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Abstract:
We present a scheme for analysing income tax perturbations, applied to a real Norwegian tax reform during 2016 - 2018. The framework decomposes the reform into a structural reform part and a tax level effect. The former consists of a distributional impact and a social e¢ ciency effect measured as the behavioural-induced change in tax revenue. Considering the overall welfare e¤ect conditional on inequality aversion, we back out the pivotal value of the decision makers' inequality aversion, according to which unfavourable redistributional e¤ects exactly cancel out a social efficiency enhancement.

Keywords: income tax, tax reform, tax perturbation, inequality aversion

JEL classification: H2, H21, H24

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Sammendrag

I diskusjoner av hvordan en kan designe et skattesystem, antas det ofte at en kan starte med blanke ark og konstruere det skattesystemet som en finner optimalt, uten hensyntaken til det allerede eksisterende skattesystemet. Men i praksis skjer mange endringer i skattesystemet ved at en justerer ulike skattesatser ved hjelp av små korreksjoner basert på hvordan dagens skattesystem ser ut. Det er det som det henvises til ved begrepet «perturbations» i tittelen på dette arbeidet. Skattereformen i Norge 2013-2019 er et godt eksempel på slike relativt begrensede korreksjoner av skattesystemet. Endringene er så moderate at de ikke alltid omtales som en reform, men betegnes som omlegginger. Følgelig henvises det til «skatteomleggingen 2013-2019». I den empiriske illustrasjonen i denne analysen tar vi ut en del av skatteendringene ved denne omleggingen, endringene i årene fra 2016 til 2018, og bruker disse for å illustrere hva som motiverer omleggingen. Skatteendringene fører til at de marginale skattesatsene reduseres på alle inntektsnivåer.

Hovedambisjonen med dette arbeidet er å utvikle og demonstrere anvendelsen av et rammeverk for hvordan slike små omlegginger av skattesystemet kan analyseres. Nærmere bestemt innebærer vårt opplegg at vi dekomponerer omleggingene i effekt på skattenivå og effekt på strukturen i skattesystemet. Det siste kan fremstilles som effekt på satsstrukturen, dvs. den marginale skattesatsen på ulike inntektsnivåer. Vi får frem effekten på strukturen i skattesystemet ved at skatteproveny-effekten tilbakeføres til skatteyterne ved at alle mottar en lik overføring eller betaler en lik skatt. Siden skatteomleggingen 2016-2018, som er vårt eksempel, innebærer redusert proveny, er justeringen i dette tilfellet en skatt.

Videre har de strukturelle endringene konsekvenser for skattenes omfordelende effekt og økonomisk effektivitet. Siden vi bare ser på marginale endringer i skattesystemet, kan fordelingseffekter av endringene måles direkte ved de mekaniske (adferdsfrie) effektene av endringene. Dette følger av å anvende det såkalte «omhyllingsteoremet», som betyr at det er ingen første-ordenseffekter på inntektene fra adferdsendringene. Adferdseffektene på skatteprovenyet bestemmer virkningene på samfunnsøkonomisk effektivitet med hensyn på allokeringen av tid på arbeid versus fritid av skatteendringene. Disse måles ved virkningene på skatteprovenyet av at personene endrer tilpasning til de nye skattereglene.

Personene arbeider mer til de reduserte skattesatsene som følge av omleggingen 2016-2018. Dette bidrar til økt samfunnsøkonomisk effektivitet. Men på den annen side svekkes skattesystemets omfordelende effekt. Vi kan med utgangspunkt i dette eksemplet bestemme i hvilken grad beslutningstakerne vegrer seg mot inntektsulikhet. Siden beslutningstakerne har valgt å gjennomføre skatteendringene 2016-2018, betyr det at de har veid effektivitetsgevinsten mot den økte inntektsulikheten. Valget gir informasjon om ulikhetsaversjonen som ligger bak disse avveiningene. Målt på denne måten finner vi en grenseverdi for ulikhetsaversjonen på 1.2. Siden omleggingen av skattesystemet faktisk er foretatt, vil ulikhetsaversjonen være mindre eller lik 1.2. Vi anser 1.2 for å være et realistisk anslag på ulikhetsaversjonen til beslutningstakere siden det er i nærheten av hva en ellers finner i litteraturen på dette feltet.

1 Introduction

There are two major strands of research in the normative tax analysis of public economics. One approach is to characterise the optimal taxes starting with a clean sheet. This is known as the tax design problem. The other is the tax reform approach, highlighted in particular by Feldstein (1976), who argued that optimal tax reform must take as its starting point the existing tax system. According to Feldstein (op.cit., p.90), "in practice, tax reform is piecemeal and dynamic in contrast to the once-and-for-all character of tax design." In the wake of Feldstein's emphasis on tax reform analysis, a series of papers addressed in a theoretical framework the effects of small commodity tax reforms, often called tax perturbations. (Diewert, 1978; Dixit, 1975; Guesnerie, 1977). Piecemeal income tax reforms have received attention only more recently, see Golosov et al. (2014), Saez (2001), Hendren (2016), Bierbrauer, Boyer and Peichl (2020). Most of the reforms that have been studied take the form of small perturbations of the initial tax function.

We present a framework for assessing income tax perturbations focussing on the distributional and social efficiency aspects of the reform. Distinguishing between tax level and tax structure has a long-standing tradition in public economics. We can think of the level as determined by the height at which the tax schedule is located, while the tax structure is determined by the shape of the tax function, i.e. the marginal taxes at various income levels. The shape of the tax function determines both the distribution of the tax burden across heterogeneous taxpayers and the extent to which taxes are distortionary and harm the social efficiency of the economy. The tax level determines the total burden imposed on the taxpayers as taxes suppress private consumption to make resources available for the public sector. Determining the tax structure and choosing the tax level are separate decisions. Politicians can have different views about either, and informing the discussion about either is important. This motivates efforts to disentangle the structural and the level part of a tax reform where it changes the tax policy in both respects.

Our contribution is to present a decomposition allowing us to study structural changes, leaving aside the choice of tax level. To separate out the structural aspect of a tax reform, we assume that any change of the aggregate burden on the taxpayers is offset by adjusting a hypothetical uniform cash transfer (or lump-sum tax) to keep the tax level unchanged. Then we have a pure structural reform that influences the distribution of the tax burden and the economic behaviour of the taxpayers. We explore the redistributive and social efficiency effects of this reform. The advantage is to achieve a clear distinction between a pure (zero-sum) redistribution and a quantifiable enlargement (or contraction) of the amount available for distribution due to a more (or less) efficient allocation. We leave aside how the actual mechanical change in aggregate tax burden may (dis)benefit the taxpayers, which is a different type of policy question.

To make a welfare assessment of the distributional effects, we apply a particular class of welfare weights that reflect the inequality aversion of the distributional preferences. Departing from a tax-distorted initial allocation, efficiency effects are determined by the pre-existing tax wedges and the behavioural responses to the tax reform. The impact on social efficiency can then be measured by the behavioural-induced change of tax revenue. We shall elaborate on these aspects below.

Suppose the outcome is a more unequal distribution and a more efficient allocation or vice versa. Then we need to place a value on the induced revenue change in order to compare it with the distributional effect and achieve an overall assessment of the structural reform. For this purpose, we assume that a behavioural-induced revenue gain is recycled as a lump-sum transfer (or a loss is covered by a lump-sum tax). Our approach implies that we consider (positive or negative) transfers to the taxpayers in two steps, first to offset the mechanical effect of the tax reform to keep the tax level unchanged (Step 1) and then to redistribute the additional tax revenue generated by enhanced efficiency (Step 2). Opting for this two-step procedure, rather than a single step, is motivated by our desire to specify the various factors that determine how the reform affects welfare.

Finally, we can describe how the overall welfare effect depends on the inequality aversion, which enables us to infer the range of distributional preferences that are implicit in political support for the reform. By exploring implicit preferences, assumed to be revealed by the tax reform, we add to the studies of implicit preferences previously based on the assumption that the actual policy is optimal, known as the inverse optimum problem (a term coined by Ahmad and Stern, 1984). We apply our procedure to piecemeal income tax reforms implemented in Norway during the period 2016-2018, enabling us to achieve results with substantial empirical content.

The studies of income tax perturbations in the literature take somewhat different approaches to highlight various reforms and various reform effects, but the papers share a number of key features. In particular, they typically distinguish mechanical effects (abstracting from behavioural responses) and behavioural effects of tax reforms, adopting the terminology of Saez (2001).

The present study connects to other contributions of the literature. Golosov et al. (2014) establish a general and rich model to characterise the welfare effects of local tax reforms in a dynamic setting. The paper identifies the various mechanical and behavioural effects of tax perturbations. It addresses tax reforms that depart from the existing tax system, such as introducing non-linear capital taxes and introducing joint taxation of various forms of income in a life-cycle setting with age-dependent behaviour. In contrast, our paper presents a more detailed analysis of a more narrow set of tax perturbations.

Bierbrauer, Boyer and Peichl (2020) are mainly concerned with the political feasibility of income tax reforms in the sense that a reform is in the self-interest of a majority of taxpayers. They consider a perturbation of the tax function, which may enhance or diminish the amount of tax revenue. The main assumption is that any additional tax revenue, whether mechanical or behavioural-induced, is recycled as a cash transfer (or a lump-sum tax makes up for a loss). To assess the reform, most of the analysis assumes that the induced revenue is transferred as a uniform lump sum to the taxpayers. (The alternative case where an increase in public goods provision compensates the agents for increased marginal tax rates is briefly discussed in the appendix.) Considering net effects of the tax and transfer changes, there may be losers and winners. The political feasibility will depend on the respective numbers of winners and losers, and the welfare effect will in general depend on the welfare weights assigned to the various agents.

With its emphasis on political feasibility, Bierbrauer et al. have a different focus than our paper. From a welfare-analytical perspective, the approaches show both similarities and differences. Bierbrauer et al. consider a revenue-neutral reform in the sense that any mechanical or behavioural-induced revenue change is offset by a lump-sum transfer. In this sense, their analysis could be perceived as addressing a structural reform where the benchmark is a fixed revenue rather than a fixed burden on the taxpayers, as in our structural reform analysis. We first ask how changing the profile of the tax schedule, while preserving the average burden on the taxpayers, impacts the distribution and affects social efficiency, where it is straightforward to measure the latter effect in terms of behavioural-induced change in tax revenue. In either study, a lump-sum transfer/tax is used as a level parameter.

Hendren (2016) expresses the reform-induced benefit to an agent as the change in net resources (lump sum transfer minus gross tax liability) transferred from the government to the agent plus the impact of the agent's change of behavior on government revenue. For a revenue-neutral reform, the net resource transfers sum to zero and can be used as one way to express distributional effects of the total reform. In our analysis we use the mechanical effects of the (structural) reform to express the distributional effects. Since the individual net resource transfers are determined both by mechanical and behavioural effects, the distributional effects in Hendren's approach include efficiency effects that we would like to separate out.

Our framework is within the strong modelling tradition of optimal non-linear and (piecewise) linear income taxation, see e.g. Mirrlees (1971), Sheshinski (1972), Dahlby (2008) and Apps and Rees (2009). The focus of this paper is mainly motivated by the application of our framework to Norwegian reforms of labour income taxes.

The current paper proceeds as follows.We describe our theoretical approach in Section 2. Section 3 presents the Norwegian tax perturbations used in the empirical analysis in Section 4. Section 5 concludes.

2 A scheme for assessing tax perturbations

2.1 Mechanical, efficiency and welfare effects

We consider a population of agents who choose labour supply for given wage rates and tax parameters. Denote the wage rate by w and labour supply by h. The tax function for labour earnings is given by $T(y,\theta)$, where y is income and θ is a vector of tax parameters ($\theta = \theta_1, \theta_2, ..., \theta_j, ...$), which may include tax rates and bracket limits of a piece-wise linear tax system. Let the initial tax function be defined by the parameter vector θ^1. We may simplify the notation by writing $T_1(y) \equiv T\left(y,\theta^1\right)$. A tax perturbation is then defined by a vector of increments, as $d\theta = d\theta_1, d\theta_2, .., d\theta_{j...}$, generating a new tax function $T_2(y) = T\left(y,\theta^2\right) = T\left(y,\theta^1 + d\theta^1\right)$. Assume there is a distribution of agents with density function $f(w)$. The tax reform will have mechanical effects, behavioural effects and distributional effects. A mechanical effect is the effect on the tax liability for unchanged behaviour, i.e. fixed labour supply and consequently fixed income. For some initial income y, the mechanical effect is $M(y) = T_2(y) - T_1(y)$. The immediate welfare effect (before any conceivable efficiency gains accrue to the consumers) on consumers is the sum of welfare-weighted real income effects of the tax reform. We note that real income losses are equal to the mechanical effects when, due to envelope properties, there are no first order effects of behavioural changes. The behavioural effect on tax payment is the change due to behavioural changes, which in this case are labour supply and corresponding income reponses. In formal terms, the behavioural effect is then $B(w) = T_1'\left(y\left(w,\theta^1\right)\right) wdh = T_1'\left(y\left(w,\theta^1\right)\right) dy$ where dh is the change in labour supply and dy is the change in taxable income that the tax reform induces.

Consider an agent with wage rate w reflecting his marginal product of labour. His marginal disutility of labour is s in monetary terms. Where the induced change in labour supply is dh, there is a social efficiency gain $[w-s]\,dh$, which is the increase in output beyond the cost of compensating the worker for the disutility of supplying the extra labour required. This is a behavioural effect. Where the tax function is differentiable, the marginal disutility of labour is equated to the after-tax marginal wage rate: $\omega = w(1-T')$, and the social efficiency gain is $[w - w(1-T')]\,dh = wT'dh = T'dy$, where dy is the change in gross income [1].

[1]We neglect any further discrepancy that might exist due to imperfect competition. A justification is

Also paying attention to the extensive margin of labour supply, we may assume that there is a cost of working, k, and a distribution of k across the population is characterised by the density $g(k)$. Assume that an agent pays the tax T_0 when not working and obtains an income net of tax $y-T$ if working. The net private gain from working is then $y-k-(T-T_0)$, which is zero for a marginal worker, while the net social gain is $y-k$. For a marginal worker induced to enter the labour force, the net social gain is $y-k=T-T_0$, which is the change in tax revenue.

A tax reform will normally affect both the tax level, and the tax structure, defined by how the marginal tax rate varies across income, typically determined by number of tax brackets, bracket limits and marginal tax rate in each bracket. In this paper, we focus exclusively on the tax structure. We are not concerned with the overall resource allocation between the public and the private sector of the economy. In accordance with our focus, we shall single out structural changes for further scrutiny. We do this in the following way. We introduce a lump sum element in the tax function allowing us to cleanse out the level effect. The new tax function is $T_3(y) = T_2(y) - \alpha$, where we can interpret α as a pure level parameter. This is a hypothetical tax schedule in the sense that it is not observed in practice. We shall set the change in α (initially set equal to zero) equal to the average mechanical effect of a tax reform. This means that for a small tax perturbation the change in tax level is measured by the average change in the burden on the taxpayers. By subtracting α in the tax function, the tax payers are on average compensated for the increased tax burden. A change in α implies a vertical shift in the tax schedule. The advantage is to have a level effect which is unaffected by the reform changes in marginal tax rates. This would not be the case where the revenue effects induced by marginal tax rate changes are offset by a lump-sum tax/transfer.

Since mechanical effects reflect the income losses of the taxpayers, assuming no aggregate mechanical effect (after adjusting α) implies that we are left with redistributive and efficiency effects. In two respects, these effects are not independent. First, the distributional profile of the tax schedule also affects how distortionary it is. Secondly, when there is a transfer from agent i to agent j, there will be income effects on behaviour that in turn will change the agents' tax payments and tax revenue for the government. Whether there is a net effect depends on whether the agents have different marginal propensities to pay tax, where an agent's marginal propensity to pay taxes is given by $T'wdh/dI$, where dh/dI is a pure income effect. As there are pre-existing distortions of labour supply a behavioural-induced rise (fall) in tax revenue is beneficial (harmful), as discussed above. We can interpret this effect of a transfer as a social efficiency effect of redistribution.[2] In

that this is less relevant when considering a small open economy

[2]It is common in tax analysis to make use of Diamond's (1975) marginal social valuation of income for an agent, which is the sum of the direct effect on the agent and the marginal propensity to pay taxes

addition, the tax reform will obviously generate substitution effects. Our aggregate measure of the social efficiency impact will be the sum of these efficiency effects. It may also be of interest to observe which households and income groups that contribute (positively or negatively) to the efficiency effect.

Now taking a formal approach, write the indirect utility function $V\left(w^i,\theta,\alpha\right)$, where i indicates agent. Simplifying the notation, we can write $V^i(\theta,I+\alpha)\equiv V\left(w^i,\theta,I+\alpha\right)$, where I allows us to define income effects. Taking θ^1 as our point of departure, we consider the tax reform $d\theta = d\theta_1, d\theta_2,...,d\theta_j...$, and $d\alpha$ to cleanse out the level effect, as discusssed above. Denote by γ^i agent i's marginal utility of income, and let $g^i = \sum_j d\theta_j \frac{\partial V^i}{\partial \theta_j}/\gamma^i + d\alpha$ be the gain in terms of income obtained by agent i due to the tax reform, defined by the increments $d\theta, d\alpha$. As discussed above, the private income gain (loss) for an agent is equal to the mechanical revenue loss (gain) for the government since both are defined absent behavioural changes.

Now write total welfare as the welfare derived from private income plus the value of government revenue in terms of welfare:

$$\sum_i V^i(\theta,I+\alpha)+\mu R\left(w^1,..,w^n,\theta,\alpha\right),$$

where μ is the shadow value of government revenue R. The welfare effect of the structural tax reform under consideration can then be expressed as[3]

$$d\Omega = \sum_i \gamma^i g^i + \mu \sum_j \frac{\partial R}{\partial \theta_j} d\theta_j + \mu \frac{\partial R}{\partial \alpha} d\alpha. \tag{1}$$

We can now distinguish the various effects of the structural reform. By our definition of constant tax level, implemented by $d\alpha$, it follows that $\sum_i g^i = 0$. However each element in the sum may be strictly positive or negative, and there will be winners and losers. The social efficiency effect, measured in terms of government revenue, is given by $\sum_j \frac{\partial R}{\partial \theta_j} d\theta_j + \mu \frac{\partial R}{\partial \alpha} d\alpha = dR_b$, which is the behavioural effect. The reason is that the mechanical effect included in the former term is offset by the latter term. The expression for the welfare effect of the structural reform is then reduced to $d\Omega = \sum \gamma^i g^i + \mu dR_b$.

Now assume that the revenue from enhanced efficiency, dR_b, is redistributed as a lump-sum transfer denoted by $d\alpha^*$. In the absence of income effects, $d\alpha^* = dR_b$. Where there are income effects on behaviour, the final transfer will have to be calculated taking income effects into account and $d\alpha^*$ may deviate from dR_b. We shall come back to this

affecting government revenue. The two effects are rarely distinguished. In our presentation we separate the two effects.

[3]We assume from the outset that the cardinalisation (in particular the concavity) of the indirect utility function is chosen such that it reflects the inequality aversion of the goverment.

later. Where any additional tax revenue is recycled to the taxpayers, the total welfare effect of the structural reform can be rewritten as $d\Omega = \sum \gamma^i \left(g^i + d\alpha^*\right)$.

2.2 A linear illustration

To provide a simple illustration of our approach, assume there is a linear income tax.[4] This is actually the simplest possible special case of a stepwise linear income tax to be considered below. Assume there is a continuum of agents with different wage rates and normalise the population to unity. The wage rate distribution is described by the distribution function $F(w)$ with density $f(w) = F'(w)$. The initial linear income tax is written as

$$T_1 = t_1 y + b.$$

Consider a "small" tax reform (perturbation) which simply increases the marginal tax rate from t_1 to t_2 :

$$T_2 = t_2 y + b.$$

We then define

$$T_3 = t_2 y + b - a,$$

and the initial tax revenue is

$$R = t_1 \int_0^\infty wh\left(\left(1 - t_1\right) w, I\right) f(w) dw + b - a.$$

Consider an increment dt_1. The aggregate mechanical effect is

$$dR^m_t = dt_1 \bar{y},$$

where $\bar{y}$ is average taxable income. The mechanical effect of increasing a is

$$dR^m_a = -da.$$

There will be an offsetting effect when

$$dR^m_t + dR^m_a = 0,$$

$$da = dt_1 \bar{y},$$

and it follows that

[4] For illustrative purposes, we abstract from other taxes in the current context.

$$T_3 = t_2 y + b - dt_1 \bar{y}.$$

The transition from T_1 to T_3 is then the change in tax structure, which in this case is simply a strengthening of the tax progressivity. The behavioural effect is

$$dR^b = \int_0^\infty \left(-t_1 wh_\omega dt_1 - t_1 wh_I \left(wh - \bar{y}\right) dt_1\right) f(w) dw$$

where $\omega = (1-t)\,w$.

We can interpret $m = twh_I$ as an agent's marginal propensity to pay tax, i.e. the additional tax revenue generated by giving an extra unit of income to the agent. The social efficiency effect reflected by the behaviour-induced change in tax revenue can then be expressed as

$$dR^b = \int_0^\infty \left(-t_1 wh_\omega dt_1\right) f(w) dw + \int_0^\infty \left(-m(w)\left(y(w) - \bar{y}\right) dt_1\right) f(w) dw,$$

and

$$dR^b = \int_0^\infty \left(\left(-t_1 wh_\omega dt_1\right) - \operatorname{cov}\left(m(w), y(w)\right) dt_1\right) f\left(w\right) dw.$$

The total efficiency effect is made up of the aggregate substitution effect,

$$\int_0^\infty \left(-t_1 wh_\omega dt_1\right) f(w) dw$$

and the efficiency effect of redistribution, $-\operatorname{cov}\sum\left(m(w), y(w)\right) dt_1$. For $dt_1 > 0$ the effect is positive if $\operatorname{cov}(m(w), y(w)) < 0$. Then people with higher income have lower marginal propensity to pay tax. They incur a loss when the marginal tax rate is increased and there is no aggregate (or average) loss or gain since the aggregate mechanical effect is zero, while people with lower incomes gain. With normal responses, high-income taxpayers supply more labour, increase their income and face a larger tax liability, whereas low-income people show opposite responses. A negative covariance implies that the losers increase their tax payments more than the winners diminish their tax payments: the efficiency gains outweigh the efficiency losses.

The welfare effect of increasing t_1 is

$$d\Omega = \int_0^{\infty} \gamma(z(w))\,(\bar{y} - y(w))\,dt_1 f(w)dw + \mu dR^b, \tag{2}$$

where the welfare weight is a function of disposable income, denoted by z. The former term on the right hand side is the gain from redistribution and the latter term is the efficiency effect.

2.3 Further specification issues

Even though we primarily study income tax perturbations, we also need to take into account effects related to indirect taxes. Behavioural responses to the reform will affect consumption, and consequently commodity taxes will influence the revenue and efficiency effects of the reform. Firstly, a commodity tax drives a wedge between the marginal valuation of a commodity and the cost of producing it. Increased demand will then yield a social efficiency gain due to the preexisting distortion. Analogous to what we found in the case of income taxation, a rise (fall) in indirect tax revenue, induced by behavioral changes, reflects a social efficiency gain (loss). The effective tax is made up of both the income tax and indirect taxes, as analysed in Edwards, Keen and Tuomala (1994), and we need to allow for changes in both sources of revenue. This will be done in the empirical part, but for ease of exposition we shall confine attention to income taxes in the theoretical discussion.

Secondly, in order to take indirect taxes into account, one has to decide how to treat savings since in a particular period the indirect tax base will be smaller the larger is the savings rate. However, a single-period perspective would be too narrow since postponed consumption will be taxed in later periods. We therefore model consumption as if there are no savings. The empirical tax reform we shall consider does not directly impact savings and taxes on savings. The reason is that Norway has a dual income tax with separate taxation of capital income and labour earnings. Any savings effect will be indirect and channelled through the impact on disposable labour income.

Our next objective is to study the welfare-weighted redistribution. In order to assess the distributional effects, we let the welfare weight be a function of disposable income, denoted by z. We choose the functional form

$$\gamma^i = \kappa \left(z^i\right)^{-\beta}, \tag{3}$$

where $\kappa > 0$. The welfare weight is decreasing in z given that $\beta > 0$. This is a widely used function for generating welfare weights (see e.g. Ahmad and Stern (1984), Evans (2005) and Layard, Mayraz and Nickell (2008)).[5] We have that $-\beta$ is the elasticity of the welfare weight with respect to disposable income: $el_{z^i}\gamma^i = -\beta$, and $\frac{\gamma^i}{\gamma^j} = \left(\frac{z^i}{z^j}\right)^{-\beta}$. We can interpret β as a measure of inequality aversion. When assigning welfare weights to different households one may want to allow for differences in household size. The standard method to compare different households is to deflate the income of larger households by using an income equivalence scale, which implies dividing the household disposable income by a factor given by $e(n)$ where n is the number of household members. Various equivalence scales can be employed. A common one is $e(n) = \sqrt{n}$. Where an equivalence scale is used, the z-variable will be disposable income adjusted for household size. The redistributional effect of the tax reform is welfare enhancing (diminishing) if $\sum_i^i \gamma^i g^i > 0$ (< 0), characterised as a distributional gain or loss. Deploying our weight function, we have that $\sum_i \gamma^i g^i = \sum_i \kappa \left(z^i\right)^{-\beta} g^i$. We note that the sign is independent of the value of $\kappa > 0$.

Having identified both distributive and efficiency effects, a final question is whether the overall welfare effect is beneficial or harmful. We then need to assign a value to the social efficiency gain (or loss) in terms of behavioural-induced rise (decline) in tax revenue.

The social value of this gain in general depends on how the government spends the revenue. An interesting option is a cash transfer to the taxpayers. It is a natural benchmark in the sense that it implies no change of the public sector's use of real resources. An alternative is to spend the revenue on publicly provided goods. In the event of political indifference between a cash transfer and real spending at the margin, it would obviously make no difference which alternative we consider, but where the policy makers place less value on real expenditure there would clearly be a stronger case for a cash transfer.[6] To get some information about the political comparison of alternatives, we could observe the tax level part of the tax reform. Where the reform involves a lowering of the tax level, it can be interpreted as evidence that less value is assigned to real government expenditure, which would establish a case for a cash transfer to redistribute any efficiency gain.

We shall now assume that government revenue could be recycled to the taxpayers through a lump sum transfer. To pursue this approach, suppose that an amount r of government funds is available for transfers to N taxpayers and denote by L a uniform lump-sum transfer. Since a lump-sum transfer will affect tax revenue through income effects we can write the behavioural effect of L on aggregate tax revenue as $\varphi(L)$. Then

[5] A more general weighting scheme is discussed by Saez and Stantcheva (2016).

[6] A third option would exist where the tax reform is considered as a partial reform enabling some other tax change, for example using labour income taxes to cut business taxes. Comparison with a cash transfer would again be an issue.

L must satisfy: $L = \frac{1}{N}r + \frac{1}{N}\varphi(L)$. This means that $\frac{dL}{dr} = \frac{1}{N} + \frac{1}{N}\varphi'\frac{dL}{dr}$, and $\frac{dL}{dr} = \frac{1}{1-\frac{1}{N}\varphi'}\frac{1}{N}$. When a lump sum transfer discourages labour supply, we have $\varphi' < 0$ and $N\frac{dL}{dr} < 1$. Since an initial positive lump sum transfer diminishes labour supply with a negative impact on tax revenue, the ultimate transfer that can be financed is less than the initial one. Thus, there is a revenue "leakage". When one unit of income is equally distributed among the taxpayers as a lump-sum transfer, each taxpayer receives $1/N$ units. Denote by m^i the additional tax that agent i will pay when receiving a one unit transfer. We call this agent i's marginal propensity to pay tax. The induced additional tax payments then amount to $\sum_i m^i \frac{1}{N} = \overline{m}$, and $\varphi' = \overline{m}$, which is the average marginal propensity to pay tax. Substituting for φ', $dL = \frac{1}{1-\frac{1}{N}\overline{m}}\frac{1}{N}dr$. We note that when a transfer to an agent has a negative impact on labour supply and shrinks the income tax base, the marginal propensity to pay tax is negative. Now letting the efficiency gain of the perturbation in our model accrue to the taxpayers as a uniform lump sum transfer, we set $dr = dR_b$, and $dL = \frac{1}{1-\frac{1}{N}\overline{m}}\frac{1}{N}dR_b$. The overall welfare effect is then

$$d\Omega = \sum_i \gamma^i g^i + N\overline{\gamma}dL = \sum_i \gamma^i g^i + \overline{\gamma}\frac{1}{1-\frac{1}{N}\overline{m}}dR_b. \tag{4}$$

We can find the cut-off value of β, denoted β^*, for which the perturbation is just welfare preserving, $d\Omega = 0$. To establish a link to the shadow value of government revenue, μ, introduced above, we see that $\mu = \overline{\gamma}\frac{1}{1-\frac{1}{N}\overline{m}}$. It is determined both by the mean value of the welfare weigths and the revenue leakage.

If we want to quantify the distributional gain (loss) or welfare effect of a perturbation for some value of β, it is convenient to normalise the welfare measure by setting the average welfare weight equal to unity, $\frac{1}{N}\sum_i \gamma^i = \overline{\gamma} = 1$. We have $\frac{1}{N}\sum_i \gamma^i = \frac{1}{N}\kappa \sum_i \left(z^i\right)^{-\beta} = 1$, implying that $\kappa = \frac{1}{\frac{1}{N}\sum_i (z^i)^{-\beta}}$. Then

$$\gamma^i = \frac{1}{\frac{1}{N}\sum_i \left(z^i\right)^{-\beta}} \left(z^i\right)^{-\beta}. \tag{5}$$

A marginal unit of income accruing to agent i is then valued as equal to γ^i units of equally distributed income.

2.4 The piecewise linear income tax

Next, we present our framework in terms of a more realistic piecewise linear income tax schedule to be applied in the empirical part below. A piecewise linear income tax, widely used in practice, is defined by three properties: number of tax brackets (steps), the bracket limits and the marginal tax rate in each bracket. Denote by Y_j the upper limit of bracket j. Assume there are J brackets and let $Y_J = \infty$. Denote by t_j the marginal tax rate in bracket j. Since a tax reform may introduce new tax brackets by splitting original ones it is helpful to let J be the number post-reform brackets. A bracket splitting may then be modelled by considering an original bracket as consisting of two parts with the same tax rate, say with $t_{j-1} = t_j$ for some j. Part of the reform may then be to differentiate t_{j-1} and t_j so that in the post-reform situation we have two proper tax brackets instead of one. In practice tax systems exhibit marginal tax progressivity in the sense that $t_{j-1} \leq t_j$ and $t_{j-1} < t_j$ for at least some values of j.

For a given number of (potential) tax brackets, a tax reform can change the properties of a bracket in two ways. It can change the bracket limits, and it can change the marginal tax rate t_j in bracket j. Suppose there is an increment dt_j. This will have three effects. It increases both the marginal and average tax rate on incomes in bracket j, and, furthermore, taxpayers in the brackets above will *cet. par.* face a lump-sum tax increase $\left(Y_{j+1} - Y_j\right) dt_j$. The rise in the marginal tax rate in bracket j will discourage labour supply through the substitution effect[7] while the increase in the average tax rate and the lump sum tax in the brackets beyond Y_j will, under standard assumptions, stimulate labour supply through the income effect.

It is common to model the tax schedule as comprising a universal transfer T_0.[8] Where there is a tax rate t at the lowest income levels, the net tax liability at income y is $ty - T_0$. This is zero for $y = \underline{y} = \frac{T_0}{t}$. T_0 will then be a further tax parameter set by the government. In case all active workers have earnings above $\underline{y}$, we can however model the tax schedule as having a zero marginal tax rate t_1 for $y < \underline{y} = Y_1$. Even if this is not strictly true, we may for simplicity confine attention to cases where we neglect workers with very low earnings and focus on the tax brackets 2, 3,..., with endpoints Y_2, $Y_{3,}$... above $\underline{y}$.

Obviously, a piece-wise linear income tax regime has kinks and is not differentiable everywhere. This means that the efficiency effect of a change in behaviour is different from the smooth case. Consider a kink where the discontinuous tax rate jumps from t_1 to t_2. Denote by S the marginal disutility in monetary terms incurred by increasing gross income by one unit, implying that $dh = 1/w$. For agents located optimally at the kink, $(1 - t_2) \leq S \leq (1 - t_1)$, otherwise moving to one of the segments on either side of the kink would be beneficial. The net social efficiency effect of a one unit increase in gross income is then $1 - S$, and $t_1 \leq 1 - S \leq t_2$. The social gain from an induced income change lies between $t_1 dy$ and $t_2 dy$ with a distribution between these limits. A rigorous

[7] An exception applies to agents located at kinks, i.e. at bracket limits where the tax rate is discontinuous.

[8] In practice people with zero or very low earnings typically receive transfers that vary according to the circumstances facing the respective taxpayers. There are unemployment benefits, disability benefits, sick benefits, welfare benefits, etc.

analysis should ideally take these circumstances into account, but can safely be neglected where no bunching at kink-points actually occurs. In the following we shall neglect the implications of kinks and assume that the Envelope Theorem applies everywhere.

A number of trade-offs will determine the optimal tax schedule.[9] A higher marginal tax rate in a bracket will increase the tax distortion but will shift more of the tax burden to those in tax brackets beyond the one we consider, and we have a standard equity-efficiency trade-off. Likewise, letting the higher tax rate kick in at a lower income level will raise the tax on agents beyond this point and will increase the marginal tax rate and associated distortion for some tax payers in the lower bracket. At the optimum, there must be indifference between alternative tax pertubations.

Within a standard optimal tax framework, welfare can obviously be enhanced by increasing the number of tax brackets, approaching a continuous tax schedule as the polar case. On the other hand, salience and avoiding complexity is often highlighted as a virtue of tax reforms.[10] In practice there is a fairly small number of tax brackets. Numerical examples also indicate that there are diminishing returns to the number of tax brackets: the welfare gain from adding another bracket rather quickly becomes small (Andrienko, Apps and Rees, 2016).

In this paper there is no assumption about optimality. Our interest is confined to the question whether a reform is efficiency or welfare enhancing. It may neither bring the schedule to its optimum nor be the most efficient step towards the optimum.

In our empirical analysis below, we shall employ a labour supply model with an extensive distribution of tax payers. Where a large-scale empirical labour supply model is not available, one may have to resort to a simplified procedure to get results. Before we proceed to the empirical part, it may therefore be of interest to consider a simpler approach that would enable an analysis similar to ours in the absence of our type of empirical apparatus. Following Dahlby (2008, ch. 5.2), we can make the simplifying assumption that all taxpayers in a given tax bracket are identical with income equal to the average income in the bracket.[11] By assumption there is nobody at the kinks in this simplified model.

We denote by y^j the (average) taxable income of taxpayers in bracket j and by n_j the number of agents in the bracket, where $j = 1, 2.....,J$. The tax liability of an agent in

[9]The optimal piecewise linear income tax with two tax brackets is characterised by Apps, Long and Rees (2014) and Andrienko, Apps and Rees (2016).

[10]Computing the actual payments that are due is hardly a concern with modern computer technology. Hence it may seem paradoxical that more tax brackets were used at the time when this may have been a concern.

[11]This simplification neglects both dispersion of within-bracket income and agents selecting a kink-point. A further, but less precise, simplification would be to assign to everybody in a given bracket an income equal to the mid-point of the bracket.

bracket 1 is then $R^1 = t_1\left(y^1 - Y_0\right)$. For an agent in bracket 2 it is $R^2 = t_1\left(Y_1 - Y_0\right) + t_2\left(y^2 - Y_1\right)$. For $j > 2$,

$R^j = t_1\left(Y_1 - Y_0\right) + t_2\left(Y_2 - Y_1\right) + ... + t_{j-1}\left(Y_{j-1} - Y_{j-2}\right) + t_j\left(y_j - Y_{j-1}\right)$. The aggregate tax revenue is

$$
\begin{aligned}
R = \sum n_i R^i &= n_1 t_1\left(y^1 - Y_0\right) + n_2 t_1 (Y_1 - Y_0) + n_2 t_1 (y^2 - Y_1) + ... \\
&+ n_J t_1 (Y_1 - Y_0) + ... + n_J t_2 (Y_2 - Y_1) + ... + n_J t_{J-1} (Y_{J-1} - Y_{J-2}) \\
&+ n_J t_J (y^J - Y_{J-1}),
\end{aligned}
$$

where n_i denotes the number of taxpayers in bracket i. The mechanical effect of a tax perturbation is

$$
\begin{aligned}
dR_m &= n_1 dt_1\left(y^1 - Y_0\right) + n_2 dt_1\left(Y_1 - Y_0\right) + n_2 dt_2\left(y^2 - Y_1\right) + ... \\
&+ n_J dt_1\left(Y_1 - Y_0\right) + n_J dt_2\left(Y_2 - Y_1\right) + ... + n_J dt_{J-1}\left(Y_{J-1} - Y_{J-2}\right) \\
&+ n_J dt_J\left(y^J - Y_{J-1}\right) + n_1 t_1\left(-dY_0\right) + n_2 t_1\left(dY_1 - dY_0\right) + n_2 t_2\left(-dY_1\right) + \\
&...... + n_J t_1\left(dY_1 - dY_0\right) + n_J t_2\left(dY_2 - dY_1\right) + ... \\
&+ n_J t_{J-1}\left(dY_{J-1} - dY_{J-2}\right) + n_J t_J\left(-dY_{J-1}\right).
\end{aligned}
$$

This formula collects a number of effects. Where a tax rate in a bracket rises this will increase the tax on the part of an agent's income that falls within the bracket in question. Where a bracket limit is extended, a higher tax rate will kick in at a higher income than before to lower the tax charged at all income levels beyond the previous limit.[12] For a fixed wage rate, choosing labour supply is equivalent to choosing income. We can therefore write gross income as a function of $1-t$ and income I: $y(1-t, I)$. A tax reform will change both t and I, where the former will induce substitution and the latter generates an income effect. We denote the compensated elasticity of agent j by $\xi_c^j = \left(\partial y^j / \partial\left(1 - t_j\right)\right)\left(1 - t_j\right)/y^j$ and the income elasticity by $\xi_I^j = \left(\partial y^j / \partial I^j\right) I^j / y^j$. The income change induced by a tax perturbation is then $dy^j = -\left(\partial y^j / \partial\left(1 - t_j\right)\right) dt_j + \left(\partial y^j / \partial I^j\right) dI^j$. It is also helpful to make use of a person's average tax rate defined as the person's total tax liability divided by her/his gross income. Denoting the average tax rate by τ_j, the agent will incur a real income loss equal to $y^j d\tau_j$. Inserting this term in the expression above, we get $dy^j = -\left(\partial y^j / \partial\left(1 - t_j\right)\right) dt_j - \left(\partial y^j / \partial I^j\right) y^j d\tau_j = y^j\left[-\left(\xi_c^j / \left(1 - t_j\right)\right) dt_j - \left(\partial y^j / \partial I^j\right) d\tau_j\right]$. We then obtain the shorter expression for the change in tax liability $dR^j = y^j d\tau_j + t_j dy^j = y^j\left[d\tau_j - \left(\xi_c^j t_j / \left(1 - t_j\right)\right) dt_j - t_j\left(\partial y^j / \partial I^j\right) d\tau_j\right]$, which replicates Dahlby (2008, formula 5.16). Also making use of the income elasticity,

[12] We assume tax rates are increasing in income.

we can write $R^j = y^j \left[d\tau_j - \left(\xi_c^j t_j / (1-t_j) \right) dt_j - t_j \left(\xi_I^j y^j / I^j \right) d\tau_j \right]$.

Using this simplified approach rather than an exact treatment of each and every individual, one can calculate the various effects used in the analysis when one knows the average income and tax rates in the various tax brackets, and has estimates of, or makes assumptions about, the elasticites at the relevant income levels. In that case the income derivatives or elasticities above will vanish and a further simplification is obtained. Moreover, one may only have a notion of the net of tax elasticity for a representative individual and may apply this at all income levels. Also neglecting income effects and setting $\xi_c^j = \xi$ (with no distinction between uncompensated and compensated elasticities), we obtain the change in aggregate tax revenue $dR = \sum dR^j = \sum y^j d\tau_j + \sum y^j \left(\xi_c^j t_j / (1-t_j) \right) dt_j$, where the former term on the right hand is the mechanical effect and the latter is the behavioural effect. How far one is willing to go in simplifying the analysis obviously depends on the extent to which one will accept crude results in the absence of detailed information. In this special case, formula (5) reduces to $d\Omega = \sum_i \gamma^i \left(\sum_j y^j d\tau_j - y^i d\tau_i \right) + \xi \sum_j y^j \left(t_j / (1-t_j) \right) dt_j$, where the former term on the right hand side expresses the distributional effect and the latter is the efficiency effect.

The formulas above show the effects at the intensive margin. Taking account of changes at the extensive margin, one will have to add how the number of agents in a bracket responds to changes in the tax imposed on the bracket income as more or fewer agents are induced to work.[13] In Norway, high participation rates limit the scope for positive responses at the extensive margin.[14] Estimates of the so-called ETI (elasticity of taxable income) are obviously relevant here, see the review in Saez, Slemrod and Giertz (2012).[15] However, we shall account for effects both at the extensive and intensive margins in our empirical illustration, presented below.

3 The Norwegian tax reform 2016–2018

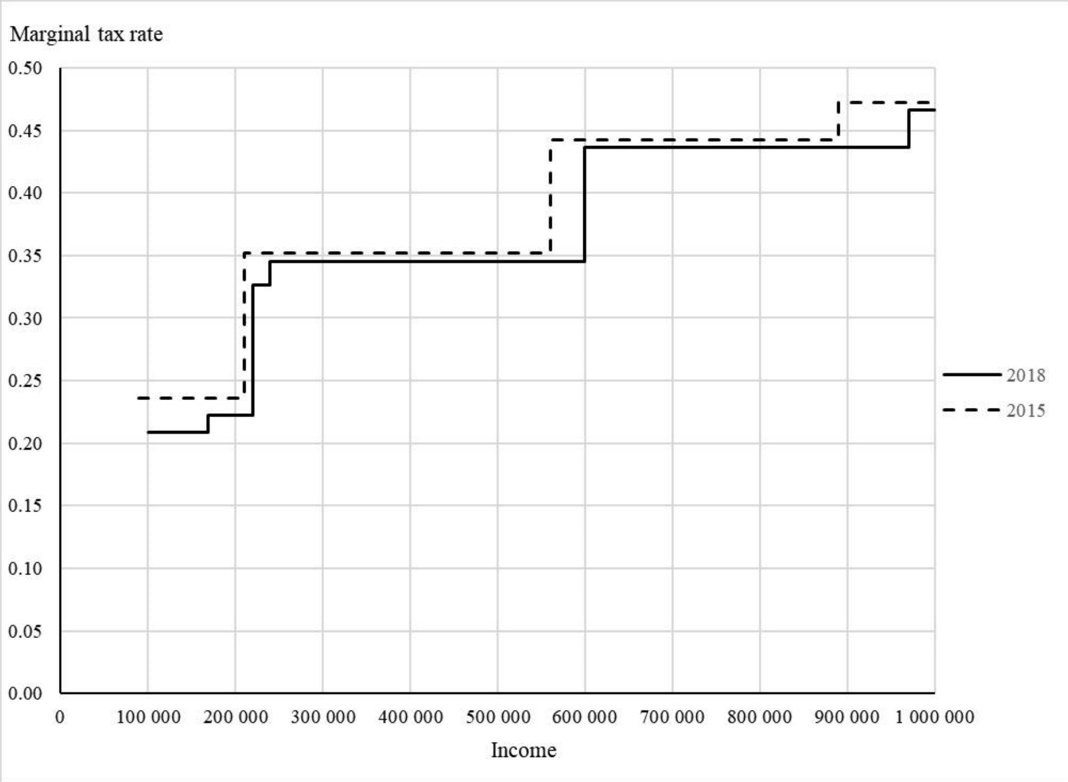
During recent decades the Norwegian tax system has undergone a number of minor and major reforms. In the current paper, we single out for analysis a particular set of reforms that can be considered as income tax perturbations. Between 2016 and 2018 the tax schedule for labour earnings in Norway was subject to to a number of perturbations. Prior to 2016, the step-wise linear income tax on labour earnings had a small number

[13] Cf. Dahlby (2008, formula 5.27)

[14] However, since participation is exclusive of persons on disability pensions, etc., the scope may be underestimated to the extent that the latter category is not entirely exogenous.

[15] In the ETI literature authors predominantly seem to neglect income effects, see Gruber and Saez (2002) and Saez, Slemrod and Giertz (2012).

Figure 1: The 2015 and 2018 tax schedules



of tax brackets, mainly characterised by a standard tax rate and two elevated tax rates referred to as "surtax" on high income. In 2016 the number of steps was increased.[16] The term "step-tax" was coined to reflect the larger number of steps distinguishing the new schedule from the previous one, and the term "surtax" was abandoned. The step-tax was then adjusted during the next couple of years. The introduction of the step-tax and the subsequent adjustments constitute the tax perturbations we study from the perspectives of social efficiency and distribution as outlined above.

In Figure 1 the schedule (as of 2018) is compared to the schedule of 2015.

4 Empirical implementation

4.1 Model tools

Our theoretical framework offers a rather general model of a population of agents supplying labour, which might comprise both wage earners and self-employed. The application in the present study is restricted by available data and estimates, and is confined to wage earners. We make use of tax simulation models developed for Norwegian policy-making,

[16]We may note that this reform direction reversed the long-term and internationally wide-spread trend towards fewer steps in the income tax.

the so-called LOTTE model system, see Aasness, Dagsvik and Thoresen (2007).

We engage the labour supply module of the model system to simulate labour supply decisions in the benchmark and in the alternative schedule, the 2015- and the 2018-system, respectively. The labour supply model is based on a discrete choice random utility framework, related to the model presented in van Soest (1995). The labour supply model employed here is a version characterised as the "job choice model", see Dagsvik et al. (2014) and Dagsvik and Jia (2016). Insofar as it gives fundamental importance to the notion of job choice, this approach differs from standard discrete choice models of labour supply, as the one in van Soest (1995). This model yields probabilities for the discrete labour supply options, both at the extensive and intensive margins.

The model is estimated by micro data from the Norwegian Labor Force Survey, deriving three separate submodules: a joint model for married couples and two separate models for single females and males. It is exploited that the labour supply module, LOTTE-Arbeid, interacts with the non-behavioural tax-benefit module, LOTTE-Skatt, which means that we have access to a detailed description of the Norwegian tax schedule. Although the theoretical framework departs from a continuous choice, we interpret the empirical model as a reasonable approximation to the theoretical one.[17]

Moreover, we shall also account for the interaction between different tax bases by also controlling for the effect working through the indirect taxation. More precisely, when we calculate the efficiency effect of the perturbation, see Equation (4), we use the module LOTTE-Konsum (Aasness, Dagsvik and Thoresen, 2007) to calculate the indirect tax part of a change in disposable income, resulting from the labour supply effects. This raises the question of the marginal propensity to consume. Here, we simply assume that agents do not save, thus the MPC is 1.[18] Revenue effects of labour supply adjustments also account for payroll tax revenues being affected. Norway has a regionally differentiated payroll tax, which means that tax rates range from 0 to 14.1 (in 2018); we apply an average tax rate, at approximately 13.2 percent.

4.2 Empirical estimates

Recall that we apply a step-wise procedure to identify the welfare effects of the reform, distinguishing between the mechanical effect, the behavioural effect, and effects on overall welfare. The first effect, the mechanical effect, as described by the first part of Equation (4), is the change in tax burden when behavioural effects are neglected by virtue of the envelope theorem. We therefore obtain estimates of the mechanical effect by keeping

[17] Alternatively, one might argue that choices are indeed discrete, and it is the theoretical model that should be perceived as an approximation to reality.

[18] See Thoresen, Aasness and Jia (2010) for further discussion on this.

labour supply behaviour fixed, as given by the tax rules of 2015, and derive individual tax burden differences caused by the reform by applying the tax rules of 2015 and 2018 on the same fixed income.[19] As the 2018-schedule diminishes the tax burden compared to the 2015-schedule, we control for the tax level effect by imposing a hypothetical lump sum tax that would offset the average tax cut. Each household would then be charged approximately NOK 6,000 lump sum[20]. We are then left with purely redistributive effects, where those given a tax relief above NOK 6,000 by the actual reform are winners, and others are losers due to the structural reform. Whereas, the (net) changes in tax burdens are measured in actual values, note that z^i of Equation (3) is measured in terms of equalized income, where we have used the square-root of the number of household members as the equivalence scale.[21]

Figure 2 presents the distribution of the net gain, defined by the difference in tax burden between the two schedules minus the lump sum tax, when households are ranked by pre-reform equivalized disposable income. It follows that for each agent the net gain is the individualised mechanical effect of the reform net of the lump sum tax that offsets the mechanical effect on average. The actual reform involves tax cuts in all parts of the piece-wise linear schedule, see Figure 1, but the substantial reductions occur at the high end of the distribution. The diagrams of Figure 2 reflect this: taxpayers with negative or small positive overall effect are predominantly found at the low end of the income distribution, whereas large gains are mostly found at the top end.

By definition, pure redistribution means that the sum of gains equals aggregate losses. We refer to the welfare effect of pure redistribution as the (total) distributional effect. Obviously, this effect is zero if all (positive and negative) income changes are given equal weight in the welfare assessment. It is trivial that this would happen only if there is no inequality aversion, i.e. the value of β is zero. We denote this threshold value by $\overline{\beta}$. For other values of β, there will be a strictly positive or negative distributional effect. The β-function shows the distributional effect of the reform for more or less inequality aversion.[22] We shall soon return to what this β-function may look like (in Figure 4).

As discussed in the theoretical part, the efficiency effects of the structural reform are determined by the labour supply effects. As above, we cleanse out the level effect to obtain estimates of the behavioural responses to structural changes. The average labour

[19]The 2018 tax rule is deflated to the 2015-level by using a wage growth index.

[20]This was equivalent to approximately 750 US dollars or 670 euros at the average exchange rates in 2015.

[21]We have also derived empirical estimates based on a framework founded on individual income; thus, no income accumulation across household members and therefore no need for equivalences scales. Of course, this gives other estimates of the inequality aversion in the benchmark case (no effects of the reform on distribution and welfare) – estimates that we soon will return to.

[22]By including negative values of β , we also show for completeness the less plausible cases where there is equality aversion.

Figure 2: Direct distributional effects of the reform

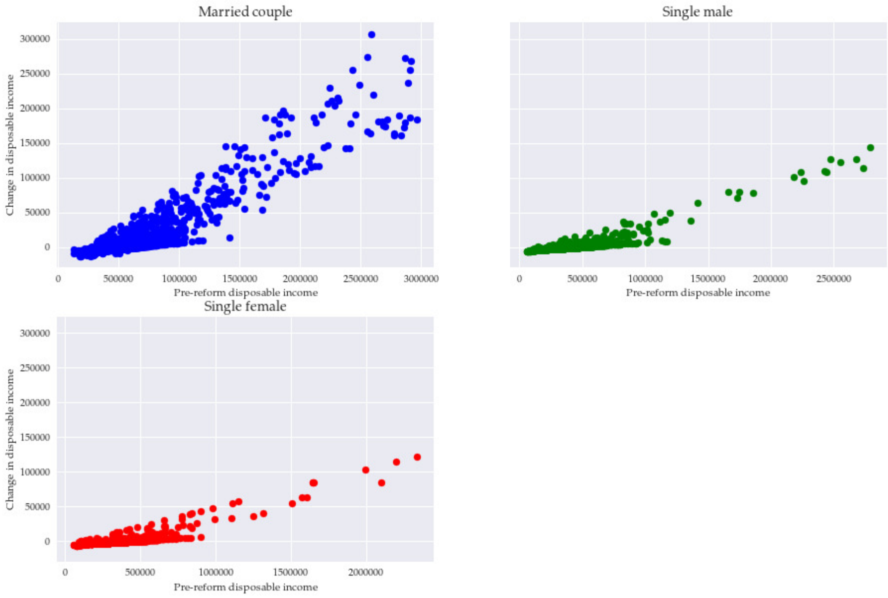


Table 1: Average predicted yearly working hours according to the respective 2015 and 2018 tax schedules

	2015	2018
Married female	1732	1747
Married male	1982	1985
Single female	1810	1822
Single male	1793	1805

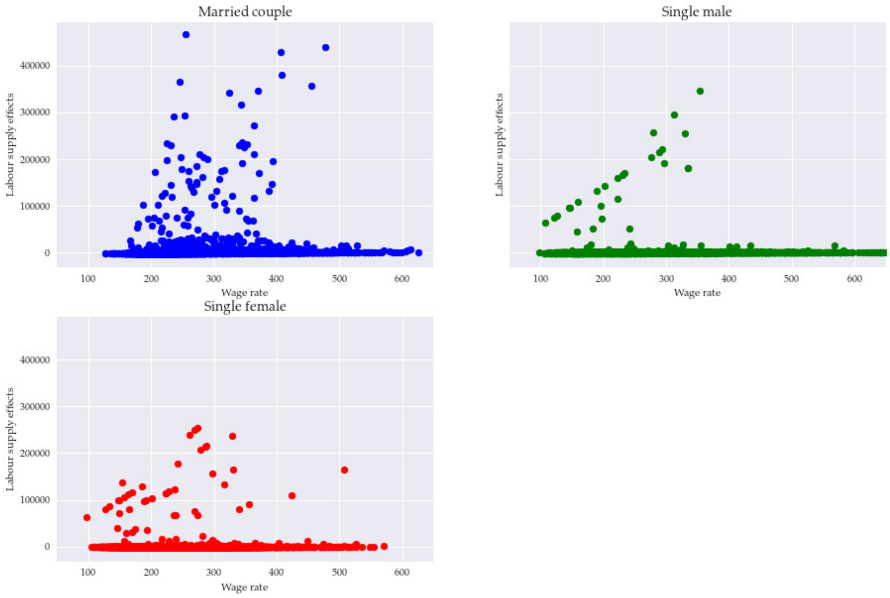
supply effects, measured in annual working hours, are presented Table 1.

In total, these effects imply that the tax revenue (from the personal income tax, the payroll tax and indirect taxation) increases by approximately NOK 2 billion. This is the behavioural-induced change in tax revenue, which is our measure of the social efficiency gain.

Furthermore, in Figure 3 we show how the gains in NOK due to labour supply responses measured by working hours in the three different subgroups. We see that most individuals do not alter their choice of working hours. All three diagrams display modest gains.[23] In the Appendix we also consider how efficiency gains vary across levels of education. We find that the effect of the reform is somewhat larger for the highly educated

[23]However, we see that there is substantial heterogeneity in the behavioural responses, in particular for married couples. The modeling approach generates such patterns by (for instance) allowing for taste-modyfying characteristics in the empirical approach, such as letting responses vary with respect to education, number of children, etc.

Figure 3: Distribution of gains from labour supply effects with respect to hourly wage



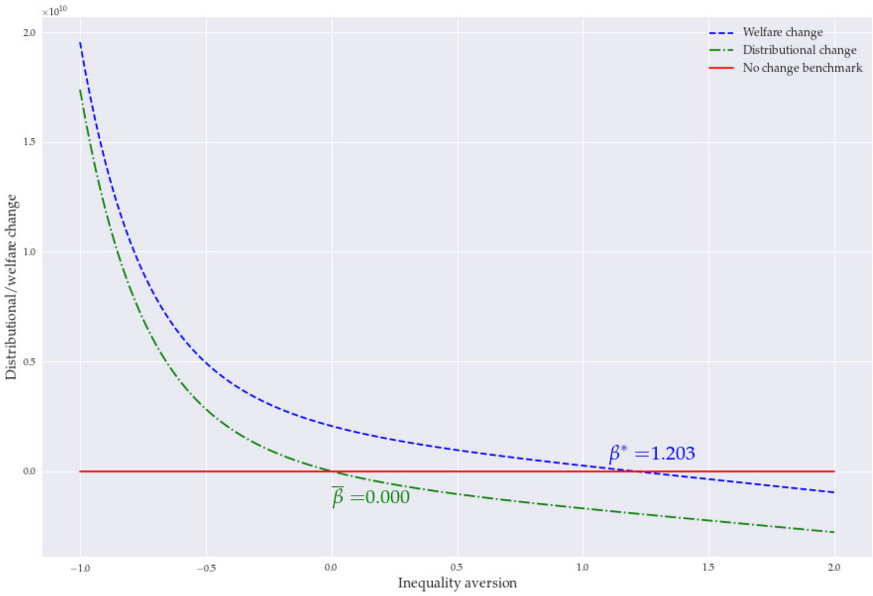
taxpayers, as shown in Figure A1.

As mentioned above, the error committed by assuming away the effects of kinks is minor where there is no bunching at the kinks. In the appendix, Figure A2 and Table A1, we consider the distribution of taxpayers around the kinks where the respective surtax rates kicked in according to the tax rules of 2015. Figure A2 shows no signs of bunching. Also taking into consideration that there are taxpayers who fail to hit the exact kinkpoint, as discussed by Chetty (2012), we see from Table A1 that the fraction of taxpayers around each of the thresholds is tiny given that there are aproximately 2.6 million individuals with wage income above NOK 50,000 (approx. 5,600 euros and 6,200 US dollars).

Finally, by using Equation (5) $d\Omega = \sum_i \gamma^i g^i + N\bar{\gamma}dL$, we combine the mechanical effect and the efficiency effect to find the cut-off value of β, denoted β^*, for which the perturbation is just welfare preserving, $d\Omega = 0$. Now, the revenue from the efficiency effect of the reform is given back to the agents in terms of lump sum transfers. In this transformation we also control for the labour supply effects working through the income effect on recipients of lump sum transfers. Figure 4 describes how an estimate of β^* is obtained, where we also display the "distributional change curve".[24] Including the effi-

[24]Here, β^* is backed out with the use of a discrete choice labour supply model, but it could also been obtained by employing a reduced form estimate of response, such as one represented by the elasticity of taxable income. However, as the elasticity of taxable income (usually) only captures intensive margin responses (Saez, Slemrod and Giertz, 2012), one has to employ other evidence or make assumptions with respect to the extensive margin responses.

Figure 4: Distributional gains and welfare gains as a function of β



ciency part implies that we obtain a new curve for the welfare change with the same shape as the distributional change curve, but moved upward by the same vertical increment all along the scale. Since the tax reform enhances social efficiency, there is a positive effect counteracting the negative effect of redistribution according to inequality-averse preferences. The allocative efficiency gain will be the overriding effect even for strictly positive inequality aversion ($\beta > 0$) as long as it falls short of a cut-off value where the welfare loss due to unfavourable redistribution and the social efficiency gain just cancel out. This "no-effect-of-the-reform" benchmark occurs for the inequality aversion $\beta^* = 1.2$.[25]

To put our result in perspective, it is of interest to note that the literature on the inequality aversion parameter has taken a number of approaches, ranging from presentation of purely illustrative examples to estimations and discussions of what may be "appropriate" values. Various strands of literature conceive of the β-parameter (in our notation) as either directly reflecting political preferences or originating from various more or less related sources, which can be pure political preferences or measures of individual utility, possibly adopted by political decision makers. In either case, β is usually interpreted as the elasticity of people's marginal (social) utility of income. Going a long way back, Dalton (1939) argued that β was greater than 1. A study of the British income taxation, reported in Stern (1977), suggested that a value around 2 seemed to give tax rates

[25]This estimate is little influenced by the choice of equivalence scale. Table A2 in the appendix reports estimates of β^* for other assumptions about equivalence scale.

not too dissimilar to those existing in the UK. Taking an inverse optimum (or implicit preference) approach, Christiansen and Jansen (1978) found a value close to 0.9 in their preferred version. Evans (2005) provides a survey of previous estimates, and itself offers an estimate of 1.4. Based on a number of surveys, Layard, Mayraz and Nickell (2008) arrived at a preferred estimate equal to 1.26. Applications in cost-benefit analyses have used many different values. The guidance of the UK Treasury has a preference for using 1, but one can find cases where analysts have used values up to 2 or 2.5, see for example Freeman, Groom and Spackman(2018), Groom and Maddison (2019) and HM Treasury (2003, 2018). With these findings in mind, we may conclude that a value of around 1.2 finds its place towards the lower end of the range of values appearing in the literature but without deviating substantially from numbers that are quite common. It follows that the considered tax reform is welfare enhancing only for a moderate inequality aversion.

We interpret the cut-off value $\beta^* = 1.2$ as conveying information about the politicians' implicit distributional preferences, where approval of the reform is taken as evidence that the decision-makers have a lower inequality aversion, and dismissal of the reform indicates a higher inequality aversion. This reform approach to reveal implicit preferences bears close resemblance to the inverse optimum approach referred to above, which is used to infer the preferences that are consistent with the actual policy, assuming that the latter is optimal given the preferences. The inference from reform analysis is less accurate since it does not yield a single estimate[26], but only conveys information about a range of preferences, such as the implicit inequality aversion being less than β^*. In either case, a number of assumptions must be satisfied for the inference to be meaningful: the underlying model and the estimates of behavioural responses derived by the analyst must be sufficiently reliable and shared by the politicians, who must also not be governed by other concerns.

5 Conclusion

We have analysed a real tax reform in Norway based on a scheme for assessing an income tax perturbation. In practice, a tax reform will consist of both a change of tax level and a change of tax structure, i.e. slope and progressivity of the tax schedule. We cleanse out the level effect by adjusting a hypothetical lump sum tax to isolate the structural change. We conceive of the impact of the structural change as consisting of distributional effects and social efficiency effects, which taken together yield an overall welfare effect. These effects are closely related to the tax perturbation effects that are referred to as mechanical effects, behavioural effects and welfare effects. Mechanical effects are effects on tax

[26]Of course, also a single estimate is "inaccurate" in the sense that it has a confidence interval.

payments and tax revenue in the absence of any change in labour supply and commodity demand. Invoking envelope properties, behavioural changes have no direct first order effects on utility, and the real income effects experienced by the taxpayers are identical to the mechanical effects. These effects will therefore reflect the distributional gains and losses of various taxpayers. To find the ensuing welfare impact one would have to assign welfare weights to the respective gains and losses.

Since there are pre-existing tax distortions, behavioural effects will affect social efficiency. Both direct and indirect taxes cause under-consumption of all commodities apart from leisure. Where a tax reform enhances labour supply and consumption of taxed commodities, a more efficient allocation is achieved. The increase in tax revenue due to behavioural changes is a measure of the allocative efficiency gain, while revenue foregone would reflect a loss. The overall welfare effect, capturing both allocative efficiency and welfare effects of redistribution, depends on the value of the use of additional tax revenue. An option is to recycle the extra tax revenue through a uniform lump sum transfer. We can then find the gains and losses of various taxpayers due to the combined distributional and efficiency effects, and we can find the welfare weights that yield a positive or negative overall welfare impact.

We have applied the theoretical approach outlined above to the actual tax perturbations implemented in Norway in the period 2016–2018. We use households as units. Household welfare is assumed to depend on disposable income per consumer unit, where the number of units is determined by an equivalence scale. The welfare weights assigned to marginal income are then determined by equivalent income. We subscribe to the widely held view that additional income is more highly valued if accruing to a larger household than if given to a smaller household with the same income. We can interpret the key parameter that determines the welfare weight corresponding to each (equivalent) income level as a measure of inequality aversion.

The structural reform in Norway redistributes income in favour of better-off households. This means that there is an equity loss according to distributional preferences exhibiting inequality aversion. On the other hand, the tax reform induces behavioural changes that increase tax revenue and enhance allocative efficiency. In this sense, we face the frequently highlighted trade-off between equity and efficiency. The combination of a distributional loss and an allocative efficiency gain obviously yields an overall welfare gain only if a sufficiently moderate inequality aversion prevails. Our empirical finding is that the overall welfare gain created by the reform is positive if the value of the inequality parameter is less than 1.2, which is considered as an inequality aversion in the medium range. One finds both lower and higher values in various contexts in the literature. Assuming that a tax reform is implemented only if it is considered beneficial according to the prevailing political preferences, we can infer, from a revealed preference perspective, that the inequality aversion underlying the political reform decision is less than the threshold value of 1.2.

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A Table and figure appendix

Table A1: Number of taxpayers around the two kink point points of the surtax schedule, 2015

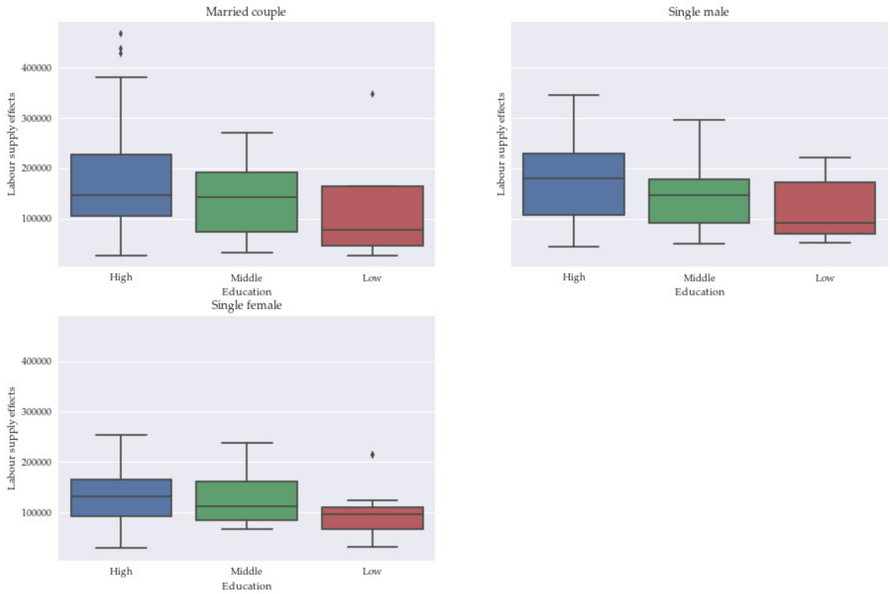
	Income interval around kink point (x)			
Kink point (x)	$(x-2000, x-1000)$	$(x-1000, x)$	$(x, x+1000)$	$(x+1000, x+2000)$
560,000	3683	3762	3792	3721
890,000	666	700	642	634

Table A2: Estimates of threshold value for inequality aversion for alternative choice of equivalence scale

	Benchmark eq. scale	Alternative equivalence scale			
Household Size	Square root scale	Per person	OECD scale	EU scale	Household
1 adult	1	1	1	1	1
2 adults	1.4	2	1.7	1.5	1
2 adults, 1 child	1.7	3	2.2	1.8	1
2 adults, 2 children	2.0	4	2.7	2.1	1
2 adults, 2 children	2.2	5	3.2	2.4	1
β^*	1.203	1.529	1.323	1.224	1.027

Note: A selection of equivalence scales used in the literature

Figure A1: Labour supply effects with respect to education



Notes: Box plot of labour supply effects of reform, measured in NOK, for individuals who have adjusted their labour supply because of the reform. Band inside the box represents the median and whisker boundaries defined by interquartile range.

Figure A2: Description of wage income density, 2015