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The marginal cost of public funds: A comment on the literature

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Abstract

The paper derives formulas for the marginal cost of public funds in a general equilibrium model. The MCF depends on how expenditure is financed, and the paper goes through a wide range of possibilities. Special emphasis is put on the most common functional forms of applied general equilibrium models. The formulas are used to explain and correct results and statements in the literature on the marginal cost of public funds. Implications for tax reform are also discussed.

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1 Introduction

This paper discusses the marginal cost of public funds (MCF). MCF is input to cost benefit analysis, where it measures what Pigou (1947) called the "indirect damage" caused by the need to finance public expenditure. MCF is also used in tax reform analysis, as the MCF's of different sources of financing imply a complete ranking of tax reforms (eg. Ahmad and Stern (1984)). For these and other reasons, Mayshar (1990) has argued that MCF should be considered the cornerstone concept of applied tax analysis.

There exists a substantial literature that measures the MCF using applied general equilibrium models[1], or partial equilibrium methods[2]. This literature is scattered with empirical observations of what the MCF "depends on". Ballard (1990) for instance finds that "the results indicate that the marginal welfare costs of additional tax-financed exhaustive government expenditure are related chiefly to uncompensated elasticities" (p.275). Ballard, Shoven and Whalley (1985) conclude that "not surprisingly, we find that the (MCF) for a given part of the tax system is greater when the taxed activity is assumed to be more elastic." (p. 128). They also observe that "in general, it appears that the (MCF)'s are greater for activities which face high or widely varying tax rates" (p.128). Hansson and Stuart (1985) summarize their simulations as follows: "We find that the marginal cost of public funds depends on three aspects of the fiscal change: (i) the nature of the expenditure effects; (ii) the specific tax instruments used; and (iii) the initial levels of the tax rates" (p. 332). Browning (1987) is skeptical to the enterprise of using general equilibrium models to single out the important effects: "one of the virtues of the partial-equilibrium approach is that is clarifies the contribution (that) key parameters make to the final estimate, something that is often obscured in large-scale equilibrium

[1]Stuart (1984), Hansson (1984). Ballard, Shoven and Whalley (1985), Hansson and Stuart (1985), Ballard (1990), Vennemo (1991).

[2]Wildasin (1984), Ahmad and Stern (1984),(1990), Decoster and Schokkaert (1990).

models" (p. 11).

Some aspects of what the MCF is all about have been clarified more rigourously. The relation between the MCF and measures of excess burden (and the superiority of the MCF) is discussed in detail in Mayshar (1990). Ballard (1990) stresses the difference between a "balanced-budget experiment", and a "differential experiment", where only the former is directly applicable to the measurement of the MCF. Stuart (1984) and others note the inappropriateness of the Harberger formula in measuring MCF, as this formula ignores income effects. Ahmad and Stern (1984, 1990) computes an expression for MCF in the case of a commodity tax.

This paper computes expressions for the MCF for a wide range of taxes within a general equilibrium setting, including the poll tax, wage income tax, commodity taxes, producer taxes and tariffs. This will clarify what the MCF "depends on". I give some examples of how simulation results reported in the literature may be explained. Further, welfare improving tax reforms are identified. Some results are given based on general functional forms, but the paper shows that radically more specific conclusions emerge one is willing to make assumptions about the functional forms of preferences. The paper also demonstrates that the MCF ranking provided by applied general equilibrium models can at least in part be explained by functional assumptions. Finally on the relation between partial equilibrium and general equilibrium, the expressions can be used to indicate the influence of the general equilibrium repercussions on results and to single out the influence of key parameters.

The paper is organized as follows: Section 2 states the model. Section 3 reviews the shadow tax methodology of Drèze and Stern (1987, 1990), that I use to derive the expressions, and shows how the methodology is modified for the present model and purpose. Section 4 contains the results. Appendix A contains the derivations necessary to state the MCF's. Appendix B contains the proofs of the propositions. In appendix C I give some observations on the relation between shadow taxes and actual taxes in the context of the model.

2 Analytical framework

The model is a simple static general equilibrium model of an open economy. I assume market power in exports. This is inspired by the applied general equilibrium models. These models either are closed economy models, or assume market power in exports (most even in imports). I leave out intermediates. There are no technical problems related to including them, but the formulas tend to be more complicated without much of substance being added. In Vennemo (1990) I discuss a model that includes intermediates in greater detail. The model of this paper is a simpler version of that model.

$$b_{dj} = e^j(q_l^j, q_k^j) \qquad j = 1 \ldots n \tag{1}$$

$$\sum_{j=1}^{n} y_j z l_j(q_l^j, q_k^j) = \sum_{h=1}^{k} l^h(p_1^c \ldots p_n^c, q_l^c, r^h) \tag{2}$$

$$\sum_{j=1}^{n} y_j z k_j(q_l^j, q_k^j) = K \tag{3}$$

$$\sum_{j=m+1}^{n} p_{aj} a_j(p_{aj}) - \sum_{j=1}^{n} b_{fj} \frac{\partial b_j^c}{\partial p_{fj}} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) = 0 \tag{4}$$

$$\frac{\partial b_j^c}{\partial p_{dj}} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) + g_j = y_j \qquad j = 1 \ldots m \tag{5}$$

$$\frac{\partial b_j^c}{\partial p_{dj}} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) + g_j + a_j(p_{aj}) = y_j \qquad j = m+1 \ldots n \tag{6}$$

where:

$$q_l^j = q_l + t_l^j \qquad j = 1 \ldots n \tag{7}$$

$$q_l^c = q_l - t_l^c \tag{8}$$

$$q_k^j = q_k + t_k^j \qquad j = 1 \ldots n \tag{9}$$

$$p_j^c = b_j^c(p_{dj}, p_{fj}) + t_j^c \qquad j = 1 \ldots n \tag{10}$$

$$p_{dj} = b_{dj} + t_{dj} \qquad j = 1 \ldots n \tag{11}$$

$$
\begin{aligned}
p_{fj} &= b_{fj} + t_{fj} & j = 1 \ldots n & \quad (12)\\
p_{aj} &= b_{dj} + t_{aj} & j = m+1 \ldots n & \quad (13)\\
r^h &= (q_k - t_k^c)K^h + \varrho^h \tau & h = 1 \ldots k & \quad (14)
\end{aligned}
$$

A brief description of the equations follows: In equation (1), b_{dj} is the output price of good j, e^j is the unit cost function, taking input prices of labour q_l^j and capital q_k^j as arguments. I assume single output producers. There are no fixed factors in production. Price equals (marginal and average) costs.

In equation (2), y_j is output of good j and zl_j is the unit input coefficient for labour (derivative of the unit cost function w.r.t. the wage paid by the producer, q_l^j). l^h is labour supply of household h, taking consumer prices $p_l^c \ldots p_n^c$, the wage rate q_l^c and lump sum income r^h as arguments. There is equilibrium in the labour market.

In equation (3), zk_j is the unit input coefficient for capital (derivative of the unit cost function w.r.t. q_k^j). K is the exogenous supply of real capital. There is equilibrium in the (single) capital market.

In equation (4), p_{aj} is the world market price of export good j. Export quantities depend on the world market price. By this assumption, the country may influence its terms of trade vis à vis the rest of the world. b_{fj} is the world market price of imports of good j, and the rather clumsy looking expression $\frac{\partial b_j^c}{\partial p_{fj}} \sum_h c_j^h$ is imports of good j, where p_{fj} is the tariff inclusive price of imports, and b_j^c is a price index of p_{fj} and the price of the domestic variety, p_{dj}. I assume all imports are for the purpose of private consumption. This is reasonable as long as there are no intermediates, investment etc. I assume that imports can be expressed as an import share $(\frac{\partial b_j^c}{\partial p_{fj}})$ times consumption of the composite good (Armington assumption). Balance of trade is assumed to close the model.

Equation (5) and (6) are the commodity balance equations. $\frac{\partial b_j^c}{\partial p_{dj}} \sum_h c_j^h$ is the domestic variety of consumption of composite j. g_j is public consumption of good j. I assume for simplicity that the public sector just purchases domestically produced goods.

The commodity balance equations may alternatively include imports. Adding and subtracting imports yields (using the standard CGE convention that prices are equal to unity in the base year of the model):

$$\sum c_j^h + g_j \, [+a_j] = y_j + i_j \quad j = 1 \ldots n$$

where i_j is imports. The bracketed term is added when $j > m$. When formulated this way, the commodity balance equations apply to composite commodities.

Equations (7) to (14) define the relation between prices. t_l^j and t_k^j are producer taxes on labour and capital, t_l^c and t_k^c are consumer taxes on labour and capital, t_j^c is an indirect tax on the consumption composite, t_{dj} is a tax on output for domestic use, t_{fj} is an import tariff, t_{aj} is an export tax, and $\varrho^h \tau$ is a lump sum grant to household h where ϱ^h is household specific and τ is a common element. An output tax is equal to a tax on output for domestic use t_{dj}, and possibly an export tax t_{aj} at an equal rate. q_l is the gross wage rate of consumers, q_k is gross capital return of consumers. K^h is capital owned by consumer h. As consumers own all the capital, $\sum_{h=1}^k K^h = K$.

There are $8n - m + k + 4$ equations, and $8n - m + k + 4$ endogenous variables b_{dj}, q_l^j, q_k^j, y_j, p_j^c, q_l^c, p_{aj}, q_l, q_k, p_{dj}, p_{fj}, r^h and one of the tax and transfer variables $t_l^j, t_l^c, t_k^j, t_j^c, t_{dj}, t_{fj}, t_{aj}, \varrho^h, \tau$. I assume government expenditures to be exogenous. The exchange rate is numeraire[3].

From the model can be obtained

[3] An alternative interpretation of the numeraire is to normalize one of the world import prices at unity and have this price as numeraire. The other import prices would stand in fixed proportion to this numeraire

$$\sum_{j=1}^{n} t_j^c c_j + t_l^c L + \sum_{j=1}^{n} t_l^j z l_j y_j + \sum_{j=1}^{n} t_k^j z k_j y_j + \sum_{j=m+1}^{n} t_{aj} a_j$$
$$+ \sum_{j=1}^{n} t_{fj} c_{fj} + \sum_{j=1}^{n} t_{dj} c_{dj} + t_k^c K - \sum_{h=1}^{k} \varrho^h \tau - \sum_{j=1}^{n} b_{dj} g_j = 0$$

where $c_j = \sum_{h=1}^{k} c_j^h$, $L = \sum_{h=1}^{k} l^h, c_{fj} = \frac{\partial b_j^c}{\partial p_{fj}} c_j, c_{dj} = \frac{\partial b_j^c}{\partial p_{dj}} c_j$. This is the government budget constraint. As producers do not invest and consumers do not save, balance in the government budget is implied by the assumption of balance of trade. The endogenous tax rate clears the government budget.

A closed economy version of the model is obtained by dropping the distinction between c_{dj} and c_{fj}, t_{dj} and t_j^c, (and naturally equation (4)). The results therefore extend to the closed economy. Dropping a tax rate as a possible financial instrument from the model is equivalent to giving it a zero value and assuming that it cannot be endogenous. The results therefore extend to the case where fewer financial instruments are available.

3 Shadow tax methodology

This section reviews the shadow tax (shadow price) methodology in terms of the model I use, and defines the MCF. By definition, MCF is integrated in the cost benefit problem of computing the change in welfare from a public project, taking account of the opportunity cost of the project. The latter is the value of the private goods that are "crowded out" by the public project. This depends on the workings of the economic model. The problem of computing the change in welfare therefore makes use of a Lagrangian formulation in which the model itself is the scarcity constraint:

$$\mathcal{L} = W\left(V^1(p_1^c, \ldots, p_n^c, q_l^c, r^1) + v^1(g_1, \ldots, g_n), \ldots, V^k(p_1^c \ldots p_n^c, q_l^c, r^k) + v^k(g_1, \ldots, g_n)\right)$$

$$
\begin{aligned}
- \quad & \mu \Bigg[\sum_{j=1}^{n} \theta_j (b_{dj} - e^j(q_l^j, q_k^j)) \\
& + \omega (\sum_{j=1}^{n} y_j z l_j(q_l^j, q_k^j) - \sum_{h=1}^{k} l^h(p_1^c \ldots p_n^c, q_l^c, r^h)) \\
& + \rho (\sum_{j=1}^{n} y_j z k_j(q_l^j, q_k^j) - K) \qquad (15) \\
& + \sum_{j=1}^{n} b_{fj} \frac{\partial b_j^c}{\partial p_{fj}} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) - \sum_{j=m+1}^{n} p_{aj} a_j(p_{aj}) \\
& + \sum_{j=1}^{n} \nu_j \left(\frac{\partial b_j^c}{\partial p_{dj}} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) + g_j - y_j \right) + \sum_{j=m+1}^{h} \nu_j a_j(p_{aj}) \Bigg]
\end{aligned}
$$

where the relation between prices and taxes is defined in eq. (7) to (14). I assume that utility functions are additive in public goods. This implies (using Roy's identity) that public goods do not enter the private demand or supply functions.

A shadow price is the increase in welfare related to a unit marginal increase in the associated quantity. ω is for instance the increase in welfare associated with a introducing a marginal unit of exogenous labour supply. A shadow tax is the difference between a market (tax inclusive) price, and shadow price. Define the following shadow taxes:

$$\hat{t}_j^c = t_j^c + b_j^c - (b_{fj} \frac{\partial b_j^c}{\partial p_{fj}} + \nu_j \frac{\partial b_j^c}{\partial p_{dj}}) \qquad j = 1 \ldots n \qquad (16)$$

$$\hat{t}_l^c = t_l^c - q_l + \omega \qquad (17)$$

$$\hat{t}_l^j = t_l^j + q_l - \omega \qquad j = 1 \ldots n \qquad (18)$$

$$\hat{t}_k^j = t_k^j + q_k - \rho \qquad j = 1 \ldots n \qquad (19)$$

$$\hat{t}_{dj} = t_{dj} + b_{dj} - \nu_j \qquad j = 1 \ldots n \qquad (20)$$

$$\hat{t}_{aj} = t_{aj} + b_{dj} - \nu_j \qquad j = m + 1 \ldots n \qquad (21)$$

$$\hat{t}_k^c = t_k^c - q_k + \rho \qquad (22)$$

$$\hat{t}_{yj} = \nu_j - b_{dj} + (e^j - b_{dj}) \frac{\theta_j}{y_j} = \nu_j - b_{dj} \qquad j = 1 \ldots n \qquad (23)$$

Rewrite shadow prices in terms of shadow taxes as

$$\omega = q_l^c + t_l^c - q_l + \omega = q_l^c + \hat{t}_l^c \tag{24}$$

$$\omega = q_l^j - t_l^j - q_l + \omega = q_l^j - \hat{t}_l^j \tag{25}$$

$$\rho = q_k^j - t_k^j - q_k + \rho = q_k^j - \hat{t}_k^j \tag{26}$$

$$\nu_j = p_{aj} - t_{aj} - b_{dj} + \nu_j = p_{aj} - \hat{t}_{aj} \qquad j = m+1 \ldots n \tag{27}$$

and observe that

$$\sum_{j=1}^{n} p_j^c c_j^h - q_l^c l^h = r^h \tag{28}$$

$$q_l^j z l_j + q_k^j z k_j = b_{dj} \tag{29}$$

Using equations (24) to (29), the Lagrangian can be written

$$\begin{aligned}\mathcal{L} &= W\left(V^1(p_1^c, \ldots, p_n^c, q_l^c, r^1) + v^1(g_1, \ldots, g_n), \ldots, V^k(p_1^c \ldots p_n^c, q_l^c, r^k) + v^k(g_1, \ldots, g_n)\right) \\ &+ \mu\left[\sum_{j=1}^{n} \hat{t}_j^c c_j + \hat{t}_l^c L + \sum_{j=1}^{n} \hat{t}_l^j z l_j y_j + \sum_{j=1}^{n} \hat{t}_k^j z k_j y_j + \sum_{j=m+1}^{n} \hat{t}_{aj} a_j + \sum_{j=1}^{n} \hat{t}_{yj} y_j + \hat{t}_k^c K - \sum_{h=1}^{k} \varrho^h \tau - \sum_{j=1}^{n} \nu_j g_j\right]\end{aligned} \tag{30}$$

Following Drèze and Stern (1987, 1990), I define the bracketed term of equation (30) as the shadow public budget constraint. μ is interpreted as the social utility (or value) of government shadow revenue. Shadow taxes account for second best effects like the impact of one tax on tax revenues from another tax, the implicit taxation of several factors and goods as a tax is carried forward, etc. Shadow taxes coincide with formal taxes in the (unlikely) case of second best optimum (see eg. Vennemo (1990)). Moreover, if the price elasticities of trade are large, at least some shadow taxes are close to actual taxes. See appendix C for more on the relation between shadow taxes and actual taxes in the model.

I also need the average of the welfare weighted marginal private utilities of income, which I denote λ. λ is defined

$$\lambda = \frac{1}{k}\sum_{h=1}^{k}\frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial r^h} = \frac{1}{k}\sum_{h=1}^{k}\beta^h, \quad \beta^h = \frac{\partial W}{\partial V^h}\lambda^h, \quad \lambda^h = \frac{\partial V^h}{\partial r^h} \tag{31}$$

λ is a natural extension to the many person case of the individual marginal utility of income, which is often used in cost benefit analyses to normalize utilities. I use λ to normalize the welfare function.

3.1 Definition of MCF

Denote the welfare function that is maximized w.r.t the endogenous variables of the model for W^*. From the envelope theorem we know that

$$\frac{dW^*}{dg_j} = \frac{\partial \mathcal{L}}{\partial g_j} \tag{32}$$

This means that if $\frac{\partial \mathcal{L}}{\partial g_j} > 0$, welfare increases if g_j is marginally increased. If $\frac{\partial \mathcal{L}}{\partial g_j} < 0$, welfare increases if g_j is marginally decreased. Note that when g_j is changed, the endogenous tax/transfer variable of the system adjusts in the background to ensure budget balance. The expression $\frac{\partial \mathcal{L}}{\partial g_j}$ therefore captures the general equilibrium effect of increasing g_j at the expense of the endogenous tax/transfer variable of the system.

Carrying out the derivation of the Lagrangian, the effect on welfare of a marginal public project is obtained as

$$\frac{dW^*}{dg_j} = \frac{\partial W}{\partial g_j} - \mu\nu_j \tag{33}$$

or

$$\frac{dW^*}{dg_j}/\lambda = \frac{\partial W}{\partial g_j}/\lambda - \frac{\mu}{\lambda}\frac{\nu_j}{p_j}p_j \tag{34}$$

where p_j is the public calculation price, the price that is used to value the public purchase. Define

$$MCF_{ij} = \frac{\mu}{\lambda}\frac{\nu_j}{p_j} \tag{35}$$

where μ/λ is the ratio of *social* utility of income over *private* utility of income. Public funds has a different value from private funds because it costs to put money in the public purse. μ/λ is thus a conversion factor between public and private funds that facilitates comparison of public costs and private gains. Likewise, the term ν_j/p_j is a conversion factor between shadow price and the price used to value public purchase. If public purchases are valued in shadow prices, this factor disappears.

When taxation is first best optimal and public purchases are valued in shadow prices, MCF = 1. The private and public utilities of income are equal, which is to say that there is no cost related to the collection of public revenue. When taxation is second best optimal, MCF is a single number (greater than unity), ie. it is independent of how expenditure is financed. When taxation is not second best optimal, both the ratio μ/λ and the ratio ν_j/p_j will depend on how government expenditure is financed. The ratio ν_j/p_j will in addition depend on what kind of public expenditure is being financed. For precision, I have written MCF_{ij} in formula (35), where i indicates financing instrument, and j the kind of expenditure. To abbreviate notation, I will however drop the subscripts on MCF in the rest of this paper.

In applied general equilibrium models, the term $\frac{\partial W}{\partial g_j}$ is most often ignored, and an estimate of MCF is obtained by dividing the welfare cost (as it now becomes) by the change in public revenue $p_j dg_j$.

If we considered a public project that was a perfect substitute for cash, income effects would enter the basic formula (33)[4]. The definition of MCF from eq. (35) would not change. But an estimate of MCF can no longer be obtained by dividing the welfare change by the value of

[4]A formula like (33) in the case of a cash project is

$$\frac{dW^*}{d\tau} = \sum_{h=1}^{k} \varrho^h \left(\beta^h - \mu + \mu(\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j^h}{\partial r^h} + \hat{t}_l^c \frac{\partial l^h}{\partial r^h}) \right)$$

Assuming a one-person economy of course simplifies considerably.

the project. This point, which has been obscured in parts of the literature, is emphasized by Ballard (1990). The same argument applies of course to projects that are imperfect substitutes for cash.

Fullerton (1991) comments that calculations of the MCF has been based on different methods for measuring welfare change. Stuart (1984) and Hansson and Stuart (1985) use the compensating surplus, Ballard, Shoven and Whalley (1985) use the equivalent variation, and Browning (1987) uses the compensating variation. The measures however coincide at the margin.

Another point that concerns the proper measurement of an item of the MCF formula, is that the estimate will depend on the price p_j used to value inputs for the project. For instance, if p_j is the market price inclusive of tax paid by the public sector, the MCF is lower than if p_j is the market price exclusive of tax. Another possibility is to use an estimate of the shadow price as calculation price. This is often done in cost benefit analysis.

Of course, the public calculation is irrelevant for the project acceptance criterion eq. (34), as it enters both the nominator and denominator[5]. It only has influence on how to divide the cost between MCF and government expenditure.

The literature on the measurement of the MCF invariably has market prices in the denominator (see eg. Fullerton (1991) or Ballard (1990) for explicit statements to this effect). This procedure implies that the calculated MCF is a condensed statistic of *both* the cost of raising revenue, and the more 'traditional' cost benefit consideration of correcting input prices of a public project for market imperfections. The latter is measured by the ratio ν_j/p_j. Interestingly, this ratio cannot be measured separately from the cost of raising revenue, as ν_j depends on the financing of the project. Taken at face value, the cost benefit analyst employing an MCF estimate from the literature should not attempt to correct market prices. The necessary

[5]If one wants to use MCF estimates to assess potential tax reforms, only shadow prices will do as calculation prices.

correction is implicit in the value of the MCF. In Vennemo (1990), I observe differences of over 15 per cent in the MCF between projects.

Given a choice of financing, expressions for the ratios μ/λ and ν_j/p_j may be found by maximizing the Lagrangian w.r.t. the endogenous variables of the model, including the endogenous tax/transfer variable. This is the procedure I use in what follows. Note that the point of this maximization is to obtain shadow prices, and not to find the highest possible value of welfare. The latter would be fairly uninteresting, as there is only one possible level of welfare given the solution of the model[6].

4 The marginal cost of public funds

4.1 Poll tax financing

Poll tax financing may not be the most popular of tax forms, but it is a key to understanding what most of the other forms of financing are about. Moreover, modern societies do have taxes that are similar to poll taxes. In Norway, municipal fees for sewage and waste is in this category. Reductions in transfers are poll taxes on large segments of the population, like the elderly.

When increased public expenditure is financed by a poll tax, the poll tax is an endogenous variable of the model, and $\frac{\partial \mathcal{L}}{\partial r} = 0$.

The expression for MCF in the case of poll tax financing is summarized in proposition 1:

Proposition 1 *The MCF of poll tax financing is equal to*

$$MCF = \frac{1}{1 - \hat{t}b} \frac{\nu_j}{p_j} \tag{36}$$

where $\hat{t}b = \sum_{j=1}^{n} \hat{t}_j^c \frac{\partial \bar{c}_j}{\partial r} + \hat{t}_l^c \frac{\partial \bar{l}}{\partial r}$, $\bar{c}_j$ *is the average demand for consumption good j, and* $\bar{l}$ *is the average labour supply.*

[6]Drèze and Stern (1987) show that maximization is formally meaningful in the context of a fully determined model.

This expression for MCF in the case of poll tax financing is independent of functional forms for demand and welfare functions. The first fraction, which is equal to μ/λ, has the interpretation of the sum, from one to infinity, of the initial increase in poll taxes of 1, plus subsequent increases to compensate for lower revenue from indirect and direct taxation. (Revenue is lost because poll taxes decrease consumer incomes). The term $\hat{t}b$ can be interpreted as the average of shadow tax *rates* $\hat{t}_j^c/p_j^c$ weighted by the marginal propensities to spend. If shadow taxes are positive and goods and leisure are normal goods, all income effects except the income effect on labour supply work towards MCF being larger than unity.

It may seem paradoxical that it is the substitution of a poll tax for indirect taxation, a non-distorting for a distorting tax, that constitutes the excess cost of the project. According to ordinary intuition, such substitution should rather yield a gain than a cost. Part of the answer is of course that the substitution is not of the same kind as if one lowered a tax rate and increased the poll tax to compensate. A more complete answer, focusing on the real economics of the issue, is that for resources of the amount p to be transferred to the government sector, the private sector must have its income/welfare reduced by $p+t$, because it pays taxes t to the government. Dividing $p+t$ by p yields a number larger than one, ie. the MCF is larger than one.

The MCF of poll taxes have been estimated by Hansson (1984), Ballard (1990) and by myself (Vennemo (1991)). In the model of Hansson (1984), MCF values are significantly below unity. This is easily explained by proposition 1, as he assumes a negative income elasticity of labour, and a very high tax rate on labour (of 72 per cent), which gives a great weight to the negative income elasticity.

Ballard (1990) also obtains MCF values below unity in the case of poll tax financing. He shows, by simulation, how the MCF of a poll tax falls as the absolute value of the labour income elasticity is increased. This too is supported by proposition 1. (Unfortunately, the discussion is made in terms of the compensated labour supply elasticity).

Ballard, Shoven and Whalley (1985) have claimed that "in general, it appears that the (MCF)'s are greater for activities which face high or widely varying tax rates. These conclusions are, in general, in accord with those drawn from a simple partial equilibrium model." (p. 128). Proposition 1 implies that it actually is the *shadow* tax rates that matter. This will be recurrent in the derivations later in the paper as well. In a closed economy, shadow taxes may be negative although actual taxes are positive (see eg. the calculations in Smith (1987)). Shadow taxes however increase in actual taxes as a first order effect, compare eq. (16) to (22).

The dispersion of tax rates referred to by the authors is not relevant as such in the case of poll tax financing (but will be later on), what matters is the product of shadow tax rates and income effects. If "luxury" goods face higher shadow tax rates than "necessary" goods, the MCF will tend to be high[7] and *vice versa.*

In the second best optimum that allows a poll tax, we may drop the hats over the t's, as shadow taxes coincide with actual taxes. Eq. (36) then gives the single value of MCF in second best optimum. Again, it is the repercussions on the public budget in the form of decreased revenue from distortionary taxation that constitutes the MCF. Neither substitution effects nor equity considerations are directly involved (but they are involved in setting the optimal tax rates, of course).

Dropping the hats over the t's is often done in *partial* analysis of the MCF (see Ahmad and Stern (1984), Decoster and Schokkaert (1990)). On that assumption, one can by the aid of proposition 1 give a response to the following issue raised by Browning (1987): "Until it is shown that the general-equilibrium models provide significantly different and more accurate estimates

[7]Note that this is irrespective of choice of welfare function. The values of the shadow taxes will depend on the choice of welfare function, however.

(for the same parameter values), the partial-equilibrium approach has some advantages. First, it is easily understood, so it is less likely that critical assumptions will be obscured... Second, it is simple for other investigators to perform sensitivity analysis by modifying the assumptions regarding parameter values if such changes seem appropriate." (p.22). In response to the call for a demonstration of differences between general equilibrium and partial equilibrium, one can use proposition 1 to compare a full general equilibrium estimate to the partial one obtained by assuming shadow and actual taxes to be equal. In Vennemo (1991) I obtain general equilibrium MCF values from 1.07–1.24 depending on type of spending. If instead I make a back of the envelope calculation assuming that indirect shadow taxes equalled the Norwegian VAT rate of 16.67, the MCF comes out at 1.03. Whether the general equilibrium calculation in this case is "significantly different and more accurate" is a matter of judgement, and the VAT is in any case not the only indirect tax in Norway. Browning's argument that general equilibrium is not as "easily understood", need however be modified in light of proposition 1. It is also clear which parameters that matter (to the first order), namely the income effects in demand.

4.2 Labour income tax financing

Much of the empirical literature on the MCF has focused on the effect of financing public goods by taxes on labour income. When increased public expenditure is financed by a tax on labour income of consumers, the tax on labour income is an endogenous variable, and $\frac{\partial \mathcal{L}}{\partial t_i^c} = 0$.

At this point it is convenient to introduce an assumption on the welfare function:

Assumption 1 *The welfare weighted marginal utility of income is equal for everyone.*

On this assumption

$$\lambda = \frac{1}{k}\sum_{h=1}^{k}\beta^h = \frac{1}{k}\sum_{h=1}^{k}\beta = \beta \tag{37}$$

I make the assumption partly to be able to derive an explicit expression for λ in the case of labour tax financing and other forms of financing, and partly because it is a maintained assumption in empirical calculations of MCF.

The following proposition may be obtained:

Proposition 2 *Given assumption 1, the MCF of a wage income tax is equal to*

$$MCF = \frac{1}{1 - \tilde{t}_l^c \varepsilon_{ll} - \sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j}{\partial q_l^c} / L} \tag{38}$$

where $\tilde{t}_l^c = \hat{t}_j^c / q_l^c$ *is the shadow tax rate, and* $\varepsilon_{ll} = \frac{\partial L}{\partial q_l^c} \frac{q_l^c}{L}$ *is the aggregate labour supply elasticity.*

This expression is in Ahmad and Stern (1984) and elsewhere with $\hat{t} = t$. As in the case of poll tax financing, the interpretation is that MCF is created by direct and general equilibrium indirect changes in public revenues brought about by the "initial" increase in taxation. Clearly, it is the *uncompensated* rather than the compensated demand effects that matter in this respect. We are interested in the actual changes in (shadow) revenue. This is emphasized by Ballard (1990) and others.

Reflecting on the empirical effect of increasing the labour supply elasticity, Ballard, Shoven and Whalley (1985) conclude that "we can generally say that the more elastic activities have higher (MCF)'s" (p.135). Proposition 2 gives the basis for this statement. As ε_{ll} increases, the MCF also increases if the shadow tax on labour is positive. However, the cross effects to consumption of commodities are also important, as is the level of labour actually supplied. These factors are ignored by the authors.

Based on his small scale simulation model, Ballard (1990) concludes that, in his model, "for any initial tax rate, the (MCF) is (unity) if the uncompensated elasticity is zero. For nonzero uncompensated elasticities, the absolute value of the (MCF) increases as the initial tax rate increases." (p. 271). Proposition 2 shows that the corresponding general statement is that the MCF is zero if the uncompensated elasticity is zero *and* the shadow tax rates on commodities also are zero. For non zero uncompensated elasticities, the generalized conclusion is that the value of the MCF increases as the *shadow* tax rate on labour increases.

Fullerton (1991) claims that the MCF is unity if commodities are untaxed and the *actual* change in labour supply resulting from the lower wage and the general equilibrium change in prices is zero. He writes, for instance "the condition for the marginal cost of funds to be 1.0 is not that the uncompensated elasticity is zero, but that actual labour supply does not change." (p. 306). If this was correct, it would be captured in the present context by the shadow taxes of proposition 2 taking values such that MCF is unity in the event that labour supply is unchanged. But the shadow taxes are influenced by other aspects of the model besides the effect of price changes on labour supply. The model of foreign trade is a case in point. See appendix C.

I will now add an assumption about the structure of preferences:

Assumption 2 *Let the income derivatives be identical for all members of society.*

This assumption requires that preferences are of the Gorman Polar Form. Most applied general equilibrium models use preference structures that satisfy assumption 2, and all the models that attempt to measure the MCF use it. Note that this assumption implies that perfect linear aggregation is possible. Assumption 1 and 2 together therefore reduce the many person economy to a one representative consumer economy.

The following proposition is obtained:

Proposition 3 *Given assumption 1 and 2 the MCF of labour income tax financing is*

$$MCF = \frac{1}{1 - \hat{t}b - m_l} \frac{\nu_j}{p_j} \tag{39}$$

$$m_l = \frac{1}{L}(\sum_{j=1}^{n} \hat{t}^c_j s_{jl} + \hat{t}^c_l s_{ll}) \tag{40}$$

where $s_{jl} = \sum_h s^h_{jl}$ and $s_{ll} = \sum_h s^h_{ll}$ are the Slutsky substitution terms.

The part $\hat{t}b$ of the denominator of equation (39) is the "revenue effect" of Atkinson and Stern (1974). The part m_l is the "distortionary effect". Note that to obtain this separation, I had to remove the many person dimension from the problem.

Atkinson and Stern use the formulas for optimal commodity taxation to derive the result that the distortionary effect m_l really is distortionary, ie. positive. When taxation is non-optimal, the result does not follow. This is interesting, as it is on the basis of the distortionary effect that Atkinson and Stern give Pigou half-right in his (general) claim that financing expenditure causes "indirect damage". Ballard (1990) has adopted this view, writing that "Atkinson and Stern (1974) show that the ratio of (μ) to (λ) can be divided into a 'distortionary effect', which *always* leads towards an (MCF larger than unity), and a 'revenue effect' which can go in either direction." (p.265, emphasis added).

For a tax reform involving a decrease in the wage income tax and an increase in the poll tax to improve welfare, it is necessary and sufficient in a one consumer setting that the distortionary effect really is distortionary, ie. that m_l is positive. It is not strange, therefore, that the tax reform literature has looked for conditions to ensure that m_l is positive. From equation (40) it can be shown that m_l is positive if labour is complementary (in the sense $s_{lj} \geq 0$) to all goods with a higher proportional shadow tax, and substitute for all others. This result is in Dixit (1975) (theorem 7). Similar sufficiency conditions are easily found, for instance is m_l positive if the shadow tax on labour is positive, and labour is complement to all goods with a positive shadow tax, and a substitute for all others.

An alternative approach to the problem of obtaining conditions for a positive m_l is pursued in the following. I make one more assumption on preferences and one on the shadow tax rates:

Assumption 3 *Let preferences be additive in labour and a function of consumption goods, ie.*

$$V^h = V_1^h(c_1^h, \ldots, c_n^h) - V_2^h(l^h)$$

This assumption covers LES, CES, Cobb Douglas and many more preference systems. All applied general equilibrium models computing MCF adopt this assumption. Note that assumptions 2 and 3 are sufficient for two stage budgeting. Pollak (1971) describe the functions that satisfy the two criteria simultaneously.

Assumption 4 *The term $\hat{t}b + \tilde{t}_l^c$ is positive, where $\tilde{t}_l^c = \hat{t}_l^c / q_l^c$ is the shadow wage income tax rate.*

This assumption allows some of the shadow tax rates to be negative. It even allows $\hat{t}b$ to be negative, ie. the MCF of poll taxation to be below unity.

I can derive the following proposition:

Proposition 4 *Given assumptions 1, 2, 3 and 4, labour income tax financing yields a higher MCF than poll tax financing.*

An implication of proposition 4 is that a tax reform consisting of more poll tax, and less labour income tax will always improve welfare. The importance of the proposition lies in the fact that all applied general equilibrium models used to calculate the MCF employ assumptions 1, 2 and 3. While the purpose of these computations often is to determine empirically what cannot be sorted out analytically, proposition 4 shows that the outcome of the computations may be determined in structure, not by measurements actually made, but by arbitrary, untested (and even unconscious) hypotheses chosen by the analyst for practical convenience, to paraphrase

Deaton (1981). Thus statements like "The marginal cost of public funds is estimated for different types of tax increases. The results suggest how the portfolio of tax instruments may be rearranged to reduce the distortionary effects of taxation for a given level of tax revenues" Hansson (1984) (p.116) are claiming too much.

A further consequence of the assumptions made thus far should be noted:

Proposition 5 *Given assumption 1,2 and 3, the MCF of labour income taxation is a function of income effects only.*

Proposition 5 is obtained because the substitution effects are functions of income effects under the assumptions of the proposition. Hansson and Stuart (1985) write: "We also investigate whether income or substitution effects are more important. (...) This is done by holding the wage elasticity constant and by increasing the total income elasticity and decreasing the substitution elasticity. (...) The outcome is a decline in the marginal cost of public funds. This indicates that the substitution effect matters most." But frankly, it seems rather futile to make simulations to determine which of the income and substitution effect is the most important determinant of the MCF, when the substitution effect depends on the income effect. Ballard (1990), who summarizes his research the following way: "the simulations suggest that the (MCF) is much more related to uncompensated elasticities than to compensated ones" (p.266) also go in this "trap". Whether the uncompensated price effect is great or small is also beside the point when the model employs assumption 2. The heart of the matter is the income effects. This objection applies, for instance to the claim of Ballard, Shoven and Whalley (1985) that the MCF is larger, the larger are the uncompensated price elasticities, and to Stuart (1984), who perform sensitivity analysis with respect to the uncompensated labour supply elasticity.

4.3 Commodity tax financing

When government expenditure is financed through taxation of a consumer commodity, the formula for the MCF is similar to labour income financing, ie. it can be written

$$MCF = \frac{1}{1 - \hat{t}b + m_j} \frac{\nu_k}{p_k}$$

$$m_j = \frac{1}{c_j} \left(\sum_{i=1}^{n} \hat{t}_j^c s_{ij} + \hat{t}_l^c s_{lj} \right)$$

As in the case of wage income taxation, general sufficiency conditions that guarantee that MCF is lower than that of a poll tax, can be derived.

I now make the following two assumptions:

Assumption 5 *Preferences over goods are additive.*

All applied general equilibrium models attempting to estimate the MCF has adopted this assumption.

Assumption 6 *Preferences are* not *of the form*

$$U(x) = -ln \left[\sum_j a_j e^{\frac{b_k - x_k}{a_k}} \right]$$

This assumption is purely for convenience, in order to get a more tractable expression for m_j in the proposition below. None of the applied general equilibrium models apply this particular preference relation.

The following proposition can be derived:

Proposition 6 *Given assumptions 1, 2, 3, 5 and 6 the MCF can be written*

$$MCF = \frac{1}{1 - \hat{t}b + m_j} \frac{\nu_k}{p_k} \tag{41}$$

where

$$m_j = \frac{(1-\frac{a_j}{c_j})(1-\frac{a_c}{c})}{1-\beta}\left(\hat{tb}-\hat{\tau}\right) + \frac{1-\frac{a_j}{c_j}}{1-\alpha}\left(\hat{\tau}-\tilde{t}^c_j\right)$$

and $a_j = \sum_h a_j^h$ *can be interpreted as committed expenditure,* a_c *is the aggregate consumption counterpart to* a_j*,* $c = \sum_h c^h$ *is the sum of aggregate consumption quantities,* $\hat{\tau} = \sum_{j=1}^n \hat{t}^c_j \frac{\partial c_j}{\partial y}$ *where* y^h *is consumption expenditure and* $\frac{\partial c_j}{\partial y}$ *is the common engel derivative, while* α *is a parameter in the preference relation for goods (see assumption 5), and* β *is its top level counterpart.* $\tilde{t}^c_j$ *is the shadow tax rate on good j.*

$\hat{\tau}$ is the average of the indirect shadow tax rates, with the marginal propensities to spend out of consumer expenditure as weights. It is thus a counterpart to $\hat{tb}$ in the second stage of budgeting. Assumptions 1, 2, 3 and 5 and 6 are again employed in all attempts to date to compute the MCF in applied general equilibrium models. Among the preference systems covered, are the CES-LES, CES-CES and CES-Cobb Douglas combinations. Proposition 6 says that if expenditure is financed by a tax that brings shadow commodity taxes closer to uniformity, the MCF is lower than if it is financed by a tax that takes commodity taxes away from uniformity. This is because the expression is symmetric in the j's, and $\hat{tb}$ and $\hat{\tau}$ are independent of j. As the MCF's can be used as indicators of tax reforms, the implication is that a reform towards uniform shadow taxation of commodities will increase welfare. This extends Deaton (1987), who assumed complete additivity and an optimal poll tax.

For a commodity tax to be cheaper than a poll tax, it suffices that $\tilde{t}^c_j$ is lower than $\hat{\tau}$ and $\hat{\tau}$ is lower than $\hat{tb}$. The latter is about upper level uniformity. It requires that the tax factor $\hat{\tau}$ on the consumption aggregate should be lower than that on consumption and labour combined. The intuition is that a tax on a consumption good pushes up the price of the aggregate of consumption goods, which is a positive thing if the price on the consumption aggregate is too low compared to the ideal of optimal taxation. The condition that $\hat{\tau}$ is lower than $\hat{t}b$, is however not likely to hold in practice if shadow tax rates are positive, because in $\hat{t}b$ there is a negative element consisting of the effect of lump sum income on labour supply. If $a_c = 0$ and $\alpha \geq \beta$, the commodity tax is cheaper than a poll tax if $\tilde{t}_j^c$ is lower than $\hat{t}b$. Other conditions for a commodity tax to be cheaper than a poll tax may be inferred by inspecting m_j of the proposition.

Ballard, Shoven and Whalley (1985) observe that "We can also see the point about high and dispersed tax rates causing large (MCF)s if we look at the results for consumer sales taxes. When we raise all sales and excise taxes including the very high taxes on alcohol, tobacco and gasoline, we have high (MCF)'s. However, when we raise only the low taxes on the other commodities, we end up with very modest (MCF)'s" (p. 136). Proposition 6 gives a precise meaning to this observation. The point is that when lower-than-average taxes are increased towards the average, the MCF will have to be low because of the preferences assumed (in their case a CES-Cobb Douglas combination).

It is of interest to compare commodity taxes to each other and to the labour income tax in order to see what is the cheapest way of financing expenditure. The result is summarized in the following proposition:

Proposition 7 *Given assumptions 1, 2, 3, 5 and 6, which of two taxes t_j^c and t_i^c that yields the lowest MCF, depends on four parameters, a_j/c_j and $\tilde{t}_j^c$ versus a_i/c_i and $\tilde{t}_i^c$. If $a_j/c_j = a_i/c_i$, the MCF of the tax with the lowest shadow tax is the lowest. Which of a commodity tax t_j^c and the tax on labour income t_l^c that yields the highest MCF depends on $a_j/c_j, a_c/c, a_l/L, \alpha$ and β. However, if $a_j = a_c = a_l = 0$ and $\alpha = \beta$, and the condition $\tilde{t}_j^c < 2\hat{t}b + \tilde{t}_l^c$ holds, the MCF of a commodity tax t_j^c is lower than that of a tax on labour income.*

We recall that "tildes" mean shadow tax *rates*. If the condition $\tilde{t}_j^c < 2\hat{t}b + \tilde{t}_l^c$ does not hold, the shadow tax rate of the commodity that finances expenditure is relatively much higher than on wage income. Proposition 7 says roughly that it *is* possible for a commodity tax to have a higher MCF than the labour income tax, but only if the shadow commodity tax already is this much higher.

The key to the proposition is the fact that, given the preference structure, all goods and (the marketed negative quantity of) *leisure* should be taxed at a uniform (shadow) rate at the optimum. The latter translates to a subsidy on labour supply. In particular, we understand why wage income taxation "always" is more expensive than commodity taxation. It is because an increase in the labour tax implies a movement away from uniformity. For a commodity tax to be equally "bad", it must be just as far away from the average level, ie. it must be very high. When comparing two commodity taxes, the condition $\tilde{t}_j^c < \tilde{t}_i^c$ says that it is better to increase the lowest of two taxes. When both are below the average, this is to say that the gain from bringing the lowest of the two towards the average, is higher than bringing the other towards the average. When both are higher than the average, the proposition says that the harm from increasing the one closest to the average, is less than increasing the other one.

An implication of proposition 7 is that a tax reform that brings the values of two (shadow) tax rates $\tilde{t}_i^c$ and $\tilde{t}_j^c$ closer together, will increase welfare *irrespective of the values of other tax-rates in the system* if their rates of committed expenditure are the same. This also means that if tax rates for some reason cannot be made fully uniform, a third best optimal solution is to make subsets of taxes uniform. This is a further extension of Deaton (1987).

Proposition 7 contradicts the Browning (1987) view that applied general equilibrium models are not suited for singling out the essential parameters in determining MCF. Given that the model obeys the preference assumptions, only four parameters matter for the evaluation of commodity taxes, namely the two shadow tax rates, and the two parameters giving the ratio of committed expenditure to actual expenditure. A limited number of parameters regulate the relation between a commodity tax and a wage income tax as well. Even in the case of general preference systems, the well defined parameters of m_j in equation (41) determine the MCF of commodity taxation in the applied general equilibrium model.

When preferences are completely additive (only one stage of budgeting or exogenous labour supply) and $\tilde{t}_l^c = 0$, $\hat{t}b = \hat{\tau}$ and the first term of m_j of equation (41) is zero. The preference for uniformity among indirect taxes still applies. In their estimation of the MCF's of indirect taxes for different demand systems, Decoster and Schokkaert (1990) use LES as a "benchmark". Externalities are not accounted for. Beverages, a good with a $\tilde{t}$ of 35 per cent and an expenditure elasticity of 1.56, then comes out with the highest MCF value. Next is tobacco, whose $\tilde{t}$ is 68 per cent and expenditure elasticity 0.41. At the lowest end is rent, whose $\tilde{t}$ is only 1 per cent. Their ranking for the LES can be explained by proposition 7. A high income elasticity of the LES implies a low committed expenditure, which tends to amplify the effect of $\tilde{t}$. This is why beverages yields a higher $\tilde{t}$ than tobacco.

4.4 Export tariff financing

So far, the formulas and propositions stated are equally true for models of closed and open economies, and for different models of foreign trade. This is because I haven't specified what the shadow taxes look like. The next two sections however concerns foreign trade explicitly. The model of foreign trade is therefore important in theses sections.

When increased public expenditure is financed by an export tariff, the tariff is an endogenous variable of the model, and $\frac{\partial \mathcal{L}}{\partial t_{aj}} = 0$.

The implication is summarized in proposition 8:

Proposition 8 *The MCF of an export tariff is written*

$$MCF = \frac{\mu}{\lambda}(1 - \frac{1}{e_j})\frac{p_{aj}}{p_j} \tag{42}$$

where $e_j = -\frac{da_j}{dp_{aj}}\frac{p_{aj}}{a_j}$ *is the (absolute value of the) price elasticity of export demand.*

Proposition 8 concerns the phenomenon of 'tax exporting'. The intuition is that when the country can influence its terms of trade, it can transfer some of the cost of increased public expenditure to foreigners.

There are two aspects of this. One is that in second best optimum, the value of MCF for a given level of public spending is lower than in a "twin" economy without such possibilities. The second is that the MCF of an export tax may be lower than that of other instruments, if tariffs are too low. This aspect is brought out here.

Wildasin (1987) writes the following: "This paper finds that the ability to export taxes need not in general lower the effective cost of public spending. A simple model is developed in which a jurisdiction optimizes the mix of taxes between those on traded and non-traded goods. Once this structure is optimally set, the jurisdiction is indifferent between exported and own-source revenues, and the marginal cost of public expenditure is unaffected by the possibility of exporting" (p. 353). He then gives an example of an economy in which the MCF of a second best optimum is the same whether or not tax exporting is possible.

Proposition 8 does not give particular support to the emphasis of this quotation. In not-so-simple models, the MCF is in general affected by the possibility to export taxes, the more so, the smaller are the price elasticities of export demand. Many applied general equilibrium models adopt values of this parameter of around 1.5-2[8]. In these cases the MCF of an export tax is considerably reduced compared to the situation of the small open economy. If the elasticity

[8] Ballard, Shoven and Whalley (1985) use 1.4 in all sectors, citing Stern, Francis and Schumacher (1976). The estimates of Lindquist (1990) for the Norwegian economy also yield values below 2.

is 2 for instance, the MCF is only one half of what it is in a small open economy. This is a significant tax exporting effect, considering that MCF of internal tax instruments normally take on values below 2.

4.5 Import tariff financing

When increased public expenditure is financed by an import tariff, the tariff is an endogenous variable, and $\frac{\partial \mathcal{L}}{\partial t_{fj}} = 0$. A general formula for the MCF can easily be derived on the basis of this condition (and assumption 1). Results however are more specific if we add the following assumption:

Assumption 7 *The b_j^c-function is of the CES-form.*

Most applied general equilibrium models apply a CES form to import demand (ie. CES import shares), justifying assumption 7. The original Armington (1969) article is based on CES functions as well.

The following result can be derived:

Proposition 9 *Given assumptions 1, 2 and 7, the MCF of expenditure financed by an import tax can be written*

$$MCF = \frac{1}{1 - \hat{t}b + m_j + n_{fj}} \frac{\nu_k}{p_k} \tag{43}$$

where

$$\begin{aligned} n_{fj} &= \sigma_j(\hat{t}v - \tilde{t}_{fj}) \\ \hat{t}v &= t_{fj}\frac{\partial c_{fj}}{\partial s} + \hat{t}_d\frac{\partial c_{dj}}{\partial s} \end{aligned}$$

where σ_j is the (absolute value of the) elasticity of substitution between foreign and domestic varieties, $\tilde{t}_{fj}$ is the tariff rate t_{fj}/p_{fj}, $s_j^h = b_j^c c_j^h$ is the income allotted to purchases of commodity j, and $\frac{\partial c_{fj}}{\partial s_j}$ and $\frac{\partial c_{dj}}{\partial s_j}$ are the (linear) income derivatives of the varieties.

Proposition 9 says that financing by means of an import tariff yields a lower MCF than financing by means of a tax on the aggregate commodity if taxes on domestic and foreign varieties become more uniform as a result. The impact on the MCF depends on the elasticity of substitution between the two varieties. This parameter regulates the quantity implication of increasing the tariffs within the composite. If it is large, the more elastic is the consumption composite index, and the more is there to gain from a reform in the direction of more uniform taxation.

When increased public expenditure is financed by a tax on a domestic variety of some good, the tax is an endogenous variable, and $\frac{\partial \mathcal{L}}{\partial t_{dj}} = 0$. An expression that is symmetric to eq. (43) is obtained.

4.6 Labour and capital input tax financing

When increased public expenditure is financed by a tax on the labour input of some industry, the tax is an endogenous variable, and $\frac{\partial \mathcal{L}}{\partial t_l^j} = 0$. An expression for the MCF in the general case (only given assumption 1) can easily be found by combining $\frac{\partial \mathcal{L}}{\partial t_l^j} = 0$ and $\frac{\partial \mathcal{L}}{\partial b_{dj}} = 0$.

I find it more fruitful to pursue the implications of the following two assumptions:

Assumption 8 *The industry in question is non-exporting.*

Assumption 9 *The production function is CES.*

Assumption 8 is purely for convenience, as there are no technical problems to adding on exports. Assumption 9, although restrictive, is satisfied by all but one (my own) of the attempts to measure the MCF by means of applied general equilibrium models. As might be guessed, assumption 9 allows us to derive another "level" where the deviation from uniform taxation is important. All of the instances of preferences for uniform taxation is contained in the following proposition:

Proposition 10 *Assume assumption 1 - 3 and 5 - 9 hold. The MCF of a labour input tax will then be*

$$MCF = \left[1 - \hat{t}b + \frac{(1-\frac{a_j}{c_j})(1-\frac{a_c}{c})}{1-\beta}\left(\hat{t}b - \hat{\tau}\right) + \frac{1-\frac{a_j}{c_j}}{1-\alpha}\left(\hat{\tau} - \tilde{t}_j^c\right) + \sigma_j(\hat{t}v - \tilde{t}_{dj}) + \sigma_{lk}(\hat{t}z - \tilde{t}_l^j)/\frac{\partial b_j^c}{\partial p_{dj}}\right]^{-1} \frac{\nu_k}{p_k}$$

where $\hat{t}z = \hat{t}_l^j \frac{\partial zl_j}{\partial e^j} + \hat{t}_k^j \frac{\partial zk_j}{\partial e^j}$.

Proposition 10 shows the issue of uniformity to enter at four levels. First, the labour input tax cet.par. lowers the MCF if the labour input tax is lower than the capital input tax. The increase in costs is carried over to an increase in the shadow tax of the domestic variety, which reduces the MCF if the shadow tax on the domestic variety is lower than the tax on the foreign variety. Third, the shadow tax on the composite commodity increases, which decreases the MCF if the shadow tax on the composite commodity is lower than the weighted average of tax rates. Finally the price of the composite commodity is increased, which is positive in the (unlikely) case that the aggregate tax on the composite is lower than the wage income tax. The conclusion is: If the labour input tax is to be used, find an industry in which labour is taxed more lightly than capital, and the shadow taxes on all stages of demand are lower than the (weighted) average. The increase in the labour input tax triggers a row of movements towards uniformity, which each contributes to the MCF going down. Only the impact on the price of aggregate consumption contribute to the MCF going up. Note how once again only a few parameters are directly relevant for the size of the MCF.

Hansson and Stuart (1985) table 5, estimate MCF of a factor tax keeping the tax on the other factor constant. The table shows that increasing a tax is cheap as long as the other factor is more heavily taxed, but not if the other factor is more lightly taxed.

Ballard (1990) performs sensitivity analyses that suggest that "the elasticity of substitution in production has only a small effect on the results. If we move from a substitution elasticity of

0.6 to 1.2, the efficiency effects previously reported are changed by less than 1 cent." (p.271)[9]. This is because he considers a labour income tax, in which the elasticity of substitution plays no direct role in determining MCF. Proposition 10 reveals that the elasticity of substitution in production however has a direct influence on the size of MCF when one considers an input tax.

When capital input is taxed, the formula is symmetric to that of labour input.

4.7 Combinations of taxes

So far in the paper, I have examined the effects of financing expenditure through increasing one tax at a time. However, many of the applied general equilibrium simulations of the MCF increase several taxes at a time to finance expenditure. I state the results for some combinations of tax rates in proposition 11:

Proposition 11 *Under assumptions 1 and 2, the MCF of a fully comprehensive VAT is*

$$MCF = \frac{1}{1 - \hat{t}b - am_l} \frac{\nu_j}{p_j}$$

where a is the labour income share. Generally, a proportional increase in labour taxes on an industry level is equal to increasing the wage income tax. A proportional increase in capital taxes on an industry level is equal to a poll tax. An increase in labour and capital taxes of an industry is equal to increasing the tax on output of the industry. An increase in taxes on labour and capital in all industries is in the closed economy version of the model equal to an increase in all consumer sales taxes. In the open economy version, it is equal to an increase in output taxes.

Some of the statements of proposition 11 are intuitive, even self-evident. Yet they seem to have gone largely unnoticed in the literature on the empirical measurement of the MCF. Ballard,

[9]See also Hansson (1984) Hansson and Stuart (1985), who find almost no effect in their "differential" experiments.

Shoven and Whalley (1985) for instance, state that "with a labour supply elasticity of zero, the labour taxes at the industry level cause relatively small amounts of marginal distortion" (p. 135). This is not surprising, considering that we are in that case talking of a poll tax.

The result on the MCF of VAT financing implies that a tax reform where the VAT is decreased and the lump sum tax is increased, will increase welfare if m_l is positive. This extends theorem 3 in Dixit (1975), who assumed a single factor of production.

5 Conclusions

The paper has presented computations of the marginal cost of public funds under different assumptions regarding the way expenditure is financed. Assumptions of preferences and technology commonly made in the literature measuring the MCF were also taken into account.

The results of the paper yield a number of implications that are summarized here in a compact form. I do not repeat the assumptions required for each result, but refer to the propositions where they are stated the first time:

- On the ranking of tax reforms

  1. A poll tax yields a lower MCF than a wage income tax (proposition 4).
  2. A commodity tax yields a lower MCF than a wage income tax (proposition 7).
  3. A VAT increase yields a lower MCF than a wage income tax, but a higher MCF than a poll tax (proposition 11).
  4. A commodity tax may yield a lower MCF than a poll tax if the initial commodity tax rate is lower than the aggregate tax rate on consumption (proposition 6).
  5. Which of two commodity taxes that yield the lowest MCF depends on the shadow tax rates and the rates of committed consumption to actual consumption for the

two goods (proposition 7).

6. An import tariff yields a lower MCF than a commodity tax on the same commodity if the import tariff is lower than the shadow tax on the domestic variety (proposition 9).

7. In a non-exporting sector, a factor tax yields a lower MCF than an output tax if the initial shadow factor tax is lower than the tax on the other factor (proposition 10).

8. An export tax yields a lower MCF, the lower is the export price elasticity (proposition 8).

9. When preferences are two level, sub-additive and engel derivatives are identical across households, a tax reform in the direction of making commodity taxes more equal will increase welfare. When two commodities have the same committed expenditure (eg. zero), a tax reform that makes the taxes on the two goods more equal will increase welfare. These results even hold for one level additive preferences.

- On other matters

  1. When an MCF value taken from applied general equilibrium models is used as input to cost benefit analysis, costs should in principle be assessed in market prices.

  2. In applied general equilibrium models, the MCF is a function of income effects only, as compensated price effects are functions of income effects. More generally, the MCF is a function of uncompensated price effects.

  3. Shadow taxes, and not actual taxes enter the formulas for the MCF.

Perhaps the most important assumption behind the results on the MCF (ie. tax reform) rankings, is the utilitarian assumption that "a dollar is a dollar".

Given this and the other assumptions, the paper has shown that a number of results on the set of desirable tax reforms can be stated on the basis of theory alone. Another merit of the paper is that it clarifies what the MCF "depends on" in general equilibrium, something which reduces the need for sensitivity analyses, illuminates the relation between general equilibrium and partial equilibrium, and reduces the tendency of applied general equilibrium modelling to treat the model as a "black box".

Appendices

A First order conditions of the Lagrangian

A.1 First order conditions that always hold

The following relations will hold no matter the financing:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial b_{dj}} &= \sum_{h=1}^{k} -c_j^h\beta^h\frac{\partial b_j^c}{\partial b_{dj}} + \mu\left\{\sum_{i=1}^{n}\hat{t}_i^c\frac{\partial c_i}{\partial p_j^c}\frac{\partial b_j^c}{\partial b_{dj}} + \hat{t}_l^c\frac{\partial L}{\partial p_j^c}\frac{\partial b_j^c}{\partial b_{dj}} + c_j\frac{\partial b_j^c}{\partial b_{dj}}\right.\\ &\left. -c_j\left(b_{fj}\frac{\partial^2 b_j^c}{\partial p_{fj}\partial p_{dj}} + \nu_j\frac{\partial^2 b_j^c}{\partial p_{dj}^2}\right) - y_j - \theta_j + \left[\hat{t}_{aj}\frac{\partial a_j}{\partial p_{aj}} + a_j\right]\right\} = 0 \qquad \text{(A1)}\end{aligned}$$

where the bracketed term only applies when j is an exporting industry. Rewriting the equation yields (using the Slutsky definition, the definitions of t_{fj}, $\hat{t}_{dj}$ and (where relevant) $\hat{t}_{aj}$, Young's and Euler's theorems on the second derivative of b_j^c, and dividing through by c_j and μ and (where relevant) $\partial a_j/\partial p_{aj}$)[10]

$$\frac{\partial\mathcal{L}}{\partial b_{dj}} \;:\; h_j = 0 \quad \text{when } j \leq m \qquad \text{(A2)}$$

$$\frac{\partial\mathcal{L}}{\partial b_{dj}} \;:\; p_{aj}(1-\frac{1}{e_j}) - \nu_j - \frac{1}{e_j}(h_j\frac{c_j}{a_j}p_{aj}) = 0 \quad \text{when } j > m \qquad \text{(A3)}$$

$$h_j = \left[\frac{1}{c_j}\left(\sum_{i=1}^{n}\hat{t}_i^c s_{ij} + \hat{t}_l^c s_{lj}\right) + \frac{\mu-\hat{\alpha}_j}{\mu}\right]\frac{\partial b_j^c}{\partial p_{dj}} + \left(t_{fj}\frac{\partial^2 b_j^c}{\partial p_{fj}\partial p_{dj}} + \hat{t}_{dj}\frac{\partial^2 b_j^c}{\partial p_{dj}^2}\right) - \frac{1}{c_j}(y_j+\theta_j)$$

$$\hat{\alpha}_j = \sum_{h=1}^{k}\frac{c_j^h}{c_j}\left(\beta^h + \mu(\sum_{j=1}^{n}\hat{t}_j^c\frac{\partial c_j^h}{\partial r^h} + \hat{t}_l^c\frac{\partial l^h}{\partial r^h})\right)$$

Two more relations that always will hold are

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial q_l} &= \sum_{h=1}^{k} l^h\beta^h + \mu\left[\sum_{j=1}^{n}\hat{t}_j\frac{\partial c_j}{\partial q_l^c} + \hat{t}_l^c\frac{\partial L}{\partial q_l^c} - L\right] \qquad \text{(A4)}\\ &+\mu\sum_{j=1}^{n}\left[\hat{t}_l^j\frac{\partial zl_j}{\partial q_l^j}y_j + \hat{t}_k^j\frac{\partial zk_j}{\partial q_k^j}y_j + (y_j+\theta_j)zl_j\right] = 0\end{aligned}$$

[10]The ":" sign means "the left hand side equal to zero is equivalent to...".

$$\frac{\partial \mathcal{L}}{\partial q_k} = \sum_{h=1}^{k} K^h \beta^h + \mu \left[\sum_{j=1}^{h} \hat{t}_j^c \sum_{h=1}^{k} K^h \frac{\partial c_j^h}{\partial r} + \hat{t}_l^c \sum_{h=1}^{k} K^h \frac{\partial c^h}{\partial r} - K \right]$$

$$+ \sum_{j=1}^{n} \left[\hat{t}_l^j \frac{\partial z l_j}{\partial q_k^j} y_j + \hat{t}_k^j \frac{\partial z k_j}{\partial q_k^j} y_j + (y_j + \theta_j) z k_j \right] = 0 \quad \text{(A5)}$$

A final set of equations that always will hold is

$$\frac{\partial \mathcal{L}}{\partial y_j} = \mu(\hat{t}_l^j z l_j + \hat{t}_k^j z k_j + \hat{t}_{y_j}) = 0 \quad \text{(A6)}$$

A.2 First order conditions of which one will hold

In addition to the relations that always hold, *one* of the following relations will hold:

A.2.1 Endogenous poll tax

$$\frac{\partial \mathcal{L}}{\partial r} = \sum_{h=1}^{k} \beta^h \frac{\partial r^h}{\partial \tau} + \mu \left[\sum_{j=1}^{n} \hat{t}_j^c \sum_{h=1}^{k} \frac{\partial c_j^h}{\partial r^h} \frac{\partial r^h}{\partial \tau} + \hat{t}_l^c \sum_{h=1}^{k} \frac{\partial l^h}{\partial r^h} \frac{\partial r^h}{\partial \tau} - \sum_{h=1}^{k} \frac{\partial r^h}{\partial \tau} \right] = 0 \quad \text{(A7)}$$

Assuming $\frac{\partial r^h}{\partial \tau}$ is constant across h (a pure poll tax) and rearranging yields (dividing by k and using the definition of λ as $\sum_h \beta^h / k$)

$$\frac{\partial \mathcal{L}}{\partial r} = \lambda + \mu \left[\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial \bar{c}_j}{\partial r} + \hat{t}_l^c \frac{\partial \bar{l}}{\partial r} - 1 \right] = 0 \quad \text{(A8)}$$

where the bars indicate average values.

A.2.2 Endogenous wage income tax

$$\frac{\partial \mathcal{L}}{\partial t_l^c} = \sum_{h=1}^{k} l^h \beta^h + \mu \left[\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j}{\partial q_l^c} + \hat{t}_l^c \frac{\partial L}{\partial q_l^c} - L \right] = 0 \quad \text{(A9)}$$

Using the Slutsky equation and dividing by μ yields

$$\frac{\partial \mathcal{L}}{\partial t_l^c} \ : \ \sum_{j=1}^{n} \hat{t}_j^c s_{jl} + \hat{t}_l^c s_{ll} - \frac{\mu - \hat{\alpha}_l}{\mu} L = 0 \quad \text{(A10)}$$

$$\hat{\alpha}_l = \sum_{h=1}^{k} \frac{l^h}{L} (\beta^h + \mu (\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j^h}{\partial r^h} + \hat{t}_l^c \frac{\partial l^h}{\partial r^h}))$$

A.2.3 Endogenous commodity tax

$$\frac{\partial \mathcal{L}}{\partial t_j^c} = \sum_{h=1}^{k} -c_j^h \beta^h + \mu \left[\sum_{i=1}^{n} \hat{t}_i^c \frac{\partial c_i}{\partial p_j^c} + c_j + \hat{t}_l^c \frac{\partial L}{\partial p_j^c} \right] = 0 \tag{A11}$$

Using the Slutsky equation and dividing by μ yields

$$\frac{\partial \mathcal{L}}{\partial t_j^c} \; : \; \sum_{i=1}^{n} \hat{t}_i^c s_{ij} + \hat{t}_l^c s_{lj} + \frac{\mu - \hat{\alpha}_j}{\mu} c_j = 0 \tag{A12}$$

A.2.4 Endogenous labour input tax

$$\frac{\partial \mathcal{L}}{\partial t_l^j} = \mu(\hat{t}_l^j y_j \frac{\partial z l_j}{\partial q_l^j} + y_j z l_j + \hat{t}_k^j y_j \frac{\partial z k_j}{\partial q_l^j} + \theta_j z l_j) = 0 \tag{A13}$$

Dividing by μ yields

$$\frac{\partial \mathcal{L}}{\partial t_l^j} : \hat{t}_l^j \frac{\partial z l_j}{\partial q_l^j} y_j + \hat{t}_k^j \frac{\partial z k_j}{\partial q_l^j} y_j + (y_j + \theta_j) z l_j = 0 \tag{A14}$$

A.2.5 Endogenous capital input tax

$$\frac{\partial \mathcal{L}}{\partial t_k^j} = \mu(\hat{t}_l^j y_j \frac{\partial z l_j}{\partial q_k^j} + \hat{t}_k^j y_j \frac{\partial z k_j}{\partial q_k^j} + y_j z k_j + \theta_j z k_j) = 0 \tag{A15}$$

Dividing by μ yields

$$\frac{\partial \mathcal{L}}{\partial t_k^j} : \hat{t}_l^j \frac{\partial z l_j}{\partial q_k^j} y_j + \hat{t}_k^j \frac{\partial z k_j}{\partial q_k^j} y_j + (y_j + \theta_j) z k_j = 0 \tag{A16}$$

A.2.6 Endogenous export tariff

$$\frac{\partial \mathcal{L}}{\partial t_{aj}} = \hat{t}_{aj} \frac{\partial a_j}{\partial p_{aj}} + a_j = 0 \tag{A17}$$

Using the definition of $\hat{t}_{aj}$ as $p_{aj} - \nu_j$ and dividing by $\partial a_j / \partial p_{aj}$ yields

$$\frac{\partial \mathcal{L}}{\partial t_{aj}} : p_{aj}(1 - \frac{1}{e_j}) - \nu_j = 0 \tag{A18}$$

A.2.7 Endogenous import tariff

$$\frac{\partial \mathcal{L}}{\partial t_{fj}} = \sum_{h=1}^{k} -c_j^h \beta^h \frac{\partial b_j^c}{\partial p_{fj}} + \mu \left[\sum_{i=1}^{n} \hat{t}_i^c \frac{\partial c_i}{\partial p_j^c} \frac{\partial b_j^c}{\partial p_{fj}} + (t_{fj} \frac{\partial^2 b_j^c}{\partial p_{fj}^2} + \hat{t}_{dj} \frac{\partial^2 b_j^c}{\partial p_{dj} \partial p_{fj}}) c_j + \frac{\partial b_j^c}{\partial p_{fj}} c_j + \hat{t}_l^c \frac{\partial L}{\partial p_j^c} \frac{\partial b_j^c}{\partial p_{fj}} \right]$$
$$= 0 \tag{A19}$$

Using the Slutsky equation and dividing through by c_j and μ yields

$$\frac{\partial \mathcal{L}}{\partial t_{fj}} : \left[\frac{1}{c_j} \left(\sum_{i=1}^{n} \hat{t}_i s_{ij} + \hat{t}_l^c s_{lj} \right) + \frac{\mu - \hat{\alpha}_j}{\mu} \right] \frac{\partial b_j^c}{\partial p_{fj}} + (t_{fj} \frac{\partial^2 b_j^c}{\partial p_{fj}^2} + \hat{t}_{dj} \frac{\partial^2 b_j^c}{\partial p_{dj} \partial p_{fj}}) = 0 \tag{A20}$$

A.2.8 Endogenous tax on domestic varieties

$$\frac{\partial \mathcal{L}}{\partial t_{dj}} = \sum_{h=1}^{k} -c_j^h \beta^h \frac{\partial b_j^c}{\partial p_{dj}} + \mu \left[\sum_{i=1}^{n} \hat{t}_i \frac{\partial c_i}{\partial p_j^c} \frac{\partial b_j^c}{\partial p_{dj}} + (t_{fj} \frac{\partial^2 b_i^c}{\partial p_{fj} \partial p_{dj}} + \hat{t}_{dj} \frac{\partial^2 b_j^c}{\partial p_{dj}^2}) c_j + \frac{\partial b_j^c}{\partial p_{dj}} c_j + \hat{t}_l^c \frac{\partial L}{\partial p_j^c} \frac{\partial b_j^c}{\partial p_{dj}} \right]$$
$$= 0 \tag{A21}$$

Using the same procedure as for an import tariff yields

$$\frac{\partial \mathcal{L}}{\partial t_{dj}} : \left[\frac{1}{c_j} \left(\sum_{i=1}^{n} \hat{t}_i s_{ij} + \hat{t}_l^c s_{lj} \right) + \frac{\mu - \hat{\alpha}_j}{\mu} \right] \frac{\partial b_j^c}{\partial p_{dj}} + (t_{fj} \frac{\partial^2 b_j^c}{\partial p_{fj} \partial p_{dj}} + \hat{t}_{dj} \frac{\partial^2 b_j^c}{\partial p_{dj}^2}) = 0 \tag{A22}$$

B Proofs of the propositions

The proofs of the propositions follow fairly directly from the first order conditions of the maximization. The idea is to obtain an expression for μ/λ that can be substituted in the general formula for MCF, $\frac{\mu}{\lambda}\frac{\nu_j}{p_j}$.

B.1 Proof of proposition 1

Use eq. (A8), which by a rearrangement of variables is written as

$$\lambda = \mu(1 - \hat{t}b) \tag{B1}$$
$$\hat{t}b = \sum_{j=1}^{n} \hat{t}_j^c \frac{\partial \bar{c}_j}{\partial r} + \hat{t}_l^c \frac{\partial \bar{l}}{\partial r}$$

An expression for μ/λ follows by a rearrangement of variables.

B.2 Proof of proposition 2

Rewriting eq. (A9) given assumption 1 yields

$$\lambda = \mu\left[1 - \hat{t}_l^c \frac{\partial L}{\partial q_l^c}/L - \sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j}{\partial q_l^c}\right] \tag{B2}$$

Noting the definition of ε_{ll} as $\frac{\partial L}{\partial q_l^c}\frac{q_l^c}{L}$, an expression for μ/λ follows by a rearrangement of variables.

B.3 Proof of proposition 3

Rewriting eq. (A10) given assumption 1 yields

$$\lambda = \mu\left[1 - \sum_{h=1}^{k} \frac{l^h}{L}(\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j^h}{\partial r^h} + \hat{t}_l^c \frac{\partial l^h}{\partial r^h})) - \frac{1}{L}(\sum_{j=1}^{n} \hat{t}_j^c s_{jl} + \hat{t}_l^c s_{ll})\right] \tag{B3}$$

Using assumption 2, I obtain

$$\lambda = \mu(1 - \hat{t}b - m_l) \tag{B4}$$

$$\begin{aligned} \hat{t}b &= \sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j}{\partial r} + \hat{t}_l^c \frac{\partial l}{\partial r} \\ m_l &= \frac{1}{L}\left(\sum_{j=1}^{n} \hat{t}_j^c s_{jl} + \hat{t}_l^c s_{ll}\right) \end{aligned}$$

Note that the average Engel derivative is equal to the identical Engel derivative under assumption 2.

B.4 Proof of proposition 4

Assumptions 1, 2 and 3 imply that (proposition 5 below)

$$MCF = \frac{1}{1 - \hat{t}b - m_l}\frac{\nu_j}{p_j} \tag{B5}$$

$$
\begin{aligned}
m_l &= \frac{1}{L}\left(\sum_{j=1}^{n} \hat{t}_j^c s_{jl} + \hat{t}_l^c s_{ll}\right) \\
s_{jl} &= \sum_{h=1}^{k} s_{jl}^h = -\sum_{h=1}^{k} \eta^h \frac{\partial c_j}{\partial r}\frac{\partial l}{\partial r} \\
s_{ll} &= \sum_{h=1}^{k} s_{ll}^h = -\sum_{h=1}^{k} \frac{\eta^h}{q_l^c}\frac{\partial l}{\partial r}(1 + q_l^c \frac{\partial l}{\partial r})
\end{aligned}
$$

Wage income taxation therefore yields a higher MCF than that of poll taxes if m_l is positive.

Inserting for s_{jl} and s_{ll} in m_l yields

$$
m_l = -\frac{\frac{\partial l}{\partial r}}{L}\sum_{h=1}^{k} \eta^h(\hat{t}b + \tilde{t}_j^c) \tag{B6}
$$

Under assumption 4, m_l is positive ($\frac{\partial l}{\partial r}$ is negative because of GPF, and η^h is a positive constant).

B.5 Proof of proposition 5

Under assumption 1 and 2 the MCF of a wage income tax is (proposition 3),

$$
MCF = \frac{1}{1 - \hat{t}b - m_l}\frac{\nu_j}{p_j} \tag{B7}
$$

$$
m_l = \frac{1}{L}\left(\sum_{j=1}^{n} \hat{t}_j^c s_{jl} + \hat{t}_l^c s_{ll}\right)
$$

The task is therefore to show that the substitution terms s_{jl} and s_{ll} in m_l are functions of income derivatives.

Since top level preferences are additive (assumption 3), it follows from equation 3.4, p. 138 in Deaton and Muellbauer (1980) that

$$
s_{ll} = \sum_{h=1}^{k} s_{ll}^h = -\sum_{h=1}^{k} \frac{\eta^h}{q_l^c}\frac{\partial l}{\partial r}(1 + q_l^c \frac{\partial l}{\partial r}) \tag{B8}
$$

It follows from assumption 2 that (compensated) demands for commodities have the form

$$
c_j^h = \frac{\partial A^h}{\partial p_j^c} + \frac{\partial P}{\partial p_j^c} c^h \tag{B9}
$$

where A^h and P are homogenous of degree one price indices of $p_1^c \ldots p_n^c$, and c^h is the quantity index of consumer goods.

Given this specification, the effect of a change in q_l^c on c_j^h must come through c^h. In particular,

$$s_{jl}^h = \frac{\partial P}{\partial p_j^c} s_{cl}^h \tag{B10}$$

where s_{cl}^h is the Slutsky derivative between labour and total consumption. As top level preferences are additive, it follows from eq. 3.3, p. 138 in Deaton and Muellbauer (1980) that

$$s_{cl}^h = -\eta^h \frac{\partial c}{\partial r} \frac{\partial l}{\partial r} \tag{B11}$$

It follows that

$$s_{jl} = \sum_{h=1}^{k} s_{jl}^h = -\sum_{h=1}^{k} \eta^h \frac{\partial c_j}{\partial r} \frac{\partial l}{\partial r} \tag{B12}$$

$$\frac{\partial c_j}{\partial r} = \frac{\partial P}{\partial p_j^c} \frac{\partial c}{\partial r}$$

The latter is a property of GDF demands. Thus s_{jl} and s_{ll} are functions of income derivatives.

B.6 Proof of proposition 6

Under assumptions 1 and 2 the MCF is (analogous to proposition 3)

$$MCF = \frac{1}{1 - \hat{t}b + m_j} \frac{\nu_k}{p_k} \tag{B13}$$

$$m_j = \frac{1}{c_j} \left(\sum_{i=1}^{n} \hat{t}_i^c s_{ij} + \hat{t}_l^c s_{lj} \right)$$

The task is to show that m_j takes the form indicated in the proposition.

Under assumptions 2, 3, 5 and 6, compensated demands are written in the following special versions of the GPF forms:

$$
\begin{aligned}
c_i^h &= a_i^h + \frac{\partial P}{\partial p_i^c} c^h && \text{(B14)} \\
c^h &= a_c^h + \frac{\partial \Psi}{\partial P} V^h && \text{(B15)} \\
l^h &= a_l^h + \frac{\partial \Psi}{\partial q_l^c} V^h && \text{(B16)}
\end{aligned}
$$

where the low level price index P is one of the two functions $\Pi_j p_j^{c\ \beta_j}$ or $[\sum_j b_j^{\frac{1}{1-\alpha}} p_j^{c\ \frac{\alpha}{\alpha-1}}]^{\frac{\alpha-1}{\alpha}}$, and the same for the high level price index Ψ. We recall c^h to be the quantity index of consumption goods, and V^h to be total utility.

The slutsky derivatives corresponding to these equations are

$$
\begin{aligned}
s_{ij}^h &= \frac{\partial^2 P}{\partial p_i^c \partial p_j^c} c^h + \frac{\partial P}{\partial p_i^c} \frac{\partial^2 \Psi}{\partial P^2} \frac{\partial P}{\partial p_j^c} V^h && \text{(B17)} \\
s_{jj}^h &= \frac{\partial^2 P}{\partial p_j^{c\,2}} c^h + \frac{\partial P}{\partial p_j^c} \frac{\partial^2 \Psi}{\partial P^2} \frac{\partial P}{\partial p_j^c} V^h && \text{(B18)} \\
s_{lj}^h &= \frac{\partial^2 \Psi}{\partial q_l^c \partial P} \frac{\partial P}{\partial p_j^c} V^h && \text{(B19)}
\end{aligned}
$$

Calculating these expressions for the two classes of admissible functional forms for the price index yields

$$
\begin{aligned}
\frac{\partial^2 P}{\partial p_i^c \partial p_j^c} c^h &= P \frac{c^h}{1-\alpha} \frac{\partial c_i}{\partial y} \frac{\partial c_j}{\partial y} && \text{(B20)} \\
\frac{\partial^2 P}{\partial p_j^{c\,2}} c^h &= -P \frac{c^h}{1-\alpha} \frac{1}{p_j^c} \frac{\partial c_j}{\partial y} (1 - p_j^c \frac{\partial c_j}{\partial y}) && \text{(B21)} \\
\frac{\partial^2 \Psi}{\partial P^2} V^h &= -\Psi \frac{V^h}{1-\beta} \frac{1}{P} \frac{\partial c}{\partial r} (1 - P \frac{\partial c}{\partial r}) && \text{(B22)} \\
\frac{\partial^2 \Psi}{\partial q_l^c \partial P} V^h &= \Psi \frac{V^h}{1-\beta} \frac{\partial c}{\partial r} \frac{\partial l}{\partial r} && \text{(B23)}
\end{aligned}
$$

where we recall y^h be consumer expenditure and $\frac{\partial c_j}{\partial y}$ the common engel derivative. Likewise $\frac{\partial c}{\partial r}$ is the common engel derivative for the consumption quantity index and $\frac{\partial l}{\partial r}$ is the common labour supply income derivative. The value of α and its top level equivalent β will vary according to which utility function is assumed. In the LES system for instance, α is zero. In the CES system, $1/(1-\alpha)$ is the (absolute value of) the elasticity of substitution.

We now possess the specific forms of the slutsky terms corresponding to the assumed preferences. Inserting in the general expression for m_j and some algebra yields desired expression for m_j.

B.7 Proof of proposition 7

Under assumptions 1, 2, 3, 5 and 6 the MCF of a commodity tax is (proposition 6)

$$MCF = \frac{1}{1-\hat{t}b+m_j \frac{\nu_k}{p_k}} \tag{B24}$$

$$m_j = \frac{(1-\frac{a_j}{c_j})(1-\frac{a_c}{c})}{1-\beta}\left(\hat{t}b-\hat{\tau}\right)+\frac{1-\frac{a_j}{c_j}}{1-\alpha}\left(\hat{\tau}-\tilde{t}_j^c\right) \tag{B25}$$

The first part of the proposition follows from comparing the MCFs of two different commodity taxes t_i^c and t_j^c (and for the same kind of project), and noting that the only parameters *not* present in both expressions are those mentioned in the proposition.

The second part is obvious by inspecting the MCFs of two commodity taxes t_j^c and t_i^c.

The third part follows from rewriting the MCF of a wage income tax (under assumptions 1, 2, 3 and 6). It can be written like eq. (B5) with

$$m_l = -\frac{1-\frac{a_l}{L}}{1-\beta}(\hat{t}b+\tilde{t}_l^c) \tag{B26}$$

The fourth part follows by noting from the assumption that a_l is zero, the MCF of a wage income tax can be written as eq. (B5) with

$$m_l = -\frac{1}{1-\beta}(\hat{t}b+\tilde{t}_l^c) \tag{B27}$$

Under the assumptions of the proof, the expression m_j for commodity tax financing reduces to

$$m_j = \frac{1}{1-\beta}(\hat{t}b-\tilde{t}_j^c) \tag{B28}$$

The MCF of commodity tax financing is lower than that of wage income tax financing iff $m_j > -m_l$. This implies the condition.

B.8 Proof of proposition 8

Rewrite eq. (A18) as follows

$$\nu_j = p_{aj}(1 - \frac{1}{e_j}) \tag{B29}$$

and insert in eq. (35).

B.9 Proof of proposition 9

Rewriting eq. (A20) under assumption 1 and 2 yields

$$\frac{\partial \mathcal{L}}{\partial t_{fj}} \quad : \quad \lambda = \mu(1 - \hat{t}b + m_j + n_{fj}) \tag{B30}$$

$$\begin{aligned} m_j &= \frac{1}{c_j}\left(\sum_{i=1}^{n} \hat{t}_i^c s_{ij} + \hat{t}_l^c s_{lj})\right) \\ n_{fj} &= \frac{1}{\frac{\partial b_j^c}{\partial p_{fj}}}\left(\frac{\partial^2 b_j^c}{\partial p_{fj}^2} + \hat{t}_{dj}\frac{\partial^2 b_j^c}{\partial p_{dj}\partial p_{fj}}\right) \end{aligned}$$

Assumption 7 implies that preferences for varieties belongs to the class of additive GPF functions. The substitution terms of these functions have the form described in the proof of proposition 6, that is

$$n_{fj} = \frac{b_j^c \frac{\partial c_{fj}}{\partial s}}{\frac{\partial b_j^c}{\partial p_{fj}}}\sigma_j\left(t_{fj}\frac{\partial c_{fj}}{\partial s} + t_{dj}\frac{\partial c_{dj}}{\partial s} - \tilde{t}_{fj}\right) \tag{B31}$$

where s^h is the income allotted to purchases of composite commodity j, and σ_j is the absolute value of the elasticity of substitution. The Engel functions are identical in CES.

Recalling that $b_j^c \frac{\partial c_{fi}}{\partial s} = \frac{\partial b_j^c}{\partial p_{fj}}$ in GPF preferences, the proposition is obtained.

B.10 Proof of proposition 10

Rewriting eq. (A2) under assumptions 1 and 8 and inserting for $y_j + \theta_j$ from eq. (A14) yields

$$\frac{\partial \mathcal{L}}{\partial b_{dj}} \quad : \quad \lambda = \mu(1 - \hat{t}b + m_j + n_{dj} + o_j) \tag{B32}$$

$$\begin{aligned}
m_j &= \frac{1}{c_j}\left(\sum_{i=1}^{n} \hat{t}_i^c s_{ij} + \hat{t}_l^c s_{lj}\right) \\
n_{dj} &= \frac{1}{\frac{\partial b_j^c}{\partial p_{dj}}}\left(t_{fj}\frac{\partial^2 b_j^c}{\partial p_{fj}\partial p_{dj}} + \hat{t}_{dj}\frac{\partial^2 b_j^c}{\partial p_{dj}^2}\right) \\
o_j &= \frac{1}{\frac{\partial b_j^c}{\partial p_{dj}}}\frac{1}{c_j}\frac{1}{zl_j}\left[\hat{t}_l^j\frac{\partial zl_j}{\partial q_l^j}y_j + \hat{t}_k^j\frac{\partial zk_j}{\partial q_l^j}y_j\right]
\end{aligned}$$

From proposition 6 it follows that under assumptions 1, 2, 3, 5 and 6:

$$m_j = \frac{(1 - \frac{a_j}{c_j})(1 - \frac{a_c}{c})}{1 - \beta}\left(\hat{t}b - \hat{\tau}\right) + \frac{1 - \frac{a_j}{c_j}}{1 - \alpha}\left(\hat{\tau} - \tilde{t}_j^c\right) \tag{B33}$$

From proposition 8 it follows that under assumptions 1, 2 and 7

$$n_{dj} = \sigma_j(t_{fj}\frac{\partial c_{fj}}{\partial s} + t_{dj}\frac{\partial c_{dj}}{\partial s} - \tilde{t}_{dj}) \tag{B34}$$

To obtain an expression for o_j, observe that the substitution terms are the same as for the function b_j^c in the proof of proposition 9 above (only the names are different). One therefore immediately obtains (recall that $y_j = c_j$ under assumption 7, and $zl_j = \frac{\partial e^j}{\partial q_l^j}$)

$$o_j = \frac{\sigma_{lk}}{\frac{\partial b_j^c}{\partial p_{dj}}}(\hat{t}_l^j\frac{\partial zl_j}{\partial e^j} + \hat{t}_k^j\frac{\partial zk_j}{\partial e^j} - \tilde{t}_l^j) \tag{B35}$$

B.11 Proof of proposition 11

VAT financing

Assume commodity prices can be written

$$p_j^c = (b_j^c + t_j^c)(1 + t) \tag{B36}$$

where t is the (ideal) VAT rate. VAT financing implies $\frac{\partial \mathcal{L}}{\partial t} = 0$ ie.

$$(1+t)\frac{\partial \mathcal{L}}{\partial t} = \sum_{j=1}^{n}\sum_{h=1}^{k} -c_j^h p_j^c \beta^h + \mu(\sum_{i=1}^{h} \hat{t}_i^c p_j^c \frac{\partial c_i}{\partial p_j^c} + \hat{t}_l^c p_j^c \frac{\partial l}{\partial p_j^c} + p_j c_j) = 0 \tag{B37}$$

Use the budget constraints,

$$\sum_{j=1}^{n} p_j^c c_j^h = q_l^c l^h + r^h \tag{B38}$$

and the homogeneity of demand and supply functions,

$$\sum_{j=1}^{n} p_j^c \frac{\partial c_i^h}{\partial p_j^c} + q_l^c \frac{\partial c_i^h}{\partial q_l^c} + r^h \frac{\partial c_i^h}{\partial r^h} = 0 \tag{B39}$$

$$\sum_{j=1}^{n} p_j^c \frac{\partial l^h}{\partial p_j^c} + q_l^c \frac{\partial l^h}{\partial q_l^c} + r^h \frac{\partial l^h}{\partial r^h} = 0 \tag{B40}$$

to get

$$\begin{aligned} & q_l^c(\sum_{h=1}^{k} l^h \beta^h + \mu(\sum_{i=1}^{n} \hat{t}_i^c \frac{\partial c_i}{\partial q_l^c} + \hat{t}_l^c \frac{\partial L}{\partial q_l^c} - L) \\ & + \sum_{h=1}^{k} r^h(\beta^h + \mu(\sum_{i=1}^{n} \hat{t}_i^c \frac{\partial c_i^h}{\partial r^h} + \hat{t}_l^c \frac{\partial l^h}{\partial r^h} - 1)) = 0 \end{aligned} \tag{B41}$$

Under assumption 1 and 2 this yields

$$MCF = \frac{1}{1 - \hat{t}b - am_l} \frac{\nu_j}{p_j} \tag{B42}$$

$$a = \frac{q_l^c L}{\sum_{j=1}^{n} p_j^c c_j}$$

In the general case, not assuming 1 and 2, we note that $(1+t)\frac{\partial \mathcal{L}}{\partial t} = q_l^c \frac{\partial \mathcal{L}}{\partial t_l^c} + \sum r^h \frac{\partial \mathcal{L}}{\partial r^h} = 0$, which implicitly determines the MCF.

Industry level tax increase on labour

Define

$$q_l^j = q_l + t_l^j + t_l \tag{B43}$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = \mu \sum_{j=1}^{n} \left[\hat{t}_l^j \frac{\partial z l_j}{\partial q_l^j} y_j + \hat{t}_k^j \frac{\partial z k_j}{\partial q_l^j} y_j + (y_j + \theta_j) z l_j \right] = 0 \tag{B44}$$

Note that $\frac{\partial \mathcal{L}}{\partial t_l} = \sum_{j=1}^{n} \frac{\partial \mathcal{L}}{\partial t_l^j}$. Combining eqs. (B44) and (A4), we find

$$\sum_{h=1}^{k} l^h \beta^h + \mu \left[\sum_{j=1}^{n} \hat{t}_j^c \frac{\partial c_j}{\partial q_l^c} + \hat{t}_l^c \frac{\partial L}{\partial q_l^c} - L \right] = 0 \tag{B45}$$

This is the condition for $\frac{\partial \mathcal{L}}{\partial t_l^c} = 0$. In other words, increasing all producer tax rates on labour input is equal to labour income tax financing of consumers.

Industry level tax increase on capital

Define

$$q_k^j = q_k + t_k^j + t_k \tag{B46}$$

$$\frac{\partial \mathcal{L}}{\partial t_k} = \mu \sum_{j=1}^{n} \left[\hat{t}_l^j \frac{\partial z l_j}{\partial q_k^j} y_j + \hat{t}_k^j \frac{\partial z k_j}{\partial q_k^j} y_j + (y_j + \theta_j) z k_j \right] = 0 \tag{B47}$$

Comparing eqs. (B47) and (A5), we see that

$$\sum_{h=1}^{k} K^h \beta^h + \mu \left[\sum_{j=1}^{n} \hat{t}_j^c \sum_{h=1}^{h} K^h \frac{\partial c_j^h}{\partial r} + \hat{t}_l^c \sum_{h=1}^{k} K^h \frac{\partial l^h}{\partial r} - K \right] = 0 \tag{B48}$$

This is to say that the tax increase is equal to a poll tax in which households are taxed according to their share of capital.

Taxing labour and capital in an industry

Define

$$q_l^j = (q_l + t_l^j)(1 + t^j) \tag{B49}$$

$$q_k^j = (q_k + t_k^j)(1 + t^j) \tag{B50}$$

This yields

$$\begin{aligned} (1 + t^j) \frac{\partial \mathcal{L}}{\partial t^j} \quad : \quad & \hat{t}_l^j q_l^j \frac{\partial z l_j}{\partial q_l^j} y_j + \hat{t}_k^j q_k^j \frac{\partial z k_j}{\partial q_l^j} y_j + (y_j + \theta_j) z l_j q_l^j \\ & + \hat{t}_l^j q_k^j \frac{\partial z l_j}{\partial q_k^j} y_j + \hat{t}_k^j q_k^j \frac{\partial z k_j}{\partial q_k^j} y_j + (y_j + \theta_j) z k_j q_k^j = 0 \end{aligned} \tag{B51}$$

Using Eulers theorem:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t^j} \quad &: \quad t_l^j(\frac{\partial zl_j}{\partial q_l^j}q_l^j + \frac{\partial zl_j}{\partial q_k^j}q_k^j) + t_k^j(\frac{\partial zk_j}{\partial q_l^j}q_l^j + \frac{\partial zk_j}{\partial q_k^j}q_k^j) \\ &+(y_j + \theta_j)b_{dj} = 0 \qquad \text{(B52)} \\ &= (y_j + \theta_j)b_{dj} = 0 \qquad \text{(B53)} \\ &\Rightarrow y_j = -\theta_j\end{aligned}$$

Inserting the result $y_j = -\theta_j$ in equation (A1) for $\frac{\partial \mathcal{L}}{\partial b_{dj}} = 0$, we observe, using eq. (11), that $\frac{\partial \mathcal{L}}{\partial b_{dj}}$ implies $\frac{\partial \mathcal{L}}{\partial t_{dj}} = 0$ if j is a non-exporting industry, and else $\frac{\partial \mathcal{L}}{\partial t_{aj}} + \frac{\partial \mathcal{L}}{\partial t_{dj}} = 0$. The interpretation is that a tax on all inputs is equal to an output tax, which is equal to t_{dj} if j non-exporting, and else equal to a tax on domestic consumption and exports.

Taxing labour and capital in all industries

Define

$$q_l^j = (q_l + t_l^j)(1+t) \qquad q_k^j = (q_k + t_k^j)(1+t) \tag{B54}$$

We find (using Eulers theorem repeatedly):

$$\frac{\partial \mathcal{L}}{\partial t} = \sum_{j=1}^{n}(y_j + \theta_j)b_{dj} = 0 \tag{B55}$$

Adding together $\frac{\partial \mathcal{L}}{\partial b_{dj}}$ over sector and using this result, we find that $\frac{\partial \mathcal{L}}{\partial t} = 0$ implies $\sum_{j=1}^{n} \frac{\partial \mathcal{L}}{\partial t_{dj}} + \sum_{j=m+1}^{n} \frac{\partial \mathcal{L}}{\partial t_{aj}} = 0$. If the economy is closed, the expression boils down to $\sum_j \frac{\partial \mathcal{L}}{\partial t_j^c} = 0$ ie. a simultaneous increase in all consumption taxes. If not, it is equal to a simultaneous increase in all output taxes.

C Shadow prices, shadow taxes and formal taxes

The goal of this appendix is to obtain expressions for shadow taxes in terms of actual taxes. This requires knowledge of the relation between shadow *prices* and actual prices. If j is an exporting industry, we know that (from (A3))

$$b_{dj} - \nu_j = \frac{p_{aj}}{e_j}(1 + h_j \frac{c_j}{a_j}) - t_{aj} = d_j \tag{C1}$$

If c_j is small as compared to a_j and h_j is close to zero (recall that it is zero in non-exporting industries) or at least not too negative, d_j will be positive.

The shadow price is approximately equal to the output price if d_j is approximately zero. This is the case if e_j is large (the industry is close to being perfectly competitive) and/or t_{aj} is sufficiently positive.

If e_j is small (approaches unity), or a_j is small as compared to c_j, the shadow price of exportables is approximately determined by domestic forces only, through h_j.

Now use eq. (A6), which can be written as

$$b_{dj} - \nu_j = (q_l^j - \omega) z l_j + (q_k^j - \rho) z k_j \tag{C2}$$

Given the shadow prices on outputs and factors, ν_j, ω, ρ, taxes t_l^j and t_k^j and the unit input demands zl_j and zk_j, equations (C2) are n equations in the $n+2$ variables b_{dj}, q_l and q_k. We need two more equations to explain market prices in terms of the other variables.

Pick two exporting industries (at random), and indicate them by 1 and 2. Two expressions relating shadow to market factor prices are (using eq. (C2))

$$q_l - \omega = \frac{1}{\frac{1}{k_1} - \frac{1}{k_2}} \left[t_k^2 - t_k^1 + \frac{t_l^2}{k_2} - \frac{t_l^1}{k_1} + \frac{d_1}{zk_1} - \frac{d_2}{zk_2} \right] \tag{C3}$$

$$q_k - \rho = \frac{1}{k_1 - k_2} \left[t_l^2 - t_l^1 + t_k^2 k_2 - t_k^1 k_1 + \frac{d_1}{zl_1} - \frac{d_2}{zl_2} \right] \tag{C4}$$

where k_1 and k_2 are capital intensities.

These expressions depend on the d's and (through the h's) on the prices and quantities of the whole system. A useful benchmark case is found by assuming $t_k^1 = t_k^2 = t_k, t_l^1 = t_l^2 =$

$t_l, \frac{d_1}{zk_1} = \frac{d_2}{zk_2}, \frac{d_1}{zl_1} = \frac{d_2}{zl_2}$ (eg. if e_j is very large or if there are export taxes or output taxes of the required size). In that case,

$$\omega = q_l + t_l = q_l^1 = q_l^2 \tag{C5}$$

$$\rho = q_k + t_k = q_k^1 = q_k^2 \tag{C6}$$

ie. the shadow factor prices are equal to the factor prices paid by the two industries.

This finding allows us to derive the shadow prices of the non-exporting industries as well. From eq. (C2), $b_{dj} - \nu_j = (t_l^j - t_l)zl_j + (t_k^j - t_k)zk_j$, ie. the deviation between market and shadow prices depends on whether factor taxes are higher in non-exporting industries than in exporting.

A tentative conclusion is that in non-exporting industries, shadow prices and market prices of outputs generally are fairly close unless the non-exporting industries are taxed markedly different or the economy is far from the small open type. In exporting industries, shadow prices generally tend to be lower than market prices.

In terms of tax rates a conclusion is that in non-exporting industries, shadow commodity taxes tend to be fairly close to formal commodity taxes. In exporting industries, shadow commodity taxes are generally lower than the formal taxes. The same reasoning applies to taxes on domestic varieties. Any import tariffs will increase the shadow commodity taxes.

Using eq. (C6), the shadow consumer wage income tax is

$$\hat{t}_l^c = t_l^c + t_l \tag{C7}$$

ie. the shadow consumer wage income tax is equal to the formal tax plus the labour input tax of the representative exporting industries.

The shadow input taxes tend to be smaller than their market equivalents. Using eq. (C6),

$$\hat{t}_l^j = t_l^j - t_l \tag{C8}$$

$$\hat{t}_k^j = t_k^j - t_k \tag{C9}$$

In the two representative exporting sectors, the shadow input taxes are zero.

The shadow tax on exports is d_j, which I have argued is positive.

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