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A linear demand system within a Seemingly Unrelated Time Series Equation framework

Abstract:
We consider a Seemingly Unrelated Time Series Equations framework for the linear Almost Ideal Demand system. The framework is applied to a consumer demand system covering nine non-durable commodities. We test for demand homogeneity within a specification where the static linear Almost Ideal Demand system is augmented by three stochastic trends and three stochastic seasonal variables. The homogeneity restriction is rejected for about half of the commodities and in the system as a whole using conventional significance levels. However, when comparing the out-of-sample predictions from a homogeneous and non-homogeneous model, we do not find that the non-homogenous model performs better than the homogeneous one. Moreover, the income and price elasticities calculated under homogeneity restrictions are all of the right sign and have reasonable magnitudes.

Keywords: Consumer demand. Linear Almost Ideal Demand system. Seemingly Unrelated Time Series Equations. Prediction.

JEL classification: C32, C51, C53, E21

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1 Introduction

The literature on consumer allocation models estimated from time series data is vast and involves a lot of difficult issues. Many functional forms have been proposed over the years, among which the linear Almost Ideal Demand (AID) system of Deaton and Muellbauer (1980) is one of the most widely used in applied work. No doubt, this is because of its simplicity; budget shares are linear functions of log prices and log real income. In applications there are, however, many issues that have to be settled.

First, there is the choice of the number of commodities that should be included in the consumer demand system. The amount of data available for estimation is often quite limited. Hence restrictive separability assumptions which permit estimation on small subsystems are often imposed. Also analyses of few and highly aggregate consumer categories are often encountered in practice.

A second issue is the question of economic interpretation. In the literature on demand systems, testing of homogeneity is a central theme. However, homogeneity is routinely rejected in applications. For a critical interpretation of this line of research, see Keuzenkamp and Barten (1995). Model misspecifications and invalid statistical inference have often been put forward as two reasons for the extensive rejection of homogeneity. One example of model misspecification is omitted dynamic effects. Anderson and Blundell (1983) emphasize, using an error correction framework, that neglected dynamics could lead to rejection of homogeneity (as well as symmetry). However, a potential problem with their inference is that they assume stationarity. In the later years, testing of homogeneity have been influenced by the cointegration literature, which explicitly distinguishes between non-stationary and stationary variables.

A third issue is related to exogeneity. In low-dimensional dynamic consumer systems, it may be possible to model all the variables simultaneously, i.e. without conditioning on prices and income. For example, Fanelli and Mazzochi (2002) and Pesaran and Shin (2002) consider demand systems with four and three commodities, respectively (including the residual group). However, due to its highly data-intensive nature, their approach is very hard to generalize to a consumer system involving many commodities. This is also acknowledged by Pesaran and Shin (2002, p. 73) who remark that only demand systems with a few commodity groups can be analyzed in this way. Attfield (1997), covering eight non-durable consumer categories, tested for long-run homogeneity using the triangular representation of Phillips (1991). As Fanelli and Mazzochi (2002) put forward, this representation is based upon the assumption that prices and total expenditure are both weakly and strongly exogenous in relation to the parameters of interest.

With nine different commodities, the present paper is, to our knowledge, the first attempt to model a large consumer demand system within the Seemingly Unrelated Time Series Equations (SUTSE) framework; see e.g. Harvey (1989). A related paper is Moosa and Baxter (2002), who model a small consumer demand system consisting of three alcoholic beverages. They demonstrate that SUTSE models are well-suited for modelling consumer demand because trend and seasonality, which are important features of the data, can be represented in a very flexible way by latent stochastic components. To analyze a situation with many commodities may seem as a rather technical extension of their paper, but in practice it is far from straightforward to do so, as we will demonstrate. In contrast to Moosa and Baxter (2002), we also consider restrictions following from demand homogeneity as well as the out-of-sample performances of different models.

More specifically, we analyze a linear AID system with nine commodities, where the budget shares depend on relative prices, real total expenditure and unobserved random variables capturing trend and seasonality. This modeling framework is applied to Norwegian quarterly national accounts data on non-durable consumption from 1966Q1 to 2001Q4.

The present paper focuses on three issues. First, we test for demand homogeneity both in the separate equations and in the system as a whole. Second, we calculate base-year income and own-price elasticities. Third, we compare the predictive ability of different models. In addition to the models with and without homogeneity, we also consider, as a benchmark, a simple model without price and income variables.

Demand homogeneity is rejected for about half of the commodities and in the system as a whole at conventional levels of significance. Thus, on a purely statistical basis, the homogeneity restriction may be hard to defend. However, since it is difficult to interpret price and income effects in a model without homogeneity, this restriction seems necessary in order for the model to be relevant for practical purposes. Under the homogeneity restriction, we find that all the elasticities have the correct sign and are of reasonable magnitudes. By comparing out-of-sample predictions, we do not find evidence that the non-homogeneous model performs better than the homogeneous one. This suggests that statistical rejection of homogeneity may not be particularly alarming for practical purposes. The implications for forecasting of imposing demand homogeneity have also been discussed elsewhere in the literature. For a recent reference, see Wang and Bessler (2002), who find mixed evidence as to whether the forecasting performance is improved when the homogeneity restriction is relaxed even when homogeneity is rejected by statistical tests.

Another interesting finding in this paper is that the (smoothed) trend estimates are quite sensitive with respect to whether homogeneity is imposed or not. The reason for this is that trending behavior in the budget shares to some extent can be captured by trends in income and price variables. Imposition of homogeneity leads to noticeable changes in the trend estimates in the equations where demand homogeneity are rejected.

The rest of this paper is organized as follows: A description of our quarterly national accounts data is given in Section 2. Section 3 presents the econometric model. Section 4 is devoted to estimation and testing of the different model specifications and reports income and own-price elasticities. Section 5 compares the different models with respect to both within- and out-of-sample predictions. Conclusions are given in Section 6, whereas some technical issues are reserved for Appendix A and B.

2 Data

The data are taken from the Norwegian quarterly national accounts and cover the period 1966Q1 to 2001Q4. We consider household's expenditure on nine non-durable consumption categories. For each consumption category we have corrected for foreigners' consumption in Norway by assuming that a fixed proportion of foreigners' total consumption is spent on a specific category. The base year is 1997, implying that the price indices are on average 1 in this year. The data for the 16 last quarters have a more preliminary status than the data in the earlier years and are used only to compare the out-of-sample predictive properties of different model specifications. An overview of the consumption categories together with their budget shares in 1997 is given in Table 1.

Table 1: Overview of consumption categories

Category #	Name	1997 budget shares
1	Food	0.191
2	Beverages and tobacco	0.106
3	Electricity	0.057
4	Running cost of vehicles	0.064
5	Other non-durable goods	0.108
6	Clothing and shoes	0.089
7	Other services	0.228
8	Transport services	0.092
9	Consumption abroad	0.065

The real per capita total expenditure variable has been constructed as follows: Let C_{jt} denote consumption in fixed prices for consumption category j in period t. Total expenditure per capita in nominal terms is then given by

$$Y_t = \frac{1}{N_t} \sum_{j=1}^{9} P_{jt} C_{jt}, \tag{1}$$

where N_t is the population size at the end of the quarter and P_{jt} is the price of commodity j. The variable Y_t is deflated by the Stone Price index [cf. Stone (1954)]

$$P_t^* = \exp \left[\sum_{j=1}^{9} s_{jt} \ln P_{jt} \right], \tag{2}$$

where s_{jt} denotes the budget share of consumption category j in period t. Real expenditure per capita, denoted R_t, is defined as:

$$R_t = \frac{Y_t}{P_t^*}. \tag{3}$$

The time series used in the analysis are depicted in Figures 1-2. Figure 1 shows the budget shares and the logarithm of total real expenditure per capita, while Figure 2 contains deflated prices for each commodity, i.e. P_{jt}/P_t^*, together with P_t^*. For several commodities the budget shares in Figure 1 show some trendlike behavior over the sample period. This is most evident for Food, for which the budget share has decreased substantially over the years (in accordance with Engel's law). Also for Clothing and shoes there is a clear negative trend, while a positive trend is evident for Other services. From the graphs of the budget shares, a non-stationary seasonality pattern is evident for all the commodities, suggesting that a stochastic seasonal vector should be included in any realistic model.

3 Econometric model

Let $\mathbf{s}_t^{(n)} = (s_{1t}, .., s_{nt})'$ be the vector of budget shares in period t and let $\mathbf{x}_t = (\ln P_{1t}, ..., \ln P_{nt}, \ln R_t)'$, see equations (1)-(3). In the deterministic and static case, the linear approximation of the AID system can be written:

$$s_{it} = \mu_i + \sum_{j=1}^{n} \pi_{ij} \ln P_{jt} + \pi_{i,n+1} \ln R_t,$$

or in matrix notation:

$$\mathbf{s}_t^{(n)} = \boldsymbol{\mu}^{(n)} + \mathbf{\Pi}^{(n)} \mathbf{x}_t, \tag{4}$$

where $\mathbf{\Pi}^{(n)} = [\pi_{ij}]_{n \times (n+1)}$, i.e. the $n \times (n+1)$ matrix with entries π_{ij}.

Various stochastic extensions of (4) have been considered in the literature; see for example Anderson and Blundell (1982). Our approach is based on the SUTSE framework, described e.g. in Harvey (1989). This framework can be used to take into account seasonal effects, changes in unobserved variables (preferences, etc.) and measurement errors through latent variables: Let $\boldsymbol{\varpi}_t^{(n)} = [\varpi_{it}]_{n \times 1}$ be an unobserved exogenous vector of variables accounting for stochastic trends and seasonal effects in the data. This will be further specified below. Furthermore, let $\boldsymbol{\varepsilon}_t = [\varepsilon_{ti}]_{n \times 1}$ be a white noise vector. In the following we shall analyze the following version of the linear AID system:

$$\mathbf{s}_t^{(n)} = \boldsymbol{\mu}^{(n)} + \mathbf{\Pi}^{(n)} \mathbf{x}_t + \boldsymbol{\varpi}_t^{(n)} + \boldsymbol{\varepsilon}_t^{(n)}. \tag{5}$$

Equation (5) must be interpreted as a conditional model, i.e. conditional on *all* prices and expenditures $(\mathbf{x}_1, ..., \mathbf{x}_T)$. The parameter matrix $\Pi^{(n)}$ is of main interest, because it can be used to obtain expressions for price and income elasticities, as we discuss later in Section 4.

To be consistent with the fact that budget shares sum to one, we impose the following

Figure 1: Budget shares and log of total expenditure per capita

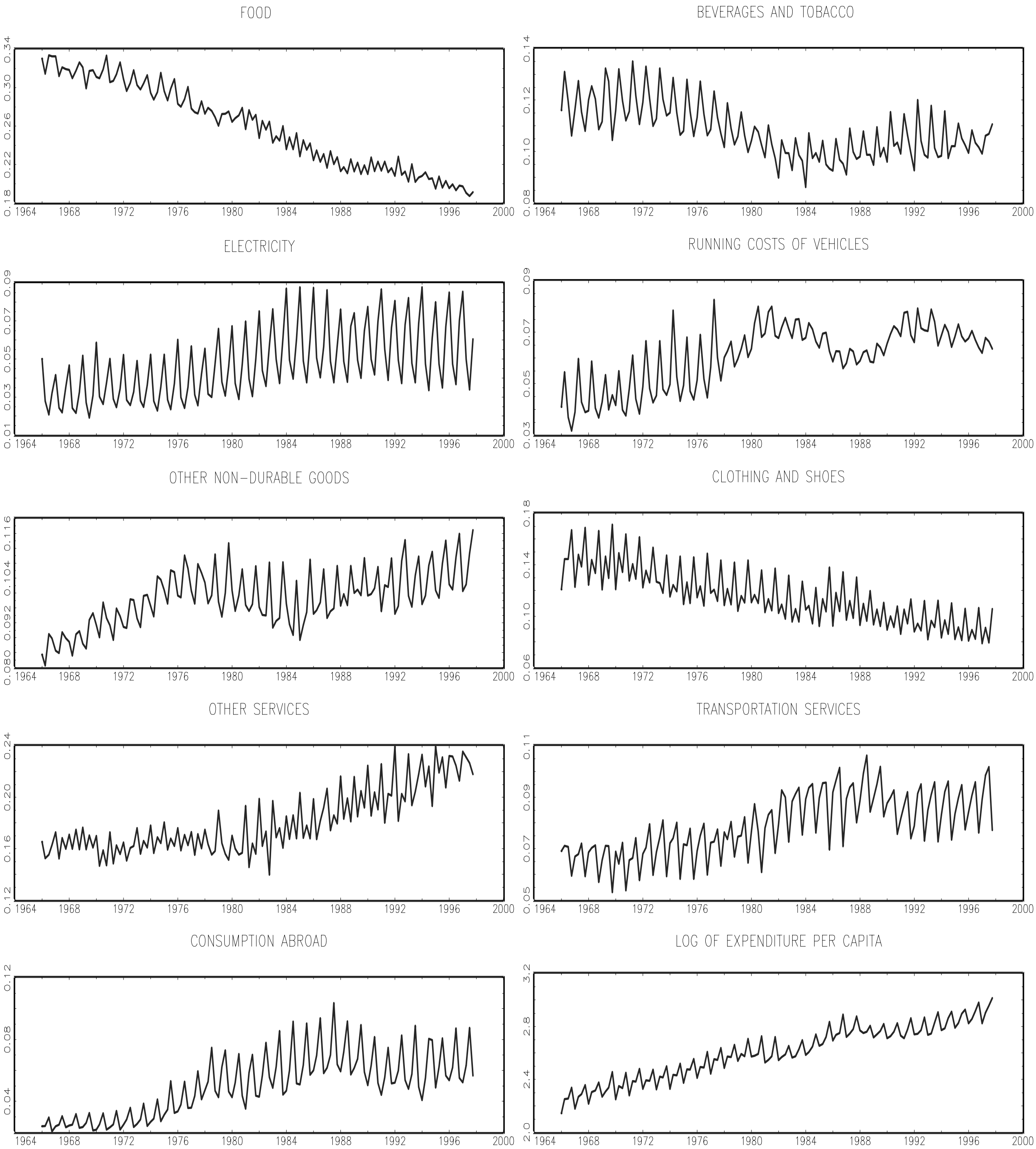
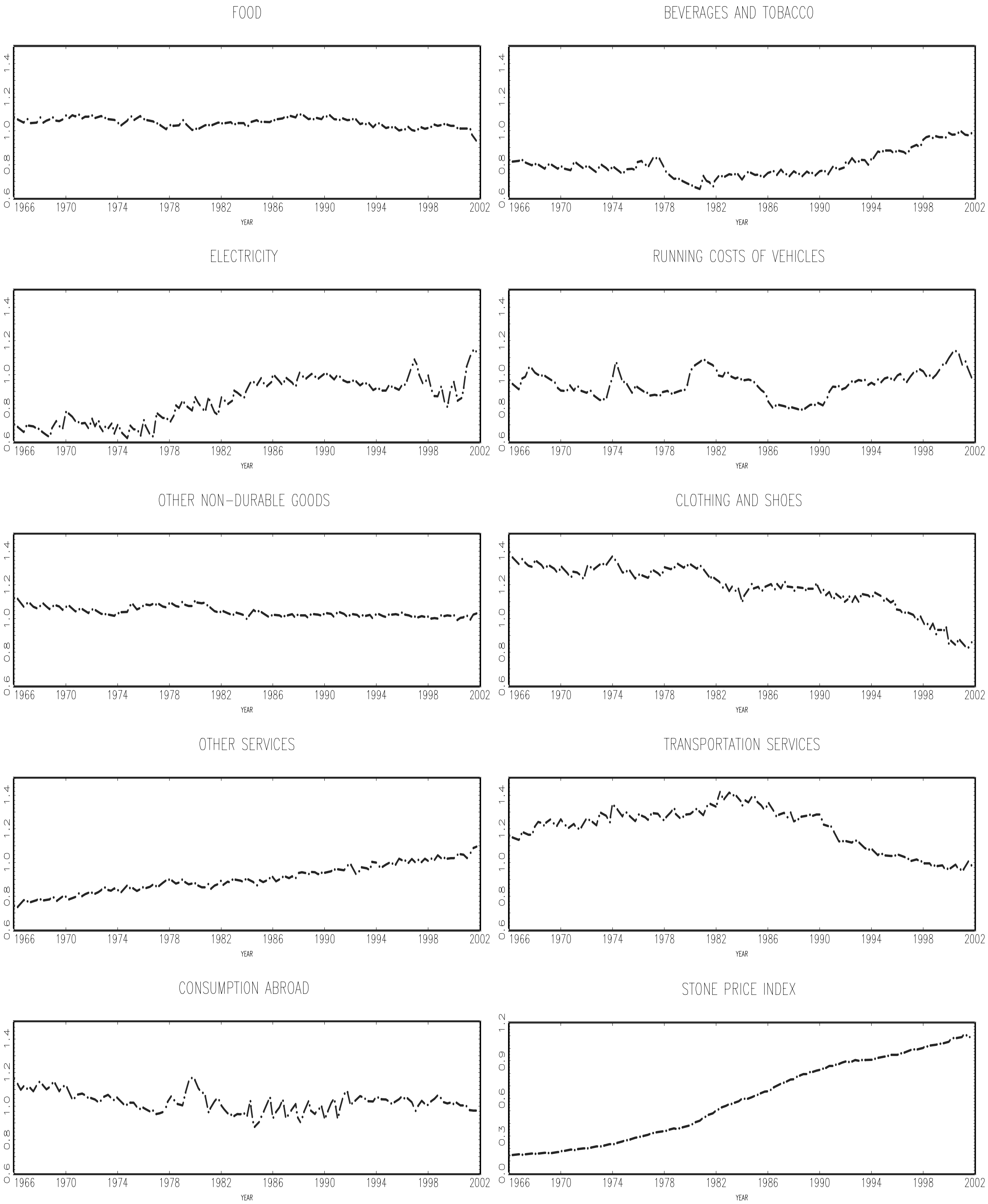


Figure 2: Deflated prices P_{jt}/P_t^* and the Stone price index, P_t^*.



set of restrictions:

$$\begin{aligned}\sum_{i=1}^{n}\mu_i &= 1 \\ \sum_{i=1}^{n}\pi_{ij} &= 0, \quad j=1,...,n+1 \\ \sum_{i=1}^{n}\varpi_{it} &= 0 \\ \sum_{i=1}^{n}\varepsilon_{it} &= 0.\end{aligned} \tag{6}$$

Hence, both $\boldsymbol{\varpi}_t^{(n)}$and $\boldsymbol{\varepsilon}_t^{(n)}$have singular covariance matrices. Following Anderson (1980), we exclude the last equation in (5) obtaining

$$\mathbf{s}_t = \boldsymbol{\mu} + \boldsymbol{\Pi}\mathbf{x}_t + \boldsymbol{\varpi}_t + \boldsymbol{\varepsilon}_t, \tag{7}$$

where $\mathbf{s}_t, \boldsymbol{\mu}, \boldsymbol{\Pi}, \boldsymbol{\varpi}_t$ and $\boldsymbol{\varepsilon}_t$ are the first $n-1$ rows of $\mathbf{s}_t^{(n)}, \boldsymbol{\mu}^{(n)}, \boldsymbol{\Pi}^{(n)}, \boldsymbol{\varpi}_t^{(n)}$ and $\boldsymbol{\varepsilon}_t^{(n)}$, respectively.

Within the SUTSE framework it is almost trivial to check that the original matrix $\Pi^{(n)}$ is identified from (7) through the summation restrictions (6). That is; it does not matter which equation which is deleted from $\Pi^{(n)}$ to obtain Π – the resulting maximum likelihood estimate of $\Pi^{(n)}$ remains the same. In contrast, for the class of VAR models, identification of $\Pi^{(n)}$ based on error correction representations with $\mathbf{s}_t - \boldsymbol{\Pi}\mathbf{x}_t$ as an assumed equilibrium, is not possible (see Anderson and Blundell, 1982).

We shall now elaborate upon the stochastic structure of the latent vector $\boldsymbol{\varpi}_t$. As in Moosa and Baxter (2002), the latent vector is assumed to be the sum of a stochastic trend and a stochastic seasonal, with rank p and q, respectively:

$$\boldsymbol{\varpi}_t = \boldsymbol{\Gamma}\widetilde{\mathbf{f}}_t + \Upsilon\widetilde{\boldsymbol{\delta}}_t, \tag{8}$$

where

$$\begin{aligned}\widetilde{\mathbf{f}}_t &= \widetilde{\mathbf{f}}_{t-1} + \boldsymbol{\eta}_t \\ \widetilde{\boldsymbol{\delta}}_t &= -\widetilde{\boldsymbol{\delta}}_{t-1} - \widetilde{\boldsymbol{\delta}}_{t-2} - \widetilde{\boldsymbol{\delta}}_{t-3} + \boldsymbol{\xi}_t,\end{aligned} \tag{9}$$

and

$$\begin{aligned}\boldsymbol{\eta}_t &\sim \mathcal{IN}_p(\mathbf{0}, \mathbf{I}) \\ \boldsymbol{\xi}_t &\sim \mathcal{IN}_q(0, \mathbf{I}).\end{aligned} \tag{10}$$

Thus, the vector $\widetilde{\mathbf{f}}_t = [\widetilde{f}_{it}]_{p\times 1}$ contains p independent Gaussian random walks with corresponding $(n-1)\times p$ loading matrix $\Gamma = [\gamma_{ij}]_{(n-1)\times p}$, while the vector $\widetilde{\boldsymbol{\delta}}_t = [\widetilde{\delta}_{it}]_{q\times 1}$ contains q independent Gaussian seasonal effects with an $(n-1)\times q$ loading matrix $\Upsilon = [\upsilon_{ij}]_{(n-1)\times q}$. Inserting (8)-(10) into (7), we obtain

$$\mathbf{s}_t = \boldsymbol{\mu} + \boldsymbol{\Pi}\mathbf{x}_t + \boldsymbol{\Gamma}\widetilde{\mathbf{f}}_t + \Upsilon\widetilde{\boldsymbol{\delta}}_t + \boldsymbol{\varepsilon}_t. \tag{11}$$

To complete the stochastic specification of the model, we will assume that $\boldsymbol{\varepsilon}_t$ is normally distributed:

$$\boldsymbol{\varepsilon}_t \sim \mathcal{IN}_{(n-1)}(\mathbf{0}, \boldsymbol{\Sigma}).$$

To obtain identification, the loading matrix $\boldsymbol{\Gamma}$ is lower triangular with positive diagonal elements if $n-1=p$. That is, if $\Gamma\Gamma'$ has full rank. However, rank restrictions will be considered. Thus we will allow $\Gamma\Gamma'$ to have arbitrary rank $p \leq n-1$. To be able to identify Γ in this case, we require that $\gamma_{ij}=0$ for $j>i$ and $\gamma_{ii}>0$ for $i \leq p$. Similarly, Υ has $\upsilon_{ij}=0$ for $j>i$ and $\upsilon_{ii}>0$ for $i \leq q$.

It will be convenient to define

$$\mathbf{f}_t = \widetilde{\mathbf{f}}_t - \widetilde{\mathbf{f}}_0 \tag{12}$$

to obtain the initial condition $\mathbf{f}_0 = 0$. Note that the initial value $\widetilde{\mathbf{f}}_0$ will be absorbed into the intercept $\boldsymbol{\mu}$ in equation (11) and hence can be treated as a constant when estimating the model.

Similarly, as we explain in Appendix A, by conditioning on the three initial realizations of $\widetilde{\boldsymbol{\delta}}_t$, i.e. $\widetilde{\boldsymbol{\delta}}_0$, $\widetilde{\boldsymbol{\delta}}_{-1}$, $\widetilde{\boldsymbol{\delta}}_{-2}$, the seasonal effect $\widetilde{\boldsymbol{\delta}}_t$ can be written as a sum of a a stochastic vector $\boldsymbol{\delta}_t$, with $\boldsymbol{\delta}_0 = \boldsymbol{\delta}_{-1} = \boldsymbol{\delta}_{-2} = \mathbf{0}$, and a deterministic season-specific vector. Let $\mathbf{d}_t$ be a vector of dummy variables which indicates the season corresponding to time t, and let $\mathbf{M}$ be the corresponding coefficient matrix. (Both $\mathbf{d}_t$ and $\mathbf{M}$ are defined explicitly in Appendix A). Then we obtain the following dynamic factor model (cf. (11)):

$$\mathbf{s}_t = \mathbf{M}\mathbf{d}_t + \boldsymbol{\Pi}\mathbf{x}_t + \boldsymbol{\Gamma}\mathbf{f}_t + \Upsilon\boldsymbol{\delta}_t + \boldsymbol{\varepsilon}_t, \tag{13}$$

where

$$\begin{aligned} \boldsymbol{\delta}_t &= -\boldsymbol{\delta}_{t-1} - \boldsymbol{\delta}_{t-2} - \boldsymbol{\delta}_{t-3} + \boldsymbol{\xi}_t, \quad \boldsymbol{\delta}_0 = \boldsymbol{\delta}_{-1} = \boldsymbol{\delta}_{-2} = \mathbf{0} \qquad (14) \\ \mathbf{f}_t &= \mathbf{f}_{t-1} + \boldsymbol{\eta}_t\ , \qquad \mathbf{f}_0 = \mathbf{0}, \end{aligned}$$

and M, Π, Γ, Υ and Σ are fixed matrix parameters to be estimated.

Our approach introduces a deterministic season in addition to the stochastic season due to the treatment of initial conditions. Note that the initialization $\boldsymbol{\delta}_0 = \boldsymbol{\delta}_{-1} = \boldsymbol{\delta}_{-2} = \mathbf{0}$ does not entail any loss of generality. However, this is not the only way to treat initial conditions. E.g. in STAMP 6.2 (see Koopman et al., 2000) initial values and as well as other parameters are stochastic (with prior distributions) and are included in the state vector when estimating the model. Unfortunately, the current version of STAMP turned out to be unable to estimate our model. The likelihood optimization program employed in this paper is described in Appendix B.

4 Results

The most general models we estimate have $q = p = 3$, i.e. three stochastic seasonal components and three stochastic trend components. Attempts to estimate models with more than three independent latent components failed because of underflow problems resulting from the white noise covariance matrix estimate $\widehat{\Sigma}$ becoming singular.

The results from estimating three different types of models with $p = q = 3$ are displayed in Table 2. The three models are: (i) the general non-homogenous model (NH) defined in (13), i.e. with no restrictions on $\mathbf{\Pi}$; (ii) the homogenous model (HM) obtained from (13) by imposing the homogeneity restriction $\sum_{j=1}^{n} \pi_{ij} = 0$ for all $i = 1, ..., n-1$ (i.e. $n-1$ linearly independent constraints); and (iii) the restricted model obtained by assuming that $\pi_{ij} = 0$ for all i and j (which amounts to $(n-1) \times (n+1)$ independent constraints). We refer to the latter model as a pure time series model (TM), since it contains no income and price variables and hence has no direct economic interpretation. Obviously, these three models are nested, with NH as the most general one and TM as the most restricted one.

4.1 Observed components

From the value of the log-likelihood function for the three models reported in Table 2 we see that both the homogeneity restrictions (8 d.f.) and the restrictions on NH entailed by the pure time series model (72 d.f.) are firmly rejected. The rejection of HM appears

Table 2: **Three different models: Eigenvalues of estimated covariance matrices with p=q=3 (scale: 10^{-5}) and the log-likelihood value**

Eigenvalues of Σ			Eigenvalues of $\Gamma\Gamma'$			Eigenvalues of $\Upsilon\Upsilon'$			log-likelihood		
NH	HM	TM	NH	HM	TM	NH	HM	TM	NH	HM	TM
3.90	3.74	5.30	.43	.63	1.78	.25	.27	.29	5389	5350	5198
1.78	1.80	2.38	.21	.32	.32	.13	.12	.19			
1.05	1.24	1.94	.14	.12	.19	.03	.05	.05			
.71	.64	.84									
.54	.57	.79									
.27	.49	.50									
.15	.15	.10									
.03	.00	.03									

less alarming, however, considering the relatively small data set and the high number of unknown parameters (190 in NH, 182 in HM and 110 in TM).

Table 3 reports the results from testing for homogeneity in each of the eight equations in the demand system using asymptotic T-values. Using critical values from the standard normal distribution, homogeneity is rejected in four of the equations at the 1 per cent significance level. These are Food, Running cost of vehicles, Clothing and shoes and Transport services.

We will later (in Section 5), examine the effect of the homogeneity restrictions on the out-of-sample predictive properties of the different models. It is shown that the non-homogeneous (general) model and the homogeneous model performs more or less equally well. However, some indications of in-sample overfit of both these models are revealed. Hence, it is still of relevance to examine the homogeneous model with regard to the price and income elasticities that can be derived from it.

There has been a discussion in the literature concerning how one should calculate price elasticities in the linear AID system. A distinction is often made between the case in which one has level information about the prices and the case in which the prices are represented by indices, as is the case in our analysis [cf. for instance Green and Alston (1990,1991)]. Asche and Wessells (1997) put forward that even if the formulae of Chalfant (1987) have been subject to criticism, they are valid in a time series context in which the prices are represented by indices as long as the elasticities are calculated at the base year in which the values of all the price indices are 1. In this study we adhere to this recommendation.

Table 4 gives the (uncompensated) own-price and income elasticities for the homogenous model. The estimated price elasticity of commodity i , $\widehat{e_i}$, and its income elasticity $\widehat{E_i}$ (evaluated at some value of the budget share, as indicated by a bar) are given by

$$\widehat{e_i} = -1 - \widehat{\pi}_{i,n+1} + \widehat{\pi}_{ii}/\bar{s} \text{ and } \widehat{E_i} = 1 + \widehat{\pi}_{i,n+1}/\bar{s}.$$

In Table 4 we use budget shares from the base year 1997. The standard errors reported in the table are calculated under the assumption that the shares are fixed. All the estimated elasticities have the a priori expected signs. The own-price elasticities are negative, whereas the income elasticities are positive. An own-price elasticity which exceeds 1 in absolute value, implies that the consumption category in question is elastic, whereas an elasticity below 1 implies that it is inelastic. Only Consumption abroad is found to be elastic (the parameter estimates of this equation are obtained from the adding up conditions (6)). On the other hand, Electricity and Running costs of vehicles, have the lowest price elasticities.

A commodity with an income elasticity above 1 is labeled a luxury, whereas a commodity with an income elasticity below 1 is a necessity. The income elasticity for Food is significantly below 1, confirming Engel's law. For Consumption abroad and Other non-durable goods, we find income elasticities significantly exceeding 1.

Table 4 also contains elasticities obtained using a deterministic benchmark model, which, besides the income and price variables, only allow for a deterministic trend and fixed seasonal effects. This model is not well-specified since the residuals are highly autocorrelated. It is interesting to compare the elasticities obtained using this model as compared to the SUTSE model. There are some apparent differences. The own-price elasticity for Energy now has the wrong sign and the absolute value of the own-price elasticity for Consumption abroad is lower than in the more sophisticated SUTSE model. Among the income elasticities the most substantial change is found for Running cost of vehicles and for Consumption abroad. The income elasticities for these two commodities are substantially higher for the benchmark model than for the SUTSE model.

Table 3: **Test of homogeneity ($\sum_{j=1}^{n} \pi_{ij} = 0$) within the general model**

Commodity (i)	Eq. #	$\sum_{j=1}^{n} \widehat{\pi_{ij}}$	St. error	T-statistic
Food	1	$-.032$	.008	-4.14
Beverages and tobacco	2	$-.008$	.005	-1.45
Electricity	3	.008	.004	1.99
Running costs of vehicles	4	.020	.007	2.79
Other non-durable goods	5	.010	.005	1.84
Clothing and shoes	6	$-.034$	.005	-5.86
Other services	7	$-.008$	.012	-0.69
Transport services	8	.017	.006	2.65
Consumption abroad	9	.026	.010	2.60

Table 4: **Estimates of elasticities of the homogeneous model (HM) and comparisons with a deterministic benchmark model (DM). Standard deviations in parentheses**

Commodity	Eq. #	Own-price elast. ($\widehat{e}_i$)		Income elast. ($\widehat{E}_i$)	
		HM	DM	HM	DM
Food	1	$-.81$ (.10)	$-.73$	.66 (.07)	.46
Beverages and tobacco	2	$-.83$ (.07)	$-.77$	.97 (.10)	.86
Electricity	3	$-.15$ (.11)	.15	.42 (.23)	.67
Running costs of vehicles	4	$-.29$ (.10)	$-.32$	.97 (.24)	1.56
Other non-durable goods	5	$-.63$ (.18)	$-.88$	1.32 (.11)	1.24
Clothing and shoes	6	$-.93$ (.13)	-1.03	1.05 (.12)	.77
Other services	7	$-.47$ (.19)	$-.64$	1.15 (.14)	1.20
Transport services	8	$-.50$ (.12)	$-.66$	1.04 (.14)	1.20
Consumption abroad	9	-1.37 (.23)	-1.00	1.94 (.08)	2.66

4.2 Unobserved components

In Figures 3 and 4 we display, for the homogeneous and non-homogeneous model, respectively, the actual budget shares and their decomposition into i) a stochastic trend, ii) seasonal effects, and iii) price and income effects. The estimated trend and seasonal variables represent the total effects of three independent random walks and three independent stochastic seasonality components. The total contribution from the price and income variables is also seen to contain trending behavior. Thus the role of the non-observed components is to pick up trend and seasonal effects not accounted for by the observed variables. In Section 4.1 we found that demand homogeneity was rejected for Food, Running cost of vehicles, Clothing and shoes and Transport services (as well as for the residual commodity Consumption abroad). When we compare these commodities in Figures 3 and 4 we find substantial changes, which is not very surprising. The trends in the budget shares are to some extent explained by the trends in the linear combinations of the log-transformed price and income variables. Without imposition of homogeneity, the implicit weights are estimated freely, whereas they are constrained under homogeneity. Rejection of homogeneity indicates that the weights are substantially altered and so are the resulting trends. A change in the trends of the observed variables will to some extent be counteracted by changes in the estimated latent trends. The seasonal pattern seems to a less degree to be influenced by the homogeneity restriction.

It is evident that changes in the seasonal pattern are taken well care of by our model. The most striking result is for Running cost of vehicles. The budget shares of this commodity is characterized by its seasonal fluctuations being much stronger in the first half than in the second half of the sample. This feature, which is not explained by the observed variables, is picked up by the seasonal component. Electricity is also an interesting commodity as far as seasonality is concerned. From Figure 3 we see that its seasonal fluctuations are much weaker at the start of the observation period than at the end of it, while the opposite is true for Running cost of vehicles. Again this pattern is well represented by the smoothed seasonal component.

Figure 3: Homogeneous model: Actual budget shares versus trend, season, and income/price effects

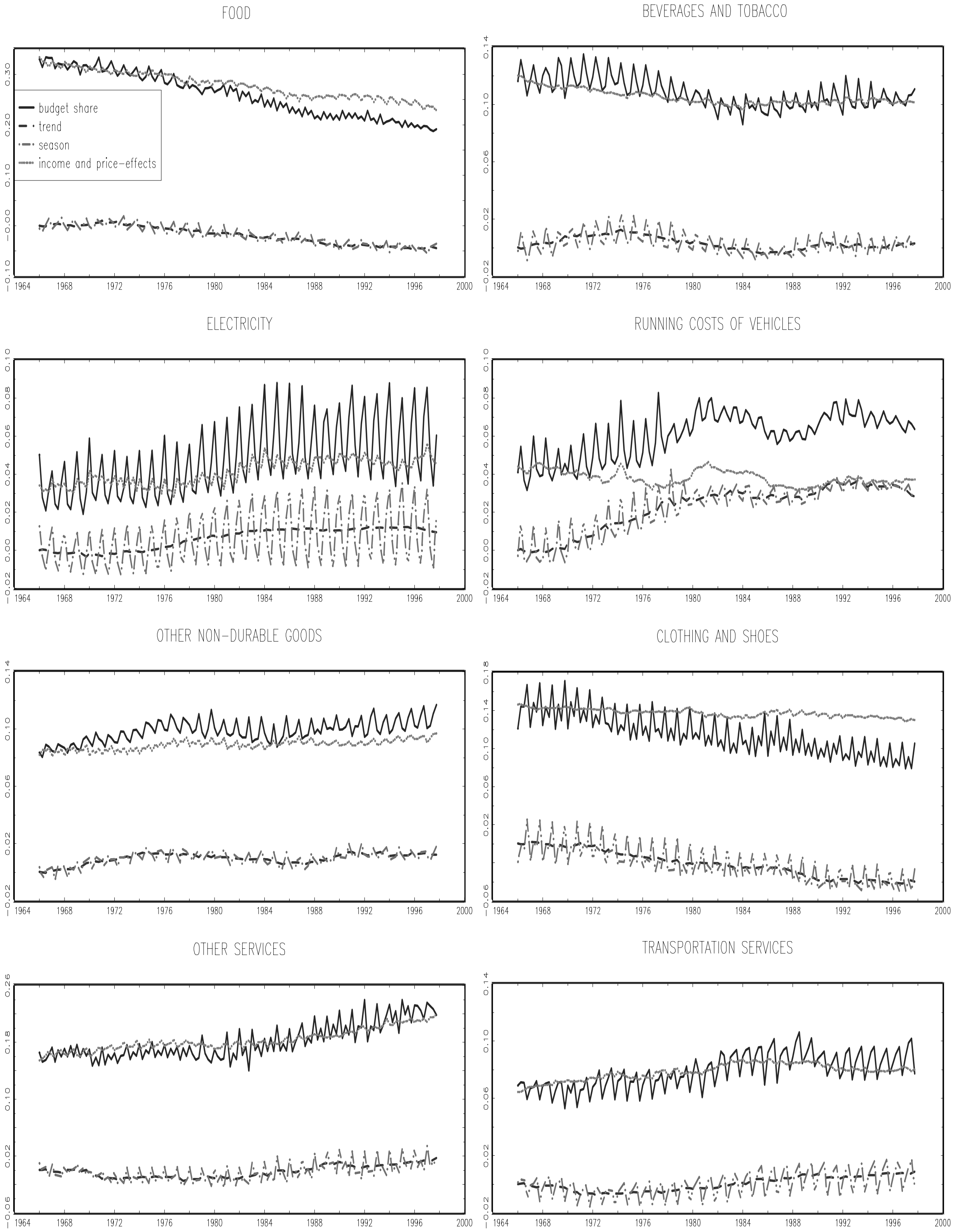
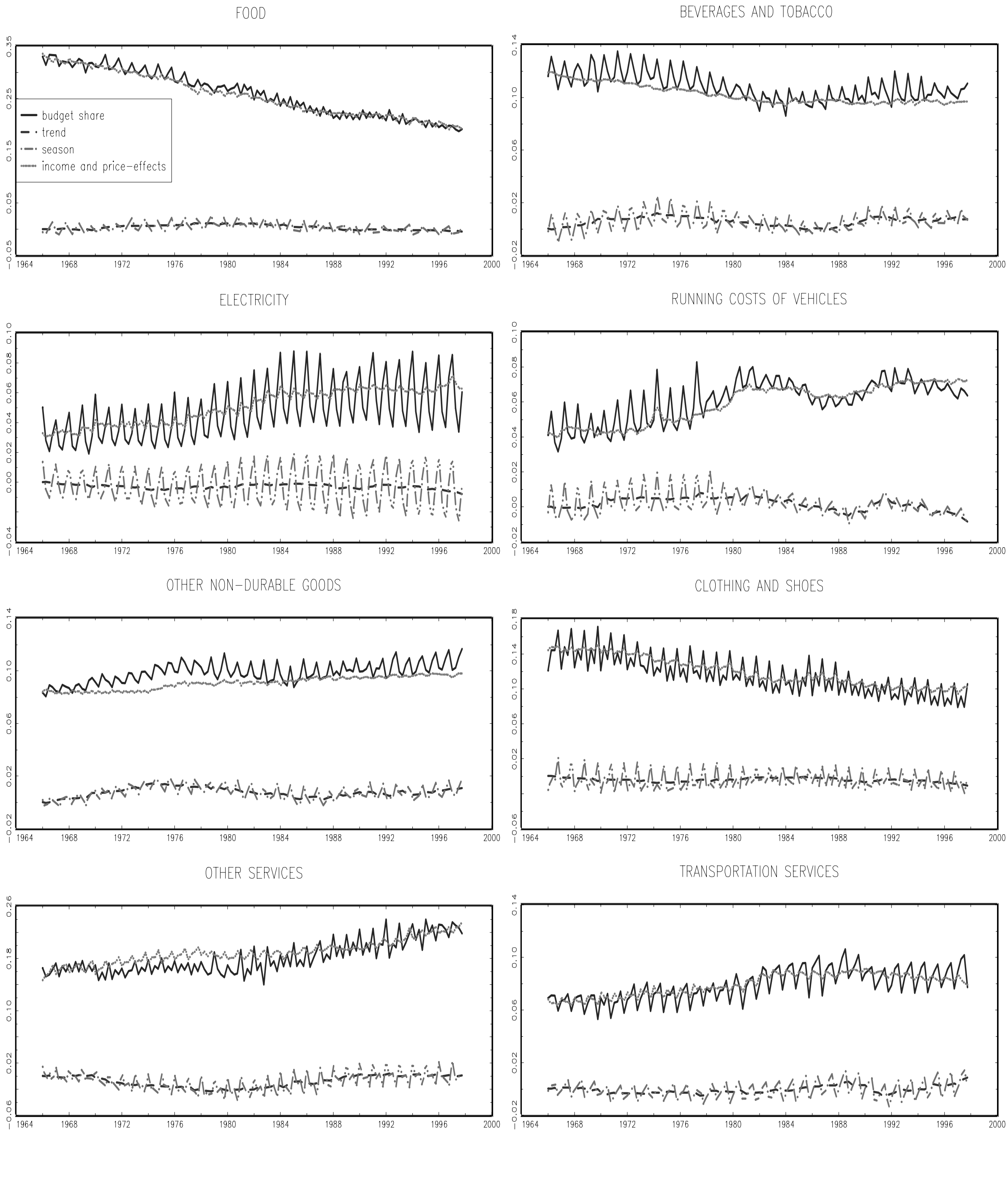


Figure 4: Non-homogeneous model: Actual budget shares versus trend, season, and income/price effects



4.3 Goodness of fit

Let $E_t(\cdot)$ denote the conditional expectation given $\mathbf{s}_1, .., \mathbf{s}_t$, and define the auxiliary residuals

$$\widehat{\varepsilon}_t = \mathbf{s}_t - E_T(\mathbf{s}_t).$$

Test statistics for skewness (S) and kurtosis (K) are defined in Harvey and Koopman (1992). Both test statistics are asymptotically $N(0,1)$ when the model is correctly specified. The results are shown in Table 5. Only the results regarding kurtosis in equation number 4 appear somewhat problematic for the assumption of normality.

In Table 6 we show the results of portmanteau tests for serial correlation in the innovations based on the statistic

$$Q = T^2 \sum_{l=1}^{s} (T-l)^{-1} tr \left\{ \widehat{\mathbf{C}}(l)' \widehat{\mathbf{C}}(0)^{-1} \widehat{\mathbf{C}}(-l) \widehat{\mathbf{C}}(0)^{-1} \right\},$$

where $\widehat{C}(l)$ is the lag l autocovariance matrix of the innovations:

$$\widehat{\mathbf{C}}(l) = T^{-1} \sum_{t=l+1}^{T} v_{t-l} v_t', \quad l = 0, 1, 2, ...,$$

with $v_t = \mathbf{s}_t - E_{t-1}(\mathbf{s}_t)$. The asymptotic distribution of Q is χ^2 with $(n-1)^2 s$ degrees of freedom when all parameters are known and both T and s tends to infinity (see Reinsel, 1993). The degrees of freedom could be adjusted for dependence among residuals caused by replacing true parameters by estimated ones. It is known in some special cases that $Q \sim \chi^2((n-1)^2 s - n^*)$, where n^* is the number of estimated parameters, except Σ. The degrees of freedom (d.f.) in Table 5 is based on the conjecture that this adjustment improves the finite sample properties of the test also in our case (with $s = 50$).We find no evidence of serial correlations, and this test result is also confirmed by empirical autocorrelation plots of the residuals (not displayed).

5 Out-of-sample predictions

The root mean squared error (RMSE) of one-step-ahead predictions for each equation is shown in Table 7: Both within-sample (cf. also Table 6) and out-of-sample results are presented: the latter are obtained for the 16 observations added to the time series

Table 5: **Innovation standard errors $\sqrt{\widehat{\mathbf{C}}_{ii}(0)}$, and test statistics for skewness and kurtosis (both distributed as $\mathcal{N}(0,1)$)**

	$\sqrt{\widehat{\mathbf{C}}_{ii}(0)}$			Skewness			Kurtosis		
Eq.# (i)	NH	HM	TM	NH	HM	TM	NH	HM	TM
1	.0048	.0051	.0057	.50	.80	−.01	1.11	.36	1.51
2	.0029	.0029	.0031	.20	.58	−.47	.28	.34	−.29
3	.0027	.0028	.0036	.59	.88	.78	.29	1.34	.16
4	.0033	.0036	.0044	2.96	1.77	.60	5.98	5.24	−.07
5	.0024	.0025	.0030	.88	−.03	01	.04	.23	−.90
6	.0038	.0039	.0042	36	.98	.89	.23	.04	−.66
7	.0063	.0062	.0082	−.99	−1.65	.74	.59	1.84	−1.05
8	.0026	.0029	.0031	−1.00	−.54	.24	−.22	−.05	−.50

Table 6: **Portmanteau test statistic (Q) for serial correlation in the innovations**

	NH	HM	TM
Q	2688	2619	3559
$s(n-1)^2$	3200	3200	3200
n^*	170	162	74
$d.f.$	3030	3038	3126

Table 7: **Three models: within- and out-of-sample RMSE for budget shares predictions**

	Within-sample			Out-of-sample		
Eq.# (i)	NH	HM	TM	NH	HM	TM
1	.0048	.0051	.0057.	.0085	.0072	0079
2	.0029	.0029	.0031	.0076	.0064	.0056
3	.0027	.0028	.0036	.0059	.0091	.0116
4	.0033	.0036	.0044	.0104	.0062	.0045
5	.0024	.0025	.0030	.0041	.0022	.0028
6	.0038	.0039	.0042	.0045	.0043	.0056
7	.0063	.0062	.0082	.0119	.0193	.0126
8	.0026	.0029	.0031	.0088	.0132	.0139

Table 8: **Three models: within- and out-of-sample RMSE for log-volume predictions**

	Within-sample			Out-of-sample		
Eq.# (i)	NH	HM	TM	NH	HM	TM
1	.0185	.0199	.0227	.0457	.0396	.0443
2	.0276	.0277	.0297	.0757	.0633	.0546
3	.0616	.0598	.0768	.1122	.1787	.2417
4	.0575	.0633	.0727	.1838	.1085	.0702
5	.0249	.0257	.0304	.0396	.0211	.0268
6	.0340	.0348	.0370	.0529	.0513	.0628
7	.0358	.0347	.0455	.0493	.0783	.0546
8	.0333	.0375	.0406	.0868	.1353	.1398

after the estimation of the model was completed, i.e. 1998Q1 to 2001Q4. While Table 6 presents prediction errors for budget shares, similar results are depicted in Table 7 for consumption volumes on logarithmic scale. The out-of-sample predictions are not forecasts in a genuine sense, since we have assumed knowledge of prices and income $\mathbf{x}_t$ for the whole prediction period. The logarithmic scaling was used in order to reduce problems with heteroscedasticity due to trends in the volume variables and hence facilitate comparisons between within- and out-of-sample results.

In contrast to the likelihood-based measures of fit in Table 2 and 3, it is striking from Table 7 that the NH model only marginally reduces the within-sample RMSE compared to HM, while both models yield substantially better fit to the data than TM. This picture is reinforced by the prediction results for log-volumes in Table 8.

The numbers change dramatically when turning to the out-of-sample predictions. Since the estimated parameters are held fixed at their estimates obtained for the period up until 1997Q4, it is not surprising that prediction errors increase. However, these are typically doubled or even tripled. These results indicate some over-fitting with respect to the models NH and HM. For the pure time series model (TM) the increase in RMSE in the out-of-sample period is much less alarming.

Another striking feature of these results is that neither of the three models appears superior to any other in terms of out-of-sample RMSE. Actually, they perform quite similarly.

6 Conclusions

This paper uses the SUTSE framework to model a linear demand system with 9 commodities. In our most general model the budget shares in levels are modeled as linear functions of the log of nominal prices, log of real income and non-observed components taking into account stochastic trends and seasonality. Demand homogeneity, which means that money illusion plays no role, is not imposed on the most general model. Trends and seasonality are each represented by three independent random components. We find this model reasonably well-specified as demonstrated by different diagnostic tests. We test for demand homogeneity within the general set-up, both for the single commodities and for all commodities simultaneously. The homogeneity restriction is rejected for about half of the commodities and for the system as a whole. Root mean squared errors (RMSE) from within-sample and out-of-sample predictions give mixed evidence with regard to the relative merits of the different model specifications. However, only the homogenous model can be given a substantial interpretation in terms of economic theory. In particular, estimates of own-price and income elasticities can be derived from this model for all commodities. All our elasticities are reasonable and have the correct sign. In particular, Consumption abroad is a luxury and is inelastic in demand, Food is a necessity, while most other commodities have income elasticities which are not significantly different from unity.

In addition to the two models described above, we also consider within- and out-of-sample predictions from a pure time series model in which the price and income effects are omitted. When predicting within-sample we find that imposition of demand homogeneity only to a very modest increases RMSE, whereas the pure time series model performs considerably worse. However, the picture becomes quite different when turning to out-of-sample predictions. First, the prediction errors are substantially larger for all model. Second, no model gives the best predictions for all commodities in terms of RMSE. For Beverages and tobacco and Running costs of vehicles the pure time series model outperforms the two other models. For Food and Other non-durable goods the model with demand homogeneity performs best.

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Appendix A: Seasonal components

From (11) we obtain $\widetilde{\boldsymbol{\delta}}_t - \widetilde{\boldsymbol{\delta}}_{t-4} = \boldsymbol{\xi}_t - \boldsymbol{\xi}_{t-1}$. Hence,

$$
\begin{aligned}
\widetilde{\boldsymbol{\delta}}_1 &= \widetilde{\boldsymbol{\delta}}_0 + \widetilde{\boldsymbol{\delta}}_{-1} + \widetilde{\boldsymbol{\delta}}_{-2} + \xi_1 \\
\widetilde{\boldsymbol{\delta}}_2 &= \widetilde{\boldsymbol{\delta}}_{-2} + \boldsymbol{\xi}_2 - \boldsymbol{\xi}_1 \\
\widetilde{\boldsymbol{\delta}}_3 &= \widetilde{\boldsymbol{\delta}}_{-1} + \boldsymbol{\xi}_3 - \boldsymbol{\xi}_2 \\
\widetilde{\boldsymbol{\delta}}_4 &= \widetilde{\boldsymbol{\delta}}_0 + \boldsymbol{\xi}_4 - \boldsymbol{\xi}_0.
\end{aligned}
$$

We can define:

$$\boldsymbol{\delta}_t = \begin{cases} \widetilde{\boldsymbol{\delta}}_t - \widetilde{\boldsymbol{\delta}}_0 - \widetilde{\boldsymbol{\delta}}_{-1} - \widetilde{\boldsymbol{\delta}}_{-2} & \text{iff } t = 4n+1,\ n \in \{0,1,2...\} \\ \widetilde{\boldsymbol{\delta}}_t - \widetilde{\boldsymbol{\delta}}_{-2} & \text{iff } t = 4n+2,\ n \in \{0,1,2...\} \\ \widetilde{\boldsymbol{\delta}}_t - \widetilde{\boldsymbol{\delta}}_{-1} & \text{iff } t = 4n+3,\ n \in \{0,1,2...\} \\ \widetilde{\boldsymbol{\delta}}_t - \widetilde{\boldsymbol{\delta}}_0 & \text{iff } t = 4n+4,\ n \in \{0,1,2...\} \end{cases}$$

with initial condition $\boldsymbol{\delta}_0 = \boldsymbol{\delta}_{-1} = \boldsymbol{\delta}_{-2} = \mathbf{0}$. Analogously to (12), we can now treat $(\widetilde{\boldsymbol{\delta}}_0, \widetilde{\boldsymbol{\delta}}_{-1}, \widetilde{\boldsymbol{\delta}}_{-2})$ as constants when estimating the model. Define

$$\begin{aligned} \mathbf{M} &= [\boldsymbol{\mu} + \widetilde{\mathbf{f}}_0,\ \boldsymbol{\Upsilon}\widetilde{\boldsymbol{\delta}}_0,\ \boldsymbol{\Upsilon}\widetilde{\boldsymbol{\delta}}_{-1}, \boldsymbol{\Upsilon}\,\widetilde{\boldsymbol{\delta}}_{-2}] \\ \mathbf{d}_t &= \begin{cases} \begin{pmatrix} 1 & -1 & -1 & -1 \end{pmatrix}' & t = 4n+1,\ n \in \{0,1,2...\} \\ \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}' & t = 4n+2,\ n \in \{0,1,2...\} \\ \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}' & t = 4n+3,\ n \in \{0,1,2...\} \\ \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix}' & t = 4n+4,\ n \in \{0,1,2...\}. \end{cases} \end{aligned}$$

Appendix B: Estimation

The main challenge in estimating our econometric model (13)-(14) is to obtain a computationally convenient representation of the log-likelihood function and its derivatives. Having achieved that, an efficient quasi-Newton algorithm can be applied to maximize the likelihood function with respect to the unknown parameters: $\boldsymbol{\beta} = (\mathbf{M}, \boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Upsilon}, \boldsymbol{\Sigma})$. A state space representation of the model, combined with a decomposition of the log-likelihood function well known from the EM (Expectation Maximization) algorithm, will provide an efficient solution to our estimation problem.

The model (13)-(14) can be restated on the following state space form:

$$\begin{aligned} &\mathbf{s}_t = \mathbf{G}\boldsymbol{\alpha}_t + \mathbf{M}\mathbf{d}_t + \boldsymbol{\Pi}\mathbf{x}_t + \boldsymbol{\varepsilon}_t \\ &\boldsymbol{\alpha}_t = \mathbf{F}\,\boldsymbol{\alpha}_{t-1} + \boldsymbol{\omega}_t \end{aligned} \qquad t = 1, ..., T \tag{15}$$

where the state vector $\boldsymbol{\alpha}_t$ has dimension $p+3q$, and is determined by the equations:

$$\begin{aligned}
\boldsymbol{\alpha}_0 &= \mathbf{0}_{p+3q} \\
\mathbf{G} &= \begin{bmatrix} \boldsymbol{\Gamma} & \boldsymbol{\Upsilon} & \mathbf{0}_{p\times q} & \mathbf{0}_{p\times q} \end{bmatrix} \\
\mathbf{F} &= \begin{bmatrix} \mathbf{I}_p & \mathbf{0}_{p\times q} & \mathbf{0}_{p\times q} & \mathbf{0}_{p\times q} \\ \mathbf{0}_{q\times p} & -\mathbf{I}_q & -\mathbf{I}_q & -\mathbf{I}_q \\ \mathbf{0}_{q\times p} & \mathbf{I}_q & \mathbf{0}_{q\times q} & \mathbf{0}_{q\times q} \\ \mathbf{0}_{q\times p} & \mathbf{0}_{q\times q} & \mathbf{I}_q & \mathbf{0}_{q\times q} \end{bmatrix} \\
\boldsymbol{\omega}_t &\sim \mathcal{IN}\left(\begin{bmatrix} \mathbf{0}_{p+q} \\ \mathbf{0}_{2q} \end{bmatrix}, \begin{bmatrix} \mathbf{I}_{p+q} & \mathbf{0}_{(p+q)\times 2q} \\ \mathbf{0}_{2q\times(p+q)} & \mathbf{0}_{2q\times 2q} \end{bmatrix} \right)
\end{aligned} \tag{16}$$

Given the state space representation (15)-(16), it is well known that the log-likelihood function can be evaluated for any given parameter value $\boldsymbol{\beta}$ by using the Kalman filter and smoother. Then

$$L(\boldsymbol{\beta}) = -\frac{1}{2}\sum_{t=1}^{T}\left(\ln\left|\mathbf{G}\mathbf{V}_{t\,|t-1}\mathbf{G}' + \boldsymbol{\Sigma}\right| + \mathbf{R}_t{}'\left[\mathbf{G}\mathbf{V}_{t\,|t-1}\mathbf{G}' + \boldsymbol{\Sigma}\right]^{-1}\mathbf{R}_t\right)$$

where

$$\begin{aligned}
&\mathbf{V}_{t\,|t-1} = E\{(\boldsymbol{\alpha}_t - \mathbf{a}_{t|t-1})(\boldsymbol{\alpha}_t - \mathbf{a}_{t|t-1})'\} \\
&\mathbf{a}_{t|t-1} = E\{\boldsymbol{\alpha}_t\,|\,\mathbf{s}_1,...,\mathbf{s}_{t-1}\} \\
&\mathbf{R}_t = \mathbf{s}_t - \mathbf{G}\mathbf{a}_{t|t-1} - \mathbf{M}\mathbf{d}_t - \boldsymbol{\Pi}\mathbf{x}_t.
\end{aligned} \tag{17}$$

For example Harvey (1989) explains in detail how the Kalman filter and smoother can be applied to the state space form to evaluate the conditional moments in (17), given $\boldsymbol{\beta}$.

While the evaluation of the likelihood function is straightforward, the main challenge is to obtain analytic expressions for the derivatives of $L(\beta)$. The task of obtaining an analytic form for $\frac{\partial L(\beta)}{\partial \beta}$ may seem prohibitive since $L(\beta)$ indirectly depends on $\boldsymbol{\beta}$ through the Kalman filter recursions.

Our solution to the problem is to make a somewhat unusual application of techniques associated with the EM (Expectation Maximization) algorithm – an algorithm originally developed by Dempster et al. (1977)..

Let $f(\mathbf{y},\boldsymbol{\alpha};\boldsymbol{\beta})$ be the joint density of the observed variables $\mathbf{y} = \{\mathbf{s}_t\}$ and the latent variables $\boldsymbol{\alpha} = \{\boldsymbol{\alpha}_t\}$. Furthermore, let $f(\boldsymbol{\alpha}|\,\mathbf{y};\boldsymbol{\beta})$ be the conditional density of $\boldsymbol{\alpha}$, given $\mathbf{y}$. The maximum likelihood estimator, $\widehat{\boldsymbol{\beta}}$, is the maximum of the log-likelihood $L(\boldsymbol{\beta})$, where

$$L(\boldsymbol{\beta}) = \ln f(\mathbf{y};\boldsymbol{\beta}). \tag{18}$$

Since

$$f(\mathbf{y};\boldsymbol{\beta}) = \frac{f(\mathbf{y},\boldsymbol{\alpha};\boldsymbol{\beta})}{f(\boldsymbol{\alpha}|\,\mathbf{y};\boldsymbol{\beta})},$$

(18) can be rewritten as

$$L(\boldsymbol{\beta}) = \ln f(\mathbf{y}, \boldsymbol{\alpha}; \boldsymbol{\beta}) - \ln f(\boldsymbol{\alpha} |\, \mathbf{y}; \boldsymbol{\beta}). \tag{19}$$

Taking the expectation of both sides in (19) with respect to $f(\boldsymbol{\alpha} |\, \mathbf{y}; \boldsymbol{\beta}^0)$, where $\boldsymbol{\beta}^0$ is an arbitrary parameter value, gives:

$$L(\boldsymbol{\beta}) = M(\boldsymbol{\beta}|\boldsymbol{\beta}^0) - H(\boldsymbol{\beta}|\, \boldsymbol{\beta}^0), \tag{20}$$

where

$$\begin{aligned} M(\boldsymbol{\beta}|\boldsymbol{\beta}^0) &= \int \ln f(\mathbf{y}, \boldsymbol{\alpha}; \boldsymbol{\beta})\, f(\boldsymbol{\alpha} |\, \mathbf{y}; \boldsymbol{\beta}^0)\, d\boldsymbol{\alpha} \\ H(\boldsymbol{\beta}|\boldsymbol{\beta}^0) &= \int \ln f(\boldsymbol{\alpha}|\mathbf{y}; \boldsymbol{\beta})\, f(\boldsymbol{\alpha} |\, \mathbf{y}; \boldsymbol{\beta}^0)\, d\boldsymbol{\alpha}. \end{aligned}$$

While the decomposition (20) is not useful in calculating $L(\boldsymbol{\beta})$, it has the following extremely important property:

$$\left.\frac{\partial L(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^0} = \left.\frac{\partial M(\boldsymbol{\beta}|\boldsymbol{\beta}^0)}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^0}, \tag{21}$$

which follows from the fact that $\boldsymbol{\beta}^0$ is the maximizer of $H(\boldsymbol{\beta}|\boldsymbol{\beta}^0)$ (by Kullback's inequality), and hence a stationary point. The derivatives $\frac{\partial L(\boldsymbol{\beta}^0)}{\partial \boldsymbol{\beta}}$ can easily be obtained by *analytic* differentiation of $M(\boldsymbol{\beta}|\boldsymbol{\beta}^0)$. Furthermore, the Hessian of $L(\boldsymbol{\beta})$ at the ML estimate $\widehat{\boldsymbol{\beta}}$ can be obtained by *numerical* differentiation of $\left.\frac{\partial M(\boldsymbol{\beta}|\widehat{\boldsymbol{\beta}})}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}}$, yielding a computationally simple estimator of the covariance matrix of $\widehat{\boldsymbol{\beta}}$ (see Oakes, 1999).

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