation up to the first order of the errors-in-variables. We compare OLS, 2SLS, and GMM estimates of the average input response elasticity (which in some cases can be interpreted as an average inverse scale elasticity), and investigate whether year specific estimates differ substantially from those obtained when data for all years are combined. The results confirm the "increasing returns to scale puzzle" for labour input (measured in three different ways), but indicate approximately constant returns to scale when we consider the material input response. This indicates non-homotheticity of the production technology.

Keywords: Errors-in-variables, panel data, labour demand, returns to scale, establishment data.

JEL classification: C23, J23

Acknowledgement: We wish to thank Bjørn Helge Vatne for help and advice with the programming of the estimation procedures in section 4.

Address: Erik Biørn, University of Oslo, Department of Economics,
P.O.Box 1095, Blindern, N-0317 Oslo. E-mail: erik.biorn@econ.uio.no

Tor Jakob Klette, Statistics Norway, Research Department,
P.O.Box 8131 Dep., N-0033 Oslo. E-mail: tjk@ssb.no

* Paper presented at the Fifth Conference on Panel Data, Paris, June 16-17, 1994.

1 Introduction

For years, many economists and econometricians analyzing production technology and producer behaviour empirically have been puzzled by the 'short run increasing returns to scale problem'. A simple example of this problem occurs in the estimation of an 'inverted short run Cobb-Douglas production function' by regressing the logarithm of a measure of the labour input on the logarithm of a measure of output. Very often, such analyses – when performed on aggregate time series data (annual, quarterly, or monthly) by means of ordinary least squares (OLS) or other methods – give an estimated coefficient of the output variable significantly below unity, indicating increasing returns to scale: One percent increase in output seems to require a less than one percent increase in labour input. [See e.g. Sims (1974).] Empirical evidence of increasing returns to scale have quite often been found also when relative price variables, for instance the logarithm of a measure of the wage rate/user cost of capital ratio, have been included in the regression – assuming output constrained cost minimization. Similar results may occur, although they have been less intensively analyzed, for other inputs, like materials, energy, and capital.

Several answers to this problem, or 'puzzle', have been proposed in the literature, *inter alia*: (i) Labour, like capital, may be a quasi-fixed factor in the short run, labour hoarding may be important. A static relationship may not be appropriate, dynamics and lags in the input adjustment process should be specified, even if we are interested in long run responses. (ii) The coefficient of output in a logarithmic 'input-output relation' should not necessarily be interpreted as an inverse scale elasticity; it may be a 'hybrid' parameter 'containing' other effects as well. (iii) Factor augmenting, or factor reducing, technical change, if omitted or improperly represented, may affect the estimated 'inverse scale elasticities'. (iv) The input and output variables and/or the relevant price variables may be inadequately measured, and the chosen econometric procedure does not take proper account of this.

In this paper, our main focus will be on the first and the fourth suggested answers above. However, our results also emphasise the relevance of the second issue, cf. (ii). We approach these issues as an errors-in-variables problem. The labour hoarding argument says that firms do not adjust their labour demand to *temporary* changes in output, but only to changes they consider *permanent*. Observed changes in output will capture both what the firm considers as temporary *and* as permanent changes. Consequently, the observed change in output is a noisy indicator for the movements in output that determine the changes in the firm's labour demand. The fourth answer considers errors-in-variables as a problem in its literal sense, corresponding, *inter alia*, to misreporting and punching errors. We can use the same methodology to deal with both issues. As we shall argue later, we seem to be able to identify which of the two issues that mostly affect our estimates.

Our approach has, to a considerable degree, been inspired by the analysis of Griliches and Hausman (1986), both with respect to methodology and empirical application. As is well known, serious identification problems may arise in errors-in-variables models when only standard data types, like pure times series or pure cross sections, are at hand. This problem can be handled if a panel data set is available, provided certain conditions are satisfied. Then *instrumental variables (IV's)* of the error-ridden regressor(s) may be obtained by transforming the variables represented in the model in a suitable manner, and these IV's may be used to obtain consistent estimates. There may, however, be problems with *potential* IV's which are (i) weakly correlated with the regressors for which they are suggested as instruments and (ii) potentially correlated with the (composite) error term(s) of the equation(s) under estimation.

In a panel data context, a multitude of potentially valid IV's, and hence a multitude of 'IV estimators', may exist [cf. e.g. Biørn (1992)]. One may then attempt to construct some sort of 'compromise estimators', by using two stage least squares (2SLS), three stage least squares (3SLS), the generalized method of moments (GMM) (which can be considered as a generalization of 3SLS), or the full information maximum likelihood (FIML) method. GMM has the attraction, in comparison with 2SLS, of enjoying a sort of efficiency even if non-restrictive assumptions are made about error autocorrelation and heteroskedasticity. GMM is also attractive as it provides an estimator which accommodate joint estimation of an equation system with different instruments for different equations.[1]. This possibility is essential in our case. Furthermore, GMM may be implemented as a stepwise procedure, using IV (or 2SLS) estimation in the introductory stage(s). [See White (1986).] Application of GMM is, computationally, far simpler than FIML.[2]

In the present paper, a set of panel data for Norwegian manufacturing firms from

[1]See Schmidt (1990) for a discussion of the shortcommings of 3SLS, as compared to GMM, in such a context.

[2]FIML is, under certain regularity assumptions, asymptotically efficient, but it may be computationally complicated, even if LISREL types of computer programs may be used for linear models, and the estimators and their properties may be sensitive to changes in assumptions, for instance about normality, error autocorrelation, and error heteroskedasticity.

the years 1975 – 1982, is used. Different measures of labour input are considered. We also present evidence on the response to changes in output of another basic input, materials. Briefly, our results indicate that the 'short run increasing returns to scale puzzle' remains for labour, and are, to some extent, in agreement with previous findings [*inter alia*, Sims (1974) and Griliches and Hausman (1986)]. There is, however, evidence of substantial differences in estimated input response elasticities between different measures of labour input. The results for the material input are significantly different, indicating approximately constant returns to scale in the equation for this input. These findings suggest non-homotheticity in the underlying production technology. This is true not only when we consider materials versus labour, but also when we compare the response to output changes of different kinds of labour. Most analyses in this field assume a homothetic technology.

The basic ingredients of the paper are the following: (i) Panel data modelling with latent variables is considered. (ii) The input response equations, based on Shephard's lemma, are specified in terms of logarithmic differences, derived from a generalized multivariate mean value theorem. (iii) In choosing IV's for observed log *differenced* output, we use *level* variables of log output for other years than those to which the difference relate. (iv) Flexible assumptions are made with respect to the second order moments of the latent variables, allowing, *inter alia*, for arbitrary heteroskedasticity of the measurement errors and of the technological differences across firms and years. (v) As a simplifying assumption, we use a random coefficients approach to represent the variation in the input response elasticity across firms and years. Flexible assumptions are made about these stochastic elements. (vi) We investigate the performance of (the inefficient) 2SLS and (the efficient) GMM methods in this context and compare them with corresponding (biased) OLS estimates. (vii) Finally, we investigate whether year specific estimates, i.e. based on log differenced inputs and output between two specific years, differ substantially from those obtained when data for all years available are combined.

The rest of the paper is disposed as follows. First, in section 2, we present the model framework, derived from output constrained cost minimization, with the firms' inputs in production as well as their outputs considered as latent variables. This framework leads to expressing the relative increases in (latent) input demand between two arbitrary years in terms of the corresponding relative increase in output volume, and an error/disturbance term, representing differences in input prices, differences in technology, etc., between the two years. We denote this equation as an *input response equation.* Next, we present, in section 3, a flexible stochastic specification of this equation. In this way, we attempt to account for the multitude of effects (transitory components in inputs and output, measurement errors, disturbances in optimization, differences in technology, firm specific differences in input prices, etc.) which are captured by the composite error and disturbance term in the equation we estimate. We argue that specifying composite errors and disturbances as homoskedastic white noise would be far too restrictive in the present context. Estimation procedures, with focus on IV and GMM, are discussed in section 4. Empirical results, with attention to the robustness issue, are presented and discussed in section 5. Concluding remarks follow in section 6.

2 Model framework

Assume that we have a balanced panel data set consisting of observations from M firms in T consecutive years. The production technology underlying the factor demand specifies N inputs. We first formulate the model of factor demand in terms of latent variables (using asterisks as superscripts to symbolize latent structural variables), and next (in section 3) respecify the model in terms of observed variables and introduce the stochastic specification. Let Q^*_{it} denote that latent volume of output, $X^*_{it} = (X^{*1}_{it}, \ldots, X^{*N}_{it})$ the latent vector of inputs, and $w^*_{it} = (w^{*1}_{it}, \ldots, w^{*N}_{it})$ the latent vector of input prices of firm i in year t ($i = 1, \ldots, M,\ t = 1, \ldots, T$). The notation w^*_{it} signalizes that we allow for variations in the input prices not only over years, but also across firms. We describe the technology by

$$(1) \qquad Q^*_{it} = \Phi_{it} F(X^*_{it}), \qquad i = 1, \ldots, M,\ t = 1, \ldots, T,$$

where $F(\cdot)$ is a production function with neo-classical properties, common to all firms, and Φ_{it} is a factor reflecting differences in the level of technology between firms and years.

We assume that the firms act as cost minimizers and price takers for given output, interpreting Φ_{it} as known constants to the firm (but interpreted as unobserved values of stochastic variables by the econometrician). The cost function dual to (1) can then, in the usual way, be written in the form [cf., e.g., Jorgenson (1986, section 5.1)]

$$(2) \qquad C^*_{it} = \sum_{j=1}^{N} w^{*j}_{it} X^{*j}_{it} = G(w^*_{it}, Q^*_{it}/\Phi_{it}) \, .$$

Using Shephard's lemma, firm i's optimal inputs of factor j in year t can be expressed as [cf. e.g. Jorgenson (1986, p. 1885)]

$$(3) \qquad X_{it}^{*j} = \frac{\partial G(w_{it}^{*}, Q_{it}^{*}/\Phi_{it})}{\partial w_{it}^{*j}} f_{it}^{j} = g_j(w_{it}^{*}, Q_{it}^{*}/\Phi_{it}) f_{it}^{j} ,$$
$$j = 1, \ldots, N, \; i = 1, \ldots, M, \; t = 1, \ldots, T,$$

where the f_{it}^{j}'s represent input specific 'errors in optimization' and other unobservable factors affecting the optimization, and $g_j(\cdot)$ is implicitly defined by (3).

Consider first the case where $F(\cdot)$ represents a *homothetic technology*, implying that its dual cost function can be separated as [cf. Jorgenson (1986, p. 1888)]

$$(4) \qquad G(w_{it}^{*}, Q_{it}^{*}/\Phi_{it}) = H(w_{it}^{*}) K(Q_{it}^{*}/\Phi_{it}) ,$$

so that (3) becomes

$$(5) \qquad X_{it}^{*j} = h_j(w_{it}^{*}) K(Q_{it}^{*}/\Phi_{it}) f_{it}^{j} , \quad j = 1, \ldots, N, \; i = 1, \ldots, M, \; t = 1, \ldots, T,$$

where $K(\cdot)$ is a monotonically increasing function and $h_j(w_{it}^{*}) = \partial H(w_{it}^{*})/\partial w_{it}^{*j}$. Applying the *generalized multivariate mean value theorem* to (5) in logarithmic form [see Berck and Sydsæter (1991, p. 11) and Klette (1993, p. 7 – 8)], it follows for any years t and s that

$$(6) \qquad x_{it}^{*j} - x_{is}^{*j} = \varepsilon_{its}(q_{it}^{*} - q_{is}^{*}) + \sum_{k=1}^{N} \gamma_{its}^{jk} \ln(w_{it}^{*k}/w_{is}^{*k}) + \ln(f_{it}^{j}/f_{is}^{j}) - \varepsilon_{its} \ln(\Phi_{it}/\Phi_{ts}),$$
$$j = 1, \ldots, N, \; i = 1, \ldots, M, \; t, s = 1, \ldots, T,$$

where $x_{it}^{*j} = \ln(X_{it}^{*j})$, $q_{it}^{*} = \ln(Q_{it}^{*})$, etc., ε_{its} is the elasticity of $K(\cdot)$ with respect to Q_{it}^{*}/Φ_{it} – which can be interpreted as the *inverse scale elasticity* – evaluated somewhere between Q_{it}^{*}/Φ_{it} and Q_{is}^{*}/Φ_{is}, and γ_{its}^{jk} is the elasticity of $h_j(\cdot)$ with respect to w_{it}^{*k}, evaluated somewhere between w_{it}^{*} and w_{is}^{*}. We denote ε_{its} as an *input response coefficient* or an *input response elasticity* in the following. We emphasise the motivation behind our use of a *mean value theorem*, rather than an approximation based on a first order logarithmic *Taylor expansion*, in the derivation above. Relative differences in inputs or outputs along the time dimension of a panel data set as in (6) may, like the differences along the cross-sectional dimension, be substantial, in particular for differences over several years.[3] Such differences might undermine the argument for using approximations by truncating Taylor expansions after the first order term. On the other hand, equations expressed in differences by using the mean value theorem in logarithmic form are valid regardless of the size of the differences of outputs and input prices, whether they are taken across time periods or across firms in the data set. [See Klette (1993).]

If, in particular, the homothetic technology is characterized by a *constant scale elasticity*, equal for all firms and years, ε denoting its inverse, we have $K(Q^*_{it}/\Phi_{it}) = (Q^*_{it}/\Phi_{it})^{\varepsilon}$. In this case, ε_{its} in (6), and in (7) and (8) below, can be replaced by the constant ε. The γ^{jk}_{its}'s in (6) will in general show variation across i, t, and s.

From (6) it follows that the optimal inputs of firm i expressed as logarithmic differences between year t and year s can be written in terms of the corresponding logarithmic difference of output as

$$(7) \qquad x^{*j}_{it} - x^{*j}_{is} = \varepsilon_{its}(q^*_{it} - q^*_{is}) + \kappa^j_{its}\,, \qquad j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T,$$

where

$$(8) \qquad \kappa^j_{its} = \sum_{k=1}^{N} \gamma^{jk}_{its} \ln(w^{*k}_{it}/w^{*k}_{is}) + \ln(f^j_{it}/f^j_{is}) - \varepsilon_{its}\ln(\Phi_{it}/\Phi_{is}).$$

The composite variables κ^j_{its} capture (changes in) technological differences, errors in optimization, input price differences, and differences in the values of the price elasticities γ^{jk}_{its} across firms and years. We discuss the stochastic specification of (7) – (8) in section 3.

We can make the following observations from the model framework described so far: (i) *A priori*, κ^j_{its} may contain both firm specific, year specific, and combined components. However, if the technological differences admit a decomposition of the form $\ln(\Phi_{it}) = a_i + b_t + c_{it}$, a_i, b_t and c_{it} being independently distributed, and if $\ln(f^j_{it})$ and $\ln(w^{*k}_{it})$ can be given similar decompositions, then all firm specific components, like a_i, will vanish from κ^j_{its} since it is constructed from logarithmic differences. (ii) κ^j_{its} may be correlated with $(q^*_{it} - q^*_{is})$, owing to 'simultaneity' in the input and output decisions which is not captured by our simplistic model with

[3] Note also that we, in our empirical implementation, perform an additional differencing along the cross-sectional dimension, since we will measure all observations from their time specific mean. (See section 3 and appendix A.)

cost minimization.[4] (iii) Serial correlation or/and heteroscedasticity of κ_{its}^{j} should be allowed for.

Consider next the more complex case where $F(\cdot)$ represents a *non-homothetic technology.* Separability of the dual cost function $G(\cdot)$, as in (4), then no longer holds. Applying the generalized multivariate mean value theorem to (3) in logarithmic form, it follows for any t and s that

$$
\begin{aligned}
(9) \qquad x_{it}^{*j} - x_{is}^{*j} &= \varepsilon_{its}^{j}(q_{it}^{*} - q_{is}^{*}) \\
&\quad + \sum_{k=1}^{N} \gamma_{its}^{jk} \ln(w_{it}^{*k}/w_{is}^{*k}) + \ln(f_{it}^{j}/f_{is}^{j}) - \varepsilon_{its}^{j} \ln(\Phi_{it}/\Phi_{is}), \\
&\qquad j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T,
\end{aligned}
$$

where ε_{its}^{j} and γ_{its}^{jk} are the elasticities of $g_j(\cdot)$ with respect to Q_{it}^{*} and w_{it}^{*k}, respectively, evaluated somewhere between $(w_{it}^{*}, Q_{it}^{*}/\Phi_{it})$ and $(w_{is}^{*}, Q_{is}^{*}/\Phi_{is})$. These equations replace (6). The corresponding versions of (7) and (8) are

$$
\begin{aligned}
(10) \qquad x_{it}^{*j} - x_{is}^{*j} &= \varepsilon_{its}^{j}(q_{it}^{*} - q_{is}^{*}) + \kappa_{its}^{j}, \\
(11) \qquad \kappa_{its}^{j} &= \sum_{k=1}^{N} \gamma_{its}^{jk} \ln(w_{it}^{*k}/w_{is}^{*k}) + \ln(f_{it}^{j}/f_{is}^{j}) - \varepsilon_{its}^{j} \ln(\Phi_{it}/\Phi_{is}), \\
&\qquad j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T.
\end{aligned}
$$

In this case, the input response elasticity, ε_{its}^{j}, cannot be interpreted as an inverse scale elasticity.

Comparing (7) – (8) with (10) – (11), we see that in the homothetic case, the 'input response elasticity' ε_{its} is a function of the output volume only (except in the case where the scale elasticity is constant, as noted above), but it is invariant to changes in the input price vector w_{it}^{*}. On the other hand, the elasticities γ_{its}^{jk} are functions of the price vector w_{it}^{*}, but are invariant to changes in the output volume. In the non-homothetic case, all these elasticities are functions of the output volume and the input prices.

3 Stochastic specification

The latent variables framework of the factor demand model outlined above contains, even for moderate N, M, and T, a large number of input, firm, and year specific

[4] For instance, $\ln(f_{it}^{j})$ may be correlated with q_{it}^{*} for any j – recall that output constrained cost minimization, as is assumed here, is necessary for full profit maximization.

parameters. In this section, we elaborate on the stochastic specification adopted in the empirical implementation of the model, taking eqs. (7) and (8), derived under *homotheticity*, as our starting point.

The triple subscript on the (long-term) 'input response coefficient' ε_{its}, reflecting its dependence on both Q^*_{it}/Φ_{it} and Q^*_{is}/Φ_{is}, is a complicating feature of the model outlined so far. However, some preliminary experimental calculations suggested that adding *levels* of the logarithm of output to regressions of logarithmic differences in the inputs on logarithmic differences in output, as in (7), did not affect the estimated 'input response coefficient' significantly. Neither did this level variable turn out to be significant in interaction with the log differenced output. For this reason, we treat, as a simplification, ε_{its} as a *random coefficient*, with a firm and year invariant mean equal to $\bar{\varepsilon}$, defining its stochastic component as

$$(12) \qquad \delta_{\varepsilon its} = \varepsilon_{its} - \bar{\varepsilon} , \qquad i = 1, \ldots, M, \ t, s = 1, \ldots, T.$$

More generally, to allow for non-homotheticity, ε_{its}, $\bar{\varepsilon}$, and $\delta_{\varepsilon its}$ can be furnished with an input superscript, j, cf. (9) – (11) above and (A.4) – (A.6) in appendix A.

We assume that the logarithms of the output and the input quantities observed are

$$(13) \qquad q_{it} = q^*_{it} + \nu_{it} , \qquad i = 1, \ldots, M, \ t = 1, \ldots, T,$$

$$(14) \qquad x^j_{it} = x^{*j}_{it} + \tau^j_{it} , \quad j = 1, \ldots, N, \ i = 1, \ldots, M, \ t = 1, \ldots, T,$$

where ν_{it} and τ^j_{it} are errors-in-variables in the wide sense, as explained in section 1, while q^*_{it} and x^{*j}_{it} are permanent components. Using (12), (13), and (14), we can reformulate the input response equations, (7), in terms of logarithmically differenced observations on output and input volumes between years t and s as

$$(15) \qquad x^j_{it} - x^j_{is} = \bar{\varepsilon}(q_{it} - q_{is}) + \theta^j_{its} ,$$
$$j = 1, \ldots, N, \ i = 1, \ldots, M, \ t, s = 1, \ldots, T,$$

where

$$(16) \qquad \theta^j_{its} = \kappa^j_{its} + \delta_{\varepsilon its}(q^*_{it} - q^*_{is}) + (\tau^j_{it} - \tau^j_{is}) - \bar{\varepsilon}(\nu_{it} - \nu_{is}),$$
$$j = 1, \ldots, N, \ i = 1, \ldots, M, \ t, s = 1, \ldots, T,$$

with κ^j_{its} defined in (8).

The error variables $\delta_{\varepsilon its}$, τ^j_{it}, and ν_{it} are all assumed to have zero expectations. The stochastic specification of the model as regards its second order moments, however, is rather flexible. It has the following four elements:[5]

(i) Realizations of $(q^*_{it}, \kappa^j_{its}, \nu_{it}, \tau^j_{it}, \delta_{\varepsilon its})$ are stochastically independent across different values of i.

(ii) The error variables $(\nu_{it},\tau^j_{it},\delta_{\varepsilon its})$ are all independent of the 'structural variables' $(q^*_{it}, \kappa^j_{its})$.

(iii) The errors in output and inputs (ν_{it}, τ^j_{it}) may have an arbitrary heteroskedasticity. In the empirical application to be discussed below, we confine attention to the cases where the errors are non-autocorrelated or follow MA(1) processes. Similar assumptions are made with respect to the random part of the 'input response coefficient' $(\delta_{\varepsilon its})$.

(iv) We allow for correlation between q^*_{it} and κ^j_{isr} not only for $s = t$ or $r = t$, but also for $t \neq s \neq r$. In the empirical application, however, we confine attention to the case where this "cross-autocorrelation" is at most of the first order.

Instead of using (15), as it now stands, as our estimating equation for the (permanent) 'input response coefficient' $\bar{\varepsilon}$, we use this equation *after having redefined the (logarithmic) inputs and output quantities by deducting their respective year specific (i.e. sectoral) means.* A similar procedure is followed in MaCurdy (1982). This transformation, although reducing the effective variation of the input and output variables in the sample, removes any additive year specific effects from the observed structural variables as well as from the composite "residual variables" θ^j_{its} – recall the definition of the latter, given by (16) and (11). Hence, these variables are more likely to be stationary after this transformation has been made than before.[6]

Letting C denote the (population) covariance operator, we introduce the short-

[5]This is elaborated in appendix A.

[6]If, for instance, κ^j_{its} admits a decomposition of the form $\kappa^j_{its} = b_{ts} + c_{its}$ (any firm specific additive effects being already eliminated by the 'within firm' difference transformation, cf. section 2), b_{ts} vanishes, and we have $\kappa^j_{its} - (1/M)\sum_{k=1}^M \kappa^j_{kts} = c_{its} - (1/M)\sum_{k=1}^M c_{kts}$.

hand notation

$$C(\nu_{it}, \nu_{is}) = \sigma^{\nu}_{its} ,$$

$$C(q^*_{it}, q^*_{is}) = \sigma^{q*}_{its} ,$$

$$C(\kappa^j_{its}, q^*_{ip}) = \sigma^{\kappa j q*}_{itsp} ,$$

$$j = 1, \ldots, N, \; i = 1, \ldots, M, \; t, s, p = 1, \ldots, T.$$

In appendix A, we show that if the number of firms, M, is sufficiently large, the variables on the right-hand side of (15), after the deduction of the time means, satisfy approximately

$$C[(q_{it} - q_{is}), q_{ip}] = \sigma^{qq}_{itp} - \sigma^{qq}_{isp} , \tag{17}$$

$$C[\theta^j_{its}, q_{ip}] = \sigma^{\theta j q}_{itsp} , \tag{18}$$

where

$$\sigma^{qq}_{itp} = \sigma^{q*}_{itp} + \sigma^{\nu}_{itp} , \tag{19}$$

$$\sigma^{\theta j q}_{itsp} = \sigma^{\kappa j q*}_{itsp} - \bar{\varepsilon}(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp}) , \tag{20}$$

$$j = 1, \ldots, N, \; i = 1, \ldots, M, \; t, s, p = 1, \ldots, T.$$

As will be shown in section 4, (17) – (20) are useful in suggesting candidates as IV's for $(q_{it} - q_{is})$ in the input response equations, (15). The zero restrictions we impose on $\sigma^{\theta j q}_{itsp}$ [cf. assumptions (iii) and (iv) above] ensure identification of the mean input response coefficient, $\bar{\varepsilon}$. If these orthogonality restrictions are not satisfied, there may be problems with some of the IV's to be used in the empirical applications (cf. sections 4.b – 4.f below).

4 Estimation procedures

In this section, we describe the procedures we use in our attempts to estimate the mean (long-term) 'input response coefficient' $\bar{\varepsilon}$ in the log-differenced equation (15), satisfying (17) – (18). A basic idea is to use variables in levels as IV's for corresponding variables in differences. Similar ideas have been followed by Anderson and Hsiao (1981, 1982), Hsiao (1986, sections 4.2 and 4.3), and Sevestre and Trognon (1992) for first order autoregressive models for panel data, and by Griliches and

Hausman (1986), for panel data models with errors in variables. See also Arellano and Bond (1991) and Biørn (1992, section 8.2.3). Our strategy is, more specifically, *to use q_{ip} as IV's for $(q_{it} - q_{is})$, for those combinations of (p,t,s) which satisfy the two conditions*

$$(21) \qquad C[(q_{it} - q_{is}), q_{ip}] = \sigma^{qq}_{itp} - \sigma^{qq}_{isp} \neq 0,$$

$$(22) \qquad C[\theta^j_{its}, q_{ip}] = \sigma^{\theta jq}_{itsp} = 0,$$

where σ^{qq}_{itp} and $\sigma^{\theta jq}_{itsp}$ are defined in (19) and (20). This can be done by using data for the M firms, either (i) for a given input j and/or a given pair of years (t,s), or (ii) for a given input j and all combinations of (t,s), or (iii) for all combinations of (t,s,j) simultaneously. This gives a substantial number of possible estimators, and we will only consider the most important ones. We assume that the number of firms in the sample, M, is so large that (i) the approximations underlying (21) and (22) can be considered as satisfactory and (ii) all realizations of $(\theta^j_{its}, q_{ip}, x^j_{iz})$ for different firms i can be considered as approximately uncorrelated (cf. appendix A).[7] Notice that joint estimation for different combinations of (t,s) or (t,s,j), requires a GMM procedure, as it involves estimation of a system of equations with different instruments for different equations. The specification of the GMM procedure will be spelt out carefully below.

4.a The basic assumption on the error structure

Let us first assume that there is *no autocorrelation of any order in the error in output*, i.e. $\sigma^{\nu}_{itp} = 0$ for all $p \neq t$, and no "cross autocorrelation" between the variables summarized in κ^j_{its} [cf. (8)] on the one hand, and the true, latent output variable q^*_{it} on the other. We then have [cf. (18) and (20)] for all i and j the orthogonality conditions

$$(23) \qquad \sigma^{\theta jq}_{itsp} = 0 \quad \text{for all} \quad t \neq s \neq p.$$

In addition, we assume for all i that

$$(24) \qquad \sigma^{qq}_{itp} \neq \sigma^{qq}_{isp} \quad \text{for all} \quad t \neq s \neq p,$$

which is satisfied if the (latent) output volume, after deduction of the time mean, is, for all i, a *non-stationary* variable, but it may be satisfied in cases of stationarity

[7] MaCurdy (1982, pp. 84, 88) makes a similar assumption for a fairly general panel data model with heteroskedasticity and autocorrelation of the disturbance terms, although with errors-in-variables disregarded.

as well. It then follows from (21) and (22) that q_{ip} is a valid IV for $(q_{it} - q_{is})$ for all $p \neq (t,s)$.

To simplify the notation in the following, we (i) omit the 'bar' symbol $^-$ on the mean input response coefficient ε, and (ii) omit the superscript j on the input variables x_{it} and the "residual term" θ_{its} whenever this omission cannot bring confusion. In the following, we also conventionally, without loss of generality, set $t > s$. An arbitrary differenced input demand equation, as given in (15), can then be written simply as

$$(25) \qquad x_{its} = \varepsilon q_{its} + \theta_{its}, \qquad i = 1,\ldots,M,\ t,s = 1,\ldots,T,\ t > s,$$

where

$$(26) \qquad q_{its} = q_{it} - q_{is},$$

$$(27) \qquad x_{its} = x_{it} - x_{is}, \qquad i = 1,\ldots,M,\ t,s = 1,\ldots,T,\ t > s.$$

We assume in the following that the difference transformations we perform in deriving (25) – one within firms and one within years – ensure that all θ_{its} have zero mean. Hence, any non-zero constant term in (25) can be disregarded.

4.b Simple IV estimators

A simple IV estimator of ε in (25), based on observations for one pair of years, (t,s), and one IV, for year p, would then, provided that (23) and (24) are satisfied, be given by [cf. Biørn (1992, p. 168)]

$$(28) \qquad \widehat{\varepsilon}_{ts,p}^{IV} = \frac{\sum_{i=1}^{M} q_{ip} x_{its}}{\sum_{i=1}^{M} q_{ip} q_{its}} = \frac{\sum_{i=1}^{M} q_{ip}(x_{it} - x_{is})}{\sum_{i=1}^{M} q_{ip}(q_{it} - q_{is})}, \quad t,s,p = 1,\ldots,T,\ t > s,\ p \neq (t,s).$$

Since the number of different ways of selecting 2 years from T is

$$(29) \qquad S = \binom{T}{2} = \frac{T(T-1)}{2},$$

and the number of admissible IV's for each pair of years is $T-2$, the number of possible simple IV estimators of the form $\widehat{\varepsilon}_{ts,p}^{IV}$ is

$$(30) \qquad R = S(T-2) = \frac{T(T-1)(T-2)}{2}.$$

In the empirical application in section 5 below, we have $T = 8$, i.e. $S = 28$ and $R = 168$. Each of these R estimators is consistent (for any T when $M \to \infty$), provided that (23) and (24) are satisfied, but obviously they are, in general, far from efficient.

Let now

$$(31) \qquad z_{its} = \left[\begin{array}{c} 1 \times (T-2) \text{ vector containing} \\ q_{ip} \text{ for } p = 1, \ldots, T, \; p \neq (t,s) \end{array} \right], \quad i = 1, \ldots, M, \; t,s = 1, \ldots, T, \; t > s.$$

Note that the year subscripts (t, s) on z indicate that output in years t and s *have been omitted from the vector.* The row vector z_{its} contains all the $T-2$ admissible IV's for q_{its}. If, for instance, $T = 4$, we have, for each i, $S = 6$ possible z_{its}'s:

$$\begin{aligned} z_{i21} &= (q_{i3}, q_{i4}), & z_{i32} &= (q_{i1}, q_{i4}), & z_{i43} &= (q_{i1}, q_{i2}), \\ z_{i31} &= (q_{i2}, q_{i4}), & z_{i42} &= (q_{i1}, q_{i3}), & z_{i41} &= (q_{i2}, q_{i3}). \end{aligned}$$

Instead of using one IV only, as in (28), a more efficient procedure is to use the complete $1 \times (T-2)$ vector z_{its} as an IV vector for the scalar q_{its} for any given (t, s), and *combine the elements of z_{its} in an "optimal" way in the 2SLS sense*, see e.g. Bowden and Turkington (1984, section 2.4). This does not mean, of course, that the resulting 2SLS estimator is optimal irrespective of the properties of the distribution of the error/disturbance term θ_{its}.

We then, in a *first stage*, (i) form the auxiliary, 'reduced form' equations relating q_{its} to z_{its},

$$(32) \qquad q_{its} = z_{its}\Pi_{ts} + \psi_{its}, \qquad i = 1, \ldots, M, \; t,s = 1, \ldots, T, \; t > s,$$

where Π_{ts} is a $(T-2) \times 1$ coefficient vector and ψ_{its} is a (scalar) disturbance term, (ii) estimate Π_{ts} by means of OLS, and (iii) compute the corresponding OLS 'predictor' of q_{its}. The two latter variables are, respectively,

$$(33) \qquad \widehat{\Pi}_{ts} = \left(\sum_{i=1}^{M} z'_{its} z_{its} \right)^{-1} \left(\sum_{i=1}^{M} z'_{its} q_{its} \right),$$

$$(34) \qquad \widehat{q}_{its} = z_{its}\widehat{\Pi}_{ts} = z_{its} \left(\sum_{i=1}^{M} z'_{its} z_{its} \right)^{-1} \left(\sum_{i=1}^{M} z'_{its} q_{its} \right), \quad i = 1, \ldots, M, \; t,s = 1, \ldots, T, \; t > s.$$

In the *second stage*, we use $\widehat{q}_{its}$ as IV for q_{its} in (25) – which, in view of the orthogonality of the OLS residuals $\widehat{\psi}_{its}$ and the OLS predictors $\widehat{q}_{its}$ [i.e. $\sum_{i=1}^{M} \widehat{\psi}_{its}\widehat{q}_{its} = 0$ for all (t,s)], is equivalent to regressing x_{its} on $\widehat{q}_{its}$. This maximizes the correlation between q_{its} and its instrument and gives the 2SLS estimator of ε

$$(35) \qquad \widehat{\varepsilon}_{ts}^{2SLS} = \left[\sum_{i=1}^{M} \widehat{q}_{its}' q_{its}\right]^{-1} \left[\sum_{i=1}^{M} \widehat{q}_{its}' x_{its}\right] = \left[\sum_{i=1}^{M} \widehat{q}_{its}' \widehat{q}_{its}\right]^{-1} \left[\sum_{i=1}^{M} \widehat{q}_{its}' x_{its}\right], \quad t,s = 1,\ldots,T,\ t > s.$$

Substituting for $\widehat{q}_{its}$ from (34) in (35) and rearranging, we find that *the 2SLS estimator of ε confined to a specific pair of years (t,s)* can be written in terms of x_{its}, q_{its}, and z_{its} as

$$(36) \qquad \widehat{\varepsilon}_{ts}^{2SLS} = \left[\left(\sum_{i=1}^{M} q_{its}' z_{its}\right)\left(\sum_{i=1}^{M} z_{its}' z_{its}\right)^{-1}\left(\sum_{i=1}^{M} z_{its}' q_{its}\right)\right]^{-1} \times \left[\left(\sum_{i=1}^{M} q_{its}' z_{its}\right)\left(\sum_{i=1}^{M} z_{its}' z_{its}\right)^{-1}\left(\sum_{i=1}^{M} z_{its}' x_{its}\right)\right]. \quad t,s = 1,\ldots,T,\ t > s.$$

The number of such estimators is S. These estimators are not, of course, independent, since we have, for instance, the identities

$$q_{i31} = q_{i21} + q_{i32}\ , \qquad q_{i41} = q_{i21} + q_{i32} + q_{i43}\ ,$$

etc., and similar relationships hold for the x_{its}'s. For the z_{its}'s, however, no such simple relationships exist, and hence it is not possible to express for instance $\widehat{\varepsilon}_{41}^{2SLS}$ as a simple and easily interpretable weighted arithmetic mean of $\widehat{\varepsilon}_{21}^{2SLS}$, $\widehat{\varepsilon}_{32}^{2SLS}$, and $\widehat{\varepsilon}_{43}^{2SLS}$.

4.c Pooling the equations for different years with different IV's

Let us next take a further step and derive an estimator of ε by *using observations from all the S pairs of years for each firm jointly.* We define, for firm i,

$$(37) \qquad Q_i = \left\{\begin{array}{c} S \times 1 \text{ vector containing all } S \text{ different} \\ q_{its} = q_{it} - q_{is} \text{ elements for } t,s = 1,\ldots,T,\ t > s \end{array}\right\},$$

$$(38) \qquad X_i = \left\{\begin{array}{c} S \times 1 \text{ vector containing all } S \text{ different} \\ x_{its} = x_{it} - x_{is} \text{ elements for } t,s = 1,\ldots,T,\ t > s \end{array}\right\},$$

$$(39) \qquad \theta_i = \left\{ \begin{array}{c} S \times 1 \text{ vector containing all } S \text{ different} \\ \theta_{its} \text{ elements for } t,s = 1,\ldots,T,\ t > s \end{array} \right\},$$

$$(40) \qquad Z_i = \left\{ \begin{array}{c} S \times S(T-2) \text{ block diagonal matrix} \\ \text{containing the } 1 \times (T-2) \text{ vector } z_{its} \\ \text{as a typical block for } t,s = 1,\ldots,T,\ t > s \end{array} \right\},$$

$$i = 1,\ldots,M.$$

The elements of these and the following vectors and matrices are ordered by the subscripts (t,s) in the following succession: $(2,1)$, $(3,2)$, $(4,3)$, ... , $(T,T-1)$, $(3,1)$, $(4,2)$, $(5,3)$, ... , $(T,T-2)$, ... , $(T-1,1)$, $(T,2)$, $(T,1)$. If $T=4$, i.e. $S=6$ and $R=12$, we have, for instance, that Q_i is the 6×1 vector

$$Q_i = \begin{pmatrix} q_{i21} \\ q_{i32} \\ q_{i43} \\ q_{i31} \\ q_{i42} \\ q_{i41} \end{pmatrix} = \begin{pmatrix} q_{i2} - q_{i1} \\ q_{i3} - q_{i2} \\ q_{i4} - q_{i3} \\ q_{i3} - q_{i1} \\ q_{i4} - q_{i2} \\ q_{i4} - q_{i1} \end{pmatrix},$$

that X_i and θ_i are similarly defined, and that Z_i is the 6×12 matrix

$$Z_i = \begin{pmatrix} z_{i21} & 0_{12} & 0_{12} & 0_{12} & 0_{12} & 0_{12} \\ 0_{12} & z_{i32} & 0_{12} & 0_{12} & 0_{12} & 0_{12} \\ 0_{12} & 0_{12} & z_{i43} & 0_{12} & 0_{12} & 0_{12} \\ 0_{12} & 0_{12} & 0_{12} & z_{i31} & 0_{12} & 0_{12} \\ 0_{12} & 0_{12} & 0_{12} & 0_{12} & z_{i42} & 0_{12} \\ 0_{12} & 0_{12} & 0_{12} & 0_{12} & 0_{12} & z_{i41} \end{pmatrix}$$

$$= \begin{pmatrix} q_{i3} & q_{i4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_{i1} & q_{i4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{i1} & q_{i2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{i2} & q_{i4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_{i1} & q_{i3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_{i2} & q_{i3} \end{pmatrix},$$

0_{12} being the 1×2 zero vector. We see that each of the S rows of Z_i contain the *level* of the output variables which are *not* represented, *in differenced form*, in the corresponding element of Q_i. The stacking of the variables in the Z_i matrix is essential.

This stacking permits the auxiliary equations, relating q_{its} to z_{its}, to differ accross different combinations of (t,s) [cf. (32)]. This flexibility in the auxillary equations is a distinctive feature of the GMM, as compared to the 2SLS and the 3SLS, and is necessary in estimating a system of equations, where different instruments are valid for different equations, as is the case here. The flexibilty in the auxiliary equations provided by GMM is also essential to obtain an efficient use of the instruments, at least in large samples.

We use all the $R = S(T-2)$ columns of Z_i as IV's for the column vector Q_i in our estimation equation, which now reads

$$(41) \qquad X_i = Q_i\varepsilon + \theta_i, \qquad i = 1,\ldots,M.$$

Letting

$$Q = \begin{pmatrix} Q_1 \\ \vdots \\ Q_M \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ \vdots \\ X_M \end{pmatrix}, \quad \theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_M \end{pmatrix}, \quad Z = \begin{pmatrix} Z_1 \\ \vdots \\ Z_M \end{pmatrix},$$

which have dimensions $MS \times 1$, $MS \times 1$, $MS \times 1$, and $MS \times S(T-2)$, respectively, we can write the equation, based on MS "observations", compactly as

$$(42) \qquad X = Q\varepsilon + \theta.$$

The *overall 1 step GMM estimator* of ε based on observations for all the M firms and the S pairs of years is

$$(43) \qquad \begin{aligned} \widehat{\varepsilon} &= [Q'Z(Z'Z)^{-1}Z'Q]^{-1}[Q'Z(Z'Z)^{-1}Z'X] \\ &= \left[\left(\sum_{i=1}^{M} Q_i'Z_i\right)\left(\sum_{i=1}^{M} Z_i'Z_i\right)^{-1}\left(\sum_{i=1}^{M} Z_i'Q_i\right)\right]^{-1} \times \\ &\qquad \left[\left(\sum_{i=1}^{M} Q_i'Z_i\right)\left(\sum_{i=1}^{M} Z_i'Z_i\right)^{-1}\left(\sum_{i=1}^{M} Z_i'X_i\right)\right]. \\ &\qquad t,s = 1,\ldots,T,\ t > s. \end{aligned}$$

The latter estimator can be interpreted as an *IV estimator* of ε in (42), utilizing the OLS 'predictor' of Q obtained by regressing Q on the complete IV matrix Z, which is $\widehat{Q} = Z(Z'Z)^{-1}Z'Q$, as IV for Q. This IV estimator, $\widetilde{\varepsilon}^{\,IV} = (\widehat{Q}'Q)^{-1}(\widehat{Q}'X)$, can, after rearrangement, be written as (43).[8]

[8] Confer the analogous derivation of (35) – (36) above and Bowden and Turkington (1984, section 2.4).

The estimator (43) can alternatively be interpreted as a *GLS estimator*. Premultiply (42) by the transposed IV matrix Z', which gives the following equation based on $R = S(T-2)$ "observations" in the matrices $Z'X$ and $Z'Q$, instead of on the "observations" X and Q as in the original form of the equation [cf. Judge et al. (1985, section 15.2.1b)],

$$(44) \qquad Z'X \;=\; Z'Q\,\varepsilon \;+\; Z'\theta.$$

Applying GLS to (44) *when proceeding as if (hypothetically) θ had a scalar covariance matrix*, so that $Z'\theta$ would have a covariance matrix proportional to $Z'Z$, we would get (43).

4.d Full (2-step) GMM estimation with error heteroskedasticity

In constructing the above 1-step GMM estimators, we have paid no regard to the second order moments of the term θ_{its}, related to the second order moments of the 'basic' stochastic elements of the model (cf. appendix A). The R simple year specific IV estimators $\widehat{\varepsilon}_{ts,p}^{IV}$, the S year specific 2SLS estimators $\widehat{\varepsilon}_{ts}^{2SLS}$, and the overall 1-step GMM estimator $\widehat{\varepsilon}$ are all consistent if (23) and (24) are satisfied, but their small sample properties will, of course, depend on the particular specification of these second order moments of θ_{its}.

This fact suggests that we take a further step in exploiting the orthogonality conditions (23) and turn to the full 2-step GMM estimator. This is a more efficient method than the 1-step GMM estimator for estimating equations with general heteroskedasticity and/or autocorrelation of the errors/disturbances. Let us, for this purpose, change the subscript notation a little, using one single time subscript (τ) to represent the number of the difference, instead of two (t,s) as follows: $\tau = 1$ denotes $(t=2, s=1)$, $\tau = 2$ denotes $(t=3, s=2)$, ..., $\tau = S$ denotes $(t=T, s=1)$.[9] We thus replace

$$q_{its}\,,\; x_{its},\; \theta_{its},\; z_{its}, \qquad i = 1,\ldots,M,\; t,s = 1,\ldots,T,\; t > s,$$

by

$$q_{i(\tau)},\; x_{i(\tau)},\; \theta_{i(\tau)},\; z_{i(\tau)}, \qquad i = 1,\ldots,M,\; \tau = 1,\ldots,S,$$

[9] Confer the ordering of the year differences described after (37) – (40).

so that the definitions (37) – (40) read

$$\begin{aligned} Q_i &= (q_{i(1)}, q_{i(2)}, \ldots, q_{i(S)})' , \\ X_i &= (x_{i(1)}, x_{i(2)}, \ldots, x_{i(S)})' , \\ \theta_i &= (\theta_{i(1)}, \theta_{i(2)}, \ldots, \theta_{i(S)})' , \\ Z_i &= diag(z_{i(1)}, z_{i(2)}, \ldots, z_{i(S)}) , \qquad i = 1, \ldots, M, \end{aligned}$$

when we recall that $z_{i(\tau)}$ is a $1 \times (T-2)$ vector. Denote the $S \times S$ covariance matrix of θ_i by Λ_i, i.e. since realizations of θ_i for different firms are assumed to be (approximately) uncorrelated,

$$(45) \qquad E(\theta_i \theta_k') = \delta_{ik}\Lambda_i , \qquad i, k = 1, \ldots, M,$$

where

$$(46) \qquad \Lambda_i = \begin{pmatrix} \lambda_{i11} & \cdots & \lambda_{i1S} \\ \vdots & \cdots & \vdots \\ \lambda_{iS1} & \cdots & \lambda_{iSS} \end{pmatrix}, \qquad i = 1, \ldots, M,$$

and hence

$$(47) \qquad E(\theta\,\theta') = \Lambda = \begin{pmatrix} \Lambda_1 & 0 & \cdots & 0 \\ 0 & \Lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_M \end{pmatrix}.$$

The specific form of Λ_i is implicitly defined in appendix A.[10] In the sequel, we treat the Λ_i's as M positive definit, but otherwise unrestricted, $S \times S$ matrices, allowing for general heteroskedasticity and autocorrelation of the elements of the error/disturbance vectors θ_i .

If Λ *were known* (possibly up to an arbitrary multiplicative constant), the *GMM*

[10] It is possibly restricted [cf. (A.23), (A.27), and (A.29)], and the relationship depends om the "basic" error structure. We do not want to exploit these possible restrictions on this matrix here, and, accordingly, neither do we want to identify and estimate the various σ's in (A.23).

estimator of ε would be

$$(48) \qquad \hat{\varepsilon}^{GMM} = [Q'Z(Z'\Lambda Z)^{-1}Z'Q]^{-1}[Q'Z(Z'\Lambda Z)^{-1}Z'X]$$
$$= \left[\left(\sum_{i=1}^{M} Q_i'Z_i\right)\left(\sum_{i=1}^{M} Z_i'\Lambda_i Z_i\right)^{-1}\left(\sum_{i=1}^{M} Z_i'Q_i\right)\right]^{-1} \times$$
$$\left[\left(\sum_{i=1}^{M} Q_i'Z_i\right)\left(\sum_{i=1}^{M} Z_i'\Lambda_i Z_i\right)^{-1}\left(\sum_{i=1}^{M} Z_i'X_i\right)\right].$$

The latter estimator can, like (43), alternatively be interpreted as a GLS estimator. Consider again the derived equation (44), whose composite disturbance, $Z'\theta$, is asymptotically uncorrelated with its matrix of right hand side variables, $Z'Q$. Applying GLS to (44) *when proceeding as if (hypothetically)* θ *had a known covariance matrix equal to* Λ, so that $Z'\theta$ would have a covariance matrix equal to $Z'\Lambda Z$, we would get (48). This explains, with reference to Gauss-Markov's theorem, intuitively, why $\hat{\varepsilon}^{GMM}$, given by (48), is more efficient than $\hat{\varepsilon}$, given by (43).

Under certain regularity conditions, it can be shown that the asymptotic variance of $\sqrt{MS}(\hat{\varepsilon}^{GMM} - \varepsilon)$ is the limit in probability of

$$(49) \qquad \left[\left(\frac{Q'Z}{MS}\right)\left(\frac{Z'\Lambda Z}{MS}\right)^{-1}\left(\frac{Z'Q}{MS}\right)\right]^{-1}$$
$$= \left[\left(\frac{\sum_{i=1}^{M} Q_i'Z_i}{MS}\right)\left(\frac{\sum_{i=1}^{M} Z_i'\Lambda_i Z_i}{MS}\right)^{-1}\left(\frac{\sum_{i=1}^{M} Z_i'Q_i}{MS}\right)\right]^{-1}.$$

[See Bowden and Turkington (1984, pp. 26, 69).]

Since in our case, Λ_i *is considered as completely unknown*, application of (48) is not feasible. We can, however, proceed as follows [cf. White (1984, pp. 132 – 142)]: Let V_M be the $S(T-2) \times S(T-2)$ covariance matrix of $M^{-1/2}Z'\theta$, i.e.

$$(50) \qquad V_M = \frac{1}{M}E(Z'\theta\theta'Z) = \frac{1}{M}E\left[\sum_{i=1}^{M}\sum_{k=1}^{M} Z_i'\theta_i\theta_k'Z_k\right]$$
$$= \frac{1}{M}\sum_{i=1}^{M} E(Z_i'\theta_i\theta_i'Z_i).$$

The last equality follows because the assumptions that $E(\theta_i\theta_k') = 0$ for all $k \neq i$ [cf. (45)] and that θ_i and Z_i are independently distributed imply $E(Z_i'\theta_i\theta_k'Z_k) = 0$ for all $k \neq i$. The $S \times 1$ vector of residuals for firm i, calculated from (41) and the

1-step GMM estimator of ε, is

$$(51) \qquad \widehat{\theta}_i = X_i - Q_i \widehat{\varepsilon}, \qquad i = 1, \ldots, M.$$

This calculation is the only use we make of the 1-step estimator in this case. Substituting (51) for θ_i in (50), we get the following estimator of V_M [cf. White (1984, sections IV.3 and VI.2)]

$$(52) \qquad \widehat{V}_M = \frac{1}{M} \sum_{i=1}^{M} Z_i' \widehat{\theta}_i \widehat{\theta}_i' Z_i \, .$$

Finally, using the latter as the estimator of $M^{-1}Z'\Lambda Z$ in (48), we get the *feasible GMM estimator of* ε,

$$(53) \qquad \begin{aligned} \widetilde{\varepsilon}^{\,GMM} &= (Q'Z\widehat{V}_M^{-1}Z'Q)^{-1}(Q'Z\widehat{V}_M^{-1}Z'X) \\ &= \left[\left(\sum_{i=1}^{M} Q_i'Z_i \right) \left(\sum_{i=1}^{M} Z_i'\widehat{\theta}_i\widehat{\theta}_i'Z_i \right)^{-1} \left(\sum_{i=1}^{M} Z_i'Q_i \right) \right]^{-1} \times \\ & \qquad \left[\left(\sum_{i=1}^{M} Q_i'Z_i \right) \left(\sum_{i=1}^{M} Z_i'\widehat{\theta}_i\widehat{\theta}_i'Z_i \right)^{-1} \left(\sum_{i=1}^{M} Z_i'X_i \right) \right] . \end{aligned}$$

This estimator is different from $\widehat{\varepsilon}^{\,GMM}$ in finite samples, but they coincide asymptotically. In this way, we do not need to have an estimate of the error/disturbance covariance matrix Λ. The estimate of the asymptotic variance of $\sqrt{MS}(\widetilde{\varepsilon}^{GMM} - \varepsilon)$ is obtained be replacing $M^{-1}Z'\Lambda Z$ in (49) by $\widehat{V}_M$, as given by (52).

4.e Remarks on the literature on GMM and IV estimators

Let us briefly add a few comments on how the estimators we have presented above, are related to the literature on efficient instrumental variable estimation. Both Hansen (1982), White (1982,1984,1986), and Bowden and Turkington (1984) have considered instrumental variable estimators which account for heteroskedastic errors, and improves efficiency along the lines suggested in section 4.d above. Bowden and Turkington (1984, chapter 3.2) and White (1986) termed this estimator the *OLS-analog.* They also discuss a related estimator, denoted as the GLS-analog, which we do not consider here.

The GMM estimator of Hansen (1982) goes beyond the issue of the distribution of the error term. The GMM estimator makes it possible to pool the estimates from a system of equations, where different instruments are valid for different equations.

In our context, different equations correespond to equations for different years. As pointed out in sections 4.c and 4.d above, this possibility is essential in our case, and is not incorporated in any of the estimators presented by White, and Bowden and Turkington.

4.f Modification in the presence of error autocorrelation

The estimation procedures discussed so far rely on the rather strong assumptions (23) – (24). Let us now relax the former and allow for *autocorrelation of the first order in the error in output*, but not of higher order, i.e. ν_{it} is, for all i, a *MA(1) process.* Using discrete time, the dating of the variables within the years may, in practice, often be somewhat arbitrary, which suggests that errors of this form may be a realistic structure. Then $\sigma^{\nu}_{itp} \neq 0$ for $p = t-1, t, t+1$ and $\sigma^{\nu}_{itp} = 0$ for all $p \neq t-1, t, t+1$. Correspondingly, we allow for first, but not higher, order "cross autocorrelation" between the variables summarized in κ^{j}_{its} and the latent output variable q^{*}_{it}. The latter assumption is strong and may be unrealistic in some situations. We then have [cf. (20) and (18)] for all i and j that the orthogonality conditions (23) are replaced by

$$(54) \qquad \sigma^{\theta jq}_{itsp} = 0 \quad \text{for all} \quad t \neq s,\ p \neq (s-1, s, s+1, t-1, t, t+1).$$

In addition, we assume for all i that (24) is replaced by the weaker condition

$$(55) \qquad \sigma^{qq}_{itp} \neq \sigma^{qq}_{isp} \quad \text{for all} \quad t \neq s, p \neq (s-1, s, s+1, t-1, t, t+1).$$

It then follows from (21) and (22) that q_{ip} is a valid IV for $(q_{it} - q_{is})$ for all $p \neq (s-1, s, s+1, t-1, t, t+1)$.

With these modifications, we can proceed as above when we only redefine the 'permissible' combinations of (p, t, s) accordingly. In the equation defining the simple IV estimators of ε, $\widehat{\varepsilon}^{IV}_{ts,p}$, (28), we should for instance replace

$$t, s, p = 1, \ldots, T,\ t > s,\ p \neq (t, s)$$

by

$$t, s, p = 1, \ldots, T,\ t > s,\ p \neq (s-1, s, s+1, t-1, t, t+1).$$

Still there are $S = T(T-1)/2$ different ways of picking two years from the T, but the number of admissible IV's corresponding to each pair is reduced. In some cases, no such IV's may exist. If $T = 3$, the vectors z_{i21}, z_{i32}, and z_{i31} are all empty, which means that $R = 0$, i.e. no permissible IV's and no 2SLS estimator exist. For $T = 4$, we find that z_{i32}, z_{i31}, z_{i42}, and z_{i41} are all empty, and

$$z_{i21} = (q_{i4}), \qquad z_{i43} = (q_{i1}),$$

which means that $R = 2$, and hence the only two existing (consistent) year specific IV estimators, which coincide with the period specific 2SLS estimators, would be

$$\widehat{\varepsilon}_{21}^{2SLS} = \widehat{\varepsilon}_{21,4}^{IV}, \qquad \widehat{\varepsilon}_{43}^{2SLS} = \widehat{\varepsilon}_{43,1}^{IV}.$$

For $T = 5$, z_{i42}, z_{i41}, and z_{i52} are empty, and

$$\begin{aligned} z_{i21} &= (q_{i4}, q_{i5}), & z_{i32} &= (q_{i5}), & z_{i43} &= (q_{i1}), \\ z_{i54} &= (q_{i1}, q_{i2}), & z_{i31} &= (q_{i5}), & z_{i53} &= (q_{i1}), \\ z_{i51} &= (q_{i3}), \end{aligned}$$

which means that the number of simple IV estimators is $R = 9$ and the number of existing period specific 2SLS estimators is 7 in this case. This changes the contents and reduces the dimensions of the vectors z_{its} and the matrices Z_i and Z, but the vectors q_{its}, x_{its}, Q_i, X_i, Q, and X are left unchanged. Otherwise, the IV, the 2SLS, and the GMM estimation procedures can be performed as described in sections 4.a – 4.e.

The above procedures can be extended, rather straightforwardly, to models with errors in output specified as *MA processes of second or higher order* and, correspondingly, with higher order "cross autocorrelation" between κ_{its}^{j} and q_{it}^{*} allowed for. The higher the order of the processes, however, the more IV's are eliminated by using the orthogonality condition (22). If, for instance, $T = 5$, while the error is, for each i, a MA(2) process, then not only z_{i42}, z_{i41}, and z_{i52} are empty [as in the MA(1) case], but also z_{i32}, z_{i43}, z_{i31}, z_{i53}, and z_{i51}. The only non-empty z_{its} vectors would in this case be

$$z_{i21} = (q_{i5}), \qquad z_{i54} = (q_{i1}).$$

This means that both the number of simple IV estimators and the number of existing period specific 2SLS estimators is $R = 2$ in this case. As before, there is one 1-step and one 2-step GMM estimator.

In the estimation procedures described so far, leads and lags are treated symmetrically when selecting the instruments. If *only predetermined IV's are allowed*, the number of possible period specific IV and 2SLS estimators are reduced. Then we should, for instance, for the case with no autocorrelation replace

$$t, s, p = 1, \ldots, T,\ t > s,\ p \neq (t, s)$$

by

$$t, s, p = 1, \ldots, T,\ t > s > p,$$

and for the MA(1) case replace

$$t, s, p = 1, \ldots, T,\ t > s,\ p \neq (s-1, s, s+1, t-1, t, t+1)$$

by

$$t, s, p = 1, \ldots, T,\ t > s,\ s-1 > p.$$

In general, we should, for the MA(q) case replace

$$t, s, p = 1, \ldots, T,\ t > s,\ |s-p| > q,\ |t-p| > q$$

by

$$t, s, p = 1, \ldots, T,\ t > s,\ s-p > q.$$

If the *errors in output, for all i, follow an AR process*, regardless of its order, then they will – since any AR process has an infinite memory – have a non-zero autocorrelation of any order. Consequently, the orthogonality condition (23) will not be satisfied for any (finite) t, s, or p, which means that no valid IV will exist. Hence, none of the estimation procedures described above will be feasible, i.e. they will all produce inconsistent estimators.

5 Data and empirical results

5.a Data description

We have extracted our samples from the annual manufacturing census carried out by Statistics Norway. The sample is balanced and covers the years 1975 – 1982. This early sample period was selected because the data for these years contain information about labour inputs broken down in more detail than for later years. In particular, our data set for these $T = 8$ years contains separate information about the number of blue versus white collar workers employed in each plant (firm), and the working hours for blue collar workers.

From this large data set two industries were chosen: Textiles (ISIC 32) and Chemicals (ISIC 35). These are the 2-digit industries with the smallest and largest plants in Norwegian manufacturing, respectively. In this sense they span the Norwegian population of manufacturing plants. Plants with less than five employees are not included in the sample. We have done some moderate cleaning: Plants were removed from the sample if they had a log of value added per worker deviating by more than 300 percent from the industry-time median of this variable. We used the same trimming criteria based on the log of value added per unit of capital. The trimming reduced our sample by 4 percent in Textiles and by 1 percent in Chemicals, giving a number of firms included equal to $M = 270$ and $M = 247$, respectively.

Our data set reports the numbers of white collar and blue collar workers as separate variables, as mentioned above. The number of working hours for blue collar workers is another labour input variable. Material input, including energy, is the final factor considered in this study. Our output measure is gross output, including net subsidies. All variables are measured as logarithmic deviations from the time (industry) average values, as explained and discussed in section 3.

5.b Results from year specific OLS and 2SLS estimation

Our first set of estimates is presented in tables 1 – 8. As indicated in the second column, the different rows correspond to separate differences and years (t and s). Rows 1 – 7 present the seven estimates based on one year differences, rows 8 to 13 present the six estimates based on two year differences, and so on. Both OLS and two IV (2SLS) estimators [cf. (36)] are reported. The instruments used for the two IV (2SLS) estimators are consistent with respectively (i) Non-autocorrelated errors in the regressor [i.e. log-differenced output, cf. (23) and (54)] and no "cross-autocorrelation" between κ^j_{its} and q^*_{it}, and (ii) MA(1) errors in the regressor, and first (but not higher) order "cross-autocorrelation" between κ^j_{its} and q^*_{it}. The standard error estimates reported in these eight tables are not robust with respect to heteroskedasticity. They are likely to exaggerate the precision of the estimates.

Some general patterns are visible throughout the various sets of estimates, as we consider the different factors of production and the two industries: (i) At least among the differences up to five years (rows 1 – 25), the OLS estimates are higher the larger is the number of years over which the differences are taken. This pattern suggests that the errors-in-variables problem might be smaller the larger is this number of years spanning the difference. One interpretation of this result is that the output variables are non-stationary at the firm level (even after subtracting the year means (!)), while the measurement errors are stationary. The increases in the estimates disappear in general when we consider differences beyond five years (rows 26 – 28). (ii) The IV estimates are higher than the OLS estimates in most cases. The latter finding is consistent with the presence of errors-in-variables in our equation, the effect of which vanish as we instrument its regressor. Some of the IV estimates, however, are very imprecise, as the instruments are poor for the years at the endpoints of our sample. [See, for instance, the results for $(t, s) = (1976, 1975)$ and $(t, s) = (1982, 1981)$ when considering the one year differences.] Lack of precision is in particular a problem for those IV estimates which are consistent with MA(1) errors in output. (iii) There are no systematic differences between the two sets of IV estimates. (iv) Finally, there are major differences between the magnitude of the estimated input response coefficients ε corresponding to different factors of production. Below, we take a closer look at these differences.

Materials

For material inputs, we find estimates that are, on the whole, close to one, see tables 1 (Textiles) and 5 (Chemicals). There is a tendency for the OLS estimates to increase as we move to longer differences (i.e. differences taken over a larger number of years) in the Textile industry, but not for Chemicals. For Textiles, the IV estimates are generally higher than the OLS estimates, while we do not find a similar pattern for the Chemical industry. We can conclude from this that there are some errors (transitory components) in our output variable determining the material input response in the Textile industry, but not in the Chemical industry. The results indicate that material input moves in proportion to output in the Chemical industry, while it responds more than in strict proportion to changes in output in the Textile industry, when we focus on the long differences for the OLS estimates or for the IV estimates (cf. rows 26 – 28 in tables 1 and 5). However, the response is not far from strict proportionality.

When interpreting these results, one should recall, however, that we have deducted the time means from both variables in the input equation. This eliminates any additive time effects, *inter alia* the major part of the price (substitution) effects and effects of the technological change, as discussed in section 2. Clearly, we do not suggest that material input and output have moved proportionally if we take account of technical change and factor substitution.

Blue collar worker hours

Let us then turn to tables 2 (Textiles) and 6 (Chemicals). Clearly, the estimated response coefficients are much smaller for blue collar worker hours than for materials. There is a substantial increase (about 50 percent) in the estimated coefficients as we move from one to seven years differences in the OLS regressions. Such increases are visible for both industries. The long difference OLS estimates are close to the IV estimates. We find these long difference OLS and the IV estimates to cluster around 0.7 for Textiles, and around 0.5 – 0.6 for Chemicals. These estimates may be interpreted as corresponding to strong "increasing returns to scale".

Finding strong "increasing returns to scale" for blue collar worker hours is surprising and perhaps disappointing. Griliches and Hausman (1986), using panel data for $N = 1242$ U.S. manufacturing firms observed over $T = 6$ years, estimated the input response of labour to be closer to unity – around 0.9 when using 2SLS and GMM to pay regard to errors-in-variables. They suggested that the presence of overhead labour could explain the slight tendency to increasing returns to scale in their estimated input equations. Blue collar worker hours may be considered as a measure of labour input (almost) without overhead labour. But we do not find anything close to an input response elasticity equal to one for this measure of labour input. Consequently, after we have removed overhead labour from our measure of the labour input and have taken account of measurement errors, the "increasing returns to scale" puzzle remains .

The large differences between the OLS estimates from short and from long differences suggest the presence of considerable errors-in-variables, since the long difference estimates may have a smaller noise/signal ratio than the short difference estimates. However, the above results indicate that there is not much measurement error, or noise, in the narrow sense, in our output variable. Measurement errors such as misreporting and punching errors should show up in the estimated input response coefficients not only for labour, but also for materials presented in tables 1 and 5. Rather, the noise seems to reflect the differences between the firms' input responses to "temporary" versus "permanent" changes in output, cf. section 1. Our interpretation is that the firms mainly change their labour input to what they consider as permanent changes in output. The observed changes in output include both temporary and permanent changes, and consequently show a stronger year-to-year variation than the permanent changes.

The number of blue collar workers

We have considered the number of blue collar workers as an alternative measure of labour input. The estimated year specific input response coefficients are given in tables 3 and 7. For brevity, we will drop the term "blue collar" in what follows. It is interesting to compare the estimated response coefficients for manhours with those of the number of workers. Such a comparison indicates the extent to which firms use changes in hours per worker to adjust to output changes.

The estimated response coefficient for the number of workers exhibits much the same pattern as the response coefficient for worker hours. In particular, they are similar if we focus on the long difference OLS estimates or on the IV estimates. Not surprisingly, we find that the firms do not adjust the number of hours per workers when there are permanent changes in output. By looking at the OLS estimates for one year differences, we find that worker hours respond more strongly to transitory changes in output than do the number of workers. The means of the permanent and the temporary input response coefficients are 0.54 and 0.39, respectively, for Textiles, 0.28 and 0.22, respectively, for Chemicals.

To summarize: First, comparing the OLS results for worker hours and the number of workers, it is evident that there is a stronger tendency to labour hoarding in the number of workers than in the number of working hours. Second, when we compare the OLS results for short and long differences, or the IV estimates, we find clear evidence of labour hoarding in both worker hours and the number of workers. Finally, considering the long differences OLS and the IV estimates, we find similar responses in the number of workers and the number of working hours, to permanent changes in output. Thus, in the long run, the length of the working day is not a margin the (individual) firm uses to adjust to changes in output. None of these findings are surprising, but it is reassuring that they show up in our data.

The number of white collar workers

Tables 4 and 8 present the corresponding estimated input response coefficients for the number of white collar workers. Its seems reasonable that this measure of labour input responds very differently to temporary and to permanent changes in output. This is confirmed when we compare the OLS estimates for the short and the long differences, or if we compare the OLS and the IV estimates for the short differences. The average response coefficient is as low as 0.15 for Textiles and 0.17 for Chemicals when we focus on one year differences in the OLS regressions only. These estimates reveal very modest responses to observed changes in output. Turning to seven years differences, we find that the OLS estimates are 0.48 and 0.44, respectively. We obtain similar estimates from the IV procedures.

The IV estimates based on the assumption of non-autocorrelated, i.e. MA(0), measurement errors increase strongly as we move from short (one year) to gradually longer differences for the Textile industry. This pattern is similar to the pattern of the OLS estimates. One interpretation we may give of this finding is that instruments based on the assumption of MA(0) errors do not give consistent estimates. These IV estimates, assuming MA(0) errors, suffer from a similar bias as the OLS estimates. The estimates based on the MA(1) specification are less sensitive to the "length" of the differences.

5.c Results based on the GMM for all years combined

Tables 9 and 10 report the main results obtained when we pool the data set underlying all the estimates in tables 1 – 8, again considering each factor or factor input measure separately and use the 1-step GMM estimator as given by (43), and the 2-step GMM estimator as given by (51) – (53). The Textile industry favours results based on the choice of instrumental variables valid under the assumption of a MA(1) process for the error term in output. This is in accordance with the results presented above. For most of the estimated input equations for Chemicals, however, the MA(0) specification seems to be an acceptable simplification. These conclusions follows from Hausman tests [Hausman (1978)]; the test statistics are presented in table 11. The GMM estimates assuming a MA(0) process for the error term, are more efficient than estimates based on a MA(1) process, if the former is correct. We can therefore use Hausman's difference formula for the variance of the difference between the estimators. A MA(1) specification of the error structure does not seem implausible since it could be generated by differences in the dating of the inputs and the output.

In the Textile industry (table 9), we find an estimated input response coefficient significantly less than one, except for materials. Our estimates indicate very similar input responses for changes in blue collar worker hours and for changes in the number of blue collar workers, which is reasonable, as remarked above, since it seems unlikely that firms adjust the number of hours per worker in response to *permanent* changes in output. For white collar workers, the GMM estimate of the input response coefficient is very low (around 0.4). When we compare the different input response elasticities, our results give a clear indication of a non-homothetic technology [confer (7) – (8) and (9) – (11) and our related comments in section 2.] This is not only true for materials as compared with labour input. The difference in the estimated input response is also striking when we compare blue and white collar workers. The low response of white collar workers to output changes confirm the view that this kind of labour input is largely a fixed factor. Its small estimated response coefficient is interesting when we recall that our latent variables modelling in combination with instrumental variable procedures (to a large extent) eliminates the effects of temporary changes in output. The low input response for this kind of labour was also recognized in Griliches and Hausman (1986, p. 108). One of the interesting results of our study is that removing this (approximately fixed) part of the labour input, and focusing on blue collar worker hours only, did not pull the estimated (permanent, long-term) input response coefficient very close to unity. Consequently, the increasing returns to scale puzzle remains, even when we consider only the response of (working hours for) blue collar workers to permanent changes in output.

The precision of the GMM estimates reported in tables 9 and 10 (estimated as explained in section 4.d) is quite good. The efficiency gains when using the pooled data set for all the $T = 8$ years, assuming MA(1) measurement errors (confer the standard error estimates in the last column of tables 1 – 8), are substantial. This conclusion is strengthened when we recall that the year specific standard errors estimates reported in tables 1 – 8 probably underestimate the correct standard errors, since they, in contrast to those in tables 9 and 10, are not corrected for heteroskedasticity.

Turning to the results for the Chemical industry (table 10), we find very low estimates of the input response coefficients. As for Textiles, the only exception is material input. The preferred estimates are based on the assumption of MA(0) measurement errors, except for blue collar worker hours. As for Textiles, we find a very similar input response for blue collar hours and the number of blue collar workers. Again, the input response for white collar workers is much lower than for blue collar workers.

6 Concluding remarks

In this paper, we have been concerned with the estimation of firms' input response – in particular with respect to labour input – to changes in the output volume, within the framework of a single equation log-linear errors-in-variables model. Our main argument for using this framework for our input response equations is not only the potential existence of measurement errors in outputs and inputs in the narrow sense, but also the hypothesis that the firms adjust their (labour) input only to output changes which they consider *permanent*, not to *temporary* changes, both permanent and temporary changes being treated as latent variables in the econometric model.

A basic idea of our estimation procedure is to use as IV's for the observed log-*differenced* output the observed log of output in *level* form for other years than those to which the differences refer. A problem with this approach is that the IV's in level form thus constructed may be *weak instruments*, in the sense that they are weakly correlated with the differenced variables for which they are used as instruments, while being *potentially* correlated with the composite error term(s) of the input response equation(s) under estimation. If a suggested IV is weakly correlated with the variable in an equation for which it is intended to serve as an instrument, it may be shown that even a small correlation between the disturbance/error term and the suggested IV may give estimates which are severely biased. [Cf. Bound, Jaeger, and Baker (1993)]. In our empirical application, leads and lags are always treated symmetrically when selecting the instruments. An alternative approach might be to use *predetermined* output variables as IV's for differenced outputs only. This is an issue which is related to the issue of weak versus strong instruments. Using only predetermined instruments might reduce the problem of correlation with the disturbance term, but the power of the instruments will also decline (often substantially according to our experience). Furthermore, the choice of instruments should be related to the assumptions which are made with respect to the *expectation mechanism* (for instance adaptive or rational expectations) for future output changes which governs the firms' observed factor adjustment in each year. These topics clearly need further research.

Conclusions which can be tentatively drawn from the results in section 5 are the following: (i) Despite our focus on the permanent input adjustment, there seems to be "increasing returns to scale". This tendency is clearly seen for the labour input. There are, however, substantial differences between the response coefficients for different measures of labour. (ii) Because of the substantial differences between the response coefficients between labour and materials on the one hand and between different kinds of labour input on the other, the common *a priori* assumption of a *homothetic technology* does not seem to be justified. (iii) The relationship between our estimated 'input response coefficients' and the underlying scale elasticity in the firms' production technology is not obvious.

The latter issue definitely deserves a closer examination, maybe within the framework of a modelling and estimation of a complete system of factor equations derived explicitly from the hypothesis of price taking and cost-minimizing firms with a *non-homothetic technology.* Throughout, we have 'swept away' from the input and output variables all additive time specific components, by measuring the variables (in log form) from their year specific means. An alternative approach is to attempt to model explicitly the time effect by means of a small number of parameters as an integral part of the model structure, instead of calculating and deducting $T = 8$ year specific means for each observable variable 'outside of' the model.

Table 1. Textiles (ISIC 32). OLS and IV estimates of the materials output relationship

Years for difference		Coefficient (standard error)		
			Consistent IV estimates	
		OLS	MA(0)	MA(1)
(1)	1975-76	0.983 (0.052)	0.977 (0.184)	1.019 (0.194)
(2)	1976-77	0.899 (0.050)	1.062 (0.087)	1.250 (0.378)
(3)	1977-78	0.974 (0.046)	1.108 (0.074)	1.144 (0.118)
(4)	1978-79	0.891 (0.047)	1.031 (0.094)	0.733 (0.144)
(5)	1979-80	0.996 (0.045)	1.138 (0.075)	0.903 (0.118)
(6)	1980-81	1.011 (0.040)	1.025 (0.066)	1.043 (0.119)
(7)	1981-82	0.991 (0.028)	1.158 (0.245)	1.135 (0.244)
(8)	1975-77	0.981 (0.048)	1.045 (0.112)	0.600 (0.457)
(9)	1976-78	1.044 (0.041)	0.972 (0.064)	1.244 (0.279)
(10)	1977-79	0.970 (0.044)	1.078 (0.076)	0.959 (0.097)
(11)	1978-80	0.983 (0.036)	1.045 (0.058)	1.003 (0.078)
(12)	1979-81	1.060 (0.040)	1.048 (0.059)	0.925 (0.142)
(13)	1980-82	1.002 (0.031)	0.990 (0.064)	1.028 (0.125)
(14)	1975-78	1.036 (0.044)	1.054 (0.058)	1.137 (0.399)
(15)	1976-79	0.943 (0.046)	1.038 (0.057)	0.975 (0.195)
(16)	1977-80	1.016 (0.033)	1.044 (0.042)	1.081 (0.055)
(17)	1978-81	1.037 (0.036)	1.052 (0.047)	1.195 (0.369)
(18)	1979-82	0.991 (0.033)	1.090 (0.048)	0.921 (0.141)
(19)	1975-79	0.955 (0.043)	1.124 (0.064)	1.133 (0.124)
(20)	1976-80	1.042 (0.034)	1.047 (0.031)	1.087 (0.075)
(21)	1977-81	1.068 (0.034)	1.097 (0.041)	1.043 (0.102)
(22)	1978-82	1.009 (0.031)	1.040 (0.043)	1.029 (0.073)
(23)	1975-80	1.064 (0.036)	1.057 (0.046)	1.165 (0.071)
(24)	1976-81	1.061 (0.033)	1.105 (0.038)	0.301 (0.513)
(25)	1977-82	1.050 (0.028)	1.077 (0.037)	1.060 (0.056)
(26)	1975-81	1.080 (0.036)	1.114 (0.043)	1.061 (0.081)
(27)	1976-82	1.063 (0.028)	1.068 (0.035)	1.102 (0.069)
(28)	1975-82	1.073 (0,032)	1.098 (0.040)	1.127 (0.055)

Table 2. Textiles (ISIC 32). OLS and IV estimates of the worker hours output relationship

		Coefficient (standard error)		
			Consistent IV estimates	
Years for difference		OLS	MA(0)	MA(1)
(1)	1975-76	0.386 (0.057)	0.517 (0.201)	0.565 (0.214)
(2)	1976-77	0.536 (0.061	0.636 (0.104)	1.353 (0.545)
(3)	1977-78	0.442 (0.052)	0.596 (0.085)	0.512 (0.132)
(4)	1978-79	0.569 (0.054)	0.742 (0.110)	0.660 (0164)
(5)	1979-80	0.386 (0.053)	0.467 (0.088)	0.529 (0.140)
(6)	1980-81	0.635 (0.047)	0.726 (0.078)	0.431 (0.145)
(7)	1981-82	0.734 (0.051)	0.992 (0.433)	1.054 (0.448)
(8)	1975-77	0.452 (0.059)	0.577 (0.139)	-0.012 (0.563)
(9)	1976-78	0.581 (0.045)	0.533 (0.070)	0.660 (0.292)
(10)	1977-79	0.596 (0.055)	0.734 (0.096)	0.585 (0.122)
(11)	1978-80	0.451 (0.046)	0.590 (0.075)	0.645 (0.103)
(12)	1979-81	0.651 (0.047	0.665 (0.069)	0.397 (0.170)
(13)	1980-82	0.785 (0.052)	1.033 (0.133)	0.703 (0.211)
(14)	1975-78	0.525 (0.048)	0.621 (0.065)	0.757 (0.454)
(15)	1976-79	0.630 (0.050)	0.664 (0.061	0.526 (0.212)
(16)	1977-80	0.514 (0.051)	0.581 (0.066)	0.589 (0.085)
(17)	1978-81	0.665 (0.046)	0.684 (0.060)	0.473 (0.468)
(18)	1979-82	0.782 (0.049)	0.837 (0.071)	0.550 (0.215)
(19)	1975-79	0.581 (0.050)	0.756 (0.075)	0.658 (0.140)
(20)	1976-80	0.585 (0.047)	0.635 (0.056)	0.543 (0.102)
(21)	1977-81	0.608 (0.051)	0.630 (0.061)	0.921 (0.163)
(22)	1978-82	0.765 (0.048)	0.836 (0.067)	0.680 (0.114)
(23)	1975-80	0.600 (0.047)	0.656 (0.060)	0.700 (0.091)
(24)	1976-81	0.652 (0.045)	0.678 (0.052)	0.561 (0.406)
(25)	1977-82	0.710 (0.049	0.716 (0.065)	0.711 (0.097)
(26)	1975-81	0.654 (0.045)	0.697 (0.054)	0.642 (0.103)
(27)	1976-82	0.740 (0.046)	0.757 (0.056)	0.622 (0.122)
(28)	1975-82	0.748 (0.046)	0.735 (0.059)	0.704 (0.080)

Table 3. Textiles (ISIC 32). OLS and IV estimates of the number of workers output relationship

		Coefficient (standard error)		
			Consistent IV estimates	
Years for difference		OLS	MA(0)	MA(1)
(1)	1975-76	0.361 (0.053)	0.427 (0.185)	0.469 (0.196)
(2)	1976-77	0.459 (0.057)	0.554 (0.096)	0.865 (0.427)
(3)	1977-78	0.341 (0.042)	0.474 (0.068)	0.513 (0.109)
(4)	1978-79	0.387 (0.048	0.659 (0.101)	0.687 (0.154)
(5)	1979-80	0.189 (0.038)	0.267 (0.062)	0.318 (0.100)
(6)	1980-81	0.533 (0.049)	0.621 (0.080)	0.393 (0.146)
(7)	1981-82	0.612 (0.049)	0.645 (0.397)	0.724 (0.405)
(8)	1975-77	0.389 (0.056)	0.558 (0.132)	-0.245 (0.582)
(9)	1976-78	0.510 (0.045)	0.453 (0.069)	0.320 (0.298)
(10)	1977-79	0.438 (0.043)	0.674 (0.078)	0.742 (0.104)
(11)	1978-80	0.433 (0.046)	0.621 (0.076)	0.660 (0.105)
(12)	1979-81	0.426 (0.043)	0.494 (0.064)	0.157 (0.160)
(13)	1980-82	0.682 (0.052)	0.921 (0.133)	0.608 (0.212)
(14)	1975-78	0.474 (0.047)	0.541 (0.064)	0.308 (0.439)
(15)	1976-79	0.505 (0.049	0.606 (0.061)	0.423 (0.208)
(16)	1977-80	0.465 (0.043)	0.565 (0.057)	0.628 (0.074)
(17)	1978-81	0.605 (0.048)	0.610 (0.063)	0.029 (0.587)
(18)	1979-82	0.614 (0.048)	0.667 (0.069)	0.318 (0.217)
(19)	1975-79	0.478 (0.048	0.676 (0.073)	0.715 (0.096)
(20)	1976-80	0.536 (0.044)	0.584 (0.053)	0.522 (0.096)
(21)	1977-81	0.549 (0.047)	0.582 (0.057)	0.873 (0.153)
(22)	1978-82	0.693 (0.049)	0.779 (0.069	0.638 (0.117)
(23)	1975-80	0.550 (0.046)	0.592 (0.057)	0.631 (0.088)
(24)	1976-81	0.583 (0.046)	0.621 (0.054)	0.655 (0.415)
(25)	1977-82	0.646 (0.048)	0.677 (0.063)	0.672 (0.095)
(26)	1975-81	0.598 (0.046)	0.630 (0.055)	0.584 (0.104)
(27)	1976-82	0.661 (0.048	0.690 (0.058	0.636 (0.115)
(28)	1975-82	0.676 (0.047	0.673 (0.059	0.638 (0.080)

Table 4. Textiles (ISIC 32). OLS and IV estimates of the number of white collar workers output relationship

		Coefficient (standard error)		
			Consistent IV estimates	
Years for difference		OLS	MA(0)	MA(1)
(1)	1975-76	0.435 (0.084)	0.233 (0.297)	0.276 (0.312)
(2)	1976-77	0.126 (0.072)	0.312 (0.123)	0.744 (0.566)
(3)	1977-78	0.103 (0.086)	0.171 (0.138)	0.415 (0.222)
(4)	1978-79	0.050 (0.072)	0.400 (0.149)	0.209 (0.217)
(5)	1979-80	0.145 (0.051)	0.215 (0.084)	0.481 (0.142)
(6)	1980-81	0.153 (0.053)	0.271 (0.088)	0.278 (0.159)
(7)	1981-82	0.312 (0.054)	0.307 (0.438)	0.310 (0.443)
(8)	1975-77	0.428 (0.078)	0.631 (0.184)	1.383 (0.838)
(9)	1976-78	0.175 (0.069	0.525 (0.122)	1.182 (0.600)
(10)	1977-79	0.245 (0.075)	0.353 (0.128)	0.501 (0.168)
(11)	1978-80	0.195 (0.069)	0.299 (0.111)	0.450 (0.154)
(12)	1979-81	0.211 (0.050)	0.313 (0.073)	0.392 (0.175)
(13)	1980-82	0.293 (0.050)	0.383 (0.105)	0.301 (0.203)
(14)	1975-78	0.461 (0.073)	0.411 (0.098)	2.067 (1.110)
(15)	1976-79	0.300 (0.069)	0.460 (0.086)	0.592 (0.303)
(16)	1977-80	0.311 (0.072)	0.361 (0.094)	0.480 (0.122)
(17)	1978-81	0.230 (0.063)	0.399 (0.084)	0.639 (0.669)
(18)	1979-82	0.311 (0.051)	0.351 (0.073)	0.509 (0.218)
(19)	1975-79	0.478 (0.063)	0.432 (0.093)	0.771 (0.184)
(20)	1976-80	0.386 (0.067)	0.505 (0.080)	0.565 (0.147)
(21)	1977-81	0.300 (0.066)	0.356 (0.079)	0.564 (0.204)
(22)	1978-82	0.343 (0.057)	0.361 (0.079)	0.613 (0.141)
(23)	1975-80	0.502 (0.065)	0.520 (0.082)	0.594 (0.125)
(24)	1976-81	0.378 (0.059)	0.467 (0.069)	1.024 (0.637)
(25)	1977-82	0.352 (0.061)	0.399 (0.080)	0.468 (0.121)
(26)	1975-81	0.497 (0.062)	0.452 (0.074)	0.519 (0.141
(27)	1976-82	0.419 (0.057)	0.459 (0.069)	0.546 (0.138)
(28)	1975-82	0.458 (0.058)	0.507 (0.074)	0.484 (0.100)

Table 5. Chemicals (ISIC 35). OLS and IV estimates of the materials output relationship

Years for difference		Coefficient (standard error)		
			Consistent IV estimates	
		OLS	MA(0)	MA(1)
(1)	1975-76	0.838 (0.032)	0.745 (0.116)	0.773 (0.115)
(2)	1976-77	0.809 (0.041)	1.032 (0.127)	0.734 (0.172)
(3)	1977-78	0.910 (0.034)	0.972 (0.064)	1.030 (0.089)
(4)	1978-79	0.928 (0.035)	0.960 (0.057)	1.042 (0.081)
(5)	1979-80	0.902 (0.055)	1.037 (0.115)	1.209 (0.184)
(6)	1980-81	1.030 (0.054)	0.886 (0.090)	0.898 (0.223)
(7)	1981-82	0.983 (0.037)	0.564 (0.389)	0.542 (0.397)
(8)	1975-77	0.899 (0.038)	1.083 (0.178)	1.099 (0.239)
(9)	1976-78	0.982 (0.042)	1.179 (0.109)	0.608 (0.381)
(10)	1977-79	0.942 (0.030)	1.029 (0.047)	1.028 (0.057)
(11)	1978-80	1.023 (0.037)	1.078 (0.064)	1.125 (0.085)
(12)	1979-81	0.851 (0.049)	0.900 (0.084)	1.299 (0.547)
(13)	1980-82	0.970 (0.038)	0.934 (0.073)	0.938 (0.219)
(14)	1975-78	0.933 (0.033)	0.958 (0.070)	0.611 (0.250)
(15)	1976-79	0.972 (0.034)	1.051 (0.048)	0.776 (0.256)
(16)	1977-80	1.014 (0.032)	1.056 (0.041)	1.009 (0.064)
(17)	1978-81	0.961 (0.038)	0.962 (0.052)	0.027 (0.859)
(18)	1979-82	0.905 (0.039)	0.971 (0.063)	2.054 (1.560)
(19)	1975-79	0.937 (0.029)	0.993 (0.054)	0.992 (0.063)
(20)	1976-80	0.998 (0.035	1.109 (0.054)	0.977 (0.083)
(21)	1977-81	0.985 (0.032)	1.016 (0.042)	0.978 (0.068)
(22)	1978-82	1.013 (0.033)	1.024 (0.046)	1.066 (0.072)
(23)	1975-80	0.937 (0.030)	1.039 (0.046)	1.030 (0.059)
(24)	1976-81	1.002 (0.037)	1.006 (0.046)	0.955 (0.097)
(25)	1977-82	1.012 (0.030)	1.058 (0.038)	0.987 (0.056)
(26)	1975-81	0.929 (0.031)	1.004 (0.046)	1.039 (0.059)
(27)	1976-82	1.012 (0.031)	1.053 (0.043)	0.932 (0.075)
(28)	1975-82	0.941 (0.028)	1.030 (0.041)	1.080 (0.058)

Table 6. Chemicals (ISIC 35). OLS and IV estimates of the worker hours output relationship

		Coefficient (standard error)		
			Consistent IV estimates	
Years for difference		OLS	MA(0)	MA(1)
(1)	1975-76	0.123 (0.037)	-0.001 (0.136)	0.022 (0.135)
(2)	1976-77	0.227 (0.057)	0.446 (0.170)	0.098 (0.238)
(3)	1977-78	0.421 (0.054)	0.602 (0.103)	0.185 (0.144)
(4)	1978-79	0.291 (0.051)	0.319 (0.083)	0.412 (0.116)
(5)	1979-80	0.213 (0.058)	0.462 (0.123)	0.381 (0.184)
(6)	1980-81	0.384 (0.062)	0.533 (0.105)	-0.221 (0.303)
(7)	1981-82	0.603 (0.060)	0.446 (0.515)	0.424 (0.519)
(8)	1975-77	0.360 (0.045)	0.163 (0.211)	-0.462 (0.416)
(9)	1976-78	0.630 (0.055)	0.895 (0.145)	0.677 (0.442)
(10)	1977-79	0.355 (0.046)	0.455 (0.070)	0.316 (0.085)
(11)	1978-80	0.390 (0.052)	0.442 (0.090)	0.432 (0.118)
(12)	1979-81	0.427 (0.052)	0.677 (0.093)	0.887 (0.576)
(13)	1980-82	0.620 (0.049)	0.620 (0.095)	0.676 (0.284)
(14)	1975-78	0.349 (0.047)	0.651 (0.108)	0.094 (0.317)
(15)	1976-79	0.526 (0.054)	0.638 (0.075)	0.486 (0.381)
(16)	1977-80	0.410 (0.045)	0.511 (0.058)	0.492 (0.090)
(17)	1978-81	0.463 (0.052)	0.495 (0.071)	-0.357 (0.895)
(18)	1979-82	0.578 (0.048)	0.677 (0.077)	1.092 (1.088)
(19)	1975-79	0.372 (0.043)	0.673 (0.088)	0.506 (0.096)
(20)	1976-80	0.564 (0.050)	0.672 (0.078)	0.568 (0.119)
(21)	1977-81	0.426 (0.048)	0.565 (0.064)	0.341 (0.104)
(22)	1978-82	0.610 (0.049)	0.618 (0.068)	0.605 (0.108)
(23)	1975-80	0.453 (0.042)	0.660 (0.066)	0.601 (0.084)
(24)	1976-81	0.645 (0.051)	0.661 (0.064)	0.417 (0.140)
(25)	1977-82	0.573 (0.046)	0.597 (0.058)	0.515 (0.087)
(26)	1975-81	0.427 (0.044)	0.687 (0.071)	0.489 (0.084)
(27)	1976-82	0.698 (0.048)	0.745 (0.066)	0.600 (0.116)
(28)	1975-82	0.558 (0.042)	0.704 (0.063)	0.642 (0.086)

Table 7. Chemicals (ISIC 35). OLS and IV estimates of the number of workers output relationship

Years for difference		Coefficient (standard error)		
			Consistent IV estimates	
		OLS	MA(0)	MA(1)
(1)	1975-76	0.162 (0.035)	-0.011 (0.130)	0.001 (0.129)
(2)	1976-77	0.102 (0.054)	0.369 (0.164)	0.201 (0.223)
(3)	1977-78	0.296 (0.049)	0.535 (0.095)	0.289 (0.124)
(4)	1978-79	0.220 (0.051)	0.265 (0.082)	0.371 (0.116)
(5)	1979-80	0.173 (0.053)	0.408 (0.112)	0.266 (0.167)
(6)	1980-81	0.388 (0.057)	0.618 (0.098)	0.068 (0.251)
(7)	1981-82	0.539 (0.062)	0.107 (0.577)	0.079 (0.585)
(8)	1975-77	0.288 (0.044)	0.301 (0.201)	-0.151 (0.316)
(9)	1976-78	0.611 (0.056)	0.994 (0.153)	0.833 (0.461
(10)	1977-79	0.251 (0.044)	0.402 (0.068)	0.273 (0.081
(11)	1978-80	0.319 (0.051)	0.409 (0.089)	0.429 (0.116)
(12)	1979-81	0.412 (0.050)	0.633 (0.089)	0.778 (0.532)
(13)	1980-82	0.598 (0.050)	0.694 (0.096)	0.948 (0.314)
(14)	1975-78	0.403 (0.044)	0.646 (0.099)	0.294 (0.285)
(15)	1976-79	0.536 (0.052)	0.604 (0.071)	0.365 (0.370)
(16)	1977-80	0.299 (0.045)	0.438 (0.058)	0.370 (0.090)
(17)	1978-81	0.406 (0.052)	0.463 (0.071)	-0.467 (0.927)
(18)	1979-82	0.541 (0.049)	0.654 (0.078)	0.170 (1.019)
(19)	1975-79	0.364 (0.043)	0.709 (0.089)	0.539 (0.096)
(20)	1976-80	0.546 (0.049)	0.605 (0.075)	0.448 (0.117)
(21)	1977-81	0.347 (0.045)	0.484 (0.061)	0.291 (0.098)
(22)	1978-82	0.570 (0.050)	0.607 (0.070)	0.587 (0.111)
(23)	1975-80	0.434 (0.042)	0.638 (0.067)	0.604 (0.085)
(24)	1976-81	0.626 (0.050)	0.632 (0.063)	0.314 (0.142)
(25)	1977-82	0.489 (0.046)	0.565 (0.058)	0.510 (0.086)
(26)	1975-81	0.433 (0.043)	0.680 (0.069)	0.501 (0.082)
(27)	1976-82	0.674 (0.049)	0.733 (0.067)	0.536 (0.119)
(28)	1975-82	0.563 (0.042)	0.704 (0.063)	0.672 (0.085)

Table 8. Chemicals (ISIC 35). OLS and IV estimates of the number of white collar workers output relationship

Years for difference		Coefficient (standard error)		
			Consistent IV estimates	
		OLS	MA(0)	MA(1)
(1)	1975-76	0.103 (0.050)	-0.248 (0.196)	-0.244 (0.196)
(2)	1976-77	0.178 (0.076)	0.194 (0.219)	-0.600 (0.373)
(3)	1977-78	0.159 (0.047)	0.321 (0.090)	0.481 (0.132)
(4)	1978-79	0.276 (0.056)	0.258 (0.090)	0.467 (0.128)
(5)	1979-80	0.123 (0.103)	0.258 (0.216)	0.461 (0.328)
(6)	1980-81	0.045 (0.080)	0.261 (0.135)	-0.315 (0.342)
(7)	1981-82	0.256 (0.073)	1,355 (0.862)	1.411 (0.886)
(8)	1975-77	0.400 (0.059)	0.437 (0.268)	0.173 (0.366)
(9)	1976-78	0.275 (0.078)	0.579 (0.201)	0.023 (0.633)
(10)	1977-79	0.195 (0.050)	0.485 (0.081)	0.362 (0.094)
(11)	1978-80	0.428 (0.072)	0.692 (0.129)	0.816 (0.173)
(12)	1979-81	0.294 (0.089)	0.498 (0.155)	-0.541 (1.002)
(13)	1980-82	0.223 (0.070)	0.257 (0.136)	0.102 (0.409)
(14)	1975-78	0.410 (0.059)	0.271 (0.127)	-0.184 (0.449)
(15)	1976-79	0.342 (0.070)	0.292 (0.096)	0.864 (0.542)
(16)	1977-80	0.370 (0.064)	0.595 (0.084)	0.635 (0.132)
(17)	1978-81	0.462 (0.074)	0.495 (0.101)	-0.264 (1.059)
(18)	1979-82	0.310 (0.077)	0.385 (0.123)	0.737 (1.536)
(19)	1975-79	0.341 (0.058)	0.335 (0.107)	0.121 (0.129)
(20)	1976-80	0.393 (0.078)	0.529 (0.120)	0.591 (0.187)
(21)	1977-81	0.329 (0.064)	0.627 (0.088)	0.540 (0.140)
(22)	1978-82	0.406 (0.068)	0.446 (0.095)	0.765 (0.158)
(23)	1975-80	0.451 (0.069)	0.521 (0.104)	0.343 (0.134)
(24)	1976-81	0.467 (0.080)	0.463 (0.101)	0.545 (0.211)
(25)	1977-82	0.346 (0.059)	0.525 (0.076)	0.642 (0.117)
(26)	1975-81	0.441 (0.067)	0.446 (0.101)	0.226 (0.130)
(27)	1976-82	0.460 (0.071)	0.448 (0.096)	0.411 (0.169)
(28)	1975-82	0.443 (0.063)	0.441 (0.092)	0.259 (0.128)

Table 9. Textiles (ISIC 32). GMM and 2SLS estimates of the output response for various factor inputs. MA(0) or MA(1) measurement errors are assumed. All years and differences are pooled

	Materials	Blue collar worker hours	Blue collar workers	White collar workers
MA(0) measurement errors				
1 step GMM	1.073	0.701	0.630	0.426
(2 step) GMM	1.044	0.692	0.520	0.281
	(0.022)	(0.030)	(0.035)	(0.045)
MA(1) measurement errors				
1 step GMM	1.087	0.782	0.737	0.390
(2 step) GMM	0.937	0.817	0.850	0.407
	(0.030)	(0.036)	(0.043)	(0.060)
Plants	270	270	270	270

Table 10. Chemicals (ISIC 35). GMM and 2SLS estimates of the output response for various factor inputs. MA(0) or MA(1) measurement errors are assumed. All years and differences are pooled

	Materials	Blue collar worker hours	Blue collar workers	White collar workers
MA(0)-measurement errors				
1 step GMM	1.023	0.615	0.590	0.461
(2 step) GMM	1.016	0.596	0.616	0.417
	(0.017)	(0.027)	(0.030)	(0.044)
MA(1)-measurement errors				
1 step GMM	1.039	0.320	0.321	0.288
(2 stpe) GMM	1.021	0.349	0.587	0.347
	(0.029)	(0.036)	(0.045)	(0.069)
Plants	247	247	247	247

Table 11. Tests for the validity of instruments based on MA(0) vs. MA(1) structure in the residual terms[1]. (See tables 9 and 10)

	Materials	Blue collar worker hours	Blue collar workers	White collar workers
Textiles (ISIC 32)	-5.25*	6.28*	13.21*	3.17*
Chemicals (ISIC 35)	0.21	-10.37*	-0.86	-1.32

Footnotes: * Significantly different from 0 at 1% significant level.
[1] The test statistics are asymptotically distributed as N(0,1) under the assumption of MA(0) residuals.

Appendix A. Stochastic specification. Details

The purpose of this appendix is to explain the derivation of eqs. (13) – (14) from (7) – (8) and discuss their stochastic specification in more detail than we did in section 3 above.

Defining

$$\text{(A.1)} \qquad \delta_{\varepsilon its} = \varepsilon_{its} - \bar{\varepsilon}\,, \qquad i = 1,\ldots,M,\ t,s = 1,\ldots,T,$$

we can reformulate (7) and (8) as

$$\text{(A.2)} \qquad x^{*j}_{it} - x^{*j}_{is} = \bar{\varepsilon}(q^*_{it} - q^*_{is}) + \xi^j_{its}\,,$$

$$j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T,$$

where

$$\text{(A.3)} \qquad \begin{aligned} \xi^j_{its} &= \kappa^j_{its} + \delta_{\varepsilon its}(q^*_{it} - q^*_{is}) \\ &= \sum_{k=1}^{N} \gamma^{jk}_{its} \ln(w^{*k}_{it}/w^{*k}_{is}) + \ln(f^j_{it}/f^j_{is}) - \bar{\varepsilon}\ln(\Phi_{it}/\Phi_{is}) \\ &\qquad + \delta_{\varepsilon its}(q^*_{it} - q^*_{is}) - \delta_{\varepsilon its}\ln(\Phi_{it}/\Phi_{is}). \end{aligned}$$

We assume that the firm and year dependent input response coefficients, ε_{its}, are distributed independently of the latent output volume q^*_{it} and of the composite variables κ^j_{its}. Since (A.2) and (A.3) are obtained by exploiting a mean value theorem (cf. section 2), this is a strong, but important assumption. The two latter variables, however, may be correlated. Below, we specify further our stochastic assumptions about (A.2) and (A.3).

Let us first, however, take a look at the relations which correspond to (A.1) – (A.3) in case of *non-homotheticity* of the production technology. We can then assume, within a corresponding simplifying random coefficients framework, that the input response elasticities ε^j_{its} can be treated as random variables with firm and year invariant, *but input specific*, means equal to $\bar{\varepsilon}^j$. Defining

$$\text{(A.4)} \qquad \delta^j_{\varepsilon its} = \varepsilon^j_{its} - \bar{\varepsilon}^j, \qquad j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T,$$

we can modify (A.2) – (A.3) to

$$\text{(A.5)} \qquad x^{*j}_{it} - x^{*j}_{is} = \bar{\varepsilon}^j(q^*_{it} - q^*_{is}) + \xi^j_{its}\,,$$

$$j = 1,\ldots,N,\ i = 1,\ldots,M,\ t,s = 1,\ldots,T,$$

where

$$\text{(A.6)} \qquad \xi^j_{its} = \kappa^j_{its} + \delta^j_{\varepsilon its}(q^*_{it} - q^*_{is})$$
$$= \sum_{k=1}^{N} \gamma^{jk}_{its} \ln(w^{*k}_{it}/w^{*k}_{is}) + \ln(f^j_{it}/f^j_{is}) - \bar{\varepsilon}^j \ln(\Phi_{it}/\Phi_{is})$$
$$+ \delta^j_{\varepsilon its}(q^*_{it} - q^*_{is}) - \delta^j_{\varepsilon its} \ln(\Phi_{it}/\Phi_{is}),$$

the formal difference from the homothetic case being that $\bar{\varepsilon}$ and $\delta_{\varepsilon its}$ now have the input superscript j.

Returning to the *homothetic* case, we assume that the logarithms of the output and the input quantities observed are [cf. (13) and (14)]

$$\text{(A.7)} \qquad q_{it} = q^*_{it} + \nu_{it}, \qquad i = 1,\dots,M,\ t = 1,\dots,T,$$

$$\text{(A.8)} \qquad x^j_{it} = x^{*j}_{it} + \tau^j_{it}, \qquad j = 1,\dots,N,\ i = 1,\dots,M,\ t = 1,\dots,T,$$

where ν_{it} and τ^j_{it} are measurement errors with zero means. The stochastic specification of the model has four elements.

(i) We assume that[1]

$$\text{(A.9)} \qquad (q^*_{it}, \kappa^j_{its}, \nu_{it}, \tau^j_{it}, \delta_{\varepsilon its}) \text{ and } (q^*_{kt}, \kappa^j_{kts}, \nu_{kt}, \tau^j_{kt}, \delta_{\varepsilon kts})$$

are stochastically independent for all firms $k \neq i$,

$$j = 1,\dots,N,\ i,k = 1,\dots,M,\ t,s = 1,\dots,T,$$

and that

$$\text{(A.10)} \qquad (q^*_{it}, \kappa^j_{isq}) \text{ are independently distributed of } (\nu_{im}, \tau^j_{in}, \delta_{\varepsilon ipr})$$
$$j = 1,\dots,N,\ i = 1,\dots,M,\ t,s,q,m,n,p,r = 1,\dots,T.$$

(ii) We specify the measurement errors as having an arbitrary heteroskedasticity and/or autocorrelation, i.e.

$$\text{(A.11)} \qquad C(\nu_{it}, \nu_{is}) = E(\nu_{it}\nu_{is}) = \sigma^{\nu}_{its},$$

$$\text{(A.12)} \qquad C(\tau^j_{it}, \tau^j_{is}) = E(\tau^j_{it}\tau^j_{is}) = \sigma^{\tau j}_{its},$$

$$j = 1,\dots,N,\ i = 1,\dots,M,\ t,s = 1,\dots,T,$$

[1]Independence of $(q^*_{it}, \kappa^j_{its})$ and $(q^*_{kt}, \kappa^j_{kts})$ may, however, be unrealistic if firms i and k belong to the same subsector and are affected by the same kind of chocks.

where C is the covariance operator and the σ's in general vary across i, t, and s. If the additional customary assumption of *stationarity* of the measurement errors for any firm i is made, then $\sigma^{\nu}_{it,t+l}$ and $\sigma^{\tau j}_{it,t+l}$ vary with j, i, and l, but are invariant to t.

(iii) Similar flexible assumptions are made for the random input response coefficients, i.e.

$$(A.13) \qquad C(\varepsilon_{its}, \varepsilon_{ipr}) \;=\; C(\delta_{\varepsilon its}, \delta_{\varepsilon ipr}) \;=\; \sigma^{\delta}_{itspr} \,,$$

$$i = 1, \ldots, M, \;\; t, s, p, r = 1, \ldots, T.$$

If the additional assumption of stationarity of the input response coefficients for any firm i is made, then $\sigma^{\delta}_{it,t+l,p,p+m}$ vary with i, l, and m, but is invariant to t and p.

(iv) We assume that q^*_{it} and κ^j_{isr}, the latter being the composite expression given by (8), may be correlated not only for $s = t$ or $r = t$, but also for $t \neq s \neq r$. Both, like the measurement errors and the random coefficients, may have an arbitrary heteroscedasticity and/or autocorrelation, i.e.

$$(A.14) \qquad C(q^*_{it}, q^*_{is}) \;=\; \sigma^{q*}_{its} \,,$$

$$(A.15) \qquad C(\kappa^j_{its}, \kappa^j_{ipr}) \;=\; \sigma^{\kappa j}_{itspr} \,,$$

$$(A.16) \qquad C(\kappa^j_{its}, q^*_{ip},) \;=\; \sigma^{\kappa j q*}_{itsp} \,,$$

$$j = 1, \ldots, N, \;\; i = 1, \ldots, M,$$

$$t, s, p, r = 1, \ldots, T.$$

Unlike $(\nu_{it}, \tau^j_{it}, \varepsilon_{its})$, stationarity of $(q^*_{it}, \kappa^j_{its})$ for any i and j will not, in general, be a realistic assumption. In the empirical application in section 5, however, we will use more restrictive assumptions about the "cross-autocorrelation" between κ^j_{its} and q^*_{ip} than given in (A.16) [cf. eqs. (23) and (54) in the main text].

Combining (A.2) with (A.7) and (A.8), it follows, under homotheticity, that the input demand equations, expressed in terms of logarithmically differenced observations on output and input volumes between years t and s, are [cf. (15) and (16)]

$$(A.17) \qquad x^j_{it} - x^j_{is} \;=\; \bar{\varepsilon}(q_{it} - q_{is}) \;+\; \theta^j_{its} \,,$$

$$j = 1, \ldots, N, \;\; i = 1, \ldots, M,$$

$$t, s = 1, \ldots, T,$$

where

$$\text{(A.18)} \qquad \theta^j_{its} = \xi^j_{its} + (\tau^j_{it} - \tau^j_{is}) - \bar{\varepsilon}(\nu_{it} - \nu_{is})$$

$$= \kappa^j_{its} + \delta_{\varepsilon its}(q^*_{it} - q^*_{is}) + (\tau^j_{it} - \tau^j_{is}) - \bar{\varepsilon}(\nu_{it} - \nu_{is}),$$

$$j = 1, \dots, N, \; i = 1, \dots, M, \; t, s = 1, \dots, T.$$

From (A.7) and (A.9) - (A.16) it follows that the variables on the right hand side of (A.17) for arbitrary years t and s, with θ^j_{its} defined in (A.18), have the following covariances: (i) with the logarithm of the observed *level* of output in an arbitrary year p

$$\text{(A.19)} \qquad C[(q_{it} - q_{is}), q_{ip}] = (\sigma^{q*}_{itp} - \sigma^{q*}_{isp}) + (\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp}) ,$$

$$\text{(A.20)} \qquad C[\theta^j_{its}, q_{ip}] = \sigma^{\kappa j q*}_{itsp} - \bar{\varepsilon}(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp}) ,$$

$$j = 1, \dots, N, \; i = 1, \dots, M, \; t, s, p = 1, \dots, T,$$

and (ii) with the observed logarithmic *change* in the observed output between two arbitrary years r and p

$$\text{(A.21)} \quad C[(q_{it} - q_{is}), (q_{ip} - q_{ir})] = (\sigma^{q*}_{itp} - \sigma^{q*}_{isp} - \sigma^{q*}_{itr} + \sigma^{q*}_{isr})$$

$$+ (\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp} - \sigma^{\nu}_{itr} + \sigma^{\nu}_{isr}) ,$$

$$\text{(A.22)} \qquad C[\theta^j_{its}, (q_{ip} - q_{ir})] = (\sigma^{\kappa j q*}_{itsp} - \sigma^{\kappa j q*}_{itsr}) - \bar{\varepsilon}(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp} - \sigma^{\nu}_{itr} + \sigma^{\nu}_{isr}) ,$$

$$j = 1, \dots, N, \; i = 1, \dots, M, \; t, s, p, r = 1, \dots, T.$$

It also follows that the autocovariances of the composite variables θ^j_{its}, for an arbitrary firm i and an arbitrary input j, are given by

$$\text{(A.23)} \qquad C[\theta^j_{its}, \theta^j_{ipr}] = \sigma^{\kappa j}_{itspr} + \sigma^{\delta}_{itspr}(\sigma^{q*}_{itp} - \sigma^{q*}_{isp} - \sigma^{q*}_{itr} + \sigma^{q*}_{isr})$$

$$+ (\sigma^{\tau}_{itp} - \sigma^{\tau}_{isp} - \sigma^{\tau}_{itr} + \sigma^{\tau}_{isr})$$

$$+ \bar{\varepsilon}^2(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp} - \sigma^{\nu}_{itr} + \sigma^{\nu}_{isr}) ,$$

$$j = 1, \dots, N, \; i = 1, \dots, M,$$

$$t, s, p, r = 1, \dots, T.$$

Now, (A.22) implies that

$$\text{(A.24)} \qquad C[\theta^j_{its}, (q_{it} - q_{is})] \;=\; 0 \text{ for any } \bar{\varepsilon}$$

$$\iff \sigma^{\kappa j q*}_{itst} - \sigma^{\kappa j q*}_{itss} \;=\; \sigma^{\nu}_{itt} - 2\sigma^{\nu}_{its} + \sigma^{\nu}_{iss} \;=\; 0,$$

$$j = 1, \ldots, N, \; i = 1, \ldots, M, \; t, s = 1, \ldots, T.$$

Since, in the presence of 'simultaneity' and/or measurement errors in output, (A.24) may be satisfied only by chance, ordinary least squares (OLS) estimation of the inverse mean scale elasticity $\bar{\varepsilon}$ from (A.17) – whether based on the M observations on $(x^j_{it} - x^j_{is})$ and $(q_{it} - q_{is})$ for given j and (t, s), or on all the $MT(T-1)/2$ possible pairs of differenced observations (for $t > s$) for given j – will in practice be inconsistent. Hence, we have the same OLS inconsistency problem as in classical errors-in-variables models. However, provided that certain additional assumptions are made, (A.19) – (A.22) are useful in suggesting candidates for instrumental variables (IV's) for $(q_{it} - q_{is})$ in (A.17). These IV's may be building blocks in the construction of consistent estimators of $\bar{\varepsilon}$ which are more efficient than simple IV estimators. This problem will be addressed in section 4.

Instead of using (A.17) as our estimating equation for the inverse mean scale elasticity $\bar{\varepsilon}$, we make one additional transformation: *We measure the (logarithmic) input and output volumes from their respective year specific means.* A similar procedure is followed in MaCurdy (1982). Hence, using ~ to symbolize deviations from year means, we express (A.17) by means of

$$\text{(A.25)} \qquad \tilde{x}^j_{it} \;=\; x^j_{it} \;-\; \bar{x}^j_{\cdot t}\,,$$

$$\text{(A.26)} \qquad \tilde{q}_{it} \;=\; q_{it} \;-\; \bar{q}_{\cdot t}\,,$$

$$\text{(A.27)} \qquad \tilde{\theta}^j_{its} \;=\; \theta^j_{its} \;-\; \bar{\theta}^j_{\cdot ts}\,,$$

where $\bar{x}^j_{\cdot t} = \sum_{i=1}^M x^j_{it}/M$, $\bar{q}_{\cdot t} = \sum_{i=1}^M q_{it}/M$, and $\bar{\theta}_{\cdot ts} = \sum_{i=1}^M \theta_{its}/M$ $(j = 1, \ldots, N,$ $t, s = 1, \ldots, T)$, respectively. This transformation, although reducing the effective variation of the input and output variables in the sample, removes any additive year specific effects, like log-linear trends, from the logarithmic structural variables as well as from the composite "residual variables" θ^j_{its}. Hence, $\tilde{x}^j_{it}$, $\tilde{q}_{it}$, and $\tilde{\theta}^j_{its}$ are more likely to be stationary than x^j_{it}, q_{it}, and θ^j_{its}.

Now, for any sequences $(a_1, b_1), \ldots, (a_M, b_M)$, we have the identities

$$\frac{1}{M}\sum_{k=1}^{M} a_k b_k \;=\; S_{ab} + \bar{a}\bar{b}$$

and

$$a_i\,b_i \;=\; \bar{a}\,\bar{b} \;+\; \widetilde{a}_i\,\bar{b} \;+\; \bar{a}\,\widetilde{b}_i \;+\; \widetilde{a}_i\,\widetilde{b}_i\;,$$

where $\bar{a} = \sum_{i=1}^{M} a_i/M$, $\bar{b} = \sum_{i=1}^{M} b_i/M$, $\widetilde{a}_i = a_i - \bar{a}$, $\widetilde{b}_i = b_i - \bar{b}$, and $S_{ab} = \sum_{i=1}^{M} \widetilde{a}_i\widetilde{b}_i/M$. Hence it follows that if the a_i and the b_i sequences are approximately uncorrelated, i.e. $S_{ab} \approx 0$, and if all $\widetilde{a}_i\,\widetilde{b}_i$ are 'second order terms' which can be ignored, then

$$a_i\,b_i - \frac{1}{M}\sum_{k=1}^{M} a_k\,b_k \;\approx\; \bar{a}\,\widetilde{b}_i + \widetilde{a}_i\,\bar{b}\;.$$

Utilizing the latter approximation, while recalling (A.10), we find that (A.17) and (A.18), after deduction of year specific means, change into

$$\text{(A.28)}\qquad \widetilde{x}^j_{it} - \widetilde{x}^j_{is} \;=\; \bar{\varepsilon}(\widetilde{q}_{it} - \widetilde{q}_{is}) \;+\; \widetilde{\theta}^j_{its}\;,$$

$$\begin{aligned}\text{(A.29)}\qquad \widetilde{\theta}^j_{its} \;\approx\;& \widetilde{\kappa}^j_{its} \;+\; \widetilde{\delta}_{\varepsilon its}(\bar{q}^*_{\cdot t} - \bar{q}^*_{\cdot s}) \;+\; \bar{\delta}_{\varepsilon\cdot ts}(\widetilde{q}^*_{it} - \widetilde{q}^*_{is}) \\ &+\; (\widetilde{\tau}^j_{it} - \widetilde{\tau}^j_{is}) \;-\; \bar{\varepsilon}(\widetilde{\nu}_{it} - \widetilde{\nu}_{is}), \\ & j = 1,\ldots,N,\; i = 1,\ldots,M,\; t,s = 1,\ldots,T,\end{aligned}$$

respectively. Note that, formally, *the variables in (A.28) are obtained by a double differencing*: a differencing within firms $[(x^j_{it} - x^j_{is})$, and $(q_{it} - q_{is})]$ is combined with a differencing within years $[(\widetilde{x}^j_{it} = x^j_{it} - \bar{x}^j_{\cdot t})$ and $(\widetilde{q}_{it} = q_{it} - \bar{q}_{\cdot t})]$.

Using (A.7), (A.9) - (A.16), and (A.26) - (A.29), we find

$$\begin{aligned}\text{(A.30)}\quad C[\widetilde{q}_{it}, \widetilde{q}_{kp}] \;=\;& \delta_{ik}(\sigma^{q*}_{itp} + \sigma^{\nu}_{itp}) \;-\; (1/M)(\sigma^{q*}_{itp} + \sigma^{\nu}_{itp} + \sigma^{q*}_{ktp} + \sigma^{\nu}_{ktp}) \\ &+\; (1/M)(\bar{\sigma}^{q*}_{\cdot tp} + \bar{\sigma}^{\nu}_{\cdot tp})\;,\end{aligned}$$

$$\begin{aligned}\text{(A.31)}\quad C[\widetilde{\theta}^j_{its}, \widetilde{q}_{kp}] \;=\;& \delta_{ik}\sigma^{\kappa jq*}_{itsp} \;-\; (1/M)(\sigma^{\kappa jq*}_{itsp} + \sigma^{\kappa jq*}_{ktsp}) \;+\; (1/M)\bar{\sigma}^{\kappa jq*}_{\cdot tsp} \\ &-\; \bar{\varepsilon}[\delta_{ik}(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp}) \;-\; (1/M)(\sigma^{\nu}_{itp} - \sigma^{\nu}_{isp} + \sigma^{\nu}_{ktp} - \sigma^{\nu}_{ktp}) \\ &+\; (1/M)(\bar{\sigma}^{\nu}_{\cdot tp} - \bar{\sigma}^{\nu}_{\cdot sp})]\;, \\ & j = 1,\ldots,N,\; i,k = 1,\ldots,M,\; t,s,p = 1,\ldots,T,\end{aligned}$$

where $\delta_{ik} = 1$ for $k = i$ and $= 0$ for $k \neq i$ and subscript $\cdot$ again denotes averaging over i, i.e. $\bar{\sigma}^{q*}_{\cdot tp} = \sum_{i=1}^{M} \sigma^{q*}_{itp}/M$, $\bar{\sigma}^{\nu}_{\cdot tp} = \sum_{i=1}^{M} \sigma^{\nu}_{itp}/M$, and $\bar{\sigma}^{\kappa jq*}_{\cdot tsp} = \sum_{i=1}^{M} \sigma^{\kappa jq*}_{itsp}/M$. In case of homoskedasticity of all variables, i.e. $\sigma^{q*}_{its} = \sigma^{q*}_{ts}$, $\sigma^{\nu}_{its} = \sigma^{\nu}_{ts}$, and $\sigma^{\kappa jq*}_{itsp} = \sigma^{\kappa jq*}_{tsp}$

for $i = 1, \ldots, M$, the latter relationships can be simplified to

$$\text{(A.32)} \qquad C[\widetilde{q}_{it}, \widetilde{q}_{kp}] = [\delta_{ik} - (1/M)](\sigma_{tp}^{q*} + \sigma_{tp}^{\nu}) \ ,$$

$$\text{(A.33)} \qquad C[\widetilde{\theta}_{its}, \widetilde{q}_{kp}] = [\delta_{ik} - (1/M)]\sigma_{tsp}^{\kappa jq*} - \bar{\varepsilon}[\delta_{ik} - (1/M)](\sigma_{tp}^{\nu} - \sigma_{sp}^{\nu}) \ ,$$

$$j = 1, \ldots, N, \ i, k = 1, \ldots, M, \ t, s, p = 1, \ldots, T.$$

Defining, in the general case with heteroskedasticity,

$$\text{(A.34)} \qquad \widetilde{\sigma}_{itp}^{qq} = (1 - 2/M)(\sigma_{itp}^{q*} + \sigma_{itp}^{\nu}) + (1/M)(\bar{\sigma}_{\cdot tp}^{q*} + \bar{\sigma}_{\cdot tp}^{\nu}) \ ,$$

$$\text{(A.35)} \qquad \sigma_{itp}^{qq} = \sigma_{itp}^{q*} + \sigma_{itp}^{\nu} \ ,$$

$$\text{(A.36)} \qquad \widetilde{\sigma}_{itsp}^{\theta jq} = (1 - 2/M)\sigma_{itsp}^{\kappa jq*} + (1/M)\bar{\sigma}_{\cdot tsp}^{\kappa jq*}$$

$$- \bar{\varepsilon}[(1 - 2/M)(\sigma_{itp}^{\nu} - \sigma_{isp}^{\nu}) + (1/M)(\bar{\sigma}_{\cdot tp}^{\nu} - \bar{\sigma}_{\cdot sp}^{\nu})] \ ,$$

$$\text{(A.37)} \qquad \sigma_{itsp}^{\theta jq} = \sigma_{itsp}^{\kappa jq*} - \bar{\varepsilon}(\sigma_{itp}^{\nu} - \sigma_{isp}^{\nu}) \ ,$$

$$j = 1, \ldots, N, \ i = 1, \ldots, M, \ t, s, p = 1, \ldots, T,$$

we then have approximately, provided that the number of firms M is sufficiently large, that

$$\text{(A.38)} \qquad C[\widetilde{q}_{it}, \widetilde{q}_{ip}] = \widetilde{\sigma}_{itp}^{qq} \approx \sigma_{itp}^{qq} \ ,$$

$$\text{(A.39)} \qquad C[\widetilde{q}_{it}, \widetilde{q}_{kp}] \approx 0 \ , \ \text{for } k \neq i \ ,$$

$$\text{(A.40)} \qquad C[\widetilde{\theta}_{its}^{j}, \widetilde{q}_{ip}] = \widetilde{\sigma}_{itsp}^{\theta jq} \approx \sigma_{itsp}^{\theta jq} \ ,$$

$$\text{(A.41)} \qquad C[\widetilde{\theta}_{its}^{j}, \widetilde{q}_{kp}] \approx 0 \ , \ \text{for } k \neq i \ ,$$

$$j = 1, \ldots, N, \ i, k = 1, \ldots, M,$$

$$t, s, p = 1, \ldots, T.$$

In case of homoskedasticity, (A.38) – (A.41) can be simplified to

$$\text{(A.42)} \qquad C[\widetilde{q}_{it}, \widetilde{q}_{kp}] = [\delta_{ik} - (1/M)]\sigma_{tp}^{qq} \approx \delta_{ik}\sigma_{tp}^{qq} \ ,$$

$$\text{(A.43)} \qquad C[\widetilde{\theta}_{its}^{j}, \widetilde{q}_{kp}] = [\delta_{ik} - (1/M)]\sigma_{tsp}^{\theta jq} \approx \delta_{ik}\sigma_{tsp}^{\theta jq} \ ,$$

$$j = 1, \ldots, N, \ i, k = 1, \ldots, M,$$

$$t, s, p = 1, \ldots, T,$$

where

$$\text{(A.44)} \qquad \sigma_{tp}^{qq} = \sigma_{tp}^{q*} + \sigma_{tp}^{\nu} ,$$

$$\text{(A.45)} \qquad \sigma_{tsp}^{\theta jq} = \sigma_{tsp}^{\kappa jq*} - \bar{\varepsilon}(\sigma_{tp}^{\nu} - \sigma_{sp}^{\nu}) ,$$

$$j = 1, \ldots, N, \; t, s, p = 1, \ldots, T.$$

In the general case with heteroskedasticity, (A.19) – (A.23) are then replaced by

$$\text{(A.46)} \qquad C[(\widetilde{q}_{it} - \widetilde{q}_{is}), \widetilde{q}_{ip}] = \widetilde{\sigma}_{itp}^{qq} - \widetilde{\sigma}_{isp}^{qq} \approx \sigma_{itp}^{qq} - \sigma_{isp}^{qq} ,$$

$$\text{(A.47)} \qquad C[\widetilde{\theta}_{its}^{j}, \widetilde{q}_{ip}] = \widetilde{\sigma}_{itsp}^{\theta jq} \approx \sigma_{itsp}^{\theta jq} ,$$

$$\text{(A.48)} \qquad C[(\widetilde{q}_{it} - \widetilde{q}_{is}), (\widetilde{q}_{ip} - \widetilde{q}_{ir})] = \widetilde{\sigma}_{itp}^{qq} - \widetilde{\sigma}_{isp}^{qq} - \widetilde{\sigma}_{itr}^{qq} + \widetilde{\sigma}_{isr}^{qq}$$

$$\approx \sigma_{itp}^{qq} - \sigma_{isp}^{qq} - \sigma_{itr}^{qq} + \sigma_{isr}^{qq} ,$$

$$\text{(A.49)} \qquad C[\widetilde{\theta}_{its}^{j}, (\widetilde{q}_{ip} - \widetilde{q}_{ir})] = \widetilde{\sigma}_{itsp}^{\theta jq} - \widetilde{\sigma}_{itsr}^{\theta jq} \approx \sigma_{itsp}^{\theta jq} - \sigma_{itsr}^{\theta jq} ,$$

$$j = 1, \ldots, N, \; i = 1, \ldots, M,$$

$$t, s, p, r = 1, \ldots, T.$$

References

Anderson, T.W. and Hsiao, C. (1981): Estimation of Dynamic Models with Error Components. *Journal of the American Statistical Association*, 76 (1981), 598 – 606.

Anderson, T.W. and Hsiao, C. (1982): Formulation and Estimation of Dynamic Models Using Panel Data. *Journal of Econometrics*, 18 (1982), 47 – 82.

Arellano, M. and Bond, S. (1991): Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. *Review of Economic Studies*, 58 (1991), 277 – 297.

Berck, P. and Sydsæter, K. (1991): *Economists' Mathematical Manual.* Berlin, Springer-Verlag, 1991.

Biørn, E. (1992): Panel Data with Measurement Errors. In *The Econometrics of Panel Data. Handbook of Theory and Applications*, ed. by L. Mátyás and P. Sevestre. Dordrecht, Kluwer Academic Publishers, 1992, pp. 152 – 195.

Bound, J., Jaeger, D.A., and Baker, R. (1993): The Cure Can be Worse than the Disease: A Cautionary Tale Regarding Instrumental Variables. National Bureau of Economic Research, Technical Paper No. 137, Cambridge, June 1993.

Bowden, R.J. and Turkington, D.A. (1984): *Instrumental Variables.* Cambridge, Cambridge University Press, 1984.

Griliches, Z. and Hausman, J.A. (1986): Errors in Variables in Panel Data. *Journal of Econometrics*, 31 (1986), 93 – 118.

Hansen, L.P. (1982): Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica*, 50 (1982), 1029 – 1054.

Hausman, J.A. (1978): Specification Tests in Econometrics. *Econometrica*, 46 (1978), 1251 – 1271.

Hsiao, C. (1986): *Analysis of Panel Data.* Cambridge, Cambridge University Press, 1986.

Jorgenson, D.W. (1986): Econometric Methods for Modeling Producer Behavior. In *Handbook of Econometrics, vol. III*, ed. by Z. Griliches and M.D. Intriligator. Amsterdam, North-Holland, 1986, pp. 1841 – 1915.

Judge, G.G., Griffiths, W.E., Hill, R.C., Lütkepohl, H., and Lee, T.C. (1985): *The Theory and Practice of Econometrics.* New York, Wiley, 1985.

Klette, T.J. (1993): Is Price Equal to Marginal Cost ? Discussion Paper No. 93 from Statistics Norway, Oslo.

MaCurdy, T.E. (1982): The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis. *Journal of Econometrics*, 18 (1982), 83 - 114.

Schmidt, P. (1990): Three-Stage Least Squares with Different Instruments for Different Equations. *Journal of Econometrics*, 43 (1990), 389 - 394.

Sevestre, P. and Trognon, A. (1992): Linear Dynamic Models. In *The Econometrics of Panel Data. Handbook of Theory and Applications*, ed. by L. Mátyás and P. Sevestre. Dordrecht, Kluwer Academic Publishers, 1992, pp. 95 - 117.

Sims, C.A. (1974): Output and Labor Input in Manufacturing. *Brookings Papers on Economic Activity*, 3, 1974, 695 - 728.

White, H. (1982): Instrumental Variables Regression with Independent Observations. *Econometrica*, 50 (1982), 483 - 499.

White, H. (1984): *Asymptotic Theory for Econometricians.* Orlando, Academic Press, 1984.

White, H. (1986): Instrumental Variables Analogs of Generalized Least Squares Estimators. In *Advances in Statistical Analysis and Statistical Computing. Theory and Applications, vol. 1*, ed. by R.S. Mariano, JAI Press, 1986, pp. 173 - 227.

Issued in the series Discussion Papers

No. 1 *I. Aslaksen and O. Bjerkholt (1985)*: Certainty Equivalence Procedures in the Macroeconomic Planning of an Oil Economy

No. 3 *E. Biørn (1985)*: On the Prediction of Population Totals from Sample surveys Based on Rotating Panels

No. 4 *P. Frenger (1985)*: A Short Run Dynamic Equilibrium Model of the Norwegian Production Sectors

No. 5 *I. Aslaksen and O. Bjerkholt (1985)*: Certainty Equivalence Procedures in Decision-Making under Uncertainty: An Empirical Application

No. 6 *E. Biørn (1985)*: Depreciation Profiles and the User Cost of Capital

No. 7 *P. Frenger (1985)*: A Directional Shadow Elasticity of Substitution

No. 8 *S. Longva, L. Lorentsen and Ø. Olsen (1985)*: The Multi-Sectoral Model MSG-4, Formal Structure and Empirical Characteristics

No. 9 *J. Fagerberg and G. Sollie (1985)*: The Method of Constant Market Shares Revisited

No. 10 *E. Biørn (1985)*: Specification of Consumer Demand Models with Stochastic Elements in the Utility Function and the first Order Conditions

No. 11 *E. Biørn, E. Holmøy and Ø. Olsen (1985)*: Gross and Net Capital, Productivity and the form of the Survival Function. Some Norwegian Evidence

No. 12 *J.K. Dagsvik (1985)*: Markov Chains Generated by Maximizing Components of Multidimensional Extremal Processes

No. 13 *E. Biørn, M. Jensen and M. Reymert (1985)*: KVARTS - A Quarterly Model of the Norwegian Economy

No. 14 *R. Aaberge (1986)*: On the Problem of Measuring Inequality

No. 15 *A.-M. Jensen and T. Schweder (1986)*: The Engine of Fertility - Influenced by Interbirth Employment

No. 16 *E. Biørn (1986)*: Energy Price Changes, and Induced Scrapping and Revaluation of Capital - A Putty-Clay Model

No. 17 *E. Biørn and P. Frenger (1986)*: Expectations, Substitution, and Scrapping in a Putty-Clay Model

No. 18 *R. Bergan, Å. Cappelen, S. Longva and N.M. Stølen (1986)*: MODAG A - A Medium Term Annual Macroeconomic Model of the Norwegian Economy

No. 19 *E. Biørn and H. Olsen (1986)*: A Generalized Single Equation Error Correction Model and its Application to Quarterly Data

No. 20 *K.H. Alfsen, D.A. Hanson and S. Glomsrød (1986)*: Direct and Indirect Effects of reducing SO_2 Emissions: Experimental Calculations of the MSG-4E Model

No. 21 *J.K. Dagsvik (1987)*: Econometric Analysis of Labor Supply in a Life Cycle Context with Uncertainty

No. 22 *K.A. Brekke, E. Gjelsvik and B.H. Vatne (1987)*: A Dynamic Supply Side Game Applied to the European Gas Market

No. 23 *S. Bartlett, J.K. Dagsvik, Ø. Olsen and S. Strøm (1987)*: Fuel Choice and the Demand for Natural Gas in Western European Households

No. 24 *J.K. Dagsvik and R. Aaberge (1987)*: Stochastic Properties and Functional Forms of Life Cycle Models for Transitions into and out of Employment

No. 25 *T.J. Klette (1987)*: Taxing or Subsidising an Exporting Industry

No. 26 *K.J. Berger, O. Bjerkholt and Ø. Olsen (1987)*: What are the Options for non-OPEC Countries

No. 27 *A. Aaheim (1987)*: Depletion of Large Gas Fields with Thin Oil Layers and Uncertain Stocks

No. 28 *J.K. Dagsvik (1987)*: A Modification of Heckman's Two Stage Estimation Procedure that is Applicable when the Budget Set is Convex

No. 29 *K. Berger, Å. Cappelen and I. Svendsen (1988)*: Investment Booms in an Oil Economy - The Norwegian Case

No. 30 *A. Rygh Swensen (1988)*: Estimating Change in a Proportion by Combining Measurements from a True and a Fallible Classifier

No. 31 *J.K. Dagsvik (1988)*: The Continuous Generalized Extreme Value Model with Special Reference to Static Models of Labor Supply

No. 32 *K. Berger, M. Hoel, S. Holden and Ø. Olsen (1988)*: The Oil Market as an Oligopoly

No. 33 *I.A.K. Anderson, J.K. Dagsvik, S. Strøm and T. Wennemo (1988)*: Non-Convex Budget Set, Hours Restrictions and Labor Supply in Sweden

No. 34 *E. Holmøy and Ø. Olsen (1988)*: A Note on Myopic Decision Rules in the Neoclassical Theory of Producer Behaviour, 1988

No. 35 *E. Biørn and H. Olsen (1988)*: Production - Demand Adjustment in Norwegian Manufacturing: A Quarterly Error Correction Model, 1988

No. 36 *J.K. Dagsvik and S. Strøm (1988)*: A Labor Supply Model for Married Couples with Non-Convex Budget Sets and Latent Rationing, 1988

No. 37 *T. Skoglund and A. Stokka (1988)*: Problems of Linking Single-Region and Multiregional Economic Models, 1988

No. 38 *T.J. Klette (1988)*: The Norwegian Aluminium Industry, Electricity prices and Welfare, 1988

No. 39 *I. Aslaksen, O. Bjerkholt and K.A. Brekke (1988)*: Optimal Sequencing of Hydroelectric and Thermal Power Generation under Energy Price Uncertainty and Demand Fluctuations, 1988

No. 40 *O. Bjerkholt and K.A. Brekke (1988)*: Optimal Starting and Stopping Rules for Resource Depletion when Price is Exogenous and Stochastic, 1988

No. 41 *J. Aasness, E. Biørn and T. Skjerpen (1988)*: Engel Functions, Panel Data and Latent Variables, 1988

No. 42 *R. Aaberge, Ø. Kravdal and T. Wennemo (1989)*: Unobserved Heterogeneity in Models of Marriage Dissolution, 1989

No. 43 *K.A. Mork, H.T. Mysen and Ø. Olsen (1989)*: Business Cycles and Oil Price Fluctuations: Some evidence for six OECD countries. 1989

No. 44 *B. Bye, T. Bye and L. Lorentsen (1989)*: SIMEN. Studies of Industry, Environment and Energy towards 2000, 1989

No. 45 *O. Bjerkholt, E. Gjelsvik and Ø. Olsen (1989)*: Gas Trade and Demand in Northwest Europe: Regulation, Bargaining and Competition

No. 46 *L.S. Stambøl and K.Ø. Sørensen (1989)*: Migration Analysis and Regional Population Projections, 1989

No. 47 *V. Christiansen (1990)*: A Note on the Short Run Versus Long Run Welfare Gain from a Tax Reform, 1990

No. 48 *S. Glomsrød, H. Vennemo and T. Johnsen (1990)*: Stabilization of Emissions of CO_2: A Computable General Equilibrium Assessment, 1990

No. 49 *J. Aasness (1990)*: Properties of Demand Functions for Linear Consumption Aggregates, 1990

No. 50 *J.G. de Leon (1990)*: Empirical EDA Models to Fit and Project Time Series of Age-Specific Mortality Rates, 1990

No. 51 *J.G. de Leon (1990)*: Recent Developments in Parity Progression Intensities in Norway. An Analysis Based on Population Register Data

No. 52 *R. Aaberge and T. Wennemo (1990)*: Non-Stationary Inflow and Duration of Unemployment

No. 53 *R. Aaberge, J.K. Dagsvik and S. Strøm (1990)*: Labor Supply, Income Distribution and Excess Burden of Personal Income Taxation in Sweden

No. 54 *R. Aaberge, J.K. Dagsvik and S. Strøm (1990)*: Labor Supply, Income Distribution and Excess Burden of Personal Income Taxation in Norway

No. 55 *H. Vennemo (1990)*: Optimal Taxation in Applied General Equilibrium Models Adopting the Armington Assumption

No. 56 *N.M. Stølen (1990)*: Is there a NAIRU in Norway?

No. 57 *Å. Cappelen (1991)*: Macroeconomic Modelling: The Norwegian Experience

No. 58 *J. Dagsvik and R. Aaberge (1991)*: Household Production, Consumption and Time Allocation in Peru

No. 59 *R. Aaberge and J. Dagsvik (1991)*: Inequality in Distribution of Hours of Work and Consumption in Peru

No. 60 *T.J. Klette (1991)*: On the Importance of R&D and Ownership for Productivity Growth. Evidence from Norwegian Micro-Data 1976-85

No. 61 *K.H. Alfsen (1991)*: Use of Macroeconomic Models in Analysis of Environmental Problems in Norway and Consequences for Environmental Statistics

No. 62 *H. Vennemo (1991)*: An Applied General Equilibrium Assessment of the Marginal Cost of Public Funds in Norway

No. 63 *H. Vennemo (1991)*: The Marginal Cost of Public Funds: A Comment on the Literature

No. 64 *A. Brendemoen and H. Vennemo (1991)*: A climate convention and the Norwegian economy: A CGE assessment

No. 65 *K. A. Brekke (1991)*: Net National Product as a Welfare Indicator

No. 66 *E. Bowitz and E. Storm (1991)*: Will Restrictive Demand Policy Improve Public Sector Balance?

No. 67 *Å. Cappelen (1991)*: MODAG. A Medium Term Macroeconomic Model of the Norwegian Economy

No. 68 *B. Bye (1992)*: Modelling Consumers' Energy Demand

No. 69 *K. H. Alfsen, A. Brendemoen and S. Glomsrød (1992)*: Benefits of Climate Policies: Some Tentative Calculations

No. 70 *R. Aaberge, Xiaojie Chen, Jing Li and Xuezeng Li (1992)*: The Structure of Economic Inequality among Households Living in Urban Sichuan and Liaoning, 1990

No. 71 *K.H. Alfsen, K.A. Brekke, F. Brunvoll, H. Lurås, K. Nyborg and H.W. Sæbø (1992)*: Environmental Indicators

No. 72 *B. Bye and E. Holmøy (1992)*: Dynamic Equilibrium Adjustments to a Terms of Trade Disturbance

No. 73 *O. Aukrust (1992):* The Scandinavian Contribution to National Accounting

No. 74 *J. Aasness, E, Eide and T. Skjerpen (1992):* A Criminometric Study Using Panel Data and Latent Variables

No. 75 *R. Aaberge and Xuezeng Li (1992):* The Trend in Income Inequality in Urban Sichuan and Liaoning, 1986-1990

No. 76 *J.K. Dagsvik and Steinar Strøm (1992):* Labor Supply with Non-convex Budget Sets, Hours Restriction and Non-pecuniary Job-attributes

No. 77 *J.K. Dagsvik (1992)*: Intertemporal Discrete Choice, Random Tastes and Functional Form

No. 78 *H. Vennemo (1993)*: Tax Reforms when Utility is Composed of Additive Functions

No. 79 *J. K. Dagsvik (1993):* Discrete and Continuous Choice, Max-stable Processes and Independence from Irrelevant Attributes

No. 80 *J. K. Dagsvik (1993):* How Large is the Class of Generalized Extreme Value Random Utility Models?

No. 81 *H. Birkelund, E. Gjelsvik, M. Aaserud (1993):* Carbon/energy Taxes and the Energy Market in Western Europe

No. 82 *E. Bowitz (1993):* Unemployment and the Growth in the Number of Recipients of Disability Benefits in Norway

No. 83 *L. Andreassen (1993):* Theoretical and Econometric Modeling of Disequilibrium

No. 84 *K.A. Brekke (1993):* Do Cost-Benefit Analyses favour Environmentalists?

No. 85 *L. Andreassen (1993):* Demographic Forecasting with a Dynamic Stochastic Microsimulation Model

No. 86 *G.B. Asheim and K.A. Brekke (1993):* Sustainability when Resource Management has Stochastic Consequences

No. 87 *O. Bjerkholt and Yu Zhu (1993):* Living Conditions of Urban Chinese Households around 1990

No. 88 *R. Aaberge (1993):* Theoretical Foundations of Lorenz Curve Orderings

No. 89 *J. Aasness, E. Biørn and T. Skjerpen (1993):* Engel Functions, Panel Data, and Latent Variables - with Detailed Results

No. 90 *Ingvild Svendsen (1993):* Testing the Rational Expectations Hypothesis Using Norwegian Microeconomic DataTesting the REH. Using Norwegian Microeconomic Data

No. 91 *Einar Bowitz, Asbjørn Rødseth and Erik Storm (1993):* Fiscal Expansion, the Budget Deficit and the Economy: Norway 1988-91

No. 92 *Rolf Aaberge, Ugo Colombino and Steinar Strøm (1993):* Labor Supply in Italy

No. 93 *Tor Jakob Klette (1993):* Is Price Equal to Marginal Costs? An Integrated Study of Price-Cost Margins and Scale Economies among Norwegian Manufacturing Establishments 1975-90

No. 94 *John K. Dagsvik (1993):* Choice Probabilities and Equilibrium Conditions in a Matching Market with Flexible Contracts

No. 95 *Tom Kornstad (1993):* Empirical Approaches for Analysing Consumption and Labour Supply in a Life Cycle Perspective

No. 96 *Tom Kornstad (1993):* An Empirical Life Cycle Model of Savings, Labour Supply and Consumption without Intertemporal Separability

No. 97 *Snorre Kverndokk (1993):* Coalitions and Side Payments in International CO_2 Treaties

No. 98 *Torbjørn Eika (1993):* Wage Equations in Macro Models. Phillips Curve versus Error Correction Model Determination of Wages in Large-Scale UK Macro Models

No. 99 *Anne Brendemoen and Haakon Vennemo (1993):* The Marginal Cost of Funds in the Presence of External Effects

No. 100 *Kjersti-Gro Lindquist (1993):* Empirical Modelling of Norwegian Exports: A Disaggregated Approach

No. 101 *Anne Sofie Jore, Terje Skjerpen and Anders Rygh Swensen (1993):* Testing for Purchasing Power Parity and Interest Rate Parities on Norwegian Data

No. 102 *Runa Nesbakken and Steinar Strøm (1993):* The Choice of Space Heating System and Energy Consumption in Norwegian Households (Will be issued later)

No. 103 *Asbjørn Aaheim and Karine Nyborg (1993):* "Green National Product": Good Intentions, Poor Device?

No. 104 *Knut H. Alfsen, Hugo Birkelund and Morten Aaserud (1993):* Secondary benefits of the EC Carbon/ Energy Tax

No. 105 *Jørgen Aasness and Bjart Holtsmark (1993):* Consumer Demand in a General Equilibrium Model for Environmental Analysis

No. 106 *Kjersti-Gro Lindquist (1993):* The Existence of Factor Substitution in the Primary Aluminium Industry: A Multivariate Error Correction Approach on Norwegian Panel Data

No. 107 *Snorre Kverndokk (1994):* Depletion of Fossil Fuels and the Impacts of Global Warming

No. 108 *Knut A. Magnussen (1994):* Precautionary Saving and Old-Age Pensions

No. 109 *Frode Johansen (1994):* Investment and Financial Constraints: An Empirical Analysis of Norwegian Firms

No. 110 *Kjell Arne Brekke and Pål Børing (1994):* The Volatility of Oil Wealth under Uncertainty about Parameter Values

No. 111 *Margaret J. Simpson (1994):* Foreign Control and Norwegian Manufacturing Performance

No .112 *Yngve Willassen and Tor Jakob Klette (1994):* Correlated Measurement Errors, Bound on Parameters, and a Model of Producer Behavior

No. 113 *Dag Wetterwald (1994):* Car ownership and private car use. A microeconometric analysis based on Norwegian data

No. 114 *Knut Einar Rosendahl (1994):* Does Improved Environmental Policy Enhance Economic Growth? Endogenous Growth Theory Applied to Developing Countries

No. 115 *Leif Andreassen, Dennis Fredriksen and Olav Ljones (1994):* The Future Burden of Public Pension Benefits. A Microsimulation Study

No. 116 *Anne Brendemoen (1994):* Car Ownership Decisions in Norwegian Households.

No. 117 *Audun Langørgen (1994):* A Macromodel of Local Government Spending Behaviour in Norway

No. 118 *Kjell Arne Brekke (1994):* Utilitarism, Equivalence Scales and Logarithmic Utility

No. 119 *Kjell Arne Brekke, Hilde Lurås og Karine Nyborg (1994):* Sufficient Welfare Indicators: Allowing Disagreement in Evaluations of Social Welfare

No. 120 *Tor Jakob Klette (1994):* R&D, Scope Economies and Company Structure: A "Not-so-Fixed Effect" Model of Plant Performance

No. 121 *Yngve Willassen (1994):* A Generalization of Hall's Specification of the Consumption function

No. 122 *Erling Holmøy, Torbjørn Hægeland and Øystein Olsen (1994):* Effective Rates of Assistance for Norwegian Industries

No. 123 *Klaus Mohn (1994):* On Equity and Public Pricing in Developing Countries

No. 124 *Jørgen Aasness, Erling Eide and Terje Skjerpen (1994):* Criminometrics, Latent Variables, Panel Data, and Different Types of Crime

No. 125 *Erik Biørn and Tor Jakob Klette (1994):*Errors in Variables and Panel Data: The Labour Demand Response to Permanent Changes in Output

Discussion Papers

Statistics Norway
Research Department
P.O.B. 8131 Dep.
N-0033 Oslo

Tel.: + 47 - 22 86 45 00
Fax: + 47 - 22 11 12 38

Statistics Norway
Research Department