

Predicting the exchange rate path

The importance of using up-to-date observations in the forecasts

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Abstract:

Central banks, private banks, statistical agencies and international organizations such as the IMF and OECD typically use information about the exchange rate some weeks before the publication date as the basis for their exchange rate forecasts. In this paper, we test if forecasts can be made more accurate by utilizing information about exchange rate movements closer to the publication date. To this end, we apply new tests for equal predictability and encompassing for path forecasts. We find that the date when the exchange rate forecast is based on is crucial and this finding should be taken into account when evaluating exchange rate forecasts. Using forecasts made by Statistics Norway over the period 2001 - 2016 we find that the random walk, when based on the exchange rate three days ahead of the publication date, encompassed the predicted path by Statistics Norway. However, when using the exchange rate two weeks before the publication deadline, which is the information used by Statistics Norway in practice when making their forecasts, the random walk path and the predicted exchange rate path by Statistics Norway have equal predictability.

Keywords: Macroeconomic forecasts; Econometric models; Forecast performance; Forecast evaluation; Forecast comparison.

JEL classification: C53, F31

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Sammendrag

Teorien om effisiente markeder tilsier at dagens valutakurs reflekterer all tilgjengelig informasjon. Det betyr ikke nødvendigvis at valutakursendringer ikke kan predikeres. Ved fravær av risikopremie må avkastningen i to land, målt i felles valuta, være lik. Hvis landene har forskjellig rente må denne avkastningsforskjellen motsvares av en tilsvarende valutakursendring. Dette kalles teorien om udekket renteparitet.

Teorien om udekket renteparitet innebærer altså at hvis Norge har høyere 3-månedersrente enn euroområdet, vil vi forvente at krona vil svekke seg de nærmeste tre månedene. Slike sammenhenger er forkastet gjentatte ganger i empiriske undersøkelser. Ofte finner man at et lands valutakurs tvert imot styrkes hvis landet har en høyere rente enn andre land.

En klassisk forskningsartikkel, Meese and Rogoff (1983), fant at kjente valutakursmodeller har problemer med å predikere valutakursendringer og at en prognose basert på en uendret kurs, ofte referert til som «tilfeldig gang» («random walk»), stort sett er bedre enn disse modellene. Dette er grunnen til at flere institusjoner som lager prognoser, som for eksempel Bank of Canada og den europeiske sentralbanken, antar uendrede valutakurser i sine framskrivninger. Bank of England utarbeider sine prognoser for det britiske pundet ut fra en mellomting mellom en uendret valutakurs og den valutakursbanen som følger av udekket renteparitet. IMF, som lager prognoser for land med ulike inflasjonsnivåer, antar uendret realvalutakurs i sine prognoser.

Det finnes en stor faglitteratur internasjonalt om valutakursprognosenes evne til å gjøre det bedre enn uendret kurs. Resultatene spriker en god del. Noen finner at modeller/prognosemakere gjør det bedre enn uendret kurs, mens andre finner at så ikke er tilfellet. For eksempel finner Ince and Molodtsova (2017) støtte for at profesjonelle prognosemakere slår «tilfeldig gang» og Rossi (2013) viser til at prediksjoner basert på Taylorregler og et lands nettofordringer overfor utlandet kan være bedre enn uendret kurs. Norges Bank og Sveriges Riksbank lager valutakursprognoser som avviker fra uendret kurs framover. SSB har også gjort det fram til desember 2018.

I Hungnes (2018, 2020a) studeres hvordan forskjellige prognosebaner kan sammenlignes. Dette benytter vi her til å sammenligne SSBs prognoser for den importveide kronkursen (I44) med en kronkursbane basert på uendret kurs. Når vi baserer den uendrede kursen på valutakursen observert i begynnelsen av perioden vi normalt utarbeider prognosene på, om lag to uker før redaksjonen setter sluttstrek, finner vi at denne og SSBs prognoser har vært om lag like gode. Sammenligner vi isteden SSBs prognoser med uendret valutakurs dagen før redaksjonsarbeidet for prognosene avsluttes, finner vi at SSBs prognose ikke har bidratt til å forbedre «tilfeldig gang»-prognosen for kronkursen. Analysen viser at det er viktig å ha så oppdaterte observasjoner av kronkursen som mulig når prognosebanen for kronkursen utarbeides, og at en antagelse om uendret kronkurs framover er vanskelig å slå.

1 Introduction

According to the efficient market hypothesis, the current exchange rate reflects all available information. The hypothesis does not necessarily imply that exchange rate changes cannot be predicted. In the absence of risk premiums, the return in two countries, measured in a common currency, must be equal: if the countries have different interest rates, this difference in return must be accounted for by an equivalent exchange rate change. This relationship between the interest rate difference and the expected exchange rate change is called the theory of uncovered interest rate parity.

According to the theory of uncovered interest rate parity, then, if Norway has a higher 3-month interest rate than the euro area, we can expect the krone to depreciate in the next three months. Such relationships have been repeatedly found to be rejected in empirical studies. On the contrary, a country's currency is often found to appreciate if the country has a higher interest rate than other countries, see, for example, Rossi (2013) and Engel et al. (2019).

A classic research article, Meese & Rogoff (1983), found that known exchange rate models have difficulty in forecasting exchange rate changes and that a forecast based on an unchanged rate, often referred to as "random walk", performs better than the alternative exchange rate models. These results are why several forecasters, like the Bank of Canada and the European Central Bank, assume unchanged exchange rates in their forecasts (Bank of Canada 2020, European Central Bank 2020). The Bank of England bases its forecasts for sterling on something in between an unchanged exchange rate and the exchange rate path that follows from uncovered interest rate parity (Bank of England 2020). The IMF, which makes forecasts for countries with various inflation levels, assumes an unchanged real exchange rate in its forecasts (IMF 2020).

However, even if these forecasters use unchanged (nominal or real) exchange rates ahead, the exchange rate used in the prediction is not the latest observed exchange rate. For example, in its forecast from January 2020, IMF uses the average real exchange rate in a period broadly covering the end of October and beginning of November 2019 as their forecast for the real exchange rate (IMF 2020). Moreover, the European Central Bank sets, in their forecast published March 12, 2020, the exchange rate equal to the average in the first half of February (European Central Bank 2020).

Some papers argue that professional forecasters provide better forecasts than what a random walk model does. Ince & Molodtsova (2017) compare random walk generated exchange rate forecasts for both developed and developing countries with forecasts from Consensus Economics and FX4Casts. Consensus Economics reports forecasts of banks, large non-financial enterprises, consulting firms, and university economists (and Statistics Norway does also report forecasts to Consensus Economics). The forecasts by FX4Cast are from large financial institutions. For forecasts of the exchange rate 12 months ahead, the forecasts from Consensus Economics are significantly better than the random walk forecasts for 6 out of 10 developed countries, whereas the forecasts from FX4Cast are significantly better than random walk for 4 out of 10 countries. For developing countries, the forecasts from Consensus Economics outperform the random walk forecasts significantly for 5 out of 23 countries; the FX4Cast outperforms random walk for 6 countries.¹

The central banks in Norway and Sweden publish a path forecast for the exchange rate that is not based on the random walk (Norges Bank 2020, Sveriges Riksbank 2020). Statistics Norway has, until the end of 2018, made forecasts of the exchange rate that deviate from the random walk (Statistics Norway

¹Ince & Molodtsova (2017) do not report test results for when random walk based forecasts beat the forecasts from professional forecasters. However, based on the reported test statistics, random walk outperforms the forecast from Consensus Economics significantly for 1 developed country and 11 developing countries, and random walk outperforms the forecasts from FX4Cast for 2 developed countries and 9 developing countries.

2018). In this paper, we investigate if the forecasts for the exchange rate made by Statistics Norway could be improved by assuming a random walk instead.

The mean squared forecast error (MSFE) and related metrics are usually used to measure the accuracy of forecasts or to compare forecasts. However, this metric is not invariant to how the forecasts are formulated (if they are formulated in levels or differences) for forecasts exceeding the one-period horizon. Therefore, [Clements & Hendry \(1993\)](#), [Hendry & Martinez \(2017\)](#), and [Hungnes \(2018, 2020a\)](#) suggest considering the full path of the forecasted variable for all forecasting horizons jointly. [Hungnes \(2018\)](#) suggests an encompassing test for path forecasts, and [Hungnes \(2020a\)](#) proposes a test of equal predictability of path forecasts. These tests are extensions of the [Diebold & Mariano \(1995\)](#) test.

The equal predictability test in [Diebold & Mariano \(1995\)](#) requires that the difference of the loss function based on the squared forecast errors between two forecasters (or models) is covariance stationary. [West \(1996\)](#) and [Clark & McCracken \(2001\)](#) consider the case where the forecasts are based on econometric models with estimated parameters, and forecast tests are conducted to compare the forecasting models (see also [Clark & McCracken 2013](#), Section 3.1). [West \(1996\)](#) shows that the distribution of the test statistic is still asymptotically normal for non-nested forecasting models. For nested models, [Clark & McCracken \(2001\)](#) show that the distribution may be non-standard. However, [Clark & West \(2006, 2007\)](#) show that under certain conditions, the distribution is either asymptotically normal or approximately normal. [Giacomini & White \(2006\)](#) consider comparisons of forecasts of models involving estimated parameters. The parameters are estimated with a rolling sample, so the estimates do not converge to their true values as more observations become available. Under this estimation scheme, the test statistic is asymptotically normally distributed. Therefore, the approach by [Giacomini & White \(2006\)](#) is more in line with the [Diebold & Mariano \(1995\)](#) test than the tests considered by [West \(1996\)](#) and [Clark & McCracken \(2001\)](#), among others. The rolling sample assumption in [Giacomini & White \(2006\)](#) is crucial for obtaining an asymptotic normal distribution of the tests (see also [Clark & McCracken 2015](#), [McCracken 2019](#)). When applying a forecasting model estimated on a recursive sample (i.e., an expanding sample with a fixed starting date), the estimation bias of both a correctly specified model and an over-fitted model will vanish, and the difference between the squared forecast errors will decrease with time, violating the assumption behind the [Diebold & Mariano \(1995\)](#) test. However, in a frequently changing economy with structural breaks in the data generating process, a rolling scheme will be more effective for detecting the data generating process at the time the forecasts are made, a point also noted by [Giacomini & White \(2006\)](#). [Hungnes \(2018, 2020a\)](#) shows, using the same assumptions as [Giacomini & White \(2006\)](#), that both the system forecast encompassing test statistic and the system forecast equal predictability test statistic are asymptotically normally distributed. Furthermore, [Hungnes \(2018, 2020a\)](#) considers some small sample corrections for the tests.

Each quarter, following the publication of new quarterly national account data, Statistics Norway publishes forecasts for the Norwegian economy 3-4 years ahead in annual terms. Among the variables Statistics Norway publishes forecasts for, is the krone exchange rate measured against a basket of currencies of Norway's most important trading partners in terms of import value (also known as the Norwegian import-weighted krone). The forecast of this exchange rate is partly based on judgemental forecasts and partly on an econometric model for the exchange rate. An early version of the exchange rate model used by Statistics Norway is documented in [Bjørnland & Hungnes \(2006\)](#), who use data from the years 1983–2002 to estimate the model. The current version of the exchange rate model, yet to be documented, uses data from 2001, and the estimation period only covers the period when Norway has had an inflation target. Therefore, the exchange rate model used for the forecasts by Statistics Norway is not a fixed specification that is recursively estimated, which would likely violate the assumption

that the difference between these forecast errors and the forecast errors of an alternative model is covariance stationary. However, since the updating of the exchange rate model is more in line with a rolling estimation scheme, as in [Giacomini & White \(2006\)](#), we believe that the assumption behind this test holds.

This paper uses the equal predictability test of path forecasts in [Hungnes \(2020a\)](#) to compare the exchange rate forecasts by Statistics Norway with the exchange rate path that follows from a random walk. For the random walk, the last observed exchange rate the day before the deadline of the publication of the forecasts is used. Although the equal predictability hypothesis is not rejected (using a two-sided test with 5 percent significance level), the results clearly indicate that random walk forecasts outperform the exchange rate forecast from Statistics Norway. This result can be either due to the superiority of the random walk or that the random walk forecasts are based on more recent observations of the exchange rate than the forecasts made by Statistics Norway. To investigate the latter, we also compare the exchange rate forecasts by Statistics Norway with a random walk based on the exchange rate 15 days before the deadline of the publication of the forecasts (as this is the day Statistics Norway usually starts its work with the forecasts). When comparing the exchange rate forecasts with the random walk forecasts based on the exchange rate 15 days before the deadline of the publication, we find that the two forecasts are equally good.

We also use the path encompassing test in [Hungnes \(2018\)](#) to test if the forecasts of the random walk model based on the exchange rate at one point in time encompass the random walk forecasts based on the exchange rate at an earlier date. The encompassing tests confirm the results from the equal predictability tests. When comparing the exchange rate forecast from Statistics Norway with the random walk based on the exchange rate the day before the deadline of the publication, we cannot reject that the latter encompasses the former.

Norway has experienced large exchange rate fluctuations in the period we are considering, which is the period after Norway started its inflation targeting at the beginning of 2001. In this period, 2001-2018, the cost of one euro has been as low as 7.22 Norwegian kroner and as high as 9.97 (when considering the official daily rates published by Norges Bank). The cost of one dollar varied between 4.96 and 9.46 Norwegian kroner in the same period. In the analysis, we consider the Norwegian import-weighted krone, which also has fluctuated much in this period. With these large fluctuations, we might get more precise results from our tests of equal predictability and encompassing for system forecasts than one could get from similar studies for other countries with less exchange rate variation.

Section 2 presents the test of equal predictability for path forecasts derived in [Hungnes \(2020a\)](#) and the encompassing test for path forecasts derived in [Hungnes \(2018\)](#). Section 3 defines the import-weighted krone exchange rate. In this section, we also apply the equal predictability test and the encompassing test to evaluate the exchange rate forecasts by Statistics Norway. Section 4 concludes.

2 Theory

Let $s_{t+h|t}^i = \log S_{t+h|t}^i$ be the forecast of the exchange rate (measured on the logarithmic scale) for period $t+h$, made in period t by forecaster (or forecasting method) i . We assume that the exchange rate for period t is not known in period t ; thus, exchange rate forecasts for the current period — also referred to as nowcasting — can be made and is denoted $s_{t|t}^i$. The forecast error for the exchange rate in period

$t + h$ for the forecast made in period t by forecaster i is given by

$$e_{t+h|t}^i \equiv s_{t+h} - s_{t+h|t}^i \quad (1)$$

where s_{t+h} is the actual exchange rate (measured on the logarithmic scale) in period $t + h$. As the exchange rate is measured on the logarithmic scale, the forecast error is approximately a measure of the percentage error (when disregarding the scaling factor).

The Mean Squared Forecast Error (MSFE) of T forecasts with forecast horizon h for forecaster i is given by

$$T^{-1} \sum_{t=1}^T \left(e_{t+h|t}^i \right)^2. \quad (2)$$

The MSFE (or the square root of it) is a widely used metric for the accuracy of forecast also for $h > 0$; see, e.g., Bjørnland et al. (2017), El-Shagi et al. (2016), Jungmittag (2016), and Kock & Teräsvirta (2016) for some recent applications. However, the MSFE for forecast accuracy when $h > 0$ depends on how the forecasts are measured, see Clements & Hendry (1993, 1998) and Hungnes (2018, 2020a). For example, the MSFE will differ depending on whether the forecasts are measured in levels or first-differences. Only in the case of $h = 0$, i.e., the forecasts for the current period, the accuracy measure based on the observed (univariate) MSFE is invariant to linear transformations of the forecasts. Furthermore, this non-invariance also implies problem of ranking of forecasts, as one forecast can be considered the best forecast when measured in levels whereas another forecast can be considered best when measured in first-differences. Ericsson (2008) illustrates this by considering two different models for forecasting the oil price, where the multi-step-ahead forecasts based on one of the models are considered better when the forecasts are examined in terms of levels, but where the forecasts of another model is considered better when the forecasts are evaluated in terms of growth rates.

Therefore, when evaluating forecasts beyond nowcasts, we must consider the full path of forecasts. We define the vector of exchange rate forecasts by forecaster i for the current and the next two years, made in year t by $\mathbf{s}_{t,H|t}^i = \left(s_{t|t}^i, s_{t+1|t}^i, s_{t+2|t}^i \right)'$, where the forecast horizon is given by $H = 2$ measured in years. The actual exchange rate path in these years is given by $\mathbf{s}_{t,2} = (s_t, s_{t+1}, s_{t+2})'$, thus the vector of forecast errors of forecaster i is given by $\mathbf{e}_{t,2|t}^i = \left(e_{t|t}^i, e_{t+1|t}^i, e_{t+2|t}^i \right)'$, where the elements are defined in (1).

Engle (1993) suggest the following loss function to evaluate a vector of forecast errors for a forecast made in year t by forecaster i :

$$\mathbf{e}_{t,2|t}^{i'} \mathbf{H} \mathbf{e}_{t,2|t}^i \quad (3)$$

where \mathbf{H} (which is an $(H + 1) \times (H + 1)$ matrix) represents the parameters in the loss function. Pesaran & Skouras (2002) present the corresponding loss-difference function between the forecast errors of forecaster A and forecaster B as

$$d_t = \mathbf{e}_{t,H|t}^A{}' \mathbf{H} \mathbf{e}_{t,H|t}^A - \mathbf{e}_{t,H|t}^B{}' \mathbf{H} \mathbf{e}_{t,H|t}^B \quad (4)$$

Capistrán (2006) suggests using $\mathbf{H} = I_{H+1}$, whereas Quaadvlieg (2019) suggests (in his weighted average loss test) using a diagonal matrix with weights along its main diagonal. However, none of these suggestions leads to a metric that is invariant to linear transformations. The lack of invariance can be seen by transforming the forecast errors \mathbf{e}^i to forecast errors in the growth rates of the forecasted

exchange rates, by

$$\mathbf{e}_{t,H|t}^{i*} = \mathbf{M}\mathbf{e}_{t,H|t}^i$$

where

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

when $H = 2$. By substituting this into the loss-difference function for the transformed forecast errors, we have

$$\begin{aligned} d_t &= \mathbf{e}_{t,H|t}^{A*}{}' \mathbf{H}^* \mathbf{e}_{t,H|t}^{A*} - \mathbf{e}_{t,H|t}^{B*}{}' \mathbf{H}^* \mathbf{e}_{t,H|t}^{B*} \\ &= \left(\mathbf{e}_{t,H|t}^A{}' \mathbf{M}' \right) \mathbf{H}^* \left(\mathbf{M} \mathbf{e}_{t,H|t}^A \right) - \left(\mathbf{e}_{t,H|t}^B{}' \mathbf{M}' \right) \mathbf{H}^* \left(\mathbf{M} \mathbf{e}_{t,H|t}^B \right), \end{aligned}$$

which is equal to the expression in (4) only if $\mathbf{H} = \mathbf{M}'\mathbf{H}^*\mathbf{M} \Leftrightarrow \mathbf{H}^* = \mathbf{M}'^{-1}\mathbf{H}\mathbf{M}^{-1}$. Therefore, if $\mathbf{H} = I_{H+1}$, then we cannot have $\mathbf{H}^* = I_{H+1}$ if the loss-difference should be the same.

Williams & Kloot (1953) introduce a general test of equal predictability for two univariate forecasts; see also Granger & Newbold (1986, Chapter 9) and Howrey (1993). Adjusted to a vector of forecasts, the regression that forms the basis of the test is given by

$$\mathbf{s}_{t,2|t} = (1 - \alpha)\mathbf{s}_{t,2|t}^A + \alpha\mathbf{s}_{t,2|t}^B + \varepsilon_t, \quad (5)$$

where the expectation of the vector ε_t is zero if the forecast of forecaster A and B are unbiased. By using the definition of forecast errors, (5) can be reformulated as

$$\mathbf{e}_{t,2|t}^A = \alpha \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right) + \varepsilon_t. \quad (6)$$

The conditional estimators for α and the covariance matrix of ε_t in (5) (when ignoring possible degrees of freedom adjustments for the covariance matrix) are given by

$$\hat{\alpha}_{(\Omega)} = \frac{T^{-1} \sum_{t=1}^T \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right)' \Omega^{-1} \mathbf{e}_{t,2|t}^A}{T^{-1} \sum_{t=1}^T \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right)' \Omega^{-1} \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right)}, \text{ and} \quad (7)$$

$$\hat{\Omega}_{(\alpha)} = \frac{1}{T} \sum_{t=1}^T \left(\mathbf{e}_{t,2|t}^A - \alpha \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right) \right) \left(\mathbf{e}_{t,2|t}^A - \alpha \left(\mathbf{e}_{t,2|t}^A - \mathbf{e}_{t,2|t}^B \right) \right)', \quad (8)$$

where the subscript in parentheses indicates that the estimates are a function of another parameter or matrix of parameters. The conditional estimators above does not account for the likely autocorrelation structure in the residual in (6) due to the overlapping forecast horizons. The quasi maximum likelihood (QML) estimates $\hat{\alpha}_{(\hat{\Omega})}$ and $\hat{\Omega}_{(\hat{\alpha})}$ can be obtained by an iterative procedure until convergence is achieved, see Oberhofer & Kmenta (1974). The estimator of α given by this iterative procedure is such that it gives the estimate of α that maximize the predictive likelihood for the weighted combination of forecast A and B with estimated weights $1 - \hat{\alpha}$ and $\hat{\alpha}$, respectively.

An alternative to obtaining the estimates of α and Ω is to apply some GLS estimators. In the hypothesis testing, we use the GLS estimators where Ω is estimated under the null hypothesis of α , denoted $\hat{\Omega}_{(\alpha_0)}$, and α is estimated conditional on $\hat{\Omega}_{(\alpha_0)}$.

Simulations conducted in Hungnes (2018) show that the test based on the GLS version of the es-

timates have very small size distortions and higher power than the test based on the QML estimates. Therefore, we apply the GLS estimates in all the tests here.

Hungnes (2020a) shows that with a correction of the estimated variance of the estimator for the parameter α (which can be seen as a correction to account for a missing intercept in (5)), the null hypothesis of testing $\alpha = \frac{1}{2}$ is identical to testing the population equivalent of \bar{d} being zero in (4), where $\bar{d} = T^{-1} \sum_{t=1}^T d_t$, when $\mathbf{H} = \hat{\Omega}_{(1/2)}^{-1}$.

The test statistic Diebold & Mariano (1995) suggest is simply

$$T^{1/2} \bar{d} \hat{q}^{-1/2},$$

where \hat{q} is the estimated variance of d_t given by

$$\hat{q} = \frac{1}{T} \left[\sum_{t=1}^T (d_t - \bar{d})^2 + 2 \sum_{l=1}^{\tau_H} \sum_{t=1}^{T-l} (d_t - \bar{d}) (d_{t+l} - \bar{d}) \right],$$

where τ_H is the truncation lag, $\tau_H \geq H$. Diebold & Mariano (1995) and Diebold (2015) show that the test statistic is asymptotically standard normally distributed under the assumption that d_t is covariance stationary. Giacomini & White (2006) confirm this asymptotic distribution for even looser restrictions on the process of d_t , provided that the forecasting models are estimated using a rolling sample, see also Hungnes (2018, 2020a). The null hypothesis that the population equivalent of \bar{d} being zero implies equal predictability.

Harvey et al. (1997) suggest a small sample correction of the test statistic by multiplying it with the square root of the factor $w_0 = T^{-1}[T - 1 - 2\tau_H + T^{-1}\tau_H(\tau_H + 1)]$. Thus, the test statistic becomes

$$T^{1/2} w_0^{1/2} \bar{d} \hat{q}^{-1/2}. \quad (9)$$

To secure that the estimated variance of d_t is positive, Hungnes (2018, 2020a) suggest applying the weighting factors by Newey & West (1987) given by $w_i = 1 - \frac{i}{\tau_H + 1}$ for $i = 1, \dots, \tau_H$. The estimated variance is then given by

$$\hat{q} = \frac{1}{T} \left[\sum_{t=1}^T (d_t - \bar{d})^2 + 2 \sum_{l=1}^{\tau_H} \sum_{t=1}^{T-l} w_l (d_t - \bar{d}) (d_{t+l} - \bar{d}) \right]. \quad (10)$$

Harvey et al. (1997) show that the test statistics in (9), in the univariate case and without the correction factors w_i for $i = 1, \dots, \tau_H$, has a distribution that is close to the t-distribution with $T - 1$ degrees of freedom. In the multivariate case of a system of forecasts with a vector of $H + 1$ forecasts, we apply the t-distribution with $T(H + 1) - 1$ degrees of freedom.

In the investigation of forecasts of the exchange rate path, we will also test if one path forecast encompasses another path forecast. The encompassing test implies testing $\alpha = 0$ or $\alpha = 1$, depending on which forecast encompasses the other. Also, for this type of test, a test statistic similar to the test statistic described by Diebold & Mariano (1995) can be applied, see Hungnes (2018). For the test of $\alpha = 0$, the definition of d_t is changed to $d_t = \left(\mathbf{e}_{t,H|t}^A - \mathbf{e}_{t,H|t}^B \right) \hat{\Omega}_{(0)}^{-1} \mathbf{e}_{t,H|t}^A$, and for the test $\alpha = 1$ we use $d_t = \left(\mathbf{e}_{t,H|t}^A - \mathbf{e}_{t,H|t}^B \right) \hat{\Omega}_{(1)}^{-1} \mathbf{e}_{t,H|t}^B$. The test statistic in (9) with \hat{q} given by (10) is also used for these encompassing tests.

3 Results

Since the beginning of 2001, Statistics Norway has, in their quarterly publication of macro economic forecasts, reported the year-to-year forecasts for the import-weighted krone (I-44) for the same year as the forecast is made and the two next years.² The data set used in this paper, along with the ox code (see Doornik 2013), is available in Hungnes (2020b).

The I-44 is calculated as a geometric weighted average of the exchange rates of 44 countries. The weights are updated on an annual basis and are calculated using Statistics Norway's statistics for imports to Norway from the 44 largest countries in terms of import value. The country composition varies. Table 1 reports the weights from used from September 4th, 2018. These weights are based on the value of import to Norway in 2017. The number of currencies in the table is less than 44 since some currencies are used in more than one country (for example, the euro).

The calculations are based on Laspeyres' index formula. It can be written as:

$$S_t = S_{t-1} \prod_{j=1}^N \left(\frac{S_t^{(j)}}{S_{t-1}^{(j)}} \right)^{\alpha_{t-1}^{(j)}}, \quad (11)$$

where S_t is the I-44 index at time t ; $S_t^{(j)}$ is the exchange rate j at time t (where we use the round brackets for the top index to not confuse it with the index of the forecaster), and $\alpha_{t-1}^{(j)}$ is the weight of the exchange rate j from time $t - 1$ ($\sum_{j=1}^N \alpha_t^{(j)} = 1$). In the expression, we assume that these weights are used from period $t - 1$ to period t .

It follows from (11) that if the best prediction of each of the individual exchange rates is no change from the previous observation, it follows that the best prediction of the I-44 index is given by the previous observed I-44 index.

The official I-44 index is published by Norges Bank. The annual figures of I-44 are given as the arithmetic average of the trading days observations of the index. Hence — if t runs over the major time period, here years, and ζ runs over the minor time period, here the trading days within year t — the annual figure of the I-44 index is

$$S_t = \frac{1}{\zeta_{max}(t)} \sum_{\zeta=1}^{\zeta_{max}(t)} S_{t,\zeta}, \quad (12)$$

where $\zeta_{max}(t)$ is the number of trading days in year t .

The random walk forecast of the I-44 index in year t made when ζ' is the latest observed trading day in year t , is

$$S_{t|\zeta'}^{RW} = \frac{\zeta'}{\zeta_{max}(t)} \left(\frac{1}{\zeta'} \sum_{\zeta=1}^{\zeta'} S_{t,\zeta} \right) + \frac{\zeta_{max}(t) - \zeta'}{\zeta_{max}(t)} S_{t,\zeta'}, \quad (13)$$

where the term in the round brackets is the average of the index from trading day 1 to trading day ζ' in year t , and the $S_{t,\zeta'}$ reflects that the best forecast under the random walk hypothesis of the exchange

²There are two exceptions: First, in the publication of the forecast made the first quarter in 2001, Statistics Norway only published forecast for I-44 for the years 2001 and 2002. In our analysis we have assumed the forecasted value of I-44 for 2003 in the forecast published in the first quarter of 2001 to be equal to the forecasted value for 2002, that is, no change in the I-44 from 2002 to 2003 on a year to year basis. Second, Statistics Norway did not publish forecasts in the third quarter of 2013. In this analysis, we have set this forecast equal to the forecast from the second quarter that year. For the forecast based on the random walk, we have also used the exchange rate equal to the exchange rate in the market relative to the time the second quarter forecast from Statistics Norway was made.

Table 1: Weights used for Import-weighted krone exchange rate, I44, based on import to Norway in the year 2017

Country, currency	Short name	weight
Bangladesh, Taka	BDT	0.003
Brazil, Real	BRL	0.015
Canada, Dollar	CAD	0.020
Switzerland, Franc	CHF	0.012
China, Yuan Renminbi	CNY	0.101
Colombia, Peso	COP	0.002
Czech Republic, Koruna	CZK	0.011
Denmark, Krone	DKK	0.056
European Union, Euro	EUR	0.325
United Kingdom, Pound	GBP	0.049
Hungary, Forint	HUF	0.004
Indonesia, Rupiah	IDR	0.002
India, Rupee	INR	0.006
Iceland, Krone	ISK	0.004
Japan, Yen	JPY	0.021
South Korea, Won	KRW	0.070
Malaysia, Ringgit	MYR	0.005
Peru, New sol	PEN	0.002
Poland, Zloty	PLN	0.035
Romania, New leu	RON	0.004
Russia, Rubel	RUB	0.019
Sweden, Krone	SEK	0.118
Singapore, Dollar	SGD	0.004
Thailand, Baht	THB	0.009
Tyrkey, Lire	TRY	0.011
Taiwan, New Dollar	TWD	0.006
United States, Dollar	USD	0.070
Vietnam, Dong	VND	0.007
Coopération Financière en Afrique Centrale, CFA-franc	XAF	0.002
South Africa, Rand	ZAR	0.004
Mexico, Peso	MXN	0.003
Sum		1

Source: Norges Bank

rate in the remaining trading days in year t is equal to the last observed value of the index. The random walk forecast of the annual value of the exchange rate index in the coming years is equal to the last observation of the index, that is

$$S_{t+h|t(\zeta')}^{RW} = S_{t,\zeta'} \text{ for } h = 1, 2, \dots \quad (14)$$

In Table 2 and Table 3 we compare the forecasts by Statistics Norway with forecasts generated by a random walk. The point of departure is (5) with $\mathbf{s}_{t+H|t}^A = \mathbf{s}_{t+H|t(q)}^{SN}$ (with SN indicating Statistics Norway and $\mathbf{s}_{t+2|t(q)}^{SN} = \left(s_{t|t(q)}^{SN}, s_{t+1|t(q)}^{SN}, s_{t+2|t(q)}^{SN} \right)'$) being the vector of I-44 forecasts by Statistics Norway up to horizon $H = 2$, where the forecasts are made in quarter q of year t ; and $\mathbf{s}_{t+H|t}^B = \mathbf{s}_{t+H|t(\zeta)}^{RW}$ (with RW indicating random walk and $\mathbf{s}_{t+2|t(\zeta')}^{RW} = \left(s_{t|t(\zeta')}^{RW}, s_{t+1|t(\zeta')}^{RW}, s_{t+2|t(\zeta')}^{RW} \right)'$) the implied forecasts by a random walk where the last observation is $t(\zeta')$, given by (13) and (14).

The forecasts by Statistics Norway are published quarterly, usually at the beginning of the third month in the quarter. The publication day is (with a few exceptions) Thursday, with a deadline of

Table 2: Equal predictability test and encompassing tests — random walk based on the exchange rate one day before the publication deadline

Projection quarter	Weight SN $1 - \alpha$	Weight RW _{1 day prior} α	t-test (two-sided)		
			$H_0 : \alpha = \frac{1}{2}$	$H_0 : \alpha = 0$	$H_0 : \alpha = 1$
Q1	-0.12	1.12	1.13 [0.26]	1.67 [0.10]	0.25 [0.80]
Q2	-0.46	1.46	1.76 [0.09]	2.30 [0.03]	0.82 [0.42]
Q3	0.15	0.85	0.65 [0.52]	1.53 [0.13]	0.28 [0.78]
Q4	0.03	0.97	0.92 [0.36]	1.77 [0.08]	0.08 [0.94]
All	0.03	0.97	1.84 [0.07]	3.26 [0.00]	0.12 [0.90]

Q1 denotes the forecasts made in the 1st quarter of the year, in the years 2001-2016. Similarly for Q2, Q3, and Q4. "All" implies that we consider the forecasts from all quarters in the analysis. The p-values in square brackets are based on a two-sided test; for the corresponding p-values for a one-sided test, the reported values must be divided by 2. In the estimation, we use $\tau_H = 2$ for the forecasts made in Q1, Q2, Q3 and Q4, and $\tau_H = 11$ for "All".

preparing the forecasts on Tuesday the same week they are published. The work with the forecasts starts (usually) Monday two and a half weeks before the publication. Thus, the forecasts are normally made in about 12 working days.³ The path for the exchange rate index is usually decided at the beginning of this period, though it can be revised during the process of making the forecasts.

Table 2 reports the results of the equal predictability test and the encompassing test when the random walk forecasts are based on the exchange rate one day before the deadline. This observation of the exchange rate is the latest official exchange rate it is possible to make use of in the forecasts as the exchange rate is published approximately at 16:00 CET. The exchange rates published by Norges Bank are identical to the ones published by the European Central Bank. In the table, we consider forecasts made in each of the four quarters of the year separately. Thus, in the row marked "Q1", forecasts made each year in the 1st quarter from 2001 to 2016 are considered. For these 1st quarter of the year forecasts, we see that the estimated weight of the forecasts by Statistics Norway is negative and close to zero. Due to the small number of observations, we cannot reject that the weights are 0.5, but the hypothesis that the forecasts by Statistics Norway encompass the random walk forecasts is close to being rejected. (The p-value just exceeds 10 percent with the two-sided test.) The last test in the row shows that we cannot reject that the random walk forecasts encompass the forecasts made by Statistics Norway.

For the forecasts made in the 2nd, 3rd and 4th quarter we see similar results as for the forecasts made in the 1st quarter of the year: the hypothesis of equal weights cannot be rejected in any of the quarters; the hypothesis that forecasts made by Statistics Norway encompass the random walk forecasts are rejected for two of the quarters (at the 10 percent level for the two-sided test which corresponds to the 5 percent level for the one-sided test); and the opposite hypothesis that the random walk forecasts encompass the forecasts made by Statistics Norway cannot be rejected in any of the quarters.

In the last row, marked "All" we have stacked all forecasts made by Statistics Norway in these years (2001-2016) after each other (in the order they were made). We have also taken into account that this will lead to autocorrelation of a higher order, as more forecasts overlap in time. we do so by using $\tau_H = 11$ when all these forecasts are considered jointly.⁴ The estimated weight on the forecasts by Statistics Norway is virtually zero (0.03), indicating that the projected exchange rate path has no value over a path given by a random walk model. The two encompassing tests confirm this finding. Though,

³Due to holidays there can be some exceptions, in particular for the forecasts made in the 2nd quarter in which there are many national holidays in the period ahead of the publications.

⁴The exchange rate path forecasts made in the 1st quarter of year t and the forecasts made in the 4th quarter of year $t + 2$ slightly overlap, as both includes forecasts of the exchange rate in year $t + 2$. The time between when these two path forecasts was made is 11 quarters.

Table 3: Equal predictability test and encompassing tests — random walk based on the exchange rate 15 days before the publication deadline

Projection quarter	Weight SN $1 - \alpha$	Weight RW _{15 days prior} α	t-test (two-sided)		
			$H_0 : \alpha = \frac{1}{2}$	$H_0 : \alpha = 0$	$H_0 : \alpha = 1$
Q1	0.52	0.48	0.04 [0.97]	0.85 [0.40]	1.26 [0.21]
Q2	0.35	0.65	0.41 [0.69]	1.36 [0.18]	1.05 [0.30]
Q3	0.45	0.55	0.15 [0.88]	1.54 [0.13]	1.11 [0.27]
Q4	0.59	0.41	0.50 [0.62]	1.19 [0.24]	1.58 [0.12]
All	0.54	0.46	0.18 [0.86]	1.74 [0.08]	2.61 [0.01]

Note: See Table 2.

the uncertainty in the estimates is substantial, so we cannot reject the hypothesis of equal weights for the two forecasts at the 5 percent level — though we are very close to doing so with a p-value of 7 percent. Nevertheless, the overall conclusion is that the exchange rate forecasts by Statistics Norway add no extra information to the future values of the exchange rate beyond what the random walk based forecasts give.

An important question is whether the random walk forecasts dominate due to the superiority of the random walk model in itself or because it uses more up-to-date observations of the exchange rate than what the forecasts made by Statistics Norway do. This question is difficult to answer. However, we can shed some light on it by comparing the forecasts by Statistics Norway with random walk based forecasts when the latter is based on the observed exchange rate on an earlier trading date. In Table 3 the random walk forecasts are based on the exchange rate 15 days before the deadline of the publication deadline for the forecasts by Statistics Norway; the exchange rate from the day Statistics Norway usually starts its work with the forecasts. Here also, we consider the forecasts made in the four different quarters of the year separately, in addition to considering the forecasts from all quarters jointly. The estimated weights are now close to 0.5, no matter what quarter the forecasts were made, and the hypothesis that the weights are 0.5 cannot be rejected in any of the considered cases. When considering forecasts from all quarters jointly, see the last row of the table, the hypothesis that random walk based forecasts encompass the forecasts by Statistics Norway is rejected at the 5 percent level, no matter whether we use a one-sided test or a two-sided test. The hypothesis that the forecasts by Statistics Norway encompass the random walk based forecasts is rejected at the 5 percent significance level when using a one-sided test.

The different estimates of the weights in Table 2 and Table 3 show the importance of the additional two weeks of exchange rate data for the forecasts. In Table 4, we compare the two random walk forecasts directly. When considering the random walk forecasts for each quarter separately, the weight on the most updated random walk forecast varies from 0.88 to 2.47. For three of the four quarters, we cannot reject that this weight is 1 (i.e., $\alpha = 0$), whereas in the random walk generated forecasts in the 1st quarter of the year, the estimated weight is significantly greater than 1. In three of the four quarters, we can also reject the hypothesis that the weight of random walk forecast based on the exchange rates 15 days ahead of the deadline for Statistic Norway’s forecasts is equal to 1 (i.e., $\alpha = 1$) when using the 5 percent significance level and a one-sided test.

Table 4 also reports the estimated weights when considering the random walk based forecasts at the time of all the publication dates of forecasts by Statistics Norway in the years 2001-2016. The estimated weight for the most recent based forecast — when considering forecasts made in all four quarters jointly — is 1.7 and exceeds 1 significantly. Based on the estimation results for the individual quarter the

Table 4: Equal predictability test and encompassing tests — random walk based on the exchange rate 1 day vs. 15 days before the publication deadline

Projection quarter	Weight RW _{1 day prior} $1 - \alpha$	Weight RW _{15 days prior} α	t-test (two-sided)		
			$H_0 : \alpha = \frac{1}{2}$	$H_0 : \alpha = 0$	$H_0 : \alpha = 1$
Q1	2.47	-1.47	3.95 [0.00]	2.70 [0.01]	4.88 [0.00]
Q2	1.45	-0.45	1.46 [0.15]	1.08 [0.29]	1.61 [0.11]
Q3	0.88	0.12	1.80 [0.08]	0.54 [0.59]	2.57 [0.01]
Q4	1.03	-0.03	1.32 [0.19]	0.11 [0.92]	2.00 [0.05]
All	1.70	-0.70	3.31 [0.00]	2.29 [0.02]	3.97 [0.00]

Note: See Table 2.

forecasts are made, we see that it is for the forecasts made in the 1st quarter that the estimated weight deviates mostly from 1. For forecasts made in the 2nd, 3rd, and 4th quarters, we cannot reject that the weight for the most recent exchange rate is 1.

4 Conclusions

We have used new tests for equal predictability and encompassing for path forecasts (Hungnes 2018, 2020a) to compare the predicted exchange rate path by Statistics Norway with a random walk forecast. The date the random walk forecast is based on, is shown to be crucial. When the random walk is generated from the exchange rate at the deadline of the publication of the forecasts made by Statistics Norway, the random walk forecast path encompasses the forecasted path by Statistics Norway. However, when the random walk forecast path is generated based on the exchange rate 15 days before the publication deadline, the random walk path and the forecasted exchange rate path by Statistics Norway have equal predictability.

Two lessons can be drawn. First, there is no indication that the exchange rate forecasts by Statistics Norway are better than a random walk forecast: the exchange rate path forecast from Statistics Norway has equal predictability as the random walk based forecast based on the exchange rates 15 days before the publication deadline. Therefore, starting from the forecasts made in the first quarter of 2019, Statistics Norway has forecasted an unchanged exchange rate based on the exchange rate close to the publication deadline (Statistics Norway 2019). Second, using the exchange rate as close to the projection deadline as possible improves the forecasts. Therefore, Statistics Norway would probably improve its exchange rate forecasts by updating the forecasts until the projection deadline.

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