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## **A Method for Improved Capital Measurement by Combining Accounts and Firm Investment Data**

A revised version

**Abstract:**

We propose a new method for estimating capital stocks at the firm level by combining business accounts information and investment data. The method also produces capital estimates at the sector or industry level by summing individual firms' capital stocks and appropriately inflating this sum to account for firms with missing data. Our approach has two major advantages compared with the much used Perpetual Inventory Method (PIM). First, long investment series are not necessary. Second, sector capital estimates are automatically adjusted for changes in the capital stock because of entry and exit of firms. While capital growth rates in Norwegian manufacturing were only 1 percent on average during 1993--2004 according to national accounts figures, our method yields much higher growth rates of 5.5 percent on average.

**Keywords:** Capital measurement, Accounts data, Firm panel data, Net capital stocks, Depreciation

**JEL classification:** C13, C23, D24, E22, M40

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# 1 Introduction

Most studies of production, including some very important topics like measurements of productivity, returns to investment, and economic depreciation, rely on measures of capital stocks and services. Although measurement of capital is one of the most controversial topics in economics (see Hicks, 1974), there exist rather well-established national accounts standards for estimating capital stocks from aggregate (e.g., sector) data using the Perpetual inventory method (PIM), see OECD (2001).<sup>1</sup> However, PIM has some well-known deficiencies, especially when applied to individual firms where one generally does not have a sufficiently long investment time series to apply this method.

Direct stock information is seldom available from micro data. Although information on book values, stock prices, and even fire insurance values have been used in combination with PIM in some studies (see e.g., Klette and Griliches, 1996), no well-documented measurement relation between these indirect observations of capital and the capital stock itself has been established. This paper proposes an alternative to existing methods for estimating capital stocks, that is based on firm-level panel data with investments and financial accounts variables.<sup>2</sup>

Accounts data are often criticized for being based on historical costs, not current prices.<sup>3</sup> Furthermore, it is often claimed that the depreciation profiles used by firms are chosen to minimize tax liabilities. Our approach addresses these criticisms. First, we propose a method for converting historical prices into current prices by combining time series of book values and investment data for each firm and adjusting the former by price indices of new capital goods. Second, financial accounts, not tax accounts, are used. The formula we apply is analytically similar to the PIM method but replaces depreciation rates with *reduction rates* that capture both ordinary depreciation, extraordinary depreciation

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<sup>1</sup>If  $K_t$  is the capital stock in year  $t$ ,  $J_t$  is gross investment and  $d_t$  is the depreciation rate, then PIM says that  $K_{t+1} = (1 - d_t)K_t + J_{t+1}$ . If one is willing to assume that  $d_t$  is time invariant, this is equivalent to geometric depreciation (see e.g. Hulten and Wykoff, 1996; and Jorgenson, 1996).

<sup>2</sup>Our paper has some resemblance with Broersma et al. (2003), who, under the assumption of linear depreciation, combine information on depreciation from accounts data with survey information on investments to obtain IT- and non-IT capital stocks at the firm level.

<sup>3</sup>For an objection to this critique see Jaffey (1990), who argues that company data in spite of being based on historical costs are informative on service lives of fixed capital assets.

and sales of fixed capital, i.e., all kinds of reductions in capital from one year to the next. The reduction rates are both firm specific and time specific. For firms established before 1993, the first year of our panel data set, the initial book values are converted into current values using cohort-specific correction factors. The correction factors are derived analytically, given (i) parameters describing the historical investment profile of a representative (“average”) firm in each cohort, (ii) price indices of capital, and (iii) estimated reduction rates. We estimate the correction factors from aggregate historical investment data. Application of the method requires data on each firm’s birth year.

Our main objective will be to measure *net capital stocks* for the individual firm. That is, the value of a firm’s tangible capital stock in a given year at the prices of similar new assets, minus depreciation. By summing over individual firms’ capital stocks, we can also obtain estimates of aggregate capital stocks. For the total manufacturing sector in Norway, this method gives larger estimates of capital growth rates than the corresponding national accounts estimates: 5.5 percent versus 1 percent average annual growth in the period 1993–2004.<sup>4</sup> One important difference between the two methods is that the average depreciation rates used by firms are larger, especially for machinery and equipment, than depreciation rates used in the national accounts. Moreover, PIM, when using low imputed depreciation rates, may almost completely smooth out variations in annual investments, whereas our method is much more responsive to fluctuations in investments over the business cycle.

A particular problem arises with PIM when applied to industry-level investment data because of reallocation and revaluation of capital caused by firm exit. It is not appropriate to assume, as a rule, that capital equipment in firms that have closed down remain operative (with an unchanged value) within the industry. Some of the equipment may be sold to firms outside the industry, in which case these sales are investments by the acquiring firms and disinvestments by the exiting firm (but not reported as such, because the firm is not operative). Other equipments may be scrapped, so that the value of the equipments should be subtracted from the capital stock of the industry. To address these problems, Harris and Drinkwater (2000) attempt to estimate the capital stock at the plant level using PIM, explicitly taking scrapping into account. They show that the effect of

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<sup>4</sup>Annual growth rates of tangible fixed capital in the national accounts are available at [http://www.ssb.no/english/subjects/09/01/nr\\_en/](http://www.ssb.no/english/subjects/09/01/nr_en/)

scrapping may be quite large for the estimates of aggregate capital stocks under periods with many plant closures. Entry of firms also poses problems: Our comparisons of the manufacturing statistics (which is the primary micro data source for the manufacturing sector) with a sample of new firms' annual reports reveal that initial capital stocks are often not reported as "investments" and hence are ignored by PIM when applied to aggregate gross investment data. In contrast, our method of aggregating individual firms' net capital stocks automatically accounts for changes in the population of operative firms.

The rest of this paper is organized as follows. Section 2 discusses the main accounting concepts related to a simple model of investment. Section 3 provides definitions of investment and depreciation, and discusses the relationship between national accounting and business accounting. Section 4 presents the formal model that is used to estimate net capital stocks at the firm-level at both current and constant prices. Section 5 discusses data and issues in the implementation of our proposed method. Section 6 uses the proposed methods to estimate the total net capital stock in the manufacturing sector for 1993–2004. Section 7 concludes.

## 2 A neoclassical model of depreciation

In order to relate accounting concepts to economic theory, we start our analysis by looking at these concepts in a familiar neoclassical setting (see e.g., Varian, 1984). Assume that the factors of production consist of fixed capital, labor, and materials, and that capital of different vintages are perfect substitutes. The production function is

$$Y_t = f(X_{K,t-1}, X_{Lt}, X_{Mt}). \quad (1)$$

Here,  $X_{K,t-1}$  refers to the capital stock at the *end* of year  $t - 1$  in physical units. The variables  $Y_t$ ,  $X_{Lt}$  and  $X_{Mt}$  are total amounts of output, labor and materials in year  $t$ , respectively. In contrast to capital, these are flow, not stock, variables.

The quasi rent (operating profit in year  $t$ , exclusive of capital costs) is given by

$$\Pi(X_{K,t-1}) = \max_{L,M} \left( pY_t - \sum_{i=L,M} q_i X_{it} \right),$$

where  $p$  is the output price and  $q_i$  is the price of input  $i$ , all for convenience assumed time invariant. The unit price of new capital is  $q_K$ . To simplify further, we assume that there

are constant returns to scale. In a competitive market, this implies a linear homogeneous profit function. Hence, the return to capital is independent of the total capital stock in the firm. We can then write

$$\Pi(X_{K,t-1}) = \pi X_{K,t-1},$$

for some constant  $\pi$ .

The net capital stock at the end of year  $t$  is the market value of  $X_{Kt}$ . We shall now analyze the change in the value of a given, original, capital stock  $X_{K0}$  (acquired at the end of year  $t = 0$ ), as it gets older and is subject to loss of productive efficiency as well as retirements (scrapping). Let  $\theta_t$  denote the reduction in efficiency (including retirements) of this capital stock during year  $t$  relative to the stock at the beginning of year  $t$ . That is

$$X_{Kt} = (1 - \theta_t)X_{K,t-1}.$$

The net present value of the capital stock at the end of  $t - 1$  is the discounted value of the remaining cash flow generated by the original investment

$$V(X_{K,t-1}) = \pi X_{K0} \sum_{s=t}^{\infty} \frac{\prod_{k=1}^{s-1} (1 - \theta_k)}{(1 + r)^{s+1-t}}; \quad t = 1, 2, \dots,$$

where  $r$  is the interest rate. In a competitive equilibrium, the marginal revenue of capital must equal its purchase price

$$V'(X_{K0}) = q_K.$$

In the case of geometric depreciation,  $\theta_k \equiv \theta$ , and we obtain the well-known user price formula of capital,  $\pi = q_K(r + \theta)$ , which says that the annual profit of one additional unit of capital should equal the cost of employing that unit of capital from the beginning of the year until the end of the year (i.e., the cost of capital services). The annual cost of capital thus has two components: depreciation,  $\theta q_K$ , and the risk-free alternative yield,  $r q_K$ .

Depreciation,  $D_t$ , is defined as the reduction in the value of the capital stock from age  $t - 1$  to age  $t$ :

$$D_t = V(X_{K,t-1}) - V(X_{K,t}).$$

Furthermore, the depreciation rate,  $d_t$ , is given by

$$d_t = \frac{D_t}{V(X_{K,t-1})}.$$

Let us consider an example. Assume that the efficiency profile is of the “one-hoss shay” type, i.e., full efficiency until the time of sudden death,  $T$ . Then

$$\theta_t = \begin{cases} 0 & t < T \\ 1 & t = T \end{cases}$$

and

$$X_{Kt} = \begin{cases} X_{K0} & t < T \\ 0 & t \geq T. \end{cases}$$

Some straightforward calculations yield

$$\begin{aligned} V(X_{Kt}) &= \frac{\pi X_{K0}}{r} [1 - (1+r)^{-(T-t)}] \\ D_t &= \pi X_{K0} (1+r)^{-(T-t+1)} \\ d_t &= \frac{r(1+r)^{-(T-t+1)}}{1 - (1+r)^{-(T-t+1)}}. \end{aligned}$$

In this case, the equilibrium price of new capital is  $q_K = \frac{\pi}{r} [1 - (1+r)^{-T}]$ . Also note the distinction between the rate of reduction in technical efficiency,  $\theta_t$ , and the depreciation rate,  $d_t$ .

This stylized model is useful from a theoretical point of view, because it clarifies the relation between some main accounting concepts but provides little guidance for calculating depreciation in practice. Imperfections in, or even lack of, secondhand markets mean that physical capital may have low opportunity costs once it has been installed, making assessment of the value and cost of capital difficult from both a practical and a conceptual point of view. Transaction costs could also be very large. An example of the latter is the putty-clay model (see Johansen, 1972), where investment expenditures are considered sunk costs (once they have been undertaken). In practice, depreciation tends to be calculated on an ex ante basis, with allowance for extraordinary write-downs. That is, the purchasing cost of a capital good is distributed throughout its expected service life (ordinary write-downs), with corrections for unexpected and significant changes in value caused by unforeseen events, such as unexpected price changes, accidents, etc. (extraordinary write-downs).

### 3 Main concepts: Firm, capital, investment and depreciation

A firm is defined as “the smallest legal unit comprising all economic activities engaged in by one and the same owner” and corresponds in general to the concept of a company

(Statistics Norway, 2000). A firm may consist of one or more establishments (plants). The establishment is the geographically local unit conducting economic activity within an industry class. The firms in our sample are all joint stock companies (limited liability companies). The firms' financial accounts used here are unconsolidated accounts, which means that they do not incorporate the ownership interests in subsidiaries (see Section 5).

The term "capital" may have different meanings (see e.g., Hicks, 1974), but in this paper, we shall concentrate on capital in the sense of a durable tangible production factor. This corresponds to fixed capital in the national accounts and tangible fixed assets in the business accounts. In this sense, capital is an input in the production process, that generates operating profits. According to accounting standards, tangible fixed assets are assets that have value beyond the current year. They consist of machines, transport vehicles, buildings, etc. Intangible fixed assets such as goodwill are not considered in this paper.

We define an investment as any acquirement of a fixed capital good (new or used) that is taken into the firm's balance sheet and depreciated over its expected lifetime. Repairs are considered as operating costs, unless they bring the asset to a higher standard so that the value of the asset is increased relative to its ex ante expected value. In the latter case, the increased value is an investment (see the discussion in McGratten and Schmitz, 1999).

Sometimes the firm does not buy the asset but pays leasing costs. There are two types of leasing: financial and operational. Financial leasing means that most of the risks and rewards are transferred to the firm that leases the tangible fixed asset. In this case, the firm that leases should capitalize the asset. Hence, financial leasing is an investment.<sup>5</sup> The other form of leasing is operational. With an operational leasing agreement, the firm that leases an asset does not capitalize it in its balance sheet but pays leasing costs.<sup>6</sup> For Buildings and land, there might be uncertainty as to whether the firm that leases the

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<sup>5</sup>However, firms that are considered to be small do not have to capitalize financially leased assets. According to Norwegian accounting law, a firm is defined as small if in the last two years it fulfills at least two of the following three criteria.

- i. Revenues less than NOK 40 millions (approximately \$6 millions)
- ii. Total assets of less than NOK 20 millions
- iii. Less than 50 employees

<sup>6</sup>According to our estimates, approximately 13 percent of annual total capital costs in manufacturing are compensation to owners of leased capital.

asset will acquire the property right, because of the longsightedness involved for these kinds of assets. In such cases, the leasing agreements will often be operational, and the risk and reward will stay with the owner.

**The business vs. national accounting view of depreciation** Business (financial) accounting and national accounting differ in several ways. While financial accounting has the purpose of providing quantitative information about a business enterprise (firm), national accounting aims to give a consistent and comprehensive overview of a country's total economy. A clarification of what is meant by business accounting is necessary, because it is important to distinguish between business accounting in the company accounts and tax accounting.<sup>7</sup> In modern accounting, these two accounts are related through the deferred tax model, where the values of e.g., “accelerated tax depreciation schemes” show up as intangible assets in the financial balance sheet (see, e.g., Hawkins, 1986, p. 72). In our discussion below, we restrict the discussion to the company accounts, which is the source used in this paper.<sup>8</sup>

Business accounting is performed according to specific laws decided on by the authorities, and certain principles, or conventions, created by the accounting community. These conventions are principles for accounting practice that are commonly agreed upon. National accounting is also performed according to certain international standards, given in the SNA. ESA is the European version of this standard. It is beyond the scope of this study to discuss all aspects of the two accounting systems. Instead we will limit our discussion to issues concerning the measurement of tangible fixed assets.

In business accounting, tangible fixed assets are valued at historic acquisition prices (book values), and depreciation is defined as the allocation of the purchase cost (historic cost) of an asset between accounting periods over the expected lifetime of the asset.

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<sup>7</sup>In Norway, assets in the tax accounts are divided into eight groups according to the expected lifetimes of the assets. Seven of the categories are for tangible fixed assets, and the eighth is goodwill. The method of depreciation is declining balance depreciation (geometric depreciation). Depreciated asset values below NOK 15 000 are fully deductible from taxable profits.

<sup>8</sup>The distinction between financial and tax accounts is not well understood even by leading economists, as is vividly displayed in the OECD manual *Measuring Capital*: “Companies will often select depreciation methods that minimize their tax liabilities regardless of whether the depreciation method used ... is a good measure of economic depreciation ... Despite these problems, several countries use depreciation reported by companies in their national accounts. Such estimates cannot even be justified as crude approximations to consumption of fixed capital ... *They are misleading statistics and have no place in the accounting system*” (OECD, 2001, p. 37).

However, in certain situations historic cost valuation is not followed. The so-called conservatism convention states that an asset shall not be over-valued. This means that if the market value is below the historic cost, market value shall be used and the valuation will deviate from the historic cost principle. On the other hand, if the market value is higher than the historic cost, historic cost valuation is applied. This implies a potential asymmetry regarding the valuation of assets in business accounting. However, the use of market prices are probably exceptional and limited to assets with well-functioning secondhand markets, such as the markets for buildings and land. On the other hand, for buildings and land rising (nominal) prices are the rule, so deviations from the historic cost principle might be of little importance in practice. In national accounting tangible fixed assets are valued at market prices, using price indices for new investments to convert last years' prices into current prices. A common method of estimating the value of the capital stock is the PIM (United Nations, 2000, p. 216). Depreciation in national accounting is defined as the value, measured at market prices, of tangible fixed assets used up during the accounting period and is also referred to as consumption of tangible fixed capital.

In both business accounting and national accounting, different methods of depreciation are allowed. Norwegian firms, though, seem mainly to use straight line depreciation in their company accounts. The straight line depreciation method, also called the linear depreciation method, means that the depreciation is allocated evenly over the lifetime of the asset. In national accounting, both geometric and straight line depreciation schemes are recommended by the SNA, and which method is used differs between countries. In measuring the lifetime of the asset, companies are evaluating for how long time the asset is to be in use. For the firm, it is economic use that guides the estimation of lifetime, which may differ from the expected physical life of the asset. In national accounting, several sources are used to decide the lifetimes of tangible fixed assets. Some of the sources used are tax lives, surveys and OECD estimates, see United Nations (2000, Appendix 3).

There is some literature concerning the depreciation patterns of individual assets (see Hulten and Wykoff, 1981a and 1981b; Jorgenson, 1996). When it comes to assessing the "true" nature of depreciation, this literature is inconclusive. Data based on transactions of used capital goods can only give a crude indication about depreciation patterns. This is partly because of imperfections in, or even absence of, secondhand markets for many

goods, and partly because of self-selection mechanisms that determine which items are sold and which are not (see OECD, 2001).

The historic cost principle is perhaps the most striking difference between business accounting and national accounting. Our method of converting book values into current prices will, in principle, lead to the same kind of valuation as in national accounting, which is a measure of the wealth value of the net capital stock. Thus our data set on firm-level net capital stocks can be used to obtain estimates of the *total* stock of tangible fixed assets in the manufacturing sector by summing over the capital stocks of individual firms. In practice, there are still differences in the way assets are measured because of differing depreciation methods and depreciation rates, so one should not in general expect that our method will give the same estimate of the net capital stock as in national accounting. Differing estimates of lifetimes and different depreciation methods will contribute to differences in the valuation of tangible fixed assets. Another factor that can cause differences between estimates from our method and national accounts estimates of the net capital stock is the effect of entry and exit of firms, which is an issue we will return to in Section 6.

## 4 Methods

As long as a single capital good acquired at a particular point in time is considered in isolation, it is possible to convert book values into net capital stocks. This is equivalent to the familiar problem of converting fixed prices into current prices. However, in practice, the situation is complicated by the fact that even narrowly defined capital categories consist of different vintages. As pointed out in Diewert (1980), the situation becomes even more unclear when  $n$  non homogeneous types of goods  $j = 1, \dots, n$  with different, and possibly time-dependent, depreciation rates  $d_{jt}$ , are lumped together into one asset category. We will now study these issues more formally.

Let  $K_{s|t}$  denote the net capital stock at the end of year  $s$  measured in year  $t$  prices, i.e., with year  $t$  as the base year. In particular,  $K_{t|t}$  is the capital measured in current prices. Then  $K_{t|t} = \sum_{j=1}^n K_{j,t|t}$ , where  $K_{j,t|t}$  is the current value of good  $j$ , total investment is

$I_t = \sum_{j=1}^n I_{jt}$ , while total depreciation, measured in current prices, is

$$\begin{aligned} D_{t|t} &= \sum_{j=1}^n D_{j,t|t} \\ &= \sum_{j=1}^n d_{jt}(K_{j,t-1|t} + I_{jt}). \end{aligned}$$

We define the *aggregate* depreciation rate,  $d_t$ , as

$$d_t = \frac{D_{t|t}}{K_{t-1|t} + I_t},$$

then  $d_t$  is a weighted average of the individual depreciation rates,  $d_{jt}$ ,

$$d_t = \sum_j w_{jt} d_{jt}, \quad \text{with } w_{jt} = \frac{K_{j,t-1|t} + I_{jt}}{K_{t-1|t} + I_t}.$$

Hence, depreciation will be time dependent even in the case of geometric depreciation ( $d_{jt} \equiv d_j$ ) for each individual capital good.

The weight,  $w_{jt}$ , given to the individual depreciation rate,  $d_{jt}$ , cannot be determined *ex ante*. We believe that depreciation is best accounted for at the micro level, for each individual asset. Hence, we must rely on the depreciation patterns designated by the firms. In this way, changes in the aggregate depreciation rates because of composition effects, extraordinary write-downs, etc. will automatically be accounted for. While the historic cost principle is often used as an argument for disregarding account statistics altogether for the purpose of capital measurement, we shall show next that this view is too pessimistic.

**A method for converting book values into current values** Obviously, for investments in new goods, book values and current values coincide. Furthermore, for the same capital good,  $j$ , acquired at a given point in time,  $t$ , the initial investment,  $I_{jt}$ , as well as all subsequent write-downs are measured on the same scale: the purchasing price,  $q_t$ . Hence, book values do say something about real depreciation when a unique capital good is considered in isolation. We will show that this conclusion can be generalized to nonhomogeneous asset categories under reasonable assumptions.

Let  $K_t$  and  $D_t$  denote the book value of the capital stock at the *end* of year  $t$  and the book value of the depreciation in year  $t$ , respectively, i.e., both are measured using

historical prices. Assume that a firm makes an investment  $I_1$  at the beginning of year 1. In this simplified model, we assume that there is only one capital good, and that no further investments take place. During period 1, the following occurs: A share  $d_1$  of the initial investment, less sale, is written down because of expected depreciation, and the book value of the depreciation is

$$D_1 = d_1(I_1 - s_1 I_1),$$

where  $s_1$  is the share of the capital good that is sold. The book value of the sale is<sup>9</sup>

$$S_1 = s_1 I_1.$$

The book value,  $K_1$ , at the end of year 1 is therefore

$$K_1 = I_1 - (D_1 + S_1) = (1 - \delta_1)I_1,$$

where  $\delta_1 = d_1 + s_1 - d_1 s_1$  is the *reduction rate* in year 1.

By recursions, we have for  $t > 1$

$$\begin{aligned} \delta_t &= \frac{D_t + S_t}{K_{t-1}} \\ K_t &= (1 - \delta_t)K_{t-1}. \end{aligned}$$

The reduction rate  $\delta_t$  does not depend on prices even if it is calculated from book values. The reason is that all book values are evaluated at the same price  $q_1$ , i.e., the purchase price. Note that the reduction rate will differ from the depreciation rate when capital goods are sold.

We now consider how the book values  $K_t$  can be converted into current prices. If

$$\rho_t = (q_t - q_{t-1})/q_{t-1} \tag{2}$$

is the relative change in the price index,  $q_t$ , then

$$K_{s|t} = K_{s|s} \prod_{u=s+1}^t (1 + \rho_u) = K_{s|s} \frac{q_t}{q_s}, \text{ for } s < t.$$

Clearly, we have

$$K_{1|1} = K_1 = (1 - \delta_1)I_1.$$

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<sup>9</sup>According to accounting principles, there is no depreciation of capital goods that are sold during the year.

Repeating the same reasoning for period  $t = 2$  and beyond, we obtain

$$\begin{aligned} K_t &= (1 - \delta_t)K_{t-1} \\ K_{t|t} &= (1 - \delta_t)(1 + \rho_t)K_{t-1|t-1} \quad \text{for } t = 2, 3, \dots \end{aligned}$$

If we define  $K_0 = K_{0|0} = 0$ , we have the general formula

$$\begin{aligned} K_t &= (1 - \delta_t)(K_{t-1} + I_t) \\ K_{t|t} &= (1 - \delta_t) \left( (1 + \rho_t)K_{t-1|t-1} + I_t \right) \\ &= (1 - \delta_t) (K_{t-1|t} + I_t) \quad \text{for } t = 1, 2, 3, \dots \end{aligned} \tag{3}$$

(recall that  $I_t = 0$  for  $t > 1$ ). The importance of these equations lies in the fact that the reduction rate  $\delta_t$  can be calculated from book values.

In the above model, an investment is made once, and only *reductions* in capital take place thereafter. These reductions are registered in the accounts using the purchase price. This is, therefore, not a realistic model for a firm, but only for a particular capital good. Hence the same type of capital good acquired at another point in time must be treated as a different good, because the purchasing price may be different.

To elaborate the model, we partition the stock of capital of a particular category into  $j = 1, \dots, N$  different capital goods. Unit  $j$  is defined by an investment in a specific type of capital made in one particular year,  $t_j$ . We assume that the same price index, with relative change  $\rho_t$ , applies to all  $N$  goods within the category. If the development in the price index for some good is different for other goods in the same category, this may cause an aggregation bias. This is further explored in the appendix.

The total book value of the firm's capital goods at the end of year  $t$  is  $K_t \equiv \sum_{j=1}^N K_{jt}$ , where  $K_{jt}$  is the book value of capital good  $j$ . Similarly, the firm's total capital stock in year  $s$ , measured in the prices of year  $t$ , is  $K_{s|t} \equiv \sum_{j=1}^N K_{j,s|t}$ . Hence, using (3), we obtain

$$\begin{aligned} \sum_{j=1}^N K_{jt} &= \sum_{j=1}^N (K_{j,t-1} + I_{jt}) - \sum_{j=1}^N \delta_{jt} (K_{j,t-1} + I_{jt}) \\ \sum_{j=1}^N K_{j,t|t} &= \sum_{j=1}^N (K_{j,t-1|t} + I_{jt}) - \sum_{j=1}^N \delta_{jt} (K_{j,t-1|t} + I_{jt}). \end{aligned}$$

The aggregate investment is  $I_t \equiv \sum_j I_{jt}$  (where  $I_{jt} = 0$  when  $t \neq t_j$ ). Hence, we can write

$$K_t = (1 - \delta_t)(K_{t-1} + I_t), \text{ where } \delta_t = \sum_{j=1}^N w_{jt} \delta_{jt} \text{ and } w_{jt} = \frac{K_{j,t-1} + I_{jt}}{K_{t-1} + I_t}$$

$$K_{t|t} = (1 - \delta_t^0)(K_{t-1|t} + I_t), \text{ where } \delta_t^0 = \sum_{j=1}^N w_{jt}^0 \delta_{jt} \text{ and } w_{jt}^0 = \frac{K_{j,t-1|t} + I_{jt}}{K_{t-1|t} + I_t}. \quad (4)$$

There is a difference between the exact aggregate reduction rate  $\delta_t^0$  (using the relative current values of the different capital goods,  $w_{jt}^0$ , as weights) and the rate  $\delta_t$  (using relative book values,  $w_{jt}$ , as weights). We can consider  $\delta_t$  as an approximation (or estimate) of  $\delta_t^0$ . This approximation will be good in two circumstances: (i) when all the  $\delta_{jt}$  are of similar magnitude, i.e., the asset categories consist of capital goods with similar life times, or (ii) when  $\delta_{jt}$  is independent of  $w_{jt}$  and  $w_{jt}^0$ . In the latter case, both  $\delta_t^0 \xrightarrow{P} \delta_t^*$  and  $\delta_t \xrightarrow{P} \delta_t^*$  when  $N$  becomes large, assuming that  $\delta_{jt} \sim i.i.d(\delta_t^*, \sigma^2)$ .

The reduction rate,  $\delta_t$ , should not be confused with a depreciation rate. However, because sales of used capital goods are relatively rare for firms that do not close down production units, then in most situations  $\delta_{jt} = d_{jt}$ . Hence, the *median* (but not necessarily the *average*) reduction rate among all firms in a given year, at least when excluding firms that report sales of capital in that year, is a useful location parameter for the distribution of the depreciation rates.

**The initial value problem** Our method for calculating net capital stocks does not address the initial value problem for firms born *before* the start of the sample period: The problem is to obtain  $K_{0|0}^i$ , the value of the capital stock of firm  $i$  in the first observation year,  $t = 0$ , measured in current prices. The problem is potentially most severe for old firms, that may have a large share of old capital. Hence the book value  $K_0$  may be a poor measure of the initial current value for these firms. We will here consider a method of correcting the initial book value observation,  $K_0^i$ , to obtain a better estimate of  $K_{0|0}^i$ . Our updating formula will have the form

$$K_{0|0}^i = \theta_c K_0^i,$$

where  $c$  is the cohort of firm  $i$  and  $\theta_c$  is the correction factor specific to cohort  $c$ . Cohort  $c$  is defined as consisting of all firms that are  $c$  years old in  $t = 0$  (i.e., they are born in

$t = -c$ ). The idea is to calculate the factors  $\theta_c$  by considering a “representative” (average) firm from each cohort. Making the correction factor cohort-specific, requires that we have data on the birth dates of each firm, enabling us to stratify firms into cohorts. The cohort-specific correction factors take into account the fact that the age distribution of capital in  $t = 0$  is different for different cohorts.

Obviously,  $\theta_c = 1$  for  $c = 0$ , so that there are no corrections of the initial book value for firms born in the first observation year (or, generally, for firms born within the observation period). To obtain an expression for  $\theta_c$  for  $c > 0$ , we first consider the bookkeeping relation

$$K_t^i = (1 - \delta_t^i) (K_{t-1}^i + I_t^i). \quad (5)$$

Assuming that  $\delta_t^i$  is uncorrelated with  $I_t^i$  and  $K_{t-1}^i$ , which is reasonable, because larger firms should not have systematically higher or lower reduction rates than smaller firms, we obtain for a representative firm from cohort  $c$

$$K_t^c = (1 - \delta)(K_{t-1}^c + I_t^c), \quad t = 0, -1, \dots, -c,$$

where the superscript  $c$  denotes the expected value of the corresponding variable taken over the cohort, and  $\delta$  is the average reduction rate in the population, assumed time-invariant for  $t \leq 0$ . Next, assume that investments “backwards in time” can be expressed on the form

$$I_t^c = \lambda_t I_0^c, \quad \lambda_0 = 1 \text{ for } t = 0, -1, -2, \dots,$$

where  $I_t^c$  is the average investment in year  $t$  ( $t \leq 0$ ) for cohort  $c$ , measured in base year ( $t = 0$ ) prices. Moreover,  $\lambda_t$  is the investment in year  $t$  relative to the investment in  $t = 0$  for a representative firm operative both in year  $t$  and in year 0. Let  $\pi_t = q_t/q_0$ , i.e., the price index of capital with  $t = 0$  as the base year. Given the  $\lambda_t$ 's, we recursively obtain

$$\begin{aligned} K_0^c &= (1 - \delta) (K_{-1}^c + \lambda_0 I_0^c) \\ &= (1 - \delta) ((1 - \delta)(K_{-2}^c + \lambda_{-1} \pi_{-1} I_0^c) + I_0^c) \\ &\quad \vdots \\ &= (1 - \delta) I_0^c [1 + (1 - \delta) \lambda_{-1} \pi_{-1} + \dots + (1 - \delta)^c \pi_{-c} \lambda_{-c}], \end{aligned}$$

where we have imposed the initial condition  $K_{-c-1}^c = 0$ . Furthermore, an analytic expres-

sion for  $K_{0|0}^c$  can be obtained by accumulating investments,  $I_t^c$ , in the usual way:

$$\begin{aligned} K_{0|0}^c &= (1 - \delta) (K_{-1|0}^c + \lambda_0 I_0^c) \\ &= (1 - \delta) I_0^c (1 + (1 - \delta)\lambda_{-1} + \dots + (1 - \delta)^c \lambda_{-c}). \end{aligned}$$

It follows that

$$\theta_c = \frac{(1 + (1 - \delta)\lambda_{-1} + \dots + (1 - \delta)^c \lambda_{-c})}{[1 + (1 - \delta)\lambda_{-1}\pi_{-1} + \dots + (1 - \delta)^c \lambda_{-c}\pi_{-c}]} \quad (6)$$

The practical implementation of this method requires that we (i) know the birth year of each firm, (ii) have a price index of capital  $\pi_t$ , and (iii) can calculate (or estimate) the relative investment rates  $\lambda_t$  (e.g., from aggregate data). We present an application of equation (6) in Section 6. Note that in the special case where  $\lambda_{-(s+1)} = (1 + \nu)^{-1}\lambda_{-s}$  and  $\pi_{-(s+1)} = (1 + \rho)^{-1}\pi_{-s}$ , we obtain

$$\theta_c \simeq \left(1 + \frac{\rho}{\delta + \nu}\right) \frac{1 - (1 - \delta - \nu)^{c+1}}{1 - (1 - \delta - \nu - \rho)^{c+1}}, \quad (7)$$

by using the approximation  $(1 - \delta)/(1 + \nu) \simeq 1 - \delta - \nu$  and the formula for finite geometric series. Thus,  $\theta_c$  goes asymptotically towards  $1 + \rho/(\delta + \nu)$  when  $c \rightarrow \infty$ .

## 5 Data and implementations

We use data from two main sources: (i) Accounts statistics for all Norwegian joint stock companies (see Statistics Norway, 2000), and (ii) structural statistics for the manufacturing sector (see Statistics Norway, 1999).<sup>10</sup> Both statistics cover the period 1993–2004. In addition, we have access to an almost complete set of annual reports for Norwegian joint stock companies for the year 2001. The latter data set is time consuming to review, because the annual reports do not have a standardized form but must be read manually from picture files. Nevertheless, annual reports are valuable sources of information about the quality of the ordinary data sources (i) and (ii). Annual reports also provide insights into accounting practices and enable us to evaluate methods for adjusting data when the investment figures in the manufacturing statistics are incompatible with information from the accounts statistics.

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<sup>10</sup>Structural statistics are also available for service industries. For construction, wholesale and retail trade and other services data, are available since 1995, and for transport and communication, hotel and restaurant, travelling and ICT data, are available since 1997.

All joint stock companies in Norway are obliged to publish a company account every year. An important distinction is between consolidated and unconsolidated financial accounts. Firms with subsidiaries, which is typical for publicly traded companies, must in addition to the (unconsolidated) account of the parent company also provide a consolidated account that treats the parent and the subsidiaries as one economic unit, i.e., one group (see Hawkins, 1986, p. 96). A group consists of legally separate units (firms) with their own unconsolidated financial statements. The Norwegian data are unconsolidated data, i.e., they are at the firm-level, not the group level.<sup>11</sup>

The accounts statistics contain data from both the income statement and the balance sheet. In particular, the accounts statistics have information about the book value of a firm's tangible fixed assets at the end of the year. The accounts statistics also have data on ordinary depreciations and write-downs. However, there are no separate data on depreciation and write-downs for tangible fixed assets. Another shortcoming of the accounts statistics is that they do not contain data on acquisitions of tangible fixed assets. The reason is that data for investments do not have a specific standard in the annual report but are given in the notes to the annual report in a format arbitrarily chosen by the firm.

The structural statistics for the manufacturing sector do, however, contain data about acquisitions of tangible fixed assets at the establishment level. The manufacturing statistics also contain information about financial leasing. Firm-level data are obtained from the manufacturing statistics by summing over all establishments within the firm. These data are matched with the data from the accounts statistics.

Both the accounts statistics and the manufacturing statistics distinguish between several groups of assets. However, to obtain consistent definitions of asset categories for the two statistic sources and over the whole observation period, we have chosen to distinguish between two classes of assets: (i) Buildings and land; and (ii) Other tangible fixed assets. The latter group consists of machinery, computers, equipment, vehicles, movables, furniture, tools, ships, rigs and aircraft, and is, hence, quite heterogeneous. However, the expected lifetimes of the assets in the first group are considerably longer than those in the second, and the between-group variation in lifetimes is much larger than the within group

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<sup>11</sup>Capital stock information is only available at the firm level. For multiplant firms, capital stock values may be allocated to the plants by using measures of, for example, employment and/or investments. This method is used by Harris and Drinkwater (2000).

variation. Averaging over all years, the median reduction rate among assets is about 5.5 percent in group (i) and about 25 percent in group (ii).

The accounts statistics are of good quality, as they contain the audited accounting figures of the firms. In a sample of about 120 annual reports, we rarely found discrepancies between the book values reported in the accounts statistics and in the annual reports. The manufacturing statistics also should be of good quality, especially for larger firms (i.e., firms with at least 10 employees), because these figures are obtained electronically from tax return forms and are later also revised by Statistics Norway.<sup>12</sup>

Denote by  $I_t^i$  and  $J_t^i$  acquisitions of tangible fixed assets (new and used) and gross investments, respectively, for firm  $i$  in year  $t$  obtained from the manufacturing statistics. Gross investments are defined as acquisitions less sales of tangible fixed assets. Furthermore, let  $K_t^i$  and  $\delta_t^i$  denote, respectively, the book value obtained from the accounts statistics and the reduction rate defined in (4) for firm  $i$  at time  $t$ . A reduction rate will always refer to one of the two categories of capital (although we suppress the capital type index in the notation, for simplicity). Because the sum of depreciations and sales cannot be negative, the lower limit on the reduction rate is  $\delta_t^i = 0$ . The upper limit is  $\delta_t^i = 1$ , which is obtained when all the firm's tangible fixed assets are depreciated or sold.

Our basic equation for estimating  $\delta_t^i$ , based on (4), is the bookkeeping relation

$$\begin{aligned} K_t^i &= (1 - \widehat{\delta}_t^i)(K_{t-1}^i + I_t^i) \\ &\Downarrow \\ \widehat{\delta}_t^i &= 1 - \frac{K_t^i}{K_{t-1}^i + I_t^i}, \end{aligned} \tag{8}$$

where we use the “hat” notation to distinguish between the “true” reduction rate and the estimated reduction rate that may be contaminated by measurement errors in the data for  $K_t^i$  and  $I_t^i$ .

From our investigation of the sample of annual reports, it seems that there are three main reasons for errors in the calculated reduction rates using (8): (i) A failure on the part of the firm to report all investments to Statistics Norway, (ii) mergers and acquisitions, and (iii) time inconsistencies in the firms' classification of their tangible fixed assets. The first type of error is by far the most common. Although quite rare, the other two of these

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<sup>12</sup>The data are mainly of good quality, but there are some problems that we will discuss later in this section.

possible sources of errors deserve special attention.

First, in the annual report, a merger or an acquisition is indicated by a revision of the tangible fixed assets at the end of the previous year to make these figures comparable with the figures at the end of the current year. In the accounts statistics, however, there is no direct information about the capital obtained through mergers or acquisitions. Because takeovers from mergers and acquisitions are not regarded as investments in the manufacturing statistics,  $\widehat{\delta}_t^i$  may even be negative: A merger is counted as a “negative reduction”. However, our method of estimating capital requires that a merger be specifically identified as an acquisition, because all means of acquiring capital, regardless of whether this is new capital or merely a change in ownership of old capital, is capitalized in the balance sheet.

Second, tangible fixed assets are divided into several categories in the balance sheet. However, sometimes a firm may not be time consistent in its classification of an asset, and the category of the asset may suddenly change. This typically leads to a negatively calculated reduction rate for the category that “gains” an asset, and a very high reduction rate in the category that “loses” the asset. Fortunately, such reclassifications are rare but may lead to large errors when they occur.

To address the problem that  $\widehat{\delta}_t^i$  may be negative, we will now consider a two-step estimator,  $\widehat{\delta}_t^i\text{-adj}$ . Let  $\delta_t^{\text{med}}$  denote the median estimate of the reduction rate in year  $t$  (for that asset category). Then  $\widehat{\delta}_t^i\text{-adj}$  is defined by the following two steps:

$$\begin{aligned} \text{step 1: } & \text{if } \widehat{\delta}_t^i \geq 0, \text{ set } \widehat{\delta}_t^i\text{-adj} = \widehat{\delta}_t^i \\ \text{step 2: } & \text{if } \widehat{\delta}_t^i < 0, \text{ set } \widehat{\delta}_t^i\text{-adj} = \delta_t^{\text{med}} \quad \text{and set } I_t^{i*} = \frac{K_t^i}{(1 - \delta_t^{\text{med}})} - K_{t-1}^i. \end{aligned}$$

In step 1, if  $\widehat{\delta}_t^i$  is non-negative, we make no corrections:  $\widehat{\delta}_t^i\text{-adj} = \widehat{\delta}_t^i$ . In step 2, if the calculated  $\delta_t^i$  is negative, whatever the reason, we set  $\widehat{\delta}_t^i = \delta_t^{\text{med}}$  and calculate the corresponding acquisition level,  $I_t^{i*}$ , that is consistent with  $K_t^i$ ,  $K_{t-1}^i$  and  $\delta_t^{\text{med}}$ . That is, we calculate an *imputed* acquisition,  $I_t^{i*}$ , by solving

$$K_t^i = (1 - \delta_t^{\text{med}})(K_{t-1}^i + I_t^{i*}). \quad (9)$$

To evaluate the two estimators,  $\widehat{\delta}_t^i$  and  $\widehat{\delta}_t^i\text{-adj}$ , we calculated their mean absolute error (MAE) and median absolute error (MdAE) in a sample of approximately 120 firms for which the correct reduction rates,  $\delta_t^i$ , could be derived from information in the annual

reports (available as picture files). For each of the two types of capital, the sample of annual reports was stratified into two groups of firms: i) firms with  $\widehat{\delta}_t^i \geq 0$  and ii) firms with  $\widehat{\delta}_t^i < 0$ . The results are given in Table 1 for Buildings and land and in Table 2 for Other tangible fixed assets. The MAE and MdAE were calculated for the two estimators,  $\widehat{\delta}_t^i$  and  $\widehat{\delta}_t^i\text{-adj}$ , in both groups of firms. Furthermore, weighted averages for both MAE and MdAE over the two groups of firms were computed using the share of tangible fixed assets in the population (*not* in the sample) as weights.

For both categories of capital, firms with  $\widehat{\delta}_t^i > 0$  make up about 70 percent of the total capital stock in the manufacturing sector and have a MdAE of zero. Hence, it seems that the overall quality of the data is quite good. In the group of firms with a negatively calculated reduction rate ( $\widehat{\delta}_t^i < 0$ ), both the MAE and MdAE of the errors are reduced quite dramatically when using  $\widehat{\delta}_t^i\text{-adj}$ . So, this way of correcting the reduction rates seems to be promising. Large firms are hugely overrepresented in the category with negative  $\widehat{\delta}_t^i$ . This suggests that a negative reduction rate could correspond to a systematic failure of these firms to report all of their investments. The problems with mergers and acquisitions discussed above is also mainly confined to very large firms, although we found no such cases in our random samples, so this does not explain the results.

## 6 Applications

The main output of our methodology is a panel data set of capital stock estimates covering the years 1993–2004 for all Norwegian joint stock companies in the manufacturing sector. We use this data set to obtain estimates of the *total* stock of tangible fixed assets in the manufacturing sector<sup>13</sup>. In this section, we apply our method to achieve two objectives. First, we obtain net capital stock estimates at the aggregate sector level by summing over the individual firms. Second, we compare our estimates with estimates obtained using PIM on our data.

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<sup>13</sup>In principle, other levels of aggregation are also possible, although, at a more disaggregate level, some of the problems we discuss at the beginning of Section 6.2 may be enlarged.

## 6.1 Initial value corrections: the calculation of $\theta_c$

Our method for calculating net capital stocks was addressed in Section 4. We also presented a method for adjusting the initial book value at the start of the observation period (in 1993), using cohort-specific correction factors  $\theta_c$ . This method requires data on the parameters  $\lambda_t$ , expressing the relative expected acquisitions in year  $1993 - s$  ( $s = 1, 2, \dots$ ), relative to 1993, for a firm operating in both years, and price indices of capital from 1993 and backwards. The median age of firms operative in 1993 was 20 years. To calculate  $\lambda_{1993-s}$ , for  $s = 1, \dots, 15$ , i.e., back to 1978, we used micro data on investments. We applied the formula

$$\lambda_{1993-s} = \frac{1}{\#c : c \geq s} \sum_{c:c \geq s} \frac{I_{1993-s}^c}{I_{1993}^c}, \quad s = 1, \dots, 15,$$

where  $I_{1993-s}^c$  denotes average acquisitions in cohort  $c$  in year  $1993 - s$ , among all firms in that cohort that were also operating in 1993. Thus, for each cohort established in  $1993 - s$ , or earlier, we calculated total investments (in fixed prices) in year  $1993 - s$ , relative to 1993. Then we took the arithmetic mean of these ratios over all the cohorts. For firms born before 1978, we used national accounts data on investments of tangible fixed assets in the period 1950–1978 to impute a common “historical” growth rate of investments,  $\nu$ , assuming that

$$\lambda_{1993-(s+1)} = \frac{1}{1 + \nu} \lambda_{1993-s}, \quad \text{for } s > 15$$

(cf. the discussion preceeding (7)). Clearly, our estimates of the “historical”  $\lambda_t$ , i.e., earlier than 1978, are uncertain and is a source of error. The historical data contain no cohort information but do contain aggregate investments of all the firms that were operative in a given year. Neither do they distinguish between different types of capital, as we do. Nevertheless, we estimated  $\nu$  to 4 percent.<sup>14</sup> Figure 1 shows that the correction factor  $\theta_c$  for Buildings rises towards 1.3 asymptotically and reaches this level for firms that were about 50 years old in 1993, while for Other tangible fixed assets, the asymptote is at just 1.05, which is reached for cohorts of firms that are 15 years or older in 1993. Thus we can say that the correction factor for Buildings and land lies between 1 and 1.3, and between 1 and 1.05 for Other tangible fixed assets. Because a large share of the capital belongs to quite old firms, we expect the effect of the initial value correction to be sizable for

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<sup>14</sup>Gross investment figures in the period 1950–1978 were collected from [http://www.ssb.ni/emner/historisk\\_statistikk/tabeller/16-16-1t.txt](http://www.ssb.ni/emner/historisk_statistikk/tabeller/16-16-1t.txt).

Buildings and land but rather small for Other tangible fixed assets.

## 6.2 Net capital stocks

Our data consist of the manufacturing joint stock companies. Firms with most of their activities in other sectors but with some activities in the manufacturing sector will be excluded. On the other hand, we include all the tangible fixed assets of firms with most of their activities in the manufacturing sector but with some activity in other sectors. Our data show that firms classified as manufacturing firms have almost negligible production outside manufacturing.<sup>15</sup> To estimate the net capital stock for the total manufacturing sector, we inflate the sample totals with appropriate inverse annual weights. Each weight is the estimated share of the sample total (i.e., the sum over all *joint stock companies* within manufacturing) relative to the sector total (i.e., the sum over all *establishments* in manufacturing). We use weights calculated as moving averages (over time) of the joint stock companies' share of the total sector, measured as the average of the share of total employment and their share of total value added (the difference between the shares of employment and value added is only 1–2 percentage points each year). These weights increase monotonically from 87 percent in 1993 to 96 percent in 2004, reflecting increased popularity of the joint stock company ownership form.

Figure 2 shows the development in the book values of Buildings and land, together with the net capital stock of Buildings and land according to our method of price correction. Results for two versions of our method are presented: (i) partial correction, i.e., the net capital stock in current prices equals the book value in 1993, and (ii) full correction; i.e., each firm's net capital stock in 1993, in current prices, equals the book value multiplied by the cohort-specific correction factor,  $\theta_c$ . The latter graph represents our final estimate of the net capital stock in current prices. We see that the price correction has some significance. With the initial value condition  $K_{1993|1993}^i = K_{1993}^i$ , the value in current prices is about 5 percent higher than the book value in 1995, rising to 12 percent in 2004. On the other hand, when choosing  $K_{1993|1993}^i = \theta_c K_{1993}^i$ , i.e., full price correction, the relationship between the net capital stock in current prices and the book value is about 1.15 throughout the entire observation period.

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<sup>15</sup>We have estimated these firms' share of capital in establishments outside the manufacturing industry to be, on average, about 0.8 percent of their total capital over the period 1993–2004.

From Figure 3, we see that for Other tangible fixed assets, the differences between book values and values in current prices are small, regardless of the initialization method. The effect of the initial value correction is an increase in current value of 3.8 percent compared with the book values. The reason for this small adjustment is that Other tangible fixed assets have much lower expected lifetimes than Buildings and land, so the replacement of these assets is more frequent. Furthermore, prices have been quite stable for this category of capital, and even decreasing in some periods. Hence, more of the stock of Other tangible fixed assets are valued at current prices or prices close to current prices.

Figures 4 and 5 compare our calculated stocks of tangible fixed assets using the method of full price correction with the results obtained from PIM. We use 2001 as the base year, with total gross investments in the manufacturing sector for the period 1993–2004 shown on the right-hand axis. Total gross investment is obtained by summing over the joint stock companies' gross investments according to the manufacturing statistics and then applying the same inverse weights described above to obtain gross investments for the total manufacturing sector (not just for the population of joint stock companies). The PIM used here can be described as hybrid PIM, because the initial value in 1993 is not obtained by PIM but is equal to the price corrected book value in 1993.<sup>16</sup> Depreciation rates are obtained from the national accounts. As before, we calculate values for Buildings and land and Other tangible fixed assets separately.

Figure 4 displays results for Buildings and land. Despite the sharp falls in gross investments in 1994, 1999–2000 and 2003–2004, the growth rate of capital as measured by PIM is largely unaffected. On the other hand, with our method there is a noticeable drop in the net capital stock during the investment slumps. The difference between the two methods is striking. While the stock of buildings and land has increased by 40 percent during 1993–2004 according to PIM, our method shows an increase of just 8 percent.

The results for Other tangible fixed assets are depicted in Figure 5. We see the same pattern as for Buildings and land, but the two methods give more equal results in this case. The growth in Other tangible fixed assets from 1993 to 2004 is still noticeably

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<sup>16</sup>Versions of hybrid PIM, where the initial value is the book value (sometimes adjusted for inflation, in some way or another), are often encountered in microeconomic studies. For a recent example, see Bloom et al. (2007). Our results illustrate the hazards of such hybrid methods.

different with the two methods: 125 percent according to PIM and 97 percent according to our method. Again, our method shows a little more responsiveness to changes in gross investments than PIM, although both methods reveal a strong monotonic increase in the stock of Other tangible fixed assets.

A partial explanation of the discrepancy between the two methods is that most businesses use depreciation rates that are well above the aggregate depreciation rates applied in the national accounts. In Figure 6 we see that the depreciation rate for Buildings and land in the national accounts is about 4 percent, while the median reduction rate, even when excluding firms with sales of assets, is around 5.5 percent. For Other tangible fixed assets, shown in Figure 7, the difference is even more striking. The depreciation rate in the national accounts is around 12–13 percent, compared with median reduction rates calculated from firm-level data that are about twice as high. This explains the high growth rates of capital for the hybrid PIM method in figures 4 and 5. The initial investment rates are far above the replacement rates of capital at this level of initial capital. This creates a strong growth impulse, that is a mere artifact of the change of depreciation method. The actual national accounts data for manufacturing have a much higher initial value in 1993 and much lower growth rates than the results based on the hybrid PIM method in figures 4 and 5. The average annual growth rate of total capital in the national accounts, when combining Buildings and land and Other tangible assets into one category, is about 1 percent, hybrid PIM gives an average annual growth rate of 7.4 percent, while our method (with full price correction) gives an average annual growth rate in the period 1993–2004 of 5.5 percent.

### **6.3 The impact of exit and entry**

Another important difference between our method and PIM is that PIM makes no corrections for firm exits, while our method only includes capital stocks of operative firms. Figure 8 illustrates the importance of exit and entry. The graph for exit capital reports the “remaining” capital in the exiting firms: i.e., the capital stock at the end of their last year. The exit therefore represents a negative investment (disinvestment) at the firm-level but is not reported as such. Similarly, the graph for entry capital contains the capital at the end of the first year of a new firm less reported gross investments during that year. That

is, entry capital is the unaccounted starting capital of an entering firm, and is defined as the amount of capital at the end of its first year less the reported investment during that year. Entry and exit are here defined as entry and exit from the population of all manufacturing firms, not simply entry or exit from our sample consisting of the joint stock companies. Because our sample is a subpopulation of the total manufacturing sector, we have inflated the aggregate data from our sample with inverse weights. The weights were calculated in a way similar to the weights used to estimate net capital stocks, described above. They are annual and are different for exit and entry.<sup>17</sup> To produce an estimate of total entry and exit capital, respectively, the aggregate data obtained from our sample, by summing over the individual stock companies, were inflated with the corresponding inverse weight. The average (over time) of the weights for exit is 0.70, while the corresponding average weight for entry is 0.84. Thus, relative to the whole population, the joint stock companies comprise a higher share of entries than of exits.

The PIM method implicitly assumes that capital equipment in firms that have closed down remains operative within the sector, either in an existing firm or in a new one. To examine this issue, a graph that measures the net effect of entry and exit is also shown in Figure 8. This is a measure of unreported gross investments because of entry and exit: entry capital less exit capital. To reduce the problem that capital may flow from an exiting firm to an entering firm with a time lag, e.g., firm A exits in year  $t - 1$ , while a new firm, B, enters the data set in  $t + 1$  with the same capital as A, we have smoothed the data by calculating moving averages over time, so that the figures for year  $t$  shown in the graphs are weighted averages of the  $t - 1$ ,  $t$  and  $t + 1$  data, with weights  $1/4$ ,  $1/2$  and  $1/4$ , respectively. All graphs are measured as the percentage of total (reported) investments in the corresponding year. Figure 8 confirms the finding of Harris and Drinkwater (2000). The net effect is large (and negative) in the period with many firm exits, which is the case at the beginning of our observation period. For example, the negative gross investments because of entry and exit constituted 20 percent of total (reported) investments in 1993.

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<sup>17</sup>The weight for entry is the average of the following two ratios (in a given year): (i) total employment in entering joint stock companies relative to total employment in all entering manufacturing firms, and (ii) the same as (i) but with value added instead of employment. Only real entries are counted as entries, e.g., excluding cases where a firm, previously not a joint stock company, becomes one and hence is a new firm in our sample, but not a new manufacturing firm. The weights for exit are defined as the weights for entry, with the obvious difference that we use exiting instead of entering firms. Again, only real exits are counted, and not, for example, cases when firms leave our sample but remain operative as non-joint-stock companies.

Later the net effect went from negative to positive. Entry capital is larger than exit capital, stabilizing at around 15 percent from 1999 onwards. Thus, the net effect of exit and entry is not only procyclical, as we would expect, but also of a very sizable magnitude regardless of cyclical variations, and thus potentially an important source of long-run distortions to the PIM method.

## 7 Conclusions

In this paper, we have proposed and explored a new method for estimating net capital stocks at the firm-level, which is based on financial accounts data for the manufacturing sector. The method converts historical acquisition prices into current prices by combining time series of book values with investment data for each firm. The book values are adjusted using price indices of new capital goods. The main output of the method is a panel database containing estimates of tangible fixed assets evaluated at both current and constant prices at the firm-level. The database can easily be updated each year as new data arrive, and it has many potential applications in the study of production and productivity at the micro, industry and macro levels.

In an application, we have compared capital stock estimates for the aggregate Norwegian manufacturing sector based on our method with a hybrid perpetual inventory method (hybrid PIM), where the initial value in 1993 is equal to the price corrected book value. We also compare our results with the official national accounts figures obtained by PIM. The average annual growth rate of total capital in the manufacturing sector is about 1 percent in the period 1993–2004 according to the national accounts, the hybrid PIM gives 7.4 percent, while our method gives an average annual growth rate of 5.5 percent. There are several reasons for discrepancies between the methods. First, the lifetimes of the assets assumed in the national accounts, especially regarding Other tangible assets, are generally much higher than in business accounting. Second, while exit of capital due to plant closures in existing firms are accounted for by our reduction rates, it is neglected by PIM. Third, our results show that entry and exit of firms lead to a procyclical gross investment pattern not captured by PIM (cf. Figure 8). We do not take a position with regard to what lifetime assumptions are the most reasonable; those of the national accounts or the business accounts. Nevertheless, our approach for calculating net capital

stocks suggests some possibilities for improving PIM. First, it is a fact that – at least in Norway – systematic surveys that collect prices in second-hand-markets or interview firms about actual depreciation rates are not carried out. By conducting such surveys on a regular basis, adequate and timely information about lifetimes for different categories of assets would become available and improved estimates of depreciation rates might be obtained. Moreover, our analysis shows that volume changes due to changes in the population of operating firms are important sources of variations in the net capital stock. On the other hand, PIM combined with low depreciation rates yields a smooth growth pattern of capital, which is insensitive to the fluctuations in investments during the business cycle. By taking the effects of entry and exit of firms (and plants) explicitly into account as "other volume changes", PIM would be improved.

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## Appendix: Aggregation of capital assets with differing price indices

Assume that a firm invests in two capital goods in period 0 and that there are no more investments in later periods. At the end of period 0 the net capital stock (and the book value) is

$$q_{1,0}X_{K_1} + q_{2,0}X_{K_2},$$

where  $q_{1,0}$  and  $q_{2,0}$  are the purchase prices of good 1 and 2, respectively (in period 0), and  $X_{K_1}$  and  $X_{K_2}$  are the quantities of the capital goods at the end of the period.<sup>18</sup> The book value of the capital stock at the end of period 1, when the two capital goods are depreciated with the rates  $d_{1,1}$  and  $d_{2,1}$ , respectively, is

$$(1 - d_{1,1})q_{1,0}X_{K_1} + (1 - d_{2,1})q_{2,0}X_{K_2}.$$

On the other hand, the value in current prices is

$$(1 - d_{1,1})q_{1,1}X_{K_1} + (1 - d_{2,1})q_{2,1}X_{K_2}.$$

Assuming that there are no sales, the depreciation rate and the reduction rate coincide. Then, from (4), the aggregate depreciation rate in period 1 based on book values is

$$d_1 = \left( \frac{q_{1,0}X_{K_1}}{q_{1,0}X_{K_1} + q_{2,0}X_{K_2}} \right) d_{1,1} + \left( \frac{q_{2,0}X_{K_2}}{q_{1,0}X_{K_1} + q_{2,0}X_{K_2}} \right) d_{2,1}. \quad (\text{A1})$$

Using current prices instead, the aggregate depreciation rate becomes

$$d_1^0 = \left( \frac{q_{1,1}X_{K_1}}{q_{1,1}X_{K_1} + q_{2,1}X_{K_2}} \right) d_{1,1} + \left( \frac{q_{2,1}X_{K_2}}{q_{1,1}X_{K_1} + q_{2,1}X_{K_2}} \right) d_{2,1}. \quad (\text{A2})$$

Thus, if the price development of the two goods is equal, i.e.,  $q_{2,0}/q_{1,0} = q_{2,1}/q_{1,1}$ , both book values and current values will give the same aggregate depreciation rate. However, they will deviate if the price development differs between the two goods. Let us assume that  $q_{1,0} = q_{1,1} = q_{2,0}$ , and that  $q_{2,1} < q_{2,0}$ . That is, the two price indices are equal in period 0, and there is no price change for good 1. On the other hand, the price of good 2 decreases from period 0 to period 1. Given these assumptions we have three main cases depending on the relative size of the depreciation rates for the two goods.

$$\begin{aligned} d_{2,1} > d_{1,1} &\Rightarrow d_1 > d_1^0 \\ d_{2,1} = d_{1,1} &\Rightarrow d_1 = d_1^0 \\ d_{2,1} < d_{1,1} &\Rightarrow d_1 < d_1^0 \end{aligned}$$

For example, if good 2 is computers and good 1 is other machinery, it may be reasonable to assume that computers have the lowest expected lifetime, so that  $d_{2,1} > d_{1,1}$ . Then the

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<sup>18</sup>We can interpret these quantities as quantities net of depreciations during period 0.

weight for computers will be overvalued when book values are used compared with using current prices

$$\frac{q_{2,0}X_{K_2}}{q_{1,0}X_{K_1} + q_{2,0}X_{K_2}} > \frac{q_{2,1}X_{K_2}}{q_{1,1}X_{K_1} + q_{2,1}X_{K_2}}. \quad (\text{A3})$$

We can illustrate this with some figures. Assume that

$$\begin{aligned} d_{1,1} &= 0.2, d_{2,1} = 0.4 \\ X_{K_1} &= X_{K_2} = 100 \\ q_{1,0} &= q_{1,1} = 1 \\ q_{2,0} &= 1, q_{2,1} = 0.9. \end{aligned}$$

From (A3) we can calculate the weight for good 2 as  $100/200 = 0.5$  using book values, and  $90/190 \approx 0.47$  using current prices. That is, the weight of good 2 (computers) will be overvalued when book values are used. Using (A1), we can calculate the aggregate depreciation rate as

$$\frac{100 * 0.2 + 100 * 0.4}{100 + 100} = \frac{60}{200} = 0.3,$$

using book values. Using current prices, from (A2), the aggregate depreciation rate is

$$\frac{100 * 0.2 + 90 * 0.4}{100 + 90} = \frac{56}{190} \approx 0.295.$$

In this case, the aggregate depreciation rate will be overvalued for computers when using book values as weights.

## Figures and tables

Table 1: **Buildings and land: Mean absolute error (MAE) and median absolute error (MdAE) for two estimators of reduction rates.** Results for two groups of firms based on a stratified sample of complete annual reports, 2001. The weights equal each group's share of total book value of Buildings and land in manufacturing in 2001

Estimator:		Firms with $\hat{\delta}_t^i < 0$	Firms with $\hat{\delta}_t^i \in [0, 1]$	Weighted average
$\hat{\delta}_t^i$	MAE	6.96	0.03	2.16
	MdAE	0.48	0.00	0.15
$\hat{\delta}_t^i$ -adj	MAE	0.03	0.03	0.03
	MdAE	0.02	0.00	0.01
Weight (share of capital)		0.31	0.69	
Share of firms in population		0.14	0.86	
Sample size		19	39	

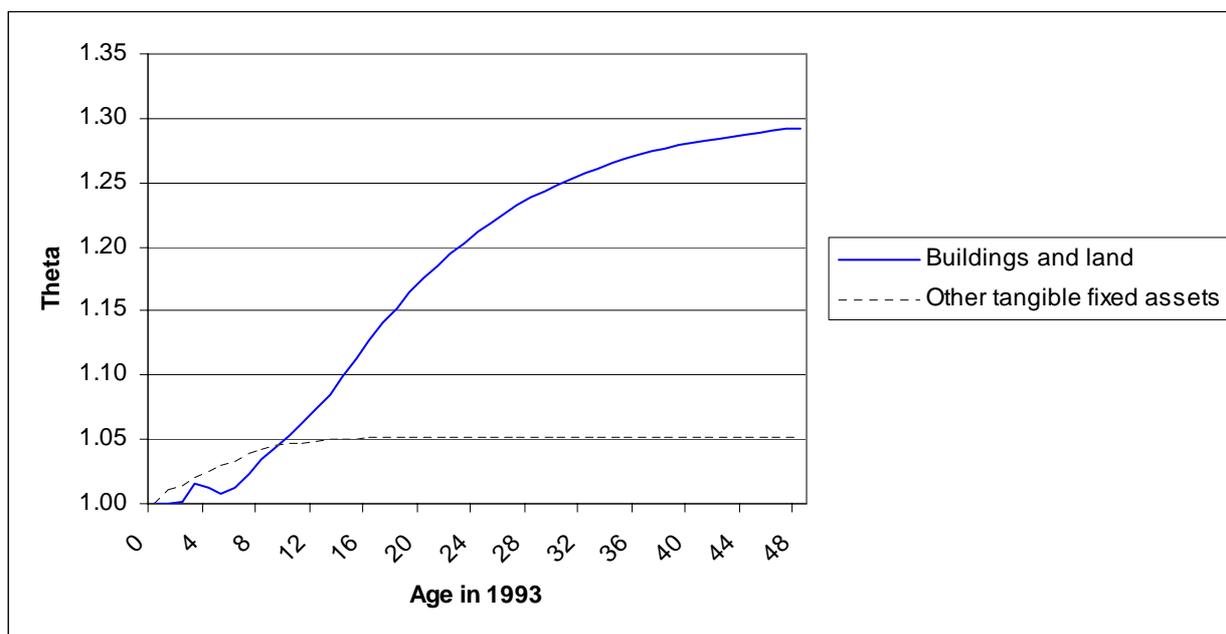


Figure 1: **Theta (adjustment factor of initial book value) as a function of the firm's birth year**

Table 2: **Other tangible fixed assets: Mean absolute error (MAE) and median absolute error (MdAE) for two estimators of reduction rates.** Results for two groups of firms based on a stratified sample of complete annual reports, 2001. The weights equal each group's share of Other tangible fixed assets in 2001

Estimator:		Firms with $\hat{\delta}_t^i < 0$	Firms with $\hat{\delta}_t^i \in [0, 1]$	Weighted average
$\hat{\delta}_t^i$	MAE	1.20	0.05	0.33
	MdAE	0.94	0.00	0.23
$\hat{\delta}_t^i$ -adj	MAE	0.15	0.05	0.07
	MdAE	0.12	0.00	0.03
Weight (share of capital)		0.25	0.75	
Share of firms in population		0.16	0.84	
Sample size		17	47	

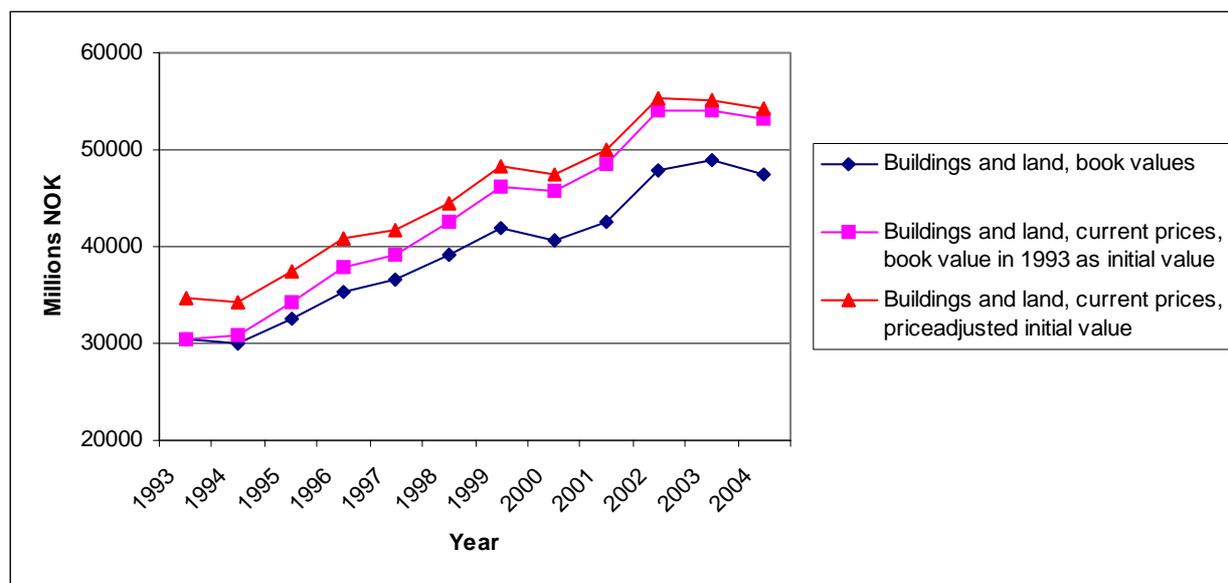


Figure 2: **Buildings and land in the Norwegian manufacturing industry 1993-2004**

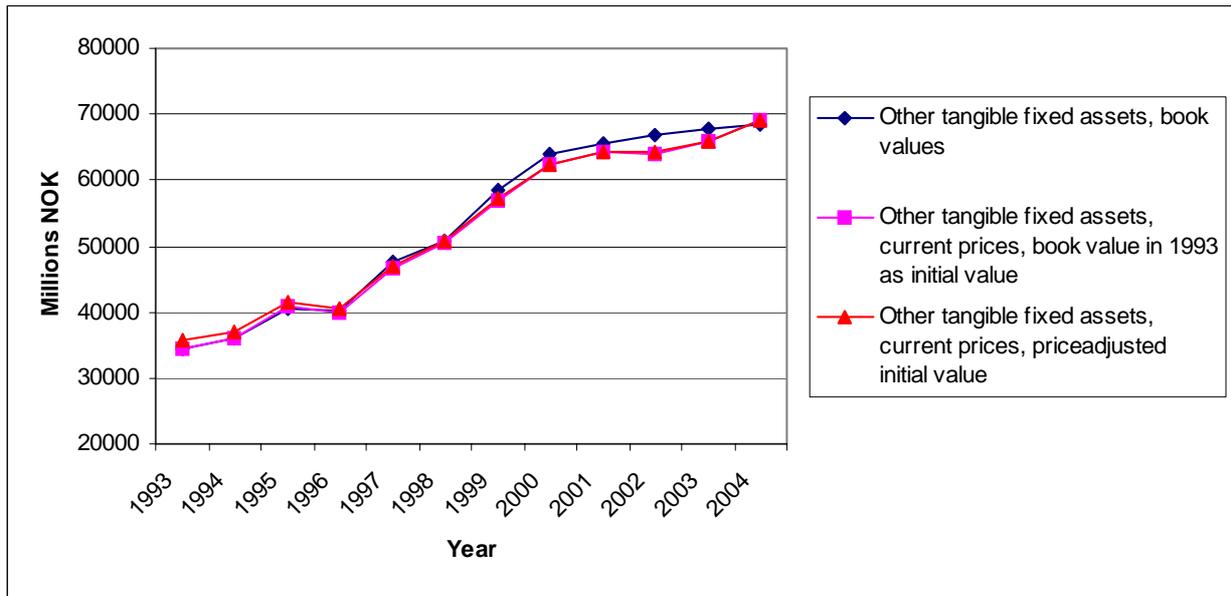


Figure 3: **Other tangible fixed assets in the Norwegian manufacturing industry 1993-2004**

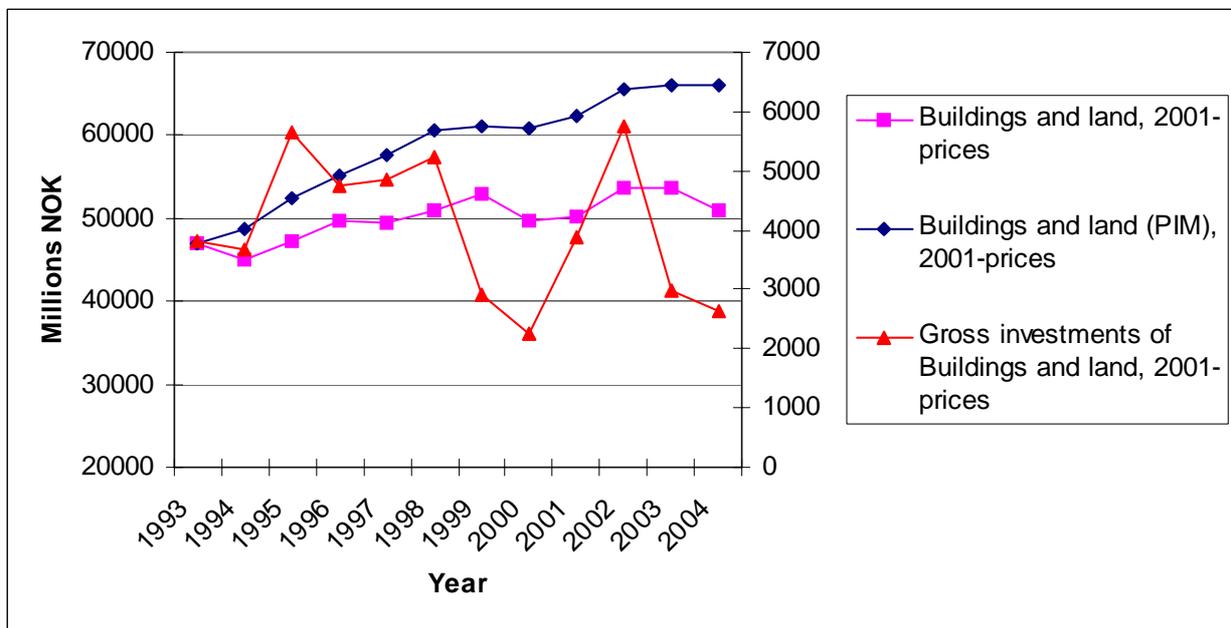


Figure 4: **Buildings and land in the Norwegian manufacturing industry calculated with two different methods.** Gross investments measured with the scale on the right-hand side, 2001-prices

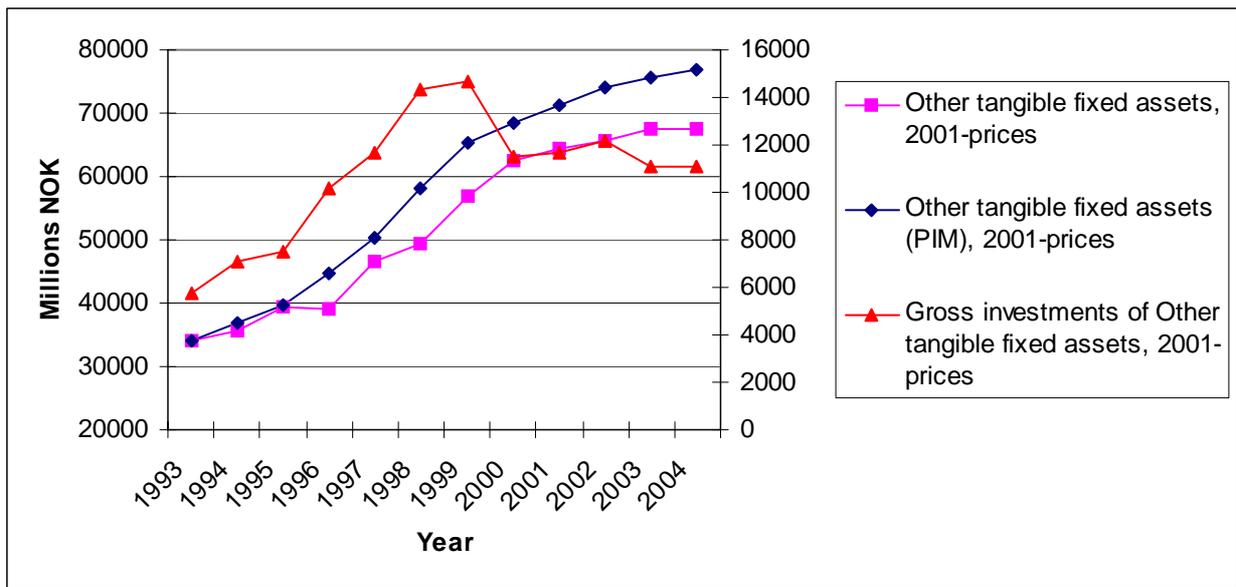


Figure 5: **Other tangible fixed assets in the Norwegian manufacturing industry calculated with two different methods.** Gross investments measured with the scale on the right-hand side, 2001-prices

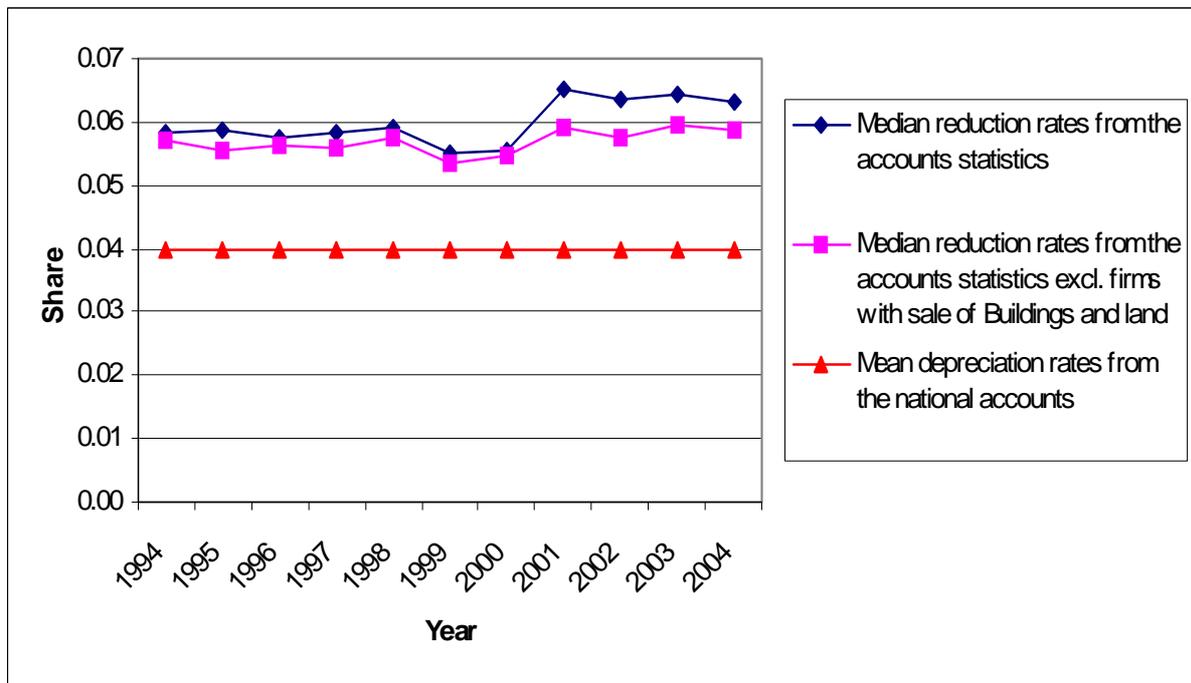


Figure 6: **Reduction rates and depreciation rates for Buildings and land in the Norwegian manufacturing industry 1994-2004**

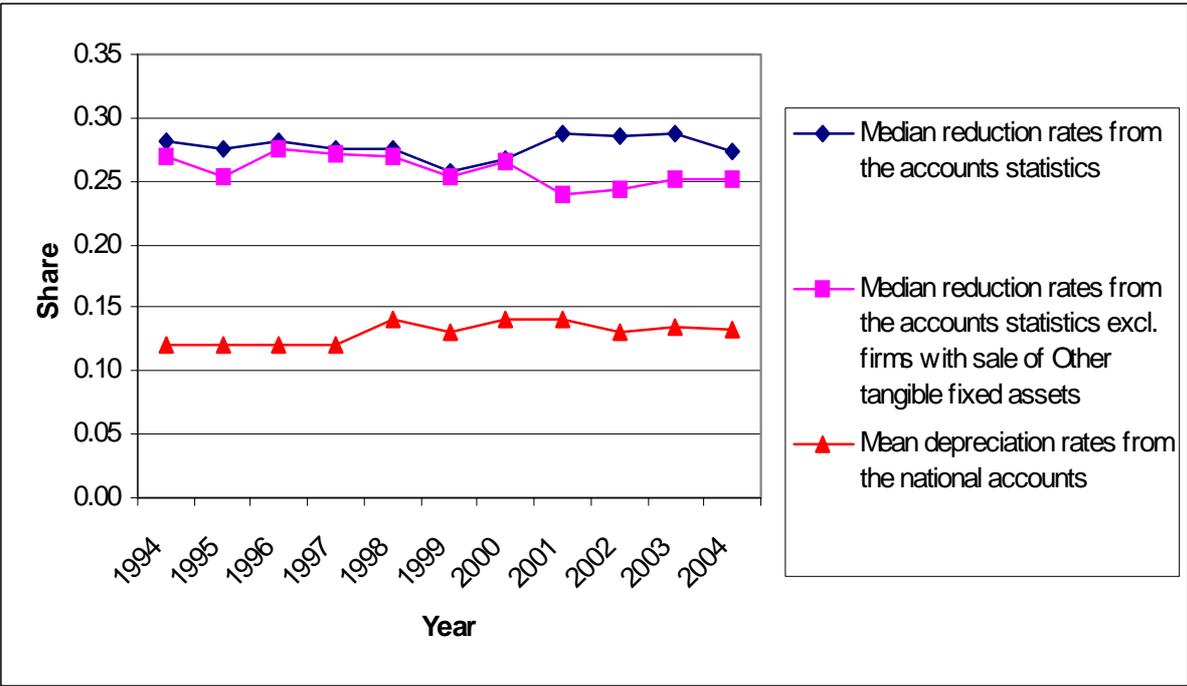


Figure 7: Reduction rates and depreciation rates for Other tangible fixed assets in the Norwegian manufacturing industry 1994-2004

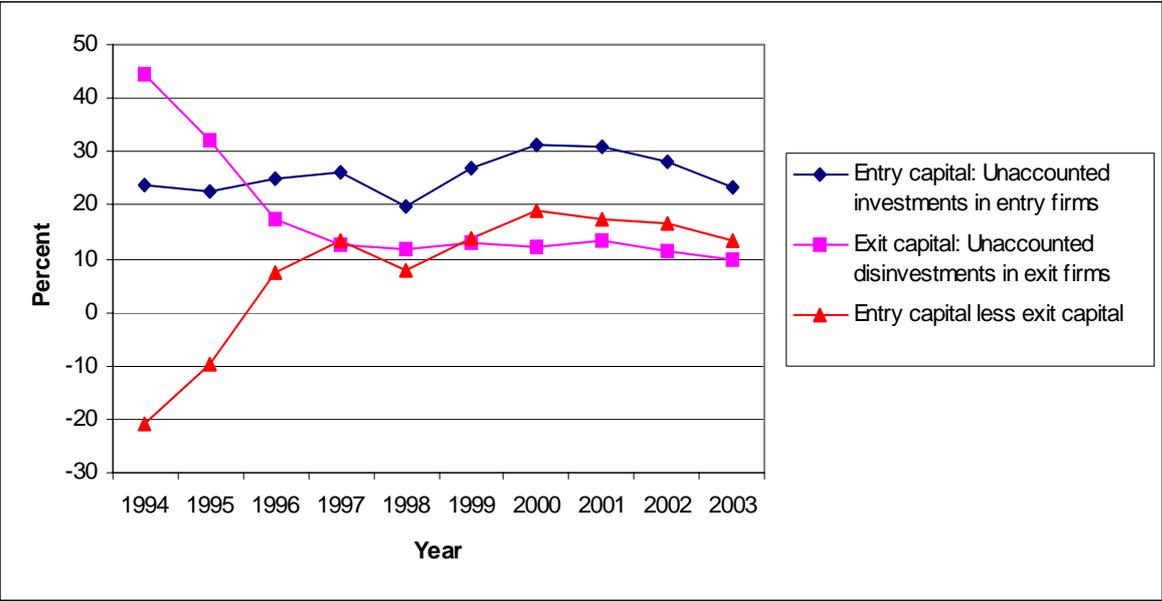


Figure 8: Entry and exit capital. In percent of total investments