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CERTAINTY EQUIVALENCE PROCEDURES IN THE MACROECONOMIC PLANNING OF AN OIL ECONOMY

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CONTENTS

ABSTRACT

The theme of the paper is how to cope with the macroeconomic exposure to risk in the Norwegian economy entailed by the increased reliance upon the extraction of petroleum resources. A framework for long-term macroeconomic planning based on optimal management of national wealth under uncertainty of future oil price and rates of return on non-oil assets is suggested, and a formal optimization model based on dynamic programming is presented. The model is solved under simplified assumptions and some properties of the solution are presented. The last part of the paper is devoted to numerical explorations in applying certainty equivalence procedures in optimizing the consumption path and capital accumulation.

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1. THE ISSUES: OIL AND UNCERTAINTY

1.1 OIL IN THE NORWEGIAN ECONOMY

Norway has been a net exporter of crude oil since 1975 and of natural gas since 1977. The current level of production of oil and gas amounts to 18 percent of GDP. The oil production at present corresponds to 3-4 times the domestic consumption of petroleum, while the production of natural gas, of which all is exported, is higher than the oil production (as measured in toe). Proven reserves amount to 35-40 times the current annual level of production, while more liberal assessments of oil and gas still in the ground indicate that there may be considerable more: 100 times the current annual production is a frequently quoted figure. With increasing production and growing real price of extracted oil and gas it is thus well within the range of possibilities that Norway may become dependent upon oil and gas production for 20-30 percent of its GDP for an extended period of time.

The theme of this paper is how to cope with the macroeconomic exposure to risk in the Norwegian economy entailed by the increased reliance upon the extraction of petroleum resources. It has been stated that the Norwegian economy has never been so dependent upon one single price as it today depends upon the international crude oil price. This may well be so, but exposure to risk is nothing new in Norwegian economic history. Over the last hundred and fifty years Norway has reaped benefits and incurred losses from such diverse circumstances as the Navigation Act, the elusive migrations of enormous shoals of herring and world wars as well as the exposure of a small, open and not very diversified economy to the ups and downs of world markets.

The current and future reliance upon extraction of petroleum resources differ from these earlier circumstances in a number of important ways:

- the long-term perspective of oil in the Norwegian economy,
- the macroeconomic importance of petroleum as measured e.g. by the share of GDP,
- the large scale of the resource base as compared e.g. with total national wealth,
- the high rent share of petroleum revenues, and
- the high government share of net revenues.

In the short-run context the rent of oil and gas production is a source of national income. In the long-run perspective the stock of oil and gas in the ground is a part of national wealth - an extraction of an amount of oil and gas represents not income, but only a running down of a large but limited stock. The real source of income connected with petroleum resources is the increase in value of these resources, (although the national accounts ignoring stocks of natural resources will tell a different story)..The rate of return on the stock of petroleum in the ground is the increase in the net price.

Most of the attention given to uncertainty in connection with the increased reliance upon petroleum extraction in the Norwegian economic and political debate has been related to short- and medium-term consequences of a volatile oil price. This has been natural in view of OPEC I and II and the downward adjustment of the oil price from 1981. (It has also played a prominent role that the government at an early stage grossly overestimated the rise of the overall production profile, but a lesson has been learnt and the importance of this incident now seems to fade). Countercyclical use of oil revenues, ratchet effects, "protection" of oil income booms from political misuse have been among the issues in this debate. Less attention has been given to uncertainty in the longer term perspectives. However, two recent reports from government appointed committees have i.a. dealt with these perspectives (NOU 1983:27, NOU 1983:37).

Our work is related to that of these two committees and may be regarded as suggestions of how the analyses could be brought further. We are well aware that answers given are very tentative to say the least, both theoretically and empirically. Our own attitude to them can be well expressed by a quote from the late Professor Leif Johansen (his share in our work is quite formidable) who wrote in the introduction to his book on the MSG model: ".... if I were required to make decisions and take actions in connection with relationships covered by this study I would (in the absence of more reliable results, and without doing more work) rely to a great extent on the data and the results presented in the following chapters." (Johansen, 1960).

In Norway macroeconomic medium- and long-term planning is based on quadrennial government White Papers presenting a four-year plan and a less detailed and less committing projection for the ensuing 20-30 years. It is in this context that the management of the long-term uncertainties of the Norwegian economy derived from the petroleum sector has its natural place. In section 1.2 we take a peek at earlier government projections of the Norwegian economy toward 2000. Section 1.3 discusses the notion of a strategic approach to long-term macroeconomic planning.

In chapter 2 we suggest a framework for overall long-term macroeconomic planning based on optimal management of national wealth under uncertainty of rates of return. A formal optimization framework based on dynamic programming in discrete time is presented and the model is solved under simplified assumptions. Some properties of the solution are discussed and some suggestions of how this framework can be applied in national economic planning are put forth.

In chapter 3 some ideas drawn from an article by Leif Johansen (1980) on certainty equivalence procedures in decision-making are applied in an attempt to analyze the implications of the projections drawn up by one of the committees referred to above when uncertainty is taken explicitly into account. The numerical explorations are based on very rough estimates of risks associated with the distribution of national wealth.

1.2 THE NORWEGIAN ECONOMY TOWARDS 2000: THE CURRENT STATE OF ANALYSIS

Official government projections for the Norwegian economy in 2000 have been presented on four occasions since their first appearance in 1973. The purpose of such projections are threefold. They shall serve

- as the basis for government policies over a wide range of issues,
- as guidelines for the development of the national economy that can be linked to sectoral, regional and other less comprehensive analyses, and
- as a general orientation about the economic prospects for the public at large.

All these projections have been elaborated by means of successive versions of the MSG model, orginally constructed by Leif Johansen in 1960. The MSG model is a large general equilibrium model which combines an overall macroeconomic framework a with considerable amount of details. The model has been extensively presented elsewhere and will not be further discussed here.

One of the more difficult tasks in the elaboration of long-term projections is to account properly for the many aspects of inherent uncertainty in the preparation and presentation of future development paths. With a time span of twenty or more years ahead there are large amounts of uncertainty with regard to many of the exogenous assumptions on which the analysis is based. Greater efforts of gathering information could probably reduce this uncertainty to some degree, but much would still remain. For a small open economy much of the uncertainty stems from abroad, such as the growth in world trade and the future crude oil price.

The traditional ways of dealing with such uncertainty are either to present alternative broad scenarios or to use sensitivity calculations varying the assumptions about exogenous influences. Such methods can give interesting illustrations of the uncertainty. But in a planning context the important question is what conclusions can be drawn for current and future planning decisions from this uncertainty. The uncertainty as it propagates from the exogenous influences must be evaluated in view of what can be governed or influenced by means of economic policy.

An illustration of the uncertainty of future prospects of the economy can be found by comparing earlier projections. In table 1 the aggregate results for the development of gross domestic product and private consumption 1980-2000 in four official projections are put together. The presentation is merely for illustrative purposes, as an adequate comparison of these projections would require a much more thorough treatment of the background and assumptions of the individual projections. The first projection had "high" and "low" alternatives while the ensuing projections had "high", "medium" and "low" alternatives. The figures given in the table are year 2000 figures as percentage increase over 1980 and average annual growth rates. The 1980 figures used are those implicit in the respective projections, for the last projection these are the preliminary national account figures available at the time (see note to table 1).

Table 1 conveys an impression of cyclical change in the assessment of the future from modest future growth rates in 1973 to a peak of optimism in the mid-1970's, and down to low prospects in 1981. The use of high-low intervals has been the method of exposing the uncertainty in these projections. Note, however, that the medium growth rate of GDP in the 1981 projection is outside the high-low interval in all the preceding projections. We shall not make too much out of these figures. They provide food for thought if one is interested in studying the confidence with which a government projects the future. One may ask whether the fluctuations in estimated long-term growth rates as revealed by table 1 reflects short-term changes in the economic climate and mood more than any real change in the evaluation of growth factors.

Looking back on earlier projections for a period that is still ahead of us, such as those included in table 1, one may search for better ways of assessing and presenting the uncertainty around the projected paths. A lot more is, of course, said about this in the respective publications. There is also a not too encouraging record of how well long-term projections have performed compared to the actual development. A survey is given in an annex to the 1981 projection.

In this article we shall focus not so much on the treatment of uncertainty in macroeconomic projections in general, but more specifically on the implications of uncertainty for the selection of "optimal" or "good" paths. In the projections referred to above no explicit welfare function or preference indicator has been applied. The projections have been presented in government papers as an annex to a medium-term programme. Usually the long-term projections are referred to as being elaborated by planning experts without the political commitments given to the medium-term programme.

Table 1. Official government projections for the Norwegian economy. Gross Domestic Product and Private Consumption in 2000 as percentage increase over 1980. (Average annual growth rates in parentheses).

Source	Year	Gross Domestic Product			Private Consumption		
		High	Medium	Low	High	Medium	Low
Long-Term Programme 1974-1977	1973	119.2 (4.0)		75.7 (2.9)	80.7 (3.0)		61.5 (2.4)
White Paper on Natural Resources and Economic Development	1975	132.1 (4.3)	106.8 (3.7)	67.1 (2.6)	136.6 (4.4)	119.1 (4.0)	60.7 (2.4)
Long-Term Programme 1978-1981	1977	99.8 (3.5)	92.5 (3.3)	85.6 (3.1)	100.2 (3.5)	85.9 (3.1)	85.9 (3.1)
Long-Term Programme 1982-1985	1981	83.1 (3.0)	59.3 (2.4)	38.3 (1.6)	92.4 (3.3)	71.7 (2.7)	49.7 (2.9)

Note: The figures are derived from published data in the following publications: St.meld.nr. 71 (1972-73), St.meld.nr. 50 (1974-75), St.meld. nr. 75 (1976-77) and St.meld. nr. 79 (1980-81). Some recalculations have been necessary to achieve comparability because of changes in the base year for volume figures and different periods of projection. The 1980 figures used are those implicit in the respective projections. For the first three projections the 1980 figures overestimated GDP in 1980 with 5,9 and 6 per cent and Private Consumption with 0,5 and 6.5 percent. A comparison of absolute year 2000 figures would thus make the 1975 and 1977 projections stand out as even more optimistic compared with the 1981 projection. In the latter projection the 1980 figures used were the preliminary national accounts figures available at the time.

Our analysis in chapter 3 is based on projections in a report called the "Perspective Analysis" (NOU 1983:37), published in 1983 by an appointed committee of experts relying to a great extent on the model tools and data sources used by the government for its projections. The committee stated views on the methodology of using macroeconomic models for long-term projections as well as presenting its own projections in the form of a reference path and alternative scenarios reflecting both uncertainty issues, policy alternatives and policy performance. The methodological part included remarks on how to cope better with uncertainty in macroeconomic projections, but refrained from introducing new procedures in the preparation and presentation of projections compared to earlier government projections. Results corresponding to those in table 1 for the reference path and four alternative projections are summarized in table 2 below.

As can be seen from the table the reference projection entails a considerable further revision downwards from the 1981 projection.

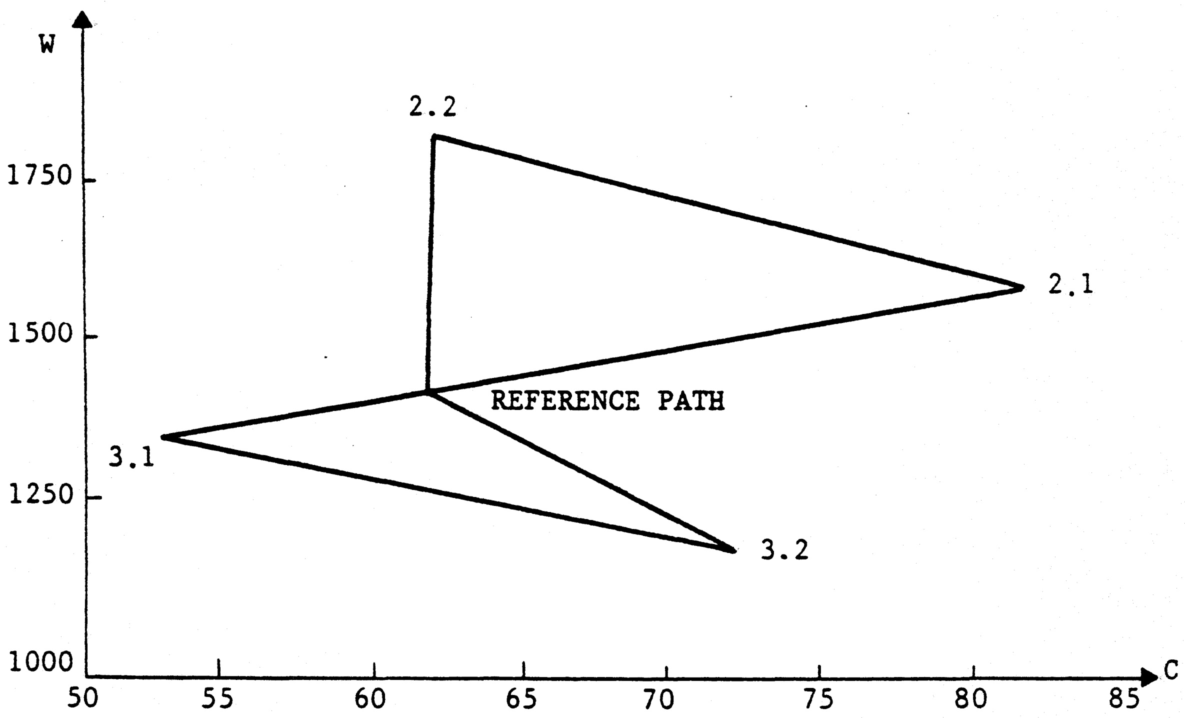
Table 2. Selected projections from the Perspective Analysis. Gross Domestic Product and Private Consumption in 2000. Percentage increase over 1980 and average annual growth rates.

	Gross Domestic Product		Private Consumption	
	Percent increase	Percent p.a.	Percent increase	Percent p.a.
1. Reference path	43.0	1.8	60.7	2.4
2. Higher petroleum income				
2.1 Increased domestic use	46.2	1.9	70.6	2.7
2.2 Increased capital exports	43.2	1.8	57.5	2.3
3. Sluggish world economy				
3.1 Tight policy	39.9	1.7	55.9	2.2
3.2 Lax policy	42.7	1.8	64.9	2.5

Note: The figures are derived from NOU 1983:37, table 7.1b. The reference path is based on full employment and an increase in the production of oil and gas reaching 80 mill. toe in year 2000. The crude oil price is in the reference path assumed to grow with 2 percent p.a. in real terms. Non-oil export grows with less than 2 percent p.a. In the two higher petroleum income scenarios the production of oil and gas is assumed to reach 90 mill. toe in year 2000, while the crude oil price grows with 3 percent p.a. In 2.1 the increased income is used to promote growth in domestic demand. Employment and the rate of technical progress increase, while in 2.2 the increased income is accumulated as foreign assets. The sluggish world economy scenarios depict developments where non-oil exports grow even less than in the reference path, only 1 percent p.a. In 3.1 the balance of payments is maintained by means of tight demand management. Employment falls off compared with the reference path. In 3.2 on the other hand priority is given to employment. Private and Government Consumption are increased with adverse consequences for the balance of payments. This table reveals, in fact, little about the differences between the alternatives. The Perspective Analysis also presented 3-4 other alternative scenarios.

These alternative projections of the Norwegian economy towards year 2000 results in different states of the economy by the end of the planning period. In the highly simplified representation of these alternatives in our further dicussion we ignore most structural and other aspects of the differences between these alternatives and focus on only two variables: consumption level (or rather increase over 1980) and wealth position. Figure 1 plots all five projections with regard to these two characteristics.

Figure 1. Selected projections from the Perspective Analysis. Percentage increase in consumption in 2000 over 1980 (C) and accumulated wealth in 2000 (W).



C = Total consumption (private and government) in 2000 as percentage increase over 1980.

W = Net foreign reserves in 2000 plus value of proven oil reserves in 2000 (see table 6).

1.3 THE CONCEPT OF STRATEGY IN LONG-TERM MACROECONOMIC PLANNING UNDER UNCERTAINTY

Long-term economic planning is undertaken by enterprises, multinational corporations, municipalities, branches of govenment and households with regard to their respective decision areas and responsibilities. Long-term macroeconomic planning is the logical counterpart for the supervisory branch of government responsible for the management of economic policy. While short-term macroeconomic planning activities are day-to-day tasks of governments exerted within well defined frameworks, long-term macroeconomic planning is a somewhat more elusive concept. Much of what is presented as long-term plans seem to be less operative and less committing than one would normally expect of a plan worthy of its name. The term "projection" is often used to convey a such more subdued intention. Sometimes government will ask expert forecasters to draw up a projection on which the government will base its long-term policy considerations. The forecasters will then, at least implicitly, have to make assumptions about what the government's decisions will be. This constitutes a puzzle which was posed and answered by the late professor Ragnar Frisch in an article written many years ago and wellknown to Norwegian economists (Frisch, 1961):

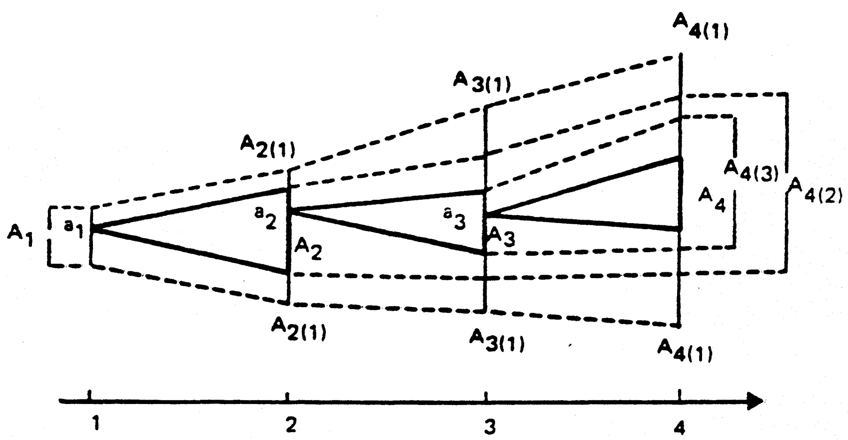
> "How can it be possible to make a projection without knowing the decisions that will basically influence the course of affairs? It is as if the policy maker would say to the economic expert: "Now you expert try to guess what I am going to do, and make your estimate accordingly. On the basis of the factual information thus received I will then decide what to do". The shift from the on-looker view-point to the decision view-point must be founded on a much coherent form of logic. It must be based on a decision model, i.e. a model where the possible decisions are built in explicitly as essential variables".

Frisch here rejects the idea that a government can adopt what he calls an on-looker point of view. It should instead adopt a decision point of view, that is, use techniques such that the analysis of the effects of government decisions are integrated in the preparation of projections.

The preparation of long-term projections is a very demanding task. It entails to bring together a large amount of data, much of which is not normally easily available, about the future course of exogenous influences. It requires, furthermore, the application to a future period of model tools representing the functioning of the economy, but which often turn out to be insufficient, inadequate and inaccurate. The length of the horizon of the projection is often longer than the time series on which the estimated coefficients are based. These issues which are quite formidable are not dealt with in the sequel.

The <u>strategic</u> problem faced by the long-term macroeconomic planner is the implications of taking sequential decisions when there is uncertainty about a number of the exogenous influences. This has two important aspects. One is that the room for possible action at a future point of time may, and normally will, be narrowed down as a consequence of earlier actions and external influences. The irreversibility of extraction of oil and gas is a case in point. A diagram, borrowed from Johansen (1977, p.117) illustrates this. In figure 2 $A_{t(s)}$ is the set of possible acti- at time t, possible before decisions taken at time s (<t).

Figure 2. Sets of possible actions in a dynamic strategy.



The other aspect is that future decisions do not have to be taken until called for. This implies that future decisions can be based on more information than is available at the time of plan preparation, in particular the realization of uncertain events in the period between plan preparation and the decision point will be known. The problem is how to integrate this dynamic flexibility into an integrated plan.

Johansen (1978, p. 326) summarizes these points:

> (1) In a dynamic context, in which there are interrelationships between what happens in the various periods, it is clearly not advisable to determine the policy for some period without at the same time thinking of which policies one should pursue in the following periods.
> (2) Since information continues to accrue in every future period, it would be inadvisable to decide already in an early period what to do in later periods; decisions should rather be postponed until they have to be taken, in order that this information, which is not available right from the beginning, can be utilized.

The answer is to search for strategies, i.e. policy functions which are decision rules stating how policy should be determined in each period on the basis of information available at the time. Perhaps the main purpose of long-term macroeconomic planning exercises should be the search for strategies. Unfortunately, the solution of this problem in the form of explicit policy functions are almost impossible to find except in very simplified cases.

Consider the following problem. An economy has at the outset an accumulated wealth given by W_0 and plans for two periods ahead. The wealth is invested abroad at a given certain interest, r. Income in each period, R_t is uncertain with known expectation and variance. The planning problem is to determine consumption, C_t. The optimization criterion is given as

$$\text{Max } EU(C_1, C_2, W_2)$$

where W_2 is the wealth remaining at the end of period 2. This problem can be solved as a static problem giving optimal solutions C_1^S and C_2^S. The dynamic optimization problem is to determine C_1 when the decision on C_2 is postponed one period. The solution to this problem entails finding the strategy for C_2, i.e. the policy function determining C_2 when R_1 is known.

Using the exponential preference functions we shall employ in chapter 2, the maximization problem above can be reformulated as a non-stochastic problem:

$$\text{Max } U(C_1, C_2, \tilde{W}_2)$$

where $\tilde{W}_2$ is an uncertainly adjusted expection of W_2, i.e. the real expectation adjusted downwards with an amount which depends both on the uncertainty of income in the two periods and the risk aversion implicit in the preference function. The problem is solved by first considering the second period decision based on a known value of W_1. This problem gives the strategy function for C_2. Then C_1 can be solved on the basis of the known strategy. How is the dynamic solution, C_1^D and C_2^D, compared with the static solution? The answer depends upon the choice of preference function. With a linear model and quadratic preference function the answer is given by the wellknown certainty equivalent result of Theil (1964): the first period decision on C_1 is the same in both cases. Using the additive exponential preference indicator of chapter 2, implying constant absolute risk aversion, it can be shown that the dynamic optimization implies higher consumption in period 1 and higher expected consumption in period 2.

Why is this so? Optimal consumption comes out higher in period 1 because less emphasis is put on the uncertainty of income in period 1. If this turns out to be different from expected income, it can to some extent be counteracted by the second period decision. The second period consumption comes out higher because this decision is based on more information: income in period 1 is not uncertain any longer. (The higher consumption in period 1 will have a slight negative influence on consumption in period 2 because of reduced interest income but not enough to counteract the effect of reduced uncertainty). Thus strategies are worth searching for.

The rationale of this approach is a major theme in Johansen (1977, 1978). After commenting on the many intractable aspects of solving planning problems in terms of strategies, he concludes (Johansen, 1978, p. 328-329):

(1) Although analytical derivation of strategies for strict optimization is usually not feasible, the general understanding of the nature of the problem and its solution may help to formulate the policy in a better way than without this understanding.
(2) Concrete questions of economic policy are often posed and debated as strategy problems, although they are not necessarily formulated in the terminology of this theory.
(3) In spite of the point made under (1) above, under certain conditions optimal strategies are really simple.

2. A MODEL FOR OPTIMAL MANAGEMENT OF NATIONAL WEALTH

2.1 NATIONAL WEALTH AND RATES OF RETURN

We assume that the national wealth is distributed over a number of assets - physical and financial assets as well as natural resources. Assets are measured in terms of the purchasing power of consumption goods. The planning horizon is divided into periods of equal length. At the beginning of each period the returns on the various assets are added up and distributed between consumption and accumulation in the same assets. For the decisions to be taken at the beginning of each period we have the following budget equation

$$(1) \qquad R_t = C_t + \sum_{i=0}^{n} I_{it}$$

where I_{it} is the investment in asset no. i and C_t is the rate of consumption in period t. Consumption is defined as the sum total of private and government consumption. All income is assumed to be capital income, accruing from investment undertaken one period earlier, hence

$$R_t = \sum_{i=0}^{n} r_{it} W_{it-1}$$

where W_{it-1} is the amount of asset no. i invested at the beginning of period t-1 and r_{it} its rate of return. In asset terms the budget equation can be written

$$(2) \qquad \sum_{i=0}^{n} W_{it-1} + R_t = C_t + \sum_{i=0}^{n} W_{it}$$

or

$$(2) \qquad G_{t-1} = C_t + \sum_{i=0}^{n} W_{it}$$

$$\text{where } G_{t-1} = \sum_{i=0}^{n} W_{it-1} + R_t = W_{t-1} + R_t$$

Total wealth G_{t-1} at the beginning of period t hence consists of stocks of assets inherited from the past as well as capital income. The rates of return are stochastic variables. We assume that when decisions are to be made at the beginning of period t the outcome of the stochastic rates of return dated t is known with certainty whereas the uncertainty regarding future periods has to be taken into account.

Oil reserves still in the ground can be considered as one type of assets although they are not usually counted as part of national wealth. The value of the oil reserves can be measured as the product of the amount of reserves S_t and the price net of marginal extraction costs, $q_t = p_t - b_t$ where p_t is the current oil price and b_t is marginal extraction cost. We assume that marginal cost is constant with respect to the rate of extraction but is a hyperbolic function of the reserve level. The rate of return on the oil reserves is equal to the rate of growth of the net oil price.

Introducing oil as an additional asset in (2) hence gives

$$(3) \qquad W_{t-1} + R_t + q_{t-1}S_{t-1} + (\frac{q_t}{q_{t-1}} - 1)q_{t-1}S_{t-1} = C_t + W_t + q_tS_t$$

Oil extraction in period t is given by

$$X_t = S_{t-1} - S_t$$

where the initial level of oil reserves S_0 is assumed known with certainty. By netting out oil terms, (3) can be stated as

$$(3') \qquad W_{t-1} + R_t + q_tX_t = C_t + W_t$$

Total wealth G_t and total stock of assets W_t are now redefined to include the oil reserves. The budget equation at the beginning of period t is thus

$$(4) \qquad G_{t-1} = C_t + W_t$$

where $G_{t-1} = \sum_{i=0}^{n} W_{it-1} + R_t + q_tS_{t-1}$

and $W_t = \sum_{i=0}^{n} W_{it} + q_tS_t$

In the numerical explorations in chapter 3 we shall distinguish between four assets apart from oil:

W_S = real capital in the sheltered sector (i.e. non-tradable goods production, protected sectors, and government)

W_E = real capital in the export sector

W_H = real capital in the import-competing sector

W_U = foreign assets

Table 3. Average rates of return. Percent.

	Estimation period	
	1962-81	1970-80
Sheltered sector (excluding government)	7.53	6.51
Import competing sector	10.00	10.24
Export sector	5.45	6.96

Foreign assets are assumed to yield a risk-free rate of return of 3 percent. The variance-covariance matrix for the estimated rates of return in the respective estimation periods is given in table 4 and 5.

Table 4. Variance-covariance matrix 1962-81.

	Sheltered sector	Import competing sector	Export sector	Real oil price
Sheltered sector	1.65685	-0.088861	-1.84331	-1.28275
Import-competing sector		2.30443	1.93291	-0.897742
Export sector			13.8807	0.621457
Real oil price				16.789

Table 5. Variance-covariance matrix 1970-80.

	Sheltered sector	Import competing sector	Export sector	Real oil price
Sheltered sector	0.18783	0.293275	-0.551394	-0.330819
Import-competing sector		2.86576	2.35785	-2.14271
Export sector			21.8722	0.242619
Real oil price				25.3738

This choice of breakdown of non-oil national wealth is - as the other specifications of the model - rather tentative. A priori we would expect capital in the non-tradeable sector to be a more certain asset (i.e. lower rate of return, but also lower variance) than investment in the tradeable sectors, while foreign reserves are assumed to be a risk-free asset.

For a small oil exporting country like Norway the oil price is exogenous, independent of domestic reserves and rate of extraction. It may not be so obvious whether the rates of return are independent of the stocks of the respective assets, and whether the stochastic rates of return on assets other than oil also are time independent as assumed in the formal model in section 2.3 below. In the following we assume that real capital by sector has constant expected rates of return as set out above. This exceedingly simplified picture of a national economy can only be defended on the ground that it serves a higher purpose!

2.2 RISK AVERSION AND PREFERENCES

The planning problem is defined here as the maximization of the sum of discounted expected utility from consumption over a planning horizon of length T, taking into consideration the discounted utility of terminal wealth. The utility of terminal wealth must be interpreted as derived from the consumption possibilities it represents beyond the planning horizon.

The objective function at the beginning of period t can thus be written as

$$(5) \qquad \sum_{\tau=t}^{T} U(C_\tau)(1+\delta)^{t-\tau} + V(G_T)(1+\delta)^{t-T} \qquad t=0,1,\ldots,T$$

where U and V are the utility functions for instantaneous consumption and terminal wealth respectively, and δ is the rate of time preference.

The decision problem at the beginning of each period is deciding on the reinvestment of total wealth and the rate of consumption to be maintained in the period. The results of earlier decisions are represented in period t through the stock of assets inherited from the previous periods. We assume that total wealth can be reallocated between assets. The decisions to be taken in the following periods up to T have to be taken into account when deciding on consumption and investment at the beginning of period t. Decisions in all periods should reflect an appropriate trade-off between consumption and investment, as well as between consumption in the planning period and terminal wealth.

For the instantaneous utility function we use the exponential function

$$(6) \qquad U(C_t) = -\theta\exp(-\beta C_t) \qquad \theta,\beta>0$$

which implies constant absolute risk aversion. The absolute risk aversion coefficient is given by $-U''/U' = \beta$. For terminal wealth we likewise assume constant absolute risk aversion, i.e

$$(7) \qquad V(G_T) = -G\exp(-\gamma G_T) \qquad G,\gamma>0$$

There is no strong apriori arguments for choosing constant absolute risk aversion as an assumption (and empirical tests of planners' preferences are hard to come by). The big advantage of the exponential utility function is that it combined with normally distributed risk factors has very pleasant properties in terms of certainty equivalence.

A well-known certainty equivalence result (e.g. Johansen, 1980) states that when x is normally distributed and f(x) is an exponential function, then

$$E\, f(x) = f(\breve{x})$$

where $\breve{x}$, the certainty equivalent of x, is given by

$$\breve{x} = EX - \frac{1}{2}\alpha \text{ var } x$$

where α, the exponential coefficient of f(x), also expresses the absolute risk aversion.

2.3 OPTIMIZATION BY MEANS OF DYNAMIC PROGRAMMING

The optimization problem given by maximization of (5) under the budget constraint (4) and given initial values of oil stock and non-oil wealth can be solved by the method of stochastic dynamic programming. For a planning horizon starting at t=0 from given values of G_0 and S_0 the optimization problem is solved by beginning at the end of the planning horizon and solving the decision problem for each period recursively. At the beginning of period T the optimal W_{iT}, S_T and C_T are determined, given the initial condition G_{T-1} and S_{T-1}. Having found the optimal solution for the last period contingent on any initial condition G_{T-1} and S_{T-1}, we solve the two-period problem for the last two periods by choosing the optimal W_{iT-1}, S_{T-1} and C_{T-1}, contingent on the initial condition G_{T-2} and S_{T-2}, and so on. In the last stage the optimal W_{i1}, S_1 and C_1 are determined, given the initial values G_0 and S_0 available at the beginning of period 1. A crucial assumption for the optimality of this procedure is stochastic independence between rates of return, including the oil price, in different periods. Our approach follows Samuelson (1969) and Chow (1975).

In the dynamic programming fashion we denote the maximum expected value of (5), contingent on G_{t-1}, by $J_t(G_{t-1})$. The decision problem at the beginning of period t can now be more precisely stated as

(8) $$J_t(G_{t-1}) = \text{Max } E\{U(C_t) + J_{t+1}(G_t)/(1+\delta)\}$$

where the maximization is with respect to W_{it} and S_t and subject to (4). Before proceeding to the solution procedure, the stochastic assumptions must be specified.

The stochastic assumptions concerning future oil prices and rates of return are of considerable importance for the optimal solution. We shall assume that the rates of return are multinormally distributed with expected values ρ_i and variances and covariances σ_{ij}, i,j=0,...,n. The oil price is assumed to be normally distributed with expected value π_t and variance τ^2. Covariances between the oil price and the rates of return on non-oil assets are given by τ_i, i=0,...,n. By the method of dynamic programming we start by solving the maximization problem given by (8) for t=T, i.e.

(9) $$J_T(G_{T-1}) = \text{Max } E\{U(C_T) + J_{T+1}(G_T)/(1+\delta)\}$$

where the maximization is with respect to W_{iT} and S_T and subject to (7). The expectation is contingent on the initial conditions G_{T-1} and S_{T-1} at the beginning of period T. The expected value operator is applied only on the second term since current consumption C_T is known once we have made our decision.

By considering (5) for t=T, (9) can be written as

$$(9') \qquad J_T(G_{T-1}) = \text{Max } E\{U(C_T) + V(G_T)\}$$

$$= \text{Max } \{U(C_T) + EV(G_T)\}$$

Applying the certainty equivalence result referred to in the section 2.2 above to (9') gives

$$(10) \qquad J_T(G_{T-1}) = \text{Max } \{U(C_T) + V(\tilde{G}_T)\}$$

where

$$\tilde{G}_T = EG_T - \frac{1}{2}\gamma \text{ var } G_T$$

$$EG_T = \sum_{i=0}^{n} W_{iT}(1+\varrho_i) + (\pi_{T+1} - b_{T+1})S_T \quad .$$

$$\text{var}G_T = \sum_{i=0}^{n} \sum_{j=0}^{n} \sigma_{ij}W_{iT}W_{jT} + \tau^2 S_T^2 + 2\sum_{j=0}^{n} \tau_j W_{jT}S_T$$

Evaluating the terminal value of the oil reserves should take into account future oil price uncertainty beyond the planning horizon. The approach of measuring the terminal value by certainty equivalent net price at the beginning of period T does not capture this future uncertainty. However, the marginal value of the terminal oil reserves is equal to the certainty equivalent net oil price, provided that the terminal level of oil reserves is optimally weighed against consumption throughout the planning period and terminal stocks of non-oil assets.

The first order conditions for the solution of (10) are

$$(11a) \qquad U'(C_T) \,.\, \delta C_T/\delta W_{iT} + V'(\tilde{G}_T) \,.\, \delta \tilde{G}_T/\delta W_{iT} = 0 \qquad i=0,\ldots,n$$

$$(11b) \qquad U'(C_T) \,.\, \delta C_T/\delta S_T + V'(\tilde{G}_T) \,.\, \delta \tilde{G}_T/\delta S_T = 0$$

or

$$\beta U(C_T) = \gamma V(\tilde{G}_T)(1+\tilde{r}_i) \qquad \text{for non-oil assets}$$

$$\beta U(C_T) = \gamma V(\tilde{G}_T)\, \tilde{q}_{T+1}/q_T \qquad \text{for the oil asset}$$

where

$$\tilde{r}_i = \frac{\delta \tilde{G}_T}{\delta W_{iT}} - 1 = \rho_i - \gamma \sum_{j=0}^{n} \sigma_{ij} W_{jT} - \gamma \tau_i S_T \qquad i=0,\ldots,n$$

and

$$\tilde{q}_{T+1} = \frac{\delta \tilde{G}_T}{\delta S_T} = \pi_{T+1} - b_{T+1} - \gamma \tau^2 S_T - \gamma \sum_{j=0}^{n} \tau_j W_{jT} - b'_{T+1}(S_T) S_T$$

$\tilde{r}_i$ is the certainty equivalent rate of return on assets no. i, i.e. the marginal increase in certainty equivalent wealth by a marginal increase in asset no. i.

$\tilde{q}_{T+1}$ is the certainty equivalent net oil price. The difference between the certainty equivalent net oil price and the expected net oil price consists of the correction terms due to the uncertainty as well a term due to the dependence of marginal cost on the reserve level. With a hyperbolic marginal cost function, $b_t = m/S_{t-1}$, cost function terms in $\tilde{q}_{T+1}$ cancel out, and $\tilde{q}_{T+1}$ appears as

$$\tilde{q}_{T+1} = \pi_{T+1} - \gamma\tau^2 S_T - \gamma \sum_{j=0}^{n} \tau_j W_{jT} .$$

To obtain an explicit solution for the optimal portfolio and consumption we make the crucial assumption that asset no. 0 is risk-free, yielding a certain rate of return r_0. Hence, $\tilde{r}_0 = r_0$ and from the first-order conditions we get

$$\text{(12a)} \qquad \tilde{r}_i = r_0 \qquad\qquad i=1,\ldots,n$$

$$\text{(12b)} \qquad \tilde{q}_{T+r}/q_T - 1 = r_0$$

Optimal accumulation in the uncertain assets is determined by the condition that certainty equivalent rate of return should be equalized for all assets. Oil extraction is determined by a modified Hotelling rule: certainty equivalent net oil price should grow at a rate of return equal to the certain rate of return.

Substituting the first order conditions into (10) using (12) gives the maximal expected utility at the beginning of period T

$$\text{(13)} \qquad J_T(G_{T-1}) = U(C_T^*)(1+\beta/\gamma(1+r_0)) = U(C_T^*)\xi_1$$

where C_T^* is optimal consumption in period T and $\xi_1 = 1 + \beta/\gamma(1+r_0)$. From (13) it is seen that optimal consumption C_T^* can be expressed as a function of total wealth G_{T-1} at the beginning of period T. The explicit solution for optimal consumption C_t^* will be derived in a similar way from the general solution for $J_t(G_{t-1})$. To realize the recursive nature of the solution, it is elucidating to consider the decision problem for t=T-1 and then derive the general solution for $J_t(G_{t-1})$ by induction. The decision problem at the beginning of period T-1 is

$$(14) \qquad J_{T-1}(G_{T-2}) = \text{Max } E\{U(C_{T-1}) + J_T(G_{T-1})/(1+\delta)\}$$

where the maximization is with respect to W_{iT-1} and S_{T-1} and subject to (4). Observing that J_T is an exponential, we apply the certainty equivalent result to (14)

$$(15) \qquad J_{T-1}(G_{T-2}) = \text{Max}\{U(C_{T-1}) + J_T(\tilde{G}_{T-1})/(1+\delta)\}$$

However, the appropriate risk aversion coefficient in the certainty equivalent procedure for G_{T-1} is not γ. J_t is an exponential function with time dependent absolute risk aversion coefficient. Differentiating (10) with respect to G_{T-1} and applying the first order condition (11) gives

$$(16) \qquad \frac{dJ_T(G_{T-1})}{dG_{T-1}} = U'(C_T^*)$$

Combining (16) and the solution for J_T given by (13) yields

$$(17) \qquad J_T'(G_{T-1}) = U'(C_T^*) = -\beta U(C_T^*) = -\beta/(1+\beta/\gamma(1+r_0)) \,.\, J_T(G_{T-1})$$

Hence,

$$(18) \qquad \frac{J_T'(G_{T-1})}{J_T(G_{T-1})} = -\beta/(1+\beta/\gamma(1+r_0)) = -\beta/\xi_1$$

The appropriate risk aversion coefficient for $\tilde{G}_{T-1}$ is thus $-\beta/\xi_1$ as given by (18) and we get

$$\tilde{G}_{T-1} = EG_{T-1} - \frac{1}{2}\beta/\xi_1 \text{ var } G_{T-1}$$

The first order conditions for the solution of (15) can hence be stated as

$$(19a) \qquad \beta U(C_{T-1}) = \beta/\xi_1(1+\delta) \quad J_T(\tilde{G}_{T-1})(1+r_0)$$

$$(19b) \qquad \beta U(C_{T-1}) = \beta/\xi_1(1+\delta) \quad J_T(\tilde{G}_{T-1})\tilde{q}_T/q_{T-1}$$

The solution for J_{T-1} is found by substituting (19) into (15)

$$J_{T-1}(G_{T-2}) = U(C_{T-1}^*)(1+(1+\beta/\gamma(1+r_0))/(1+r_0)) = U(C_{T-1}^*)\xi_2$$

Comparing the solutions for J_T and J_{T-1} the recursiveness of the solution for J_t appears through the coefficient ξ_{T-t}, which is recursively determined by the difference equation.

$$\xi_{T-t} = 1 + \frac{\xi_{T-t-1}}{1+r_0}$$

The solution for ξ_{T-t} is given by

$$\xi_{T-t} = \left(\frac{1}{1+r_0}\right)^{T-t} \left(\beta/\gamma - \frac{1+r_0}{r_0}\right) + \frac{1+r_0}{r_0}$$

with $\xi_0 = \beta/\gamma$

By induction it can be shown that the generalizations of (13), (16) and (18) are

$$\text{(20)} \qquad J_t(G_{t-1}) = U(C_t^*)\xi_{T-t+1}$$

$$\text{(21)} \qquad J_t'(G_{t-1}) = U'(C_t^*)$$

$$\text{(22)} \qquad \frac{J_t'(G_{t-1})}{J_t(G_{t-1})} = -\beta/\xi_{T-t+1}$$

From (22) it is seen that J_t is an exponential and it then follows from (21) that C_t^* is a linear function of G_{t-1}, i.e.:

$$\text{(23)} \qquad C_t = a_t G_{t-1} + b_t$$

a_t is easily found to be $1/\xi_{T-t+1}$, while b_t can be solved from the difference equation

$$b_{t+1} = (\xi_{T-t+1}/(\xi_{T-t+1} - 1))\, b_t + \alpha/\beta$$

to give

$$(24) \qquad b_t = -\left\{ \frac{1+r_0}{r_0}\frac{\alpha}{r_0} + \frac{1}{(1+r_0)^{T-t}}\left[-\beta/\gamma(1+r_0)(\ln(\beta/\gamma)-\alpha-\ln(1+\delta)) + \frac{T-t}{1+r_0}\left(\frac{\beta}{\gamma} - \frac{1+r_0}{r_0}\right)\alpha - \frac{1+r_0}{r_0}\frac{\alpha}{r_0}\right]\right\}/\beta\xi_{T-t+1}$$

where $\alpha = \ln\frac{1+r_0}{1+\delta} - X$

and

$$X = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(\varrho_i - r_0)(\varrho_j - r_0)\bar{\sigma}_{ij} + \left(\frac{\bar{q}_{t+1}}{q_t} - 1 - r_0\right)\sum_{j=1}^{n}(\varrho_j - r_0)\bar{\tau}_j + \frac{1}{2}\left(\frac{\bar{q}_{t+1}}{q_t} - 1 - r_0\right)^2\bar{\tau}^2$$

$\bar{\sigma}_{ij}$, $\bar{\tau}_j$ and $\bar{\tau}^2$ are the elements of the inverse of the variance-covariance matrix of σ_{ij}, τ_j and τ^2, and $\bar{q}_{t+1}$ is the expected net price (equal to $\pi_{t+1} - b_{t+1}$). One can use the approximation $\ln((1+r_0)/(1+\delta)) = r_0 - \delta$, which means $\alpha = r_0 - \delta^*$, where $\delta^* = \delta + X$. Thus as an implication of the certainty equivalence procedure, the stochastic parameters appear only in the risk-adjusted time preference rate δ^*.

In the solution of b_t the coefficients B and G in (6) and (7) are assumed equal to one. The marginal propensity to consume out of current wealth is the reciprocal of the recursion coefficient ξ_{T-1}. By rewriting the expression for ξ_{T-1} it is easily seen that

$$(25) \qquad \xi_{T-t} = \beta\left[\frac{1/(1+r_0)^{T-t}}{\gamma} - \frac{1-1/(1+r_0)^{T-t}}{\beta\frac{r_0}{1+r_0}}\right]$$

β/ξ_{T-t} can be interpreted as a weighted harmonic average of the terminal wealth risk aversion coefficient γ and the risk aversion coefficient β adjusted by the term $r_0/1+r_0$. As the time interval from the present date until the planning horizon is increasing, the effect of γ on current consumption is diminishing. In the limiting case where $T-t \to \infty$, ξ is a constant given by

$$(26) \qquad \xi = \frac{1+r_0}{r_0}$$

In this case the marginal propensity to consume is independent of γ as well as β. However, γ and β appear in the constant term of the consumption function.

When the optimization problem has been solved step by step, optimal consumption is implemented by recording actual development and inserting, period by period, the outcome of the stochastic rates of return, i.e. G_{t-1}, in the consumption function (24). The optimal solution can thus be interpreted as a <u>strategy</u>; decision rules for optimal consumption are calculated initially, whereas actual consumption decisions are postponed until current wealth is known with certainty.

This consumption strategy is consistent with a long-term consumption path given by

$$(27) \qquad C_t = \frac{r_0-\delta}{\beta} t + C_0$$

were C_0 is initial consumption.

The first order conditions for the optimization problem given by (8) combined with (21) gives a relation between marginal utility of consumption in two successive periods,

$$U'(C_t) = \frac{1+r_0}{1+\delta} U'(C_{t+1})$$

hence the optimal C_t is derived by taking logarithms on both sides and solving the resulting difference equation for C_t.

Given the optimal consumption, the accumulation in the uncertain assets is determined as a one-period portfolio problem.

$$(28) \qquad W_{it} = \frac{\xi_{T-t+1}}{\beta}\{\sum_{j=1}^{n} (\varrho_j - r_0)\bar{\sigma}_{ij} + \bar{\tau}_i(\pi_{t+1} - (1+r_0)q_t)\}$$

$$(29) \qquad S_t = \frac{\xi_{T-t+1}}{\beta}\{\sum_{j=1}^{n} (\varrho_j - r_0)\bar{\tau}_j + \bar{\tau}^2(\pi_{t+1} - (1+r_0)q_t)\}$$

Hence, optimal oil extraction in period t is given by

$$(30) \qquad X_t = S_{t-1} - S_t$$

where S_t is determined by (29) and S_{t-1} is given from the previous period.

Due to the strong assumptions regarding the utility function and the stochastic parameters as well as the production structure and the cost function for oil extraction we have thus obtained explicit solutions with intuitive interpretations.

3. *NUMERICAL EXPLORATIONS IN APPLYING CERTAINTY EQUIVALENCE PROCEDURES IN OPTIMIZING THE NORWEGIAN ECONOMY*

3.1. PREFERENCE FUNCTIONS DERIVED FROM MACROECONOMIC PROJECTIONS

The intended application of the stochastic optimization framework outlined in this article is mainly as a means for evaluating and corroborating long-term projections from the MSG model. Although stochastic elements are not included in the MSG model, the model is a valuable means for illustrating the wide range of possible long-term projections under alternative oil price assumptions. Model calculations are performed with alternative oil price scenarios and exogenously stipulated oil and gas production profiles. The consequences of alternative oil revenue scenarios are traced out by model calculations. These long-term projections illustrate the considerable impact on sectoral development and accumulated foreign reserves under alternative oil price assumptions. A consistent evaluation of these long-term equilibrium growth paths under uncertainty requires a stochastic optimization framework.

In order to apply the stochastic optimization model outlined above, we have to make on assessment of the risk aversion coefficients β and γ. Before facing this cumbersome task, a quote from an early paper on certainty equivalent procedures by Freund (1956) may be appropriate: "The estimation of the risk aversion constant a is a purely subjective task, and any chosen value is exceedingly difficult to defend". However, in our approach to applying certainty equivalence procedures to long-term macroeconomic planning, the estimation of the risk aversion coefficients should <u>not</u> be based on subjective judgements, but rather reflect current political preferences.

The analysis of this chapter is based on the MSG projections in the report of the Perspective Analysis (NOU 1983:37). As stated in chapter 1, these long-term projections are elaborated by a group of experts without the political commitments that are given to the projections presented in e.g. the medium-term programme. However, for our purpose it may not be totally misleading to interpret them as reflecting current political preferences. The projections of the Perspective Analysis do not easily lend themselves to the assessment of preferences. Little is said about the evaluation of the alternative projections, and no precise guidelines are given for the trade-off between consumption and wealth accumulation.

The present analysis is based on the reference path and the four alternative projections which are summarized in table 2. These five projections illustrate a wide range of possibilities for the choice between consumption and accumulation of foreign assets. The two triangles in figure 3 indicate the feasible sets under the assumptions of either higher petroleum income (2.1 and 2.2) or sluggish world economy (3.1 and 3.2). Little is said about the choice between increased domestic use and increased capital exports in the case of higher petroleum income, and the choice between tight and lax policy in the case of a sluggish world economy.

Based on the information provided in the report of the Perspective Analysis we have however established the following crucial assumptions.

Consider the following stochastic experiment with two possible outcomes: Either the outcome of higher petroleum income is realized, where the feasible set is represented by the line segment between 2.1 and 2.2, or the outcome of a sluggish world economy is realized, where the feasible set is represented by the line segment between 3.1 and 3.2. These two outcomes are assumed to have an equal probability. The alternatives 2.1 or 2.2 and 3.1 or 3.2 thus represent extreme policies in view of the uncertainty, and to reveal the optimal policy we state the following assumptions:

a) <u>Sluggish world economy</u>: Given a feasible set of all points between 3.1 and 3.2 the best choice is to pursue a policy aiming at a result midway between the two extreme policies.

b) <u>Higher petroleum income</u>: Given a feasible set of all points between 2.1 and 2.2 the best choice is to pursue a policy aiming at a result slightly closer to 2.1 than the midpoint.

c) <u>Reference path:</u> The reference path is considered as the <u>certainty equivalent</u> of the stochastic experiment described above. Given the two optimal policies described in a)-b) the expected utility of these two outcomes is equal to the utility of the reference path.

These assumptions are formulated in view of a preference function given by

(31) $U(C,W) = -B\exp(-bC) - G\exp(-gW)$

C = Total consumption (private and government) in 2000 as percentage increase over 1980.

W = Net foreign reserves in 2000 plus value of proven oil reserves in 2000 (see Table 6).

To simplify the estimation of the risk aversion coefficients, the preference function (31) has been formulated as a static analogy to the multi-period preference function (5) of the dynamic optimization problem. In (31) preferences are attached to the percentage increase in consumption over the planning horizon, rather than the sum of discounted utility of consumption in each period. However, this reformulation does not alter the main conclusions for the trade-off between consumption and terminal wealth under uncertainty. The numerical estimate for the risk aversion coefficient b will differ from the risk aversion coefficient β of the multi period preference function, and the appropriate estimate for β will finally be derived.

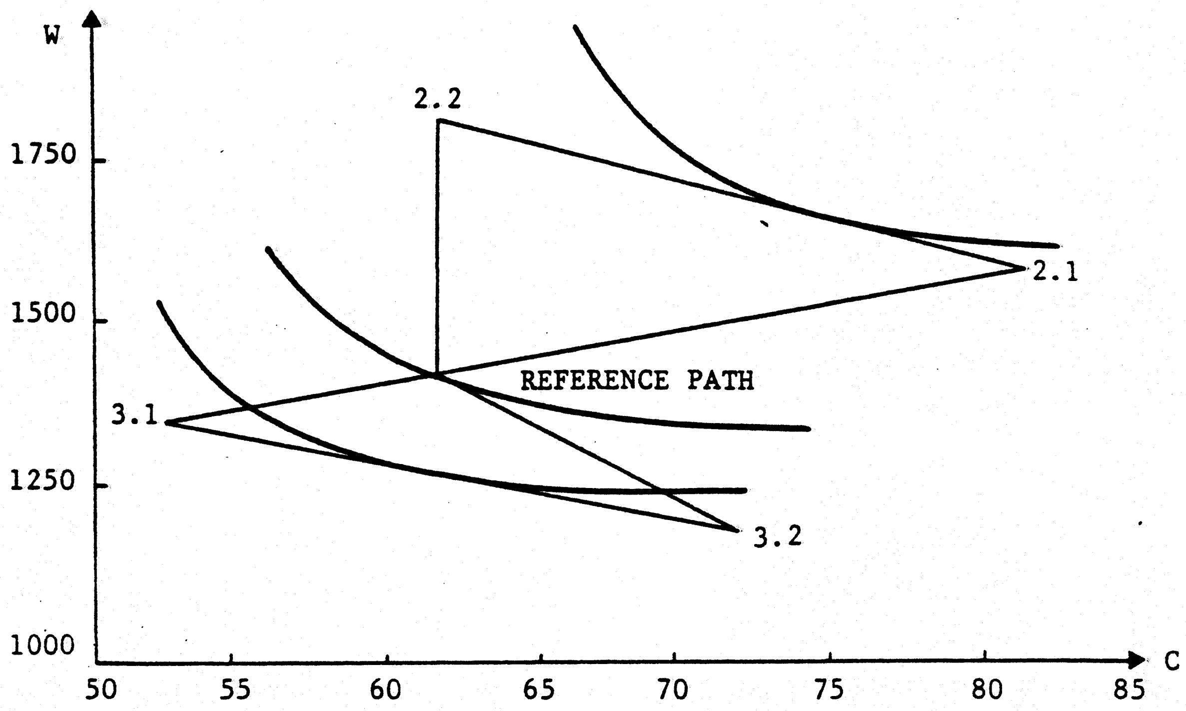
The wealth concept W defined as net foreign reserves plus the value of the oil reserves is highly tentative, to say the least. It does not properly reflect the concept of national wealth as defined in the optimization model. According to the preference function (5), consumption should be weighed against total wealth at the end of the planning period, i.e. production capital, financial assets and natural resources. The role of terminal wealth in the preference function is to represent the production potential for future consumption beyond the planning horizon. The discussion of the Perspective Analysis is however more explicitly related to the trade-off between consumption growth and net foreign reserves at the end of the planning period. The point of foreign reserves in this connection seems to be as a safeguard against the risk inherent in the oil reserves. In order to accommodate the views expressed in the report as a guideline for our estimation of the risk aversion coefficients, the value of petroleum reserves and net foreign reserves are included in our wealth concept here while other assets are disregarded. This is perhaps a dubious interpretation and inclusion of real capital would have given different estimation results.

The assumptions a)-c) give three relationships to determine the parameters b, g and G/B. The level of utility is arbitrarily chosen by setting B=1. Furthermore, the parameter values are adjusted to yield G=B=1. The following parameter values are thus obtained:

$b = 0.1426$

$g = 0.00589$

Figure 3. Indifference curves with b=0.1426 and g=0.00589



Given these parameter values, the numerical application will be pursued in two directions. First, the indifference curve approach outlined in this section will be elaborated as an illustration of a more general certainty equivalence procedure developed by Leif Johansen (1980). The idea of this approach is to incorporate uncertainty in the decision-making by a modification of the parameters of the objective function. The effect of uncertainty on the trade-off between consumption growth and terminal wealth is thus clearly exposed. Based on our estimates of the risk aversion parameters and the standard deviations, the optimal trade-off between consumption growth and terminal wealth under uncertainty will be discussed.

Secondly, the relationship between the preference function given by (31) and the multi-period preference function of chapter 2 will be established, and some tentative numerical results will be given with regard to the optimal consumption path.

3.2. THE JOHANSEN APPROACH TO CERTAINTY EQUIVALENCE PROCEDURES IN DECISION-MAKING UNDER UNCERTAINTY

In this section the idea of certainty equivalence will be further elaborated in order to illustrate the consequences of uncertainty. A simple certainty equivalence procedure was introduced in section 2.2 as a means for solving the stochastic dynamic optimization problem of chapter 2. Based on the assumption of constant absolute risk aversion and normally distributed stochastic elements, this procedure permitted a transforation of a stochastic optimization problem to an optimization in terms of certainty equivalents. The certainty equivalent of a stochastic variable is the expected value minus a correction term, which is proportional to the variance and the risk aversion coefficient.

In this section the preference function (31) will be examined in terms of certainty equivalence. It may be elucidating to analyze the consequences of uncertainty in the static analogy to our dynamic optimization problem, before proceeding to illustrate the consequenses of uncertainty on the decisions taken year by year in the planning period.

This section is based on some ideas from an article by Leif Johansen (1980). In economic theory certainty equivalence procedures have mainly been elaborated in the case of a quadratic objective function combined with a linear structural model. One of the many contributions of Leif Johansen in this field is the generalization of the usual certainty equivalence procedure to the case of an objective function expressed in terms of combinations of exponential functions. The idea of this approach is to modify the original parameters of the objective function in order to incorporate the variances and covariances of the stochastic elements. This parametric certainty equivalence procedure, as developed in Johansen (1980), gives a procedure which is the same as under certainty, but the decisions actually taken will generally be different under uncertainty than under certainty because the modified parameter values will depend on the probability distribution.

Consider the preference function given by (31). If there were no uncertainty involved, we would choose from the feasible points so as to maximize (31). The indifference curves of (31) are characterized by the marginal rate of substitution between C and W given by

$$(32) \quad \frac{dW}{dc} = -\frac{bB}{gG}\exp(gW - bC)$$

Given the assumption of normal probability distributions, the parametric certainty equivalence procedure entails the following transformation of (31)

$$(33)\quad U(EC,EW) = -\tilde{B}\exp(-b\,EC) - \tilde{G}\exp(-g\,EW)$$

where $\tilde{B} = B\exp(1/2\; b^2\sigma_c^2)$

and $\tilde{G} = G\exp(1/2\; g^2\sigma_w^2)$

The standard deviation of C and W is denoted by σ_c and σ_w respectively. The certainty equivalence procedure consists in choosing EC and EW so as to maximize (33). The marginal rate of substitution is now expressed as

$$(34)\quad \frac{dEW}{dEC} = -\frac{bB}{gG}\exp\left(1/2(b^2\sigma_c^2 - g^2\sigma_w^2\right)\exp(gEW - bEC)$$

First it can be noted that uncertainty has no effect on the actual decisions in the case where

$$b\sigma_c = g\sigma_w$$

If this is not the case, the indifference curves of (33) will be twisted as a consequence of uncertainty. Furthermore, a partial increase in σ_w will make the indifference curve flatter while a partial increase in σ_c will make the indifference curve steeper. This will in general mean that a larger σ_c tends to induce a change in the decision in the direction of a larger value of EC, while a larger value of σ_w tends to induce a change in the decision in the direction of a larger value of EW.

To illustrate the parametric certainty equivalence procedure, a tentative calculation is performed based on the risk aversion coefficients derived above and the stochastic parameters estimated over the period 1962-81, cf. the data given in chapter 2. The following values are used:

$b = 0.1426$

$g = 0.00589$

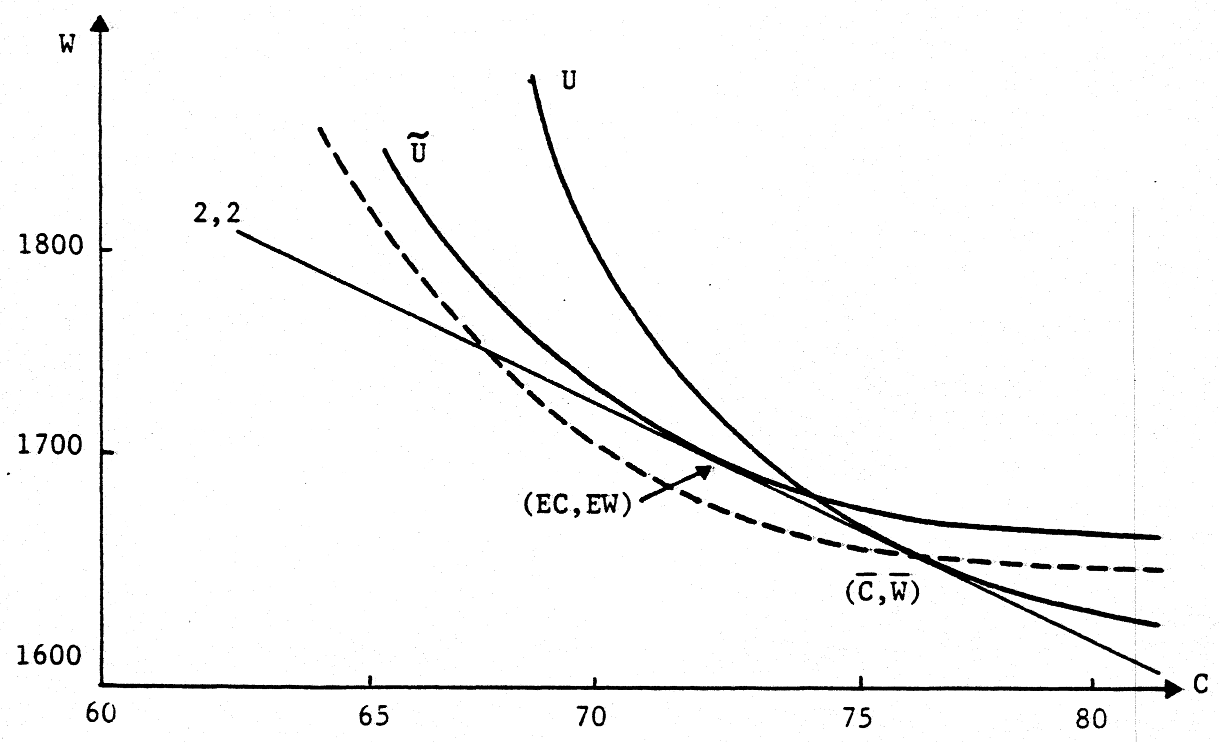
$\sigma_c = 12.5$

$\sigma_w = 364.2$

which implies that $g\sigma_w > b\sigma_c$.

Our parameter values indicate that the risk adjustment term of terminal wealth is the larger, which implies a flatter indifference curve compared to the indifference curve when uncertainty is disregarded. In figure 4 the consequences of uncertainty are illustrated in the case where the feasible set is given by a line corresponding to the upper line of figure 3. This means that we consider a situation with higher petroleum revenue in 2000 than in the reference path, and the question is how the uncertainty should influence the trade-off between consumption growth and terminal wealth. The extreme alternatives 2.1 and 2.2 represent respectively increased domestic use and increased capital exports.

Figure 4. Consequences of uncertainty when $g\sigma_w > b\sigma_c$.



The indifference curve U(0) corresponds to the case where uncertainty is disregarded, and in this case $(\bar{C}, \bar{W})$ represents the optimal trade-off between consumption growth and terminal wealth. The dotted curve illustrates how the indifference curve is twisted due to the certainty equivalence transformation of the parameters. However, we assume that the feasible set of (C,W) combinations is not influenced by the uncertainty, and the relevant indifference curve is thus $\tilde{U}(1)$. A flatter indifference curve thus entails a change in the decision in the direction of a larger expected value of terminal wealth and a smaller expected value of consumption growth. This is indicated by the point (EC,EW) in figure 4, which represents the optimal trade-off between consumption growth and terminal wealth in terms of certainty equivalence. The parametric certainty equivalence procedure implies that the decision maker will tend to safeguard against uncertainty by taking a decision which implies a higher expected value of the variable which has the higher uncertainty, i.e. uncertainty as measured by the product of the standard deviation and the risk aversion coefficient.

3.3 A STRATEGY FOR OPTIMAL CONSUMPTION UNDER UNCERTAINTY. NUMERICAL RESULTS AND TENTATIVE CONCLUSIONS

In this section an empirical application of the dynamic programming model in chapter 2 will be outlined. Based on the stochastic parameters given in section 2.1 and the risk aversion coefficients derived in section 3.1, the conclusions of the model will be tested against an actual long-term projection. The reference path of the Perspective Analysis is chosen as our point of departure. The questions we are addressing are the following:

- To what extent should current and future consumption be influenced when the value of the oil reserves is considered a part of national wealth?
- How should the uncertainty regarding the value of the oil reserves - as well as the non-oil assets - influence current and future consumption?
- To what extent should a shift in the variance and the expected oil price influence current consumption?
- How should the trade-off between terminal wealth and consumption be affected by uncertainty?

In these tentative calculations, the consequences of uncertainty are examined only with regard to the consumption path and the trade-off between total consumption and terminal wealth. At present we have not made any attempts of estimating an optimal oil extraction path under uncertainty. In our model the principle of estimating an optimal oil extraction path under uncertainty is straightforward: Certainty equivalent net oil price should grow at a rate equal to the risk-free rate of return. However, we have not yet resolved the difficulties in making appropriate cost assumptions. Hence, we have applied the oil extraction path and the accumulation in non-oil assets as given by the reference path.

First of all we need to establish the correspondence between the preference function (5) of the dynamic model and the static analogy given by (31). In the dynamic model which is to be applied now, preferences are formulated in terms of the sum of dicounted utility from consumption over the planning period, whereas in the static preference function (31), the relevant concept is percentage increase in consumption over the planning horizon. In order to find the appropriate risk aversion coefficient in a dynamic context, we make the assumption that the sum of discounted utility from consumption over the planning period is equal to the utility of the percentage increase of consumption. The annual growth rate of consumption in the reference path is 2.4 percent. We assume that the time preference rate is 1 percent. Given the estimate of b=0.1426 an estimate of β=0.0352 is obtained for the risk aversion coefficient of the dynamic model. The estimate of the risk aversion coefficient g=0.00589 is calibrated in order to include the production capital. An estimate of γ= 0.0027 is thus obtained.

The optimal consumption path of the model in chapter 2 is given by the consumption function (24), where optimal consumption in each period is related to wealth. As an implication of the certainty equivalence procedure, uncertainty regarding future income influences the current consumption decision through a risk-adjustment of the time preference rate, which enters the constant term of the consumption function. This consumption function can be expressed as a strategy in the sense discussed in secction 1.3. According to the strategy, decision rules for consumption are elaborated at the beginning of the planning period, whereas actual consumption decisions are implemented by recording the outcome of the stochastic rates of return and inserting, period by period, actual wealth in the strategy function (24). Under uncertainty there is a gain in elaborating a strategy where consumption decisions can be revised, when more information is available, instead of determining the consumption path at the beginning of the planning period.

An increase in uncertainty has the effect of reducing expected consumption in all periods to safeguard against future income loss. A partial increase in a standard deviation implies less risk-adjustment of the time preference rate and thus a partial decrease in the consumption path. The consequence of uncertainty for the optimal consumption path is illustrated by the following calculation. Consider an increase in the oil price uncertainty, which is measured by the standard deviation of the trend-adjusted real oil price. In the estimation over the period 1962-81, the standard deviation is 4 dollar/barrel. The question is now how current consumption is affected by a 100 percent increase in the standard deviation from 4 dollar/barrel to 8 dollar/barrel. The stochastic parameters are given in table 3 and 4, and the expected growth rate of the real oil price is set at 4 percent. This growth rate is exceedingly high, but still small compared to the risk-free rate of return which is set at 3 percent.

The marginal propensity to consume out of current wealth is time dependent and depends on the risk aversion coefficients and the risk-free rate of return, cf. (25). It is estimated to 0,04 by the beginning of the planning period and increases as the planning horizon is approached.

The main assumptions of the calculations are given in table 6.

Table 6. Assumptions for calculating optimal consumption. Billion 1980-Nkr.

	1980	2000
Production capital................	938.9	1726.6
Net foreign reserves	-97.3	58.3
Estimated value of oil reserves .	2782.5	2407.5
Gross domestic product	282.4	403.7
Balance of interest and transfers	-12.1	-0.3
Total wealth	3894.4	4595.8
Optimal consumption	196.9	328.1
Actual consumption/projected consumption in the reference path of the Perspective Analysis ...	186.8	303.0

The risk adjustment terms of the consumption path is illustrated in figure 5. A 100 percent increase in the standard deviation of the real oil price hence has the effect of reducing optimal consumption at the beginning of the planning period by approximately 10 billion Nkr.

Figure 5. Risk adjustment of the optimal consumption path.

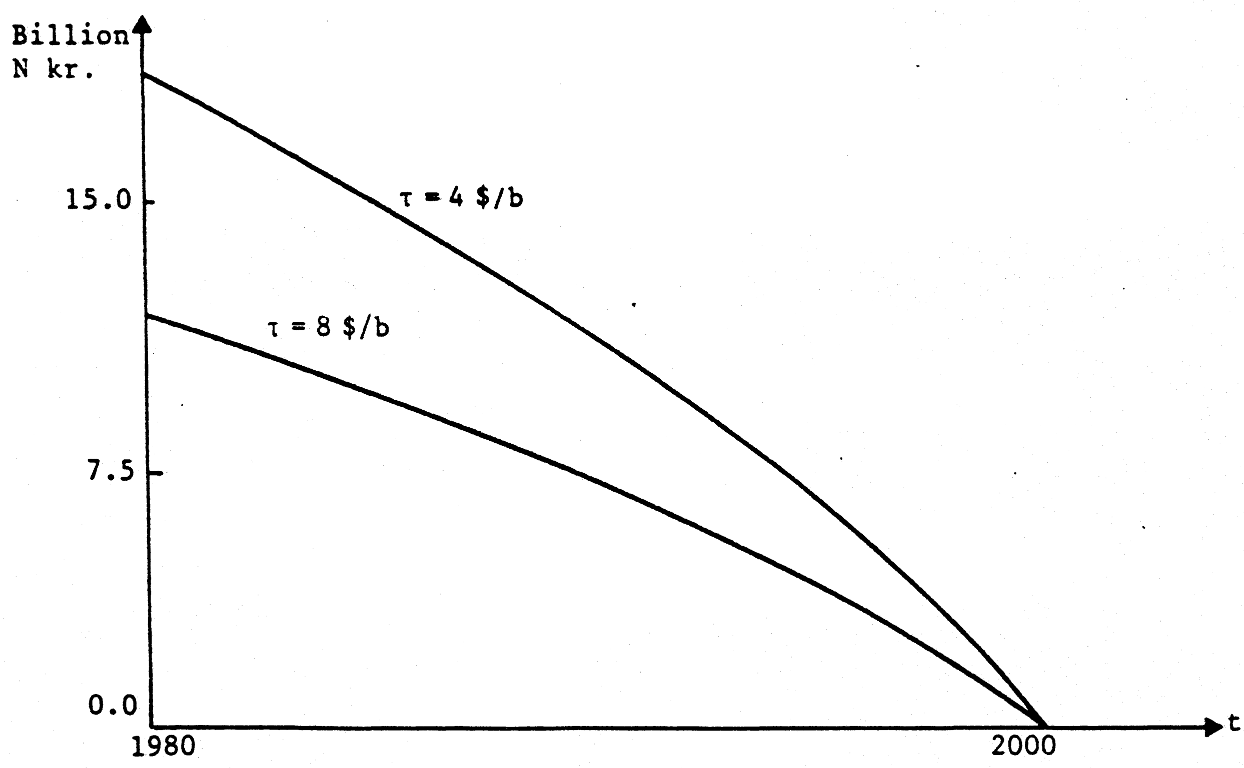


Figure 6. Optimal consumption path.

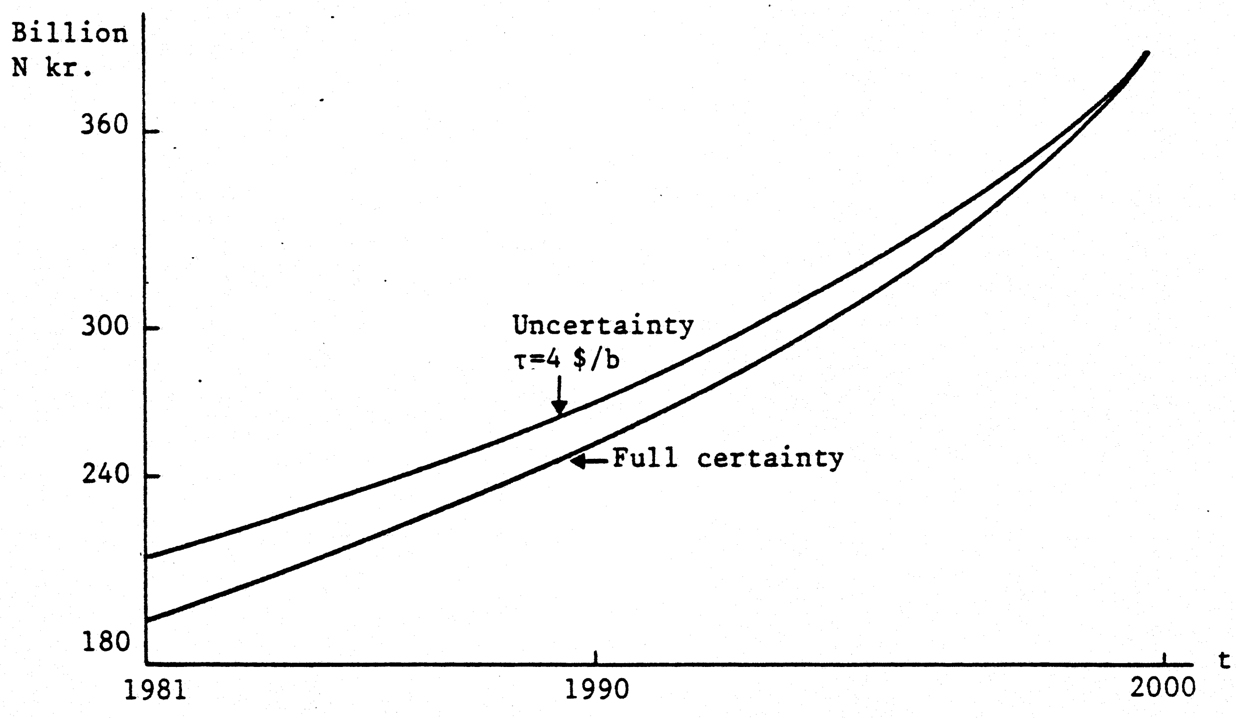


Figure 6 illustrates the optimal consumption path under the assumptions in table 6, compared to an optimal consumption path where uncertainty is disregarded.

The interpretation of the strategy implies that only the current consumption decision is to be influenced by the uncertainty. The indicated value for consumption in 2000 has the interpretation as an estimate of a future decision. The idea of formulating a consumption strategy under uncertainty is that future decisions can be based on more information about the realization of uncertain events than is available at the time of plan preparation. However, the initial decisions should be adjusted to account for future uncertainty.

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