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A LABOR SUPPLY MODEL FOR MARRIED COUPLES WITH NON-CONVEX BUDGET SETS AND LATENT RATIONING[3]

BY

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Abstract

The basic assumption in this paper is that individuals make their choices from a set of latent discrete alternatives, called matches. Given the match, hours of work, wages and non-pecuniary characteristics follow. This model allows for very general budget specifications as well as restrictions on job opportunities and hours of work. The model is estimated on Norwegian data from 1979. Some of the results are summarized in wage elasticities and it is demonstrated that they are in the range of what others have obtained. Moreover, aggregate elasticities which reflect observed as well as unobserved heterogeneity are calculated. We also report estimates derived from alternative specifications of the budget set, ranging from ignorance of taxes at all to a detailed specification of all sorts of taxes and transfers. The results of tax policy simulations are included in the final section of the paper.

1. Introduction

This paper presents a new econometric framework for analyzing labor supply. By labor supply we mean the decision of whether or not to work and the decision of how many hours to work, or from the econometrician's point of view, the hours of work distribution which includes participation probabilities.

The hours of work distribution is derived from a random utility model. The utilities are perceived as random because as econometricians we do not observe all the variables that influence the individual's decisions. These variables are assumed to be known to the individual and we do not consider the case where the individual is uncertain about say, tax rates, wages and job opportunities.

The principal purpose of our study is to model and estimate the hours of work distribution when all details of the tax system are taken into account. This is a demanding task also because the Norwegian tax system, as the tax system in most of the other countries in the Western world, implies a non-convex budget set. The option of joint and separate taxation, social security rules and tax allowances turn an otherwise progressive tax structure into a structure which is not uniformly progressive, rather partially regressive. These features of the tax structure create a non-convex budget set. Fixed costs of working contribute to this non-convexity as job-specific wage tariffs also do (see Moffitt [23]). Therefore, the traditional marginal calculus is no longer sufficient to simulate optimal behavior. Global rather than local criteria have to be applied. Of course, an important question is whether a detailed budget specification really matters. We argue that even if the model specification is robust with respect to approximations of the budget constraints, an exact treatment of the tax system is of considerable interest in the context of policy simulations. This is so because it enables us to specify quite general tax regimes without having to "translate" these regimes into the corresponding approximate ones that enter the model.

Most of the labor supply studies have used the counterfactual assumption of a convex smooth budget set (cf. early contributions such as Rosen [27], Nakamura and Nakamura [24], Wales and Woodland [24] and more recent contributions by Blundell et al [6], Kohlase [20] and Ransom [26]). Only recently there have been attempts to take the non-convexity properties of the tax structure into account. These attempts are usually versions of the approach suggested by Burtless and Hausman [7] (cf. Arrufat and Zabalza [2], Blomquist [4], Hausman [11], [12], [13], Hausman and Ruud, [15]. However, from an econometric point of view the Hausman approach is not ideal. When the number of tax brackets gets large, the Hausman model seems complicated to estimate. For example, the corresponding likelihood function is not in general globally concave in the unknown parameters. When the detailed tax structure for married couples is taken into account (with the options of joint taxation for some incomes, as in Norway and the UK, or with joint or split taxation, as in France and West-Germany), the Hausman approach is likely to be extremely cumbersome unless quite restrictive simplifying assumptions on functional forms are introduced such as linear or log-linear labor supply curves. These functional forms exclude apriori the backward bending case. Also for simulation purposes the Hausman approach seems complicated to apply.

In contrast to the traditional approach in the analysis of labor supply (see Killingsworth [19] for a review of models) we have adopted a theoretical framework in which some of the unobservables are interpreted as choice variables. Specifically, the choice environment is assumed to consist of a set of opportunities, called matches, where each match corresponds to a particular combination of individual abilities offered, skills required to perform certain tasks or activities and other non-skill attributes of jobs such as working in polluting environments, etc. Apart from wage and hours of work, the quality of a match, relative to the individual, depends on the "tension" between the abilities offered and skills demanded as well as of non-pecuniary attributes related to these activities. For a given match we assume throughout this paper that the wage rate and hours of work are fixed. Thus the individual maximizes his utility with respect to discrete latent alternatives (matches) characterized by wage rates, hours of work and non-pecuniary attributes. Our theoretical model is related to the matching models of Tinbergen [28], Hartog [10] and Heckman and Sedlacek [18]. The implied hours of work distribution has a similar form as the continuous logit model and it is particularly convenient for dealing with general budget constraints.

The set of matches available to the individual is, as mentioned above, latent and it is represented by a probability distribution. Specifically, this distribution describes the fraction of market matches with a particular combination of hours and wages that is feasible to the individual. Thus, the framework is consistent with the notion of rationing with respect to job-offers as well as with respect to the allowed amount of hours worked in different jobs. Thus the framework allows us to take into account the fact that the fraction of full-time jobs is higher than the fraction of other jobs. Of course, the consentration of jobs around full-time jobs can be due to preference, but most likely this is not the whole truth. Regulations enforced by firms or by the authorities may restrict the set of feasible jobs to full-time, part-time jobs, etc. Since the framework allows for a rationing of job-offers it means that unemployment can be accounted for in the estimation.

The assumption that hours of work is fixed for a given match implies that the model satisfies the assumption of "independence from irrelevant alternatives" (IIA). In Section 6 of this paper we report the results of a series of tests of the IIA- property. In Dagsvik [8] the more general case is considered in which the worker is free to choose hours of work from a match-specific choice set. The corresponding hours of work distribution is a continuous version of a generalized extreme value random utility model, see McFadden [22]. However, this version of the model is complicated to estimate.

The empirical part of our paper deals with labor supply of married couples in Norway 1979. In our approach we assume that the couple's desicions concerning labor supply are made simultaneously. All previous studies of labor supply with taxes, except for Hausman and Ruud [15] and Ransom [26] assume that the wife takes the husband's income as given.In a labor supply study without taxes Ashenfelter and Heckman [13] found that the cross elasticities were significantly different from zero. There is no a priori reason to expect these cross elasticities to vanish when taxes are taken into account.

This paper is organized as follows. In the next section we present the individual decision model which includes a characterization of the stochastic properties of the unobservables. In Section 3 the probability distribution function for the labor supply and the realized wage rate is derived for one-person households and in Section 4 this model is extended to two-person households. Data and the Norwegian tax rules are described and discussed in Section 5 and Appendix , respectively. The data set contains detailed information about taxes and income since it is based on filled-in tax returns that are in principle checked by local taxation authorities. In Section 6 we present the estimation results. Wage elasticities are reported in Section 7. In Section 8 we report the results from estimating the model under alternative specifications of the budget set and in Section 9 we give the results of tax policy simulations.

2. Random utilities and latent choice variables.

In recent econometric analyses of labor supply one starts by specifying either the direct or the indirect utility function. From this specification and from the budget set the labor supply function is derived. This function is perceived as random since many of the variables that affect the individual preferences are not observed by the econometrician. Usually the specification of the distributional properties of the random terms are made ad. hoc. One reason for this seems to be that the random elements of the model are believed to be of minor importance. Another reason is that it may be difficult to provide theoretical arguments to support the choice of distribution functions because economic theory seldom gives any guidance in that matter.

Our point of departure is that some of the unobservables are choice variables and that the individual's choice of optimum values for the unobserved choice variables are not made independently of the level of consumption and hours worked. These two variables are the only choice variables that are observed.

A formal discussion and interpretation of the latent choice environment is given in Dagsvik [8]. Here we shall consider some main points.

Important examples of unobserved choice variables are type of job and type of leisure or non-market activities such as schooling, sports, household activities, etc. By type of job we understand the specific tasks performed at the job, the type of qualifications or skills demanded to perform these tasks and other non-skill attributes of the job like working conditions, location, commuting distance, etc. Similarly, non-market alternatives may be identified in an analogues way. Non-market alternatives also demand certain skills to perform the tasks associated with the different types of activities.

The individual's set of available opportunities depends on his skills or abilities. These are a mixture of inherited abilities and qualifications obtained through education and training. Following Tinbergen [28] the individual's choice of market and non-market positions is a process in which the individuals try to obtain the best match of personal abilities and skills required to perform certain activities. We extend Tinbergen's approach by assuming that positions and non-skill attributes of the different activities may have a direct influence on preferences. We call a particular combination of skills offered, skills required to perform certain tasks, and non-skill attributes associated with these tasks a match. We assume that the individual finds the optimal match, among the set of feasible matches, by evaluating how well he is fit for a particular task jointly with his taste for that task and for the non-skill attributes of these tasks.

For the moment suppose that the set of skills is finite and can be defined and numbered from 1,2,...,n. Skills offered and skills required in each task are both to be found on this list. For example, one combination could be offered skill no j and requirede skill no j. In this case there is a perfect match of skills. A <u>mismatch</u> may be due to the fact that some individuals prefer to have a job with skill requirements different from what follows from education and training. In the short run offered skills as well as required skills are given.

The total number of combinations of offered and required skills is n^2. In addition we define m attributes of tasks which can be distinguished from skill requirements. Examples are working conditions, location, commuting distance, etc. Altogether this gives $N \equiv mn^2$ combinations of skills and non-skills attributes. Each combination is called a match and the universe of matches is enumerated by a discrete variable, $z = 1,2,...,N$. In the econometric model developed later we assume, however, that this universe is infinite.

Throughout this paper we assume a match-specific wage rate, $W(z)$. The wage rate thus depends on offered skills, required skills and non-skill attributes of different tasks. This assumption differs slightly from Tinbergen [28] who prices out skills and from Heckman and Sedlacek [18] who price out tasks. In the latter paper a sector-specific function is defined that maps individual skills into sector-specific amounts of tasks performed. Tasks are priced out according to the value of marginal productivity in the different sectors. In our framework each individual has to choose his match from his set of feasible matches. The wage rates that are feasible to each individual depend on this choice set. Later, individual characteristics such as education and experience will be introduced to parametrize these individual choice sets. Thus, in our framework neither tasks nor skills are priced out, but matches. Wages might therefore differ according to offered skills, required skills and non-skill attributes of jobs. Sectoral specifications are not introduced, but these can easily be incorporated.

As noted above we assume that hours of work is given when the match is given. This assumption means either that different tasks require a fixed amount of hours or that hours of work is regulated by the authorities, through negotiations between employers and unions, etc. The econometrics of the more general case when hours are allowed to vary for a given match is outlined in Dagsvik [8].

Let H(z) denote hours of work related to match z and let T(z) represent the non-pecuniary attributes of match z. Thus, the choice environment is described by a distribution of wages, W(z), hours, H(z), and non-pecuniary attributes, T(z). This multidimensional distribution is 'objective' in the sense that it is the same for all individuals. As noted in the introduction, we assume that all individuals have a perfect knowledge of this distribution.

Individual choices are assumed to follow from the maximization of utility given the budget constraint and the match- and hour-constraints.

Let U(C,h,z) denote the utility for a one-person household where C is annual consumption, h is hours worked a year and z is the match. The reason why z enters the utility function is of course not because utility depends on the numbering of alternatives, but because the argument z in the utility function takes care of other aspects of the match than C and h. The individuals' decision problem is to choose between discrete alternatives, i.e., matches, z, characterized by W(z) and T(z) subject to the following two constraints:

(2.1)	$C = f(hW(z) + I)$	: Budget constraint
(2.2)	$h = H(z),\ z \in B,$	: Constraint on hours worked and the choice set of matches.

where f is the function that transforms gross income into consumption. The form of the function f depends on the tax system and rules of social security payments, etc. It may be non-differentiable, non-concave and even discontinuous at some points. This corresponds to how the tax systems are in many countries. I is non-labor income and B is the set of feasible matches. This set varies across individuals and it depends on education and training.

The key issue in the present paper is that we do not observe the discrete alternatives or matches, z. As econometricians we are forced to consider these alternatives as latent. An essential assumption in the present paper is that these latent variables are choice variables and that the utility function has the structure

$$U(h,C,z) = v(h,C,T(z)) + \varepsilon(z), \tag{2.3}$$

where $v(\cdot)$ is a deterministic function in the sense that for given values of h, C and T, v is a constant. $\{T(z), \varepsilon(z)\}$ is an enumeration of the points of the bivariate Poisson process on $[0,\infty)X(-\infty,\infty)$ with intensity measure

$$\lambda(t)dte^{-x}dx \ , \quad \int_0^\infty \lambda(t)dt < \infty. \tag{2.4}$$

where $\lambda(t)$ is a positive function.

This means that the probability that there is a match for which

$$(T(z)\varepsilon(t,t + dt)) \cap (\varepsilon(z)\varepsilon(x,x + dx))$$

is equal to

$$\lambda(t)dte^{-x}dx + o(dtdx).$$

Moreover, the expected fraction of matches for which $T(z)<t$ is given by

$$G(t) = \frac{\int_0^t \lambda(y)dy}{\int_0^\infty \lambda(y)dy}. \tag{2.5}$$

In order to facilitate interpretation of the random points $\{T(z), \varepsilon(z)\}$ and to establish the link to discrete choice models assume for a moment that the size of the universe of matches is given and equal to N (say). Then an equivalent representation of the utility function is

$$v(h,C, T(z)) + \eta(z)$$

where $\eta(z)$, $z=1,2,\ldots,N$, are independent draws from the extreme value distribution, $\exp(-e^{-\eta})$. The variables $\eta(z)$ account for the fact that for a given match the taste for this match varies across individuals and it is thus perceived as random. However, for a given z the attribute value, $T(z)$, is the same relative to every individual and it is therefore non-stochastic. But the set of feasible matches B varies across individuals which implies that the set of attribute values for the feasible matches varies from one individual to another. Consequently, we may interpret the set of feasible matches as random and thus their respective attribute values becomes random. Since the conditional distribution of $\eta(z)$ for given z is independent of z the unconditional distribution of $\eta(z)$ across individuals and matches will also be $\exp(-e^{-\eta})$.

In the general case N is stochastic and since by (2.4) the intensity measure of $\{T(z)\}$ is $\lambda(t)dt$ it follows that N is Poisson distributed with expected number of points given by

$$EN = \int_0^\infty \lambda(t)dt$$

which is finite by assumption. However the expected number of points of $\{T(z),\varepsilon(z)\}$ is infinite which means that there may occur several values of $\varepsilon(z)$ to one value of $T(z)$. We may then adopt the rule that whenever multiplisity of $\varepsilon(z)$ occurs then the largest value is used. It is clear that this rule does not alter the results of the following section.

Assumptions that yield (2.3) are

(i) The utilities are stochastically independent and identically distributed across matches.

(ii) The individual selects the optimal match according to the axiom "independence from irrelevant alternatives". (IIA)

Proofs as well as equivalent assumptions are given in Dagsvik [8]. Assumption (i) is a standard assumption that states that preferences are purely random across matches. Assumption (ii) is the famous Luce axiom, Luce [21]. Since the empirical content of a match is not specified, assumption (ii) is quite weak. We might in fact define the different types of matches so as to obtain IIA.

Let

$$U^*(h,C) = \max_z U(h,C,z).$$

Then U^* is the utility of the observed "commodities" (h,C).

3. The distribution of the realized wage and hours of work

In this section we consider the distribution of the individual's realized wage and labor supply.

For expository simplicity we shall consider the derivation of the hours of work distribution for the case where B is given so that H(z), W(z) and T(z) are non-stochastic.

After inserting the hours and budget constraint, the utility can be written

$$(3.1) \qquad u_z = \alpha(H(z),W(z),T(z)) + \varepsilon(z) .$$

where

$$\alpha(H(z),W(z),T(z)) = v(H(z),f(H(z)W(z) + I),T(z))$$

and where $\varepsilon(z)$ are independent draws from the extreme value distribution. It is well known that the corresponding choice probabilities have the form (see McFadden [22])

$$(3.2) \qquad P(u_z = \max u_j) = \frac{\exp[\alpha(H(z),W(z),T(z))]}{\sum_{j\varepsilon B} \exp[\alpha(H(j),W(j),T(j))]} .$$

This model is often called the Luce model (cf. Luce [21]). Let $A(h,w,t)$ denote the set of feasible matches for which $H(z) = h$, $W(z) = w$, $T(z) = t$. Then we realize that the probability, $p(h,w,t)$, of selecting a match with attribute values (h,w,t) is given by

$$(3.3) \quad p(h,w,t) = \sum_{z\varepsilon A(h,w,t)} P(u_z = \max u_j) = \frac{n(h,w,t)\exp(\alpha(h,w,t))}{\sum_{x,y,r} n(x,y,r)\exp(\alpha(x,y,r))} .$$

where $n(h,w,t)$ is the number of matches in $A(h,w,t)$.

Let

$$q(h,w,t) = \frac{n(h,w,t)}{\sum_{x,y,r} n(w,y,r)} .$$

Thus, $q(h,w,t)$ is the relative number of available matches with attributes (h,w,t).

From 3.3) we get

$$(3.4) \quad p(h,w) \equiv \sum_t p(h,w,t) = \frac{\sum_t q(h,w,t)\exp(\alpha(h,w,t))}{\sum_{x,y,r} q(x,y,r)\exp(\alpha(x,y,r))} ,$$

where $p(h,w)$ is the probability density that the optimal job has wage w and hours of work h. In other words, $p(h,w)$ is the joint density of the realized wage and hours of work.

Now, let us turn to a more general case and let, analogously to $G(t)$ in (2.5), $G_2(w,t,h)$ be the (expected) fraction of feasible matches for which $(W(z) \leq t,\ 0 < H(z) \leq h)$. In other words, $G_2(w,t,h)$ is the probability that a randomly selected match, z, satisfies $(W(z) \leq w,\ T(z) \leq t,\ 0 < H(z) \leq h)$. Assume that the density of G_2, g_2, exists and let g_1 be the (expected) fraction of feasible matches for which $H(z) \varepsilon K$ where K is the set of feasible hours. Furthermore, let

$$g_0 = 1-g_1$$

and let

$$g_4(w,h) = \int g_2(w,t,h)dt.$$

The probabilities g_1 and g_0 represent the shares of feasible market and non-market opportunities, respectively. Specifically, g_1 is the probability that a random draw from the set of feasible matches is a market match. The density g_4 represents the frequency of market matches with hours h and wages w. When the number of matches is random and generated by the (positive) Poisson distribution described above then it can be demonstrated that the continuous version corresponding to (3.4) is

$$\text{(3.5)} \qquad \phi(h,w,K) = \frac{g_1 V(h,w)}{g_0 V_0 + g_1 V_1(K)},$$

for $h > 0$, $h \varepsilon K$, and

$$\text{(3.6)} \qquad \phi(0,K) = \frac{g_0 V_0}{g_0 V_0 + g_1 V_1(K)}$$

where

$$\text{(3.7)} \qquad V(h,w) = \int \exp(v(h,f(hw+I),t))g_2(w,t,h)dt,$$

$$\text{(3.8)} \qquad V_1(K) = \int_{\substack{x>0 \\ x \varepsilon K}} V(x,y)dxdy,$$

$$\text{(3.9)} \qquad V_0 = \int \exp(v(0,f(I),t))g_3(t)dt$$

and $g_3(t)$ is the marginal density of $T(z)$, given that $H(z)=0$.

Notice that (3.5) allows a "frequency type" interpretation: The numerator of $\phi(h,w,K)$ is the mean utility across the suitable matches with attributes (h,w) and is thus the expected value of the favourable outcomes. The denominator is the mean utility across all available matches and it is therefore the expected value of all the possible outcomes.

From (3.5) and (3.6) we immediately realize that the odds ratios of $\phi(h,w,K)$ are independent of the choice sets, e.g.,

$$\frac{\phi(h_1,w_1,K_1)}{\phi(h_2,w_2,K_1)} = \frac{\phi(h_1,w_1,K_2)}{\phi(h_2,w_2,K_2)}, \quad (h_1,h_2)\varepsilon K_1 \cap K_2 .$$

As is wellknown, this property is equivalent to Luce choice axiom, also called "independence from irrelevant alternatives". As mentioned above this property makes it possible to carry out an empirical test of the structure (3.5) and (3.6). Note however that if also g_2 depends on K, then IIA does not hold true. A rejection of IIA can therefore only be interpreted as a rejection of either (i), (ii), (2.2) or the functional forms for v and g_2.

For the purpose of empirical implementation we shall simplify (3.5) and (3.6). Let

$$(3.10) \quad \exp(\phi(h,C,w)) = E\{\exp(v(h,C,T(z)))|H(z) = h, W(z) = w)\}$$

$$= \int \exp(v(h,C,t)) \frac{g_2(w,t,h)}{g_4(w,h)} dt$$

for $h>0$, $h \varepsilon K$

and

$$(3.11) \quad \exp(\phi(0,C)) = E\{\exp(v(0,C,T(z)))|H(z) = 0\}$$

$$= \int \exp(v(0,C,t)) \frac{g_3(t)dt}{g_0} .$$

Eqs. (3.10) and (3.11) define the

mean utility across all matches conditional on $H(z) = h$, $W(z) = w$, and $H(z) = 0$, respectively. Note that $\psi(h,C,w)$ may depend on w since $T(z)$ and $W(z)$ may be interdependent. In the case where $T(z)$ and $W(z)$ are independent ψ becomes a function of (h,C) solely.

From (3.7)-(3.11) it follows that

$$V(h,w) = \exp(\psi(h,f(hw+I),w))g_4(w,h)$$

and

$$V_0 = \exp(\psi(0,f(I))).$$

We assume that $W(z)$ and $H(z)$ are independent, i.e.,

$$g_4(w,h) = g_5(w)g_6(h) \quad . \tag{3.12}$$

This does not imply, however, that there is no dependency between observed wages and observed hours worked. On the contrary, the conditional distribution of wages, given hours of work, is under the hypothesis of (3.12), given by

$$\phi_1(w,K|h) = \frac{\phi(h,w,K)}{\int\phi(h,y,K)dy} = \frac{\exp(\psi(h,f(hw+I),w))g_5(w)}{\exp(\psi(h,f(hy+I),y))g_5(y)dy} \quad . \tag{3.13}$$

for $h > 0, h\varepsilon K$.

From (3.13) we realize that we can write

$$E(W|K) = \frac{E[W \exp(\psi(h,f(hW+I),W))]}{E \exp(\psi(h,f(hW+I),W))} \quad ,$$

where the expectation operator on the right hand side is taken with respect to g_5. By a first order Taylor approximation of the denominator and the numerator we get, assuming that $T(z)$ and $W(z)$ are independent,

$$E(W|h) \approx Var(W) \frac{\partial\psi(h,f(hEW+I))}{\partial f} f'(hEW+I)h$$

provided f(•) is differentiable. In the case where f is linear and hEW+I is kept constant $E(W|h)$ becomes linear in h with a positive slope provided $\partial\psi/\partial f > 0$. This demonstrates that the empirical "evidence" of the wage rate being dependent on hours of work may be explained by selectivity.

Furthermore, the conditional wage distribution given that $h > 0$, is given by

$$(3.14) \qquad \phi_2(w,K|h>0) = \frac{\int\limits_{x>0,\,x\varepsilon K} \exp(\psi(x,f(xw+I),w))g_5(w)g_6(x)dx}{\int\limits_{\substack{x>0\\ x\varepsilon K}} \exp(\psi(x,f(xy+I),y))g_5(y)g_6(x)dydx} .$$

Formulas (3.13) and (3.14) correspond to the wellknown selectivity bias problem, Heckman [16], namely that in general the conditional wage distribution given that the individual works, $\phi_2(\cdot)$, differs from the unconditional wage distribution, g_5.

In some studies only working individuals are analyzed and observed wage rates are applied when estimating the model. A study of this type is reported in Anderson et al [1]. The conditional distribution of hours of work given the wage and given that the individual works, $\phi_3(h,K|w)$, is given by

$$(3.15)\, \phi_3(h,K|w) = \frac{\phi(h,w,K)}{\int\limits_{\substack{x>0\\ x\varepsilon K}}\phi(x,w,K)dx} = \frac{\exp(\psi(h,f(hw+I),w))g_4(w,h)}{\int\limits_{\substack{x>0\\ x\varepsilon K}} \exp(\psi(x,f(xw+I),w))g_4(w,x)dx},$$

for $h > 0$, $h \varepsilon K$.

Note that while the traditional labor supply models are silent about rationing of jobs, the structure of (3.5) and (3.6) allows variations in market opportunities to be explicitly accounted for through the ratio g_1/g_0. Similarly to Blundell et al [6] we may for example specify this ratio as a parametric function of labor market indicators such as regional unemployment rates, and individual characteristics such as education and training.

4. Extension of the model to two-person households (married couples).

The decision problem of a married couple is to decide the labor supply of the wife and the husband as well as the level of consumption of the household subject to the budget and hours constraints.

Let $U(h_F, h_M, C, z)$ denote the household's utility function where h_F, h_M denotes the wifes and the husbands hours of work, respectively. C is total consumption of the household and $z = (z_F, z_M)$ is an index of the matches of the wife, z_F, and husband, z_M, respectively.

The constraints are given by

$$(4.1) \qquad C = f(h_F W_F(z_F),\ h_M W_M(z_M), I) \ ,$$

$$(4.2) \qquad (h_F,\ h_M) = (H_F(z_F),\ H_M(z_M)),\ z \in B,$$

where $H_F(z_F)$, $W_F(z_F)$, $H_M(z_M)$ and $W_M(z_M)$ are the match-specific hours of work and wages for the wife and husband, respectively. Under assumptions that are straight forward extensions of the assumptions of the preceding section we can write

$$(4.3) \qquad U(h_F,h_M,C,z) = v(h_F,h_M,C,T(z)) + \varepsilon(z)$$

where $\{T(z),\ \varepsilon(z)\}$ is an enumeration of the points of the Poisson process on $[0,\infty)X(-\infty,\infty)]$ with intensity as described in Section 2. We define g_{11} as the (expected) fraction of the feasible market matches for the couple that satisfies $H_F(z)\varepsilon K$, $H_M(z)\varepsilon K$. g_{01} is the (expected) fraction of the feasible matches that are market matches for the male with $H_M(z)\varepsilon K$ and non-market matches for the wife. g_{10} is defined analogously (by replacing husband with wife) and

$$g_{00} = 1-g_{11} - g_{10} - g_{01} \ .$$

Let G_{22} (w_F, w_M, h_F, h_M) be the (expected) fraction of the feasible market matches for which

$$(W_F(z)<w_F,\ W_M(z)<w_M,\ H_F(z)<h_F,\ H_M(z)<h_M)$$

and let $G_{2M}(w_M, h_M)$ be the expected fraction of feasible market matches for the husband and non-market matches for the wife for which $(W_M(z)<w_M, H_M(z)<w_M)$ and similarly $G_{2F}(w_F, h_F)$ for the wife. As in Section 3 let

$$(4.4)\quad \exp\{\phi(h_F,h_M,C,w_F,w_M)\} = E\{\exp(v(h_F,h_M,C,T(z)))\mid H_F(z) = h_F, H_M(z) = h_M, W_F(z) = w_F, W_M(z) = w_M\},$$

$$(4.5)\quad \exp\{\phi(h_F,0,C,w_F)\} = E\{\exp(v(h_F,0,C,T(z)))\mid H_M(z) = 0, H_F(z) = h_F, W_F(z) = w_F\},$$

$$(4.6)\quad \exp(\phi(0,h_M,C,w_M)) = E\{\exp(v(0,h_M,C,T(z)))\mid H_F(z) = 0, H_M(z) = h_M, W_M(z) = w_M\}$$

and

$$(4.7)\quad \exp(\phi(0,0,C)) = E\{\exp(v(0,0,C,T(z)))\mid H_F(z) = H_M(z) = 0\}.$$

Similarly to Section 3 we can now express the joint density of hours and wages. Here we shall only consider the particular case where $H_F(z)$, $H_M(z)$, $W_F(z)$ and $W_M(z)$ are assumed independent.

In this case

$$(4.8)\quad g_{22}(w_F,w_M,h_F,h_M) = g_{4F}(w_F)g_{4M}(w_M)g_{5F}(h_F)g_{5M}(h_M),$$

(4.9) $g_{2F}(w_F,h_F) = g_{4F}(w_F)g_{5F}(h_F)$,

(4.10) $g_{2M}(w_M,h_M) = g_{4M}(w_M)g_{5M}(h_M)$,

and the corresponding probability densities are given by

$$(4.11)\quad \phi(h_F,h_M,w_F,w_M,K) = \frac{g_{11}V(h_F,h_M,w_F,w_M)}{g_{00}V_0+g_{10}V_{0F}(K)+g_{01}V_{0M}(K)+g_{11}V_1(K)}$$

for $h_F>0$, $h_M>0$ and $(h_F,h_M)\varepsilon K$,

$$(4.12)\quad \phi\ (h_F,0,w_F,K) = \frac{g_{1\ 0}(h_F,0,w_F)}{g_{00}V_0+g_{10}V_{0F}(K)+g_{01}V_{0M}(K)+g_{11}V_1(K)}\ ,$$

for $h_F>0$ and $h_F\varepsilon K$,

$$(4.13)\quad \phi(0,h_M,w_M,K) = \frac{g_{01}V(0,h_M,w_M)}{g_{00}V_0+g_{10}V_{0F}(K)+g_{01}V_{0M}(K)+g_{11}V_1(K)}\ ,$$

for $h_M>0$ and $h_M\varepsilon K$,

$$(4.14)\quad \phi(0,0,K) = \frac{g_{00}V_0}{g_{00}V_0+g_{10}V_{0F}(K)+g_{01}V_{0M}(K)+g_{11}V_1(K)}\ ,$$

where

$V(h_F,h_M,w_F,w_M) =$

$\exp(\psi(h_F,h_M,f(h_Fw_F,h_Mw_M,I),w_F,w_M))g_{4F}(w_F)g_{4M}(w_M)g_{5F}(h_F)g_{5M}(h_M)$,

$$V_1(K) = \int_{\Omega_{11}} V(x_1,x_2,y_1,y_2)dx_1dx_2dy_1dy_2\ ,$$

$$V_{0F}(K) = \int_{\Omega_{10}} \exp(\psi(x_1,0,f(x_1y_1,0,I),y_1))g_{5F}(x_1)g_{4F}(y_1)dx_1dy_1,$$

$$V_{0M}(K) = \int_{\Omega_{01}} \exp(\psi(0,x_2,f(0,x_2y_2,I),y_2))g_{4M}(y_2)g_{5M}(x_2)dx_2dy_2\ ,$$

$$V_0 = \exp(\psi(0,0,f(I))),$$

and

$$\Omega_{11} = \{(x_1,x_2,y_1,y_2) \mid x_1 > 0,\ x_2 > 0,\ (x_1,x_2)\ \varepsilon K\}\ ,$$

$$\Omega_{10} = \{(x_1,y_1) \mid x_1 > 0,\ x_1 \varepsilon K\}\ ,$$

$$\Omega_{01} = \{(x_2,y_2) \mid x_2 > 0,\ x_2\ \varepsilon K\}\ .$$

5 Data

The data are obtained from two different data sources with information about married couples in Norway, 1979. The first source is a questionnaire which contains data on hours worked, wage rates and socio-economic variables such as the number and age of children and education level. The other source is based on filled in and approved tax reports and yield detailed information about all sorts of reported income, legal deductions, taxes paid and transfer payments received. The two sets of data are linked on the basis of personal identification numbers. The Central Bureau of Statistics has been responsible for collecting and preparing the data sets.

The data based on the tax reports have been used to check the answers on the wage rates and hours worked given in the questionnaire. For around 90 per cent of those working the reported wage rate has been used. For the remaining individuals, including some reported working, observations are drawn from an estimated wage distribution. The quality of the hours observations was considered to be so poor (data for hours last week) that instead we have calculated the hours worked per year by dividing the reported labor income per year by the reported or predicted wage rate.

In table I we report some statistics for the average of the sample. The sample selection rules are as follows.

Only couples where the age of the husband is less than 66 years and the age of the wife is between 27 and 66 years are included. Both wife and husband are wage earners or not working. Couples for which the wife's or husband's hours of work is above 3600 hours per year is excluded. When the female wage rate is below 15- or above 56 NOK it is replaced by a draw from the estimated wage distribution mentioned above. The same procedure is followed when the male wage rate is below 24- or above 74 NOK. The resulting sample size is 778. In the simulation experiments reported below the sample size is, however, 815 because the couples with high reported hours of work have been included.

Notice that not working is defined to be less than or equal to 60 hours worked per year.

[table I]
[Figure 1]
[Figure 2]

6. Estimation results

The densities in (4.11)-(4.14) are used to construct the likelihood function applied in the estimation of the model. A maximum likelihood procedure has been used. The consumption function is defined by

$$(6.1)\quad C = f(w_M h_M, w_F h_F, I) = \sum_{j=M,F} w_j h_j + I - S(w_F h_F,\ w_M h_M, I)$$

where I denotes capital income and S(•) is the tax function.

In the calculation of f(•) for alternative values of h_j, j=M,F, all details concerning the tax structure of 1979, as outlined in Appendix , are taken into account.

In order to estimate the model we need to specify functional forms for $\psi(h_F, h_M, C, w_F, w_M)$, $g_{4F}(w_F)$, $g_{4F}(h_F)$ and $g_{5M}(h_M)$. We have chosen ψ to be quadratic, separable in consumption and leisure after having performed a preliminary estimation with a general quadratic specification. Only the leisure terms are assumed to depend on household characteristics. The densities $g_{4F}(w_F)$ and $g_{4M}(w_M)$ are chosen to be log-normal densities. The densities associated with the latent rationing assumptions related to hours are assumed to be of the form

$$g_{5F}(h_F) = d_F \exp(-(h_F-\bar{h}_F)^2 a_F + b_F D(h_F))$$

where d_F, a_F, $\bar{h}_F$, and b_F are parameters, and $D(h_F)$ is an indicator function defined by

$$D(h_F) = \begin{cases} h_F & \varepsilon[2040, 2200] \\ 0 & \text{otherwise} . \end{cases}$$

The indicator function D allows for a peak at "full-time" hours. When $a_F = b_F = 0$ the rationing distribution reduces to the uniform distribution. When $a_F = 0$ the distribution is uniform apart from a peak at full time hours.

For the males the corresponding density is assumed to be of the form

$$g_5(h_M) = d_M \exp(-(h_M-\bar{h}_M)^2 a_M + b_M D(h_M)).$$

We have experimented with a peak at part-time hours of work, but it turned out to be of minor importance.

Since ψ is a quadratic form we get that

$$\begin{aligned}\psi^* &\equiv \psi(h_F,h_M,C,w_F,w_M) + \log g_{5F}(h_F) + \log g_{5M}(h_M) - \log(d_F d_M) \\ &= \alpha_1 C + \alpha_2 C^2 + \alpha_3 L_F^2 + \alpha_4 L_F + \alpha_5 L_F \log A_F \\ &+ \alpha_6 L_F L_M + \alpha_7 L_F BU6 + \alpha_8 L_F BO6 + \alpha_9 L_M^2 + \alpha_{10} L_M \\ &+ \alpha_{11} L_M \log A_M + b_F D(h_F) + b_M D(h_M)\end{aligned}$$

where L_j is leisure time per year, $L_j = 8000-h_j$, A_j is age, BU6 and BO6 are number of children below and above 6 years of age, j=M,F.

Moreover the log odds ratio of feasible non-market matches to market matches are parametrized in the following way

$$\log\left(\frac{g_{10}}{g_{11}d_M}\right) = \alpha_{12} + \alpha_{13}E_F, \quad \log\left(\frac{g_{01}}{g_{11}d_F}\right) = \alpha_{14}$$

and

$$\log\left(\frac{g_{00}}{g_{11}d_F d_M}\right) = \alpha_{12} + \alpha_{13}E_F + \alpha_{14} + \alpha_{15},$$

where E_F is years of education for the female. We have also tried males education but it turned out to have no significant influence. Unemployment variables have not been included due to the low rate of unemployment in Norway in 1979 (below 2 per cent).

The estimation is based on a procedure suggested by McFadden [22] which yields results that are close to the full information maximum likelihood method. We are not able to use the exact likelihood function to estimate the model because the evaluation of the integrals in (4.11)-(4.14) would be to costly and cumbersome. The estimation procedure applied replaces the continuous four-tuple integral in the denominators of the densities by a sum over 70 and alternatively 30 random points where each term is adjusted by appropriate weights. In other words, the continuous logit model (4.11)-(4.14) is replaced by a discrete logit version with 70 (30) alternatives. McFadden [22] has demonstrated that this method yields consistent and asymptotically normal parameter estimates.

It should be kept in mind that we are not able to separate the structural coefficients in the mean utility function from some of the parameters of the densities g_{5F} and g_{5M}. However, if we keep the parameters of these densities as well as the parameters of ϕ fixed we are able to perform any simulations including the calculation of wage elasticities.

The results of the estimation are shown in table II and

III. Two alternatives are presented. In the first alternative we have used 70 draws or discrete alternatives for each couple and in the second, 30 draws. Moreover, in the first alternative tax deductions are set equal to the maximum of observed deductions and standard deductions. Of course, for the observed hours of work actual deductions will be the maximum (which for some individuals might be equal to standard deductions). The model requires, however, that marginal taxes as well as taxes paid and consumption are evaluated for all other feasible alternatives. In the first estimation alternative tax deductions are equal to the maximum of observed deductions, related to the observed hours of work, and standard deductions. This alternative is thus to be interpreted as a conditional model given the observed deductions. A model closer to an unconditional model is one for which deductions vary with income. In Norway deductions are highly correlated with income since interest payments are equal to the maximum of standard deductions and predicted deductions based on an estimated tax deduction function. The estimated tax deduction function is reported in the Appendix.

The results show that the difference between these two estimation alternatives is small. The 5 per cent confidence interval overlap for all coefficients. In what follows we will refer to alternative 2 as the base case and all comments given below are related to this case. Elasticities will be calculated on the basis of this case, as well.

[Table II]
[Table III]

Except for the cross leisure term all variables have a significant influence on the hours of work and wage distribution. The 'mean utility' function is estimated to be a strictly concave function in C and L, and the mean 'marginal utility' of consumption is positive for all admissible values of C.

The estimates imply that the female's marginal utility of leisure increases with age and number of children. If we take the youngest female in the sample (27 years of age) without children as a reference case, her 'marginal utility' of working is positive when $h < 423$. If we consider a woman with one child below 6, then the 'marginal utility' of working is negative for all h and if we consider a woman with one child between 6 and 18, then the 'marginal utility' of working becomes negative for $h < 227$. A woman of 42.5 years of age or older, without children, has a negative marginal utility of working for all h.

Results not reported here show that for men the number of children has no impact on the marginal utility of leisure.

For both sexes age affects behavior directly through the utility of leisure. Moreover, experience and therefore age has an impact on behavior through the choice set B and therefore through the wage equation. Wage rates are estimated to be a concave function of experience with a peak at 31.5 years of experience for men and at 30.9 years for women.

The wife's education turns out to affect the fraction of feasible market matches such that a higher educated woman has more job opportunities than a less educated. Moreover, education is estimated to have a positive impact on wages and it seems that education has a stronger relative impact on female wage rates than on wage rates among men.

We end this section with the results of the tests of the assumption of independence from irrelevant alternatives.

As stated above the IIA property implies that the ratio of the probabilities of choosing any two alternatives is independent of the other alternatives in the choice set. The continuous logit model we have developed has the property that the mean utility function for alternative (h,w) depends only on the attributes associated with this alternative, namely $C=C(h_F,h_M,w_F,w_M)$ where $C(h_F,h_M,w_F,w_M)$ means that consumption is evaluated at hours and wages (h_F,h_M,w_F,w_M). Accordingly it satisfies IIA. In general the mean utility at (h_F,h_M,w_F,w_M) could also depend on consumption values at other hours and wages. An implication of IIA is that the parameters remain unchanged when the model is estimated conditional on a restricted subset of the full choice set. This is the basis for the Hausman-McFadden specification test (HMT) [14]. We have performed several versions of the test. They show that in most cases (13 out of 16 tests) the IIA-hypothesis is not rejected. The details of the estimation procedure as well as of the HMT tests will be reported in a forthcoming paper.

7. Wage and income elasticities

In labor supply studies it is common to report individual elasticities or elasticities for mean sample values, or for subsamples of individuals who are grouped according to some socio-demographic characteristics. This is meaningful when the error terms are assumed to be independent of C and h. This is not the case in our model since

$$\max_{z \in B} U(h_F, h_M, f(h_F(z_F) W_F(z_F), h_M W_M(z_M), I), z)$$

has the same distribution as

$$U^*(h_F, h_M) \equiv \psi^*(h_F, h_M) + \varepsilon(\bar{z}_F(h_F), \bar{z}_M(h_M)) \tag{7.1}$$

where $\bar{z}_j(h_j)$, $j=F,M$, denotes the optimal match, given h_j.

The conditional expected utility, $\psi(\cdot)$, evaluated for mean sample values of the variables is the utility concept that comes closest to the one used by others in the calculation of elasticities. However, the utility expression appearing in the probabilities in the likelihood function is not ψ, but ψ^*, which is a mixture of ψ and the densities, g_{5j}, reflecting 'rationing' on hours. We are not able to separate ψ from g_{5j} without introducing further assumptions. But if a shift in an exogeneous variable does not change the 'rationing' densities, then elasticities calculated on the basis of ψ^* might approximate supply elasticities for the 'mean sample' individual.

With these reservations in mind we have calculated local elasticities for the mean sample individual given that he or she works, on the basis of the following set of equations:

$$(7.2)\qquad C = \Sigma\, m_j h_j + \hat{I}\ ,$$

$$(7.3)\qquad \frac{\partial\phi^*(\cdot)}{\partial L_M} - \frac{\partial\phi^*(\cdot)}{\partial C}\, m_M = 0\ ,$$

$$(7.4)\qquad \frac{\partial\phi^*(\cdot)}{\partial L_F} - \frac{\partial\phi^*(\cdot)}{\partial C}\, m_F = 0\ ,$$

where

$$m_j = w_j(1-S_j') = \text{marginal rate};\ j=M,F,$$

$$\hat{I} = I(1-S_M') + \Sigma\, d_{jb} = \text{virtual income},\ j=M,F,$$

$$d_{jb} = \sum_{k=1}^{jb} t_k \bar{R}_k - \sum_{k=1}^{jb} t_k \bar{R}_{k-1}$$

and where $\bar{R}_k - \bar{R}_{k-1}$ denotes the size of the tax-bracket k measured in NOK. t_k is the marginal tax rate on tax segment k. jb indicates that the optimal tax bracket for the representative individual is jb. Note that the tax rules in Norway imply that capital income is allocated to the spouse with the highest income, in most cases the husband.

Sample averages are used to calculate these elasticities. The elasticities are denoted mean utility, mean sample elasticities and the following ones are reported in table IV:

- uncompensated elasticities, hours h_j with respect to w_j (Cournot elasticities)
- compensated or utility constant elasticities (Slutsky)
- Total income elasticities (Cournet minus Slutsky)
- virtual income elasticities (which are the elasticities of h_j with respect to $\hat{I}$
- consumption constant elasticities (Frisch elasticities).

[Table IV]

Compared to what others have obtained (see Killingsworth [19] for a review) our calculated elasticities are all of a reasonable magnitude and of the expected sign. Females are calculated to be more wage-responsive than males and Slutsky-elasticities show that substitution effects are strong, especially for women.

Another set of elasticities arise when we consider how the distribution of labor supply is affected by changes in say, wage levels. These elasticities are denoted aggregate ones since they take into account the unobserved and observed heterogeneity in the population. Moreover, they do not require, as the above calculation of mean utility elasticities does, that behavior is determined by local criteria. They also permit marginal utilities of working to be positive at the point of adjustment which might be optimal if the individual is constrained.

For the sake of expository simplicity let $\phi(h,w,r)$ denote the labor supply distribution conditional on the observed vectors of household characteristics, r, and let

$$\phi(h,w) = \int \phi(h,w,r)\Gamma(r)dr$$

be the aggregate (per capita) labor supply distribution function where $\Gamma(r)$ is the density of r. Since the sample is representative with respect to r an estimate of the aggregate distribution is obtained by

$$\hat{\phi}(h,w) = \sum_i \phi(h,w,r_i) \ , \tag{7.5}$$

where r_i denotes the enumeration of the sample. The aggregate (marginal) hours of work distribution can be estimated by

$$(7.6) \qquad \hat{\phi}(h) = \sum_i \phi(h, w_i, r_i) \, .$$

Similarly, we can estimate various conditional aggregate distributions as for instance the marginal distribution of the male supply, given that the wife works.

Table V gives the aggregate elasticities. Three types of elasticities are shown.

Let $\hat{\phi}_{ij}(0)$ denote the probability that individual i, sex j is not working and let

$$(7.7) \qquad P_{ij} = 1 - \hat{\phi}_{ij}(0) \, .$$

Then

$$(7.8) \qquad N_j = \sum_{i=1}^{N} P_{ij} \, , \qquad j=F,M,$$

is equal to the expected number of participating individuals of sex j in the sample.

The first line in table V gives the elasticity of N_j with respect to wage levels. N is the total number of households in the sample.

The second and third line give the elasticities of the conditional and unconditional expectation of hours worked, respectively. With the simplified notation introduced above the unconditional expectation of hours supplied in the population, denoted H_j, is estimated by

$$(7.9) \qquad H_j = \sum_{i=1}^{N} \sum_{x=0}^{3600} x \, \phi(x, w_{ij}, r_{ij}), \qquad j=M,F,$$

and the conditional expectation (conditional on working) is

$$(7.10) \qquad H_j^c = \sum_{i=1}^{N} \sum_{x>0}^{3600} x \, \frac{\phi(x, w_{ij}, r_{ij})}{P_{ij}} \, , \qquad j=M,F,$$

where 3600 is the upper limit on hours worked per year.

[Table V]

These elasticities give the impact on the labor supply aggregates specified in (7.8)-(7.10) from a 1 per cent increase in the wage levels for all individuals and they are the result of a simulation by using the model with these new wage levels replacing the initial ones.

The results show that female participation is slightly more elastic than hours supplied, conditional on working. For men the opposite is true. Moreover, hours supplied, conditional on working, is almost inelastic.

The elasticities in the second line can be compared with the Cournot elasticities given in table IV. We observe that the cross terms are not only of the same sign, but they are nearly identical. Since the aggregate elasticities are the most reliable ones, we observe that the mean utility elasticities <u>overestimate</u> the own-wage response to a great extent.

The last line of table V is approximately equal to the sum of the first and second line, since for each individual the unconditional expectation of hours supplied equals the product of the participation probability and the expected hours worked, conditional on participation. The total supply elasticities imply that a partial 1 per cent wage increase in the male wage rates will increase expected hours supplied by men by 0.33 per cent and reduce expected hours supplied by women by 0.54 percent.

Women are more wage responsive than men since a 1 per cent increase in the female wage rates will increase expected hours supplied by 1.2 per cent. The negative impact on the males labor supply is rather weak, 0.13 per cent.

An overall wage increase of 1 per cent can be found by adding own-wage and cross-wage elasticities. The positive impact of an overall wage increase on female labor supply is substantially lower than a partial increase, a fact that should be kept in mind when cross-section estimates are compared with estimates based on partial time-series studies in which male and female wage rates grow almost at the same rate.

8. Estimation under alternative budget specifications

In addition to the estimation alternatives reported in table II we have applied the McFadden estimation method (with 30 draws) to the following alternative specifications of the budget constraint:

i) Smoothed tax function without kinks, one for separate and another one for joint taxation. The applied functions are described in Appendix ("Smoothed tax functions").

ii) Standard tax deductions instead of actual, observed tax deductions ("Standard deductions").

iii) No marginal taxes, taxes paid are constant and equal to the observed taxes paid by the household for all h ("No marginal taxes").

The results are given in table VI.

A striking result is that counterfactual specifications of the budget set such as a smoothed tax-function rather than the full representation of all kinks in the tax system, and standard deductions rather than the actual and observed deductions, have small impact on the estimates of the coefficients. These results are important since the present labor study is the first one with access to filled in tax returns. In a majority of empirical labor supply studies US data have been used in which one is forced to use standard deductions to evaluate effective tax rates. Moreover, until recently the non-convexities in the budget sets generated by a non-uniform progressiveness of the tax system have also been ignored. Our results indicate that these misspecifications might not yield quite misleading results.

Another, but expected finding is that the ignorance of marginal taxes, the "No marginal taxes" case, give estimates quite different from the other alternatives. The coefficients α_1, α_2, α_3, α_4, α_6, α_9, α_{10} and α_{15} are all estimated to be significantly different from what we obtained in the base case. If the parameters, erroneously, are considered as estimates of the utility function, then the results imply that the marginal utility of leisure for most of the females are underestimated and that the cross term in the utility function is overestimated.

The ignorance of marginal taxes implies quite different elasticities from what is obtained in the other cases. Own-wage responses are substantially lowered and the numerical values of the cross elasticities are higher. The upward bias in cross response of female labor supply is particularly strong which might be one explanation why previous studies have reported strong cross terms, see Ashenfelter and Heckman [3]. The downward bias in own-wage responses, when marginal taxes are ignored, supports the theoretical conclusions drawn by Blomquist [5].

[Table VI]

9. Policy simulations

To demonstrate how the model can be used in policy simulations we have simulated the impact of six changes of the tax system.

The simulation experiments are based on the representation (7.1). The set of feasible hours is a continuum but it can be demonstrated that we obtain a good approximation to the aggregate predictions by drawing a few feasible hours for each individual. We have drawn 15 values of h for each spouse. In addition each individual has the option of not working (hours worked less than 60 hours a year). Finally, 256 corresponding values are drawn from the extreme value distribution for each couple to simulate realizations of the error terms. The present simulations are performed conditional on the observed wages as the reference case. However, the model allows us to draw wage observations that corresponds to the feasible hours.

Simulations have been performed on a larger sample than the one used in estimating the model, since those excluded due to low quality (or missing) of the observations of the endogeneous variables have been included here.

Column I of Table VII gives the base predictions. All variables are on a per capita basis.

The following policy simulations have been performed:

I. A 10 per cent reduction in all marginal tax rates.

II. Removal of the option of joint taxation.

III. As II, but total tax revenue is kept constant. Since the removal of joint taxation increases tax revenue, a uniform cut in all tax rates is carried out in order to keep tax revenue constant. The model is used to find this reduction and it amounts to a 16.7 per cent cut in all tax rates (not percentage points).

IV. The flat tax rate on gross earnings is increased from 5 percentage points to 10 percentage points. In a first alternative marginal taxes levied on net, taxable income are cut uniformly in order to keep tax revenue constant. The needed reduction is 33.3 per cent. In a second alternative aggregate consumption is kept constant which implies a cut of 14.3 per cent. Both alternatives imply that tax rates in the progressive part of the schedule are reduced and proportional tax elements are expanded and this alternative is therefore called reduced progressiveness.

All simulation are partial in the sence that the impact on wage levels are not considered. An equilibrium exercise of that kind is left for the future.

[Table VII]

As seen from table VII, a 10 per cent across-the-board-cut in all marginal taxes, but keeping the progressive tax structure as of 1979 unchanged, stimulates labor supply. Women are more responsive than men. Tax revenue is lowered which might disappoint those who argue that revenue can be raised through tax cuts with a reference to supply side factors. A point to worry about is that consumption increases more than gross earnings which indicate increased imports, deteroriation of the balance of payments and a future need for raising taxes. It is beyond the scope of the present paper to analyse how expectations of future tax increases are formed and how they might influence present behavior.

A removal of joint taxation has a strong impact on female labor supply. When the changes are decomposed into participation effects and effects on hours supplied, conditional on working, we find that female participation rates are increased by 9.7 percentage points; that is an increase from the initial participation rate of 69.7 per cent to 79.4 per cent. Hours supplied, conditional on working, is increased by 6.3 per cent. An interesting point is that the male labor supply supply is negatively affected. Thus, the results meet the expectation that the tax systems in countries like Norway, UK, France and West-Germany imply strong disincentives for women in the labor market.

Table VIII reports the relative number of transitions between participation and non-participation, given that the non-pecuniary attributes, T(z), as well as the unobservables affecting preferences, $\varepsilon(z)$, remain unchanged. Off-diagonal elements indicate changes and the most noteworthy result is the transition of 33 per cent of the females from the category where initially only the males were working to a category where both are working.

[Table VIII]

The removal of joint taxation increases tax revenue and a tax neutral change is shown in column III of table VII. The important point is that female labor supply gets further stimulated and the negative impact on labor supply is turned into a positive effect.

The last two columns of table VII give the effect of reduced progressiveness. In a tax neutral simulation labor supply and gross earnings, as well as consumption, are substantially increased and the results show that progressive tax rates might cause serious efficiency losses. Also in this case the increase in consumption is a point to worry about.

The last column gives the result of a simulation when aggregate consumption is kept constant. Although the effects are drastically reduced they still show the negative effects of progressive taxes on labor supply.

10. Conclusion.

The basic idea in this paper is to adapt the framework of discrete choice models to the analysis of household labor supply. This is done by introducing latent choice opportunities called matches. Given the match, then wages, hours of work and non-pecuniary characteristics follow. This framework has the advantage of being consistent with latent rationing on hours and job-opportunities and it is also well suited for taking into account general budget constraints.

The model is estimated on Norwegian data from 1979. The model allows for a detailed specification of the tax system.

This system, as well as the tax system of most countries, is not uniformly progressive. This creates a non-convex budget set and the model is designed to allow for this.

Some of the results are summarized in wage elasticities and these are shown to be in the range of what others have obtained. We calculate aggregate elasticities in which observed as well as unobserved heterogeneity are taken into account and we argue that these elasticities are more reliable than the traditional individual elasticities.

In the final section we report the results of some policy simulations. Noteworthy results are in the first place the strong and positive impact on female labor supply of removing joint taxation. Second, the results show some strong negative effects of progressive taxes on labor supply.

Appendix . Norwegian tax rules as of 1979

As mentioned in the introduction, the data set contains observations based on filled in tax returns with information about actual deductions and taxes paid by the households. The model is specially designed to include tax and deduction rules in a detailed way. This is in contrast with all other studies in this field. Here we give a brief description of the Norwegian tax rules as of 1979.

In a condensed form the tax rules can be desribed as follows:

Let R_j, Y_j, Q_j denote the net and gross income and deductions for spouse j, j=F,M, respectively. Taxes are levied on net income according to the tax function $S_1(\cdot)$ when the spouses are jointly taxed, and by $S_2(\cdot)$ when they are taxed separately. A minor part of the taxes are based on gross income according to the rule denoted by the function $S_G(\cdot)$. Thus, taxes paid by the household, S, is defined as

$$(A.1) \quad S(R_M,R_F,Y_M,Y_F) = \begin{cases} S_1(\sum_j R_j)+\sum_j S_G(Y_j) & \text{when } (R_M,R_F)\varepsilon J \\ \sum_j [S_2(R_j)+S_G(Y_j)] & \text{when } (R_M,R_F)\varepsilon\, R_+^2-J \end{cases}$$

where $R_+^2 = [0,\infty)x[0,\infty)$ and J is defined as the region of R_+^2 for which

$$(A.2) \qquad R_j < R_0 \text{ for at least one } j,$$

$$(A.3) \qquad R_j = Y_j - Q_j$$

and where R_0 is given by the tax rules.

It is up to the household to decide whether they prefer to be taxed separately or jointly. In 1979 the level of R_j that minimized the total taxes paid by the household was NOK

22,000. The tax regions are shown on figure A1.

[Figure A1]

The region of joint taxation can be divided into sub-regions as marked on figure A1. Each sub-region constitutes a tax-bracket. A similar division of the region of separate taxation is indicated in the figure.

Deductions applied in the estimation of the model are defined as

$$Q_j = \max \left[Q_{min}, Q_j^*\right] \tag{A.4}$$

where Q_{min} is a minimum tax allowance that every taxpayer has the right to deduct. However, expenses such as interest on loans, union fees, travel expenses over and above a given limit are also deductible. Q_j^* denotes the actual deductions legitimately claimed by the taxpayer.

The minimum allowance, Q_{min}, depends on gross income according to rules set out in table A1.

[Table A1]

We observe the actual deductions claimed by the taxpayers and approved by the IRS. However, in principle, the econometric model outlined in Section 2 requires that we predict the deductions for all permissible values of the wage income for all individuals in the sample. In one estimation alternative Eq. (A.4) is used to evaluate deductions outside the observed point of adjustment. In another alternative Q_j^* is replaced by an estimated deduction function, $\hat{Q}_j$. This function captures the variation in deductions with income and with other variables.

The estimated tax deduction function is the following: (t-values in parentheses)

$$(A.5) \qquad Q_j^* = \underset{(-4.3)}{-229,323} + \underset{(7.5)}{0.109}(wh)_j + \underset{(3.9)}{0.188}(IP)_j + \underset{(19.4)}{0.865}(IK)_j$$

$$+\underset{(4.6)}{134,403} \log A_j - \underset{(-4.9)}{19,210} (\log A_j)^2$$

where wh = wage income,

IP = pensions,

IK = capital income,

A = age.

The estimates are OLS-estimates. We observe that the marginal propensity to deduct related to wage income is 0.109, whereas the marginal propensity to deduct related to capital income is much higher; 0.865. Deductions are estimated to vary with age with a maximum at the age of 33.1.

Taxes related to net income follow from the rules reported in table AII.

[Table AII]

Taxes levied on gross income are given by the following rule.

[Table AIII]

In addition to the deduction and tax rules outlined so far there are some special transfer payments related to the number and age of children in the household. For children below 17 years of age the parents received (in 1979) NOK 900 per child and NOK 1,200 for children between 17 and 20.

To illustrate the implications of the tax rules and deduction decisions on the effective marginal tax rates, we have calculated the marginal tax rates for a married women with socioeconomic characteristics near the average of the sample (age 35, 1 child under 13 years of age, her husband's annual income is NOK 75,000 and her wage rate is NOK 40 per hour).

[Table AIV]

Table AIV shows that there are 24 tax-brackets altogether of which the top 4, or perhaps 5 or 6, are not feasible since they require an unrealistic high working effort. Although the woman considered has a wage rate per hour near the sample average, her wage rate is still too low to make the highest tax-brackets feasible with a realistic maximum amount of hours worked per year. Table AIV makes it clear that effective marginal tax rates are not uniformly increasing with income, or, given the wage rate, with hours worked. The budget set is therefore non-convex.

In Section 8 we have estimated a version of the model with smoothed tax functions replacing the tax rules. Taxes levied on gross income, S_G, are kept according to rules. the tax functions $S_1(R_M+R_F)$ and $S_2(R_j)$ specified in table AII are replaced by the following two functions:

Joint taxation:

$$S_1(R_M+R_F) = \log(1.46)10^{-4} + 1.667 \log(R_M+R_F) \ .$$

Separate taxation:

$$S_2(R_j) = \log(4.66)10^{-4} + 1.584 \log R_j \ , \qquad j=M,F.$$

We observe that the tax elasticity is slightly higher under joint taxation than under separate taxation.

The marginal tax rates that correspond to rules and smoothed function are given in figur A2.

[Figure A2]

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Table I: Sample Values - Married Couples, Norway 1979.

	Averages	Standard deviations	Min. values	Max. values
Hours worked per year by wife	919	859	0	3 368
Hours worked per year by husband	2 059	740	0	3 572
Female wage rate, NOK per hour	31.30	6.10	15.50	55.80
Male wage rate, NOK per hour	41.60	9.4	24.00	73.90
Female labor income, NOK per year	30 021	29 914	0	152 497
Male labor income, NOK per year	84 911	35 701	0	185 988
Female pension income, NOK per year	1 247	5 477	0	51 539
Male pension income, NOK per year	2 538	10 410	0	86 988
Other female income, NOK per year	132	1 746	0	34 480
Other male income, NOK per year	802	3 957	0	35 338
Capital income of the household, NOK per year	2 536	7 842	0	162 734
Wife's education in years	10.5	1.7	9.0	17.5
Husband's education in years	11.4	2.5	9.0	18.0
Age of the wife	43.6	11.3	27	66
Age of the husband	46.1	11.5	25	66
Number of children below 6	0.36	0.66	0	4
Number of children 7-20	1.01	1.55	0	6
Female participation rate, per cent	70.3	45.7	-	-
Male participation rate, per cent	92.8	25.9	-	-

Table II. Estimates of the mean utility function. Married couples in Norway 1979. Age of wife is between 27 and 66 years.

Variables	Coefficients	Estimation alternative 1. 70 draws. Tax deductions equal to max [observed ded, standard ded].		Estimation alternative 2. 30 draws, Tax deductions equal to max [predicted ded, standard ded].	
		Estimates	Standard errors	Estimates	Standard errors
$10^{-4}C$	α_1	1.4447	0.14	1.2194	0.14
$10^{-9}C^2$	α_2	-0.1196	0.06	-0.1801	0.06
$10^{-6}L_F^2$	α_3	-0.5366	0.08	-0.5747	0.08
$10^{-2}L_F$	α_4	0.4563	0.14	0.4751	0.14
$10^{-3}L_F\log A_F$	α_5	1.1547	0.24	1.0774	0.24
$10^{-6}L_FL_M$	α_6	0.0546	0.09	0.0690	0.09
$10^{-3}L_F BU6$	α_7	1.0184	0.12	1.0137	0.12
$10^{-3}L_F BO6$	α_8	0.2323	0.05	0.2262	0.05
$10^{-6}L_M^2$	α_9	-1.5667	0.12	-1.6693	0.12
$10^{-2}L_M$	α_{10}	1.7988	0.17	1.8128	0.17
$10^{-3}L_M\log A_M$	α_{11}	0.5299	0.22	0.6518	0.22
$10K_F$	α_{12}	1.0114	0.07	1.0465	0.07
E_FK_F	α_{13}	-0.2724	0.07	-0.2990	0.07
$10K_M$	α_{14}	1.3779	0.06	1.3834	0.06
$10K_FK_M$	α_{15}	-0.9729	0.04	-1.0177	0.04
D_F	b_F	0.8749	0.21	0.9145	0.21
D_M	b_M	0.7257	0.13	0.7193	0.13

C = household consumption, L_j = leisure time per year, $h_j = 8000-L_j$

BU6, BO6 = number of children below/above 6 years of age

A_j = age

$$K_j = \begin{cases} 1 & \text{if } h_j \leq 60 \\ 0 & \text{otherwise} \end{cases}$$

$$D_j = \begin{cases} 1 & \text{if } h_j \text{ is a "full-time load", } h_j \in [2040, 2200] \\ 0 & \text{otherwise} \end{cases}$$

Table III. Estimates of the wage equation

Alternative 2.

	Males		Females	
Variables	Estimates	Standard error	Estimates	Standard error
Variance	0.0313	0.0016	0.0233	0.0012
Intercept	2.8127	0.0604	2.6852	0.0560
Years of schooling	0.0440	0.0030	0.0526	0.0036
Experience	0.0204	0.0032	0.0118	0.0027
Experience squared/100	-0.0324	0.0054	-0.0190	0.0046

Experience is defined as age minus years of schooling minus seven.

Table IV. Mean utility (mean sample) elasticities, Norway 1979. Elasticities of hours with respect to wage levels and income.

	Males		Females	
Type of elasticity	Own	Cross	Own	Cross
Cournot	0.19	-0.04	1.05	-0.25
Slutsky	0.23	-0.02	1.17	-0.02
Total income	-0.04	-	-0.12	-
Virtual income	-0.01	-	-0.01	-
Frisch	0.24	0.01	1.33	0.03

Table V. Aggregate labor supply elasticities, Norway 1979

	Male elasticities		Female elasticities	
Type of elasticity	Own wage elast.	Cross elast.	Own wage elast.	Cross Elast.
Elasticity of expected of participating persons, N_j	0.27	-0.09	0.66	-0.31
Elasticity of conditional exp. of total supply of hours, H_j^c	0.07	-0.04	0.60	-0.26
Elasticity of unconditional exp. of total supply of hours, H_j	0.33	-0.13	1.20	-0.54

Table VI. Estimates and asymptotic standard deviations under alternative specifications of the budget constraint. McFadden estimation method with 30 draws to represent possible choices of hours and error terms for each partner in the couple.

		Base case, "Predicted deductions"		"Smoothed tax"		"Standard deductions"		"No marginal taxes"	
Variables St.dev.	Coef-ficients	Estimates	St.error	Estimates	St.error	Estimates	St.error	Estimates	St.error
$10^{-4}C$	α_1	1.2194	0.1391	1.1788	0.1379	1.3277	0.1530	2.1359	0.1278
$10^{-9}C^2$	α_2	-0.1801	0.0557	-0.1702	0.0571	-0.1403	0.0778	-0.9264	0.0528
$10^{-6}L_F^2$	α_3	-0.5747	0.0760	-0.5594	0.0757	-0.5489	0.0765	-0.2738	0.0754
$10^{-2}L_F$	α_4	0.4751	0.1392	0.4504	0.1382	0.4546	0.1385	-0.5666	0.1455
$10^{-3}L_F \log A_F$	α_5	1.0774	0.2363	1.0580	0.2363	1.1891	0.2366	1.3280	0.2625
$10^{-6}L_F L_M$	α_6	0.0690	0.0860	0.0791	0.0859	0.0161	0.0876	0.9212	0.0915
$10^{-3}L_F BU6$	α_7	1.0137	0.1192	1.0125	0.1190	1.0152	0.1185	0.8986	0.1240
$10^{-3}L_F Bo6$	α_8	0.2262	0.0474	0.2276	0.0470	0.2270	0.0474	0.1521	0.0498
$10^{-6}L_M^2$	α_9	-1.6694	0.1224	-1.6919	0.1224	-1.6193	0.1232	-1.1355	0.1213
$10^{-2}L_M$	α_{10}	1.8128	0.1735	1.8262	0.1735	1.7601	0.1733	0.4985	0.1846
$10^{-3}L_M \log A_M$	α_{11}	0.6518	0.2198	0.6472	0.2203	0.7437	0.2190	0.8685	0.2547
$10\ K_F$	α_{12}	1.0465	0.0716	1.0494	0.0715	1.0274	0.0719	1.0574	0.0725
$E_F K_F$	α_{13}	-0.2989	0.0691	-0.3028	0.0691	-0.2941	0.0693	-0.3164	0.0701
$10\ K_M$	α_{14}	1.3834	0.0597	1.3971	0.0598	1.3805	0.0600	1.3279	0.0592
$10\ K_F K_M$	α_{15}	-1.0177	0.0396	-1.0393	0.0396	-0.9844	0.0398	-1.0971	0.0427
D_F	b_F	0.9144	0.2046	0.9218	0.2045	0.8942	0.2047	0.9945	0.2116
D_M	b_M	0.7193	0.1324	0.7230	0.1325	0.7222	0.1323	0.6893	0.1365

Individual elasticities:	Base case	Smoothed tax	Standard deductions	No marginal taxes
Direct cournot				
Male	0.19	0.19	0.25	-0.09
Female	1.05	1.06	1.38	0.21
Direct Slutsky				
Male	0.23	0.22	0.29	0.20
Female	1.17	1.18	1.48	0.62
Cross Cournot				
Male	-0.04	-0.06	-0.04	-0.13
Female	-0.25	-0.23	-0.23	-1.12

Table VII. Policy simulations, Norway 1979, married couples.

Variables, aggregation of individual responses divided by total number of individuals/households	Predicted values, base case	Simulation results, percentage changes from base case				
		I.	II.	III.	IV.	
		10 per cent reduction in all tax rates	Removal of joint taxation	As II, but tax neutral	Reduced progressiveness: Tax neutral	Reduced progressiveness: Constant consumption
Unconditional supply of hours a year						
Male	2080	3.8	- 0.9	4.9	7.7	1.1
Female	936	7.5	19.7	28.7	12.4	1.6
Total	3003	4.9	5.5	12.2	9.1	
Gross earnings (NOK per year)						
Male	99152	4.6	- 1.3	6.1	10.1	1.9
Female	37519	6.8	16.6	24.8	11.7	1.7
Total	136671	5.3	3.6	11.3	10.6	1.8
Household consumption (NOK per year)	95494	8.9	1.5	16.1	15.1	-
Tax revenue (NOK per year)	41176	-2.6	8.5	-	-	6.0

Table VIII. Simulated transition probabilities between categories of households.

From \ To	Joint taxation removed			
	$h_M = h_F = 0$	$h_M = 0,\ h_F > 0$	$h_M > 0,\ h_F = 0$	$h_M > 0,\ h_F > 0$
Base case $h_M = h_F = 0$	1.00	0	0	0
Base case $h_M = 0\ h_F > 0$	0	0.83	0	0.17
Base case $h_M > 0,\ h_F = 0$	0	0.01	0.66	0.33
Base case $h_M > 0,\ h_F > 0$	0	0	0	1.00

Table AI: Intervals for

Gross income (NOK) Y	Minimum tax allowance (NOK) Q_{min}
0- 2,000	Y
2,000- 9,500	0.4Y + 1,200
9,500-10,000	5,000
10,000-16,000	0.04Y + 4,600
16,000-17,500	0.14Y + 3,000
17,500-31,000	0.10Y + 3,700
31,000-	6,800

Table AII:

Separate taxation Intervals for net income (NOK) R_j	Marginal tax rates (per cent) $S'_2(R_j)$	Joint taxation Intervals for net income (NOK) $R_M + R_F$	Marginal tax rates (per cent) $S'_1(R_M + R_F)$
0- 7.000	0	0- 14.000	0
7.000- 32.000	27.4	14.000- 48.000	27.4
32.000- 41.000	33.4	48.000- 60.000	33.4
41.000- 58.000	38.4	60.000- 77.000	38.4
58.000- 69.000	43.4	77.000- 88.000	43.4
69.000- 79.000	49.4	88.000- 98.000	49.4
79.000- 89.000	55.4	98.000-108.000	55.4
89.000-106.000	60.4	108.000-125.000	60.4
106.000-136.000	65.4	125.000-155.000	65.4
136.000-186.000	69.4	155.000-205.000	69.4
186.000-286.000	73.4	205.000-305.000	73.4
286.000-	75.4	305.000-	75.4

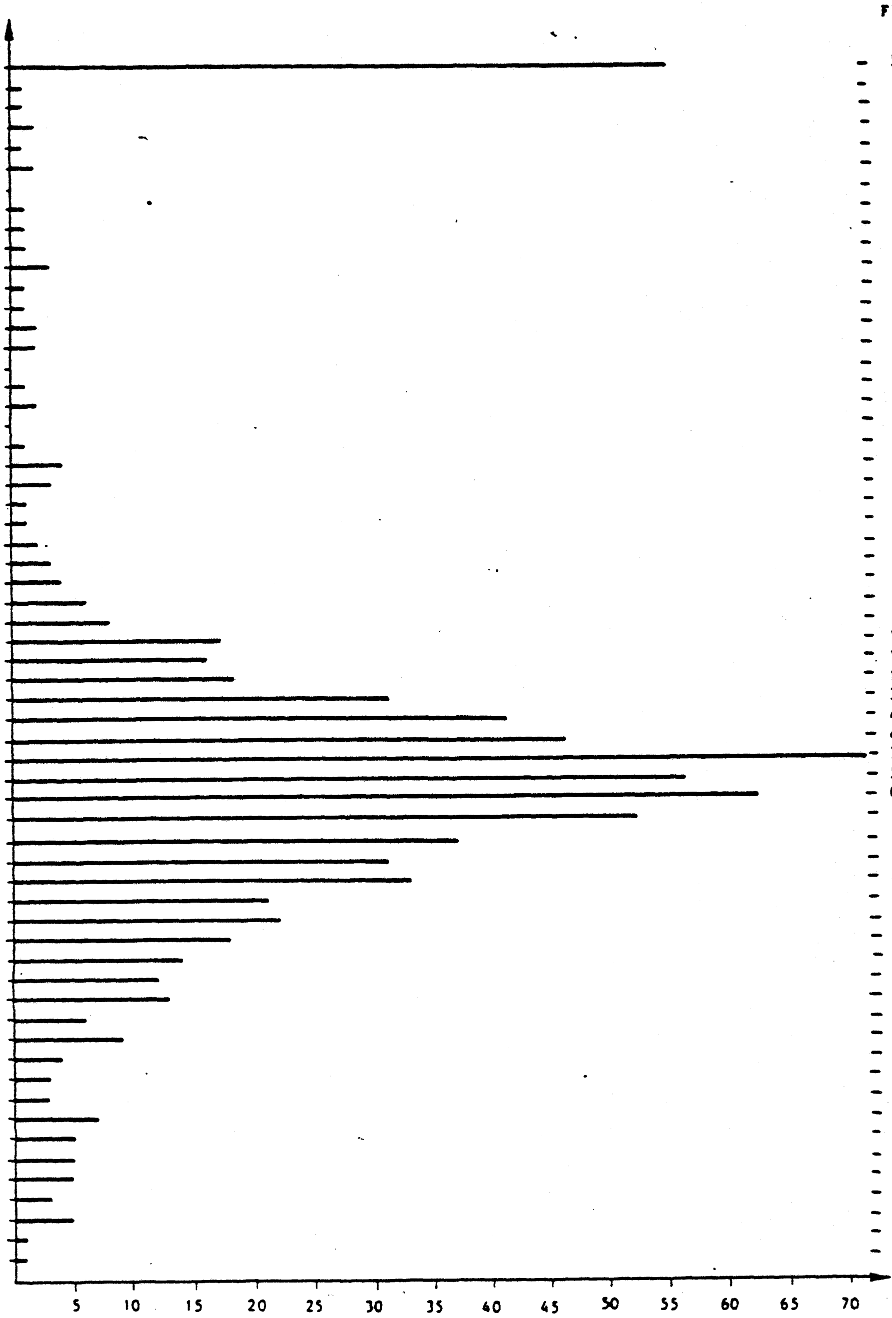
Table AIII

Intervals for gross income (NOK)	Taxes paid (NOK)
Y	$S_G(Y)$
0- 9,000	0
9,000- 11,500	0.25Y - 2,250
11,599-182,400	0.05Y
182,400-	9,120

Table AIV: Effective total marginal tax rates for a married woman age 35, one child under 13 years of age, her wage rate is NOK 40 per hour and her husband's income is NOK 75,000 per year, near average sample values). Norway 1979.

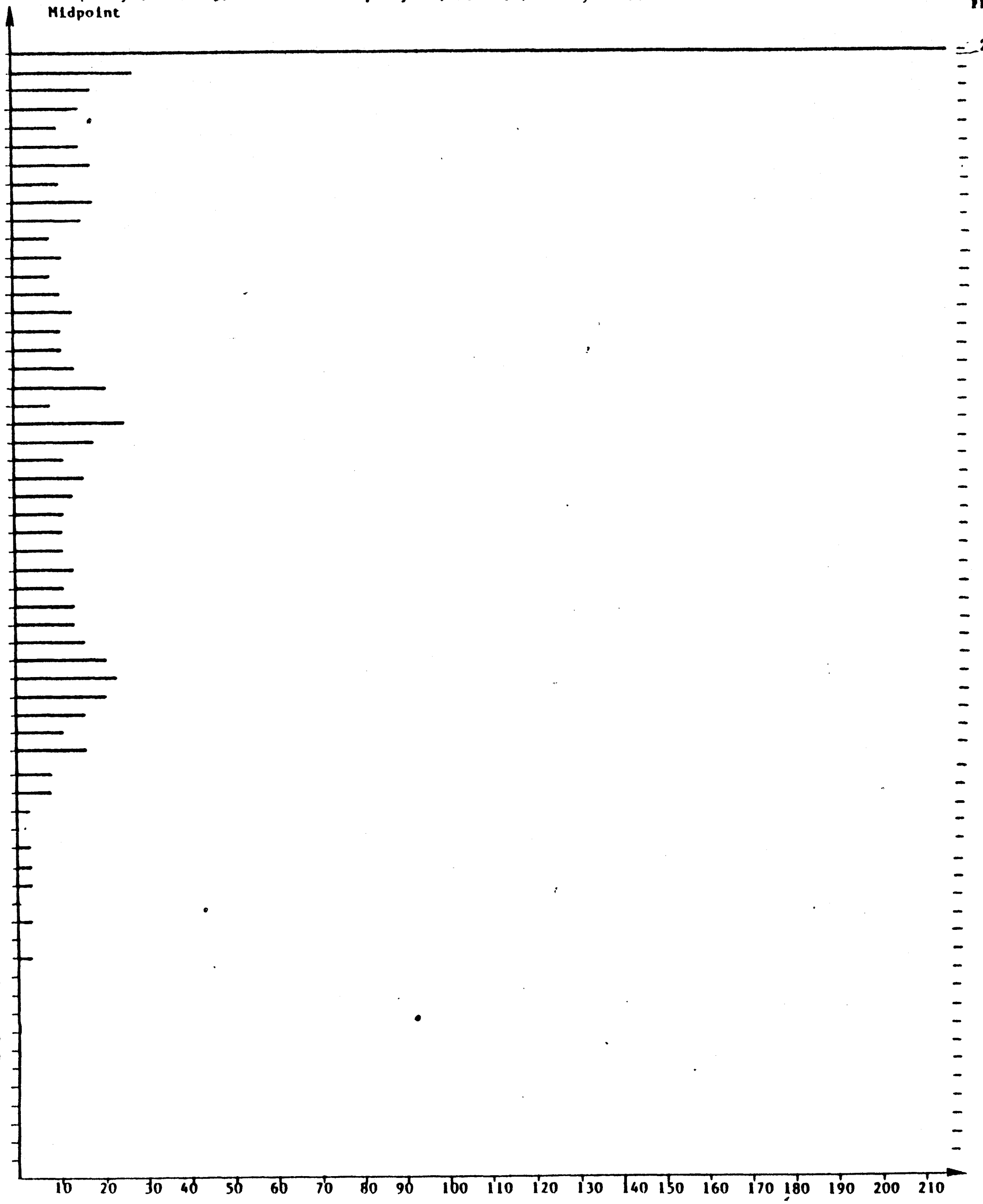
Gross income NOK per year	Hours worked per year	Effective marginal tax rates per cent	Remarks
0- 2,000	0- 50	0	
2,000- 9,000	50- 225	23.0	Joint taxation minimizes taxes paid by the household
9,000- 9,500	225- 238	48.0	
9,500- 10,000	238- 250	63.4	
10,000- 10,833	250- 270	61.9	
10,833- 11,250	270- 281	66.6	
11,250- 16,000	281- 400	46.7	
16,000- 17,500	400- 438	42.3	
17,500- 22,777	438- 570	44.1	
22,777- 24,313	570- 607	49.1	
24,313- 31,000	607- 775	29.7	Separate taxation minimizes taxes
31,000- 38,800	775- 970	32.4	
38,800- 47,800	970-1,195	38.4	
47,800- 64,800	1,195-1,620	43.4	
64,800- 75,000	1,620-1,877	48.4	
75,000- 75,941	1,877-1,899	43.6	Separate taxation, but capital income and capital expenses are allocated to the wife's income and expenses
75,941- 87,705	1,899-2,192	49.0	
87,705- 99,470	2,192-2,487	54.3	
99,470-119,470	2,487-2,987	58.8	
119,470-154,764	2,987-3,869	63.3	
154,764-182,400	3,860-4,560	66.8	Hours worked per year not feasible
182,400-213,588	4,560-5,339	61.8	
213,588-331,235	5,339-8,280	65.4	
331,235-350,400	8,280-8,760	67.2	

Figure 1. Frequency bar chart, Hours of work per year, males, Norway 1979



Midpoint	Freq	Cum. freq	Percent	Cum. percent
0	55	55	7.07	7.07
60	1	56	0.13	7.20
120	1	57	0.13	7.33
180	2	59	0.26	7.58
240	1	60	0.13	7.71
300	2	62	0.26	7.97
360	0	62	0.00	7.97
420	1	63	0.13	8.10
480	1	64	0.13	8.23
540	1	65	0.13	8.35
600	3	68	0.39	8.74
660	1	69	0.13	8.87
720	1	70	0.13	9.00
780	2	72	0.26	9.25
840	2	74	0.26	9.51
900	0	74	0.00	9.51
960	1	75	0.13	9.64
1020	2	77	0.26	9.90
1080	0	77	0.00	9.90
1140	1	78	0.13	10.03
1200	4	82	0.51	10.54
1260	3	85	0.39	10.93
1320	1	86	0.13	11.05
1380	1	87	0.13	11.18
1440	2	89	0.26	11.44
1500	3	92	0.39	11.83
1560	4	96	0.51	12.34
1620	6	102	0.77	13.11
1680	8	110	1.03	14.14
1740	17	127	2.19	16.32
1800	16	143	2.06	18.38
1860	18	161	2.31	20.69
1920	31	192	3.98	24.68
1980	41	233	5.27	29.95
2040	46	279	5.91	35.86
2100	71	350	9.13	44.99
2160	56	406	7.20	52.19
2220	62	468	7.97	60.15
2280	52	520	6.68	66.84
2340	37	557	4.76	71.59
2400	31	588	3.98	75.58
2460	33	621	4.24	79.82
2520	21	642	2.70	82.52
2580	22	664	2.83	85.35
2640	18	682	2.31	87.66
2700	14	696	1.80	89.46
2760	12	708	1.54	91.00
2820	13	721	1.67	92.67
2880	6	727	0.77	93.44
2940	9	736	1.16	94.60
3000	4	740	0.51	95.12
3060	3	743	0.39	95.50
3120	3	746	0.39	95.89
3180	7	753	0.90	96.79
3240	5	758	0.64	97.43
3300	5	763	0.64	98.07
3360	5	768	0.64	98.71
3420	3	771	0.39	99.10
3480	5	776	0.64	99.74
3540	1	777	0.13	99.87
3600	1	778	0.13	100.00

Figure 2: Frequency bar chart. Hours of work per year, females, Norway 1979.



Midpoint	FREQ	CUM. FREQ	PERCENT	CUM. PERCENT
0	214	214	27.51	27.51
60	27	241	3.47	30.98
120	17	258	2.19	33.16
180	14	272	1.80	34.96
240	9	281	1.16	36.12
300	14	295	1.80	37.92
360	18	313	2.31	40.23
420	9	322	1.16	41.39
480	17	339	2.19	43.57
540	16	355	2.06	45.63
600	7	362	0.90	46.53
660	10	372	1.29	47.81
720	8	380	1.03	48.84
780	11	391	1.41	50.26
840	12	403	1.54	51.80
900	9	412	1.16	52.96
960	11	423	1.41	54.37
1020	13	436	1.67	56.04
1080	21	457	2.70	58.74
1140	8	465	1.03	59.77
1200	26	491	3.34	63.11
1260	18	509	2.31	65.42
1320	11	520	1.41	66.84
1380	14	534	1.80	68.64
1440	13	547	1.67	70.31
1500	10	557	1.29	71.59
1560	11	568	1.41	73.01
1620	10	578	1.29	74.29
1680	13	591	1.67	75.96
1740	10	601	1.29	77.25
1800	12	613	1.54	78.79
1860	12	625	1.54	80.33
1920	16	641	2.06	82.39
1980	20	661	2.57	84.96
2040	23	684	2.96	87.92
2100	19	703	2.44	90.36
2160	14	717	1.80	92.16
2220	10	727	1.29	93.44
2280	14	741	1.80	95.24
2340	7	748	0.90	96.14
2400	8	756	1.03	97.17
2460	2	758	0.26	97.43
2520	1	759	0.13	97.56
2580	2	761	0.26	97.81
2640	3	764	0.39	98.20
2700	2	766	0.26	98.46
2760	1	767	0.13	98.59
2820	3	770	0.39	98.97
2880	1	771	0.13	99.10
2880	2	773	0.26	99.36
2940	1	774	0.13	99.49
3000	1	775	0.13	99.61
3060	0	775	0.00	99.61
3120	1	776	0.13	99.74
3180	1	777	0.13	99.87
3240	0	777	0.00	99.87
3300	1	778	0.13	100.0
3360	0	778	0.00	100.0
3420	0	778	0.00	100.0
3480	0	778	0.00	100.0
3540	0	778	0.00	100.0
3600	0	778	0.00	100.0

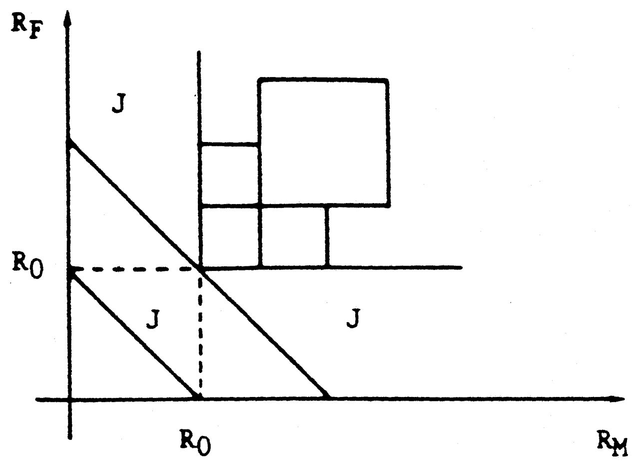


Figure A1. Regions of joint (J) and separate taxation

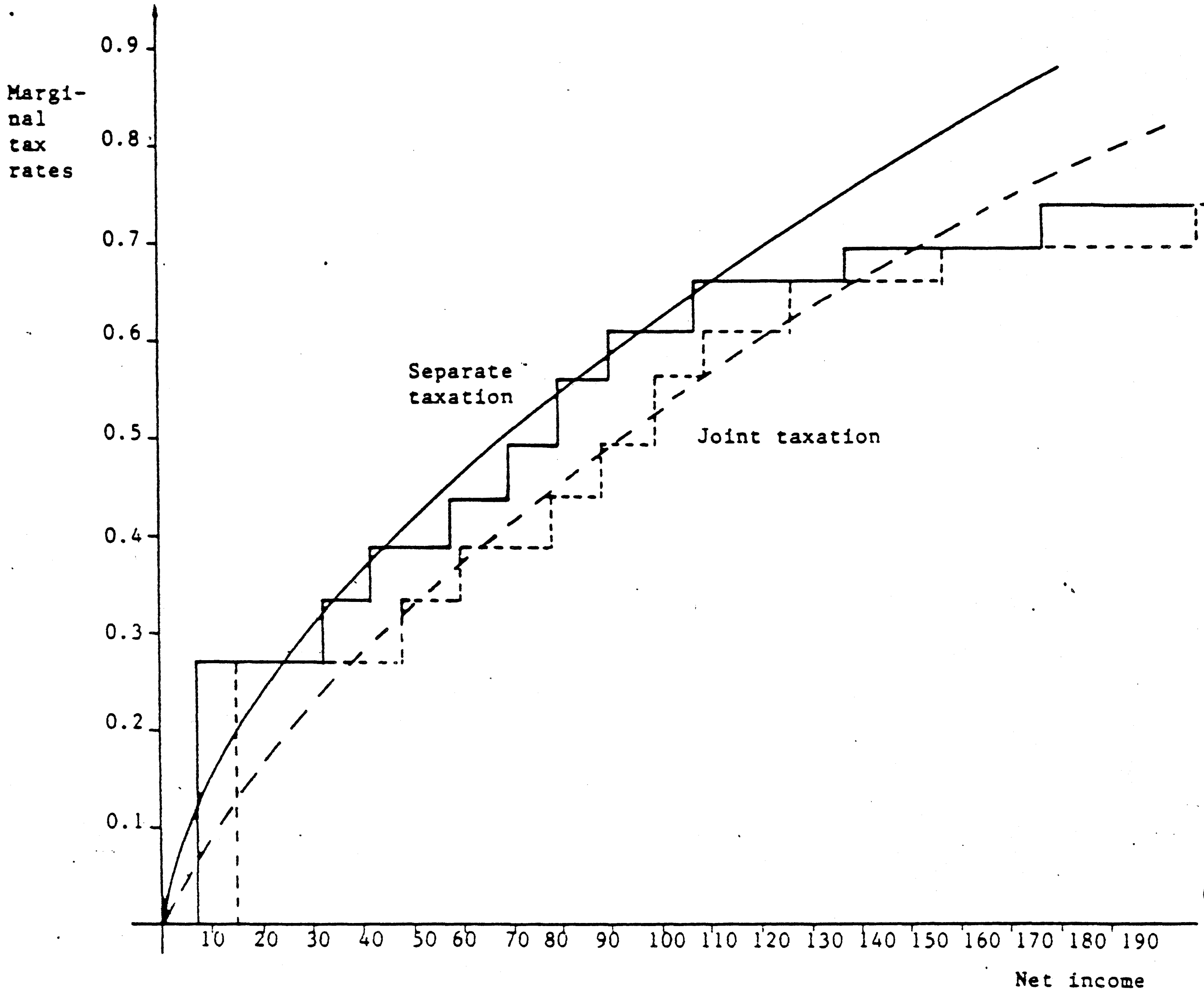


Figure A2 Marginal tax rates that correspond to rules and smoothed tax functions.

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