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DISCUSSION PAPERS

967

Discussion Papers No. 967, November 2021
Statistics Norway, Research Department

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Abstract:
The recent boom in house prices in many countries during the Covid-19 pandemic and the possibility of household financial distress are of concern among some central banks. We revisit the empirical modelling of house prices and household debt with a policy-oriented perspective using Norwegian data over the last four decades within the cointegrated VAR model. Our findings suggest, in line with previous work, a long-run mutually reinforcing relationship between these financial magnitudes, and thus the potential for the build-up of financial instabilities and spillover effects to the real economy. Applying a control analysis, we find that both house prices and debt are controllable magnitudes to some pre-specified target levels through the mortgage interest rate, which enables the central bank to reduce large fluctuations and bubble tendencies in the housing market. The present control analysis thus provides some useful policy implications from empirically relevant representations of two important financial factors entering the decision process of the policy maker.

Keywords: House prices, household debt, econometric modelling, cointegrated VAR, policy control analysis, simulation

JEL classification: C32, C53, E52, R21

Acknowledgements: We are grateful to seminar participants at the Centre for Applied and Theoretical Econometrics at BI Norwegian Business School, Ragnar Nymoen, Victoria Sparrman and Genaro Succarat in particular, for helpful discussions, and to Thomas von Brasch, Ådne Cappelen, Eilev Jansen, Terje Skjerpen and Anders Rygh Swensen for comments and suggestions on earlier drafts. Takamitsu Kurita gratefully acknowledges financial support from JSPS KAKENHI 18K01600. The usual disclaimer applies.

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ISSN 1892-753X (electronic)

Sammendrag

Mange land har under Covid19-pandemien opplevd sterk boligprisvekst og tilhørende økning i husholdningenes gjeld. Det kan gi sentralbanker utfordringer i avveiningen mellom å sette opp rentene i takt med en gradvis normalisering av realøkonomien og holde igjen renteøkningene for ikke å øke rentekostnadene kraftig for husholdningene med de ringvirkningene det kan få på både boligpriser og realøkonomi. I denne artikkelen ser vi nærmere på sammenhengen mellom boligpriser og husholdningenes gjeld i Norge. Det gjør vi med utgangspunkt i statistiske metoder og kvartalsvise data som går tilbake til begynnelsen av 1980-tallet. Vi finner, som i tidligere studier på norske data, en gjensidig forsterkende sammenheng mellom boligpriser og gjeld på lang sikt og således muligheten for oppbygging av finansielle ubalanser og negative spillover effekter til realøkonomien. Samtidig indikerer våre analyser at både boligpriser og gjeld er kontrollerbare størrelser rundt noen spesifikke målnivåer gjennom boliglånsrentene som husholdningene står overfor. Våre funn tyder på at sentralbanken til en viss grad kan redusere store fluktuasjoner og bobletendenser i boligmarkedet gjennom rentepolitikken.

1 Introduction

The substantial deregulation of housing markets in the 1980s enhanced the exposures from imbalances in these markets to the real economy in many countries. The role of the housing market in the real economy depends *inter alia* on the transmissions of wealth effects to private consumption, see e.g. Aron *et al.* (2012) and Mian *et al.* (2013), the interactions with the credit market and the households' access to mortgage credit, see e.g. Fitzpatrick and McQuinn (2007) and Gimeno and Martinez-Carrascal (2010), and the residential investments of entrepreneurs, see e.g. Leamer (2015) and Aastveit *et al.* (2019). For an overview of main transmission channels from the housing market to the real economy of monetary policy, see e.g. Boivin *et al.* (2011).

Over the past three decades, there are several examples in which imbalances in the housing market have substantially affected the real economy. The banking crisis in Norway that took place in a five years period from 1988 is a clear example where a collapse in house prices was followed by a long-lived recession in the real economy and more than a doubling of the unemployment rate, see Vale (2004) and Eitrheim *et al.* (2016, chapter 14). The global financial crisis in 2008 with a significant bust in house prices in many countries and a subsequent deep downturn in the world economy is another example emphasising the spillovers from the housing market to the real economy, see e.g. Mian *et al.* (2013). Recently, a house price boom is experienced in many economies during the Covid-19 pandemic and some central bankers worry about financial distress among households and thereby the possibility of amplifying effects on the ongoing real economic downturn.[1]

Against this background, the monetary authorities' ability to contribute to a more stable development in financial magnitudes, including house prices and household debt, is potentially important in reducing fluctuations in the real economy. The question then is whether the authorities can influence the housing market, say by means of the key policy rate, in order to stabilise both house prices and household debt as a part of stabilising inflation and GDP growth. We investigate this question empirically using a control analysis within the context of a cointegrated VAR model and Norwegian data over the last four decades. The question of empirical controllability may not after all be obvious from the authorities point of view given the literature showing that changes in the fundamentals cannot account for the large fluctuations in house prices in many countries, see e.g. Bolt *et al.* (2019) and the references therein. We therefore believe it is worthwhile to show quantitatively whether house prices and household debt are controllable variables by analyzing real-life time series data.

As in many other countries, Norway has an independent central bank. During the last two decades, from 2001, the monetary policy in Norway has been geared to stabilising inflation. The monetary policy shall also contribute to high and stable production and employment and counteract the build-up of financial imbalances. In

[1] See https://www.economist.com/graphic-detail/2021/04/09/house-prices-boom-despite-the-pandemic.

its decisions on the key policy rate, Norges Bank often monitors the development in house prices and household credit to assess the status of the financial stability conditions, see e.g. Norges Bank (2021). Since housing wealth constitutes the main part of households' total wealth, the home-ownership rate is around 80 per cent and the share of floating interest rates on mortgage to households is close to 95 per cent in Norway, fluctuations in house prices, debt and interest rates usually translate into corresponding changes in private consumption, see Boug *et al.* (2021).[2] The central bank's ability to control house prices and household debt is therefore essential for the wealth effects on consumption, and thereby on GDP.

The empirical literature on the housing market, and in particular on the fundamental drivers of house prices, is overwhelming, see Duca *et al.* (2021) for a recent and comprehensive overview. The Norwegian housing market has been studied in Bjørnland and Jacobsen (2010), Anundsen and Jansen (2013), Robstad (2018) and Anundsen (2019, 2021) among others. The econometric approaches, sample periods and data sets vary across these studies. Applying a structural VAR in GDP, inflation, domestic and foreign interest rates, the real exchange rate and the real house prices for the period from 1983 to 2006, Bjørnland and Jacobsen (2010) find that house prices react immediately and strongly to a temporary monetary policy shock of a one percentage point increase in the interest rate. After about ten years, house prices are reduced by approximately 1.5 per cent due to the monetary policy shock. Robstad (2018) extends the analysis in Bjørnland and Jacobsen (2010) by including household credit in real terms as an extra variable in the model. Overall, this study covering the period from 1994 to 2013 supports the findings in Bjørnland and Jacobsen (2010), while the household credit response to a monetary policy shock is relatively modest. By way of contrast, Anundsen and Jansen (2013) find a mutual long-run dependency between real house prices and debt over the period from 1985 to 2008 in the context of a partial cointegrated VAR which also includes household real disposable income, housing stock and the real after-tax interest rate. Anundsen and Jansen (2013) show that a permanent increase in the interest rate of one percentage point leads to a decrease in house prices of about 10 per cent in the long run when the housing stock is fixed. Likewise, by linking real house prices to household real disposable income, the housing stock and the real after-tax interest rate in a cointegrated VAR covering the period from 1986 to 1999, Anundsen (2019) finds a somewhat higher long-run negative house price response with respect to the interest rate of about 14 per cent.[3]

In this paper, we revisit the empirical modelling of house prices and debt in Anundsen and Jansen (2013) with a policy-oriented perspective. Our contribution is threefold. First, we set up a theoretical model similar to Anundsen and Jansen (2013), but extend the data set by one decade and model house prices, debt, dispos-

[2]See https://www.ssb.no/en/bygg-bolig-og-eiendom/bolig-og-boforhold/statistikk/boforhold-levekarsundersokelsen and https://www.ssb.no/en/bank-og-finansmarked/finansinstitusjoner-og-andre-finansielle-foretak/statistikk/renter-i-banker-og-kredittforetak (Table 2).

[3]In a similar model set-up, but with both household disposable income and housing stock in per capita terms, Anundsen (2021) finds that the long-run house price (semi) elasticity with respect to the interest rate is 11 per cent.

able income, the housing stock and the interest rate jointly using a full cointegrated VAR. Applying likelihood-based methods, we identify two cointegrating relationships which are interpreted as a long-run house price relation and a debt relation similar to those in Anundsen and Jansen (2013). In particular, due to the mutual long-run dependency between house prices and debt, we find that a permanent increase in the interest rate of one percentage point leads to a decrease in house prices of about 15 per cent when the housing stock is fixed. Hence, extending the sample period by ten years after the financial crisis with relatively low, and sometimes even negative interest rates (in real terms) on mortgage credit, pushes the long-run house price elasticity upwards by about 5 percentage points compared to the findings in Anundsen and Jansen (2013).

Then, we calculate the so-called persistence profiles for the two cointegrating vectors, relying on the methods in Pesaran and Shin (1996), to provide estimates of the speed with which the housing market returns to its equilibrium state after a system-wide shock. As pointed out by Pesaran and Shin (1996), the persistence profile approach is invariant to the way shocks in the underlying cointegrated VAR are orthogonalized, which is not true of the traditional impulse response analysis using a structural VAR. We find that around 80 per cent and 90 per cent of the adjustments towards equilibrium after a system-wide shock are made after six years in the cases of the house price relation and the debt relation, respectively. The slow speed of adjustment may be attributed to costly information gathering of housing opportunities in the market and government-imposed restrictions on housing trade by *inter alia* lending criteria based on payment-to-income ratios. Nevertheless, the fact that both cointegrating relationships eventually converge to zero provides further evidence that the house price relation and the debt relation indeed represent cointegrating relationships in line with the theory.

Finally, we apply the control analysis of non-stationary time series, as originally proposed by Johansen and Juselius (2001), see also Kurita (2018), to analyse whether the interest rate (real after-tax) can be used as an instrument in controlling house prices and debt within the cointegrated VAR model. Our control analysis thus addresses cointegration properties among non-stationary time series, an issue which is neglected in existing studies of the Norwegian housing market using a standard-type impulse response analysis of monetary policy based on a structural VAR. We are, of course, fully aware that the real after-tax interest rate cannot be treated as an explicit policy instrument by Norges Bank in the real world. However, Hungnes (2015) shows that a long-run one-to-one relationship exists between the money market rate, which is closely related to the key policy rate, and the interest rate on mortgage credit. Hence, the key policy rate in our modelling framework works implicitly through both components of the real interest rate; the nominal interest rate on household loans and the inflation rate. We demonstrate that both house prices and debt are controllable magnitudes to some pre-specified target levels through the real after-tax interest rate, which enables the central bank to reduce large fluctuations and bubble tendencies in the housing market. The present control analysis thus provides some useful policy implications from empirically relevant representations of two important financial factors entering the decision process of Norges Bank.

The rest of the paper is organised as follows: Section 2 outlines the theoretical background, Section 3 presents the data, Section 4 reports the findings from the cointegration analysis and Section 5 examines the empirical controllability of house prices and debt and conducts the policy simulation study. Section 6 provides some conclusions.

2 Theoretical background

In this section, we present an economic model for the interaction between house prices and debt, which serves as an impetus for the study of the Norwegian data. We follow Anundsen and Jansen (2013) in the formulation of the model, which is presented here as a simplified linear version so as to be able to directly test the validity of theoretical long-run economic relationships in the empirical framework in Section 4.

Based on the commonly used life-cycle model of housing, see e.g. Meen (1990), Mullbauer and Murphy (1997, 2008) and Anundsen (2019), augmented with the presence of credit constraints and operationalised with measures for unobservable theoretical variables, Anundsen and Jansen (2013) arrived at the following inverted demand function $f(\cdot)$ for real housing stock K_t:

$$P_t = f(D_t, Y_t, K_t, R_t), \tag{1}$$

where P_t denotes house prices, D_t and Y_t represent household debt and disposable income, respectively, and R_t is a tax-adjusted interest rate on mortgage credit, while noting that all the variables in (1) are expressed in real terms.

Given the real housing stock supply, we may also interpret (1) as the price level that clears the housing market, depending on the remaining factors D_t, Y_t and R_t. Anundsen and Jansen (2013) argue the partial derivatives

$$\frac{\partial f}{\partial D_t} > 0, \quad \frac{\partial f}{\partial Y_t} > 0, \quad \frac{\partial f}{\partial K_t} < 0 \quad \text{and} \quad \frac{\partial f}{\partial R_t} \lesseqgtr 0,$$

which say that house prices are increasing in both household debt and disposable income, and decreasing in the housing stock. The sign of the partial derivative with respect to the interest rate is ambiguous as the substitution effect between consumption of housing and other consumer goods underlying the life-cycle model may be either positive or negative. In view of the empirical analysis conducted below, we allow for the possibility that some of these derivatives are zero.

The log-linearised version of (1), augmented with a stochastic disequilibrium error υ_t, is given as

$$p_t = c_p + \theta_d d_t + \theta_y y_t - \theta_k k_t - \theta_R R_t + \upsilon_t, \tag{2}$$

where lowercase letters indicate that the variables, except the interest rate, are log-transformed, c_p is a constant, the coefficients θ_d, θ_y and θ_k are expected to be *non-negative* and the sign of θ_R is, as aforementioned, theoretically ambiguous.[4] This log-linearised equation embodies a static long-run equilibrium, which can be seen as a candidate for a cointegrating combination between the observable variables. We are therefore justified in the analysis of the Norwegian data in exploring the possibility that

$$p_t - \theta_d d_t - \theta_y y_t + \theta_k k_t + \theta_R R_t - c_p = \upsilon_t \sim I(0), \tag{3}$$

where $I(j)$ denotes j-th order of integration, so that $I(0)$ implies a stationary process.

Furthermore, Anundsen and Jansen (2013), inspired by Fitzpatrick and McQuinn (2007) among others, introduced the equilibrium condition for D_t, the log-linearised version of which can be presented, after some simplifications, as

$$d_t = c_d + \phi_p p_t + \phi_y y_t + \phi_k k_t - \phi_R R_t + \nu_t, \tag{4}$$

where c_d is a constant, ν_t is a stochastic error term and ϕ_p, ϕ_y, ϕ_k and ϕ_R are all expected to be *non-negative*.[5] Accordingly, banks may agree to provide more mortgage if households have more collateral, higher income or face lower interest expenses. Again, this equation is subject to cointegration analysis, in which we investigate whether or not

$$d_t - \phi_p p_t - \phi_y y_t - \phi_k k_t + \phi_R R_t - c_d = \nu_t \sim I(0). \tag{5}$$

Since we have a set of two candidates for the underlying long-run relationships, there are several directions conceivable in the cointegration analysis. If test statistics indicate a single cointegrating combination, for example, we will then find it important to check whether the relationship can be interpreted as the empirical counterpart of either (3) or (5). In this case, the sign of the coefficient for k_t and also for R_t if positive will play critical roles in the identification of (3) or (5), since they are opposite in these candidate long-run conditions. The revealed structure of adjustment towards a long-run relationship will also shed some light on the identification problem, see Johansen and Juselius (1994).

If the tests show evidence for two cointegrating combinations, as in Anundsen and Jansen (2013), we will have to explore various coefficient restrictions in such a way that the empirical relationships can be identified and interpreted as (3) and (5). We note that neither (3) nor (5) is distinguishable in a system of equations as they stand, since no identifying restrictions are pre-imposed on the coefficients at this stage. The arguments so far lead us to consider a vector of variables, X_t, defined as

$$X_t = (p_t, d_t, y_t, k_t, R_t)',$$

[4] R_t is not log-transformed as this variable can take negative numbers during the sample period.

[5] Anundsen and Jansen (2013) also included housing turnover as an additional explanatory variable in the equilibrium condition for D_t. We assume here that effects from changes in sales turnover are reflected in both house prices and debt.

which is to be modelled as a full VAR as a point of departure for the cointegration analysis instead of a partial VAR employed from the outset by Anundsen and Jansen (2013). We thus assign importance to the fact that all the variables in X_t are inherently endogenous in an overall macroeconomic system. By opting for a full system if feasible in the empirical analysis, we follow the procedure recommended by Juselius (2006, p. 198) when estimating the long-run parameters of interest. Adopting the joint-model framework is also advantageous in that we can naturally estimate a long-run impact matrix required for the subsequent cointegrated VAR-based control analysis. Such an analysis enables us to examine the controllability of policy target variables through instrument variables and allows us to conduct various policy simulations. Further details will be provided in Section 5. Having established the theoretical background for the empirical analysis, we now turn to an overview of the data with particular attention to any observed relationships between p_t, d_t, y_t, k_t and R_t and their time series properties.

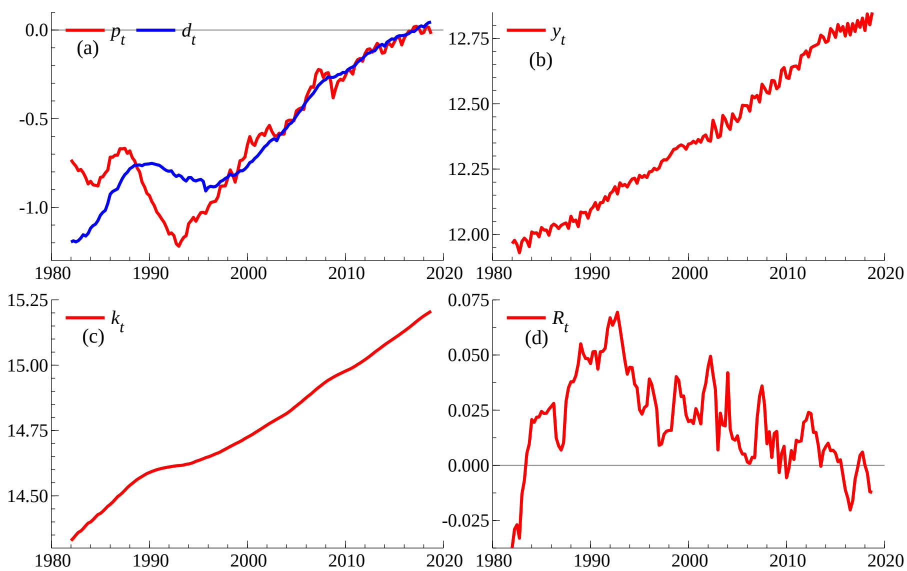
3 Overview of data

The empirical analysis is based on quarterly, seasonally unadjusted data, published by Statistics Norway, that span the period $1982q1 - 2018q4$. We measure the real house prices, p_t, by the overall price index for residential buildings in the second-hand market; the household real debt, d_t, by the total amount of outstanding gross household debt at the end of the quarter; and the household real disposable income, y_t, by the disposable income excluding dividend payments. All of these variables in nominal terms are adjusted by the consumption deflator in the National Accounts. The real housing stock in fixed 2018-prices, k_t, measures the total stock of housing at the end of the quarter and is in the National Accounts calculated by means of the perpetual inventory method. Finally, we measure the real after-tax interest rate, R_t, by the average nominal interest rate paid by households on loans in private financial institutions net of the capital tax rate and adjusted by the fourth-quarter change in the consumer price index.[6]

Our chosen sample period is justified on several grounds. As described in Krogh (2010), the housing and capital markets were heavily regulated during the 1970s and early 1980s, which likely prevented the existence of a self-reinforcing relationship between house prices and credit. The housing market regulations, both with regard to quantities and prices, ended during the first half of 1982. The period of liberalisation of the credit markets took place in several steps between 1982 and 1985 to allow for competition among lending institutions in the household segment. As a result, an incipient boom in the real estate market was evident during the deregulation period, which was made possible and financed through credit expansion to households. We therefore argue that a self-reinforcing relationship between house prices and household debt was appearing during the deregulation period, and choose $1982q1$ as the effective starting point for estimation purposes. Noticeably, our

[6]See the Appendix for further details on variable definitions, data descriptions and sources.

Figure 1: Real house prices (p_t), household real debt (d_t), real disposable income (y_t), real housing stock (k_t) and real after-tax interest rate (R_t)



Notes: Sample period: $1982q1 - 2018q4$. The household real debt matches mean and range to the real house prices (panel a). Source: Statistics Norway.

conclusions from the cointegration analysis in Section 4 remain intact with different choices of starting date during the deregulation period. Since 2018 represents the last year of final figures in the present National Accounts, we choose $2018q4$ as the data end point of the sample period. Accordingly, we extend the sample period in Anundsen and Jansen (2013) by ten years or by a total of 40 quarterly observations.

Figure 1 shows the time series X_t over the selected sample period. We observe that the aforementioned boom in the real estate market initiated by the deregulation period, was followed by a huge drop in the real house prices (panel a) during the banking crisis in Norway between 1988 and 1993. After the banking crisis, the real house prices have increased almost steadily until 2018, only interrupted by a significant, but short-lived, fall in the aftermath of the financial crisis in 2008. Much the same picture can be said about the development in the household real debt, suggesting a close interdependency between p_t and d_t during the sample period. We also notice that the upward trending behaviour from the mid 1990s of both the real house prices and the household real debt is associated with increasing real disposable income (panel b) and real housing stock (panel c). These similarities in the trending behaviour of p_t, d_t, y_t and k_t may indicate the presence of co-trending among these time series, such that inclusion of a linear deterministic trend in the underlying VAR model of the cointegration analysis may not be necessary. The real after-tax interest rate (panel d) for its part reached a historically high level in the early 1990s in the wake of the huge boom in the real estate market. Since then the real after-tax interest rate has shown a downward trend and has reached negative levels as in the early 1980s by the end of the sample period. Hence, it seems to be a negative association between R_t and p_t and between R_t and d_t in accordance with the predictions from the theory outlined in Section 2.

The fact that the time series, overall, exhibit a clear trending behaviour with no apparent mean-reversion property suggests that X_t is a vector of non-stationary, $I(1)$, time series. In addition, plotting the time series properties of $\Delta X_t = (\Delta p_t, \Delta d_t, \Delta y_y, \Delta k_t, \Delta R_t)'$, the first difference of the variables may be judged to be stationary, $I(0)$, see the Appendix.[7] Therefore, we shall in line with Anundsen and Jansen (2013) treat X_t as $I(1)$, such that a reduced rank VAR is a candidate as an empirical model.

4 Cointegration analysis[8]

In this section, we carry out a multivariate cointegration analysis within the context of Johansen (1988, 1991, 1995). First, we apply the trace test for cointegrating rank determination based on an estimated full VAR in X_t. Then, we use likelihood ratio tests for various restrictions on the estimated cointegrating vectors in order to find a theory-consistent long-run structure between house prices and debt. Finally, we examine the persistence profiles of the estimated cointegrating vectors by means of Pesaran and Shin (1996) to shed light on the speed of adjustment towards equilibrium after a system-wide shock on the underlying cointegrated VAR.

4.1 Rank determination

As opposed to Anundsen and Jansen (2013), who worked with a partial VAR where the real housing stock and the real after-tax interest rate were conditioned upon from the outset, our point of departure is a p-dimensional unrestricted VAR of order k expressed as

$$X_t = \sum_{i=1}^{k} \Pi_i X_{t-i} + \vartheta + \rho_0 t + \delta D_t + \varepsilon_t, \quad \text{for} \quad t = 1, \dots, T, \tag{6}$$

where $p = 5$ is the number of variables in X_t, $\Pi_1, ..., \Pi_k$ are autoregressive coefficients, ϑ represents a vector of constant terms, ρ_0 includes coefficients of a linear deterministic trend t, δ includes coefficients of seasonal dummies D_t, $\varepsilon_{k+1}, ..., \varepsilon_T$ are independent Gaussian innovations with expectation zero and variance-covariance matrix Ω and T is the total number of observations (excluding lags). The initial observations $X_{-k+1}, ..., X_0$ are kept fixed. Under the assumption that X_t is $I(1)$,

[7] A battery of Augmented Dickey-Fuller tests have also been conducted in order to examine the orders of integration of the time series. Based on these tests, albeit such tests are only indicative as pointed out by *inter alia* Juselius (2006, p. 297), X_t may be treated as $I(1)$.

[8] The econometric modelling in this section was carried out with PcGive 15/OxMetrics 8, see Doornik and Hendry (2018), and Microfit 5.5, see Pesaran and Pesaran (2009).

Table 1: Misspecification tests for the VAR(6)

	p_t	d_t	y_t	k_t	R_t	Vector
AR[F(5,93)]	0.96[0.45]	1.09[0.37]	0.99[0.43]	1.91[0.10]	0.69[0.63]	
ARCH[F(4,134)]	1.64[0.17]	2.97[0.02]*	1.38[0.24]	1.31[0.27]	0.10[0.98]	
HET[F(65,67)]	1.14[0.30]	1.23[0.20]	0.80[0.82]	1.46[0.06]	1.39[0.09]	
NORM[$\chi^2(2)$]	3.90[0.14]	8.35[0.02]*	1.21[0.55]	1.82[0.40]	0.31[0.86]	
AR[F(125,344)]						0.90[0.75]
HET[F(325,319)]						1.21[0.04]*
NORM[$\chi^2(10)$]						21.6[0.02]*

Notes: Sample period: $1982q1-2018q4$. Test statistics are rounded to one or two decimal places. AR[·] are F-tests for 5th-order serial correlation, ARCH[·] are F-tests for 4th-order autoregressive conditional heteroscedasticity, HET[·] are F-tests for heteroscedasticity and NORM[·] are joint chi-squared tests for normality (no skewness and excess kurtosis), see Doornik and Hendry (2018, p. 165-170) for further details. Figures in square brackets are p-values. * denotes rejection of the null hypothesis at the 5 per cent significance level.

the presence of cointegration implies $1 \leq r \leq p-1=4$, where r denotes the rank or the number of cointegrating vectors of the impact matrix $\Pi = \Pi_1 + \cdots + \Pi_k - I$. The null hypothesis of r cointegrating vectors can be formulated as H_0: $\Pi = \alpha\beta'$, where α and β are matrices of adjustment coefficients and cointegration coefficients, respectively, and $\beta' X_t$ comprises r cointegrating linear combinations. For future reference, we write the equilibrium correction form of (6), the cointegrated VAR, as

$$(7) \quad \Delta X_t = \alpha \begin{pmatrix} \beta \\ \rho \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Delta\Gamma_i X_{t-i} + \vartheta + \delta D_t + \varepsilon_t, \quad \text{for} \quad t = 1, \ldots, T,$$

where the linear trend is restricted to lie within the cointegrating space, $\rho = \alpha\rho_0$, thereby addressing the observed trending behaviour and possibly co-trending behaviour in the time series, and the deterministic components (constant terms and seasonal dummies) are unrestricted in (7). The parameters of the model are $\alpha, \beta \in \mathbf{R}^{p\times r}$ for $r < p$, $\Gamma_i \in \mathbf{R}^{p\times p}$, $\vartheta \in \mathbf{R}^p$, $\rho \in \mathbf{R}^r$ and $\delta \in \mathbf{R}^{p\times 3}$; all of them vary freely. The variance-covariance matrix $\Omega \in \mathbf{R}^{p\times p}$ is positive definite.

Cheung and Lai (1993) point out that the trace test is rather sensitive to under-parametrization and not so to over-parametrization in the lag length of the VAR. According to both Akaike's information criterion, likelihood ratio tests of sequential model reduction and misspecification tests of the residuals, the VAR in our case should include six lags ($k = 6$) as the premise for the cointegration analysis. Otherwise with fewer lags, the VAR suffers from severe autocorrelation in the residuals, particularly in the equations for the real house prices and the real housing stock. Also, the sixth lag of p_t, d_t and k_t are all strongly significant in the model. We notice that nine dummy variables being 1 for large outliers in the residuals in $1986q1$, $1987q1$, $1987q3$, $1995q4$, $2002q3$, $2003q1$, $2003q2$, $2008q4$ and $2015q2$, and 0 otherwise, are added unrestrictedly to the VAR(6). The dummy variables may be associated with the abnormal upswing in the real estate market in 1986 and 1987, the revisions of household disposable income in the National

Table 2: Tests for cointegration rank

H_0	λ_i	λ_{trace}	p-value
$r = 0$	0.275	104.69	0.002**
$r \leq 1$	0.244	68.72	0.017*
$r \leq 2$	0.164	37.44	0.160
$r \leq 3$	0.104	17.41	0.393
$r \leq 4$	0.045	5.13	0.585

Notes: Sample period: $1982q1 - 2018q4$. The underlying VAR is of order 6 with $X_t = (p_t, d_t, y_t, k_t, R_t)'$ as modelled variables, a linear trend as a restricted variable and constants, seasonal dummies and nine dummy variables for outliers as unrestricted deterministic terms. r denotes the rank order of $\Pi = \alpha\beta'$ in (7), λ_i are the eigenvalues from the reduced rank regressions, λ_{trace} are the trace test statistics adjusted for degrees of freedom and p-value are the significance probabilities based on the approximations to the asymptotic distributions derived by Doornik (1998). ** and * denote rejection of the null hypothesis at the 1 and 5 per cent significance level, respectively.

Accounts in 1995, 2002 and 2015,[9] the substantial fluctuations in electricity prices, and hence in the consumption deflator, in 2003, and the financial crisis in 2008. We emphasise that using the dummy variables for large outliers does not alter the conclusions from the cointegration analysis.

Table (1) displays a battery of misspecification tests for the VAR(6). Although the debt-equation still has significant, but not very strong, ARCH-effects and non-normality in the residuals, we judge the model to be well-specified and thus to be a valid statistical representation of the data. Generally speaking, cointegration analysis is quite robust against ARCH-effects, according to Hansen and Rahbek (1998), and non-normality caused by excess kurtosis, according to Gonzalo (1994).

Based on the VAR(6) model, we now turn to the cointegration rank determination of the impact matrix $\Pi = \alpha\beta'$ in (7). Table 2 reports trace test statistics for a sequence of null hypotheses of the rank order r. We conclude that $r = 0$ can be rejected at the 1 per cent significance level, whereas $r = 2$ can be accepted at the 5 per cent significance level. Evidence of two cointegrating vectors may be supportive of a self-reinforcing relationship between real house prices and household real debt. We next examine the underlying long-run structure of the two cointegrating vectors relying on the theoretical set-up in Section 2.

4.2 Long-run structure

Having established that $r = 2$, we need to exactly identify the two cointegrating vectors before testing overidentifying restrictions in order to reveal the underlying long-run structure between p_t and d_t. For this purpose, we write out the cointegrat-

[9]See Helliesen *et al.* (2021, Appendix A) for an overview of benchmark revisions in the Norwegian National Accounts.

ing part of (7) as

$$(8)\quad \alpha \begin{pmatrix}\beta \\ \rho\end{pmatrix}' \begin{pmatrix}X_{t-1} \\ t\end{pmatrix} = \begin{pmatrix}\alpha_{p,1} & \alpha_{p,2} \\ \alpha_{d,1} & \alpha_{d,2} \\ \alpha_{y,1} & \alpha_{y,2} \\ \alpha_{k,1} & \alpha_{k,2} \\ \alpha_{R,1} & \alpha_{R,2}\end{pmatrix} \begin{pmatrix}\beta_{p,1} & \beta_{d,1} & \beta_{y,1} & \beta_{k,1} & \beta_{R,1} & \rho_1 \\ \beta_{p,2} & \beta_{d,2} & \beta_{y,2} & \beta_{k,2} & \beta_{R,2} & \rho_2\end{pmatrix} \begin{pmatrix}X_{t-1} \\ t\end{pmatrix},$$

and set $\beta_{p,1} = 1$, $\beta_{d,2} = 1$, $\beta_{R,1} = 0$ and $\beta_{y,2} = 0$ as the identification scheme, which is motivated by the findings in Anundsen and Jansen (2013, Table 4, panel 5). In particular, $\beta_{p,1} = 1$ and $\beta_{d,2} = 1$ normalise the first and second cointegrating vector on p_t and d_t, respectively. The restriction $\beta_{R,1} = 0$ may be justified by the notion that the effects of interest rates feed into house prices through disposable income and through the cost of financing households' debt. Likewise, the restriction $\beta_{y,2} = 0$ may be motivated by the notion that the effects of disposable income on household debt work through a self-reinforcing relationship between house prices and debt. The chosen identification scheme provides the following estimates of α, β and ρ:

$$(9)\quad \hat{\alpha} \begin{pmatrix}\hat{\beta} \\ \hat{\rho}\end{pmatrix}' = \begin{pmatrix}\underset{(0.052)}{-0.201} & \underset{(0.013)}{-0.068} \\ \underset{(0.029)}{0.0513} & \underset{(0.007)}{-0.003} \\ \underset{(0.040)}{-0.007} & \underset{(0.010)}{-0.002} \\ \underset{(0.002)}{-0.001} & \underset{(0.0004)}{-0.0004} \\ \underset{(0.021)}{0.036} & \underset{(0.005)}{0.006}\end{pmatrix} \begin{pmatrix}\underset{(-)}{1} & \underset{(0.14)}{-0.66} & \underset{(0.31)}{-1.33} & \underset{(1.00)}{-0.82} & \underset{(-)}{0} & \underset{(0.005)}{0.012} \\ \underset{(0.54)}{-1.21} & \underset{(-)}{1} & \underset{(-)}{0} & \underset{(2.90)}{10.19} & \underset{(2.54)}{6.24} & \underset{(0.015)}{-0.056}\end{pmatrix},$$

together with estimated standard errors in parentheses. Overall, the unrestricted estimates of the cointegration coefficients and the adjustment coefficients are in accordance with the underlying theory. In particular, the evidence of significant feedback effects with respect to p_t and d_t, and less so with respect to y_t, k_t and R_t, points to assigning importance to the roles of p_t and d_t in the long-run relationships. Moreover, the estimates of $\beta_{d,1}$, $\beta_{y,1}$, $\beta_{p,2}$ and $\beta_{R,2}$ are all strongly significant with signs as expected from the theory. We do notice that the signs of the estimates of $\beta_{k,1}$ and $\beta_{k,2}$ contradict the theory underlying the chosen identification scheme. An alternative identification scheme would be to replace $\beta_{y,2} = 0$ with $\beta_{k,2} = 0$. However, this scheme yields estimated signs of $\beta_{k,1}$, $\beta_{y,2}$ and $\beta_{R,2}$ that contradict the theory and that $\hat{\alpha}_{p,1} > 0$ and $\hat{\alpha}_{p,2} > 0$. Accordingly, we judge the identification scheme in (9) as a satisfactory point of departure for testing overidentifying restrictions on the two cointegrating vectors.[10]

[10]It turned out difficult using other identification schemes, for instance replacing $\beta_{y,2} = 0$ with $\beta_{p,2} = \beta_{k,2}$ as in Anundsen and Jansen (2013), to find reasonable cointegrating vectors in line with the theory. Using the data set and codes in that study, available at www.andre-anundsen.com,

After stepwise testing of various theory-consistent restrictions by means of likelihood ratio tests, both individually and jointly, we end up with the following estimates of α, β and ρ:

$$(10)\qquad \hat{\alpha}\begin{pmatrix}\hat{\beta}\\ \hat{\rho}\end{pmatrix}' = \begin{pmatrix} \underset{(0.040)}{-0.237} & \underset{(0.014)}{-0.082} \\ \underset{(-)}{0} & \underset{(0.006)}{-0.022} \\ \underset{(-)}{0} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(0.004)}{-0.011} \end{pmatrix} \begin{pmatrix} \underset{(-)}{1} & \underset{(0.02)}{-0.65} & \underset{(0.21)}{-1.49} & \underset{(-)}{1.49} & \underset{(-)}{0} & \underset{(-)}{0} \\ \underset{(-)}{-1} & \underset{(-)}{1} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(1.80)}{8.13} & \underset{(-)}{0} \end{pmatrix},$$

together with estimated standard errors in parentheses. The hypothesis of the joint restrictions is not rejected at the 5 per cent significance level according to a likelihood ratio test statistic of $\chi^2(11) = 12.78$ with a p-value of 0.31. We observe that the imposed restrictions in (10), except the one related to $\beta_{k,1}$, do not change the remaining estimates of α and β much compared to those in (9). A preliminary restriction of $\beta_{k,2} = 0$ produces a likelihood ratio test statistic of $\chi^2(1) = 1.53$ with a p-value of 0.22 and theory-consistent cointegrating vectors where $\beta_{k,1}$ now is significantly negative. In addition, the preliminary joint restrictions of $\beta_{k,2} = 0$ and $\rho_1 = \rho_2 = 0$ provide a likelihood ratio test statistic of $\chi^2(3) = 4.06$ with a p-value of 0.26 and still a significantly negative estimate of $\beta_{k,1}$. The results from these preliminary tests thus justify that the real housing stock is excludable from the second cointegrating vector and that the linear trend is excludable from both vectors. The evidence for $\rho_1 = \rho_2 = 0$ may reflect a dominant property of co-trending between p_t, d_t, y_t and k_t over the sample period, as addressed in Section 3. The stepwise testing of restrictions on β further justifies the additional two restrictions of $\beta_{p,2} = 1$ and $\beta_{k,1} = -\beta_{y,1}$ according to a likelihood ratio statistics of $\chi^2(5) = 5.38$ with a p-value of 0.37. Finally, imposing the additional six restrictions of $\alpha_{d,1} = \alpha_{y,1} = \alpha_{y,2} = \alpha_{k,1} = \alpha_{k,2} = \alpha_{R,1} = 0$ produces our preferred empirical long-run structure reported in (10).[11] Notably, the zero restrictions on α associated with y_t and k_t imply that these two variables are weakly exogenous for the parameters of interest β, see Engle *et al.* (1983).

We are now able to interpret the empirical long-run structure between real house prices and household real debt in light of the theory. The estimates in (10) imply the following two restricted cointegrating vectors:

$$(11)\qquad p_t = \underset{(0.02)}{0.65}d_t + \underset{(0.21)}{1.49}y_t - 1.49k_t$$

shows that $\hat{\beta}_{R,1}$ is not statistically different from zero and that $\hat{\beta}_{y,2}$ has the wrong sign in the exactly identified cointegrating vectors in Anundsen and Jansen (2013, Table 4). These two coefficients are thus set to zero in their final empirical long-run structure between p_t and d_t, which we as mentioned echo in our identification scheme.

[11]These additional zero restrictions on α are also individually not rejected according to $\chi^2(6)$.

$$d_t = p_t - \underset{(1.80)}{8.13} R_t, \tag{12}$$

where all estimated coefficients have their expected signs and are significant at the 5 per cent level. The cointegrating linkages in (11) and (12) are consistent with (2) and (4) in Section 2, and are thus interpretable as long-run equations for p_t and d_t, respectively. Our findings also support the hypothesis of a long-run mutual dependency or a self-reinforcing relationship between p_t and d_t as the former enters the equation for the latter and vice versa. Although there is no direct linkage between real house prices and the real after-tax interest rate in (11), an increased interest rate still causes house prices to fall through the self-reinforcing relationship between p_t and d_t. Similarly, due to the mutual dependency between p_t and d_t, household debt will respond positively to an increase in disposable income. Hence, both house prices and debt are linked to income, the housing stock and the real-after tax interest rate in the long run. These findings match those in Anundsen and Jansen (2013). However, by combining (11) and (12), the cointegrating linkage for house prices becomes

$$p_t = \underset{(0.60)}{4.29} y_t - 4.29 k_t - \underset{(3.40)}{15.37} R_t, \tag{13}$$

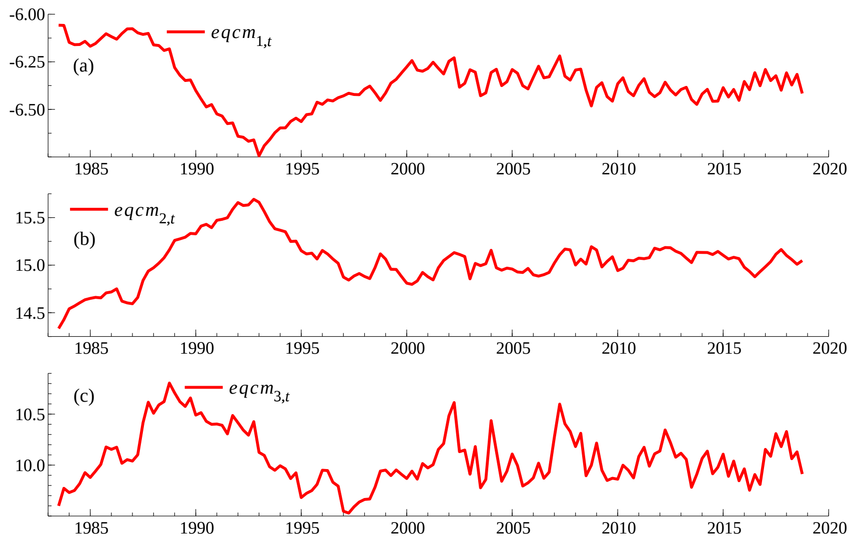
which shows that the reduced form magnitude of the long-run (semi) elasticity of house prices with respect to the real after-tax interest rate is, in the context of Johansen (2005), around 15 per cent compared to around 10 per cent in Anundsen and Jansen (2013).[12] Accordingly, extending the sample period by ten years after the financial crisis increases the long-run responsiveness of house prices to changes in the interest rate by around 5 percentage points.[13] A possible interpretation may be that relatively low, and sometimes even negative, real after-tax interest rates after the financial crises, has pushed the average elasticity upwards over the sample period. The fact that the house prices have increased by more than 10 per cent during the Covid-19 pandemic with a zero key policy rate supports this conjuncture.

The estimated adjustment coefficients for p_t are -0.24 (t-value of -5.93) and -0.08 (t-value of -5.86), which imply that house prices adjust steadily to deviations from its equilibrium as well as to deviations from household debt equilibrium. The single estimated adjustment coefficient for d_t, on the other hand, is -0.02 (t-value of -3.67), and indicates that household debt only reacts to its disequilibrium errors. These findings are also in line with those in Anundsen and Jansen (2013). Finally, as opposed to disposable income and the housing stock, the real after-tax interest rate is not weakly exogenous with respect to the coefficients in the long-run equation for household debt. That said, the feedback effects with respect to R_t are relatively weak since $\hat{\alpha}_{R,2} = -0.01$ (t-value of -2.75). Thus, the evidence for relatively strong and

[12] The standard errors in (13) are calculated by setting the t-value of the estimated coefficient of y_t in (11) equal to the t-value of the estimated coefficient of y_t in (13) and then solving for the corresponding standard error. The standard error associated with the estimated coefficient of R_t is calculated likewise by means of (12) and (13).

[13] Admittedly, the magnitude of the interest rate elasticity in Anundsen and Jansen (2013) is closely within (minus) two times the standard errors of our estimated elasticity.

Figure 2: Estimates of equilibrium correction terms based on the VAR(6)



Notes: Sample period: $1982q1 - 2018q4$. The first equilibrium correction term (panel a), $eqcm_{1,t} = p_t - 0.65d_t - 1.49y_t + 1.49k_t$, the second equilibrium correction term (panel b), $eqcm_{2,t} = d_t - p_t + 8.13R_t$ and their combination (panel c), $eqcm_{3,t} = p_t - 4.29y_t + 4.29k_t + 15.37R_t$ are based on (11), (12) and (13), respectively.

highly significant feedback effects with respect to p_t and d_t makes the normalising of the two cointegrating vectors on these variables justifiable.

Figure 2 shows the two equilibrium correction terms, $eqcm_{1,t}$ and $eqcm_{2,t}$ based on (11) and (12), along with their combination, $eqcm_{3,t}$ based on (13), over the sample period. The two equilibrium correction terms are clearly mean-reverting stationary series from the mid-1990s. The mean-reversion property is, however, relatively slow during the aforementioned banking crises.[14] We may argue that the self-reinforcing effects between house prices and household debt were particularly strong during the banking crises due to the steady increase in the unemployment rate from 2.6 per cent in 1987 to a record-high 6.5 per cent in 1993 and the associated increased uncertainty about the prospects for the households' economy. Intuitively, it may be the case that such a big and persistent shock to the system causes the huge disequilibrium in the two cointegrating vectors and thus that mean-reversion takes longer time during the banking crisis. The bumps of the two equilibrium correction terms during the banking crisis, however, cancel somewhat out in their combination, implying a somewhat more stationary time series during the entire sample period.[15]

[14] Applying the data set used in Anundsen and Jansen (2013), reveals similar mean-reversion properties of the two equilibrium correction terms in that study, cf. Table 4, panel 5.

[15] Indeed, Augmented Dickey-Fuller tests reveal that $eqcm_{1,t}$ (borderline case) and $eqcm_{2,t}$ are stationary time series at the 5 per cent level and that $eqcm_{3,t}$ is a stationary time series at the 1 per cent level.

4.3 Persistence profiles

Given the evidence of relatively slow mean-reversion property during the banking crisis, it is of interest to provide estimates of the speed with which the housing market returns to its equilibrium state after a system-wide shock on the two cointegrating vectors. For this purpose, we estimate the so-called persistence profiles for the two cointegrating vectors, as originally proposed by Pesaran and Shin (1996).

The scaled persistence profile of the effect of a system-wide shock on the j-th cointegrating relationship is in our case defined as

$$h(\beta_j' X_t, n) = \frac{\beta_j' A_n \Omega A_n' \beta_j}{\beta_j' \Omega \beta_j}, \tag{14}$$

for $j = 1, 2$ and $n = 0, 1, 2, ...$ and where the $p \times p$ matrix A_n is the coefficient matrix for the residual vector lagged n periods in a Granger representation form of the cointegrated VAR in (7).[16] The value of the persistence profile is equal to unity on impact as $A_0 = I_p$, but tends to zero as n, the horizon of the profile, approaches infinity under the assumption that β_j is a cointegrating vector. As such, $h(\beta_j' X_t, n)$ as a function of n provides insightful information on the speed of adjustment towards equilibrium after a system-wide shock on the cointegrating relationship, $\beta_j' X_t$.

To facilitate estimation of the persistence profiles using the econometric package Microfit, we re-estimate (10) with no restrictions on α, but retain the same restrictions imposed on β and ρ. The estimates of α, β and ρ then become

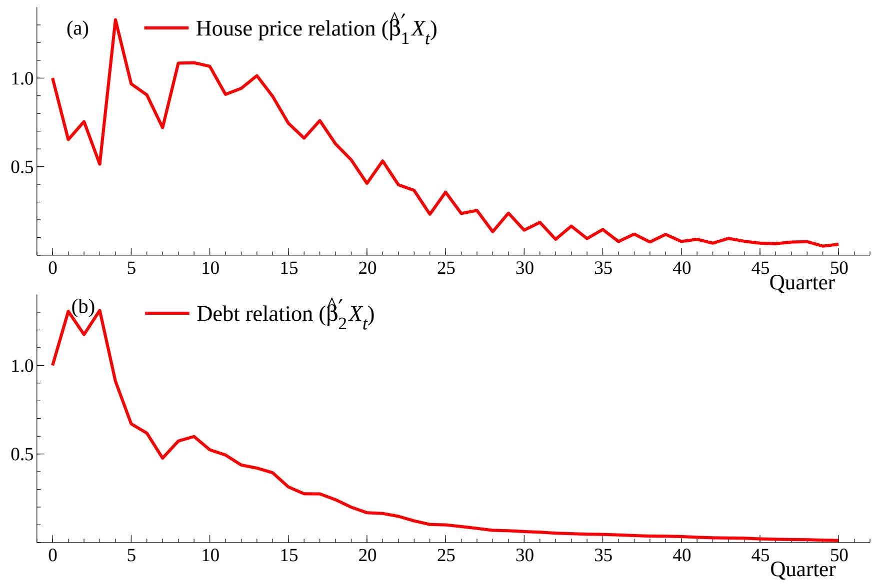
$$\hat{\alpha}\begin{pmatrix}\hat{\beta}\\ \hat{\rho}\end{pmatrix}' = \begin{pmatrix} \underset{(0.045)}{-0.189} & \underset{(0.012)}{-0.055} \\ \underset{(0.026)}{0.030} & \underset{(0.007)}{-0.013} \\ \underset{(0.034)}{0.0009} & \underset{(0.0088)}{0.0027} \\ \underset{(0.0014)}{-0.0012} & \underset{(0.0004)}{-0.0007} \\ \underset{(0.017)}{0.023} & \underset{(0.0044)}{-0.0052} \end{pmatrix} \begin{pmatrix} \underset{(-)}{1} & \underset{(0.02)}{-0.63} & \underset{(0.22)}{-1.65} & \underset{(-)}{1.65} & \underset{(-)}{0} & \underset{(-)}{0} \\ \underset{(-)}{-1} & \underset{(-)}{1} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(2.39)}{11.63} & \underset{(-)}{0} \end{pmatrix}, \tag{15}$$

recalling that the five restrictions imposed in (15) are justified by a likelihood ratio statistics of $\chi^2(5) = 5.38$ with a p-value of 0.37. A comparison of (10) and (15) shows that the estimates, overall, are of the same magnitudes, which further confirms the validity of imposing the additional six restrictions on α in (10).

Figure 3 displays the estimates of the scaled persistence profiles for the two cointegrating relationships, the house price relation $(\hat{\beta}_1' X_t)$ in panel a and the debt relation $(\hat{\beta}_2' X_t)$ in panel b, estimated by means of (14). The estimates of the persistence profiles clearly show that both the house price relation and the debt relation converge to zero, but quite slowly. About 80 per cent and 90 per cent of the adjustments towards equilibrium after a system-wide shock are made after six years

[16] By pre-multiplying the Granger representation form of the cointegrated VAR with one of the two cointegrating vectors, we have $\beta_j' X_t = \mathbf{E}\left[\beta_j' X_t\right] + \beta_j' \sum_{n=0}^{\infty} A_n \varepsilon_{t-n}$, see Pesaran and Shin (1996) for details.

Figure 3: Estimates of persistence profiles based on the VAR(6)



Notes: The first cointegrating vector (panel a), $\hat{\beta}_1' X_t = \begin{pmatrix} 1 & -0.63 & -1.65 & 1.65 & 0 \end{pmatrix} X_t$, and the second cointegrating vector (panel b), $\hat{\beta}_2' X_t = \begin{pmatrix} -1 & 1 & 0 & 0 & 11.63 \end{pmatrix} X_t$, are the house price relation and the debt relation, respectively, based on (15). $n = 0, 1, ..., 50$.

in the cases of the house price relation and the debt relation, respectively. Although completely different markets and not comparable as such, it is interesting that the estimates of the persistence profile for the UK purchasing power parity (PPP) relation in Pesaran and Shin (1996) give a similar picture of the speed of adjustment. Johansen and Juselius (1992), who also study the PPP relation for the UK, point out that "Whatever the true case, there can hardly be any doubt that if the PPP holds as a long-run relation, the speed of adjustment has to be very slow due to costly information gathering, product heterogeneity, government-imposed barriers to trade, etc." Likewise, if the house price and debt relations are cointegrating vectors, the slow speed of adjustment may be attributed to costly information gathering of housing opportunities in the market and government-imposed restrictions on housing trade by *inter alia* lending criteria based on payment-to-income ratios. Because the estimates of both persistence profiles eventually converge to zero provides further evidence that both the house price relation and the debt relation indeed represent cointegrating relationships restricted by the theory. Our findings from the cointegration analysis set the stage for the subsequent control analysis of house prices and debt in the context of Johansen and Juselius (2001).

5 Controllability of house prices and debt[17]

In this section, we explore the controllability of d_t and p_t in the cointegrated VAR model. We first give a brief review of cointegrated VAR-based control theory by referring to Johansen and Juselius (2001). The theory is then applied to an analysis of the Norwegian data so as to check the empirical controllability of the two financial magnitudes. Finally, we conduct a simulation study using a class of estimated parameters with a view to drawing useful policy implications.

5.1 Cointegrated VAR-based control theory

The control theory of Johansen and Juselius (2001) was developed to study controllability of target variables by using instrument variables in a cointegrated VAR. A target variable is said to be controllable in this framework if it can be made (trend) stationary around a pre-specified target value by adjusting the instrument variable.

When ignoring the deterministic variables in D_t, the cointegrated VAR in (7) can be reparameterized as

$$\Delta X_t - \gamma = \alpha(\beta' X_{t-1} - \rho(t-1) - \mu) + \sum_{i=1}^{k-1} \Gamma_i(\Delta X_{t-i} - \gamma) + \varepsilon_t, \quad \text{for} \quad t = 1, \ldots, T,$$

see e.g. Engle and Granger (1987) and Hansen (2005), where the new parameters of the model are $\gamma \in \mathbf{R}^p$ and $\mu \in \mathbf{R}^r$; all of them vary freely. For future reference, we introduce $\alpha_\perp$ and $\beta_\perp$, the orthogonal complements of α and β, respectively; these complements enable us to define the impact matrix $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$ for $\Gamma = I_p - \sum_{i=1}^{k-1}\Gamma_i$, which boils down to $C = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp = I_p - \alpha(\beta'\alpha)^{-1}\beta'$ when $k = 1$.

For the rest of this sub-section, we assume $k = 1$ and $\rho = \gamma = 0$ to simplify the arguments, so that the above model is reduced to

$$\Delta X_t = \alpha(\beta' X_{t-1} - \mu) + \varepsilon_t. \tag{16}$$

The solution to this equation is

$$X_t = (I_p + \alpha\beta')^t X_0 + \textstyle\sum_{j=1}^{t-1}(I_p + \alpha\beta')^j(\varepsilon_{t-j} - \alpha\mu).$$

The long-run expected value of X_t derived from this solution, see Johansen and Juselius (2001) and Johansen (2005), is

$$X_\infty = \lim_{t\to\infty} \mathbf{E}(X_t \,| X_0) = CX_0 + \alpha(\beta'\alpha)^{-1}\mu,$$

which satisfies $\beta' X_\infty = \mu$ since $\beta' C = 0$.

In order to consider a policy control rule, we define here a class of selection matrices $a, b \in \mathbf{R}^{p\times s}$ for $s + r < p$, a policy intervention $v_t \in \mathbf{R}^s$ and a target value

[17] The econometric analysis in this section was carried out by means of an Ox code available from the authors upon request.

$b^* \in \mathbf{R}^s$. Instrument variables are represented by $a'X_t$ while target variables are represented by $b'X_t$. A policy intervention replaces $a'X_t$ with $a'X_t + \upsilon_t$ with the aim of turning $b'X_t$ into a stationary series with mean b^*. The controlled value of X_t in this context, X_t^{ctr}, is expressed as

$$X_t^{ctr} = X_t + a(a'a)^{-1}\upsilon_t, \tag{17}$$

so that we can then infer the system dynamics will deliver a sequence of new series X_{t+h}^{new} for $h = 1, 2, \ldots$ from the initial value X_t^{ctr}. Hence, we choose υ_t such that the desired target value b^* equals its long-run expected value:

$$b^* = b' \lim_{h\to\infty} \mathbf{E}\left(X_{t+h}^{new} \,\middle|\, X_t^{ctr}\right) = b'\left\{C\left[X_t + a(a'a)^{-1}\upsilon_t\right] + \alpha(\beta'\alpha)^{-1}\mu\right\}. \tag{18}$$

Under the assumption that

$$det(b'Ca) \neq 0, \tag{19}$$

and substituting $C = I_p - \alpha\left(\beta'\alpha\right)^{-1}\beta'$ into (18), yields

$$\upsilon_t = -a'a(b'Ca)^{-1}\left[(b'X_t - b^*) - b'\alpha(\beta'\alpha)^{-1}(\beta'X_t - \mu)\right].$$

Plugging back this into (17) leads to

$$X_t^{ctr} = X_t - a(b'Ca)^{-1}\left[(b'X_t - b^*) - b'\alpha(\beta'\alpha)^{-1}(\beta'X_t - \mu)\right],$$

where $b'X_t - b^*$ is a difference between the actual target and the desired target at t, whereas $\beta'X_t - \mu$ is a disequilibrium error at t. If a sequence of interventions takes place at all time points, so that the control rule is compatible with the system (16) which now generates X_t^{new}, we are able to express X_t^{ctr} as

$$X_t^{ctr} = X_t^{new} - a(b'Ca)^{-1}\left[(b'X_t^{new} - b^*) - b'\alpha(\beta'\alpha)^{-1}(\beta'X_t^{new} - \mu)\right]. \tag{20}$$

In practice, it is important to verify (19) so as to make the control rule feasible. Hence, (19) is referred to as the controllability condition in this study.

The economic system (16) then derives the value of X_{t+1}^{new} given X_t^{ctr}, that is,

$$X_{t+1}^{new} = (I_p + \alpha\beta')X_t^{ctr} - \alpha\mu + \varepsilon_{t+1}. \tag{21}$$

Combining (20) with (21), we arrive at the overall cointegrated VAR for X_{t+1}^{new}:

$$\Delta X_{t+1}^{new} = [\alpha, (I_p + \alpha\beta')a]\begin{pmatrix}\beta'X_t^{new} - \mu \\ \kappa'X_t^{new} - \kappa_0\end{pmatrix} + \varepsilon_{t+1}, \tag{22}$$

where

$$\kappa' = -(b'Ca)^{-1}b'[I_p - \alpha\left(\beta'\alpha\right)^{-1}\beta'] \quad \text{and} \quad \kappa_0 = -(b'Ca)^{-1}[b^* - b'\alpha(\beta'\alpha)^{-1}\mu],$$

and Johansen and Juselius (2001) proved

$$\kappa'X_t^{new} - \kappa_0 \sim I(0)$$

under (19). Since $\kappa' X_t^{new} - \kappa_0$ consists of $b' X_t^{new} - b^*$ and the cointegrated combinations, $b' X_t^{new}$ is also judged to be stationary around its mean b^*, or interpreted as additional cointegration. Note that the equilibrium $b' X_t^{new} = b^*$ is not attained even as $t \to \infty$, but there is an additional stability or an attractor set in this system in that deviations of $b' X_t^{new}$ from b^* are partially corrected by the adjustment coefficient $-(I_p + \alpha\beta')a(b'Ca)^{-1}$, as shown in (22). As a result, $b' X_t^{new}$ is stabilised and tends to move around the desired target level b^* without departing from it. In practice, the underlying short-run dynamics have various influences on the adjustment process, so that it may take time for disequilibrium errors to be corrected.

We shall apply this theory to the empirical cointegrated VAR for the Norwegian housing market with the objective of conducting a policy simulation analysis. Given the cointegrated VAR specification employed in the preceding empirical analysis and a set of actual observations for the starting values of simulation, the processes X_t^{ctr} and X_t^{new} are recursively simulated as

$$\begin{aligned} X_t^{ctr} &= X_t^{new} - a(b'\hat{C}a)^{-1}\left[b'(X_t^{new} - \hat{\gamma}t) - b^* + b'(\hat{C}\hat{\Gamma} - I_p)\hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\right. \\ &\qquad \left.\times(\hat{\beta}' X_t^{new} - \hat{\rho}t - \widehat{\mu}) + b'\hat{C}\textstyle\sum_{i=1}^{5}\hat{\Gamma}_i(X_t^{new} - X_{t-i}^{ctr} - \hat{\gamma}i)\right], \end{aligned} \tag{23}$$

and

$$X_{t+1}^{new} = X_{t+1} + (I + \hat{\alpha}\hat{\beta}')(X_t^{ctr} - X_t) + \sum_{i=1}^{5}\hat{\Gamma}_i(\Delta X_{t+1-i}^{ctr} - \Delta X_{t+1-i}). \tag{24}$$

For further details of the formulation of (23) and (24) for simulation purposes, see Johansen and Juselius (2001, Section 5.3).

5.2 Analysis of empirical controllability

We have now reached a stage where selection of policy instrument and target variables from $X_t = (p_t, d_t, y_t, k_t, R_t)'$ can be made. The selection of R_t as a policy instrument is justifiable, given the variable set X_t, on the grounds that a long-run one-to-one relationship exists between the money market rate, which is closely related to the key policy rate by Norges Bank, and the interest rate on mortgage credit, see Hungnes (2015). Hence, we assume here that the key policy rate works implicitly in our model through both components of the real after-tax interest rate; the nominal interest rate on household loans and the inflation rate. Although the purpose of monetary policy depends on the economies and times which are to be studied, it is generally the case that asset prices belong to a class of important target variables in the implementation of monetary policy. This reasoning allows us to pick out p_t and d_t as policy target variables in the study of the Norwegian housing market. Hence, the selection matrices should be specified as

$$a = (0,0,0,0,1)', \quad b^{(p_t)} = (1,0,0,0,0)' \quad \text{and} \quad b^{(d_t)} = (0,1,0,0,0)',$$

so that

$$a' X_t = R_t, \quad b^{(p_t)\prime} X_t = p_t \quad \text{and} \quad b^{(d_t)\prime} X_t = d_t.$$

According to the condition in (19), the controllability of p_t and d_t by using R_t as a policy instrument is guaranteed by

$$b^{(p_t)\prime}Ca \neq 0 \quad \text{and} \quad b^{(d_t)\prime}Ca \neq 0, \tag{25}$$

respectively. These conditions can be tested formally, see Paruolo (1997) for further details of inference on the C matrix. In terms of consistency with the economic theory outlined in Section 2, we find it necessary to verify not solely (25), but also

$$b^{(p_t)\prime}Ca < 0 \quad \text{and} \quad b^{(d_t)\prime}Ca < 0, \tag{26}$$

which indicate that raising R_t brings about a decrease in p_t and d_t, so that stabilising policies are judged to be effective.

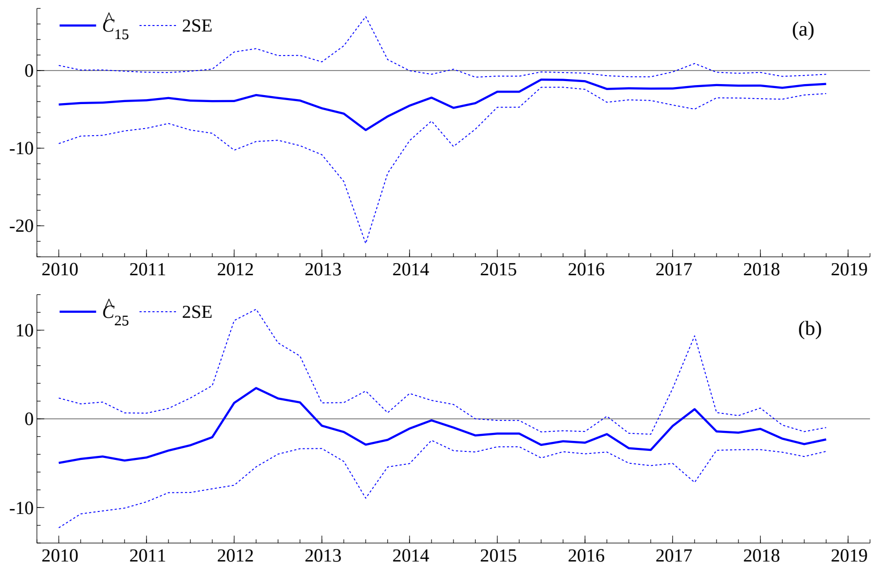
Let us get back to the empirical analysis of the Norwegian time series data. We have already confirmed that the cointegrating rank is two, or $r = 2$, which enables us to obtain the maximum likelihood estimates of the model's parameters using reduced rank regression. The estimated impact matrix is

$$\hat{C} = \begin{bmatrix} \underset{(0.243)}{-0.113} & \underset{(0.339)}{0.605} & \underset{(0.327)}{0.990} & \underset{(14.372)}{-15.746} & \underset{(0.621)}{-\mathit{1.720}} \\ \underset{(0.263)}{-0.099} & \underset{(0.366)}{0.807} & \underset{(0.353)}{0.366} & \underset{(15.526)}{-25.710} & \underset{(0.671)}{-\mathit{2.321}} \\ \underset{(0.086)}{0.015} & \underset{(0.120)}{-0.026} & \underset{(0.116)}{0.489} & \underset{(5.099)}{-1.597} & \underset{(0.221)}{0.177} \\ \underset{(0.059)}{-0.082} & \underset{(0.083)}{0.128} & \underset{(0.080)}{0.121} & \underset{(3.505)}{4.161} & \underset{(0.152)}{-0.513} \\ \underset{(0.091)}{0.128} & \underset{(0.127)}{-0.221} & \underset{(0.123)}{-0.065} & \underset{(5.389)}{-5.731} & \underset{(0.233)}{0.877} \end{bmatrix},$$

in which each figure in parentheses denotes a standard error. The figures in italics on the upper right corner of the matrix, $\hat{C}_{15}$ and $\hat{C}_{25}$, correspond to $b^{(p_t)\prime}\hat{C}a$ and $b^{(d_t)\prime}\hat{C}a$, respectively. Both of the estimates are judged to be significantly negative at the 5 per cent level, so that the conditions in (26) are empirically satisfied and we can argue for the effectiveness of monetary stabilisation policy concerning p_t and d_t. Figure 4 records a set of recursive estimates for $b^{(p_t)\prime}\hat{C}a$ and $b^{(d_t)\prime}\hat{C}a$, indicating some fluctuations, but the estimates are negative and significant in most cases over the last decade of the sample period. Interpretation of these coefficients in the context of long-run impulse responses may be as follows: a one percentage point increase in the real after-tax interest rate leads to a decline in house prices of about 1.7 per cent, whereas household debt is reduced by approximately 2.3 per cent. We can go further from this interpretation in the context of the control analysis reviewed in the previous sub-section. As shown in (20), the inverses of these two coefficients play critical roles in the feedback mechanism required for the stability of the controlled system. We will assess this mechanism quantitatively by using the method of data-based simulation in the next sub-section.

As a possible caveat, we should recall that R_t was not judged to be weakly exogenous for the cointegrating parameters; the rejection of weak exogeneity implies the absence of super exogeneity, a property which is required in the context of policy analysis to counteract the Lucas critique, see Lucas (1976) and Engle *et al.* (1983)

Figure 4: Recursive unrestricted estimates of $\hat{C}_{15}$ and $\hat{C}_{25}$



Notes: Sample period: $2010q1 - 2018q4$ after initialisation of observations for recursive estimation. $r = 2$.

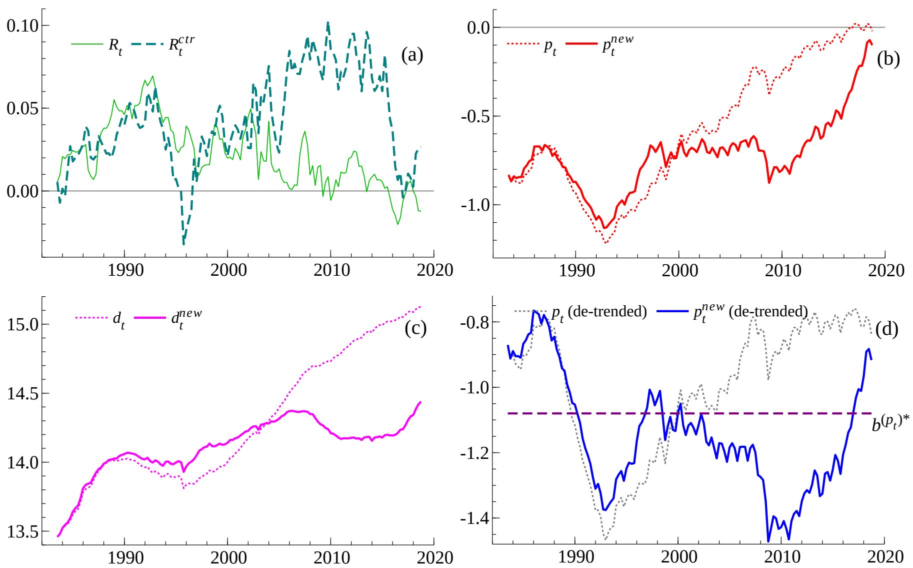
for further details. Although our analysis may be subject to this type of criticism, we assign importance to the fact that all of the conditions in (25) and (26) are empirically satisfied, thereby justifying the conduct of a policy simulation study in the context of Johansen and Juselius (2001).

5.3 Simulation study and policy implications

The finding of empirical controllability of p_t and d_t leads us to a class of policy simulation studies using the data-based parameter estimates. For this purpose, we follow Johansen and Juselius (2001) and set up a dynamic system consisting of (23) and (24), in which a set of restricted estimates $\hat{\alpha}$ and $\hat{\beta}$ obtained in Section 4 is used. For the derivation of the impact matrix to be used in the control rule, the orthogonal complements, $\hat{\alpha}_\perp$ and $\hat{\beta}_\perp$, are calculated as the null spaces of $\hat{\alpha}$ and $\hat{\beta}$, respectively, and Γ is estimated by regression given $\hat{\alpha}$ and $\hat{\beta}$. Since the policy target b^* is indeterminate yet in the system, it is necessary to specify its level according to p_t and d_t. Checking the initial observations of p_t and d_t given the sample period, we judge the following is a reasonable selection of target levels: $b^{(p_t)*} = -1.08$ and $b^{(d_t)*} = 13.3$.

First, we focus on the control of p_t by treating R_t as a policy instrument. Figure 5 (panel a) records the series of R_t, together with those of R_t^{ctr} given the target value $b^{(p_t)*} = -1.08$. It is evident that R_t^{ctr} has risen significantly since the mid of the sample period and stayed at a higher level than that of R_t. This distinguishing feature is interpreted as a policy response, driven by the underlying control rule,

Figure 5: Stabilising p_t by a series of interventions affecting R_t



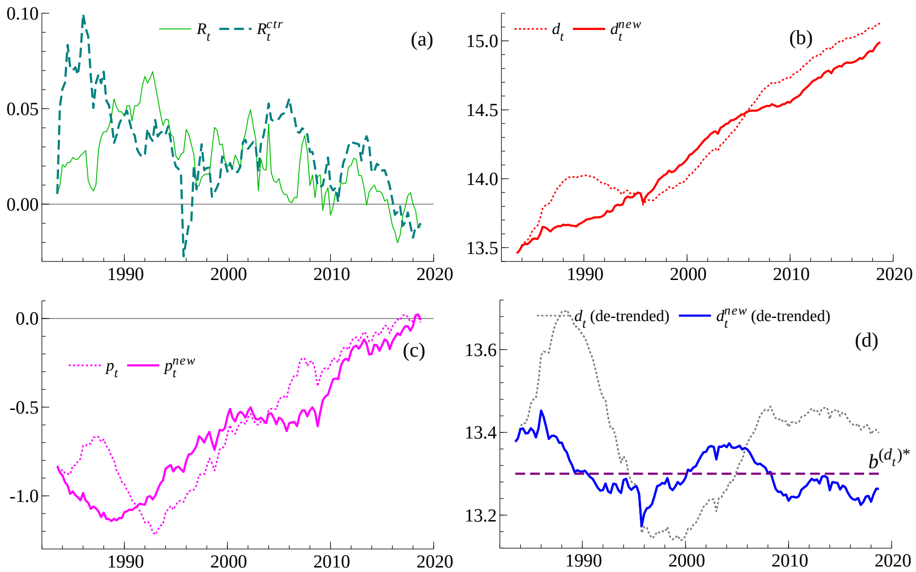
Notes: Sample period: $1982q1 - 2018q4$. The simulations are based on (23) and (24).

for the objective of achieving the selected target level $b^{(p_t)*}$. The behaviour of p_t^{new}, as shown in Figure 5 (panel b), appears to be relatively stabilised compared to p_t, in that the flattening of p_t^{new} is observed around the middle of the sample period. This feature is clearly attributable to a rise in R_t^{ctr}, a reflection of the evidence for controllability $b^{(p_t)\prime}\hat{C}a < 0$ revealed in the previous sub-section. Similarly, d_t^{new} in Figure 5 (panel c) has been damped down in a manner similar to p_t^{new}, suggesting the presence of significant influences from the spike of R_t^{ctr}. Lastly, Figure 5 (panel d) displays a de-trended version of p_t^{new}; its mean-reverting characteristics are fairly obvious as compared with p_t^{new} in Figure 5 (panel b). The mean of this de-trended series is judged to correspond to $b^{(p_t)*}$ as a result of effective control policy.

Next, controlling d_t by utilising the policy instrument R_t is considered. Figure 6 has the same structure as Figure 5, apart from the fact that d_t is the policy target variable aiming at the target level $b^{(d_t)*} = 13.3$. According to Figure 6 (panel a), R_t^{ctr} increased significantly in the first half and second half of the sample period. This behaviour can be recognised as a policy reaction towards the attainment of the target level. Figure 6 (panel b) shows, as expected, that the behaviour of d_t^{new} appears to be more stable than that of d_t, in that the bumps of the original series have been removed by the increase of R_t^{ctr}. Similarly, Figure 6 (panel c) depicts the series p_t^{new}, which also seems to be stabilised by R_t^{ctr}. In terms of the attainment of the target level, Figure 6 (panel d) is noteworthy in that the de-trended series of d_t^{new} exhibits mean-reverting features around $b^{(d_t)*}$.

The results of the simulation-based study are consistent with the evidence for controllability in the estimated C matrix in the previous sub-section. We can,

Figure 6: Stabilising d_t by a series of interventions affecting R_t



Notes: Sample period: $1982q1 - 2018q4$. The simulations are based on (23) and (24).

therefore, argue that the policy maker is capable of achieving the pre-specified target levels for p_t and d_t through the instrument R_t. Overall, our findings suggest that monetary policy may be effective in reducing large fluctuations and bubble tendencies in the housing market.

6 Conclusions

We have revisited the empirical modelling of house prices and household debt in Norway with a policy-oriented perspective. Our point of departure has been an operational representation of the life-cycle model of housing in which equilibrium real house prices are determined by households' real disposable income, household real debt, the housing stock and the real after-tax interest rate on mortgage credit. To account for a self-reinforcing mechanism between house prices and debt, typically observed in the Norwegian housing market, this representation is augmented by an equilibrium real debt relationship with real house prices as one of the key determinants in the long run.

We have then confronted the theoretical set-up empirically over the last four decades by means of a full cointegrated VAR in real house prices, real disposable income, household real debt, the housing stock and the real after-tax interest rate. Our findings suggest the existence of two cointegrating relationships which, we argue, can be interpreted as long-run relations for house prices and debt with mutual dependency between them. More specifically, we find that a permanent increase in the interest rate of one percentage point leads to a long-run decrease in house prices of about 15 per cent when the housing stock is fixed. The existence of the two cointegrating relationships is reinforced by means of estimated persistence profiles which eventually converge to zero, albeit quite slowly, after a system-wide shock. We argue that the slow speed of adjustment may be attributed to costly information gathering of housing opportunities in the Norwegian market and government-imposed restrictions on housing trade by *inter alia* lending criteria based on payment-to-income ratios. Finally, we have demonstrated by means of a control analysis of the cointegrated VAR that both house prices and debt are controllable magnitudes to some pre-specified target levels through the interest rate. These findings suggest that Norges Bank to some extent is capable of reducing large fluctuations and bubble tendencies in the housing market, which from a monetary policy perspective is potentially important when trying to prevent spillover effects from financial instabilities to the real economy.

We have not in this paper, given our information set, pursued a standard-type impulse response analysis of monetary policy based on a structural VAR. However, we believe that the present study provides theoretically understandable and empirically relevant representations of house prices and debt which are two financial magnitudes entering the decision process of Norges Bank. We should emphasise that our modelling framework is based on the underlying premise that the housing stock representing the supply side of the housing market is fixed. The implications of extending the VAR analysis with observable variables for the purpose of modelling a flexible supply side are left for future research.

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Appendix

The original data series used in the empirical analysis are all seasonally unadjusted and measured on a quarterly frequency over the period $1982q1 - 2018q4$. Variable definitions, data descriptions and sources are listed below.

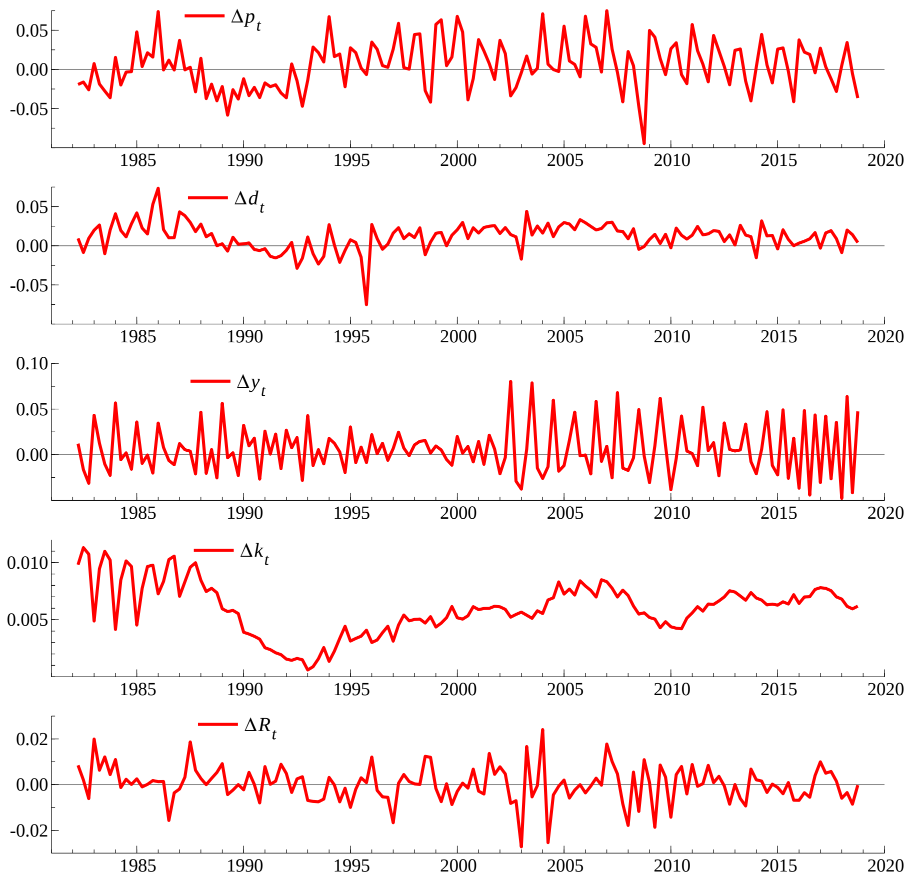
- p_t The log of real house prices measured by the overall price index for residential buildings in the second-hand market and adjusted by the consumption deflator, PC_t, in the National Accounts. Source: Statistics Norway.
- d_t The log of household real debt measured by the total amount of outstanding gross household debt at the end of the quarter and adjusted by PC_t. Source: Statistics Norway.
- y_t The log of household real disposable income measured by disposable income excluding dividend payments and adjusted by PC_t. Source: Statistics Norway.

k_t The log of total housing stock at the end of the quarter in fixed 2018-prices calculated in the National Accounts by means of the perpetual inventory method. Source: Statistics Norway.

R_t The real after-tax interest rate calculated by $\frac{1+4\times I_t\times(1-\Upsilon_t)}{CPI_t/CPI_{t-4}}-1$, where I_t, Υ_t and CPI_t are the average nominal interest rate (quarterly) paid by households on loans in private financial institutions, the capital tax rate and the consumer price index for all commodities, respectively. Source: Statistics Norway.

The data are available from the authors upon request.

Figure 7: First difference of real house prices (Δp_t), household real debt (Δd_t), real disposable income (Δy_t), real housing stock (Δk_t) and real after-tax interest rate (ΔR_t)



Notes: Sample period: $1982q1 - 2018q4$.