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Terje Skjerpen and Anders Rygh Swensen

Testing for long-run homogeneity in the Linear Almost Ideal Demand System

An application on Norwegian quarterly data for non-durables

Abstract:
We consider testing for long-run homogeneity within a dynamic consumer demand system allowing for non-stationarities. The static long-run solution is assumed to follow the Linear Almost Ideal Demand System and we test for long-run homogeneity within this system utilizing a triangular representation. Both long-run homogeneity in single equations and in the entire consumer system are considered. Consistent with the general triangular representation the deduced share equations depend on both current, lagged and leaded differenced explanatory variables (besides level variables and deterministic variables capturing trend and seasonality). Before testing for long-run homogeneity we test whether prices and real total expenditure are Granger-caused by the share variables. In the case with Granger-causality the leaded differenced variables can be omitted from the equation to be estimated. Thus Granger non-causality can be tested by looking at the joint significance of all leaded variables in all share equations simultaneously. The methodological framework is applied on non-durable consumption data from 1966:1 to 1997:4 taken from the unadjusted Norwegian National Accounts. We find that the null of no Granger-causality are clearly rejected and in light of this the differenced variables are retained when testing for long-run homogeneity. When testing for long-run homogeneity we consider different specifications of the deterministic trend and of the lags/leads specification. We find that the homogeneity-tests are substantially influenced by the specification of the deterministic trend. In one of the specifications long-run homogeneity is neither rejected for the single equations nor for the entire system. By imposing long-run homogeneity long-run Engel- and price elasticties can be calculated. Generally, the long-run elasticities seem reasonable.

Keywords: Consumer demand, LAIDS, Homogeneity, Non-Stationarity.

JEL classification: C12, C32, E21.

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Address: Terje Skjerpen, Statistics Norway, Research Department. E-mail: terje.skjerpen@ssb.no

Anders Rygh Swensen, Statistics Norway and University of Oslo.

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Statistics Norway
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Telephone: +47 62 88 55 00
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1. Introduction

There is a long tradition in consumer econometrics to test for homogeneity in demand systems. The homogeneity restriction follows from economic theory where agents are assumed to maximise utility given a fixed budget and given the prices of the commodities. Homogeneity (of degree zero) in demand systems implies the absence of money illusion: a proportional increase in the budget and all nominal prices will leave consumption unchanged. An implication of this is that consumer demand can be formulated as functions of *relative* prices. Since models in which homogeneity holds are nested within a model which include effects of nominal prices and total outlay, the homogeneity restriction can easily be tested within a parametric framework. The homogeneity hypothesis has frequently been rejected in empirical analyses. An important issue is how one should interpret a statistical rejection of the homogeneity restriction. As emphasised by Keuzenkamp and Barten (1995), statistical rejection of the homogeneity restriction has not led to rejection of the underlying economic hypothesis because of the presence of many auxiliary hypotheses. In their view it is more fruitful to interpret testing of homogeneity in demand systems as a specification search rather than as an attempt to test economic theory.

Anderson and Blundell (1983,1984), building on the linear approximation of the AIDS model of Deaton and Muellbauer (1980)), emphasised that the homogeneity restriction from economic theory should be viewed as a long-run restriction and that neglecting dynamics could lead to an invalid rejection of homogeneity. However, they did not take explicitly account of potential non-stationarities in the data. Ng (1995) and Attfield (1997) test for long-run homogeneity within a cointegrating framework utilising the triangular representation of Phillips (1991), which seems well suited for non-stationarities in consumer demand systems containing a substantial number of commodities. Both Ng and Attfield utilises seasonally adjusted data, whereas we are conducting our analysis on seasonally unadjusted data. The seasonality is represented by dummy variables.

Ng (1995) adresses the question of long-run homogeneity within a single equation framework and thus does not test for long-run homogeneity in the complete system. Attfield (1997) tests for long-run homogeneity both for single commodities and in the system as a whole. However, in contrast to the analysis of Ng and our analysis the explanatory variables are at the outset assumed to follow finite autoregressions, which implies that the conditioning variables are not Granger-caused by the budget shares variables. Whether Granger non-causality can be imposed or not are important for formal testing of long-run homogeneity. When Granger non-causality is invalid, the exact distribution result for the Wald-statistic, which Attfield relies on, can no longer be used for testing long-run homogeneity.

In our paper we find that Granger non-causality is strongly rejected and in accordance with this we use the Chi-square form of the Wald-test to make inference about long run-homogeneity in the single equations as well as in the system as a whole.

In analyses of consumer demand systems it is common to include a deterministic trend. Several arguments can be given for including a deterministic trend in the demand equations. First a deterministic trend may capture the effects of omitted variables, for instance dispersion effects caused by aggregating over consumers at the micro level. Second drift in the model parameters may result in significant trend effects. Lastly the deterministic effects may pick up omitted non-linear effects since the share equations are assumed to be linear in total real outlay. Since the deterministic trend may pick up several distinctive features, it may be too restrictive to stick to a linear trend.

We operate with different deterministic trend functions in the estimated share equations and find that the specification of it has a substantial influence on the tests for long-run homogeneity. When the trend is assumed to be linear, we find that long-run homogeneity is rejected for some of the included commodities and for the consumer system as a whole. Especially in the equations for *Food* and *Beverages and tocacco* we find clear evidence against long-run homogeneity. Ng(1995) found it necessary to include a quadratic trend function for some of the commodities in order not to reject long-run homogeneity. However, we still find evidence against long-run homogeneity when such a trend is used. Our most general trend specification consists of a quadratic trend augmented by one additional term, which is the cross product between the linear trend and the natural logarithm of the linear trend. To take account of short-run effects lagged and leaded difference variables are included in the level share equations. Two distinctive specifications are investigated, depending on the number of lags/leads and on a priori parameter restrictions. In both specifications the lags and leads are treated symmetrically. The number of lags/leads is set to 2 in the first specification, and 4 in the second specification. However, in the latter specification the parameters connected to lags/leads 2 and 3 are constrained to zero in order to limit the number of parameters. Using the former specification combined with the most flexible trend specification long-run homogeneity in the entire consumer system can not be rejected. For the single commodities the lowest significance probability, 0.08, is found for *Tobacco and Beverages.*

Long-run elasticities are calculated for different models after imposition of long-run homogeneity. In the reference model all elasticities have the a priori expected sign and seem reasonable in magnitude. Moreover, the expenditure elasticities are rather robust over the different model specifications.

The rest of the paper is organised as follows. In Section 2 we establish the modelling framework. Data issues are covered in section 3. The empirical analysis is carried through in section 4. The conclusions are given in section 5.

2. Modelling framework

If $\underline{s}^{(n)} = (s_1, \quad \cdots \quad, s_n)'$ is the vector of budget shares and $\underline{p} = (\log(P_1), ..., \log(P_n), \log(R_e))$ is the vector of explanatory variables (where P_i is the price index for commodity i and R_e is real total expenditure per capita), the linear approximation of the AIDS model may be written as

(2.1) $$s_i = \pi_{i0} + \sum_{j=1}^{n} \pi_{ij} \log(P_j) + \pi_{i,n+1} \log(R_e).$$

The price index of Stone (1954), cf. equation (3.2) below, has been used to deflate the nominal total expenditure variable (per capita). In matrix notation equation (2.1) becomes

(2.2) $$\underline{s}^{(n)} = \underline{\Pi}_0^{(n)} + \Pi^{(n)} \underline{p}.$$

A natural stochastic and dynamic extension of (2.2) is (when also an explicit time index has been added)

(2.3) $$\Delta \underline{s}_t^{(n)} = \Pi_1^* \Delta p_t - \Pi_2^* \left(\underline{s}_{t-1} - \underline{\Pi}_0^{(n)} - \Pi^{(n)} \underline{p}_{t-1} \right) + \underline{\varepsilon}_t^{(n)},$$

where Π_1^* and Π_2^* are suitably defined matrices and Δ is the difference operator. The form (2.3) is not identifiable and Anderson and Blundell (1983) therefore proposed to consider an identifiable version given by

(2.4) $$\Delta \underline{s}_t = \Pi_1 \Delta p_t - \Pi_2 \left(\underline{s}_{t-1} - \underline{\Pi}_0 - \Pi \underline{p}_{t-1} \right) + \underline{\varepsilon}_t,$$

where $\underline{s}_t$, $\underline{\Pi}_0$, Π and $\underline{\varepsilon}_t$ are the first (n-1) rows of $\underline{s}_t^{(n)}$, $\underline{\Pi}_0^{(n)}$, $\Pi^{(n)}$ and $\underline{\varepsilon}_t^{(n)}$, respectively.

The problem with a specification like (2.4) is that weak exogeneity assumptions have been imposed a priori. A suitable general representation, which is less rigid in this respect, is the triangular form considered by Phillips (1991) and Stock and Watson (1993), which, in our case with added deterministic variables, is given by

(2.5a) $$\underline{s}_t = B\underline{p}_t + D\underline{d}_t + \underline{u}_{1,t}$$

and

(2.5b) $$\Delta\underline{p}_t = \underline{u}_{2,t},$$

where $\underline{d}_t$ is a vector of n_d deterministic variables. These can be bounded variables or polynomial trends. The process $\underline{u}_t = (\underline{u}_{1t}', \underline{u}_{1t}')'$ is a stationary stochastic process with full rank spectral density. Since there are no exogeneity assumptions involved in the model (2.5), both leads and lags have to be taken into account when considering the conditional process $\{\underline{u}_{1,t}\}$ given $\{\underline{u}_{2,t}\}$.Thus $E\left[\underline{u}_{1t} \middle| \{\underline{u}_{2t}\}\right] = d(L)\underline{u}_{2,t}$, where *d(L)* is the infinite matrix polynomial given by $d(L) = \sum_{j=-\infty}^{\infty} d_j L^j$. *E*quations (2.5a) and (2.5b) may now be written as

(2.6a) $$\underline{s}_t = B\underline{p}_t + D\underline{d}_t + d(L)\Delta\underline{p}_t + \underline{v}_t,$$

(2.6b) $$\Delta\underline{p}_t = \underline{u}_{2,t},$$

where $\underline{v}_t = \underline{u}_{1t} - E\left[\underline{u}_{1t} \middle| \{\underline{u}_{2t}\}\right]$. Since $\{\underline{v}_t\}$ and $\{\underline{u}_{2t}\}$ are now independent, by construction, Stock and Watson (1993) noticed that B may be estimated from (2.6a) under the assumptions that (i) the data are generated from (2.5a) and (2.5b), (ii) $\{\underline{u}_t\}$ is stationary, Gaussian and with full rank spectral density matrix and (iii) *d(L)* is approximated by the finite matrix polynomial $\sum_{j=-k}^{k} d_j L^j$. The parameters we consider are now defined in terms of (2.6), that is B, D, the coefficients of the polynomial d(L) and the parameters of the distribution of the process $\{\underline{v}_t\}$ in the conditional model (2.6a) and the parametes of the distribution of the process $\{\underline{u}_{2t}\}$ in the marginal model (2.6b). Since $\{\underline{v}_t\}$ and $\{\underline{u}_{2t}\}$ are independent, the likelihood factorizes. The parameters of the conditional and marginal models also vary freely. Hence the parameters in (2.6a) can be estimated from the conditional part. As showed by Stock and Watson, the OLS estimator of B is now asymptotically equivalent to the maximum likelihood estimator conditional on the initial and terminal values. They also show that the two-sided triangular representation is valid under weaker conditions than mentioned above. In particular, the assumption that $\{\underline{u}_t\}$ is Gaussian may be dropped. We therefore estimate the long-run parameters with OLS utilising the following set of regressions

(2.7) $$\underline{s}_t = B\underline{p}_t + F\underline{q}_t + \underline{v}_t,$$

where $\underline{q}_t$ is the column vector $\left(\Delta\underline{p}'_{t-k},...,\Delta\underline{p}'_{t+k},\underline{d}'_t\right)'$ of dimension $n_1 = (2k+1)(n+1)+n_d$. The coefficient matrix F may be partioned correspondingly as

(2.8) $$F = \left[F_{-k}...F_0...F_k\ F_d\right],$$

where F_j is of dimension $(n-1)\times(n+1)$ for $j=-k,...,0,...k,$ and F_d is of dimension $(n-1)\times n_d$.

For testing long-run homogeneity it is convenient to introduce the variable vector $\underline{\xi}_t$ defined by

$\underline{\xi}_t = \left(\xi_{1,t},...,\xi_{n-1,t},\log\left(P_{n,t}\right),\log\left(R_{e,t}\right)\right)'$,

where $\xi_{j,t} = \log\left(\frac{P_{j,t}}{P_{n,t}}\right),\ \ j=1,...,n-1,$ and express (2.7) as

(2.9) $$\underline{s}_t = H\underline{\xi}_t + F\underline{q}_t + \underline{v}_t,$$

for a suitable definition of *H*.

Testing long-run homogeneity in single equations now amounts to testing whether the coefficients $h_{j,n} = 0\left(j=1,...,n-1\right),$ where $h_{j,n}$ denotes the element of the j'th row and n'th column of *H*. Overall homogeneity in the demand system means that they are simultaneously zero.

By transposing (2.7) and thereafter stacking over all (effective) sample periods we obtain

(2.10) $$S = XH' + ZF' + V,$$

where *S* is a $T\times(n-1)$ matrix with budget shares, *X* a $T\times(n+1)$ matrix with explanatory variables in levels, *H* an $(n-1)\times(n+1)$ matrix of long-run coefficients, *Z* a $T\times n_1$ matrix of differenced and deterministic variables, *F* an $(n-1)\times n_1$ matrix with short run and deterministic coefficients and *V* a $T\times(n-1)$ matrix with errors.

The least squares estimator for the long-run parameters are now, utilising partioned regression,

(2.11) $$\hat{H} = S^{/} Q_Z X (X^{/} Q_Z X)^{-1},$$

where

(2.12) $$Q_Z = I_T - Z(Z^{/} Z)^{-1} Z^{/}.$$

The covariance matrix of the errors are estimated by

(2.13) $$\hat{\Omega} = T^{-1} \hat{V}^{/} \hat{V},$$

where $\hat{V}$ is the matrix with the least squares residuals. Using a well-known formula for Kronecker products equation (2.9) may alternatively be written as

(2.14) $$\underline{s}_t = \left[\left(\underline{\xi}_t', \underline{q}_t' \right) \otimes I_{n-1} \right] vec\left(H\,F \right) + \underline{v}_t .$$

The restrictions for overall homogeneity is given by $H\underline{e}_{n,n+1} = \underline{0}_{n-1}$, where $\underline{e}_{n,n+1}$ is the $n+1$ unit vector having the n'th element equal to unity. These restrictions can also alternatively be written as

(2.15) $$\left(R \otimes I_{n-1} \right) vec(H\,F) = \underline{0}_{n-1},$$

where $R = \left[\left(\underline{e}_{n,n+1}', \underline{0}_{n_1}' \right) \right]$. As explained in Appendix A, using the results in Stock and Watson (1993) these restrictions may be tested using the Wald-statistic

(2.16) $$W = \frac{\underline{e}_{n,n+1}^{/} \hat{H}^{/} \hat{\Omega}^{-1} \hat{H} \underline{e}_{n,n+1}}{\underline{e}_{n,n+1}^{/} (X' Q_z X)^{-1} \underline{e}_{n,n+1}}.$$

Homogeneity in single equations may be tested using a similiar expression. This is explained in more detail in Appendix A. The homogeneity restrictions only involve the long-run parameters. Hence according to Stock and Watson (1993) the Wald statistic based on the OLS estimation, and possibly modified for serial correlation in $\{\underline{v}_t\}$ (cf. Newey and West (1987), is asymptotically χ^2 distributed with $(n-1)$ degrees of freedom.

In fact, under stronger assumptions, more can be said about the distribution in finite samples. Phillips (1994) showed that if (2.5b) was specified as a finite autoregression (ignoring the deterministic variables)

(2.17) $$\Delta p_t = \sum_{i=1}^{k} \Phi_i \Delta p_{t-i} + u_{2,t} \ ,$$

where $\{\underline{u}_{2,t}\}$ is a series of uncorrelated Gaussian variables and if in addition $\underline{u}_t = \left(\underline{u}_{1,t}', \underline{u}_{2,t}'\right)'$ are independent N(0,Σ), the augmented regression only involves current and lagged variables

(2.18) $$\underline{s}_t = B\underline{p}_t + \sum_{i=0}^{k} \Phi_i \Delta \underline{p}_{t-i} + \underline{v}_t \ .$$

In this case the distribution of the statistic $\frac{T - n_1 - (n-1) + 1}{T(n-1)}W$ is exactly F-distributed with $(n-1)$ degrees of freedom in the numerator and $\left(T - n_1 - (n-1) + 1\right)$ degrees of freedom in the denominator. Under these assumptions Attfield (1997) tested for long-run homogeneity on UK consumption data.

3. Data

The data are taken from the quarterly norwegian national accounts and cover the period 1966:1 to 1997:4. We model household's expenditure on nine consumption categories; only non-durable goods and services are considered. For each consumption category we have corrected for foreigners consumption in Norway by assuming that a fixed proportion of foreigners total consumption is spent on a specific category An overview of the different consumption categories together with their average budget shares over the sample is given in Table 3.1 below.

Table 3.1. Overview of consumption categories

Consumption category number	Consumption category	Average budget shares
1	Food	0.252
2	Beverages and tobacco	0.108
3	Energy	0.059
4	Running cost of vehicles	0.087
5	Other non-durable goods	0.098
6	Clothing and shoes	0.114
7	Other services	0.147
8	Transport services	0.081
9	Consumption abroad	0.054

The real per capita total expenditure variable has been constructed as follows. Let C_{jt} denote consumption in fixed prices for consumption category j in period t. Total per capita expenditure in nominal terms is then given by

$$Y_t = \frac{\sum_{j=1}^{9} P_{j,t} C_{j,t}}{N_t}, \tag{3.1}$$

where N_t is the population of Norway at the end of the quarter in 1000 persons. The variable Y_t is deflated by the Stone Price index [cf. Stone (1954)]:

$$P_t^* = \exp\left[\sum_{j=1}^{n} s_{j,t} \log(P_{j,t})\right], \tag{3.2}$$

where $s_{j,t}$ denotes the budget share of consumption category j in period t. The variable that is used in the empirical application is $R_{e,t}$ which is now defined as

$$R_{e,t} = Y_t / P_t^*. \tag{3.3}$$

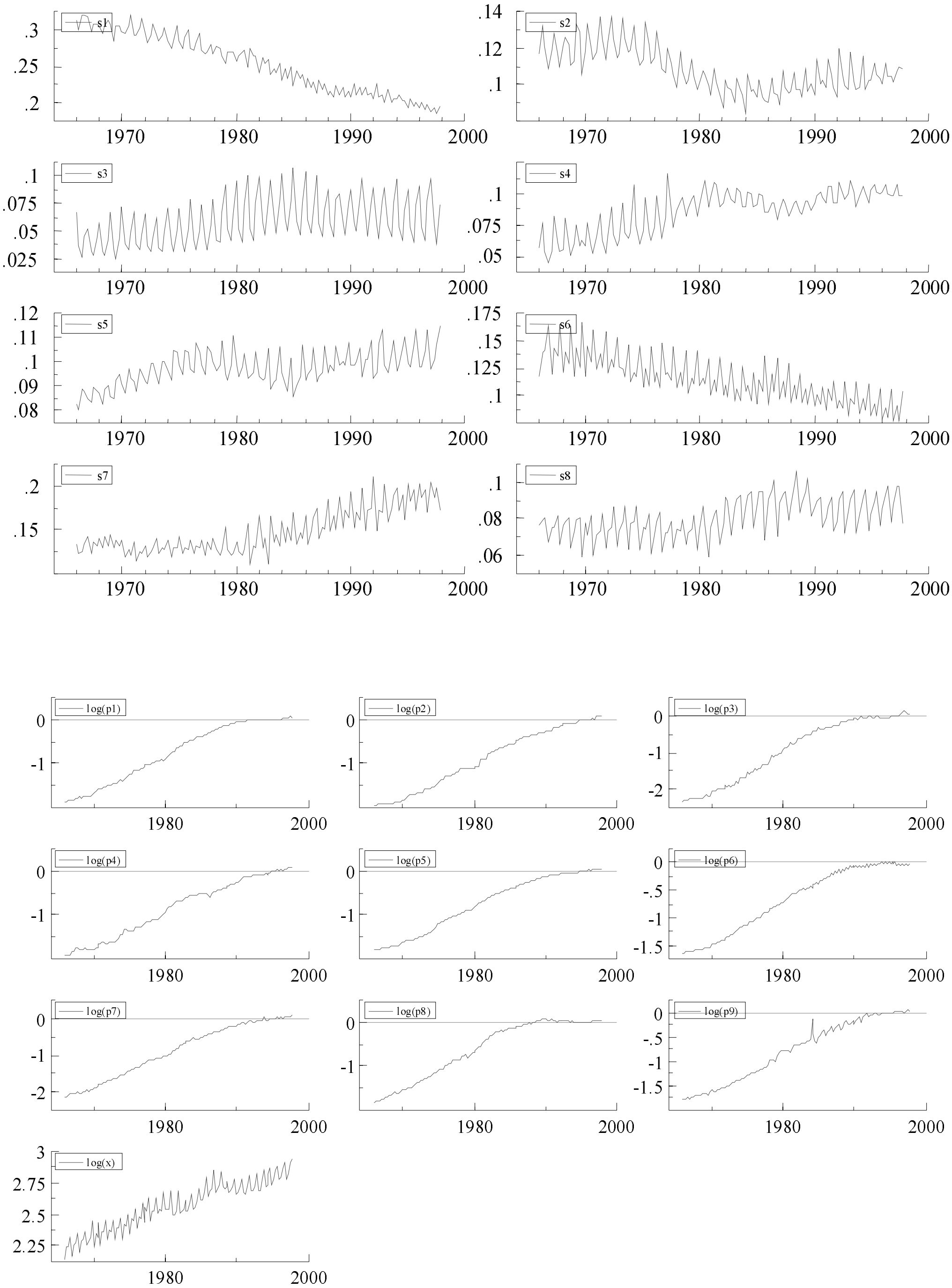
Our effective sample (after allowing for lags and leads) starts in 1967:2 and ends in 1996:4. An overview of the deterministic variables is given in Table 3.2. The first four variables are a constant term and three seasonal dummy variables. Due to a change in the quarterly national accounts, there is a break in measured seasonality after 1977:4. We thus include specific seasonal effects for each of these two subsamples, cf. the variables $d_{5,t} - d_{8,t}$. The three last deterministic variables are due to the deterministic trend. In all our models we include a linear trend variable. However, we also estimate models with a more complex deterministic trend structure in which a quadratic trend and a trend variable defined as the crossproduct between t and $\ln(t)$ are included.

Table 3.2. An overview of deterministic variables used in the empirical analysis

Variable	Definition
$d_{1,t}$	The value 1 (corresponding to the constant term)
$d_{1+j,t}$; j=1,2,3	Seasonal dummy variable which is 1 in the j'th quarter, -1 in the fourth quarter and zero in the two remaining quarters
$d_{5,t}$	Dummy variable which is 1 until 1977:4 and 0 thereafter
$d_{5+j,t}$; j=1,2,3	$d_{1+j,t}d_{5t}$
$d_{9,t}$	A linear trend which is 1 in 1966:1
$d_{10,t}$	$(d_{9,t})^2/100$
$d_{11,t}$	$(d_{9,t})\ln(d_{9,t})$

The time series used in our analysis are displayed in Figure 3.1. None of the variables look stationary, and our implicit assumption that the level variables contain a unit root seems plausible.

Figure 3.1 Time series used in the empirical analysis



4. Empirical analysis

4.1. Regression diagnostics and model reduction

Since a rather large system is considered a lot of parameters have to be estimated. When for instances the number of lags of the differenced explanatory variables increases by one $(n-1)\times(n+1)$ new coefficients are included, which with $n=9$ implies 80 coefficients. Thus we have to use short leads and lags or to make other a priori restrictions. The error vectors ought to be white noise, however this will usually not be fulfilled if the dynamics are too restrictive. We allow for Granger-causality which implies that leads of the differenced explanatory variables occur as regressors. Thus our treatment of lags and leads are from the outset symmetric. We distinguish between two types of models corresponding to the choice of which leads/lags to include. In the first type of model we allow for 2 lags and leads, which implies that $E_{-k}=E_k=0$ for $k>2$ [cf. equation (2.8)], whereas in the second type of models we assume apriori $E_{-k}=E_k=0$ for $k>4$ and $k=2,3$. Since we have quarterly data it may be important to include variables at the four period lag/lead. The former specification is labelled LL12 and the latter is labelled LL14.

In addition to the assumption with regard to the inclusion of leads and lags we operate with four different specifications of the deterministic trends (cf. section 3). These specifications are labelled T1, T2, T3 and T4. The specification T4 means that only a linear trend is included in the set of regressions. T3 corresponds to the specification when the term tln(t) is added. In the specification T2 we use a linear and a quadratic trend variable. The most general specification of the trend is contained in T1. In this specification we include a linear trend, a quadratic trend and the above mentioned crossproduct.

Remark that the specification T1 and T3 are not of the form covered by the models described in section 2 since ln(*t*) is not a polynomial trend. We do not think that the finite sample distribution of the Wald statistic based on a trend specification including terms of the form tln(t) are radically different from not including it, so we use the same method for evaluating all these four models. To substantiate this assertion Monte Carlo simulations using the estimated model is a possibility.

Combining the two specifications we get eight models. All the models have been estimated by PcGive 9.2 (cf. Doornik and Hendry (1997). In appendix Tables B1-B6 we give some single equation and system diagnostics utilising the ordinary least squares residuals. Using the LL14 specification no evidence is found for serial correlation in the errors. In the LL12 specification no evidence of serial correlation is found in the case with a linear trend. Under the three other trend specifications significant error correlation are found for consumption group number 7, that is *Other services*.

Table B2 shows that the test statistic for normality is not significant at the 5 per cent significance level for any of the consumption categories when using the LL14 specification together with a linear trend specification. Using the more complex trend specifications we mainly find sign of non-normality in the equation for consumption groups 4 and 8, which is *Running cost of vehicles* and *Transport services* respectively. Under the lags/leads specification LL12 significant non-normality is found for O*ther services*, irrespective of the trend specification. Using one of the more general trend specifications significant non-normality is also detected for *Running cost of vehicles*. None of the 64 ARCH statistics in Table B3 are significant at the 5 percent significance level. Thus we find no evidence for conditional heteroskedasticity.

The results from the Reset tests are given in Table B4. For consumption group 8, *Transport services*, functional misspecification are found for all models. Under LL14 the Reset statistics are significant also for consumption group 3, *Energy*, and consumption group 4, *Running cost of vehicles*, but in the latter case it depends on the trend specification. Disregarding *Transport services*, none of the Reset statistics are significant at the 1 percent test level, but some are significant at the 5 percent significance level.

In Table B5 we also give some diagnostic statistics for the complete model. With the lags/leads assumption LL14 and the most general trend assumption, T1, the statistic for vector autocorrelation is insignificant at the 1 percent level whereas vector normality is rejected at the same significance level. Under a linear trend none of these statistics are significant at the 1 percent significance level. Using the lags/leads assumption LL12 the general tendency is that the two vector statistics are more significant than under the LL14 assumption.

Totally it seems that the model with LL12 are less well-specified than the model with LL14, but how clear-cut this conclusion is depends on the deterministic trend specification. Especially the LL14 models seem to perform slightly better with regard to serial autocorrelation in the residuals.

Regarding the specification of deterministic effects we have also conducted two tests reported in Table B7. First, it is tested whether one can simplify to a linear trend, that is the linear trend specification T4 is tested against the most general trend specification T1. The result from this test depends on the specification of lags/leads. Under the LL14 specification the hypothesis is clearly rejected, whereas the conclusion is less clear-cut under the alternative specification LL12, the significance probability being about 4 percent. Second, we test whether the a priori assumed structural break is insignificant. This hypothesis is strongly rejected under both LL12 and LL14, which supports the inclusion of the variables capturing a break in the seasonality.

4.2 Testing for absence of leads

As explained in the introduction the primary goal of the present paper is to investigate the homogeneity property of the long-run share equations. This question only involves the parameters of the cointegrating relations, and the other parameters are nuisance parameters in this respect. This does not mean that they are of no interest however. It is of substantial importance to test for the joint significance of the differenced variables with leads. If these variables are insignificant a lot of degrees of freedom are saved since 160 parameters can then be constrained to zero.

The results, based on Wald-tests, are given in Table B6. In all eight cases the hypothesis that the coefficients of the leads are insignificant is heavily rejected. Thus the finite autoregression representation (2.18), which was used by Attfield (1997), is not valid in our case. In view of this the leaded variables are retained in the system when we later test for long-run homogeneity.

The interpretation of the presence of the leads is as pointed out by Stock and Watson (1993), that there is Granger causality between the innovations of the shares and the innovations of prices and income. This means that there is a feedback from the past shares to the present values of prices and income.

The reasons for this feedback can be many. One is that there is a real causality present. We feel that this is a bit farfetched in this context and is more inclined to ascribe the result to aggregation effects or due to omission of relevant explanatory variables.

In view of the above results we will in the next section test for long-run homogeneity under different assumptions with regard to lags/leads and deterministic trend when differenced variables taking account of Granger causality and deterministic variables are included as additional regressors.

4.3. Testing for long-run homogeneity

The results from testing long-run homogeneity using Wald-tests are given in Tables 4.1a and 4.1b. The conclusion seems to depend both on the assumption with regard to lags/leads and trend. Only in one case we can not reject the hypothesis of long-run homogeneity in the entire system, cf. the bottom line in Tables 4.1a and 4.1b, at the 5 percent test level. This occurs for the model characterised by LL12 and T1. At the 1 percent test level we can in addition not reject any of the two T3-models.

Looking at the individual commodities, long-run homogeneity is clearly rejected for *Beverages and Tobacco* in all the T2-T4 models. However the significance probabilities are generally larger for T3 than for T2. This may indicate that inclusion of the term tln(t) is more important for assuming homogeneity than the quadratic term. In the LL14/T1-model long-run homogeneity is not rejected at the 1 percent, but at the 5 percent level. Besides *Beverages and Tobacco* some evidence against long-run homogeneity is present for *Food*. In the T2- and T4-models long-run homogeneity is rejected at the 1 percent level, and in the model LL12/T3 long-run homogeneity is rejected at the 5 percent level. For all the other consumption groups we find one additional case of rejection of long-run homogeneity, that is for *Other non-durable goods* in the model obtained by combining LL14 and T1.

As commented on in section 4.1 there was some sign of residual serial correlation in the share equations for the T2-T4 models under the lags/leads specification LL12. This may suggest that one in the Wald-tests should replace the usual estimate of the covariance matrix of the errors, given by equation (2.13), with a Newey-West formula (cf. Newey and West (1987)) which takes account of the serial correlation. This is done by Attfield (1997).

Table 4.1a. Tests of long-run homogeneity in the single equations and all share equations together in different models assuming LL12. Significance probabilities[a]

	Trend assumption[b]			
Hypothesis	T1	T2	T3	T4
$h_{1,9}=0$	0.5276	0.0004	0.0374	0.0009
$h_{2,9}=0$	0.0801	0.0001	0.0042	0.0001
$h_{3,9}=0$	0.7708	0.8467	0.9580	0.8483
$h_{4,9}=0$	0.3135	0.4490	0.0849	0.4634
$h_{5,9}=0$	0.9195	0.3821	0.9340	0.3966
$h_{6,9}=0$	0.5063	0.3272	0.4913	0.3254
$h_{7,9}=0$	0.8882	0.3064	0.9432	0.3356
$h_{8,9}=0$	0.8104	0.3443	0.5635	0.4028
$h_{j,9}=0 \ \forall \ j=1,\ldots,8$	0.4168	0.0003	0.0142	0.0005

[a] The single equation statistics are χ^2-distributed with 1 degree of freedom, whereas the multi-equation statistics are χ^2-distributed with 8 degrees of freedom.

[b] The different assumptions with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table 4.1b. Tests of long-run homogeneity in the single equations and all share equations together in different models assuming LL14. Significance probabilities[a]

Hypothesis	Trend assumption[b]			
	T1	T2	T3	T4
$h_{1,9}=0$	0.1009	0.0003	0.1438	0.0080
$h_{2,9}=0$	0.0461	0.0000	0.0005	0.0000
$h_{3,9}=0$	0.4259	0.6527	0.4402	0.6351
$h_{4,9}=0$	0.2736	0.4800	0.1273	0.4664
$h_{5,9}=0$	0.0401	0.4337	0.9961	0.4791
$h_{6,9}=0$	0.7469	0.5474	0.6694	0.5505
$h_{7,9}=0$	0.8226	0.0838	0.7243	0.1496
$h_{8,9}=0$	0.3094	0.3120	0.4503	0.4777
$h_{j,9}=0\ \forall\ j=1,\dots,8$	0.0084	0.0003	0.0281	0.0034

[a] The single equation statistics are χ^2-distributed with 1 degree of freedom, whereas the multi-equation statistics are χ^2-distributed with 8 degrees of freedom.
[b] The different assumption with regard to the trend is T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

4.3. Calculation of long-run elasticities

In Tables 4.2a and 4.2b we report Engel-elasticities and direct price elasticities for the eight models when we have imposed long-run homogeneity. The conclusion from the tests in the preceding section was that long-run homogeneity was only valid for one model at the 5 percent test level, namely LL12/T1. Thus imposing long-run homogeneity in the other cases is in a strict sense not valid. However, to investigate how robust the long-run elasticties are over the different specifications, Engel- and price elasticities are also calculated for these models. In the following we will refer to LL12/T1 as our reference model. The estimated coefficients for this reference model are given in Appendix C. Especially, the coefficients needed for the long-run elasticity-calculations are reported in Table C1. With one exception all the elasticities in Tables 4.2a and 4.2b have the a priori expected sign. For consumption category 3, *Energy*, we obtain a positive (but small) price elasticity in all the four LL14-models. However, in the LL12 specification the elasticity is, as expected, negative.

The calculated long-run Engel-elasticities are very robust, especially over the different trend specifications. A commodity with an Engel elasticity above 1 is labelled a luxury commodity, whereas a commodity with an Engel elasticity below 1 is labelled a necessity. Engels law which claim that *Food* is a necessity is confirmed in all cases, the estimated Engel-elasticity being somewhat higher in the LL12-models. In the LL12-models *Running cost of vehicles*, *Other non-durable goods* and *Transport services* are found to be luxuries, whereas in the LL14-models only the two last mentioned consumption categories are classifies as luxuries. However, in the LL14-models *Other services* we are on the border area between necessities and luxuries.

The price elasticities show somewhat more variation than the Engel-elasticities. A direct price elasticity, which is above 1 in absolute value, implies that the consumption category in question is elastic, whereas an elasticity below 1 implies that it is inelastic. In the LL12-models *Other services* is found to be price elastic irrespective of the choice of trend specification. The estimate is rather similar for the T1-T3 models, and in absolute value substantially smaller than for the T4 model where the estimated elasticity is around -3. For *Beverages and Tobacco* and *Clothing and shoes* the price elasticity is about one in absolute value, being higher than one in some cases and lower than one in some other cases. As in the LL12-models *Other services* is price elastic in the LL14-models. For *Clothing and shoes* we obtain an estimated price elasticity which exceeds one in absolute value in all the four cases. However, in contrast to what was the case in the LL12-models we do not find that *Beverages and Tobacco* are price elastic in any of the models.

Table 4.2a. Engel- and uncompensated own price-elasticties[a] in the LL12-models. Long-run homogeneity imposed

	Engel elasticities (E_i)				Price elasticities (Pr_i)			
	Trend[b]				Trend[b]			
	T1	T2	T3	T4	T1	T2	T3	T4
1	0.828	0.821	0.814	0.853	-0.719	-0.682	-0.686	-0.724
2	0.609	0.595	0.577	0.634	-1.155	-1.057	-0.849	-1.473
3	0.581	0.579	0.580	0.606	-0.186	-0.228	-0.188	-0.302
4	1.013	1.019	1.058	1.119	-0.243	-0.268	-0.311	-0.332
5	0.644	0.641	0.643	0.669	-0.293	-0.308	-0.299	-0.148
6	1.375	1.379	1.383	1.366	-1.093	-1.027	-1.088	-0.954
7	0.530	0.537	0.528	0.454	-1.835	-1.984	-1.840	-3.002
8	1.477	1.482	1.466	1.380	-0.490	-0.434	-0.555	-0.787

[a] The Engel elasticity is calculated as $E_i = 1 + B\left[i, R_e\right] / \overline{s}(i)$, where B[i,R_e] denotes the coefficient in front of the log of total real expenditure in the equation for the i'th share and where $\overline{s_i}$ denotes the empirical mean of the share variable for the i'th commodity over the sample 1967(2) to 1996(4). The uncompensated own price elasticity is calculated as $\Pr_i = -1 - B[i, R_e] + B[i,i] / \overline{s}(i)$, where B[i,i] is the coefficient in front of the log of the relative own price in share equation i. The formulae are taken from Chalfant (1987).

[b] The different assumption with regard to the trend is T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table 4.2b. Engel- and uncompensated own price-elasticties[a] in the LL14-models. Long-run homogeneity imposed

	Engel elasticties (E_i)				Price elasticties (Pr_i)			
	Trend[b]				Trend[b]			
	T1	T2	T3	T4	T1	T2	T3	T4
1	0.665	0.646	0.657	0.680	-0.748	-0.644	-0.706	-0.787
2	0.606	0.560	0.563	0.590	-0.865	-0.594	-0.550	-0.949
3	0.489	0.495	0.506	0.518	0.159	0.199	0.234	0.157
4	0.743	0.761	0.787	0.811	-0.336	-0.385	-0.431	-0.418
5	0.722	0.710	0.716	0.730	-0.377	-0.352	-0.377	-0.242
6	1.746	1.753	1.753	1.749	-1.408	-1.351	-1.375	-1.336
7	0.984	1.012	0.990	0.948	-1.130	-1.192	-1.121	-1.880
8	1.277	1.289	1.257	1.209	-0.276	-0.221	-0.383	-0.805

[a] The Engel elasticity is calculated as $E_i = 1 + B\left[i, R_e\right] / \overline{s}(i)$, where B[i,R_e] denote the coefficient in front of the log of total real expenditure in the equation for the i'th share and where $\overline{s}_i$ denotes the empirical mean of the share variable for the i'th commodity over the sample 1967(2) to 1996(4). The uncompensated own price elasticity is calculated as $\Pr_i = -1 - B[i, R_e] + B[i,i] / \overline{s}(i)$, where B[i,i] is the coefficient in front of the log of the relative own price in share equation i. The formulae are taken from Chalfant (1987).

[b] The different assumptions with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

6. Conclusions

Within a consumer system covering 9 non-durable commodities we test for long-run homogeneity using a triangular error correction representation. In our most general models the long-run coefficients are estimated together with short-run parameters and effects of deterministic variables. The latter parameters are considered as nuisance parameters in the analysis. Since Granger non-causality is not imposed a priori, the differenced variables are also included with leads. With regard to leads/lags we operate with two alternatives because of the degrees of freedom problem caused by the high dimension of the consumer demand system. The estimated share equations for these general models seem reasonably well-specified.

Before testing for long-run homogeneity we test whether the model may be simplified, that is we investigate whether Granger non-causality may be imposed. Granger non-causality means that the differenced variables with leads are omitted from all the share equations. Based on formal testing we find clear evidence that imposing Granger non-causality is not valid. A break in the seasonal pattern is also imposed a priori, based on a substantial argument. The joint significance of the deterministic variables representing the break is tested and we find that the hypothesis of no break is heavily rejected. Because of this the structural break variables are retained in the model when testing for long-run homogeneity.

It turns out that the outcomes of the long-run homogeneity tests rely on the treatment of deterministic trends. Thus it is of interest to test whether the simplest trend specification may be rejected against the most complex trend specification. This means testing the linear trend specification against the most flexible trend specification, which in addition contains a quadratic term and a term which is the product between the linear trend and the logarithm of the linear trend. Using a specification where the differenced variables at lags/leads 1 and 4, besides the current differenced variables, are included in the share equations, the linear trend specification is clearly rejected. Using the alternative specification where the differenced variables at lags/leads 1 and 2 are included, a simplification to linear trend can be rejected at the 5 percent significance level, but not at the 1 percent significance level.

In one case we do not reject long-run homogeneity in the entire system at the five percent significance level. This is in the model characterised by combining 2 lags/leads with the most flexible trend specification. At the 1 percent significance level long-run homogeneity can not be rejected in some additional models. Focusing on the individual consumption categories absence of long-run homogeneity in the models with a restricted trend specification seems most evident for *Beverages and tobacco* and *Food.* However, in in the models where the term tln(t) is added to the linear trend the lack of long-run homogeneity is much less pronounced.

Long-run Engel- and own uncompensated direct price elasticites are calculated for the eight estimated non-durable commodities. With one exception all the long-run elasticities have the right sign and reasonable magnitudes. Moreover the calculated Engel-elasticities are very robust over the different models, especially over the trend specifications. The price elasticities show more substantial variations. For *Energy* the estimated direct price elasticity is positive in the LL14-models, whereas the a priori expected sign is obtained when using the LL12-models.

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Derivation of the Wald statistic for testing long-run homogeneity

We shall explain the derivation of the Wald statistic (2.16) fot testing the joint homogeneity hypothesis. The equation (2.14) corresponds to the equation (3.6) in Stock and Watson (1993) where the regressors have stochastic and deterministic trends of different order. The n'th element of $\underline{\xi}_t$ corresponds to an $I(1)$ variable. Hence, the restriction $(R \otimes I_{n-1}) vec(H \quad F) = \underline{0}_{n-1}$ does not involve stationary regressors. According to (4.4) in Stock and Watson (1993) the Wald statistic based on the OLS estimators may then be written as

$$W = vec\left(\hat{H} \quad \hat{F}\right)^{/} (R \otimes I_{n-1})^{/} \left\{ \left(R \otimes I_{n-1}\right) \left[A^{-1} \otimes \hat{\Omega}\right] \left(R \otimes I_{n-1}\right)^{/} \right\}^{-1} (R \otimes I_{n-1}) vec\left(\hat{H} \quad \hat{F}\right),$$

where

$$A = \begin{pmatrix} X^{/}X & X^{/}Z \\ Z^{/}X & Z^{/}Z \end{pmatrix}.$$

Since the upper left block of A^{-1} may be written as

$$(X^{/}X - X^{/}Z(Z^{/}Z)^{-1}Z^{/}X)^{-1} = \left(X^{/}Q_Z X\right)^{-1},$$

it follows that

$$\left(R \otimes I_{n-1)}\right)\left(A^{-1} \otimes \hat{\Omega}\right)\left(R \otimes I_{n-1}\right)^{/} = \underline{e}_{n,n+1}^{/} \left(X^{/}Q_Z X\right)^{-1} \underline{e}_{n,n+1} \otimes \hat{\Omega} = \underline{e}_{n,n+1}^{/} \left(X^{/}Q_Z X\right)^{-1} \underline{e}_{n,n+1} \hat{\Omega}.$$

Using that $(R \otimes I_{n-1}) vec\left(\hat{H} \quad \hat{F}\right) = \hat{H} \underline{e}_{n,n+1}$, W may be written as

$$W = \frac{\underline{e}_{n+1}^{/} \hat{H}^{/} \hat{\Omega}^{-1} \hat{H} \underline{e}_{n+1}}{\underline{e}_{n+1}^{/} (X^{/}Q_Z X)^{-1} \underline{e}_{n+1}},$$

which is the same as equation (2.16).

The same arguments may be used to derive the Wald statistic to test for homogeneity in the i'th equation. The restriction is, using the notation introduced in the main text,

$$\underline{e}_{i,n-1}^{/} H \underline{e}_{n,n+1} = 0$$

or

$$\underline{e}_{i,n-1}^{/}\left(HF\right)\begin{pmatrix} e_{n,n+1} \\ \underline{0}_{n_1} \end{pmatrix} = \underline{e}_{i,n-1}^{/}\left(HF\right)R^{/},$$

which also equals

$$(R \otimes \underline{e}_{i,n-1}^{/})vec\left(HF\right) = 0.$$

By the same argument as earlier the Wald statistic may be written as

$$\frac{\underline{e}_{n,n+1}^{/}\hat{H}^{/}\underline{e}_{i,n-1}\left(\underline{e}_{i,n-1}^{/}\hat{\Omega}\underline{e}_{i,n-1}\right)^{-1}\underline{e}_{i,n-1}^{/}\hat{H}\underline{e}_{n,n+1}}{\underline{e}_{n,n+1}^{/}\left(XQ_Z X\right)^{-1}\underline{e}_{n,n+1}}$$

since

$$\left(R \otimes \underline{e}_{i,n-1}^{/}\right)\left(A^{-1} \otimes \hat{\Omega}\right)\left(R \otimes \underline{e}_{i,n-1}^{/}\right)^{/} = \underline{e}_{n,n+1}^{/}\left(X^{/}Q_Z X\right)^{-1}\underline{e}_{n,n+1}e_{i,n-1}^{/}\hat{\Omega}\underline{e}_{i,n-1}.$$

Regression diagnostics (Homogeneity not imposed)

Table B1. Single equation regression diagnostics. Significance probabilities of autocorrelation tests[a]

Consum. category	Assumption with regard to lags/leads[b]							
	LL12				LL14			
	Trend assumption[c]				Trend assumption[c]			
	T1	T2	T3	T4	T1	T2	T3	T4
1	0.0997	0.1852	0.2020	0.5110	0.4994	0.5000	0.5003	0.2096
2	0.3852	0.6897	0.6651	0.2547	0.6202	0.5966	0.5651	0.4064
3	0.2990	0.3909	0.3840	0.3430	0.1865	0.1646	0.1702	0.1986
4	0.0440	0.2112	0.2871	0.5219	0.1257	0.2250	0.2372	0.3647
5	0.1638	0.1815	0.1704	0.1063	0.2634	0.2538	0.2494	0.2928
6	0.2597	0.3161	0.3284	0.3317	0.7960	0.7546	0.7615	0.7143
7	0.0049	0.0061	0.0064	0.1405	0.2255	0.2249	0.2083	0.3933
8	0.5515	0.6946	0.7999	0.7166	0.2671	0.3625	0.4239	0.2360

[a] The statistics are F-distributed with 5 degrees of freedom in the numerator and 46-n_t degrees of freedom in the denominator, where n_t is the number of included trend variables.

[b] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions).

[c] The different assumptions with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table B2. Single equation regression diagnostics. Significance probabilities of normality tests[a]

	Assumption with regard to lags/leads[b]							
	LL12				LL14			
Consum.	Trend assumption[c]				Trend assumption[c]			
category	T1	T2	T3	T4	T1	T2	T3	T4
1	0.7939	0.7937	0.7971	0.9593	0.9007	0.8729	0.8472	0.8104
2	0.0343	0.0492	0.0573	0.1092	0.3010	0.2858	0.2882	0.4992
3	0.7480	0.7898	0.7895	0.6522	0.3909	0.3821	0.3732	0.5651
4	0.0001	0.0007	0.0014	0.0854	0.0009	0.0034	0.0051	0.1000
5	0.8212	0.8640	0.8411	0.7952	0.4175	0.7125	0.6932	0.8617
6	0.1390	0.1420	0.1434	0.1486	0.1627	0.1268	0.1265	0.1584
7	0.0000	0.0000	0.0001	0.0000	0.0401	0.2249	0.0499	0.5085
8	0.9306	0.9318	0.9454	0.7218	0.0220	0.0223	0.0232	0.0903

[a] The statistics are χ^2-distributed with 2 degrees of freedom.
[b] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions).
[c] The different assumption with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table B3. Single equation regression diagnostics. Significance probabilities of ARCH tests[a]

	Assumption with regard to lags/leads[b]							
	LL12				LL14			
Consum.	Trend assumption[c]				Trend assumption[b]			
category	T1	T2	T3	T4	T1	T2	T3	T4
1	0.8378	0.8202	0.8453	0.8904	0.9205	0.8037	0.7207	0.8253
2	0.8232	0.9021	0.9213	0.9763	0.8136	0.6779	0.6614	0.9779
3	0.0536	0.0809	0.0722	0.1301	0.6466	0.7250	0.7138	0.7588
4	0.5188	0.3951	0.3736	0.4663	0.3829	0.2925	0.2662	0.2371
5	0.7465	0.7107	0.7253	0.8454	0.8491	0.8316	0.8235	0.9697
6	0.9682	0.9697	0.9697	0.9640	0.8706	0.7783	0.7897	0.7148
7	0.9662	0.9650	0.9643	0.9806	0.9707	0.9679	0.9703	0.9018
8	0.9200	0.9141	0.8877	0.9822	0.8782	0.9261	0.9467	0.7543

[a] The statistics are F-distributed with 4 degrees of freedom in the numerator and 43-n_t degrees of freedom in the denominator, where n_t is the number of included trend variables.
[b] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions)
[c] The different assumptions with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table B4. Single equation regression diagnostics. Significance probabilities of Reset tests[a]

Consum. category	Assumption with regard to lags/leads[b]							
	LL12				LL14			
	Trend assumption[c]				Trend assumption[c]			
	T1	T2	T3	T4	T1	T2	T3	T4
1	0.8433	0.8234	0.8978	0.1069	0.0518	0.1402	0.2754	0.0131
2	0.3358	0.3174	0.3033	0.1298	0.0450	0.0763	0.0818	0.0076
3	0.0350	0.0320	0.0328	0.0251	0.0001	0.0002	0.0001	0.0003
4	0.0372	0.0421	0.0488	0.2687	0.0008	0.0024	0.0036	0.1150
5	0.9257	0.9011	0.9168	0.6984	0.3793	0.6319	0.6097	0.8321
6	0.0412	0.0462	0.0450	0.0358	0.4187	0.3825	0.3874	0.3544
7	0.0421	0.0405	0.0393	0.0618	0.3490	0.3277	0.3606	0.6933
8	0.0091	0.0085	0.0086	0.0063	0.0020	0.0016	0.0015	0.0087

[a] The statistics are F-distributed with 1 degrees of freedom in the numerator and 50-n_t degrees of freedom in the denominator, where n_t is the number of included trend variables.
[b] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions)
[c] The different assumption with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).

Table B5. Multivariate regression diagnostics. Significance probabilities of different tests

Statistics	Assumption with regard to lags/leads[a]							
	LL12				LL14			
	Trend assumption[b]				Trend assumption[b]			
	T1	T2	T3	T4	T1	T2	T3	T4
Auto-correl.[c]	0.0025	0.0395	0.0797	0.0031	0.0351	0.0049	0.4065	0.0895
Normal.[d]	0.0000	0.0017	0.0000	0.0019	0.0000	0.0027	0.0000	0.0206

[a] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions)
[b] The different assumptions with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).
[c] The statistics are F-distributed with 320 degrees of freedom in the numerator and 49-8n_t degrees of freedom in the denominator, where n_t is the number of included trend variables.
[d] The statistics are χ^2-distributed with 16 degrees of freedom.

Table B6. Testing of Granger non-causality. Significance probabilities[a]

Lags/ Leads[c]	Trend[b]			
	T1[c]	T2[d]	T3[d]	T4[e]
LL12	0.0001	0.0001	0.0001	0.0000
LL14	0.0002	0.0021	0.0026	0.0027

[a] The statistics are χ^2-distributed with 160 degrees of freedom.
[b] The different assumption with regard to the trend are T1 (Quadratic trend augmented by tln(t)), T2 (Quadratic trend), T3 (Linear trend augmented by tln(t)) and T4 (Linear trend).
[c] The different assumptions with regard to lags/leads are LL12 (Differenced variables at the current period and the two first lags and leads are included in the set of regressions) and LL14 (Differenced variables at the at the current period and at lags and leads 1 and 4 are included in the set of regressions).

Table B7. Testing for deterministic effects within the most general models. Significance probabilities

Hypotheses	Lags/Leads assumption			
	LL12		LL14	
	χ^2-statistic	Significance prob.	χ^2-statistic	Significance prob.
Linear trend	26.969	0.041874	83.505	0.000000
No structural break	201.630	0.000000	168.650	0.000000

[a]χ^2-distributed with 16 degrees of freedom.
[b]χ^2-distributed with 32 degrees of freedom.

Estimated parameters in the reference model, LL12/T1

Table C1. Estimates of the non-constrained elements in the H-matrix. Standard errors in parentheses

0.0601 (0.0428)	-0.0249 (0.0451)	-0.0279 (0.0322)	0.0306 (0.0281)	0.0530 (0.0429)	-0.0351 (0.0398)	-0.0340 (0.0763)	-0.0351 (0.0299)	-0.0434 (0.0314)
0.0762 (0.0329)	-0.0213 (0.0346)	0.0008 (0.0247)	0.0010 (0.0216)	-0.0322 (0.0329)	0.0303 (0.0306)	0.0371 (0.0586)	-0.0710 (0.0229)	-0.0422 (0.0241)
-0.0359 (0.0449)	0.0146 (0.0473)	0.0466 (0.0338)	-0.0102 (0.0295)	0.0254 (0.0450)	-0.0635 (0.0417)	0.0021 (0.0801)	0.0140 (0.0313)	-0.0247 (0.0330)
-0.1206 (0.0461)	0.0094 (0.0486)	-0.0110 (0.0347)	0.0658 (0.0303)	0.0780 (0.0462)	-0.0329 (0.0429)	0.0263 (0.0822)	-0.0052 (0.0322)	0.0011 (0.0339)
-0.0445 (0.0227)	-0.0109 (0.0239)	-0.0059 (0.0170)	-0.0425 (0.0149)	0.0657 (0.0227)	0.0081 (0.0211)	0.0444 (0.0404)	-0.0259 (0.0158)	-0.0348 (0.0166)
0.0036 (0.0399)	-0.0039 (0.0420)	0.0018 (0.0300)	-0.0701 (0.0262)	-0.0653 (0.0400)	-0.0057 (0.0371)	0.0539 (0.0711)	-0.0104 (0.0278)	0.0429 (0.0293)
0.0479 (0.0722)	-0.0091 (0.0761)	-0.0599 (0.0543)	0.0080 (0.0474)	0.0328 (0.0724)	0.0710 (0.0672)	-0.1333 (0.1288)	0.0688 (0.0504)	-0.0692 (0.0530)
0.0005 (0.0335)	-0.0246 (0.0353)	0.0390 (0.0252)	0.0075 (0.0220)	-0.1183 (0.0336)	0.0048 (0.0312)	0.0812 (0.0598)	0.0442 (0.0234)	0.0385 (0.0246)

Table C2. Estimates of the elements in the E_2-matrix. Standard errors in parentheses

-0.0859 (0.0503)	0.0129 (0.0287)	-0.0173 (0.0212)	0.0177 (0.0185)	0.0205 (0.0612)	0.0841 (0.0511)	0.0168 (0.0328)	0.0062 (0.0228)	-0.0013 (0.0097)	0.0290 (0.0275)
-0.0045 (0.0386)	0.0083 (0.0220)	-0.0109 (0.0163)	-0.0074 (0.0142)	0.0622 (0.0470)	-0.0566 (0.0393)	0.0005 (0.0252)	0.0043 (0.0175)	-0.0065 (0.0074)	0.0087 (0.0211)
0.0578 (0.0527)	-0.0107 (0.0301)	-0.0264 (0.0223)	0.0133 (0.0194)	-0.0393 (0.0642)	0.0165 (0.0536)	-0.0042 (0.0345)	0.0013 (0.0239)	-0.0066 (0.0101)	0.0110 (0.0289)
-0.0281 (0.0542)	-0.0103 (0.0309)	0.0123 (0.0229)	-0.0098 (0.0199)	-0.0861 (0.0659)	0.0055 (0.0551)	0.0331 (0.0354)	-0.0303 (0.0246)	-0.0073 (0.0104)	-0.0246 (0.0296)
-0.0149 (0.0266)	0.0207 (0.0152)	-0.0020 (0.0112)	0.0170 (0.0098)	0.0062 (0.0324)	0.0263 (0.0271)	-0.0352 (0.0174)	0.0343 (0.0121)	0.0095 (0.0051)	0.0022 (0.0146)
0.0191 (0.0468)	-0.0171 (0.0267)	0.0234 (0.0198)	0.0206 (0.0172)	-0.1061 (0.0570)	-0.0081 (0.0476)	-0.0107 (0.0306)	0.0179 (0.0213)	-0.0109 (0.0090)	0.0057 (0.0256)
0.1178 (0.0849)	0.0114 (0.0484)	0.0059 (0.0358)	-0.0087 (0.0312)	-0.0319 (0.1033)	-0.0385 (0.0863)	0.0043 (0.0555)	-0.1079 (0.0385)	0.0056 (0.0163)	0.0194 (0.0465)
-0.0373 (0.0394)	0.0012 (0.0225)	-0.0042 (0.0166)	-0.0051 (0.0145)	0.0824 (0.0480)	-0.0279 (0.0401)	-0.0241 (0.0257)	0.0186 (0.0179)	0.0007 (0.0076)	-0.0077 (0.0216)

Table C3. Estimates of the elements in the E_1-matrix. Standard errors in parentheses

0.0240 (0.0491)	0.0328 (0.0293)	-0.0276 (0.0226)	-0.0031 (0.0213)	0.0217 (0.0644)	0.0066 (0.0543)	0.0460 (0.0397)	0.0372 (0.0266)	0.0033 (0.0123)	0.0180 (0.0327)
-0.0159 (0.0378)	0.0247 (0.0225)	-0.0046 (0.0174)	0.0091 (0.0164)	0.0025 (0.0495)	-0.0474 (0.0417)	-0.0532 (0.0305)	0.0143 (0.0204)	0.0039 (0.0095)	-0.0076 (0.0252)
0.0110 (0.0516)	-0.0161 (0.0307)	-0.0309 (0.0237)	0.0114 (0.0223)	0.0503 (0.0675)	0.0208 (0.0570)	0.0058 (0.0417)	-0.0212 (0.0279)	-0.0046 (0.0129)	0.0263 (0.0343)
0.0398 (0.0530)	-0.0194 (0.0315)	0.0021 (0.0244)	-0.0017 (0.0230)	-0.1170 (0.0693)	0.0018 (0.0585)	0.0678 (0.0428)	0.0071 (0.0286)	-0.0073 (0.0133)	-0.0401 (0.0353)
0.0195 (0.0260)	0.0219 (0.0155)	0.0143 (0.0120)	0.0080 (0.0113)	-0.0107 (0.0341)	-0.0013 (0.0288)	-0.0410 (0.0210)	0.0172 (0.0141)	0.0072 (0.0065)	0.0035 (0.0173)
-0.0243 (0.0458)	-0.0052 (0.0273)	0.0054 (0.0211)	0.0519 (0.0199)	0.0703 (0.0600)	0.0077 (0.0506)	0.0368 (0.0370)	0.0422 (0.0248)	-0.0248 (0.0115)	0.0367 (0.0305)
0.0114 (0.0830)	-0.0858 (0.0494)	0.0293 (0.0382)	-0.0445 (0.0360)	0.0663 (0.1087)	0.0406 (0.0917)	0.0134 (0.0671)	-0.0713 (0.0449)	0.0088 (0.0208)	0.0377 (0.0553)
0.0026 (0.0385)	-0.0065 (0.0229)	-0.0004 (0.0177)	-0.0278 (0.0167)	0.0200 (0.0504)	-0.0530 (0.0426)	-0.0717 (0.0311)	0.0168 (0.0208)	0.0100 (0.0096)	-0.0416 (0.0257)

Table C4. Estimates of the elements in the E_0-matrix. Standard errors in parentheses

-0.0404 (0.0535)	0.0647 (0.0349)	0.0084 (0.0232)	-0.0001 (0.0221)	0.0895 (0.0620)	-0.0052 (0.0525)	-0.0145 (0.0447)	0.0311 (0.0312)	-0.0023 (0.0158)	0.1080 (0.0350)
-0.0276 (0.0411)	-0.0637 (0.0268)	0.0118 (0.0179)	0.0028 (0.0170)	-0.0036 (0.0476)	0.0403 (0.0403)	-0.0587 (0.0344)	0.0000 (0.0240)	-0.0046 (0.0121)	0.0048 (0.0269)
0.0287 (0.0561)	-0.0500 (0.0366)	-0.0168 (0.0244)	0.0140 (0.0231)	-0.0119 (0.0650)	0.0485 (0.0551)	0.0099 (0.0469)	-0.0166 (0.0327)	-0.0025 (0.0166)	-0.0269 (0.0367)
0.1111 (0.0576)	-0.0215 (0.0376)	-0.0097 (0.0250)	0.0226 (0.0238)	-0.1748 (0.0668)	0.0500 (0.0566)	0.0242 (0.0482)	-0.0024 (0.0336)	-0.0064 (0.0170)	-0.0527 (0.0377)
-0.0240 (0.0283)	0.0241 (0.0185)	0.0007 (0.0123)	0.0080 (0.0117)	0.0145 (0.0328)	0.0100 (0.0278)	-0.0145 (0.0237)	0.0086 (0.0165)	0.0106 (0.0084)	-0.0006 (0.0185)
-0.0348 (0.0499)	0.0770 (0.0325)	0.0070 (0.0216)	0.0470 (0.0206)	-0.0409 (0.0578)	-0.0318 (0.0489)	0.0336 (0.0417)	0.0322 (0.0291)	-0.0311 (0.0147)	0.0570 (0.0326)
0.0603 (0.0903)	0.0155 (0.0590)	0.0050 (0.0392)	-0.0347 (0.0373)	-0.0406 (0.1047)	-0.0365 (0.0886)	0.0535 (0.0755)	-0.0494 (0.0527)	0.0056 (0.0267)	0.0714 (0.0591)
0.0113 (0.0419)	-0.0148 (0.0274)	-0.0184 (0.0182)	-0.0251 (0.0173)	0.0446 (0.0486)	0.0110 (0.0412)	-0.0785 (0.0351)	0.0255 (0.0244)	0.0178 (0.0124)	-0.0665 (0.0274)

Table C5. Estimates of the elements in the E_{-1}-matrix. Standard errors in parentheses

-0.0398 (0.0448)	0.0474 (0.0312)	0.0013 (0.0266)	0.0279 (0.0219)	-0.0154 (0.0659)	-0.0339 (0.0459)	-0.0133 (0.0435)	-0.0008 (0.0263)	0.0170 (0.0118)	0.0610 (0.0320)
0.0341 (0.0345)	-0.0142 (0.0239)	-0.0027 (0.0204)	0.0109 (0.0168)	-0.0287 (0.0506)	-0.0182 (0.0353)	0.0082 (0.0334)	-0.0331 (0.0202)	-0.0042 (0.0091)	-0.0156 (0.0246)
0.0000 (0.0470)	0.0308 (0.0327)	-0.0248 (0.0279)	0.0067 (0.0230)	0.0035 (0.0691)	-0.0297 (0.0482)	0.0097 (0.0456)	-0.0257 (0.0276)	0.0028 (0.0124)	-0.0242 (0.0336)
-0.0217 (0.0483)	0.0204 (0.0336)	-0.0070 (0.0286)	0.0215 (0.0236)	-0.0019 (0.0710)	0.0142 (0.0495)	0.0547 (0.0468)	-0.0252 (0.0284)	-0.0001 (0.0127)	-0.0313 (0.0345)
-0.0262 (0.0237)	-0.0216 (0.0165)	0.0127 (0.0141)	-0.0265 (0.0116)	0.0679 (0.0349)	-0.0142 (0.0243)	0.0533 (0.0230)	0.0053 (0.0139)	0.0076 (0.0063)	-0.0200 (0.0170)
0.0683 (0.0418)	0.0030 (0.0290)	-0.0016 (0.0247)	-0.0420 (0.0204)	-0.1147 (0.0614)	0.0070 (0.0428)	0.0992 (0.0405)	0.0476 (0.0245)	0.0222 (0.0110)	0.0979 (0.0298)
-0.0021 (0.0757)	-0.0483 (0.0526)	0.0150 (0.0448)	-0.0086 (0.0370)	0.0124 (0.1113)	0.0824 (0.0776)	-0.1673 (0.0734)	0.0068 (0.0444)	-0.0102 (0.0199)	-0.0954 (0.0540)
-0.0022 (0.0351)	-0.0309 (0.0244)	-0.0143 (0.0208)	0.0166 (0.0172)	0.0172 (0.0516)	0.0016 (0.0360)	0.0050 (0.0341)	0.0174 (0.0206)	-0.0157 (0.0093)	-0.0011 (0.0251)

Table C6. Estimates of the elements in the E_{-2}-matrix. Standard errors in parentheses

-0.0648 (0.0444)	-0.0092 (0.0302)	-0.0163 (0.0227)	0.0279 (0.0202)	-0.0556 (0.0611)	0.0924 (0.0466)	0.0421 (0.0329)	-0.0153 (0.0233)	-0.0104 (0.0101)	0.0409 (0.0237)
0.0698 (0.0341)	0.0003 (0.0232)	-0.0118 (0.0174)	-0.0037 (0.0155)	0.0454 (0.0469)	0.0323 (0.0358)	0.0260 (0.0253)	-0.0052 (0.0179)	0.0028 (0.0078)	0.0081 (0.0182)
-0.0112 (0.0466)	-0.0057 (0.0317)	-0.0202 (0.0238)	0.0166 (0.0212)	0.0546 (0.0641)	-0.0168 (0.0489)	0.0250 (0.0345)	0.0016 (0.0244)	-0.0066 (0.0106)	-0.0109 (0.0249)
-0.0484 (0.0479)	0.0040 (0.0326)	0.0045 (0.0244)	0.0109 (0.0218)	0.0087 (0.0658)	-0.0240 (0.0502)	0.0693 (0.0355)	-0.0553 (0.0251)	-0.0032 (0.0109)	-0.0234 (0.0255)
0.0072 (0.0235)	0.0125 (0.0160)	-0.0071 (0.0120)	-0.0073 (0.0107)	-0.0236 (0.0324)	-0.0183 (0.0247)	-0.0021 (0.0174)	0.0062 (0.0123)	0.0001 (0.0053)	-0.0167 (0.0126)
-0.0274 (0.0414)	0.0351 (0.0282)	-0.0082 (0.0211)	-0.0256 (0.0189)	-0.1564 (0.0569)	-0.0303 (0.0434)	0.0277 (0.0307)	0.0488 (0.0217)	0.0075 (0.0094)	0.0371 (0.0221)
0.0628 (0.0750)	-0.0215 (0.0510)	0.0368 (0.0383)	-0.0558 (0.0342)	0.0565 (0.1032)	-0.0115 (0.0787)	-0.1170 (0.0556)	0.0062 (0.0393)	0.0156 (0.0171)	-0.0889 (0.0400)
0.0383 (0.0348)	-0.0402 (0.0237)	-0.0094 (0.0178)	0.0107 (0.0159)	0.0569 (0.0479)	-0.0091 (0.0365)	-0.0202 (0.0258)	0.0216 (0.0183)	-0.0079 (0.0079)	0.0021 (0.0186)

Table C7. Estimates of the elements in the E_d-matrix. Standard errors in parentheses

0.3725 (0.0726)	0.0027 (0.0034)	-0.0089 (0.0039)	0.0056 (0.0042)	-0.0103 (0.0061)	0.0010 (0.0032)	-0.0115 (0.0028)	0.0045 (0.0033)	0.0054 (0.0016)	0.0011 (0.0006)	-0.0015 (0.0004)
0.2020 (0.0558)	-0.0023 (0.0026)	0.0017 (0.0030)	0.0040 (0.0033)	0.0050 (0.0047)	-0.0024 (0.0024)	0.0041 (0.0021)	0.0017 (0.0025)	0.0047 (0.0012)	0.0011 (0.0004)	-0.0012 (0.0003)
0.1349 (0.0761)	0.0214 (0.0036)	-0.0087 (0.0041)	-0.0178 (0.0044)	-0.0068 (0.0064)	-0.0082 (0.0033)	0.0061 (0.0029)	0.0031 (0.0034)	0.0006 (0.0017)	0.0000 (0.0006)	-0.0001 (0.0005)
0.0686 (0.0782)	-0.0032 (0.0037)	0.0047 (0.0042)	0.0011 (0.0046)	-0.0029 (0.0066)	-0.0036 (0.0034)	0.0182 (0.0030)	-0.0058 (0.0035)	-0.0005 (0.0017)	-0.0012 (0.0006)	0.0005 (0.0005)
0.1575 (0.0384)	-0.0031 (0.0018)	-0.0028 (0.0021)	-0.0001 (0.0022)	0.0024 (0.0032)	0.0019 (0.0017)	-0.0024 (0.0015)	0.0026 (0.0017)	0.0013 (0.0009)	0.0000 (0.0003)	-0.0002 (0.0002)
0.0556 (0.0676)	-0.0110 (0.0032)	0.0019 (0.0037)	-0.0142 (0.0039)	0.0027 (0.0057)	-0.0003 (0.0030)	-0.0045 (0.0026)	-0.0015 (0.0030)	-0.0021 (0.0015)	-0.0003 (0.0005)	0.0003 (0.0004)
0.2176 (0.1225)	0.0184 (0.0058)	0.0085 (0.0066)	-0.0111 (0.0072)	0.0041 (0.0103)	-0.0014 (0.0054)	-0.0065 (0.0046)	0.0071 (0.0055)	-0.0023 (0.0027)	0.0001 (0.0010)	0.0007 (0.0007)
0.0325 (0.0569)	-0.0023 (0.0027)	0.0010 (0.0031)	0.0104 (0.0033)	0.0067 (0.0048)	-0.0024 (0.0025)	-0.0004 (0.0022)	0.0008 (0.0026)	-0.0023 (0.0013)	0.0003 (0.0004)	0.0003 (0.0003)