Discussion Papers No. 518, October 2007
Statistics Norway, Research Department

Erik Biørn

Measuring and Decomposing Capital Stock and Service Values: Should Capital Quality be given a Place?

Abstract:
Aggregation of tangible capital assets across vintages and decomposition of value aggregates into quantities and prices are considered. Focus is on both capital stock values and capital service values. If the definitions and ways of measuring prices and quantities are not conformable, a third component, denoted a quality component, may have to be included as a buffer. Should it then be suppressed by allocating it to either of the price and quantity components, or should quality be accounted for separately? In discussing these issues, five quantity variables and five price variables are involved. Some of them are observable from market data without large efforts, some are essentially unobservable, and some can be quantified only if certain (often non-testable) assumptions are made. Illustrations based on parametric functions are given.

Keywords: Capital measurement, Aggregation, Capital service price, Capital quality, Arbitrage, Neo-classical theory

JEL classification: C43, C82, D24, E22

Acknowledgement: Based on presentations at a research seminar at Statistics Norway, December 1, 2005, and an econometric seminar at the Department of Economics, University of Oslo, September 19, 2006. The style of the seminar presentation is kept in the paper. I thank Terje Skjerpen for comments. An elaboration of the argument is given in Biørn (2007). A part of this work was done when the author was employed at Statistics Norway.

Address: University of Oslo, Department of Economics, P.O. Box 1095 Blindern, 0317 Oslo, Norway. Email: erik.biorn@econ.uio.no

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

Abstracts with downloadable Discussion Papers
in PDF are available on the Internet:
http://www.ssb.no
http://ideas.repec.org/s/ssb/dispap.html

For printed Discussion Papers contact:

Statistics Norway
Sales- and subscription service
NO-2225 Kongsvinger

Telephone: +47 62 88 55 00
Telefax: +47 62 88 55 95
E-mail: Salg-abonnement@ssb.no

1 Introduction and background

The measurement and aggregation of capital quantities, capital prices, and capital values are important problems in applied economic research, including national income and wealth accounting, productivity measurement, research on technology and behaviour of firms, construction of price and quantity indices. This problem is closely attached to the problem of measuring net national product and net national income; see *e.g.*, Weitzman (1976) and Hulten (1992). Capital goods are less homogeneous than 'ordinary' perishable goods – which cannot be stocked and have 'service lives' close to zero – since they have a vintage dimension. Aggregation procedures then have to consider that goods belonging to different vintages have some properties which are similar – for example their ability to generate services during a given time interval – while other properties are dissimilar – for example their values when traded in a market. Briefly, the aggregation problem for capital has a higher dimension than for non-durable goods. Items which appear as homogeneous in some respects, *e.g.*, as generators of capital service inputs, may be heterogeneous in other respects, *e.g.*, as wealth assets. I draw two consequences from this: (A) The problem of measuring capital prices, or price indexes, should be considered alongside the problem of measuring capital quantities, or quantity indexes. (B) When constructing and using a specific capital price index, one should be explicit about the quantity index to which it corresponds – and *vice versa*.

In this note I will discuss the measurement of capital and its price, describe the relationship between these variables, explain why several capital quantity and capital price concepts may be needed, and discuss why the extent to which they can be observed, differ. Some variables can be easily observed and some may be quantified if the analyst is willing to make certain, to some extent testable, assumptions. Some are hardly measurable at all and should be treated as latent. In doing this I will discuss whether *capital quality* should have a place as a third component in capital value aggregates, and if so, how it should be treated.

By 'capital' I will mean tangible productive assets in firms (production capital) and tangible service-generating assets in households (consumption capital). Capital has several dimensions, and it is important to specify whether our interest is in a measure for capital as an input in explaining output or for productivity research, or a measure of capital as a wealth asset, reflecting its ability to produce services to its owner or user today and in the future.

2 Capital measurement: Three quotations

I will start by quoting how three prominent economists have characterized the problem of measuring tangible capital:

John R. Hicks: "The measurement of capital is one of the nastiest jobs that economists have set to statisticians." [Hicks (1981, p. 204)]

Joan Robinson: "The student of economic theory is taught to write $O = f(L, C)$ where L is a quantity of labour, C a quantity of capital and O a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labour; he is told something about the index-number problem involved in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units C is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next." [Robinson (1953, p. 81)]

Dale W. Jorgenson: "As a consequence of the rapid assimilation of the results of Hulten and Wykoff, depreciation has been transformed from one of the most contentious and problematic areas in economic measurement to one of the best understood and most useful." [Jorgenson (1996, p. 24)]

3 Measuring values versus measuring quantities and prices

Measuring values of capital goods may be easier than measuring prices and quantities. We will need, in price and quantity terms, not only capital stocks, but also capital service flows. For each capital item, as well as for indices constructed for aggregates, we should, ideally, have:

Capital Stock Value	=	Capital Stock Price	×	Capital Stock Quantity
Capital Service Value	=	Capital Service Price	×	Capital Service Quantity

If our definitions and/or methods of measurement are not conformable, we may need to extend the relationships by inserting a 'buffer', called quality (for lack of a better term), between price and quantity as follows:

Value = Price × Quality × Quantity

In order to separate capital values properly into prices and quantities, one may come out with one of three solutions:

(a) suppress quality altogether,
(b) allocate quality to capital's price component,
(c) allocate quality to capital's quantity component.

4 Weight functions and capital quantities

Some notation will be needed to expose the argument. We let t denote running time, a denote age, and $t-a$ denote vintage, all considered as continuous, and first introduce three quantity variables:

$J(t)$: *Gross* INVESTMENT QUANTITY *at time* t.
$K(t,a)$: GROSS CAPITAL *of age* a *at time* t*:* capital measured in efficiency units.
$G(t,a)$: PHYSICAL CAPITAL *of age* a *at time* t*:* capital measured in physical units.

Two weight functions describe the relationship between J, G and K:

$S(a)$: SURVIVAL FUNCTION:
Share of investment, in physical units, which remains at age a.

$E(a)$: EFFICIENCY FUNCTION:
Share of efficiency of one physical unit existing at age a which remains at this age.

These weight functions are time-invariant and are assumed to satisfy

$$\begin{array}{lll} S'(a) \leq 0, & S(0) = 1, & S(\infty) = 0, \\ E'(a) \leq 0, & E(0) = 1, & E(\infty) = 0. \end{array}$$

Capital measured in physical units and in efficiency units at age a at time t can then be expressed as, respectively,

$$G(t,a) = S(a)\,J(t-a), \tag{1}$$

$$K(t,a) = E(a)\,G(t,a), \tag{2}$$

and therefore

$$K(t,a) = S(a)E(a)J(t-a). \tag{3}$$

It follows that

$$\begin{array}{ll} K(t,0) = G(t,0) = J(t), & \forall\, t, \\ K(t,a) \leq G(t,a) \leq J(t-a), & \forall\, t \& \forall\, a > 0. \end{array}$$

Example:

$G(2000,5)$ = Number of machines, 5 years old in year 2000.
$K(2000,5)$ = Number of efficiency units which are contained in $G(2000,5)$.

The relevant quantity variable for *productivity analysis* is gross capital, K, since it is the *generator of capital services.* I will stick to the usual assumption that one *unit of efficiency capital generates one unit of services per unit of time.* The variables G and K coincide numerically only under particular circumstances. For *wealth accounting*, neither G nor K is, in general, the relevant capital quantity variable, which will be elaborated in Section 8.

5 Three basic problems

I will define three basic problems related to the measurement of capital.

THE FIRST PROBLEM: It is much easier to measure $J(t)$ and $G(t,a)$ for $a>0$ than it is to measure $K(t,a)$ for $a>0$.

We now define STOCK VALUES AND STOCK PRICE VARIABLES:

$q(t)$: PRICE PER NEW CAPITAL UNIT at time t.
$V(t,a)$: VALUE OF CAPITAL STOCK of age a at time t.
$p(t,a)$: *Price per* PHYSICAL *capital unit* of age a at time t.
$r(t,a)$: *Price per capital* EFFICIENCY *unit* of age a at time t.

I use the term VINTAGE PRICE for $p(t,a)$ $(a>0)$ and $r(t,a)$ $(a>0)$. Vintage prices may be difficult (and expensive) to measure.

The value of capital value which belongs to a certain vintage can be split into a price and a quantity in two ways:

$$V(t,a) = p(t,a)\,G(t,a) = r(t,a)\,K(t,a). \tag{4}$$

This, in combination with (2) implies

$$p(t,a) = E(a)r(t,a), \tag{5}$$

so that the capital price and the two vintage prices satisfy:

$$\begin{aligned} p(t,0) = r(t,0) = q(t), \qquad & \forall\, t \\ p(t,a) \leq r(t,a) \leq q(t), \qquad & \forall\, t \& \forall\, a > 0. \end{aligned}$$

Example:

$p(2000,5)$ = Price of one machine, 5 years old in year 2000.
$r(2000,5)$ = Price of one efficiency-corrected machine, 5 years old in year 2000.

THE SECOND PROBLEM: If vintage prices are observable, what we are likely to observe is $p(t,a)$ for $a>0$, not $r(t,a)$ for $a>0$.

Prices of the latter kind arguably are the most interest vintage price concepts for accounting and research.

In Equation (4), the decomposition $V(t,a) = p(t,a)\,G(t,a)$ lends itself more easily to statistical quantification than $V(t,a) = r(t,a)\,K(t,a)$. With respect to observability of $p(t,a)$ and $G(t,a)$, three situations can occur:

(i) We can observe $p(t,a)$ and are able to count the corresponding number of physical units, $G(t,a)$, and then obtain $V(t,a)$ as their product.
(ii) We can observe $V(t,a)$ and $G(t,a)$, and then obtain $p(t,a)$ as their ratio.
(iii) We can observe $V(t,a)$ and $p(t,a)$, and then obtain $G(t,a)$ as their ratio.

It follows from (2) and (4) that the two sets of quantities and prices are related by:

$$(6) \qquad \frac{K(t,a)}{G(t,a)} = \frac{p(t,a)}{r(t,a)} = E(a) \qquad \forall\, t \;\&\, \forall a > 0.$$

This implies that only if we are assured that capital has a constant efficiency, *i.e.*, $E(a)=1\ \forall\, a > 0$, the problem of non-observability of $K(t,a)$ and $r(t,a)$ vanishes. However, several reasons can be given why $E(a)=1$ lacks realism.

THE THIRD PROBLEM: How construct the missing link in the price system for capital?

Precisely, which mathematical relationship can be established between $p(t,a)$ and $r(t,s)$ on the hand and $q(t)$ on the other? A related problem, also to be addressed, is: How can a capital service price be obtained?

My answer is:

> *A firm, or a household, purchases and uses a capital item, because it is interested in the future services that the item (is expected to) generate(s). A firm or household which buys one efficiency unit of capital, pays (should pay) the same price per unit of prospective discounted capital services as a firm or household which buys one new unit, regardless of the age of the unit.* THIS COMMON RATIO IS THE CAPITAL SERVICE PRICE.

This criterion is related to other criteria in the literature, but its precise formulation is different. The mathematical formalization to be given in Equations (11) through (14) below is related to the condition frequently postulated as an equilibrium condition in the capital market literature, saying that the acquisition price of a capital asset should equal the (present value of the) its (expected) future rental prices weighted by the relevant remaining efficiency [see, *e.g.*, Hotelling (1925), Hicks (1973, Chapter II), Jorgenson (1989, Section 1.2), and Diewert (2005, Section 2)]. In Biørn (2007), this issue is elaborated.

The rationale for the above *arbitrage condition* is the neo-classical *malleability assumption* for capital in conjunction with the existence of PERFECT MARKETS for new and used capital goods. However, several reasons can be given why this condition – which resembles theoretical conditions for determination of prices of financial assets – can be expected to hold only approximately: market imperfections, uncertainty, imperfect information, imperfect malleability, putty-clay effects, indivisibility of capital goods, etc.

6 Formalizing the argument

We now formalize the argument, in three steps:

STEP 1: The *discounted service flow from one new capital unit which is generated after age a* is defined as

$$\omega(a) = \int_a^\infty e^{-\rho(s-a)} S(s)E(s)ds, \tag{7}$$

where ρ is interpreted as a (real) rate of discount [nominal interest rate minus rate of increase of the capital price $q(t)$].

STEP 2: The *discounted service flow from one capital efficiency unit which has attained age a and which is generated from age onwards* then becomes:

$$\phi(a) = \frac{\omega(a)}{S(a)E(a)}. \tag{8}$$

STEP 3: The *capital service price* for age a at time t can then be defined in two alternative ways, by normalizing capital vintage prices $r(t,a)$ or $p(t,a)$ against the service flow indicator $\phi(a)$:

$$c(t,a) = \frac{r(t,a)}{\phi(a)} = c(t), \tag{9}$$

$$d(t,a) = \frac{p(t,a)}{\phi(a)} = E(a)d(t,0), \qquad \forall\, t \,\&\, \forall\, a \geq 0 \tag{10}$$

Equations (9)–(10) represent my first way of stating the arbitrage condition, and can be interpreted as giving either normative or positive statements:

NORMATIVE: These equalities SHOULD HOLD for all a and t.
POSITIVE: These equalities HOLDS, approximately, for any a and t.

7 Implications of the arbitrage condition

I next consider four implications of the arbitrage condition (9)–(10).

IMPLICATION 1: Expressed in terms of vintage price PER EFFICIENCY UNIT:

$$r(t,a) = \frac{\phi(a)}{\phi(0)} q(t). \tag{11}$$

IMPLICATION 2: Expressed in terms of vintage price PER PHYSICAL UNIT:

$$p(t,a) = E(a)\frac{\phi(a)}{\phi(0)}q(t) = \frac{1}{S(a)}\frac{\omega(a)}{\omega(0)}q(t). \tag{12}$$

This equation, combined with observations on $p(t,a)/q(t)$, can be used to make inference on the form of the efficiency function $E(a)$ and possibly also – but not always, see *e.g.* Biørn (1998) – on the form of the survival function $S(a)$ – provided that the arbitrage condition holds. Alternatively: With suitable parameterizations of $S(a)$ and $E(a)$, the latter equation could be a vehicle for testing the arbitrage condition econometrically. See also Biørn (2005, 2007).

IMPLICATION 3: Expressed in terms of SERVICE PRICES:

$$c(t,a) = c(t), \tag{13}$$

$$d(t,a) = \frac{E(a)}{\phi(0)}q(t) = E(a)c(t). \tag{14}$$

IMPLICATION 4: The service price $c(t)$ and the vintage prices $r(t,a)$ and $p(t,a)$ (should) – for any age a – change in proportion to $q(t)$, *cet. par.* The time path of $q(t)$ is unrestricted. WHEN THE PRICE PER NEW CAPITAL UNIT INCREASES BY α PER CENT AND $S(a)$, $E(a)$, AND ρ ARE UNCHANGED, THEN THE VINTAGE AND SERVICE PRICES ARE PREDICTED TO INCREASE BY α PER CENT. This also is a testable implication, subject to the availability of data.

8 Capital stock value, service value and net capital

We define *net capital stock* as the value of the capital stock deflated by the current capital price. The vintage specific net capital of age a at time t can therefore be expressed as

$$H(t,a) = \frac{\omega(a)}{\omega(0)}J(t-a), \tag{15}$$

which also represents the *initial investment weighted by the share of services generated after age a*. Hence, the vintage specific capital stock value of age a at time t can be written as

$$V(t,a) = q(t)H(t) = q(t)\frac{\omega(a)}{\omega(0)}J(t-a), \tag{16}$$

which means

CAPITAL STOCK VALUE $=$ REPLACEMENT VALUE OF INVESTMENT $[q(t)J(t-a)]$ $\times$ SHARE OF SERVICES FROM ONE **NEW** CAPITAL UNIT GENERATED AFTER AGE a $=$ PRICE PER NEW CAPITAL UNIT $\times$ INITIAL INVESTMENT WEIGHTED BY SHARE OF SERVICES GENERATED AFTER AGE a

We define the *vintage specific capital service value* of age a at time t as the product of the capital service price and the capital stock volume – both of which are expressed either in terms of efficiency units or in terms of physical units, *i.e.*,

$$(17) \qquad W(t,a) = c(t)K(t,a) = d(t,a)G(t,a) = \frac{q(t)}{\omega(0)}\, S(a)E(a)\, J(t-a),$$

which means

CAPITAL SERVICE VALUE $=$ REPLACEMENT VALUE OF INVESTMENT PER UNIT OF TOTAL SERVICE FLOW $[q(t)J(t-a)/\omega(0)]$ $\times$ SURVIVAL FUNCTION $\times$ EFFICIENCY FUNCTION.

9 Aggregation across vintages

So far, only one vintage of a specific homogeneous capital category has been considered. We now turn to aggregation across vintages.

First, define TOTAL CAPITAL STOCK VALUE and TOTAL CAPITAL SERVICE VALUE as, respectively:

$$(18) \quad V(t) = \int_0^\infty V(t,a)\, da = \int_0^\infty q(t)H(t,a)\, da = \int_0^\infty q(t)\tfrac{\omega(a)}{\omega(0)}J(t-a)\, da,$$

$$(19) \quad W(t) = \int_0^\infty W(t,a)\, da = \int_0^\infty c(t)\, K(t,a)\, da = \int_0^\infty c(t)S(a)E(a)J(t-a)\, da.$$

This aggregation is one aspect of 'the index problem' for capital goods. The other aspect is the more common 'index problem' of aggregating across different good categories, including different capital categories, non-durable goods, services, etc.

Next, define AGGREGATE GROSS CAPITAL STOCK and AGGREGATE NET CAPITAL STOCK as, respectively:

$$K(t) = \int_0^\infty K(t,a)\,da, \qquad \text{where} \quad K(t,a) = S(a)E(a)\,J(t-a), \tag{20}$$

$$H(t) = \int_0^\infty H(t,a)\,da, \qquad \text{where} \quad H(t,a) = \frac{\omega(a)}{\omega(0)}\,J(t-a). \tag{21}$$

$K(t)$ and $H(t)$ may be considered TWO QUANTITY INDEXES for capital stocks – serving different purposes. Then, using (18)–(21), the two value aggregates can, *in principle*, be decomposed as

$$\begin{aligned} V(t) &= q(t)H(t), \\ W(t) &= c(t)K(t), \end{aligned} \tag{22}$$

which means

STOCK VALUE (INDEX)	=	PRICE PER NEW CAPITAL UNIT (INDEX)	×	NET CAPITAL (INDEX)
SERVICE VALUE (INDEX)	=	SERVICE PRICE (INDEX)	×	GROSS CAPITAL (INDEX)

10 Parametric survival and efficiency functions

How should value aggregates be decomposed in practice? To answer this question, we have to address the issue of *capital quality*, referred to in Section 3, and to select appropriate parametric forms for the survival function $S(a)$ and the efficiency function $E(a)$. This is the topic of this and the next section.

Regarding the quality dimension of capital stocks and capital services, my position is that what I denote as a quality component should be accounted for only when the definitions and ways of measuring prices and quantities are not conformable.

For the $S(a)$ and $E(a)$ functions, we will consider three examples with two, one and three parameters, respectively. They are also discussed in Biørn (2007), where more details are given.

EXAMPLE 1: Exponentially declining survival and efficiency:

$$S(a) = e^{-\beta a} \quad (\beta \geq 0), \qquad E(a) = e^{-\gamma a} \quad (\gamma \geq 0).$$

EXAMPLE 2: No retirement until age N. Constant efficiency:

$$S(a) = E(a) = 1, \qquad a \in [0, N].$$

EXAMPLE 3: Two-parametric functions with maximal age = N:

$$S(a) = \left(1 - \frac{a}{N}\right)^{\mu} \quad (\mu \geq 0), \qquad E(a) = \left(1 - \frac{a}{N}\right)^{\nu} \quad (\nu \geq 0), \qquad a \in [0, N].$$

For $\mu = \nu = 0$, Example 3 degenerates to Example 2. Example 3 is rather flexible with respect to curvature:

- ∘ $S(a)$ is convex/linear/concave $\iff$ $\mu \gtreqless 1$, respectively.
- ∘ $B(a)$ is convex/linear/concave $\iff$ $\nu \gtreqless 1$, respectively.
- ∘ $S(a)E(a)$ is convex/linear/concave $\iff$ $\tau = \mu + \nu \gtreqless 1$, respectively.

Hence, the combined survival-efficiency function $S(a)E(a)$ can be convex even if one of its components, or both, are concave. An example is $\frac{1}{2} < \mu < 1$, $\frac{1}{2} < \nu < 1$.

11 Decompositions of stock and capital service values

In this section, we illustrate, for the parametric survival functions described in Section 10, the decompositions of the aggregate capital stock and service values described for general survival and efficiency functions in Sections 8 and 9. For simplicity we will assume that *no discounting of future services* ($\rho = 0$) is performed. This assumption can be relaxed rather easily for the exponential decay Example 1, and with somewhat more difficulty for Examples 2 and 3, where finite service life of the capital is assumed. In Biørn (2007) a detailed derivation of the decompositions below and a description of the generalization with $\rho > 0$ are given.

The primary purpose of this section is to illustrate cases in which *capital quality* should be entered as a buffer between the price and the quantity component and when it is not needed. In the latter case the definition of quantity and price are said to be conformable, in the former case they are not.

CAPITAL VALUE DECOMPOSITIONS. EXAMPLE 1

STOCK VALUE:

VERSION 1.1: $$V(t) = \int_0^\infty \underbrace{q(t)e^{-\gamma a}}_{p(t,a)} \underbrace{e^{-\beta a}J(t-a)}_{G(t,a)} da$$

VERSION 1.2: $$V(t) = \int_0^\infty \underbrace{q(t)}_{r(t,a)} \underbrace{e^{-\gamma a}e^{-\beta a}J(t-a)}_{H(t,a)=K(t,a)} da$$

SERVICE VALUE:

VERSION 1.3: $$W(t) = \int_0^\infty \underbrace{c(t)}_{c(t,a)} \underbrace{e^{-\gamma a}e^{-\beta a}J(t-a)}_{K(t,a)=H(t,a)} da$$

VERSION 1.4: $$W(t) = \int_0^\infty \underbrace{c(t)e^{-\gamma a}}_{d(t,a)} \underbrace{e^{-\beta a}J(t-a)}_{G(t,a)} da$$

Capital value decompositions. Example 2

Stock value:

$$\text{Version 2.1:}\quad V(t) = \int_0^N \underbrace{q(t)\left(\tfrac{N-a}{N}\right)}_{p(t,a)=r(t,a)} \underbrace{J(t-a)}_{G(t,a)=K(t,a)} da$$

$$\text{Version 2.2:}\quad V(t) = \int_0^N \underbrace{q(t)}_{q(t)} \underbrace{\left(\tfrac{N-a}{N}\right) J(t-a)}_{H(t,a)} da$$

Service value:

$$\text{Version 2.3:}\quad W(t) = \int_0^N \underbrace{c(t)}_{c(t,a)} \underbrace{J(t-a)}_{G(t,a)=K(t,a)} da$$

$$\text{Version 2.4:}\quad W(t) = \int_0^N \underbrace{c(t)}_{d(t,a)=r(t,a)} \underbrace{J(t-a)}_{G(t,a)=K(t,a)} da$$

Capital value decompositions. Example 3

Stock value:

$$\text{Version 3.1:}\quad V(t) = \int_0^N \underbrace{q(t)(\tfrac{N-a}{N})(\tfrac{N-a}{N})^{\nu}}_{p(t,a)} \underbrace{\left(\tfrac{N-a}{N}\right)^{\mu} J(t-a)}_{G(t,a)} da$$

$$\text{Version 3.2:}\quad V(t) = \int_0^N \underbrace{q(t)\left(\tfrac{N-a}{N}\right)}_{r(t,a)} \underbrace{\left(\tfrac{N-a}{N}\right)^{\nu}(\tfrac{N-a}{N})^{\mu} J(t-a)}_{K(t,a)} da$$

Service value:

$$\text{Version 3.3:}\quad W(t) = \int_0^N \underbrace{c(t)}_{c(t,a)} \underbrace{\left(\tfrac{N-a}{N}\right)^{\nu}\left(\tfrac{N-a}{N}\right)^{\mu} J(t-a)}_{K(t,a)} da$$

$$\text{Version 3.4:}\quad W(t) = \int_0^N \underbrace{c(t)\left(\tfrac{N-a}{N}\right)^{\nu}}_{d(t,a)} \underbrace{\left(\tfrac{N-a}{N}\right)^{\mu} J(t-a)}_{G(t,a)} da$$

We can write the stock value and the service values of capital, as given in (18) and (19) for the general case, as

$$V(t) = \int_0^\infty q(t)\Lambda(a)J(t-a)da, \tag{23}$$

$$W(t) = \int_0^\infty c(t)\Psi(a)J(t-a)da. \tag{24}$$

where $\Lambda(a) = \frac{\omega(a)}{\omega(0)}$ and $\Psi(a) = S(a)E(a)$. The capital value decompositions in Examples 1–3 above – each having two versions of the stock value decomposition and two versions of the service value decomposition – can be summarized by the following table:

	$\Lambda(a)$	$\Psi(a)$
Example 1	$e^{-(\gamma+\beta)a}$	$e^{-(\gamma+\beta)a}$
Example 2	$\left(\frac{N-a}{N}\right)$	1
Example 3	$\left(\frac{N-a}{N}\right)^{\nu+\mu+1}$	$\left(\frac{N-a}{N}\right)^{\nu+\mu}$

Example 1 has the property that $\Lambda(a) = \Psi(a)\ \forall\, a$. This is not the case for Examples 2 and 3. The only price-quantity constellations which are conformable for the *stock* value are $[p(t,a), G(t,a)]$ and $[r(t,a), K(t,a)]$; confer Versions 1.1 and 1.2. The only price-quantity constellations which are conformable for the *service* value are $[c(t), K(t,a)]$ and $[d(t,a), G(t,a)]$; confer Versions 1.3 and 1.4.

Then, consider *Example 2*. If we for all vintages entering the capital value aggregates represent the capital *service* price by $c(t)$ and the quantity by $J(t-a)$, these components are conformable for the *service* value; confer Versions 2.3.–2.4. This is reflected by $\Psi(a) = 1\ \forall a$, which means that the quality component can be neglected. On the other hand, if we for all vintages represent the capital *stock* price by $q(t)$ and the quantity by $J(t-a)$, the price and quantity components are not conformable for the *stock* value. This follows because $\Lambda(a) = \left(\frac{N-a}{N}\right)$, and therefore we cannot neglect the quality components, but have to account for it by this age-specific $\Lambda(a)$ function; confer Versions 2.1 and 2.2. In general, for all the three examples, we can interpret $\Lambda(a) = \frac{\omega(a)}{\omega(0)}$ and $\Psi(a) = S(a)E(a)$ as quality adjustment functions which must be included if we insist on representing the capital quantity by $J(t-a)$ and the price by $q(t)$ and by $c(t)$, respectively.

Consider next *Example 3*. Here the price $c(t)$ and the quantity $J(t-a)$ are not conformable for the *service* value and the price $q(t)$ and the quantity $J(t-a)$ are not conformable for the *stock* value, when at least one of ν or μ are positive. The only price-quantity constellations which are conformable for the *service* value are $[c(t), K(t,a)]$ and $[d(t,a), G(t,a)]$; confer Versions 3.3 and 3.4. The only price-quantity constellations which are conformable for the *stock* value are $[r(t,a), K(t,a)]$ and $[p(t,a), G(t,a)]$; confer Versions 3.1 and 3.2.

Nine cases belonging to Example 3, in which *quality components* are included – six for the stock value (A through F) and three for the service value (G through I)– are displayed in the table below. It is obvious that the magnitude of the quality component – for given price and quantity indicators – will change when at least one of the parameters N, ν or μ changes. An interesting question, not to be discussed here, is to which extent such parameter changes will be shifted in the market observations on prices and quantities.

	VALUE	PRICE	QUALITY	QUANTITY
A	$V(t,a)$	$q(t)$	$\left(\frac{N-a}{N}\right)^{\nu+\mu+1}$	$J(t-a)$
B	$V(t,a)$	$p(t,a)$	$\left(\frac{N-a}{N}\right)^{\mu}$	$J(t-a)$
C	$V(t,a)$	$r(t,a)$	$\left(\frac{N-a}{N}\right)^{\nu+\mu}$	$J(t-a)$
D	$V(t,a)$	$q(t)$	$\left(\frac{N-a}{N}\right)^{\nu+1}$	$G(t,a)$
E	$V(t,a)$	$r(t,a)$	$\left(\frac{N-a}{N}\right)^{\nu}$	$G(t,a)$
F	$V(t,a)$	$q(t)$	$\left(\frac{N-a}{N}\right)$	$K(t,a)$
G	$W(t,a)$	$c(t)$	$\left(\frac{N-a}{N}\right)^{\nu+\mu}$	$J(t-a)$
H	$W(t,a)$	$d(t,a)$	$\left(\frac{N-a}{N}\right)^{\mu}$	$J(t-a)$
I	$W(t,a)$	$c(t)$	$\left(\frac{N-a}{N}\right)^{\nu}$	$G(t,a)$

12 Conclusions

The conclusions put forth in this paper, as well as some reflections they motivate, can be summarized in the following seven points:

[**1**] We need several measures of capital prices and quantities depending on the purpose for which they are to be used: $G(t,a)$, $K(t,a)$, and $H(t,a)$ differ conceptually and in general also numerically.

[**2**] If our definitions of prices and quantities for old capital goods are not conformable, it will be necessary to introduce quality as a separate component in order to make their product equal to the capital value.

[**3**] Often, the only variables observable from collected statistical data are $J(t)$ and $q(t)$. We can specify parametric functional forms for the survival and efficiency functions $S(a)$ and $E(a)$, like those in the illustrative examples, but a lot has to be done before their unknown parameters [like (β,γ,N,μ,ν) in the examples] can be quantified properly.

[4] To be able to quantify (the unknown parameters of) $S(a)$ and $E(a)$, we may need information on maximal service lives (scrapping ages) and vintage prices for physical capital goods, $p(t, a)$, for approximately homogeneous items. Information on quantiles (*e.g.*, the first, second and third quartiles) in the survival and efficiency functions) would also be valuable pieces of information.

[5] Such information would put researchers in a position to *estimate* by econometric methods parameters of the survival and efficiency functions and/or to *test* arbitrage conditions for capital on which capital accounting should be based. In reality, it has been very difficult to motivate statistics-producing agencies to give collection of such information high priority. This has for long been the situation in Norway, and, to my knowledge, it seems not to be radically different in most other countries. The methods exist, but the core data needed to implement them are lacking.

[6] The correct way of decomposing capital values (indices) into prices and quantities (indices) will change when the shapes of the survival and efficiency functions [represented by $(\beta, \gamma, N, \mu, \nu)$ in the illustrative examples] change. An assertion often made is that many consumer durable goods have become gradually less long-lived than before. This raises interesting and important questions for econometric research and policy, *inter alia*, for the construction and use of price and quantity indices.

[7] It it not difficult to give examples to illustrate that improper measurement of survival and efficiency functions due to lack of data can bias quantitative conclusions which may have serious consequences for economic analysis and policy. I will finish by giving two:

Example 1: Calculation of rates of return to capital:
By definition,

$$\text{Rate of return} = \frac{\text{Gross operating surplus} - \text{ Value of depreciation}}{\text{Value of capital stock}}$$

The assessment of the value of depreciation (in the numerator) and the capital value (in the denominator) both depend on the forms of the survival and efficiency functions. If the latter cannot be well quantified owing to lack of adequate data and 'guesstimates' therefore take their place, one runs the risk of obtaining severely biased measures of rates of return.

Example 2: Estimation of Elasticities of substitution:
Often elasticities of substitution between inputs, say labour and capital, are estimated by regressing (say) the log labour/capital ratio on the log wage rate/capital service price ratio. If the forms of the survival and efficiency functions cannot be well quantified owing to data unavailability, then the series for both the capital stock and the capital service price are likely to contain substantial errors. As a consequence, the estimates of this important parameter may be severely biased, and the scarcity of our information may prevent further in-depth research.

References

Biørn, E. (1998): Survival and Efficiency Curves for Capital and the Time-Age-Profile of Vintage Prices. *Empirical Economics* **23**, 611–633.

Biørn, E. (2005): How are Survival-Efficiency Profiles Related to Retirement Rates and Capital Vintage Prices? In *Focus on Economic Growth and Productivity*, ed. by L.A. Finley. New York: Nova Science Publishers, pp. 143–186.

Biørn, E. (2007): Decomposing Capital Values into Prices and Quantities. What about Quality? Unpublished paper.

Diewert, E. (2005): Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes, and Interest Rates. In *Measuring Capital in the New Economy*, ed. by C. Corrado, J. Haltiwanger, and D. Sichel. Chicago: The University of Chicago Press.

Hicks, J. (1973): *Capital and Time.* Oxford: Clarendon Press.

Hicks, J. (1981): *Wealth and Welfare: Collected Essays in Economic Theory.* Cambridge, MA: Harvard University Press.

Hotelling, H.S. (1925): A General Mathematical Theory of Depreciation. *Journal of the American Statistical Association* **20**, 340–353.

Hulten, C.R. (1992): Accounting for the Wealth of Nations: The Net versus Gross Output Controversy and its Ramifications. *Scandinavian Journal of Economics* **94,** Supplement, 9–24.

Jorgenson, D.W. (1989): Capital as a Factor of Production. In *Technology and Capital Formation*, ed. by D.W. Jorgenson and R. Landau. Cambridge, Mass: The MIT Press, pp. 1–35.

Jorgenson, D.W. (1996): Empirical Studies of Depreciation. *Economic Inquiry* **34**, 24–42.

Robinson, J. (1953): The Production Function and the Theory of Capital. *Review of Economic Studies* **21**, 81–106.

Weitzman, M.L. (1976): On the Welfare Significance of National Product in a Dynamic Economy. *Quarterly Journal of Economics* **90**, 156–162.