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When can micro properties be used to predict aggregate demand?

Abstract:
Heterogeneity in consumer behaviour creates differences in demand responses, which may create problems with aggregation across consumers. If aggregation problems exist, results from estimation based on aggregate data may prove difficult to interpret. Using estimation results from micro data to predict aggregate demand responses may also create disaggregation bias (the reverse aggregation problem). The aim of this paper is to discuss potential problems of using micro data to predict aggregate demand, and how such problems relate to the linear and non-linear aggregation problem. We also expand the theories of linear and non-linear aggregation to the case in which prices vary across agents. We formulate and test criteria for aggregation by using data on Norwegian household electricity consumption. We find clear evidence of aggregation problems, as heterogeneity in both price and income derivatives are significant. We thus expect to experience problems with aggregation when analysing Norwegian household electricity consumption.

Keywords: Aggregation problems, Electricity demand, Microeconometrics.

JEL classification: C43, D1, C21

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1. Introduction

Most political goals are concerned with aggregate indicators of the economy. In order to achieve these political goals, economic policy instruments are used to change individual consumption. The effectiveness of these instruments depends on the aggregate response of all consumers. We know from the aggregation literature that unless consumers are homogeneous in their demand responses, we may experience aggregation problems. The main concerns in the existing aggregation literature have been: what restrictions on the micro functions are necessary for aggregate demand to exist as a function of aggregate income; when can aggregate demand be given a behavioural interpretation; and when does aggregate demand have welfare implications?[1] The early work on aggregation is dominated by Gorman's and Muellbauer's theories of the representative agent (Gorman 1953, Blackorby and Shorrocks 1995, Deaton and Muellbauer 1980a, 1980b). Later, Lau developed the concept of exact aggregation (Lau 1981, Lau and Wu 1987, 1996). The main finding in this literature is that if consumers are heterogeneous in their responses to income change, we can give the macro function neither a behavioural nor a welfare interpretation, and it may not even be meaningful to assume the existence of aggregate demand as a function of aggregate income. If it is inappropriate to assume the existence of an aggregate demand function, the interpretation of price and income derivatives from an aggregate demand function estimated using macro data is problematic.

Alternatively, we may estimate individual demand by using micro data, and then use estimated micro properties to predict aggregate demand responses. Analyses based on micro data can always be given a behavioural interpretation. However, it is not obvious that the properties of the micro functions transfer to aggregate demand (see, for example, Mas-Colell, Whinston and Green 1995 for a discussion). This creates differences between micro and macro elasticities.[2] If consumers are heterogeneous, we may experience disaggregation bias when using estimated micro properties to predict aggregate demand responses (hereafter referred to as the reverse aggregation problem). The criteria for the existence of an aggregate demand function may differ from the criteria for reverse aggregation. This is because the existence of an aggregate demand function only requires homogeneity across consumers with respect to income changes, whereas reverse aggregation also requires homogeneity with respect to price changes (see the discussion in Section 2).

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[1] See, for example, Deaton and Muellbauer (1980), Mas-Colell, Whinston and Green (1995), and Forni and Brighi (1991) for discussion.

[2] See, for example, Deaton and Muellbauer (1980), Stoker (1986), Buse (1992) and Denton and Mountain (2001, 2004) for discussion of the differences between micro and macro elasticities.

Potential disaggregation bias from using estimated micro properties to predict aggregate demand responses, i.e., the reverse aggregation problem, is not often discussed in the literature. Blundell, Pashardes and Weber (1993) compare estimation results based on micro data with estimation results based on aggregated micro data (15 years of the British Family Expenditure Survey), for which they allow for heterogeneity in income derivatives. They find significant differences in the estimation results,[3] and conclude in the abstract that "aggregate data alone are unlikely to produce reliable estimates of structural price and income coefficients". In another interesting paper, Blundell, Meghir and Weber (1993) present an empirical methodology for testing whether aggregate data can be used to estimate micro parameters. However, in neither of the papers do the authors discuss potential disaggregation bias from using micro estimates to predict aggregate demand. Denton and Mountain (2001, 2004) discuss differences between aggregated and disaggregated elasticities. They calculate aggregated elasticities based on estimates of micro elasticities by using the results in Blundell, Pashardes and Weber (1993). Their aim is to explore the degree of disaggregation bias. They find that the disaggregation bias is relatively small. However, they do not discuss the effect on the results of heterogeneity in price derivatives and price variation across consumers.[4] The criteria under which it is appropriate to assume reverse aggregation have not, to this author's knowledge, previously been discussed in the literature.

The aim of this study is to explore the conditions under which estimated micro properties can be used to predict aggregate demand responses without generating disaggregation bias. That is, we aim to find the criteria for reverse aggregation. We also discuss how the criteria for reverse aggregation relate to the existing literature on aggregation problems, and in particular to Gorman's and Muellbauer's work on linear and non-linear aggregation. We expand the theory of linear and non-linear aggregation to study how price variation across consumers affects the transfer of micro properties to the macro function, and discuss how these changes affect the potential for aggregation bias. We also expand the existing empirical literature on differences between micro and macro elasticities by discussing the effect on aggregation of price variations across consumers. In the literature, most tests for the existence of a representative consumer are based on macro data (see, for example, Russell, Breunig and Chiu 1998 for a discussion). In this study, we test all hypotheses about aggregation by using micro data. The case of Norwegian household electricity demand is used as an example.

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3 Similar results are found by Halvorsen, Larsen and Nesbakken (2001).

4 In most of the aggregation literature, it is assumed that all consumers face equal prices. One exemption is Lau and Wu's discussion of exact aggregation (Lau and Wu 1996).

The outline of this paper is as follows. In Section 2, we give a brief summary of the aggregation problem. The main criteria for linear and non-linear aggregation are stated and expanded to the case in which prices may vary across consumers. Then, the criteria for reverse aggregation are discussed and compared to criteria for linear and non-linear aggregation. In Section 3, we analyse Norwegian residential electricity consumption. We describe our data and estimation results, and formulate and test hypotheses of linear and reverse aggregation. In Section 4, concluding remarks are made. A detailed discussion of the aggregation properties of the generalized linear functional form (non-linear aggregation) is given in the appendix.

2. The aggregation problem

Mas-Colell, Whinston and Green (1995) distinguish between three types of questions concerning aggregate demand. The first is whether aggregated demand *exists* as a function of prices and aggregated wealth or income. If consumers respond differently to income, a redistribution of wealth between consumers changes aggregate demand even though aggregate income is constant. The second question is whether aggregate demand can be given a *behavioural* interpretation (as a demand response). For some behavioural properties, aggregation is automatic; examples include continuity, homogeneity and Walras' law. By contrast, the weak axiom (WA) of revealed preference does not transfer automatically to aggregate demand. The idea behind the WA is that if x is revealed to be at least as good as y, then y cannot be revealed preferred to x. This is a basic assumption of consistency in choice behaviour. Finally, Mas-Colell, Whinston and Green (1995) discuss the existence of a *representative consumer*. To assume a representative consumer, aggregate demand must be inferred from utility maximization by a representative consumer. This implies that aggregate demand must satisfy the strong axiom of revealed preference, which generates the symmetry of the Slutsky matrix (also referred to as integrability). If we can assume a representative consumer, aggregate demand has welfare implications. Another question (not discussed by Mas-Colell, Whinston and Green) is whether the properties of the micro functions can help to determine aggregate demand responses to a policy instrument when there are aggregation problems.

The main aim of this section is to describe the criteria that ensure reverse aggregation and to discuss how these criteria relate to the criteria for linear and non-linear aggregation. We start by describing the criteria for linear aggregation (Gorman's representative consumer) and non-linear aggregation (Muellbauer's representative consumer). Then, we discuss the criteria for reverse aggregation and explain how these are related to the criteria for linear and non-linear aggregation. All aggregation criteria are discussed under the assumption that prices are constant and under the assumption that prices vary across consumers. That is, the theories of linear and non-linear aggregation are extended to apply when prices vary across consumers. As discussed by Forni and Brighi (1991), the behavioural interpretation of aggregate demand is lost in Lau's concept of exact aggregation. Since our aim is to predict aggregate demand responses, we need a behavioural interpretation. Thus, we do not discuss Lau's theory of exact aggregation further in this paper.

2.1. Linear aggregation

In this section, we discuss the criteria for the existence of an aggregate demand function and Gorman's representative consumer. This is known as the case of *linear aggregation*. In addition to assuming that prices are equal across agents, as Gorman did, we also assume that prices are allowed to vary.

2.1.1. Equal prices

Assume that individual *h*'s consumption of good *i* (q_i^h) is a function of income (y^h) and a vector of prices (p), which is assumed to be equal for all consumers. The demand function is given by $q_i^h = f_i^h\left(p, y^h\right)$. Assume further that the income distribution is exogenous to the choice of whether to consume good *i*. Aggregate demand for good *i* (Q_i) is defined as the sum of all individual consumption demands. Aggregate consumption is thus a function of the price vector and individual incomes: $Q_i = \sum_{h=1}^{H} f_i^h\left(p, y^h\right) = G_i\left(p, y^1, \ldots\ldots, y^H\right)$, where H denotes the number of individuals.

To assume the existence of an aggregate demand function (F_i) as a function of prices (p) and aggregate income ($Y = \sum_{h=1}^{H} y^h$), the sum of all individual consumption demands must equal the aggregate demand function:

(1) $$Q_i = G_i\left(p, y^1, \ldots\ldots, y^H\right) = F_i(p, Y).$$

This is the case if individual income is a fixed proportion of aggregate income for all consumers (that is, if $y^h = r^h Y$), or if the micro functions are given by:

(2) $$q_i^h = a_i^h(p) + b(p) y^h .$$

Gorman proved that the micro functions in (2) imply the *existence* of the aggregate demand function in (1). That is, for the macro function to exist, all consumers must have the same demand response to income changes. If prices do not vary across consumers, heterogeneity is allowed in the constant term, a_i^h. Furthermore, the income derivative cannot be a function of income because income must appear linearly in the demand function. This is the same as assuming linear and parallel Engel curves for all individuals. In this case, the aggregate demand function is given by:

(3) $$Q_i = F_i(p, Y) = \sum_{h=1}^{H} a_i^h(p) + b(p)Y = a_i(p) + b(p)Y \; .$$

In order to assume a *representative consumer*, we also need to impose integrability on the aggregate demand function. To transfer the adding-up condition to the macro function and to ensure the non-negativity of demand quantities for all goods, the income–expansion paths must pass through the origin for all consumers ($a_i^h(p) = 0$) and utility-maximizing agents must have homothetic preferences.[5] Thus, for Gorman's representative agent to exist, the individual demand function must be given by $q_i^h = b(p)y^h$. This function implies the following aggregate demand function for the representative consumer: $Q_i = b(p)Y$. That is, all consumers must have the same income response but not the same price response, because the price response is a function of income. See, for example, Forni and Brighi (1991) for more information.

2.1.2. Varying prices

The theory of the existence of an aggregate demand function and of Gorman's representative consumer is discussed in the context in which all consumers face the same prices (*p*). However, actual prices often vary across consumers. This is particularly true of Norwegian household electricity consumption, which we use as an example to test for aggregation problems. Thus, we must find the criteria for the existence of an aggregate demand function and of Gorman's representative consumer when prices vary across consumers. The problem involves finding the criteria for the existence of mean (aggregate) consumption as an integrable function of mean prices and mean income, when prices, as well as income, vary across consumers.

[5] To allow for quasi-homothetic preferences, we must assume that the income–expansion paths are only defined above a given indifference surface, for example, above a minimum consumption level.

In order for the aggregate demand function to exist as a function of aggregate income and prices, we must assume either that: (i) prices and income are proportional to aggregate (mean) prices and income for all consumers, that is, $y^h = r^h Y$ and $p_i^h = r_i^h \overline{p_i}$; or (ii) the micro functions are linear in prices and income. The only heterogeneity is in the constant term. In case (ii), the micro demand function must be given by (4) for the aggregate demand function to exist as a function of aggregate prices and income:

(4) $$q_i^h = a_i^h + \sum_k b_k p_k^h + c_i y^h \ .$$

In this case, the consumption of the aggregate consumer (expressed in mean terms) is given by:

(5) $$\overline{q_i} = \overline{a_i} + \sum_k b_k \overline{p_k} + c_i \overline{y} \ .$$

For this macro consumer to be representative, that is, to transfer the adding-up condition to the macro function and to impose the non-negativity of demand quantities for all goods, the income–expansion paths must pass through the origin for all consumers ($a_i^h = 0$). That is, when prices are allowed to vary across consumers, the micro demand functions are given by $q_i^h = \sum_k b_k p_k^h + c_i y^h$, and the demand function of a representative mean consumer is given by $\overline{q_i} = \sum_k b_k \overline{p_k} + c_i \overline{y}$.

2.2. Non-linear aggregation

Linear aggregation is relatively restrictive on individual consumption. Instead of assuming that the consumer is representative in terms of *consumption*, we may assume that the consumer is representative in terms of *expenditure shares*. This is the case of non-linear aggregation. This type of aggregation is less restrictive than linear aggregation because it allows the derivatives of the demand function with respect to expenditures to differ across consumers (see, for example, Deaton and Muellbauer 1980 and Forni and Brighi 1991; see also the appendix for a discussion). We discuss the criteria for the existence of an aggregate expenditure share function and of Muellbauer's representative consumer. This case is discussed when prices are equal, and when prices are allowed to vary across agents. We also discuss what happens when we relax the assumption that the distribution of income is exogenous in the consumption decision.

2.2.1. Exogenous income distribution and equal prices

First, suppose that prices are equal across consumers and that the distribution of income is exogenous in the consumption decision. Under what conditions on the micro expenditure share functions (w_i^h) does an aggregate expenditure share function (W_i) exist as a function of aggregate income (*Y*)? That is:

(6) $$W_i = \frac{Q_i p_i}{Y} = W_i\left(p, g_i\left(p, y^1, \ldots, y^H\right)\right) = \sum_h \frac{y^h}{Y} w_i^h ,$$

where $g_i\left(p, y^1, \ldots, y^H\right)$ is a function that describes how the distribution of income affects aggregate consumption. Non-linear aggregation implies that the expenditure shares are linear functions of each other when aggregate wealth changes at given prices, even if the Engel curves are not linear. This property is known as generalized linearity (GL). Muellbauer has shown that the necessary and sufficient conditions for non-linear aggregation are that the micro and macro demand relations are given by:

(7) $$q_i^h = f_i\left(p, y^h\right) = b_i(p) y^h + c_i(p) g_i^h\left(y^h, p\right)$$

and

(8) $$Q_i = X_i\left(p, g_i\left(y^1, \ldots, y^H\right)\right) Y = \left[b_i(p) + c_i(p) \frac{\sum_h g_i^h\left(p, y^h\right)}{Y} \right] Y = \left[b_i(p) + c_i(p) g_i\left(p, y^1, \ldots, y^H\right) \right] Y ,$$

where $g_i\left(p, y^1, \ldots, y^H\right) = \sum_h g_i^h\left(p, y^h\right) \big/ Y$. Heterogeneity in micro behaviour is incorporated in the form of different functions, g_i^h . However, the coefficients b_i and c_i must be identical across agents.

Muellbauer has shown that, given this structure on the micro functions, the properties of the micro function transfer to the macro relationship. Hence, equation (8) can be interpreted as the demand of a representative consumer. The income of the representative consumer (Y^o) is implicitly defined by the equation $g_i\left(p, Y^o\right)Y = \sum_h g_i^h\left(p, y^h\right)$. The function g_i, and thus also Y^o, depends on prices and the distribution of income across consumers; that is, $Y^o = Y^o\left(p, y^1, ..., y^H\right)$.

The existence of Muellbauer's representative agent allows the quantity demanded by the representative agent to differ from aggregate demand. This is because, when the problem is defined in terms of expenditure shares, the income of the representative agent may differ from aggregate income (see Appendix A1 for more information). This has important implications for our purposes. As noted by Forni and Brighi (1991), this implies that even if the micro properties aggregate to the macro level in this case, the properties do not necessarily relate to market demand. This is because it is the demand of the representative agent, not the actual macro demand relationship, that is integrable.

2.2.2. Endogenous income distribution and varying prices

Deaton and Muellbauer (1980) do not discuss the criteria for non-linear aggregation when consumers face different prices. They do, however, relax the assumption of the exogenous distribution of expenditures. They find that if all consumers receive equal prices for labour (for example, equal hourly wages), non-linear aggregation is possible. However, when consumers have different hourly wages, non-linear aggregation is no longer possible unless there is separability in the consumption of goods and leisure. Deaton and Muellbauer (1980) note that this result implies that every good whose price varies between consumers must be separable from the consumption of other goods for non-linear aggregation to be allowed.

In practice, prices on many goods vary across consumers. So, what are the criteria for non-linear aggregation when prices vary? Is Deaton and Muellbauer's argument about separability in consumption sufficient to guarantee the existence of a representative consumer when prices vary across agents? To investigate this, we assume that the function g_i^h is independent of prices and only depends on income. In addition, we define the function m_k^h, which represents the heterogeneity in the price derivative with respect to the price of good *k*. In addition, we assume that all coefficients in the demand function are equal across consumers. This yields the following micro demand function:

(9) $$q_i^h = f_i\left(P^h, y^h\right) = \sum_k a_{ik} p_k^h + b_i y^h + c_i g_i^h\left(y^h\right) + \sum_k d_{ik} m_{ik}^h\left(p_k^h\right).$$

Equation (9) implies that price and income derivatives differ across consumers because the functions $g_i^h\left(y^h\right)$ and $m_{ik}^h\left(p_k^h\right)$ vary across consumers.[6] However, is the micro function in (9) consistent with the existence of an aggregate expenditure share function as a function of the mean prices and aggregate income when prices, as well as income, vary across consumers? That is, can one guarantee the following?

$$\text{(10)} \qquad W_i = \frac{Q_i p_i}{Y} = W_i\left(p_1\left(p_1^1,...,p_1^H\right), ..., p_K\left(p_K^1,...,p_K^H\right), g_i\left(y^1,...,y^H\right)\right) = \sum_h \frac{y^h}{Y}\frac{\overline{p_i}}{p_i^h} w_i^h$$

The micro expenditure share function associated with the demand function in (9) is given by:

$$\text{(11)} \qquad w_i^h = \frac{p_i^h}{y^h}\sum_k \left[a_{ik} p_k^h + d_{ik} m_{ik}^h\left(p_k^h\right)\right] + b_i p_i^h + \frac{c_i g_i^h\left(y^h\right) p_i^h}{y^h}.$$

Inserting equation (11) into the criterion for the existence of an aggregate expenditure share function in equation (10) yields:

$$\text{(12)} \qquad W_i = \sum_{k=1}^{I} \frac{a_{ik}\overline{p_i} P_k}{Y} + b_i \overline{p_i} + c_i g_i\left(y^1,....,y^H\right) + \sum_{k=1}^{I} d_{ik} m_k\left(p_k^1,...,p_k^H\right),$$

where $g_i\left(y^1,....,y^H\right) = \frac{\overline{p_i}\sum_{h=1}^{H} g_i^h\left(y^h\right)}{Y}$, $m_{ik}\left(p_k^1,...,p_k^H\right) = \frac{\overline{p_i}\sum_{h=1}^{H} m_{ik}^h\left(p_k^h\right)}{Y}$ and $P_i = \sum_{h=1}^{H} p_i^h$. It can be shown that the expenditure share function in (11) is consistent with the existence of an aggregate expenditure share function (see Appendix A2). The aggregate expenditure share may either be written as a function of individual prices and income, or as a function of a representative price (p_i^o) and a representative income (Y^o). The representative price and income is found implicitly by solving the functions $g_i\left(Y^o\left(y^1,....,y^H\right)\right) = \frac{\overline{p_i}\sum_{h=1}^{H} g_i^h\left(y^h\right)}{Y}$ and $m_{ik}\left(p_k^o\left(p_k^1,...,p_k^H\right)\right) = \frac{\overline{p_i}\sum_{h=1}^{H} m_{ik}^h\left(p_k^h\right)}{Y}$ with respect to p_i^o and Y^o.

[6] See Appendix A2 for more information.

2.3. Reverse aggregation

The most commonly used method for predicting aggregate demand from micro estimates is to use the properties of the mean consumer (the consumer with mean properties).[7] So, what are the main criteria for avoiding disaggregation bias when applying this method? To avoid disaggregation bias when using micro estimates to predict aggregate demand, the consumption of the mean consumer multiplied by the number of consumers must equal aggregate consumption. We may also want to use micro estimates to predict the aggregate effect on consumption of price and income changes. That is, we want the predicted change in consumption for the mean consumer multiplied by the number of consumers to equal the aggregate demand response. For these criteria to be fulfilled, restrictions on the properties of the micro functions are required. These restrictions may depend on whether consumers pay equal or different prices for the same good.

2.3.1. Equal prices

Individual *h*'s consumption of good *i* (q_i^h) is assumed to be a function of his or her income (y^h) and a vector of prices $p = \{p_1, \ldots\ldots, p_I\}$ that is assumed to be equal across consumers; that is, $q_i^h = f_i^h(p, y^h)$. The consumption of the mean consumer is given by $f_i(p, \bar{y})$, where $\bar{y}$ is mean income. In order to use the consumption of the mean consumer to predict aggregate demand, the consumption of the mean consumer multiplied by the number of consumers should equal aggregate consumption (Q_i):

(13) $$f_i(p, \bar{y})H = \sum_{h=1}^{H} f_i^h(p \;, y^h) = Q_i \,.$$

That is, the consumption of the mean consumer must equal mean consumption. To use micro estimates to predict the aggregate demand response from price and income changes, the derivatives of the mean consumer's consumption multiplied by the total number of consumers (*H*) must equal aggregate price and income responses:

(14) $$\frac{\partial f_i(p, \bar{y})}{\partial p_i} H = \sum_{h=1}^{H} \frac{\partial f_i^h(p \;, y^h)}{\partial p_i} \;,$$

[7] This is the method applied by, for example, Denton and Mountain (2001, 2004) to test for disaggregation bias.

(15) $$\frac{\partial f_i(p,\bar{y})}{\partial \bar{y}} H = \sum_{h=1}^{H} \frac{\partial f_i^h(p\ \ ,y^h)}{\partial y^h} \ .$$

Equation (13) implies that the properties of the mean consumer can be used to predict the aggregate price response if the price response of the mean consumer is equal to the mean response of all consumers. Similarly, the properties of the mean consumer can be used to predict the aggregate income response if the mean consumer's income response equals the mean response of all consumers.

For equations (13) to (15) to be valid, micro demand functions must be linear in income and prices, as in equation (4). This is because heterogeneity in price and/or income derivatives may cause the derivative of the mean consumer to differ from the mean derivative of all consumers.

2.3.2. Varying prices

When prices vary across consumers, the mean consumer is characterized by both mean income and mean prices. In this case, the criteria for avoiding problems with reverse aggregation are given by:

(16) $$f_i(\bar{p},\bar{y})H = \sum_{h=1}^{H} f_i^h(p^h,y^h) = Q_i ,$$

(17) $$\frac{\partial f_i(\bar{p},\bar{y})}{\partial \bar{p}_i} H = \sum_{h=1}^{H} \frac{\partial f_i^h(p^h,y^h)}{\partial p_i^h} \ ,$$

(18) $$\frac{\partial f_i(\bar{p},\bar{y})}{\partial \bar{y}} H = \sum_{h=1}^{H} \frac{\partial f_i^h(p^h,y^h)}{\partial y^h} \ ,$$

where p^h is a vector of prices specific to consumer *h* and $\bar{p}$ is a vector of all mean prices over all consumers.

The essential criterion for avoiding problems with reverse aggregation is that all partial derivatives must be equal across consumers irrespective of consumers' initial incomes and prices. If this is not the case, disaggregation bias might arise when using estimates for the mean consumer from micro data to predict aggregate demand responses. A sufficient condition for this criterion to be fulfilled is that the micro demand functions are linear in both income and prices, as in equation (4).

The criterion for the existence of reverse aggregation is thus irrespective of whether prices are equal or vary across consumers. The reason is that we want to avoid disaggregation bias not only when using micro estimates to predict the aggregate consumption level, but also when using micro estimates to predict aggregate changes in consumption following price and income changes. If we only wanted to predict aggregate consumption (not changes in consumption), the criterion for reverse aggregation could be relaxed when prices are equal because prices need not enter the micro functions in a linear form. In this case, reverse aggregation is equivalent to the existence of an aggregate demand function (discussed in section 2.1.1) that incorporates heterogeneity in price derivatives.

2.4. Reverse versus linear and non-linear aggregation

From the above discussion, linear aggregation only ensures reverse aggregation when prices are allowed to vary across consumers. In this case, the criteria for the existence of an aggregate demand function are the same as the criteria for reverse aggregation. However, this is not the case when prices are the same for all consumers. This is because linear aggregation requires income derivatives, but not price derivatives, to be equal across consumers. If Gorman's representative consumer is valid, we may still experience problems with reverse aggregation when prices are equal, even if the homogeneity and adding-up conditions remove some of the heterogeneity in the price derivatives through the constant terms in the micro functions. This is because the price derivatives are functions of income, and thus vary across consumers.

Non-linear aggregation cannot ensure reverse aggregation whether or not prices vary across consumers. This is because the criteria for non-linear aggregation allow heterogeneity in price and income derivatives through the functions g_i^h and m_{ik}^h (when prices vary across consumers). Furthermore, since the income of the representative agent may differ from aggregate income, the consumption of a representative agent may differ from aggregate demand. Thus, the existence of Muellbauer's representative agent does not ensure reverse aggregation. This means that if we have estimated an AIDS by using micro data, it may not be appropriate to use these estimates to predict the effects on aggregate demand of policy instruments, even if the AIDS model is consistent with Muellbauer's representative agent.

Whether the criteria for reverse aggregation, the existence of an aggregate demand function or an aggregate expenditure share function, and the existence of Gorman's or Muellbauer's representative agent are met is an empirical question, which needs to be tested in each individual case. In the next section, we formulate and test hypotheses about linear and reverse aggregation by using estimation results based on micro data on Norwegian household electricity consumption. We do not test hypotheses about non-linear aggregation because non-linear aggregation is not sufficient to avoid disaggregation bias.

3. Norwegian residential electricity demand

Norwegian household electricity consumption is a useful example of a good for which aggregation problems are relevant. Norwegian households are likely to be heterogeneous in their electricity demand. This, among other reasons, is because the substitution possibilities vary considerably across households, as the stock and capacity of heating equipment vary (see Table 1). In this section, we estimate Norwegian household demand for electricity by using micro data, taken primarily from the Norwegian Survey of Consumer Expenditures (SCE), to test the assumptions behind reverse aggregation and the existence of an aggregate demand function.

3.1. The data

The data used in the analysis come from different sources and contain information on 3,511 individual households in 1993, 1994 and 1995. The main data source is the annual Norwegian SCE (see Statistics Norway 1996).[8] Our data include information on annual electricity expenditure, electricity suppliers, the ownership of durables, heating technology and other household and dwelling characteristics. Information on electricity prices was collected from households' individual electricity suppliers and the Norwegian Water Resources and Energy Directorate. If price information for a household is missing, the mean price of all power suppliers distributing to the household's area of residence (municipality) is allocated to the household.[9] Individual prices on paraffin, fuel oil and firewood were obtained from the SCE, and calculated as expenditure divided by quantity. These prices are averaged by county and applied to households in that county for which neither a positive expenditure on, nor an amount of, firewood, paraffin and/or fuel oil is recorded. The Norwegian Institute of Meteorology provides annual information on regional variations in temperature for all municipalities included in the SCE.

Table 1 reports the mean, minimum and maximum values of key variables in the data set. First, note that there are considerable variations in energy prices and income. This is particularly true for prices on firewood. In addition, households are relatively heterogeneous in terms of household and residence characteristics. For example, the number of household members ranges from one to 12, and households range from 12 m^2 rented net floor spaces to owner-occupied 550 m^2 houses. This implies large differences in the need for electricity for room and water heating and thus probably creates heterogeneity in price and income derivatives. In addition, substitution possibilities between electricity and other energy sources vary considerably. For example, 80 per cent of the households in the sample

[8] The Norwegian SCE is supplemented by additional information on energy use and the stock of heating equipment in the years under study in this analysis.

[9] In this period, most households used their local power distributor.

can use firewood and electricity for heating. The number of electric heaters, stoves and rooms with floor heating also varies considerably. This variation is also likely to create heterogeneity in the household responses to price and income changes.

Heterogeneity across households may affect the demand function in several ways. Consider two groups of households: one may only use electricity for heating, whereas the other may use both electricity and fuel oil in a central heating system. The former will, on average, consume more electricity and their electricity consumption will be less sensitive to price and income changes, *ceteris paribus*, because they cannot substitute other energy goods for electricity when electricity prices rise. Furthermore, the variation in observed electricity consumption will be higher for the other group because some households will use electricity only whereas others will use only fuel oil or a combination of electricity and fuel oil for heating.

Table 1. Descriptive statistics of key variables in the data set based on the sample from the Norwegian Consumer Expenditure Survey 1993–1995

	Mean	*Minimum*	*Maximum*
Electricity consumption (kWh)[a)]	22 860	0	89 149
Paraffin consumption (kWh)[b)]	821	0	109 150
Fuel-oil consumption (kWh)[b)]	690	0	122 277
Firewood consumption (kWh)[b)]	3 113	0	227 500
Electricity price (øre/kWh)	44	25	59
Paraffin price (øre/kWh-utilized)	48	4.58	123
Fuel-oil price (øre/kWh-utilized)	39	0.35	76
Firewood price (øre/kWh-utilized)	51	0.96	154
Household annual gross income (NOK)	373 271	95	8 950 900
Number of household members	3.20	1	12
Net floor space (m^2)	128	12	550
Heating degree days during winter season	3 012	2 239	4 291
Opportunity to use paraffin (0, 1)	0.24	0	1
Opportunity to use fuel oil (0, 1)	0.04	0	1
Opportunity to use firewood (0, 1)	0.80	0	1
Number of electric heaters	5.05	0	30
Number of rooms with electric floor heating	1.48	0	12
Number of firewood stoves	1.07	0	11
Number of paraffin stoves	0.13	0	3

[a)] The reason some households (2.5%) have a zero electricity consumption is that they are registered with zero expenditure in the SCE. The two main reasons are that the electricity bill is included in the rent (in the case of tenants) or is paid by the employer.

[b)] Acquired quantities (not consumption) calculated in utilized kWh.

3.2. Econometric specification

To illustrate the problem of aggregating demand over heterogeneous households, we estimate a linear demand function in which price and income derivatives and the standard deviation of the error term depend on household characteristics. We assume that the consumption of electricity in household *h* (q^h) comprises a deterministic component (μ^h) and a stochastic component (ε^h), and in which the deterministic component is approximated by a linear function.[10] Electricity consumption in household *h* is given by:

$$(19) \qquad q^h = \mu^h + \varepsilon^h = \alpha^h + \sum_j^J \gamma_j^h p_j^h OE_j^h + \beta^h y^h + \varepsilon^h ,$$

where y^h is annual real income in household *h* and p_j^h is the price of energy good *j* for household *h*. To ensure that only the prices of the goods that the household can consume enter the individual demand function, we multiply the price variables by a dummy variable, OE_j^h, which equals unity if the household can consume good *j*, and equals zero otherwise. We assume that the stochastic component is independently and normally distributed with a zero mean, $E(\varepsilon^h)=0$, and has a variance that depends on the characteristics of the household, σ_h^2.

In order to test for heterogeneity in the demand response, we assume that the constant term (α^h), the price derivative (γ_j^h), the income derivative (β^h) and the stochastic component (σ^h) are functions of observed household characteristics (θ^h), given in equation (20). These characteristics include the stock and capacity of heating equipment and other characteristics of the household and residence, such as the number of household members, the type of residence (detached house, block of flats, farmhouse) and the stock of household appliances. A complete list of the variables that have significant effects is given in Table 2.

[10] The linear demand function facilitates testing of the hypotheses of reverse aggregation and the existence of an aggregate demand function, but is sufficiently complex to illustrate the problems of aggregation.

$$
(20)\qquad
\begin{aligned}
&\alpha^h = \alpha_0 + \sum_{n=1}^{N} \alpha_n \theta_n^h, \\
&\gamma_j^h = \gamma_0^j + \sum_{f=1}^{F} \gamma_f^j \theta_f^h \qquad\qquad \forall\ j = 1,2,3,4, \\
&\beta^h = \beta_0 + \sum_{k=1}^{K} \beta_r \theta_r^h + \beta_y y^h \\
&\sigma^h = \sigma_0 + \sum_{s=1}^{S} \sigma_s \theta_s^h + \sigma_y y^h.
\end{aligned}
$$

We allow all parameters to differ across households, including the variance of the error term. We also allow the income derivative to vary with income. This is done to test the assumption that income enters linearly into the demand function (see hypothesis (ii) below). This heterogeneity in the estimated parameters (α^h, γ_j^h, β^h and σ^h) ensures that there is a unique demand function for each individual household. In the next section, we specify testable hypotheses for reverse aggregation, the existence of an aggregate demand function and Gorman's representative consumer.

3.3. Tests for reverse and linear aggregation

To be able to assume the existence of an aggregate demand function, Gorman's representative consumer and reverse aggregation under the assumption that prices vary across agents (as is the case for Norwegian household electricity consumption), the individual demand function must have three properties. First, income must enter linearly into the demand functions; that is, the income derivatives must be independent of income:

$$
\text{(i)}\qquad
\begin{aligned}
&H_0 : \beta_y = 0 \\
&H_1 : \beta_y \neq 0
\end{aligned}.
$$

This follows from Gorman's polar form, in which income enters linearly into the micro function. Second, we need to test if the partial derivatives with respect to income are equal across agents. If they are not, a redistribution of income between two households changes aggregate demand without changing aggregate income, and hence an aggregate demand function does not exist as a function of aggregate income. Thus, we need to test the hypothesis:

$$
\text{(ii)}\qquad
\begin{aligned}
&H_0 : \beta_r = 0 \\
&H_1 : \beta_r \neq 0
\end{aligned}
\qquad \text{for all } r.
$$

If these two criteria are met, we can assume the existence of an aggregate demand function when prices are the same for all households. In addition, to assume the existence of Gorman's representative consumer, all constant terms must be zero: $\alpha_0 = 0 = \alpha_n$, $\forall\ n$.

When prices vary across households, the partial derivatives with respect to prices must be the same for all households if an aggregate demand function, reverse aggregation and/or Gorman's representative agent are to exist. That is:

(iii) $\begin{matrix} H_0 : \gamma_f^j = 0 \\ H_1 : \gamma_f^j \neq 0 \end{matrix}$ for all combinations of *j* and *f*.

We need to test whether these criteria are met for demand functions estimated using micro data when we allow heterogeneity across households with respect to price and income derivatives and when we allow the income derivative to be a function of income. If the criteria (i) to (iii) hold, we can assume reverse aggregation and can use microeconometric estimates for the mean household to predict the aggregate demand response. We can also estimate an aggregate demand function as a function of aggregate prices and income to determine the aggregate demand response. If the null hypotheses are rejected, we may experience disaggregation bias when predicting the aggregate demand response by using the properties of the mean household's electricity demand. If hypotheses (i) and (ii) are rejected, the existence of an aggregate demand function (and of Gorman's representative consumer) cannot be guaranteed even if the prices are equal across agents. To be able to assume Gorman's representative agent, additional restrictions on the constant term must also be imposed; to be specific, the constants in α^h cannot be significantly different from zero.

If none of these criteria are met, it may still be reasonable to assume the existence of Muellbauer's representative consumer. Since we focus on when it is appropriate to use the estimated properties of the micro function to calculate the properties of the macro function (reverse aggregation), testing for non-linear aggregation is beyond the scope of our analysis.

3.4. Estimation results and tests

To test the hypotheses (i) to (iii), we estimate the demand functions in (19) and (20) by using the maximum likelihood estimation procedures in NLOGIT 3.0 and Limdep 8.0 (Greene 2002) on the data from the Norwegian SCE. The estimation results are presented in Table 2. In the first column, we list the parameter being estimated; in the second column, we list the variables that have a significant effect on the parameters or those whose effect on the parameter we want to test for significance (such as income); in the third column, we report the estimated coefficients; in the fourth column, we report the t-statistics; and in the last column, we report the estimated p-values.

Table 2. Results from ML estimation of the model for Norwegian household electricity demand. 100 000 kWh

Parameter	*Variable*	*Coefficient*	*t-value*	*p-value*
Constant term (α^h)	Constant	0.2719	17.58	0.0000
	Number of household members	0.0142	8.61	0.0000
	Block of flats (1, 0)	–0.0249	–5.41	0.0000
	Cottage ownership (1, 0)	0.0202	7.30	0.0000
	Moved during the last 12 months (1, 0)	–0.0257	–5.04	0.0000
	Tenant (1, 0)	–0.0203	–5.75	0.0000
	Collective central heater ownership (1, 0)	–0.0397	–3.36	0.0008
	Electricity as the main energy source (1, 0)	0.0218	7.16	0.0000
	Number of washing machines	0.0139	2.33	0.0199
	Number of dishwashers	0.0117	4.10	0.0000
	Number of tumble dryers	0.0079	2.96	0.0031
Electricity price (γ_1^h)	Constant	–0.0045	–13.10	0.0000
(øre per kWh)	Number of electric heaters	0.0001	6.25	0.0000
	Number of rooms with electric floor heating	0.0001	6.39	0.0000
	Net floor space (m^2)	0.0000	3.61	0.0003
Paraffin price (γ_2^h)	Constant	–0.0008	–2.28	0.0227
(øre per kWh)	Number of heating degree days (1000 degree days)	0.0002	1.89	0.0588
	Capacity of electric heating equipment (0, 1, 2, 3, 4)	0.0001	2.61	0.0091
Fuel-oil price (γ_3^h)	Constant	–0.0010	–5.03	0.0000
(øre per kWh)	Capacity of electric heating equipment (0, 1, 2, 3, 4)	0.0003	3.75	0.0002
Firewood price (γ_4^h)	Constant	0.0001	1.37	0.1702
(øre per kWh)	Number of firewood stoves	0.0001	2.69	0.0071
	Capacity of wood stoves (0, 1, 2, 3, 4)	–0.0001	–2.34	0.0195
Household income (β^h)	Constant	0.0001	0.15	0.8810
(10,000 NOK)	Net floor space (m^2)	0.0000	2.66	0.0078
	Number of children	–0.0002	–3.39	0.0007
	Detached house (1, 0)	0.0007	6.38	0.0000
	Household income (10,000 NOK)	0.0000	–1.38	0.1673
Standard deviation (σ^h)	Constant	–3.0981	–135.94	0.0000
	Net floor space (m^2)	0.0028	15.82	0.0000
	Capacity, oil-based heating equipment (0, 1, 2, 3, 4)	–0.0563	–5.28	0.0000
	Household income (10,000 NOK)	0.0053	9.20	0.0000

The table indicates that the constant terms are significant. Thus, we cannot assume the existence of Gorman's representative consumer. The price derivatives indicate significant heterogeneity across households for all energy prices. In particular, variation in the stock and capacity of heating equipment generates heterogeneity in the price derivatives across households. There is also significant heterogeneity in the income derivative. Hence, one cannot guarantee that there is a unique aggregate demand function for Norwegian household electricity consumption as a function of aggregate income. One may also experience problems with reverse aggregation. However, the income derivative does not depend significantly on income at a 10 per cent level. Thus, the assumption that income enters linearly into the demand function (as is assumed in Gorman's polar form) is not rejected.

Given these results, it is difficult to evaluate the effects of policy instruments on aggregate electricity consumption in Norwegian households. This is because it may be appropriate neither to estimate an aggregate demand function nor to use the properties of the estimated demand function for the mean household based on micro data to predict the aggregate demand response. To calculate the aggregate effect, we must sum up the predicted demand responses from each individual household whilst allowing for heterogeneity in the individual demand responses. To do this, we need to build a micro simulation model of Norwegian household electricity consumption.

4. Concluding remarks

In this paper, we focused on the similarities and differences between reverse aggregation and the existence of an aggregate demand function and a representative consumer. First, the perspective differs. The theory of the representative consumer is concerned with the existence of an aggregate demand function and whether it may be given behavioural and welfare implications. The problem of reverse aggregation relates to the potential disaggregation bias that arises when microeconometric estimates are used to predict aggregate demand responses and aggregation problems are not accounted for. We are concerned with reverse aggregation because, although it has been shown theoretically that elasticities from micro and macro relationships differ if consumers are heterogeneous,[11] elasticities based on micro estimates are often used in macro models. Furthermore, when micro estimates for the mean household are multiplied by the population size to obtain an estimate of the aggregate demand response, there is no discussion of potential disaggregation bias.

[11] See, for example, Deaton and Muellbauer (1980), Stoker (1986), Buse (1992) and Denton and Mountain (2001, 2004).

Strong restrictions must be imposed on the properties of the micro function to be able to use micro estimates for the mean consumer to predict the aggregate demand response in a theoretically consistent way.[12] These restrictions are stronger than those needed to ensure the existence of an aggregate demand function and Gorman's and Muellbauer's representative consumers when prices are equal across agents. When prices vary across agents, the criteria for reverse aggregation are the same as those for an aggregate demand function. The criteria for an aggregate expenditure share function allow for more heterogeneity in the micro functions. Thus, using micro estimates to predict aggregate demand and aggregate demand responses may cause disaggregation bias unless the micro functions are linear in income and prices. Heterogeneity across agents is only allowed in the constant term. If there is heterogeneity in the price and income derivatives, the assumption behind reverse aggregation may not be appropriate. With respect to linear aggregation, heterogeneity in price derivatives is allowed when prices are equal. In the case of non-linear aggregation, heterogeneity is allowed in both the income and price derivatives through the functions describing the distribution of prices and income (m_{ik}^h and g_i^h), even if prices vary across consumers.

Why use micro rather than macro data to estimate aggregate demand if stronger assumptions must be made when using micro data? The answer relates to the existence of the representative consumer, which is necessary if price and income derivatives estimated from macro data are to be interpreted as behavioural response to policy instruments. If agents are too heterogeneous, assuming the existence of a representative consumer is not appropriate. It may not even be meaningful to estimate an aggregate demand function if agents differ in their income responses, since a redistribution of wealth changes aggregate demand. In this case, the macro function does not necessarily exist, and the only relevant option is to use micro data. Thus, we must test whether reverse aggregation is possible and whether the assumption of a representative consumer is appropriate. If neither test is met, which is the case for Norwegian household electricity consumption, micro data must be used to estimate price and income responses. Heterogeneity across consumers must be allowed for. Then, individual responses are summed to obtain the aggregated response.

In general, all forms of non-linearity in the demand function create problems with reverse aggregation. The reason is that non-linearity creates heterogeneity in price and income derivatives across agents. We may also experience problems with reverse aggregation when many consumers record zero consumption, as in the case of several energy goods, for example firewood. The effect that

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[12] Whether these restrictions are satisfied in practice is an empirical question.

observations of zero have on aggregating micro properties to the macro level is beyond the scope of this paper. In future work, we aim to build a micro simulation model based on estimating Norwegian household electricity consumption by using data from the Norwegian Survey of Consumer Expenditures to illustrate and quantify the disaggregation bias.

Aggregation and generalized linearity

In this appendix, we explain why a consistent aggregation of budget shares is possible in the case of generalized linearity (GL) when prices are equal across households, and discuss whether non-linear aggregation is possible when prices differ across consumers.

A.1 Equal prices across consumers

Consider the case in which prices are equal across consumers and the distribution of income is exogenous in the consumption decision. Under what conditions on the micro expenditure share functions (w_i^h) does an aggregate expenditure share function (W_i) exist as a function of aggregate income (Y)? That is:

(A1) $$W_i = \frac{Q_i p_i}{Y} = W_i\left(p, g_i\left(y^1, \ldots.., y^H\right)\right) = \sum_h \frac{y^h}{Y} w_i^h \ .$$

Non-linear aggregation implies that the expenditure shares are linear functions of each other when aggregate wealth changes at given prices, even if the Engel curves are not linear. This property is known as GL. Muellbauer has shown that necessary and sufficient conditions for the $W_i\left(P, g_i\left(y^1, \ldots.., y^H\right)\right)$ function to exist are that the micro relations are given by:

(A2) $$q_i^h = f_i\left(p, y^h\right) = a_i^h(p) + b_i(p) y^h + c_i(p) g_i^h\left(y^h, p\right),$$

where either (i) $\sum_h a_i^h(p) = 0$, or (ii) $c_i(p) = 0$. The second case is equal to linear aggregation, whereas the first is the case of non-linear aggregation. Heterogeneity in micro behaviour is allowed in the form of different constant terms, (a_i^h), and different functions, g_i^h . However, the coefficients b_i and c_i must be identical across agents. In order for a macro function based on the micro function in (A2) to be integrable, which implies the existence of a representative agent, we also need to assume that $\sum_i a_i^h(p) p_i = 0$, $\sum_i b_i(p) p_i = 1$ and $\sum_i c_i(p) p_i = 0$. The corresponding macro demand function is:

(A3) $$Q_i = X_i\left(p, g_i\left(y^1,...,y^H\right)\right)Y = \left[b_i(p) + c_i^*(p) g_i\left(p, y^1,.....,y^H\right)\right]Y \ ,$$

where $g_i^h\left(p, y^1,..., y^H\right) = \frac{\sum_h g_i^h\left(p, y^h\right)}{Y}$. In case (i) $c_i^*(p) = c_i(p)$, whereas in case (ii) $c_i^*(p) = \sum_h a_i^h(p)$ and $\sum_h g_i^h\left(p, y^1,.....,y^H\right) = 1$. A special case of the GL function in (A3) is the price-independent generalized linear (PIGL) function, where g_i^h is a function of income only.

The micro and macro expenditure shares are given by equations (A4) and (A5), respectively, in the case of non-linear aggregation (that is, in case (i)):

(A4) $$w_i^h \equiv \frac{q_i^h p_i}{Y} = b_i(p)p_i + c_i(p)p_i \frac{g_i^h\left(p, y^h\right)}{y^h},$$

(A5) $$W_i \equiv \frac{Q_i p_i}{Y} = b_i(p)p_i + c_i(p)p_i \frac{\sum_h g_i^h\left(p, y^h\right)}{Y} = b_i(p)p_i + c_i(p)p_i g_i\left(p, y^1,....,y^H\right).$$

To see that the GL structure ensures the existence of an aggregate expenditure share function as a function of aggregate income, we insert the micro expenditure shares in equation (A4) into (A1), which yields the aggregate expenditure share function in equation (A5).

Muellbauer has shown that, given the GL structure on the micro functions, the properties of the micro function transfer to the macro relationship, in which case, equation (A3) can be interpreted as the demand of a representative consumer. The aggregated functions in (A3) and (A5) can be written either as a function of all individual incomes or as a function of the income of a representative consumer (Y^o): $W_i \equiv W_i\left(p, Y^o\right)$. The income of a representative consumer is implicitly defined by the equation $g_i\left(p, Y^o\right)Y = \sum_h g_i^h\left(p, y^h\right)$. That is, the income of the representative consumer is the aggregate income that makes the properties of the micro function transfer to the macro relationship.

The representative income may differ from observed aggregate income; that is, $Y^o \neq Y$.[13] Thus, the existence of Muellbauer's representative agent allows the quantities demanded by the representative agent to differ from aggregate demand. This is because, when the problem is defined in terms of expenditure shares, aggregate consumption is given by $Q_i = W_i Y / p_i$, whereas the consumption of the representative agent is $Q_i^o = W_i^o Y^o / p_i$, which is different when $Y^o \neq Y$.

A.2 Is non-linear aggregation possible when prices vary across agents?

Does an aggregate expenditure share function exist when prices vary across consumers and can we interpret the aggregate expenditure share as the share of a representative consumer? That is, under what conditions on the micro expenditure share functions (w_i^h) does an aggregate expenditure share function (W_i) exist as a function of aggregate income (Y) and the mean price ($\overline{p_i}$)? That is:

(A6) $$W_i = \frac{Q_i p_i}{Y} = W_i\left(p_1\left(p_1^1,\ldots,p_1^H\right),\ldots,p_K\left(p_K^1,\ldots,p_K^H\right),g_i\left(y^1,\ldots,y^H\right)\right) = \sum_h \frac{y^h}{Y}\frac{\overline{p_i}}{p_i^h} w_i^h .$$

We focus on the case of non-linear aggregation. We try to generalize case (i) in equation (A2) to apply when prices, in addition to income, vary across consumers. We assume the function g_i^h is independent of prices (cf. the discussion of the consequences for aggregation of an endogenous leisure decision in Deaton and Muellbauer 1980a), and depends only on income. Additionally, we define a function, m_k^h, which describes the heterogeneity in the price derivative with respect to the price of good *k*. In addition, we assume that all coefficients in the demand function are equal across consumers:[14]

(A7) $$q_i^h = f_i\left(P^h, y^h\right) = \sum_k a_{ik} p_k^h + b_i y^h + c_i g_i^h\left(y^h\right) + \sum_k d_{ik} m_{ik}^h\left(p_k^h\right).$$

Can the micro function in (A7) ensure the existence of an aggregate expenditure share function as a function of the mean price and aggregate income when prices, as well as income, vary across consumers? The micro expenditure share function associated with the demand function in (A7) is given by:

---

[13] See, for example, Forni and Brighi (1991) for a discussion.

[14] In order for the micro function in (A7) to be integrable, and in particular for it to satisfy the adding-up condition, we also need to assume that $\sum_i b_i p_i = 1$ and $\sum_i c_i p_i = 0$.

$$\text{(A8)} \qquad w_i^h = \frac{p_i^h}{y^h}\sum_k \left[a_{ik} p_k^h + d_{ik} m_{ik}^h\left(p_k^h\right)\right] + b_i p_i^h + \frac{c_i g_i^h\left(y^h\right) p_i^h}{y^h}.$$

Checking the right-hand side of equation (A6) by inserting equation (A8) into the criterion for the existence of an aggregate expenditure share function, we obtain:

$$\begin{aligned}
W_i &= \sum_{h=1}^{H} \frac{y^h}{Y}\frac{\overline{p_i}}{p_i^h} w_i^h = \sum_{h=1}^{H} \frac{y^h}{Y}\frac{\overline{p_i}}{p_i^h}\left\{\frac{p_i^h}{y^h}\sum_k \left[a_{ik} p_k^h + d_{ik} m_{ik}^h\left(p_k^h\right)\right] + b_i p_i^h + \frac{c_i g_i^h\left(y^h\right) p_i^h}{y^h}\right\} \\
\text{(A9)} \qquad &= \sum_{k=1}^{K} \frac{a_{ik}\overline{p_i}}{Y}\sum_{h=1}^{H} p_k^h + \frac{b_i \overline{p_i}}{Y}\sum_{h=1}^{H} y^h + \frac{c_i \overline{p_i}\sum_{h=1}^{H} g_i^h\left(y^h\right)}{Y} + \sum_{k=1}^{K}\frac{d_{ik}\overline{p_i}\sum_{h=1}^{H} m_{ik}^h\left(p_k^h\right)}{Y}, \\
&= \sum_{k=1}^{I} \frac{a_{ik}\overline{p_i} P_i}{Y} + b_i \overline{p_i} + c_i g_i\left(y^1,\ldots,y^H\right) + \sum_{k=1}^{I} d_{ik} m_k\left(p_k^1,\ldots,p_k^H\right)
\end{aligned}$$

where $g_i\left(y^1,\ldots,y^H\right) = \frac{\overline{p_i}\sum_{h=1}^{H} g_i^h\left(y^h\right)}{Y}$, $m_{ik}\left(p_k^1,\ldots,p_k^H\right) = \frac{\overline{p_i}\sum_{h=1}^{H} m_{ik}^h\left(p_k^h\right)}{Y}$ and $P_i = \sum_{h=1}^{H} p_i^h$.[15] To determine whether equation (A6) holds, which enables us to write the aggregate expenditure share as a function of aggregate income and mean prices, we check whether the right-hand side of the equation equals the left-hand side. To do this, we need to find the aggregate demand function when prices are allowed to vary across consumers. Hence, we have:

$$\begin{aligned}
Q_i &= \sum_{h=1}^{H} q_i^h = X_i\left(p_1\left(p_1^1,\ldots,p_1^H\right),\ldots,p_K\left(p_K^1,\ldots,p_K^H\right), g_i\left(y^1,\ldots,y^H\right)\right)\frac{Y}{\overline{p_i}} \\
\text{(A10)} \qquad &= \sum_{h=1}^{H}\left\{\sum_k a_{ik} p_k^h + b_i y^h + c_i g_i^h\left(y^h\right) + \sum_k d_{ik} m_{ik}^h\left(p_k^h\right)\right\}. \\
&= \frac{Y}{\overline{p_i}}\left[\sum_k \frac{a_{ik} P_k \overline{p_i}}{Y} + b_i \overline{p_i} + \frac{c_i \overline{p_i}\sum_h g_i^h\left(y^h\right)}{Y} + \sum_k \frac{d_{ik}\overline{p_i}\sum_h m_{ik}^h\left(p_k^h\right)}{Y}\right]
\end{aligned}$$

Inserting (A10) into the right-hand side of the aggregate expenditure share function in (A6) yields:

---

[15] The term $a_{ik} P_k = \sum_h a_{ik} P_k^h$ is the effect on aggregate consumption of good *i* of the price on good *k*.

(A11) $$W_i = \frac{Q_i \overline{p_i}}{Y} = \sum_k \frac{a_{ik} P_k \overline{p_i}}{Y} + b_i \overline{p_i} + \frac{c_i \overline{p_i} \sum_h g_i^h\left(y^h\right)}{Y} + \sum_k \frac{d_{ik} \overline{p_i} \sum_h m_{ik}^h\left(p_k^h\right)}{Y}.$$
$$= \sum_{k=1}^{I} \frac{a_{ik} \overline{p_i} P_k}{Y} + b_i \overline{p_i} + c_i g_i\left(y^1,....,y^H\right) + \sum_{k=1}^{I} d_{ik} m_k\left(p_k^1,...,p_k^H\right)$$

Since (A9) equals (A11), the micro expenditure function in (A8) ensures the existence of an aggregate expenditure share function as a function of mean prices and aggregate income. The aggregate expenditure share can either be written as a function of individual prices and income, or as a function of a representative price, (p_i^o), and a representative income, (Y^o).The representative price and income is found implicitly by finding the values of p_i^o and Y^o that satisfy the functions

$$g_i\left(Y^o\left(y^1,....,y^H\right)\right) = \frac{\overline{p_i} \sum_{h=1}^{H} g_i^h\left(y^h\right)}{Y} \text{ and } m_{ik}\left(p_k^o\left(p_k^1,...,p_k^H\right)\right) = \frac{\overline{p_i} \sum_{h=1}^{H} m_{ik}^h\left(p_k^h\right)}{Y}.$$

Equation (A9) shows that price and income derivatives differ across consumers because the functions $g_i^h\left(y^h\right)$ and $m_{ik}^h\left(p_k^h\right)$ vary across consumers. That is, assuming the existence of an aggregate expenditure share function (and Muellbauer's representative agent) when prices are allowed to vary across consumers does not ensure reverse aggregation.

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