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Income Taxation, Tuition Subsidies, and Choice of Occupation

Abstract:
Differentiated tax rates on labor and capital income are found to be optimal in this study, where agents choose occupation based on lifetime income net of tuition costs. Efficient revenue raising in a case where the government can not observe educational effort implies that the government should trade off efficiency in production for efficiency in intertemporal consumption. The subsequent wage difference between high and low-skilled occupations is increased compared to a production efficient outcome, which is in contrast to previous results in the literature.

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1. Introduction

This study analyzes optimal dual income taxation and tuition subsidies in the presence of general equilibrium effects on wages due to occupational choices. The existing literature on optimal taxation and choice of education does not consider general equilibrium effects on skill-specific wage rates; see e.g. Bovenberg and Jacobs (2005), Nielsen and Sørensen (1997), and Nerlove et al. (1993). The common practice is to assume that labor skill-types are perfect substitutes, where educational attainment generates a productivity increase equivalent to the resulting higher pre-tax wage rate in the labor market. Hence, wage effects due to changes in the supply of labor skill-types are neglected. However, the effects on the supply of labor skill-types are important because they are crucial to the determination of the pre-tax wage premium between a high and a low-skilled occupation in an economy where labor skill-types are imperfect substitutes, and where wage rates balance supply and demand for labor skill types (see Heckman et al., 1998 and Dur and Teulings, 2004). A fixed pre-tax wage gap between high and low-skilled occupations can be justified by the factor price equalization theorem (Heckscher-Ohlin). However, Katz and Autor (1999) find that pre-tax wage differences between occupations to a large extent are driven by local demand and supply factors. Hence, general equilibrium effects in wage rates between occupations are likely to affect incentives to choose occupation, and hence education, and should be considered when the design of the tax system is analyzed.

This study responds to the criticism above, and analyzes optimal income taxation in the presence of occupational choices that determine the skill-formation in an economy where wage rates balance supply and demand for labor skill-types. Previous results in favor of a dual income tax system are confirmed within a framework where general equilibrium effects on wages are incorporated. However, results on the desirability for production efficiency emerge, as income taxation affects the skill-formation. The Diamond and Mirrlees (1971) production efficiency theorem brakes down when the government is unable to remove the entire distortion in choices of occupation by employing tuition subsidies. The pre-tax wage gap between high and low-skilled occupations is increased compared to a production efficient outcome, which is in contrast to previous results in the literature.

A new model is presented where general equilibrium effects on wages due to occupational choices are incorporated in an optimal taxation framework. The occupational choices generate adjustments in the occupation-specific wage rates, to satisfy a non-arbitrage condition, which states that present value lifetime income net of taxes and tuition costs is equal across occupations. However, the government is

unable to adjust the supply of labor skill-types by employing occupation specific instruments, as the government is assumed not to observe the skill-type of individuals. The analysis is restricted to residence based linear taxation of capital income, labor income, and tuition costs. The study further distinguishes between the case where the government has access to education subsidies that can correct distortions in choice of occupation, and a case where the use of education subsidies is restricted. The last case is interesting because governments generally lack full information on educational efforts. The restriction on the use of education subsidies is employed to represent this restriction on information.

When the government has full access to education subsidies to offset distortions on choices of occupation, efficient revenue raising implies that the tuition subsidy rate is set equal to the labor income tax rate and that there is no tax on capital income. This result strengthens the result in Bovenberg and Jacobs (2005), as general equilibrium effects due to occupational choices are allowed for in this study. The intuition is that the gain of choosing a high-skilled occupation, which consists of receiving a higher wage income, is reduced by the same amount as the cost of choosing a high-skilled occupation, which consists of paying higher tuition costs. Hence, the government is able to raise revenue without distorting the allocation of labor skill-types. There is no tax on capital income, because such taxation would distort the allocation of consumption over time.

When the access to the tuition subsidy rate is restricted to be lower than the labor income tax rate, due to lack of information on educational effort, the government cannot offset the distortions from the labor tax on occupational choices by means of educational subsidies. The government faces a trade-off between two distortions; Labor income taxation distorts the choice of occupation, and hence, the composition of skill-types, while capital income taxation distorts financial saving, and hence, the allocation of consumption over time. A positive tax on capital income becomes optimal. Hence, the result in Jacobs and Bovenberg (2005) holds in the presence of general equilibrium effects of occupational choices. The intuition is that the tax on capital income contributes to reverse the distortion in choice of occupation, as choosing the high-skilled occupation implies less financial saving compared to choosing the low-skilled occupation. Introducing a marginal tax on capital income is welfare improving, because the first-order distortion in composition of labor skill types is diminished at the cost of creating a second-order distortion in the allocation of consumption. However, it is not welfare improving to increase the capital income tax rate to a level that completely neutralizes the distortion in choice of occupation, as the first-order distortion in consumption is enhanced at the expense of correcting a second-order distortion in the allocation of labor-skill types. Efficient revenue

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raising implies, in line with Ramsey principles, a balance between the marginal welfare cost from the two sources of distortions in this economy.

This study also investigates whether the results derived imply a compression of the *pre-tax* wage gap between high and low-skilled occupations at the expense of efficiency in production. The production efficient allocation of labor skill types is desirable when the government has full access to education subsidies to offset distortions on choices of occupation. This result strengthens the Diamond and Mirrlees (1971) production efficiency theorem, because this theorem holds in a case where skill-specific taxation is excluded. The same conclusion is derived in Saez (2004). However, he employs occupational choices based on after tax (labor) income, while this study employs occupational choices based on net lifetime income. Production efficiency is no longer desirable when the tuition subsidy rate is restricted to be lower than the labor income tax rate. In fact, efficient revenue raising to finance government consumption implies that the wage gap between high and low-skilled occupations should be increased relative to a production-efficient outcome of wage rates. The higher wage-gap is generated by a lower relative supply of high-skilled labor. This lower relative supply is a result of the desirable distortion in choices of occupations. In contrast, Naito (1999) shows that indirect taxation aimed at compressing the wage-gap between high and low-skilled occupations is welfare enhancing even in the presence of a non-linear income tax. However, this result does not hold in the long run when agents choose occupation based on after-tax (labor) income (see Saez, 2004). Both Saez (2004) and Naito (1999) exclude skill-specific policy instruments in an economy where labor skill-types are imperfect substitutes in production.

Section 2 states the general assumptions. Efficient taxation is analyzed in section 3. Section 4 concludes and suggests some extensions for further research.

2. General Assumptions

Consider a small, open economy taking the interest rate in the rest of the world as given. The economy produces one tradable good, which is either invested as tuition costs or consumed. The price is set to unity. Individuals live for two periods. At the beginning of the first period, individuals choose a high-skill or a low-skill occupation. The supply of labor effort is normalized so that individuals in occupation *i* supply one unit of their skill type in the second period. These types of skill are imperfect substitutes in production. Labor is internationally immobile. International trade in the consumer good is not sufficient to determine factor prices in this economy, as one traded good is insufficient to determine factor prices on two non-traded input factors.

The education required for each occupation involves resources. z^i denotes the amount of resources needed per student in occupation i. The high-skilled occupation is assumed to require more resources per student than the low-skilled occupation, i.e. $z^2 > z^1$.

The following notation is followed throughout the paper.

$$n^{occupation}_{timeperiod}$$

Superscript denotes type of occupation (low-skill = 1, high-skill = 2) for variable n (= number of individuals). Subscript indicates time period. Superscript will be ignored for variables not characterized by skill type. Subscripts will be skipped for variables that are not characterized by either age or time of involvement in the market.

2.1 Households

Individuals are endowed with the same initial abilities, and maximize an identical Cobb-Douglas utility function[1] w.r.t. first and second period consumption, constrained by the present value of consumption being equal to their present value of lifetime income.

(1) $\qquad \max \quad c_1^{\gamma} c_2^{(1-\gamma)}$

w.r.t c_1, c_2, constrained by their budget constraint

$$c_1 + \left(\frac{1}{1+r(1-\tau_r)} \right) c_2 = W^i (1-\tau) \left(\frac{1}{1+r(1-\tau_r)} \right) - (1-s) z^i .$$

Where τ_r is the capital income tax rate, τ is the labor income tax rate, s is the tuition subsidy rate, and W^i is the occupation specific wage rate. The capital income tax is residence based hence individuals are taxed with the same rate on foreign and domestic-source income. A non-arbitrage condition for investments implies that the pre-tax return on domestic investment equals the interest rate abroad. The relative price between first and second period consumption is determined by capital income taxation and the given interest rate. A zero tax rate on capital income implies that the rate of substitution between first and second period consumption equals the rate of transformation. Present

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[1] The Cobb-Douglas functional form is chosen for both utility and production to simplify the calculations.

value lifetime income consists of the discounted second period wage income, minus first period tuition costs. The indirect utility function is given by

$$(2) \qquad \gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}\left[W^i(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-s)z^i\right],$$

and first period consumption is given by

$$c_1 = \gamma\left[W^i(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-s)z^i\right].$$

Individuals are assumed to choose occupation to maximize their expected lifetime income. In a market solution where the population is divided between both occupations, the expected wage rates have to adjust so that expected lifetime income net of taxes and tuition costs is equal in both occupations. Rational expectations are assumed, hence

$$(3) \qquad W^1(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-s)z^1 = W^2(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-s)z^2.$$

Since tuition cost is higher in the high-skilled occupation, it follows that the high-skilled wage rate exceeds the low-skilled wage rate. This represents the return on the extra tuition costs in the high-skilled occupation. Forgone earning is not included as a cost of education, as individuals are assumed to earn all wage income in their second period of life. However, including first period wage income to take account of forgone earnings would not change the functioning of this economy, as the non-arbitrage condition for education implies that the extra wage income earned by a high-skilled equals the extra tuition costs. Hence, results are not likely to be affected by this simplification.

The schools will supply the amount of education that the individuals demand, given that the individuals cover tuition costs net of tuition subsidies, $(1-s)z^i$.

2.2 Producers
The production function is given by

(4) $Y = n^{1^\alpha} n^{2^{1-\alpha}}$,

where n^i is the occupation specific supply of labor used in production. The number of individuals is fixed, and set to unity, and divided between the two types of occupations.

(5) $n^1 + n^2 = 1$.

Firms maximize present value of profits, taking factor prices as given.

The problem is

(6) $\max n^{1^\alpha} n^{2^{1-\alpha}} - W^1 n^1 - W^2 n^2$

w.r.t. n^1, n^2 .

The first order conditions are

(7) $\alpha \left(\dfrac{n^1}{n^2} \right)^{-(1-\alpha)} = W^1$,

(8) $(1-\alpha) \left(\dfrac{n^1}{n^2} \right)^{\alpha} = W^2$,

Due to the residence principle of taxation, the pre-tax return on domestic investments equals the foreign interest rate, and hence, is unaffected by taxation. Introduction of real capital into this model would not alter the first order conditions for labor due to this feature. Hence, real capital can be excluded from the production function without affecting any of the results obtained in this study.

2.3 General Equilibrium

By inserting the first order condition from the producer side, (7) and (8), into the non-arbitrage condition for education, (3), and substituting n^2 with $1 - n^1$, I get

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$$(9) \quad \alpha \left(\frac{n^1}{(1-n^1)} \right)^{-(1-\alpha)} (1-\tau) \left(\frac{1}{1+r(1-\tau_r)} \right) - (1-\tau)gz^1$$

$$= (1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha} (1-\tau) \left(\frac{1}{1+r(1-\tau_r)} \right) - (1-\tau)gz^2.$$

One immediate implication is that there are no distributional effects from taxation in this economy. Any tax change that favors one occupation generates a reallocation of individuals towards that occupation which results in wage rate adjustments that neutralize this favorable tax effect.

The effect of each of the policy parameters is as follows:
Uniform tuition subsidies generate a first round benefit to individuals that choose the high-skilled occupation relative to individuals that choose the low-skilled occupation, because high-skilled pay more tuition costs. However, this benefit is neutralized as more individuals choose the high-skilled occupation. The high-skilled wage rate is reduced, while the low-skilled wage rate is increased.

Proportional taxation of labor income generates a first round loss to individuals in the high-skilled occupation relative to individuals in the low-skilled occupation because high-skilled earn more labor income. However, more individuals are allocated into the low-skilled occupation, and the occupation specific wage rates are adjusted to neutralize this loss.

Capital income taxation generates a first round gain to individuals in the high-skilled occupation relative to individuals in the low-skilled occupation as high-skilled earn more second period income which becomes more valuable because it allows them to save less financially. Hence, more individuals are allocated into the high-skilled occupation, and the wage rates are adjusted to neutralize this gain. Note that the allocation of consumption over time is distorted by a tax on capital income.

2.4 The Government
The government chooses tax policy rates to maximize the indirect utility function of a high-skilled individual, constrained by a public budget constraint and the specification of the economy. Tax and subsidy changes are introduced in the beginning of the first period in the model. The public sector collects taxes to finance public consumption, G, and tuition subsidies. The non-arbitrage condition for education, (3), is included as a constraint implying that all individuals receive the same utility. The indirect utility function is maximized with respect to n^1 in addition to the tax rates due to this constraint. Constraining the tuition subsidy rate to be lower than the labor income tax rate is a second

best approach for incorporating a restriction on the government's ability to observe educational effort. Such a constraint is included into this framework by setting the share of tuition costs paid by individuals, $1-s$, proportionally to the share of wage income received by individuals, i.e. $1-s = (1-\tau)g$, where $g > 1$.

$$(10) \qquad MAX \ \gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2\right]$$

w.r.t τ, τ_r, n^1, so that

$$\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^1$$

$$= (1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2$$

and

$$\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^1 - \gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^1\right]\right]n^1$$

$$+\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^2 - \gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2\right]\right](1-n^1)$$

$$+\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\tau\left(\frac{1}{1+r}\right)-(z^1-(1-\tau)gz^1)\right]n^1$$

$$+\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\tau\left(\frac{1}{1+r}\right)-(z^2-(1-\tau)gz^2)\right](1-n^1) = G.$$

The general equilibrium effects of the occupation-specific wage rates are incorporated into the government maximization problem by replacing the occupation-specific wage rates with their marginal products from the producer side of the economy, (7) and (8). These marginal products are determined by the allocation of individuals between the occupations. The non-arbitrage condition for education, which constitutes the first constraint of the maximization problem, determines the

allocation of individuals between occupations for a given set of tax rates. The government budget constraint, which constitutes the second constraint of the maximization problem, states that the present value of tax revenue minus subsidies for tuition equals present value of government consumption (G). The first and second term are (present value) tax revenues from capital income taxation of individuals in occupation 1 and 2, respectively. First-period financial saving of individuals in occupation i, is multiplied by the number of individuals in occupation i, and by the interest rate, and the capital income tax rate, and then discounted. The third and forth term are tax revenues from labor income minus subsidies for tuition from individuals in each of the occupations.

2.5 The Foreign Sector

The current account is assumed to balance in this economy. This assumption is not represented by a separate equation in the model. However, domestic budget constraints and the market clearing conditions of the economy imply that the foreign budget constraint is binding. This is illustrated as follows: The second period production is given by the production function, (4). The Euler sentence and the first order conditions from the producer side, (7) and (8), imply that second period production equals the total factor income.

(11) $Y = W^1 n^1 + W^2 n^2$

Adding present value of consumption for all domestic individuals to the present value of government consumption and applying their budget constraints and the equilibrium conditions in the economy gives

$$\left[c_1 + z^1 n^1 + z^2 n^2 + G \right](1+r) + c_2 = Y .$$

Hence, the foreign budget is balanced when first period consumption plus tuition costs are financed by foreign loans that are repaid with interest in the second period.

3. Optimal Taxation

3.1 The First-best

The tax system is constrained to linear instruments, and does not include instruments that rely on variables that are individual-specific and/or non-observable to the government. First, pre-tax wage

income differences only rely on choice of occupation in this study. However, such differences are unobservable to the government in real life due to individual-specific variables. To avoid tax rules that rely on such individual-specific and non-observable variables, progressive taxation of labor income is excluded from the study. Second, individuals are only categorized by choice of occupation in this study. Hence, occupation specific tuition subsidies are equivalent to individual-specific lumpsum subsidies in this framework. Such individual-specific instruments are excluded from the study by only including uniform tuition subsidies. Third, uniform lumpsum taxation is excluded due to its unfortunate distributional effects.

Taxation creates two kinds of distortions in this economy. First, intertemporal consumption is distorted by capital income taxation as it reduces the return to financial saving. Second, the allocation of individuals between the two occupations is distorted by the tax parameters. A zero tax rate on capital income eliminates the distortion in intertemporal consumption.

The production efficient allocation of individuals between occupations is obtained by maximizing present value of production net of tuition costs

$$(12) \qquad \frac{n^{1\alpha}(1-n^1)^{1-\alpha}}{1+r} - z^1 n^1 - z^2(1-n^1)$$

w.r.t. n^1.

The first order conditions imply that

$$(13) \qquad \frac{\alpha\left(\dfrac{n^1}{1-n^1}\right)^{-(1-\alpha)}}{1+r} - z^1 = \frac{(1-\alpha)\left(\dfrac{n^1}{1-n^1}\right)^{\alpha}}{1+r} - z^2.$$

This condition states that the discounted value of one extra high-skilled individual minus tuition costs for this individual is equal to the discounted value of one extra low-skilled individual minus tuition cost for this individual.

The design of the tax system that generates production efficiency is found by inserting (13) into the non-arbitrage condition for education, where the first order conditions from the producer side are inserted, (9). The production efficient design is

$$(14) \qquad \frac{1-\tau}{1+r(1-\tau_r)} = \frac{1-s}{1+r} .$$

By implementing a zero tax rate on capital income into (14), it follows that a tuition subsidy rate equal to the labor income tax rate gives a production-efficient allocation of individuals. The labor income taxation leads to a larger tax payment from high-skilled individuals, as they earn more present value labor income. However, this extra tax burden is exactly offset by the extra tuition subsidies received due to a larger tuition cost. Hence, the government is able to implement the first-best allocation, and raise tax revenue at the same time.

3.2 Taxation in a Second-best Setting

The exclusion of factors liable to affect the design of the tax/ subsidy system leaves room for interesting extensions. Non-verifiable learning is not included in this study. The presence of such learning is likely to limit the government's ability to neutralize tax distortions on choice of occupation by employing tuition subsidies. The limited ability to neutralize tax distortions by employing tuition subsidies can be represented by constraining the tuition subsidy rate to be lower than the labor income tax rate (see Jacobs and Bovenberg, 2005). This constraint reduces the incentive to choose the high-skilled occupation. The incentive is reduced because individuals pay a larger share of the tuition cost, and the tuition cost is higher in the high-skilled occupation. The subsequent reallocation of individuals from the high-skilled occupation, to the low-skilled occupation, restores the non-arbitrage condition for education by increasing the high-skilled wage rate, and lowering the low-skilled wage rate.

This preset distortion in choice of occupation forces the government to consider a tradeoff between the two sources of distortions in the economy. The government is left with capital income taxation as a means to reduce the distortion in choice of occupation. Since individuals in the low-skilled occupation borrow less financially, they harvest a smaller first round gain as a result of capital income taxation, compared to individuals in the high-skilled occupation. To restore the non-arbitrage condition for education, more individuals choose the high-skilled occupation, and the wage rates are adjusted. However, as second-period consumption becomes more expensive relative to first-period

consumption, individuals trade off second-period consumption for first-period consumption, hence, intertemporal consumption is distorted.

The first task is to find out whether capital income should be taxed in this second best setting. The tradeoffs involved are given by the first order conditions from the Lagrangian of the government maximization problem. The mechanism that generates the optimal dual income tax structure is complex, as the trade-offs are numerous. However, an investigation of the first order condition with respect to the capital income tax rate provides some useful insight. The condition states that (see appendix):

(15)

$$
\begin{aligned}
\frac{\partial L}{\partial \tau_r} &= \gamma^\gamma (1-\gamma)^{(1-\gamma)}(1-\gamma)\left(1+r(1-\tau_r)\right)^{-\gamma} r(1-\tau)gz^2 \\
&+ \gamma^\gamma (1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{-\gamma} \gamma(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{r}{1+r(1-\tau_r)}\right) \\
&- \mu\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{r}{\left(1+r(1-\tau_r)\right)^2}\right) - (1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{r}{\left(1+r(1-\tau_r)\right)^2}\right)\right] \\
&- \lambda\left[\frac{r}{(1+r)}\left[-(1-\tau)gz^1 - \gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right) - (1-\tau)gz^1\right]\right]n^1 \right. \\
&- \frac{r\tau_r}{(1+r)}\gamma\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{r}{\left(1+r(1-\tau_r)\right)^2}\right)n^1 \\
&+ \frac{r}{(1+r)}\left[-(1-\tau)gz^2 - \gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right) - (1-\tau)gz^2\right]\right](1-n^1) \\
&- \left.\frac{r\tau_r}{(1+r)}\gamma(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{r}{\left(1+r(1-\tau_r)\right)^2}\right)(1-n^1)\right] = 0
\end{aligned}
$$

The first two terms on the right hand side of (15) constitute the direct effect on the utility of a high skilled individual. This effect is positive, as financial saving is negative. Capital income taxation operates like a subsidy on financial loans, and hence, increases the utility. The term in the third line of (15) incorporate effects generated by the non-arbitrage condition for education. The Lagrangian parameter connected to the non-arbitrage condition for education, μ, is multiplied with the effect on the non-arbitrage condition. This effect is negative because a marginal increase in the capital income tax rate favors the high-skilled occupation. The subsequent general equilibrium effects on wage rates

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result in a direct loss of utility for high-skilled individuals. The reallocation of individuals from the low-skilled to the high-skilled occupation contributes to lower the capital income tax revenue as a high-skilled individual borrows more financially, to increase the labor income tax revenue as a high-skilled earns more labor income, and to increase tuition subsidy payments as a high-skilled receives more tuition subsidies. The subsequent wage decrease for high-skilled individuals generates less labor income tax revenue, while the negative financial saving is reduced. The subsequent wage increase for low-skilled individuals generates more labor income tax revenue, and more negative financial saving. These general equilibrium effects are accounted for by the term in the third line. The terms on the fourth, fifth, sixth and seventh line of (15) constitutes the direct effect on the government budget of a marginal increase in the capital income tax rate. The term on the fourth and sixth line constitute the drop in tax revenue as individuals in occupation 1 and 2, respectively, borrow financially. The term on the fifth and seventh line represents a drop in the tax revenue as individuals in occupation 1 and 2, respectively, increase their first period consumption, and hence, increase their financial loans. Hence, a marginal increase in the capital income tax rate generates a first round loss of tax revenue within this model. The first order condition is satisfied when these negative tax revenue effects, weighted with the Lagrangian parameter associated with the government budget, λ, are identical to the positive utility effect plus the factor accounting for the general equilibrium effects.

The labor income tax rate, and hence, the share of tuition costs paid by individuals, are adjusted to balance the government budget. This adjustment has a direct utility effect. Nevertheless, the allocation of individuals between occupations is unaffected, as the distortion in choice of occupation remains unaffected. This can be illustrated formally by removing the term $(1 - \tau)$ in the non-arbitrage condition for education. This feature of the model is related to the restriction on the tuition subsidy rate set to represent the lack of information on educational effort. The distortion due to lack of information is assumed to be unaffected by the labor income tax level, as formal educational support can be adjusted according to the income tax level.

The question, whether capital income should be taxed, is answered by investigating whether the first order conditions from the Lagrangian of the government maximization problem are violated when a zero tax rate on capital income is implemented into these conditions. The derivative of the Lagrangian with respect to the capital income tax rate becomes:

$$(16) \qquad \frac{\partial L}{\partial \tau_r} = \frac{-r\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}(1+r)^{1-\gamma}g(z^1-z^2)^2 n^1(1-g)(1-n^1)^2}{\alpha(1-\alpha)\left(\dfrac{n^1}{1-n^1}\right)^{\alpha-1}}$$

(see appendix for calculations).

This expression is positive when $g > 1$ (i.e. when $\tau > s$). Hence, a zero tax rate on capital income is not optimal. By treating the capital income tax rate as an exogenous parameter, and employing the envelope theorem on the Lagrangian of the government maximization problem, it follows that imposing a positive marginal tax on capital income is welfare enhancing. The intuition is that the welfare cost of introducing a marginal distortion in intertemporal consumption must be smaller than the welfare gain of reducing the infra-marginal distortion in choice of occupation created by a tuition subsidy rate lower than the labor income tax rate. Capital income taxation generates a revenue loss as a result of financial loans held by individuals. Hence, the result holds despite of this negative tax revenue effect. The case for a positive tax on capital income is strengthened by this result, as the argument in favor of a tax on capital income given by Jacobs and Bovenberg (2005) holds in the presence of occupational choices that determine the skill-formation in an economy where wage rates balance supply and demand for labor skill-types. The result also supports the conclusion in Erosa and Gervais (2002), which demonstrates that optimal capital taxes are positive if leisure and consumption are more compliments later in life than they are earlier in life. Another strand of literature finds that capital income should be taxed as it contributes to correct an incomplete capital and/ or insurance market (see Aiyagari, 1995 and Golosov, Kocherlakota and Tsyvinski, 2003).

The subsequent marginal cost of public funds, defined as $-\dfrac{\lambda}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}}$, where λ is the Lagrangian parameter for the government budget constraint, and $\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}$ is the marginal utility of present value income, equals unity when the capital income tax rate is zero.

The second task is to investigate whether the capital income tax rate should be increased to the level where production efficiency in labor skill-types is implemented. This question is equivalent to the question of whether general equilibrium effects of occupation-specific wage rates should be exploited to raise revenue. When the production efficient allocation of individuals between occupations, given by (13), is implemented into the first order conditions for a maximum, the derivative of the Lagrangian with respect to the capital income tax rate becomes:

(17)
$$\frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}g}{1+r(1-\tau_r)}(\gamma-1)\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^{\alpha}\frac{1}{1+r}-z^2\right]\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]$$

(see appendix for calculations).

This expression is negative when $g > 1$ (which imply that $\tau_r > 0$), since $(\gamma-1)$ is the only negative factor. Hence, it is not optimal to increase the tax rate on capital income to a level that implements the production efficient allocation of labor skill types. This conclusion is opposite to that in Saez (2004).

Note that the design of the tax system that generates production efficiency, (14), becomes a condition on the capital income tax rate:

(18)
$$\frac{1}{1+r(1-\tau_r)} = \frac{g}{1+r}$$

Hence, production efficiency can be implemented by fixing the capital income tax rate. The envelope theorem implies that the welfare is increased by a marginal reduction in this fixed capital income tax rate level. The intuition for this result is that the welfare effect from the production side becomes marginal as the allocation of individuals is approaching the production-efficient allocation. However, imposing a capital income tax required to obtain the production efficient allocation, generates a welfare cost due to the distortion in intertemporal consumption that exceeds the marginal effect from the producer side.

As it is optimal to reduce the allocation of high-skilled individuals relative to the production efficient allocation, it follows that the wage difference between high and low-skilled should be increased compared to a production-efficient outcome. This conclusion is opposite to that in Naito (1999), which finds that occupation specific wage rates should be compressed to achieve redistribution. The result is also related to Stiglitz (1982), and Allen (1982), who find that production efficiency may no longer be desirable when labor supply is elastic.

The subsequent marginal cost of public funds is larger than unity in this case, as

$$\lambda = -\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}\left(\frac{1+r}{1+r-r\tau_r(1-\gamma)}\right),$$

where the expression in the last bracket is larger than unity.

4. Conclusion and Possible Extensions

This study introduces occupational choices based on lifetime income in a framework where labor skill types are imperfect substitutes in production. A tuition subsidy rate equal to the labor income tax rate, combined with a zero tax rate on capital income, is found to be optimal. Optimal taxation in the case where the tuition subsidy rate is constrained to be lower than the labor income tax rate, induces the government to trade efficiency in production against distortions in intertemporal consumption. The capital income tax rate is used as a second-best instrument to reduce the distortion in choice of occupation. Hence, the result in Jacobs and Bovenberg (2005) holds in a framework where general equilibrium effects on occupation-specific wage rates are incorporated. The solution implies increased wage differences between high and low-skilled occupations compared to a production-efficient outcome. This conclusion is opposite to that in Saez (2004) and Naito (1999).

The analysis does not cover all aspects of optimal income taxation and subsidies for tuition. The transition period, i.e. the period it takes to adjust the relative supply of high and low-skilled labor, is not considered in this study. However, an increase in the supply of low-skilled labor is likely to occur within a relatively short period of time, as more young unskilled individuals can enter directly into the labor force. Adjustments in the supply of high-skilled labor is likely to occur more gradually, as individuals need to spend time in school to become high-skilled workers. Introducing liquidity constraints is an interesting extension. Another interesting aspect is how the allocation of high and low-skilled labor affects the growth rate. Romer (1990) argues that wage rates for high-skilled labor are lower than their productivity because of positive external effects from employing them in the research sector. Hence, in a second-best solution, where the government has no direct instrument to affect the allocation of high-skilled into the research sector, the education of high-skilled individuals should be subsidized. Further, including ability differences and distributional considerations could reveal interesting tradeoffs between efficiency and equity.

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The Lagrangian of the government maximization problem is

(A1)

$$L = \gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2\right]$$

$$-\mu\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^1-\left((1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2\right)\right]$$

$$-\lambda\left[\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^1-\gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^1\right]\right]n^1$$

$$+\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^2-\gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)()-(1-\tau)gz^2\right]\right](1-n^1)$$

$$+\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\tau\left(\frac{1}{1+r}\right)-(z^1-(1-\tau)gz^1)\right]n^1$$

$$+\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\tau\left(\frac{1}{1+r}\right)-(z^2-(1-\tau)gz^2)\right](1-n^1)-G],$$

where μ and λ denote the Lagrange multipliers of the non-arbitrage condition for educational choices and the government budget constraint respectively.

The first order conditions for a maximum are given by:

$$\frac{\partial L}{\partial \tau} = -\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\left(\frac{1}{1+r(1-\tau_r)}\right)-gz^2\right]$$

$$-\mu\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\left(\frac{(-1)}{1+r(1-\tau_r)}\right)+gz^1+(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\left(\frac{1}{1+r(1-\tau_r)}\right)-gz^2\right]$$

$$-\lambda\left[\frac{r\tau_r}{(1+r)}\left[gz^1+\gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\left(\frac{1}{1+r(1-\tau_r)}\right)-gz^1\right]\right]n^1\right.$$

(A2)

$$+\frac{r\tau_r}{(1+r)}\left[gz^2+\gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\left(\frac{1}{1+r(1-\tau_r)}\right)-gz^2\right]\right](1-n^1)$$

$$+\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\left(\frac{1}{1+r}\right)-gz^1\right]n^1$$

$$+\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\left(\frac{1}{1+r}\right)-gz^2\right](1-n^1)\right]=0$$

(A3)

$$\frac{\partial L}{\partial n^1} = \gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} \alpha(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha-1} (1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)\frac{1}{(1-n^1)^2}$$

$$-\mu\left[-(1-\alpha)\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)-1}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)\frac{1}{(1-n^1)^2}-\alpha(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha-1}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)\frac{1}{(1-n^1)^2}\right]$$

$$-\lambda\left[\frac{r\tau_r}{(1+r)}\gamma(1-\alpha)\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)-1}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)\frac{n^1}{(1-n^1)^2}\right.$$

$$+\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^1-\gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^1\right]\right]$$

$$-\frac{r\tau_r}{(1+r)}\gamma\alpha(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha-1}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)\frac{1-n^1}{(1-n^1)^2}$$

$$-\frac{r\tau_r}{(1+r)}\left[-(1-\tau)gz^2-\gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right)-(1-\tau)gz^2\right]\right]$$

$$-(1-\alpha)\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)-1}\tau\left(\frac{1}{1+r}\right)\frac{n^1}{(1-n^1)^2}$$

$$+\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}\tau\left(\frac{1}{1+r}\right)-(z^1-(1-\tau)gz^1)$$

$$+\alpha(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha-1}\tau\left(\frac{1}{1+r}\right)\frac{1-n^1}{(1-n^1)^2}$$

$$\left.-(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\tau\left(\frac{1}{1+r}\right)+z^2-(1-\tau)gz^2\right]=0$$

(A4)

$$\frac{\partial L}{\partial \tau_r} = -\gamma^\gamma (1-\gamma)^{(1-\gamma)}(1-\gamma)\big(1+r(1-\tau_r)\big)^{-\gamma} r\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha (1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right) - (1-\tau)gz^2\right]$$

$$+\gamma^\gamma (1-\gamma)^{(1-\gamma)}\big(1+r(1-\tau_r)\big)^{(1-\gamma)}(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha (1-\tau)\left(\frac{r}{(1+r(1-\tau_r))^2}\right)$$

$$-\mu\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{r}{(1+r(1-\tau_r))^2}\right) - (1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha (1-\tau)\left(\frac{r}{(1+r(1-\tau_r))^2}\right)\right]$$

$$-\lambda\left[\frac{r}{(1+r)}\left[-(1-\tau)gz^1 - \gamma\left[\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right) - (1-\tau)gz^1\right]\right]n^1\right.$$

$$-\frac{r\tau_r}{(1+r)}\gamma\alpha\left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)}(1-\tau)\left(\frac{r}{(1+r(1-\tau_r))^2}\right)n^1$$

$$+\frac{r}{(1+r)}\left[-(1-\tau)gz^2 - \gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha (1-\tau)\left(\frac{1}{1+r(1-\tau_r)}\right) - (1-\tau)gz^2\right]\right](1-n^1)$$

$$\left.-\frac{r\tau_r}{(1+r)}\gamma(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha (1-\tau)\left(\frac{r}{(1+r(1-\tau_r))^2}\right)(1-n^1)\right] = 0$$

Proof That Capital Income Taxation Is Optimal.

Insert $\tau_r = 0$ and the non-arbitrage condition for education into the first order condition for optimum to check whether they are violated. (A2) becomes

(A5)
$$\frac{\partial L}{\partial \tau} = -\gamma^\gamma (1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} - gz^2\right]$$
$$-\lambda\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} - gz^2\right] = 0$$

hence

(A6) $\lambda = -\gamma^\gamma (1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}$

Inserting the solution for λ in (A6), $\tau_r = 0$, and the non-arbitrage condition for education into the foc. for individuals, (A3), gives

$$(A7) \quad \frac{\partial L}{\partial n^1} = \frac{1}{(1-n^1)^2} \left[\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1+r)^{(1-\gamma)} \alpha(1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \frac{(1-\tau)}{1+r} \right.$$

$$+ \mu \alpha (1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \frac{(1-\tau)}{1+r} \frac{1}{n^1}$$

$$+ \gamma^\gamma (1-\gamma)^{(1-\gamma)} (1+r)^{(1-\gamma)} \left[-(1-\alpha)\alpha \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \tau \left(\frac{1}{1+r} \right)(1-n^1) \right.$$

$$+ \alpha(1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \tau \left(\frac{1}{1+r} \right)(1-n^1)$$

$$\left. \left. - (1-g)(z^1 - z^2)(1-n^1)^2 \right] \right] = 0$$

hence

$$(A8) \quad \frac{\partial L}{\partial n^1} = \frac{1}{(1-n^1)^2} \left[\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1+r)^{(1-\gamma)} \alpha(1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \frac{(1-\tau)}{1+r} \right.$$

$$+ \mu \alpha (1-\alpha) \left(\frac{n^1}{(1-n^1)} \right)^{\alpha-1} \frac{(1-\tau)}{1+r} \frac{1}{n^1}$$

$$\left. - \gamma^\gamma (1-\gamma)^{(1-\gamma)} (1+r)^{(1-\gamma)} (1-g)(z^1 - z^2)(1-n^1)^2 \right] = 0$$

hence

$$(A9) \quad \mu = -\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1+r)^{(1-\gamma)} n^1 \left[1 - \frac{(1-g)(z^1 - z^2)(1-n^1)^2}{\alpha(1-\alpha) \left(\frac{n^1}{1-n^1} \right)^{\alpha-1} \frac{(1-\tau)}{1+r}} \right]$$

Inserting the solution for μ in (A9), λ in (A6), $\tau_r = 0$, and the non-arbitrage condition for education into the foc. for capital income taxation, (A4), gives

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(A10)

$$\frac{\partial L}{\partial \tau_r} = \frac{r}{1+r}\left[-\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1-\gamma)(1+r)^{1-\gamma}\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha\frac{(1-\tau)}{1+r}-(1-\tau)gz^2\right]\right.$$

$$+\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha\frac{(1-\tau)}{1+r}$$

$$+\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}n^1\left[1-\frac{(1-g)(z^1-z^2)(1-n^1)^2}{\alpha(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^{\alpha-1}\frac{(1-\tau)}{1+r}}\right](1-\tau)g(z^1-z^2)$$

$$\left.+\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}\left[-(1-\tau)g(z^1-z^2)n^1-(1-\tau)gz^2-\gamma\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha\frac{(1-\tau)}{1+r}-(1-\tau)gz^2\right]\right]\right]$$

hence

(A11)
$$\frac{\partial L}{\partial \tau_r} = \frac{r}{1+r}\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}\left[n^1\left[1-\frac{(1-g)(z^1-z^2)(1-n^1)^2}{\alpha(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^{\alpha-1}\frac{(1-\tau)}{1+r}}\right](1-\tau)g(z^1-z^2)\right.$$

$$\left.-(1-\tau)g(z^1-z^2)n^1\right]$$

hence

(A12)
$$\frac{\partial L}{\partial \tau_r} = -\frac{r\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r)^{(1-\gamma)}g(z^1-z^2)^2 n^1(1-g)(1-n^1)^2}{\alpha(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^{\alpha-1}}$$

This expression is larger than zero, as there is a minus sign in front of the expression, and $(1-g)$ is the only negative expression.

Proof That Production Efficiency Is Not Part of an Optimal Solution.

The first order condition for production efficiency implies that

$$\text{(A13)} \qquad \alpha\left(\frac{n^1}{1-n^1}\right)^{-(1-\alpha)} - (1-\alpha)\left(\frac{n^1}{1-n^1}\right)^{\alpha} = (z^1 - z^2)(1+r)$$

Inserting this solution into the non-arbitrage condition for education gives

$$\text{(A14)} \qquad \frac{1}{1+r(1-\tau_r)} = \frac{g}{1+r}$$

These solutions and the non-arbitrage condition for education are inserted into the first order condition for maximum to check whether production efficiency is part of an optimal solution. (A2) becomes

$$
\begin{aligned}
\text{(A15)} \qquad \frac{\partial L}{\partial \tau} = &-\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)} g\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\frac{1}{1+r} - z^2\right] \\
&-\lambda\left[\frac{r\tau_r}{(1+r)}\left[gn^1(z^1-z^2) + \gamma g\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\frac{1}{1+r} - z^2\right] + gz^2\right] \\
&+(z^1-z^2)n^1 - g(z^1-z^2)n^1 \\
&+(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}(\frac{1}{1+r}) - gz^2\right] = 0
\end{aligned}
$$

Inserting from the production efficient tax combination, (A14), into (A15) gives

$$
\begin{aligned}
\text{(A16)} \qquad \frac{\partial L}{\partial \tau} = &-\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)} g\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\frac{1}{1+r} - z^2\right] \\
&-\lambda\left[(1+\frac{r\tau_r\gamma g}{(1+r)})\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^{\alpha}\frac{1}{1+r} - z^2\right]\right] = 0
\end{aligned}
$$

hence

$$\text{(A17)} \qquad \lambda = \frac{-\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}(1+r(1-\tau_r))^{(1-\gamma)} g}{(1+\frac{r\tau_r\gamma g}{1+r})} = -\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}\frac{(1+r)}{(1+r-r\tau_r(1-\gamma))}$$

Inserting for the production efficient allocation, (A13), and the non-arbitrage condition for education into (A3) gives

$$
\begin{aligned}
\frac{\partial L}{\partial n^1} = \frac{1}{(1-n^1)^2} &\left[\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} \alpha(1-\alpha) \left(\frac{n^1}{(1-n^1)}\right)^{\alpha-1} (1-\tau) \left(\frac{1}{1+r(1-\tau_r)}\right) \right. \\
&+ \mu(1-\alpha)\alpha \left(\frac{n^1}{(1-n^1)}\right)^{-(1-\alpha)} (1-\tau) \left(\frac{1}{1+r(1-\tau_r)}\right) \frac{1}{n^1} \\
&\left. + \lambda \left[\frac{r\tau_r}{(1+r)}(1-\tau)g(z^1-z^2) + (1-\tau)(z^1-z^2) - (1-\tau)g(z^1-z^2) \right] \right] = 0
\end{aligned}
$$

(A18)

Inserting from the production efficient tax combination, (A14), into (A18) gives

(A19) $\qquad \mu = -\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} n^1$

Inserting from the production efficient tax combination, (A14), the non-arbitrage condition for education, and the production efficient allocation, (A13), into (A4) gives

$$
\begin{aligned}
\frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)} &\left[-\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1-\gamma) \left(1+r(1-\tau_r)\right)^{-\gamma} \left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha - (1+r)z^2 \right] \right. \\
&+ \gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} (1-\alpha) \left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r(1-\tau_r)} \\
&- \mu \frac{1}{1+r(1-\tau_r)}(z^1-z^2)(1+r) \\
&+ \lambda \left[(z^1-z^2)n^1 + \gamma \left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} - z^2 \right] + z^2 \right. \\
&+ \frac{r\tau_r\gamma}{1+r(1-\tau_r)}(z^1-z^2)n^1 \\
&\left. \left. + \frac{r\tau_r\gamma}{1+r}\left(\frac{1}{1+r(1-\tau_r)}\right)(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \right] \right] = 0
\end{aligned}
$$

(A20)

Inserting for the production efficient tax combination, (A14), λ in (A17), and μ in (A19), into (A20) gives

$$(A21) \quad \frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)} \left[-\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1-\gamma)(1+r(1-\tau_r))^{-\gamma} \left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha - (1+r)z^2 \right] \right.$$

$$+\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} (1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r(1-\tau_r)}$$

$$+\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} g(z^1-z^2)n^1$$

$$\frac{-\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} g}{\left(1+\dfrac{r\tau_r\gamma g}{1+r}\right)} \left[\gamma\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha \frac{1}{1-r} - z^2 \right] + z^2 \right]$$

$$+\left(1+\frac{r\tau_r\gamma g}{1+r}\right)(z^1-z^2)n^1$$

$$\left. +\frac{r\tau_r\gamma g}{1+r}(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} \right]$$

Inserting from the production efficient tax combination, (A14), into (A21) gives

$$(A22) \quad \frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)} \left[-\gamma^\gamma (1-\gamma)^{(1-\gamma)} (1-\gamma)\left(1+r(1-\tau_r)\right)^{1-\gamma} g \left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} - z^2 \right] \right.$$

$$+\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} g(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r}$$

$$+\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} g(z^1-z^2)n^1$$

$$\frac{-\gamma^\gamma (1-\gamma)^{(1-\gamma)} \left(1+r(1-\tau_r)\right)^{(1-\gamma)} g}{\left(1+\dfrac{r\tau_r\gamma g}{1+r}\right)} \left[\gamma\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha \frac{1}{1-r} - z^2 \right] + z^2 \right]$$

$$+\left(1+\frac{r\tau_r\gamma g}{1+r}\right)(z^1-z^2)n^1$$

$$\left. +\frac{r\tau_r\gamma g}{1+r}(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha \frac{1}{1+r} \right]$$

hence

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$$\frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)}\left[\gamma^\gamma(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}gz^2\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]\right.$$

$$+\gamma^\gamma(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}g\gamma\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha\frac{1}{1+r}-z^2\right]\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]$$

(A23)

$$+\gamma^\gamma(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}g(z^1-z^2)n^1\left[1-\frac{1+\frac{r\tau_r\gamma g}{1+r}}{1+\frac{r\tau_r\gamma g}{1+r}}\right]$$

$$\left.-\gamma^\gamma(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{(1-\gamma)}g(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha\frac{1}{1+r}\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]\right]$$

hence

$$\frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)}\gamma^\gamma(1-\gamma)^{(1-\gamma)}\left(1+r(1-\tau_r)\right)^{1-\gamma}g\left[\gamma\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha\frac{1}{1+r}-z^2\right]\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]\right.$$

(A24)

$$\left.-\left[(1-\alpha)\left(\frac{n^1}{(1-n^1)}\right)^\alpha\frac{1}{1+r}-z^2\right]\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]\right]$$

hence

(A25)

$$\frac{\partial L}{\partial \tau_r} = \frac{r(1-\tau)}{1+r(1-\tau_r)}\gamma^\gamma(1-\gamma)^{(1-\gamma)}(1+r(1-\tau_r))^{1-\gamma}g(\gamma-1)\left[(1-\alpha)\left(\frac{n^1}{1-n^1}\right)^\alpha\frac{1}{1+r}-z^2\right]\left[1-\frac{1}{1+\frac{r\tau_r\gamma g}{1+r}}\right]$$

$\frac{\partial L}{\partial \tau_r}$ is negative when $g>1$ (which implies a strictly positive τ_r) because all factors are positive except $(\gamma-1)$ which is strictly negative. Hence, production efficiency is not part of a maximum.

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