Discussion Papers No. 456, April 2006
Statistics Norway, Statistical Methods and Standards

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On some common practices of systematic sampling

Abstract:
Systematic sampling is a widely used technique in survey sampling. It is easy to execute, whether the units are to be selected with equal probability or with probabilities proportional to auxiliary sizes. It can be very efficient if one manages to achieve favourable stratification effects through the listing of units. The main disadvantages are that there is no unbiased method for estimating the sampling variance, and that systematic sampling may be poor when the ordering of the population is based on inaccurate knowledge. In this paper we examine an aspect of the systematic sampling that previously has not received much attention. It is shown that in a number of common situations, where the systematic sampling has on average the same efficiency as the corresponding random sampling alternatives under an assumed model for the population, the sampling variance fluctuates much more with the systematic sampling. The use of systematic sampling is associated with a risk that in general increases with the sampling fraction. This can be highly damaging for large samples from small populations in the case of single-stage sampling, or large sub-samples from small sub-populations as in the case of multi-stage sampling.

Keywords: Statistical decision; second order Bayes risk; Robust design; Panel survey.

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1 Introduction

Systematic sampling has a long tradition in survey sampling (e.g. Madow and Madow, 1944; Madow, 1949, 1953). When applied to a list of units it is known as the *every kth* rule, where k refers to the sampling interval. Where the ordering of the units is conceivably uncorrelated with the survey variable of interest, or contains at most a mild stratification effect, the systematic sampling is generally considered as a convenient substitute for simple random sampling "with little expectation of a gain in precision" (Cochran, 1977, p. 229). The same holds for sampling within strata or sub-sampling under a multi-stage sampling design. By a modification (Madow, 1949) where the sampling interval is calculated in terms of an accumulated auxiliary total, the systematic sampling can be used to select a πps sample with great ease.

In situations where auxiliary information is available for partial ordering of the population, it is more natural to compare systematic sampling with stratified random sampling. The systematic sampling is more convenient especially because it is neither subjected to restrictions on the number of auxiliary variables, nor the number of levels each of them may take. So there is less need for variable selection as may be necessary in the case of stratified random sampling. By using many auxiliary variables the systematic sampling can introduce greater balance into the sample, although a more parsimonious stratified sampling design may be just as efficient.

It is important to be clear that when speaking of the efficiency of the systematic sampling above, we are referring to its sampling variance *in expectation*. Take for instance the case where systematic sampling is applied to a fixed, conceivably random ordering of a given population. The sampling variance, denoted by V_{sys}, is based on only k possible systematic samples, and is either larger or smaller than that of the simple random sampling for the given population, denoted by V_{srs}. There are two results which show that V_{sys} may be equal to V_{srs} in expectation. In the first case, considering the fixed ordering to be randomly chosen from all $N!$ possible permutations of the N units of a finite population, Madow and Madow (1944) showed that $E(V_{sys}) = V_{srs}$, where the expectation is taken over all permutations. Notice that V_{srs} is a constant of the permutations. Example 3.4.2 of Särndal, Swensson, and Wretman (1992) provides a simple illustration on how greatly V_{sys} may vary for different population orderings. In the second case, we regard the ordering of the population as fixed, and the associated values of interest as realizations of independent random variables with constant mean (Cochran, 1977, Theorem 8.5). It can then be shown that $E(V_{sys}) = E(V_{srs})$, with the expectation being over all possible finite populations under the assumed model.

To clarify the choice between systematic and simple random sampling in the situation above, we rephrase it as a decision problem. Let $\theta = (y_1, ..., y_N)^T$ be the vector of variables of interest, where $U = (1, ..., N)$ is a particular ordering of the units prepared for the systematic sampling. Given θ (and U), we can choose to draw a systematic sample, or we can choose to draw a simple random sample (which does not depend on U). These are the two decision rules, or actions, available to us, denoted by δ = SYS and δ = SRS, respectively. Let the sampling variance of the sample mean be our loss function, denoted by $L(\theta, \delta) = V_\delta(\bar{y}_s)$, where s denotes the selected sample and $\bar{y}_s$ the sample mean. Notice that this is a no-data problem, so that the frequentist risk of δ is equal to the loss function (Berger, 1985), denoted by $R(\theta, \delta) \equiv L(\theta, \delta)$. Now, depending on the actual θ (and U), $R(\theta, \mathsf{SYS}) = V_{sys}$ may be greater or less than $R(\theta, \mathsf{SRS}) = V_{srs}$, i.e. none of them is R-better than the other for all θ. Indeed, $R(\theta, \delta)$ can be arbitrarily large as long as there is no limit on how much variation θ can have, such that e.g. the minimax principle is not well defined without further restrictions.

It is however possible to invoke other decision principles. For instance, denote by $r(\pi, \delta) = E_\theta(V_\delta(\bar{y}_s))$ the Bayes risk of δ with respect to some assumed distribution of θ, denoted by $\pi(\theta)$, i.e. the expected sampling variance induced by δ in this case. Then, according to the Bayes risk principle, the decision rule SYS may be preferred to SRS if $r(\pi, \mathsf{SYS}) < r(\pi, \mathsf{SRS})$. However, as we have seen, $r(\pi, \mathsf{SYS}) = r(\pi, \mathsf{SRS})$ under the two models of θ above so that the two actions are equivalent w.r.t. the Bayes risk. Thus, the decision can not be based on the Bayes risk principle alone, but an additional criterion of cost, or easiness in execution, is invoked to motivate the choice of SYS in practice. Notice that, since the conditional Bayes decision principle gives the same answer as the Bayes risk principle to a no-data problem (Berger, 1985), there is no difference between a Bayesian and a frequentistic treatment of the problem here.

Now, there are at least two reasons for which a reconsideration of the choice of SYS in the situation above may be appropriate. In the first place, due to the development in computational power and alternative random sampling techniques, easiness in execution is no longer a valid argument in favor of the systematic sampling. Using a computer one can draw a simple random sample as easily as a systematic sample. The same goes for πps sampling. For instance, the sequential Poisson sampling (SPS, Ohlsson, 1998) is easy to implement, yielding an approximate πps sample with a fixed sample size. Secondly, easiness in execution counts only if there are no other more important decision principles that can be used to distinguish between the two actions. So the choice can not be settled before we have considered the following question: Is there any other reasonable decision principle that we may follow in this case, apart from the minimax principle, the Bayes risk principle and the conditional Bayesian decision principle?

The situation we are considering here has an analogy in the Utility theory. Suppose that one is offered a 50-50 lottery between 0 and 100 pounds. The expected utility is 50 pounds. It is unlikely, however, that one is entirely indifferent between accepting the lottery and accepting 50 pounds for sure. In the Utility theory, a decision maker is risk averse if he prefers to accept the 50 pounds for sure than to enter the lottery; whereas he is risk prone if he prefers to enter the lottery instead (French, 1986). For statistical decisions, however, we can motivate the same kind of distinction without reference to the lottery scenario. Let

$$d(\pi, \delta) = V_\theta(R(\theta, \delta)) \tag{1}$$

be the *second order Bayes risk* of a decision rule δ w.r.t. $\pi(\theta)$. While the (first order) Bayes risk is the expectation of the risk w.r.t. $\pi(\theta)$, the second order Bayes risk is its variance. It is non-negative by definition. In the case of zero second order Bayes risk, the risk of a decision rule is the same no matter the value of θ. The smaller the second order Bayes risk, the more robust a decision rule is as θ varies. A decision rule δ is preferred to another δ' according to the *robust decision principle* if

$$r(\pi, \delta) = r(\pi, \delta') \qquad \text{and} \qquad d(\pi, \delta) < d(\pi, \delta') \tag{2}$$

That is, provided two rules have the same expected risk, we will choose the one that has less variation around the expected risk, on the ground of its robustness towards θ.

In the situation above, we have two sampling designs to choose from, which have the same Bayes risk under the assumed $\pi(\theta)$. The second order Bayes risk is $d(\pi, \delta) = V_\theta(V_\delta(\bar{y}_s))$. It follows that if we choose between SYS and SRS according to the robust decision principle, we will have tighter control over the actual sampling variance over all possible θ. Notice that the second order Bayes risk is a measure of robustness *given* $\pi(\theta)$. It is different from robustness towards mis-specification of $\pi(\theta)$, which is a standard robustness concept in the statistical decision theory. Thus, a decision rule δ may be preferred to another δ' according to the robust decision principle provided the conditions in (2) hold based on the assumed $\pi(\theta)$. Whereas what happens to the choice as $\pi(\theta)$ varies is another robustness concern, i.e. robustness towards mis-specification of $\pi(\theta)$. A numerical illustration will be provided later where both types of robustness are brought into consideration at the same time.

In the rest of the paper we will mainly be dealing with two issues. Firstly, we will show theoretically as well as by simulations that the systematic sampling has greater second order Bayes risk than the corresponding random sampling alternatives in all the situations mentioned at the beginning of this introduction, where the former is commonly preferred on the ground of easiness in execution. Our approach is based on the population models, i.e. we fix the ordering of the population and consider the values of interest as realized random variables under some assumed population model. This seems to be more in accordance with the practice of systematic sampling where the ordering is typically given once and for all. Moreover, we investigate the possible consequences of ignoring the robust decision principle, i.e. to choose the systematic sampling in spite of knowing that it has greater second order Bayes risks. In particular, by simulations based on census Labour Force data, we show that the use of systematic sampling in panel surveys causes the estimates of changes in a timely auto-correlated population to vary considerably in precision over time, which we consider to be a fault that can not be overlooked in panel surveys. A summary will be given in the end.

2 Homogeneous populations

Consider first equal-probability systematic sampling from a fixed population ordering that may be considered as uncorrelated with the variable of interest. Let the sample size be n, and let the sampling interval be k. For simplicity we assume that k is naturally an integer satisfying $N = nk$. Denote by s_m the mth systematic sample, i.e. $s_m = \{m, m+k, m+2k, ..., m+(n-1)k\}$. Let $\bar{y}_m$ be the corresponding sample mean, which is an unbiased estimator of the population mean, denoted by $\bar{Y} = \sum_{i\in U} y_i/N$. The sampling variance of $\bar{y}_m$ is given as $V_{sys} = k^{-1}\sum_{m=1}^{k}(\bar{y}_m - \bar{Y})^2$, which may or may not exceed the variance of the simple random sample mean, i.e. $V_{srs} = (n^{-1} - N^{-1})\sigma^2$, where $\sigma^2 = (N-1)^{-1}\sum_{i\in U}(y_i - \bar{Y})^2$.

As mentioned before, there are two results which show that SYS and SRS have the same Bayes risk, i.e. $E_\theta(V_{sys}) = E_\theta(V_{srs})$. We now proceed to find their second order Bayes risks under the following homogeneous population model

$$E(y_i) = \mu \quad \text{and} \quad E((y_i - \mu)^r) = \mu_r \text{ for } i \in U \quad \text{and} \quad E(y_i y_j) = \mu^2 \text{ for } i \neq j \in U \tag{3}$$

where, for simplicity, we write E instead of E_θ. It follows that $E(V_{sys}) = (1/n - 1/N)\mu_2 = E(V_{srs})$. This is a special case of the more general Theorem 8.5 of Cochran (1977), where the model variance of y_i is allowed to vary over the units. The exact second order Bayes risk of SYS is given in Appendix A. Here we have, approximately,

$$V_{sys} = \frac{1}{k}\sum_{m=1}^{k}(\bar{y}_m - \mu)^2 - (\bar{Y} - \mu)^2 \doteq \frac{1}{k}\sum_{m=1}^{k}(\bar{y}_m - \mu)^2.$$

Let $\mu_{r,n}$ denote the rth central moment of $\bar{y}_m$ w.r.t. the model. We have

$$E(V_{sys}) \doteq \mu_{2,n} \quad \text{and} \quad E(V_{sys}^2) \doteq \frac{\mu_{4,n}}{k} + \frac{k-1}{k}\mu_{2,n}^2 \quad \text{and} \quad V(V_{sys}) > \frac{2}{k}\mu_{2,n}^2$$

since $\mu_{4,n} = \mu_4/n^3 + 3\mu_{2,n}^2 > 3\mu_{2,n}^2$. Meanwhile,

$$V_{srs} \doteq \frac{1}{Nn}\sum_{i\in U}(y_i - \mu)^2 \quad \text{and} \quad V(V_{sys}) \doteq \frac{1}{Nn^2}(\mu_4 - \mu_2^2) = \frac{1}{Nn^2}V\{(y_i - \mu)^2\}$$

It follows that the coefficients of variation (CV) of V_{sys} and V_{srs} under the model are, respectively,

$$CV(V_{sys}) > \sqrt{\frac{2}{k}} = \sqrt{\frac{2n}{N}} \qquad \text{and} \qquad CV(V_{srs}) \doteq \sqrt{\frac{1}{N}\frac{V\{(y_i - \mu)^2\}}{\{V(y_i)\}^2}} = O(\frac{1}{\sqrt{N}}) \tag{4}$$

It is seen that the actual systematic sampling variance may considerably deviate from its expectation. The lower bound of CV(V_{sys}) is proportional to the squared root of the sampling fraction. This can be highly damaging for large samples taken from a small population. For instance, the overall sampling fraction is about $1/140$ in the Norwegian Labour Force Survey (LFS), such that by (4) the lower bound for CV(V_{sys}) is about 12%. In comparison, the second order Bayes risk of the simple random sampling is negligible. Drawing systematic samples from a seemingly random, but fixed list of population is a haphazard business without expectation of gains compared to simple random sampling. One simply has less control over the actual sampling variance. The same obviously holds for stratified systematic sampling compared to stratified simple random sampling. In two-stage sampling where the systematic sampling is used for sub-sampling of units within a primary sampling unit (PSU), what counts for the second order Bayes risk is the within-PSU sampling fractions. It follows that the systematic sampling can easily have a large second order Bayes risk in the case of multi-stage sampling, even if the overall sampling fraction may be low.

3 Ratio regression populations

Consider now the situation for systematic πps sampling. In this case the "every kth" rule is applied to the cumulated total of an auxiliary variable, denoted by x_i for $i \in U$. Any fixed list U can be used. For simplicity we assume that x_i is an integer. Let $X = \sum_{i \in U} x_i$. The interval length is then given by $k = X/n$, where again we assume that k is naturally an integer. Looked the other way around, equal probability systematic sampling becomes systematic πps sampling with $x_i \equiv 1$. The unit i may appear in x_i different systematic samples. We assume that the inclusion probability is such that $\pi_i = nx_i/X < 1$ for all $i \in U$. Base on any systematic πps sample, denoted by s_m for $m = 1, ..., k$, the estimator of Y is

$$\hat{Y}_m = \sum_{i \in s_m} \frac{y_i}{\pi_i} = \frac{X}{n} \sum_{s_m} b_i = X\bar{b}_m \quad \text{for} \quad b_i = \frac{y_i}{x_i} \quad \text{and} \quad \bar{b}_m = \sum_{i \in s_m} \frac{b_i}{n}.$$

We have $E_{sys}(\bar{b}_m) = Y/X$, and $E_{sys}(\bar{y}_m) = Y$, and

$$V_{sys}(\hat{Y}_m) = X^2 V_{sys}(\bar{b}_m) = X^2 \{\frac{1}{k} \sum_{m=1}^{k} (\bar{b}_m - Y/X)^2\}$$

Now, $\bar{b}_m$ is the best linear unbiased estimator (BLUE) of β under the following model

$$y_i = x_i\beta + x_i\epsilon_i \qquad \text{where} \quad E(\epsilon_i) = 0 \quad \text{and} \quad V(\epsilon_i) = \mu_2 \quad \text{and} \quad Cov(\epsilon_i, \epsilon_j) = 0 \qquad (5)$$

for $i \neq j \in U$, i.e. a ratio regression model with residual variance proportional to x_i^2. We have

$$V_{sys}(\bar{b}_m) = \frac{1}{k}\sum_{m=1}^{k}(\bar{b}_m - \beta)^2 - (Y/X - \beta)^2 \doteq \frac{1}{k}\sum_{m=1}^{k}(\bar{b}_m - \beta)^2 = \frac{1}{k}\sum_{m=1}^{k}\bar{\epsilon}_m^2$$

where $\bar{\epsilon}_m = \sum_{i\in s_m}\epsilon_i/n$. It follows that

$$E(V_{sys}(\bar{b}_m)) \doteq \mu_2/n \qquad \text{and} \qquad \{V_{sys}(\bar{b}_m)\}^2 \doteq k^{-2}(\sum_{m=1}^{k}\bar{\epsilon}_m^4 + \sum_{p\neq m}\bar{\epsilon}_m^2\bar{\epsilon}_p^2)$$

Notice that $\bar{\epsilon}_m$ and $\bar{\epsilon}_p$ are not necessarily independent of each other here, because some units may appear both in s_m and s_p. However,

$$E(\bar{\epsilon}_m^2\bar{\epsilon}_p^2) = n^{-4}\sum_{(i_1,i_2)\in s_m,(j_1,j_2)\in s_p} E(\epsilon_{i_1}\epsilon_{i_2}\epsilon_{j_1}\epsilon_{j_2})$$

where $E(\epsilon_{i_1}\epsilon_{i_2}\epsilon_{j_1}\epsilon_{j_2})$ is not zero, indeed positive, only if it is of the form $E(\epsilon_i^4)$ or $E(\epsilon_i^2\epsilon_j^2)$. More specifically, let s_{mp} denote the joint set of s_m and s_p. Let s_m^p denote the units of s_m that are not included in s_p, and let s_p^m denote the units of s_p that are not included in s_m. We have

$$\begin{aligned}
E(\bar{\epsilon}_m^2\bar{\epsilon}_p^2) &= \frac{1}{n^4}E(\sum_{i\in s_{mp}}\epsilon_i^4 + \sum_{i\in s_{mp},g\in s_p^m}\epsilon_i^2\epsilon_g^2 + \sum_{i\in s_{mp},h\in s_m^p}\epsilon_i^2\epsilon_h^2 + \sum_{g\in s_p^m,h\in s_m^p}\epsilon_g^2\epsilon_h^2) \\
&\geq \frac{1}{n^4}(\sum_{i\in s_{mp}}\{E(\epsilon_i^2)\}^2 + \sum_{i\in s_{mp},g\in s_p^m}\mu_2\mu_2 + \sum_{i\in s_{mp},h\in s_m^p}\mu_2\mu_2 + \sum_{g\in s_p^m,h\in s_m^p}\mu_2\mu_2) \\
&= \frac{1}{n^4}(\sum_{i\in s_m}\mu_2)(\sum_{i\in s_p}\mu_2) = \mu_2^2/n^2.
\end{aligned}$$

with equality if s_{mp} is empty. Denote by $\mu_{4,n}$ the fourth central moment of $\bar{\epsilon}_m$. We have

$$V(V_{sys}(\bar{b}_m)) \geq \frac{\mu_{4,n}}{k} + (\frac{k-1}{k} - 1)\frac{\mu_2^2}{n^2} > \frac{2}{k}\frac{\mu_2^2}{n^2}$$

and

$$CV(V_{sys}(\hat{Y}_m)) = CV(V_{sys}(\bar{b}_m)) > \sqrt{\frac{2}{k}} \tag{6}$$

Meanwhile, there are a variety of alternative random πps sampling methods available. It is easily shown that in the case of Poisson sampling (PS), the CV of $V_{ps}(\hat{Y})$ is of the order $O(1/\sqrt{N})$ under the model (5). Provided a fixed-sized πps sampling design has smaller sampling variance than the PS, the corresponding CV should not exceed the same order.

4 A numerical illustration

For a numerical illustration of the results (4) and (6), let us consider sampling of 10 units from a population of 100, denoted by $U = \{1, 2, ..., 100\}$. The auxiliary variables are simply given as $x_i = i$. The survey variables y_i are to be simulated under the following ratio regression model,

$$y_i = x_i + x_i^a \epsilon_i \qquad \text{where} \quad \epsilon_i \overset{iid}{\sim} N(0, \sigma^2) = 0 \quad \text{and} \quad 0 \leq a \leq 1 \tag{7}$$

The conditional variance of y_i given x_i is thus equal to $x_i^{2a}\sigma^2$. In the case of $a = 0$, $y_i - x_i$ follows the homogeneous model (3). Whereas in the case of $a = 1$, we have the model (5) with $\beta = 1$, which can be used to motivate the πps sampling.

Consider first the πps sampling. Let $a = 0, 0.25, 0.5, 1$ and $\sigma = 0.001, 0.01, 0.1$. Notice that σ can not be too large before negative y-values can be generated with non-negligible probabilities, in which case the rationale for πps sampling would be doubtful. For any (a, σ), we generate a population $\theta = (y_1, ..., y_{100})^T$, for which three sampling variances are calculated. The first one is the sampling variance of systematic πps sampling. The second one is the variance of the SPS, which is an approximate random πps sampling method. This is calculated by simple Monte Carlo. Finally, we calculate the asymptotic theoretical sampling variance of systematic πps sampling, with random permutation of θ before a systematic sample is drawn, i.e.

$$V_{asy} = \sum_{i=1}^{100} \pi_i (1 - \frac{n-1}{n}\pi_i)(\frac{y_i}{\pi_i} - \frac{Y}{n})^2$$

(Hartley and Rao, 1962), which can be used to benchmark the efficiency of the other two.

Table 1: Simulation results in percentage: Systematic πps *vs.* random πps sampling

		Relative Efficiency		CV of Sampling Variance		
Design		Systematic	SPS	Systematic	SPS	Theoretical
$a = 1$	$\sigma = 0.001$	100	100	44	15	15
	$\sigma = 0.01$	98	100	41	16	16
	$\sigma = 0.1$	101	100	41	16	16
$a = 0.5$	$\sigma = 0.001$	100	99	32	14	14
	$\sigma = 0.01$	101	100	34	15	15
	$\sigma = 0.1$	99	99	34	15	15
$a = 0.25$	$\sigma = 0.001$	99	99	29	18	18
	$\sigma = 0.01$	100	99	28	18	18
	$\sigma = 0.1$	99	99	29	18	18
$a = 0$	$\sigma = 0.001$	99	99	40	37	37
	$\sigma = 0.01$	100	99	37	36	35
	$\sigma = 0.1$	101	99	40	38	37

The simulations are repeated for 1000 independently generated θ. The results are given in Table 1, where the relative efficiency (RE) refers to the ratio $E(V_\delta)/E(V_{asy})$. We notice the following. (I) It is seen that both the systematic πps sampling and the SPS achieve RE around 100%, such that the two sampling methods are equivalent w.r.t. the Bayes risk principle. (II) Under the model (5), i.e. $a = 1$, the systematic πps sampling has much greater second order Bayes risk than the random πps sampling alternatives. Clearly, this is due to the fact that the systematic πps sampling is based on a fixed population list, because the variance fluctuation is greatly reduced in the theoretical case, where the systematic sampling is applied after random permutation of the population. Due to the covariance between the possible systematic samples, the CV of the systematic πps sampling greatly exceeds the lower bound depicted by (6), which is $\sqrt{2/k} = \sqrt{2/505} = 6.3\%$ in this case. Indeed, it is rather close to $\sqrt{2f} = 44.7\%$ by (4), where $f = n/N$ is the usual sampling fraction. (III) The second order Bayes risk varies little over σ given a. For $0.25 < a < 1$ the second order Bayes risk of random πps sampling is almost a constant, and is considerably lower than that of the systematic πps sampling. The second order Bayes risks of the random πps sampling methods increase quickly as a gets close to 0, but remain lower than that of the systematic πps sampling. In summary, random πps sampling is preferred according to the robust decision principle (2) under the model (5), and the choice is robust towards departures from the assumption $a = 2$, i.e. mis-specification of $\pi(\theta)$.

Consider next equal probability systematic sampling. There is a general result which states that systematic sampling is more efficient than SRS provided that the within-sample variance is larger than the population variance, due to the following decomposition

$$\sum_{i\in U}(y_i - \bar{Y})^2 = \sum_{m=1}^{k}\sum_{i\in s_m}(y_i - \bar{y}_m)^2 + \sum_{m=1}^{k} n(\bar{y}_m - \bar{Y})^2,$$

i.e. the variation within the k systematic samples and the variation between the systematic samples. Since V_{sys} is proportional to the second component, it is minimized for a given θ when the first component is maximized. Based on the corresponding ordering of the units, systematic sampling could potentially lead to gains in efficiency over simple random sampling. For instance, suppose the extreme case under the model (7) with $\sigma = 0$, i.e. $y_i = x_i$. The optimal ordering for a systematic sample of 10 units is to alternate between increasing and decreasing order every 10 units in the population (Särndal, Swensson, and Wretman, 1992, Example 3.4.2), denoted by $U_{opt} = (1, ..., 10, 20, ..., 11, 21, ..., 30, 40, ..., 31, ..., 100, ..., 91)$, in which case $V_{sys}(\bar{y}_s) = 0$.

In practice, of course, one never knows y_i exactly. However, the ordering U_{opt} remains optimal under the model (7) with $a = 0$, now that $\bar{x}_m = X/N$ is a constant of sampling. The estimator based on an equal-probability systematic sample drawn from U_{opt} is given by

$$\hat{Y}_m = X + N\bar{\epsilon}_m \qquad \text{where} \quad X = \sum_{i=1}^{N} x_i \quad \text{and} \quad \bar{\epsilon}_m = \sum_{i\in s_m}(y_i - x_i)/n$$

Table 2: Simulation results in percentage: Systematic sampling based on U_{opt} *vs.* combined use of simple random sampling and difference estimator

	$a = 0$			$a = 0.5$			$a = 1$		
	RE	CV$_{sys}$	CV$_{srs}$	RE	CV$_{sys}$	CV$_{srs}$	RE	CV$_{sys}$	CV$_{srs}$
$\sigma = 0.001$	102	46	16	110	47	17	111	48	20
$\sigma = 0.01$	104	47	16	107	48	18	112	48	19
$\sigma = 0.1$	99	49	16	107	47	17	112	47	20

which is the same as the difference estimator (Särndal, Swensson, and Wretman, 1992, Chapter 6.3) based on a simple random sample. In other words, the efficiency of systematic sampling based on U_{opt} can as well be achieved by the combined use of simple random sampling and difference estimator. Of course, the second order Bayes risk of the latter strategy is only of order $O(1/\sqrt{N})$. The situation is illustrated in Table 2, where RE refers to $E(V_{sys})/E(V_{srs})$, and CV_δ is the CV of the actual sampling variance induced by $\delta =$ SYS and SRS, respectively. We notice the following. (I) As expected, the combined use of simple random sampling and difference estimator is as efficient as the optimal systematic sampling under the model (7) with $a = 0$. The systematic sampling becomes slightly less efficient under departures from the assumption $a = 0$, i.e. as a moves from 0 towards 2. (II) The second order Bayes risk of systematic sampling is much greater than that of the simple random sampling under the assumption $a = 0$. There is little variation in both second order risks as a moves from 0 to 2. In summary, the combined use of simple random sampling and difference estimator is preferred to the optimal systematic sampling according to the robust decision principle under the model (7) with $a = 0$, and the choice is robust towards departures from the assumption $a = 2$, i.e. mis-specification of $\pi(\theta)$.

5 Systematic sampling for several occasions

A systematic sample, once drawn, may be used for several subsequent occasions. Such a sample may constitute a group in a rotating panel design, such as that of the LFS in most countries. It can also form the core of a panel survey, with supplementary units added to the sample from time to time, designed to account for natural regenerations of the population. To simplify the discussion we assume here that a single systematic sample is drawn on the first occasion and used for all the subsequent occasions before it is abandoned, and that the population U remains the same throughout the whole period.

The results (4) and (6) apply then directly to the entire active period of the panel. More explicitly, let $\mathbf{y}_i = (y_{i1}, ..., y_{it}, ..., y_{iT})^T$ be the variables of interest associated with $i \in U$ for the period $t = 1, ..., T$. The results (4) and (6) apply directly to any function of $\mathbf{y}_i$. For instance, suppose that $\mathbf{y}_i$ consists of 4 employment status measured in each of the 4 quarters of a calender year from the LFS. The average yearly employment is given by the average of y_{i1} to y_{i4}. By drawing a systematic sample on the first occasion, one risks a variance fluctuation in the estimator of the average yearly employment rate as well as in any single quarter. Moreover, an important use of panel data is to estimate changes in the population. Let $\delta_i = (\delta_{i2}, ..., \delta_{iT})^T$, where $\delta_{it} = y_{it} - y_{i,t-1}$ for $t = 2, ..., T$, be the changes from one period to the next for $i \in U$. Again, the results (4) and (6) apply directly to any δ_{it}, such that the estimation of change may have a high second order Bayes risk due to systematic sampling.

Considerations above do not take into account possible strong auto-correlation among $\mathbf{y}_i$, which one often finds in natural populations. A conditional examination is needed in addition. Consider the simplest setting where $T = 2$, and y_{it} is a categorical variable such as the employment status. As a simple model of the dependence between y_{i1} and y_{i2} we assume (Markov) transition probability p_{ab} for $y_{i2} = b$ given $y_{i1} = a$, independent for $i \neq j \in U$. This amounts to a homogeneous population model (3) conditional on $y_{1i} = a$. The systematic sample mean of $\delta_i = y_{i2} - y_{i1}$ is given as

$$\bar{\delta}_m = \sum_{a;y_{i1}=a} \frac{n_a}{n} \bar{\delta}_{m,a}$$

where n_a is the number of units with $y_{i1} = a$ and $\bar{\delta}_{m,a}$ is the mean of change among them. Closed expression of the conditional variance $V_{sys}(\bar{\delta}_m | \{y_{i1}; i \in U\})$ appears intrackable in general.

Instead, consider any ordering where the units are segmented according to the value of y_{i1}. Assume that N_a/k is naturally an integer for all a, where N_a is the number of units with $y_{i1} = a$ in the population. Both n_a and $\bar{y}_{m,t=1}$ become then constants of sampling, such that the variance of $\bar{\delta}_m$ is simply the variance of $\bar{y}_{m,t=2}$. The result (4) can now be applied to $\bar{y}_{m,a,t=2}$, i.e. within each segment of y_{i1} under the Markov transition model, such that the second order Bayes risks of $\bar{y}_{m,a,t=2}$ given $\{y_{i1}; i \in U\}$ carries straight over to $\bar{\delta}_m$. Consideration of this special situation suggests that the second order Bayes risk of systematic sampling can be high for estimators of change in auto-correlated populations, also when the variance is evaluated conditionally.

6 Simulation: Labour market dynamics

We simulate the labour market dynamics using data from the Norwegian Census 2001 and the Norwegian LFS as follows. From the Census 2001, we obtain the employment status, classified as "Employed", "Unemployed" and "Not in the labour force", which are to be treated as the variable of interest in the population at $t = 1$. Next, from the LFS of the last quarter in 2004 and the first quarter in 2005, we observe a 3×3-transition matrix for the employment status between the two quarters. Using these Markov transition probabilities, we are able to simulate an employment status in the population at $t = 2$. The population within each of the 19 counties in Norway is sorted by municipality, age, sex, and the personal identification number (PIN), where the PIN may be considered as uncorrelated with the employment status of interest.

We consider four different strategies: (1) equal probability systematic sampling at $t = 1$ and estimation based on direct weighting, denoted by Sys-Dir, (2) simple random sampling at $t = 1$ and estimation based on direct weighting, denoted by SRS, (3) proportionally allocated stratified random sampling w.r.t. sex and age (22 groups) followed by stratified estimation, denoted by Str-SRS, and (4) equal probability systematic sampling followed by post-stratified estimation, with the 22 age-sex groups as the post-strata, denoted by Sys-Pst.

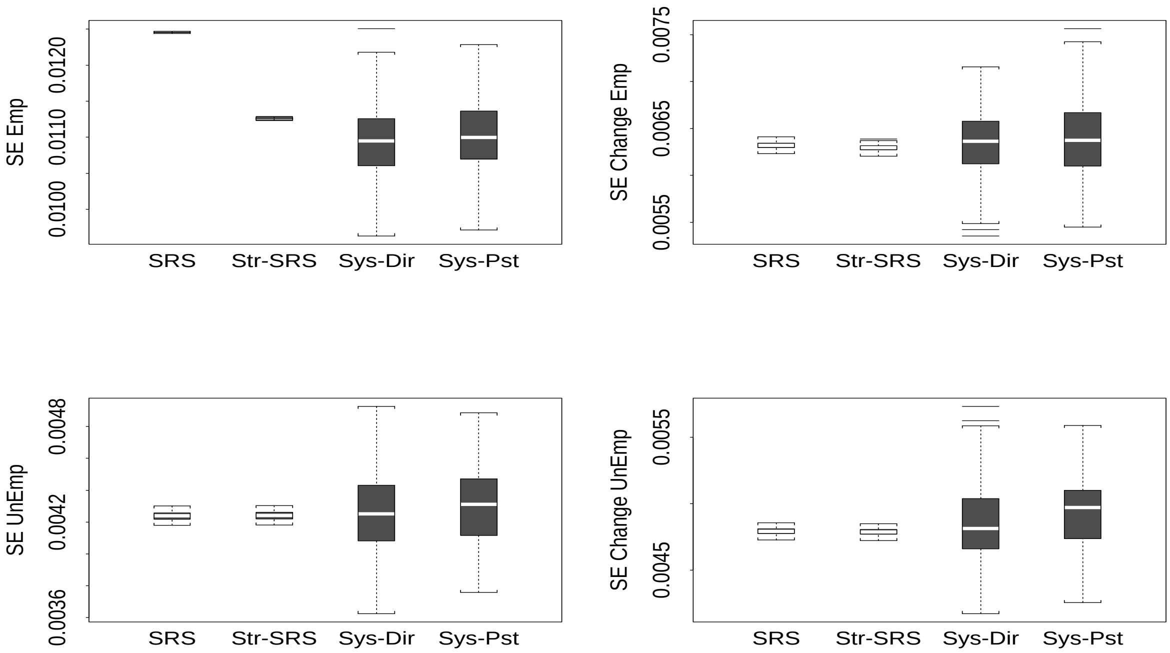

Figure 1: Boxplot of standard error (SE) of employment rate at $t = 2$ (Emp), change in employment rate (Change Emp), unemployment rate at $t = 2$ (UnEmp) and change in unemployment rate (Change UnEmp) for county Østfold: direct weighting following simple random sampling (SRS), stratified random sampling with proportional allocation (Str-SRS), systematic sampling (Sys-Dir), and post-stratified estimation following systematic sampling (Sys-Pst)

The simulations are carried out separately within each of the 19 counties of Norway, reflecting the stratified design of the Norwegian LFS. A sample selected at $t = 1$ is also used at $t = 2$, and the within-county sample sizes are taken from the Norwegian LFS. The results are very similar in all the counties. Here we show only the situation for Østfold in Figure 1. Systematic sampling can in this case be considered as implicit stratification w.r.t. municipality, sex and age. The stratification effects are notable only for employment rate at $t = 2$ (Emp), giving about 20% variance reduction compared to SRS. Most of the effects, however, can be achieved through stratification w.r.t. sex and age alone. Notice that stratification w.r.t. municipality in addition is unpractical due to the large number of strata. In all the other cases, no gains of efficiency can be expected from using systematic sampling.

It is seen that, while the second order Bayes risks of SRS and Str-SRS are negligible for a population of this size (about 179 thousand persons), they are appreciable under systematic sampling also when the variances are evaluated conditionally as it is done here. The CV of V_{sys} is 11.0% for Emp, 15.8% for Change Emp, 16.4% for UnEmp, and 15.4% for Change UnEmp, which are comparable to the lower bound of the unconditional CV, i.e. $\sqrt{2f} = \sqrt{2/134.6} = 12.2\%$ for Østfold. On certain occasions, therefore, the variance fluctuation may completely cancel out the expected stratification effects on the estimation of Emp. Notice that the second order Bayes risk of systematic sampling can not be reduced by means of post-stratification.

In particular, for the estimation of changes which is our primary concern here, the CVs of the systematic sampling variances are about the same as in the case of level estimators. Thus, the use of systematic sampling may cause the actual sampling variance of a change estimator to vary greatly over time. For instance, if the actual variance is 15% above the expectation between the first and second quarters, and it is 15% below the expectation between the second and third quarters, then the two change estimates have a difference of 30% in their sampling variances, caused by the use of systematic sampling alone. Now that the CV for the variance of either change estimator is about 15% here, this is hardly an unusual scenario. In the more extreme case of 2 standard deviations up or down from the expected sampling variance, the actual variances of two subsequent change estimates can have almost 100% difference compared to each other. It is certainly undesirable to keep this as a feature of the sampling design.

7 Summary

In the above we introduced the concept of second order Bayes risk and the robust decision principle. We have considered a number of situations where systematic sampling is commonly used as a substitute for alternative random sampling methods that are equally efficient. It is shown that the practice can induce large second order Bayes risks, i.e. fluctuations in the actual sampling variance, both in cross-sectional and longitudinal survey sampling. This can be highly damaging for large samples taken from small populations, or large sub-samples from small sub-populations. The use of systematic sampling for convenience is in such situations a haphazard business without any expectation of gains in efficiency. Given the availability of computer-aided random sampling, one certainly needs to reconsider whether the practice is worth keeping.

Systematic sampling is also frequently applied outside the situations that we have considered. Cochran (1977) cited several examples. A case in point is the use of one or two dimensional systematic sampling in forestry and land surveys. Such situations can be studied similarly as it has been done here, but will require rather special population models containing both correlations over time and space, which are beyond the scope of this paper.

Finally, we have studied the systematic sampling from a statistical decision point of view, where the loss function is defined as the sampling variance of the survey estimator. For the model-based inference where the variance of an estimator is evaluated w.r.t. the population model alone, the second order Bayes risk of systematic sampling does not differ from that of an alternative random sampling method, provided the sampling is non-informative in both cases. Indeed, there the systematic sampling is considered to be useful as a first step in constructing various balanced samples (Valliant, Dorfman, and Royall, 2000, Chapter 3). Whether this observation constitutes an argument against the irrelevance of the design-based inference, or the lack of robustness of the model-based inference is another discussion beyond the scope of this paper.

A Second order Bayes risk of systematic sampling

For equal probability systematic sampling under the homogeneous population model (3), we have

$$V_{sys} = \frac{1}{k}\sum_{m=1}^{k}(\bar{y}_m - \bar{Y})^2 = \frac{1}{k}\sum_{m=1}^{k}(\bar{y}_m - \mu)^2 - (\bar{Y} - \mu)^2$$

such that

$$V_{sys}^2 = \frac{1}{k^2}\{\sum_{m=1}^{k}(\bar{y}_m - \mu)^4 + \sum_{p\neq m}(\bar{y}_m - \mu)^2(\bar{y}_p - \mu)^2\} - \frac{2}{k}\sum_{m=1}^{k}(\bar{y}_m - \mu)^2(\bar{Y} - \mu)^2 + (\bar{Y} - \mu)^4$$

Now, $(\bar{y}_m - \mu)^2(\bar{Y} - \mu)^2$ can be written as

$$\begin{aligned}(\bar{y}_m - \mu)^2\{\frac{1}{k}\sum_{p=1}^{k}(\bar{y}_p - \mu)\}^2 = \frac{1}{k^2}\{&(\bar{y}_m - \mu)^4 + \sum_{p\neq m}(\bar{y}_m - \mu)^2(\bar{y}_p - \mu)^2 \\ &+ (\bar{y}_m - \mu)^2\sum_{p\neq q\neq m}(\bar{y}_p - \mu)(\bar{y}_q - \mu) + (\bar{y}_m - \mu)^3\sum_{p\neq m}(\bar{y}_p - \mu)\}\end{aligned}$$

such that

$$E\{(\bar{y}_m - \mu)^2(\bar{Y} - \mu)^2\} = \frac{\mu_{4,n}}{k^2} + \frac{k-1}{k^2}\frac{\mu_2^2}{n^2} \quad \text{for} \quad \mu_{4,n} = E\{(\bar{y}_m - \mu)^4\} = \frac{\mu_4}{n^3} + \frac{3\mu_2^2}{n^2}$$

where $\mu_{4,n}$ denotes the fourth central moment of $\bar{y}_m$ about its mean. We now have

$$\begin{aligned}
E(V_{sys}^2) &= \frac{1}{k^2}\{k\mu_{4,n} + k(k-1)\frac{\mu_2^2}{n^2}\} - \frac{2}{k}k\{\frac{\mu_{4,n}}{k^2} + \frac{k-1}{k^2}\frac{\mu_2^2}{n^2}\} + \mu_{4,N} \\
&= \frac{k-2}{k^2}\mu_{4,n} + \mu_{4,N} + \frac{(k-1)(k-2)}{k^2}\frac{\mu_2^2}{n^2} \\
V(V_{sys}) &= \frac{k-2}{k^2}\mu_{4,n} + \mu_{4,N} - \frac{k-1}{k^2}\frac{\mu_2^2}{n^2} > \frac{2k-5}{k^2}\frac{\mu_2^2}{n^2} \\
CV(V_{sys}) &= \frac{SD(V_{sys})}{E(V_{sys})} > \frac{\sqrt{2k-5}}{k-1} \doteq \sqrt{\frac{2}{k}}
\end{aligned}$$

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