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## *GROSS AND NET CAPITAL, PRODUCTIVITY, AND THE FORM OF THE SURVIVAL FUNCTION - SOME NORWEGIAN EVIDENCE*

*BY*

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### *ABSTRACT*

The paper deals with the empirical measurement of capital stocks from data on gross investment. Two capital concepts are involved: gross capital - representing the capital's capacity dimension - and net capital - representing its wealth dimension. Their interpretation is briefly discussed.

The data base consists of long series of Norwegian national accounts data for gross investment at a fairly disaggregate level of sector classification and for 1-3 capital categories within each sector. Survival functions - representing the process of retirement and decline in efficiency of capital units over time - with different curvature (concave, convex) and non-zero interest rates for the discounting of future capital service flows are considered. The effects of these parameters on the calculated gross and net capital stocks in the years 1956-1982 as well as on the implied replacement and depreciation rates, measures of capital productivity, and rates of return are discussed.

GROSS AND NET CAPITAL, PRODUCTIVITY, AND THE FORM OF THE SURVIVAL FUNCTION - SOME NORWEGIAN EVIDENCE

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# 1. INTRODUCTION

The question of how to measure real capital stocks and flows of capital services are frequently discussed in economic literature (Johansen and Sørsveen (1967), Hall (1968), Hicks (1969), and Jorgenson (1974)). The reason is obviously that estimates of these variables are required in important fields of economic research, such as studies of productivity and producer behaviour, analyses of profitability, and national accounting. Closely related to the estimation of capital stocks are the problems of measuring capital services, capital value, capital prices, and depreciation. As recognized by several authors, real fixed capital has at least two "dimensions". First, it may be interpreted as a capacity measure, i.e. a representation of the potential volume of capital services which can be "produced" by the existing capital stock at a given point of time. Second, it is a wealth concept; capital has a value because of its ability to produce capital services today and in the future. In Biørn (1983a) a theoretical framework is introduced which distinguishes formally between the two capital concepts: Gross capital measures the instantaneous productive capacity of the capital objects, whereas net capital indicates the accumulated prospective capacity of the capital stock. Gross capital can be constructed straightforwardly by aggregating data for gross investments, with the corresponding values of the assumed survival function - representing the remaining capacity of each capital vintage - used as weights. Figures for net capital can be calculated from investment data in a similar way, by applying a different, but related, weighting scheme. The weight attached to a capital unit of a specific age measures the total, discounted flow of services over the remaining life time of this capital unit relative to the corresponding total flow of services provided by a new capital object.

Directly related to the two capital concepts are the derived variables retirement - which is related to gross capital and measures physical wear and tear - and depreciation - which is related to net capital and measure the rate of decline of the capital value.

In this paper, we present series of gross and net capital stocks and derived variables calculated by applying the above mentioned theoretical framework on investment data from the Norwegian national accounts. These accounts contain rather long series of investment (back to the nineteenth century for some categories) at a fairly detailed level of sector classification, distinguishing also between different categories of capital. In our calculations, we have aggregated the investment data somewhat to obtain consistency with the level of aggregation in the Norwegian macroeconomic models MSG and MODAG, which specify 32 production sectors and 3 (main) capital categories. Here, however, we will only be concerned with selected capital items in order to demonstrate some main empirical features and draw some preliminary conclusions. Specifically, we will focus on how the estimated gross and net capital stocks and related variables, like rates of retirement and depreciation, vary with the assumption made about physical wear and tear of the capital units. For this purpose, we distinguish between four different non-exponential survival profiles.

Furthermore, we use the estimated capital data to calculate some important derived economic measures, such as productivity and rates of return to capital (profitability). Again, a main purpose is to investigate the sensitivity of these measures with respect to the form of the survival function - a problem that, to our knowledge, has not received much attention in the literature.

The discussion in this paper focuses on the distinction between gross and net capital and related variables. Other difficult problems in the field of capital measurement, such as problems of aggregation of different capital categories in different sectors, are not addressed. We will only be concerned with aggregation across vintages.

One basic point concerning the interpretation of our results in this paper should be mentioned: Although the distinction between different capital vintages is of major importance in our theoretical model, this does by no means imply that the presented formal framework fits into any type of "vintage production model", e.g. the putty-clay model suggested by Johansen (1959). A vintage production model is characterized by the fact that there is (i) a specific technology attached to each vintage of capital goods and (ii) limited substitutability between capital goods belonging to different vintages. The concepts and assumptions in this paper are, on the contrary, implicitly related to or derived from a <u>neoclassical</u> production framework, in which (i) only the total, accumulated capital stock is specified as argument in a production function for the sector as a whole, reflecting the underlying assumption of perfect substitutability between capital vintages, and (ii) perfect markets exist for both new and old capital objects.

The paper now proceeds as follows: In the next section, the theoretical framework and the basic concepts are established. The four chosen profiles describing capital outwear are presented in section 3, while section 4 gives an overview of the investment data used in the empirical part of the paper. The empirical results are reported in section 5, and finally, section 6 contains concluding remarks and some suggestions for further research.

## 2. THEORETICAL BACKGROUND

The concept of gross capital can be defined straightforwardly from a sequence of gross investment figures and assumptions of how the productive capacities of the capital objects decline over time. In the following, $J(t)$ denotes the quantity of capital invested at time $t$, measured in physical units or as a quantity index so that different vintages may be compared (cf. the remarks made above). In order to simplify the presentation, time is throughout this paper treated as continuous. However, the formulas actually implemented to calculate capital figures are, of course, converted to the annual periodicity of the investment data. (Confer section 5.) The physical wear and tear of the capital units is described in a rather traditional way (see e.g. Johansen and Sørsveen (1967) and Jorgenson (1974)) by introducing the technical survival function $B(s)$, expressing the proportion of an investment made s periods ago which still exists as productive capital. The function $B(s)$ is assumed to represent both the loss of efficiency of existing capital units and the physical retirement of old capital goods. The following restrictions are imposed on this function:

$$(1) \qquad 0 \leq B(s) \leq 1 \quad , \quad \frac{dB(s)}{ds} \leq 0 \qquad \text{for all} \quad s \geq 0,$$

$$B(0) = 1 \quad , \quad \lim_{s \to \infty} B(s) = 0.$$

The volume of capital which is s years of age at time t, $K(t,s)$, is now defined by the following relation:

$$(2) \qquad K(t,s) = B(s)J(t-s), \qquad s \geq 0.$$

The <u>gross capital</u> stock at time t is obtained simply by aggregating over capital vintages, i.e.

$$(3) \qquad K(t) = \int_0^\infty K(t,s)ds = \int_0^\infty B(s)J(t-s)ds.$$

In accordance with the definition of $B(s)$, gross capital is a technical concept; $K(t)$ represents the current productive capacity of the total capital stock at time t. Thus, gross capital - or the services produced by this stock - is the relevant concept to be represented e.g. as argument in a neoclassical production function. Its age distribution, as represented by K(t,s), is irrelevant to the description of the technology.

Related to the gross capital stock is the volume of retirement (or replacement) at time t, D(t), which by definition is the difference between gross investment and the rate of increase of the (gross) capital stock. An expression for D(t) can be found by differentiating (3) with respect to t, which gives

$$(4) \qquad D(t) = J(t) - \dot{K}(t) = \int_0^{\infty} b(s)J(t-s)ds,$$

where the 'dot' denotes the derivative with respect to time and

$$b(s) = -\frac{dB(s)}{ds} .$$

The function b(s) indicates the structure of the scrapping process, and will be called the retirement (replacement) function in the sequel.

Formulas for gross capital and retirement similar to (3) and (4) can be found in e.g. Jorgenson (1974) and Hulten and Wykoff (1980). Unfortunately, the terminology does not seem to be quite consistent in the literature. Some authors (e.g. Steele (1980) and Johansen and Sørsveen (1967)) define gross capital as the cumulated volume of past gross investment flows, i.e. without adjusting the remaining stock for physical outwear or efficiency loss. This definition is of course equivalent to (3) if the survival profile is of the simultaneous retirement ("one-horse-shay") type, i.e. if the productive capacity of the capital units actually remain constant (and full) over their lifetime (see below). The definition of gross capital given in (3) is, however, a more general and for empirical purposes more interesting one, since it may also encompass other structures of capital retirement.

While gross capital expresses the current productive capacity of the capital stock, net capital is related to the value dimension of the capital. The value concepts to be introduced are implicitly based on the assumption that there exist well organized markets for capital goods, where both new and old capital goods may be bought and sold. The market value of the capital objects will, in general, reflect the cost of producing new capital goods on the one hand, and the producers' expectations about future productivity on the other. For old capital units, it is the service flow that they are expected to produce during their remaining life time that matters; thus, it is reasonable to assume that capital prices are decreasing functions of the age of the capital objects.

The price of a capital unit which is s periods old at time t is in the following denoted by $q(t,s)$. For new capital installed at time t the simplifying notation $q(t) = q(t,0)$ is applied. The value of capital of age s, $V(t,s)$, may then be written as

$$(5) \qquad V(t,s) = q(t,s)K(t,s),$$

and cumulation over all vintages gives the total value of the capital stock at time t, i.e.

$$(6) \qquad V(t) = \int_0^\infty V(t,s)ds = \int_0^\infty q(t,s)K(t,s)ds = \int_0^\infty q(t,s)B(s)J(t-s)ds.$$

Equation (6) defines the current value of the capital stock. This expression reveals a basic difference between the value dimension and the capacity dimension, represented by the gross capital concept: while the latter measures the instantaneous service flow at time t, the market value depends moreover on the decline in <u>future</u> productivity of the capital stock. For this reason, the prices, $q(t,s)$, will in general decrease with s for every given t. However, we want to decompose the current market value into a price and a quantity component in order to obtain a measure for the capital value that is not influenced by changes in the general price level. For practical purposes it is then necessary to introduce specific assumptions of how capital prices vary with age, s, i.e. to eliminate the s index in the price variable, $q(t,s)$. In this paper, following Biørn (1983a), the specific assumption made is that the relative prices of capital units of different ages perfectly reflect the differences in their prospective service flows. More precisely, the price per unit of the discounted future flow of capital services is assumed to be the same for all capital vintages at each given point of time. The discounted future service flow per capital unit which is s years old, is given by

$$(7) \qquad \phi_\rho(s) = \frac{\int_s^\infty e^{-\rho(z-s)} B(z)dz}{B(s)},$$

where $\rho$ is the rate of discount.

Formally, our assumptions regarding relative capital prices can then be expressed as follows:

$$(8) \qquad \frac{q(t,s)}{\phi_\varrho(s)} = \frac{q(t)}{\phi_\varrho(0)} \qquad \text{for all } t \text{ and all } s > 0.$$

This equation implies a sort of 'law of indifference' to hold between different capital vintages: since the prices per unit of (discounted) prospective capital services are the same, a firm will be indifferent between investing in new and old equipment.[1)]

The common price of per unit of (discounted) capital services is

$$(9) \qquad c(t) = \frac{q(t)}{\phi_\varrho(0)} = \frac{q(t)}{\int_0^\infty e^{-\varrho s}B(s)ds}.$$

This is a <u>general</u> expression for the user cost of capital in a neoclassical model of producer behaviour, in the absence of taxes. Traditionally, the user cost of capital is interpreted as a shadow price, obtained from the first-order conditions for maximization of the present value of future cash flow (see e.g. Jorgenson (1963) or Biørn (1983b)). The interpretation given here has a more intuitive basis, since it is derived directly from (8).

Combining (6) and (8), the value of the capital stock may be written as

$$(10) \qquad V(t) = q(t) \int_0^\infty \frac{\phi_\varrho(s)B(s)}{\phi_\varrho(0)} J(t-s)ds.$$

If we furthermore choose the current investment price, $q(t)$, as the price component of the market value, the quantity component becomes[2)]

$$(11) \qquad K_N(t) = \frac{V(t)}{q(t)} = \int_0^\infty G_\varrho(s)J(t-s)ds,$$

$$\text{where} \qquad G_\varrho(s) = \frac{\phi_\varrho(s)\,B(s)}{\phi_\varrho(0)}.$$

This is the variable which we shall refer to as the <u>net capital</u> stock in the following. It is seen that $K_N(t)$, like $K(t)$, is constructed by aggregating previous investment flows, but the weighting system is different; the weight assigned to investment made $s$ years ago in $K_N(t)$, $G_\varrho(s)$, is the share of the total discounted service flow produced by a capital unit <u>after it is s years old</u>, whereas the calculation of $K(t)$ is based on the technical survival profile, $B(s)$. It is easily seen that $G_\rho(s)$ has the same mathematical properties as $B(s)$, cf. (1).

The conceptual difference between gross and net capital can be explained in a slightly different way: Let the elements of the net capital at time t which belongs to vintage t-s be denoted as $K^*_\rho(t,s)$, i.e.

$$(12) \qquad K^*_\rho(t,s) = G_\rho(s)\, J(t-s) = \frac{\Phi_\rho(s)}{\Phi_\rho(0)}\, K(t,s).$$

While the gross capital stock is defined by simply adding (integrating over s) all $K(t,s)$, the net capital stock is calculated in a similar way after having first multiplied these vintages by the ratio $\Phi_\rho(s)/\Phi_\rho(0)$, which expresses the remaining (discounted) flow of services per unit from "old" (age s) capital vintages relative to the corresponding service flow produced by new capital. When compared with the gross capital, the net capital is thus adjusted for the fact that old capital objects generally are less productive in terms of <u>future</u> cumulated services than new ones, even if they are equivalent in terms of <u>instantaneous</u> service flows.

From these interpretations it may be concluded that estimates of net capital will normally be lower than corresponding figures for gross capital (strictly, the inequality $K_N(t) < K(t)$ always holds).[3)]

Net capital, in contrast to gross capital, is dependent on the rate of discount, $\rho$. This is due to the fact that it reflects prospective capital service flows. The net capital stock will, in general, increase with increasing discounting rate. This may be explained intuitively as follows: The calculation of net capital stock is a way of "correcting" the gross capital stock for the presence of "old" capital units when focusing on its value dimension. When the rate of discount increases, the total service flows from both "new" and "old" capital units decrease, but the reduction in "cumulated future productivity" is relatively strongest for the former units, i.e. for those which have the longest remaining life time. Returning to (12), we see that this result is reflected by the fact that $\Phi_\rho(s)$ approaches $\Phi_\rho(0)$ as $\rho$ increases, regardless of the form of the survival function $B(s)$. With a very high rate of discount it does not matter much whether the capital stock consists of new or old capital units, since a very small weight is given to the service flows in future periods by the discounting procedure. For $\rho \to \infty$ we find $K_\rho(t) \to K(t)$.

The final concept to be defined in this section is depreciation. This variable has the same formal relationship to the net capital stock as retirement has to the gross capital stock. This means that depreciation in volume terms is the difference between the gross investment quantity and the increase in the net capital stock. Proceeding in a similar way as when deriving retirement (eq. (4) above) <u>depreciation</u> at time t, $D_N(t)$, can be expressed as follows:

$$(13) \qquad D_N(t) = J(t) - \dot{K}_N(t) = \int_0^\infty g_\rho(s)J(t-s)ds ,$$

$$\text{where} \qquad g_\rho(s) = -\frac{d\,G_\rho(s)}{ds} .$$

The $g_\rho(s)$ function indicates the structure of depreciation, in the same way as the b(s) function represents the retirement process.

Finally, we will call attention to an interesting relationship between the variables introduced above. It can be shown (cf. Biørn (1983a, section 6)) that depreciation, net capital and gross capital satisfy the following equation identically in $\rho$ and gross investment:

$$(14) \qquad D_N(t) + \rho K_N(t) = \frac{K(t)}{\Phi_\rho(0)} .$$

Combining (9) and (14), we obtain

$$(15) \qquad q(t)D_N(t) + \rho q(t)K_N(t) = c(t)K(t).$$

Recalling the interpretation of c(t) as the user cost (per unit) of capital, this relation expresses that the current "user value" of the capital stock equals the sum of the value of depreciation and a term which represents interests imposed on the capital value. If we had replaced $D_N(t)$ and $K_N(t)$ in (15) by D(t) and K(t), respectively, the resulting relation would have been identical to the expression for the user value of capital found in many textbooks describing static producer behaviour. However, decomposing the user value additively in this way - on the basis of the gross capital concept - implicitly presupposes an exponential retirement structure of the capital stock (see e.g. Biørn (1983b, section 7) or Holmøy and Olsen (1985)). If another type of survival function is in effect, it is no longer possible to separate the user value additively into an interest term attached to <u>gross</u> capital and a term which is the product of the volume of <u>retirement</u> and the current investment price. Equations (14) and (15) show that such an additive formula exists as an identity only between the "value" related concepts depreciation and net capital.

The relationships (14) and (15) may be used to support the common practice applied in many countries when calculating net operating surplus as a residual, i.e. as what is left from gross factor income when wages and the value of "depreciation" is deducted. When this is done, as e.g. in national accounting in Norway, the procedure may be said to be consistent with the framework and concepts presented above, since it is reasonable to assume that national accounting calculations are intended to represent the value dimension rather than the capacity dimension of the capital stock.[4)]

Equation (14) may also - because of its simple form - be used to facilitate the computation of gross and net capital figures from investment data, or to check the consistency of the resulting series. In our calculations, this relation turned out to be useful both for the latter type of application and in searching for the "best" discrete time approximation to the framework established in continuous time above (see section 5).

## 3. SOME PARAMETRIC SURVIVAL FUNCTIONS

In this section, we present two classes of parametric survival functions which we consider useful for empirical applications. Each class is characterized by two parameters; the first representing the maximal life time of the capital, the second indicating the "curvature" of the survival profile. When performing the calculations for this paper the latter parameter was varied in such a way as to produce four different survival functions within these broad classes of parametrizations.

Class I: Convex

Consider the following parametric form for B(s):

$$(16) \qquad B(s) = B^I(s;N,n) = \begin{cases} (1-\frac{s}{N})^n & \text{for } 0 \leq s \leq N, \\ 0 & \text{for } s > N, \end{cases}$$

where N is the maximal life time of the capital objects and n is a positive integer constant. The notation $B^I(s;N,n)$ is introduced in order to indicate the two-parametric nature of this class of survival functions. The corresponding retirement function is

$$(17)\qquad b(s) = b^I(s;N,n) = \begin{cases} \frac{n}{N}\,(1-\frac{s}{N})^{n-1} & \text{for } 0 \leq s \leq N, \\ 0 & \text{for } s > N. \end{cases}$$

General expressions and a recursive procedure of deriving numerically the weighting functions for net capital and depreciation from this class of technical survival functions are presented in Biørn (1983a, section 7), and they will not be repeated here. An interesting result for the case when the interest rate, $\rho$, is zero may, however, be mentioned:

$$(18)\qquad G_0(s) = (1-\frac{s}{N})^{n+1} = B^I(s;N,n+1),$$

$$(19)\qquad g_0(s) = \frac{n+1}{N}\,(1-\frac{s}{N})^{n} = b^I(s;N,n+1).$$

When no discounting of future capital services is performed, there is thus in this case a very simple relationship between the weighting function of the gross capital and that of the net capital: we only have to replace n by n+1 to go from the former to the latter.

By varying the parameter n, the class of survival functions (16) generates several specifications discussed in the literature as special cases. Three of these will be considered in this paper. We first consider the case when <u>n=0.</u> This is the <u>simultaneous retirement</u> case[5] in which the capital objects are assumed to retain their full productive capacity during N periods and are then completely scrapped. The survival function (16) then equals unity for all vintages up to the scrapping age. The retirement function, b(s), is not formally defined in this case. In the special case of zero interest rate it is easily seen from (18) and (19) that the weighting functions for net capital and depreciation then are given by

$$(20)\qquad G_0(s) = 1 - \frac{s}{N},$$

$$(21)\qquad g_0(s) = \frac{1}{N}.$$

When <u>n = 1</u>, the survival function is a <u>linearly decreasing</u> func-function of s, i.e.

$$(22)\qquad B(s) = 1 - \frac{s}{N}$$

$$\qquad\qquad\qquad 0 \leq s \leq N.$$

$$(23)\qquad b(s) = \frac{1}{N}$$

Comparing (22), (23) with (20), (21) exemplifies the general property specified in equations (18) and (19): when the technical survival function is of the simultaneous retirement type, the net capital stock in the zero-interest case is depreciated linearly; the net capital figures estimated with this structure will thus be identical to figures for gross capital calculated when the <u>technical</u> survival function is assumed to be linear. This is the second survival function we shall consider in this paper.

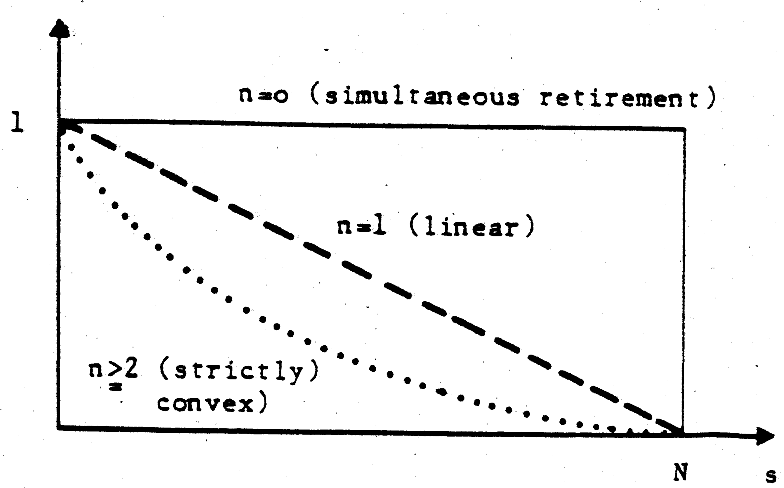
The reason why we denote this class of survival functions as "convex" is the fact that for n > 2 eq. (16) defines a set of <u>strictly convex</u> functions. Formally this follows from

$$(24) \qquad \frac{db(s)}{ds} = -\frac{n(n-1)}{n^2}\left(1-\frac{s}{n}\right)^{n-2} < 0$$

whenever n > 2.

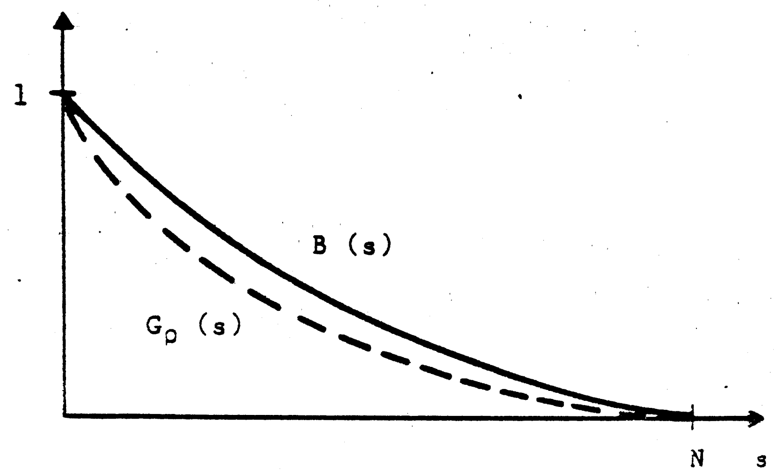
In the calculations presented in this paper for the specific survival profile denoted as "convex", which is the third profile we shall consider, n is set equal to 5. A typical (but arbitrarily drawn) strictly convex survival profile is depicted in figure 1, where we have also included the linear profile and the simultaneous retirement profile.

Figure 1: Three special cases of survival functions within class I.



As emphasized above, the net capital stock will normally be lower than the corresponding gross capital, and the weighting function for the former will lie below that for the latter. With a strictly convex survival function this property is illustrated in figure 2.

Figure 2: A strictly convex survival function and a corresponding weighting function for the net capital




Class II: Concave
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The second class of survival functions with which we shall be concerned has the following general form:

$$(25) \qquad B(s) = B^{II}(s;N,m) = \begin{cases} 1-(\frac{s}{N})^m & \text{for } 0 \leq s \leq N, \\ 0 & \text{for } s > N, \end{cases}$$

where $m$ is a positive, integer constant and N, as before, is the maximal life time of the capital units.

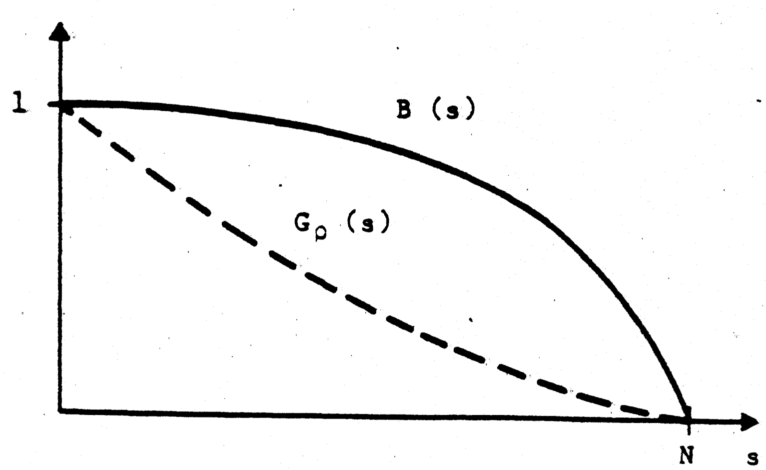
Again, the "curvature" parameter $m$ may be varied in order to "produce" specific survival profiles. First, it is seen that when $m = 1$, we are back again at the linear survival function. Second, we note that if $m \to \infty$, this model degenerates to the simultaneous retirement case, since $\lim_{m \to \infty} (s/N)^m$ is zero when $s < N$ and one when $s = N$.

The fourth specific survival function used for empirical investigations in this paper is a strictly concave one, i.e. a specification within the subset of class II where $m > 2$. In our calculations, we have chosen to illustrate this case by setting $m = 5$. The survival profile is seen to be strictly concave in $s$ for $m > 2$, since then

$$(26) \qquad \frac{db(s)}{ds} = \frac{m(m-1)}{N^2}\left(\frac{s}{N}\right)^{m-2} > 0 \qquad (m>2).$$

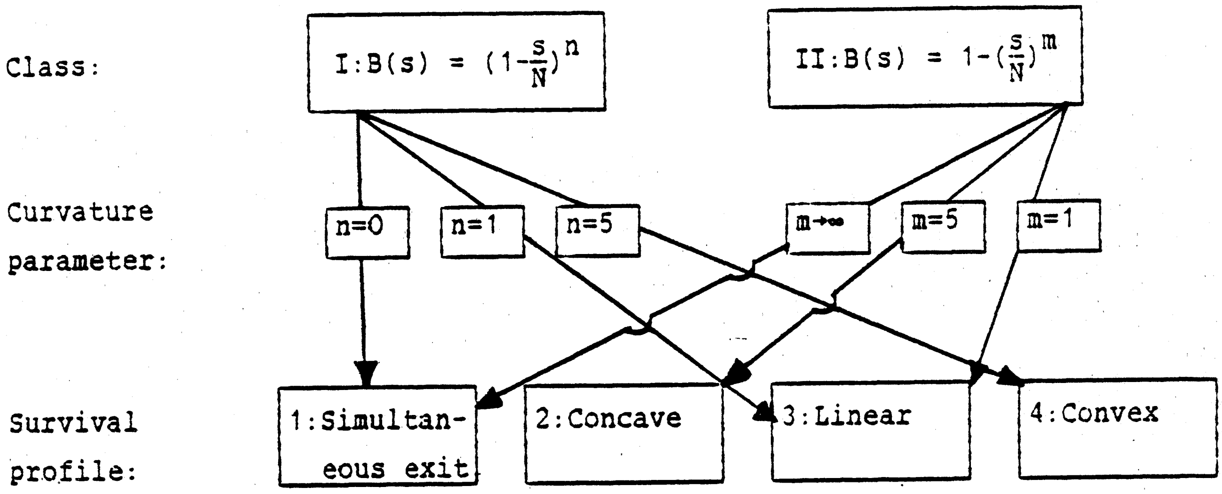
The derived weighting function for the net capital, $G_\rho(s)$, lies below the technical survival function, $B(s)$. It should be noted that when the latter is of the strictly concave type - i.e. when the rate of physical outwear increases with age - it may very well be the case that the corresponding decline in the net capital is represented by a convex function - i.e. that depreciation decreases with age.[6) ] An example of this type of structure is indicated in figure 3.

Figure 3: A strictly concave survival function and the corresponding weighting function for the net capital



As a conclusion, it is useful to illustrate the relationship between the two classes of survival functions, and the specific survival profiles applied in the present study. This is done in figure 4.

Figure 4: An overview of the two classes of survival functions and the specific parameter values chosen in this study.




## 4. DATA

The practical procedure for constructing time series for gross and net capital stocks, as it follows from sections 2 and 3, consists in cumulating past series for gross investment at constant prices over a period of length equal to the capital's maximal life time, by application of two different weighting schemes. The weights are defined once the survival function, B(s), and the rate of discount, $\rho$, have been specified, and once an algorithm for conversion from continuous to discrete time has been constructed. The latter problem, which is mainly of a technical nature, will be discussed briefly in section 5.

Needless to say, this aggregation across vintages places a rather strong claim on the length of the historic gross investment series: If the maximal life time of the equipment is assumed to be N years and we want our capital stock series to start in year $T_0$, then investment series back to at least year $T_0$-N should be available. For capital items with long service lives, notably buildings and structures, this may, in particular, be felt as a severe limitation.

This problem cannot (or, at least, should not) be considered as separated from the choice of level of aggregation for the capital items.

The researcher frequently faces a conflict between the desire to work with reasonably homogeneous capital categories - i.e., homogeneous with respect to normal service lives, assumed retirement patterns, and other technical characteristics - and the need to have sufficiently long investment series to make the vintage aggregation approach work. Often, long series exist only for large categories like machinery and equipment in total manufaturing, office buildings and structures in private companies, dwellings, etc.; disaggregation can only be achieved at the expense of shorter time series.

The data base for the present paper is Norwegian national accounts data for gross investment at constant (1975) prices, which permits us to go longer in the direction of disaggregating capital by sector and kind than is usually possible. With a few exceptions, our gross investment series go as far back in time as to permit - with the values of the maximal life time specified (see below) - the computation of capital stock series to start in the year 1956 (at the latest). For our purpose, we have aggregated the detailed investment data in the national accounts to a sector classification with 26 sectors - corresponding, with minor discrepancies, to the one used in the present version of the Norwegian multi-sectoral growth model MSG (see Longva, Lorentsen and Olsen (1985)). For most of the sectors we specify three capital types:

1. Buildings and structures,
2. Transport equipment,
3. Machinery, and other equipment, etc.,

which makes a total of 71 different capital items.

The gross investment concept (gross fixed capital formation) includes, according to the United Nations' "System of National Accounts" (1968) (SNA)

> "The outlays of industries and general government on additions of new durable goods to their stocks of fixed assets less their net sales of similar second-hand and scrapped goods. The item includes acquisitions of reproducible and non-reproducible durable goods except land (but costs in connection with purchases and sales of land etc. is included), and less mineral deposits, timber tracts and the like for civilian use. The item also includes work in progress on construction projects, significant capital repairs, outlays on land improvement and changes in the stock of breeding animals, dairy cattle and the like.............

The general rule is that acquisitions of assets with a lifetime of at least one year shall be included in gross fixed capital formation. Repair and maintenance expenses are generally counted as intermediate consumption, but are considered gross fixed capital formation if the repairs or maintenance work are so sizeable that the lifetime of the asset is expected to be extended or result in higher productivity". (Fløttum (1981), pp. 14 and 65.)

This definition is not ideal for our purpose: In particular, the recommended treatment of transactions in second-hand goods between sectors and of repairs and maintenance, may violate our basic assumption that the form of the survival function is time-invariant and independent of the composition of the gross investment flow. With respect to repairs, an additional problem is created by the fact that the time series for gross investment in the accounts are recalculated according to the new SNA only back to 1967, which means that earlier figures include repairs and maintenance to a larger extent than investment figures after this point of time. On the other hand, the recommended SNA procedure for treating transactions in second hand markets is not fully adopted in the construction of the present investment data: For most categories, the figures are not adjusted for sales and purchases of old capital objects, which implies that our data actually measure investments in new capital equipment. Unfortunately, we were unable, from the information available, to obtain more homogenous series.

From the 71 capital items in the data file, we have selected the following sectors and capital types for empirical investigation in this paper:

| Capital item | Maximal life time, years, N |
|---|---|
| Manufacture of textiles etc. | |
| Machinery etc. .......................................... | 25 |
| Buildings and constructions .......................... | 60 |
| Manufacture of metals | |
| Machinery etc. .......................................... | 25 |
| Buildings and constructions .......................... | 60 |
| Petroleum sector | |
| Constructions ............................................ | 16 |
| Dwellings | |
| Buildings ................................................ | 90 |

The values assumed for the maximal life time, N, are the same as those presently used for the calculation of capital stocks and depreciation flows in the Norwegian national accounts. Confer also OECD (1982, section II).

## 5. EMPIRICAL RESULTS

The theoretical framework, developed in sections 2 and 3, has, for matematical convenience, been expressed in continuous time. The conversion to discrete time required for confrontation with our annual gross investment data is carried out as follows:

(i) Let year t (with discrete time) be defined as the interval between time t and time t + 1 (with continuous time), let $J_t$ be the gross investment effectuated during year t and $K_t$ the gross capital stock at the end of year t. Assuming a maximal life time of N years, we then have

$$(27) \qquad K_t = \sum_{i=0}^{N-1} B_i J_{t-i},$$

where

$$B_i = \int_0^1 B(i+\tau)d\tau \qquad (i=0,1,\dots,N-1).$$

This formula can be shown to hold exactly from the formula in continuous time, (3), if the investment is effectuated at a constant rate during each year; otherwise, it represents a more or less good approximation. Retirement in year t is calculated as

$$(28) \qquad D_t = J_t - (K_t - K_{t-1}).$$

The values of $B_i$ are calculated from the parametric survival functions given in section 3 by numerical integration.

Net capital and depreciation are similarly treated.

(ii) The rate of interest, $\varrho$, used for the discounting of prospective service flows expressed in continuous time is specified to be related to the interest rate on a per annum basis, $\varrho'$, as follows:

$$\varrho = \log(1+\varrho').$$

All interest rates reported below are given as annual rates.

(iii) We have tested the accuracy of the approximations (i) and (ii) by checking - for selected capital items - the definitional consistency between the annual series for gross capital and replacement on the one hand, and net capital and depreciation on the other. (Confer eq. (14).) The largest approximation errors were found for capital items with (a) short service lives, (b) very volatile investment series (e.g. investment in oil production), and/or (c) strongly convex survival functions. On the whole, however, we judged the accuracy as satisfactory.

We now want, in particular, to focus on the following issues:

(a) the effects of the assumed form and curvature of the survival profile on gross capital stocks in sectors with different development of gross investment,

(b) the relationship between gross and net capital stocks, and the dependence of the net capital stock on the rate of discount of future capital services,

(c) the effect of the assumed form and curvature of the survival profile on the retirement rate,

(d) the relationship between the retirement and depreciation rates, and finally,

(e) the effect of the form and curvature of the survival profile on capital productivity, and the effect of the assumed rate of discount on the estimated rate of return to capital.

The sensitivity of the gross capital stock to the form of the survival function for three selected industries is illustrated in figures 5, 6 and 7. We see that not only the level of the capital stock, but also its growth profile is strongly dependent on the structure of retirement. There are, however, notable differences between the production sectors in this respect: Oil production, which has been the most outstanding growth sector in Norway in the last decade, shows a sharply increasing capital stock

Figure 5 : Gross capital for 4 different survival functions,
in 100 million 1975 kroner.
Manufacture of metals. Machinery etc.
Maximal lifetime 25 years.

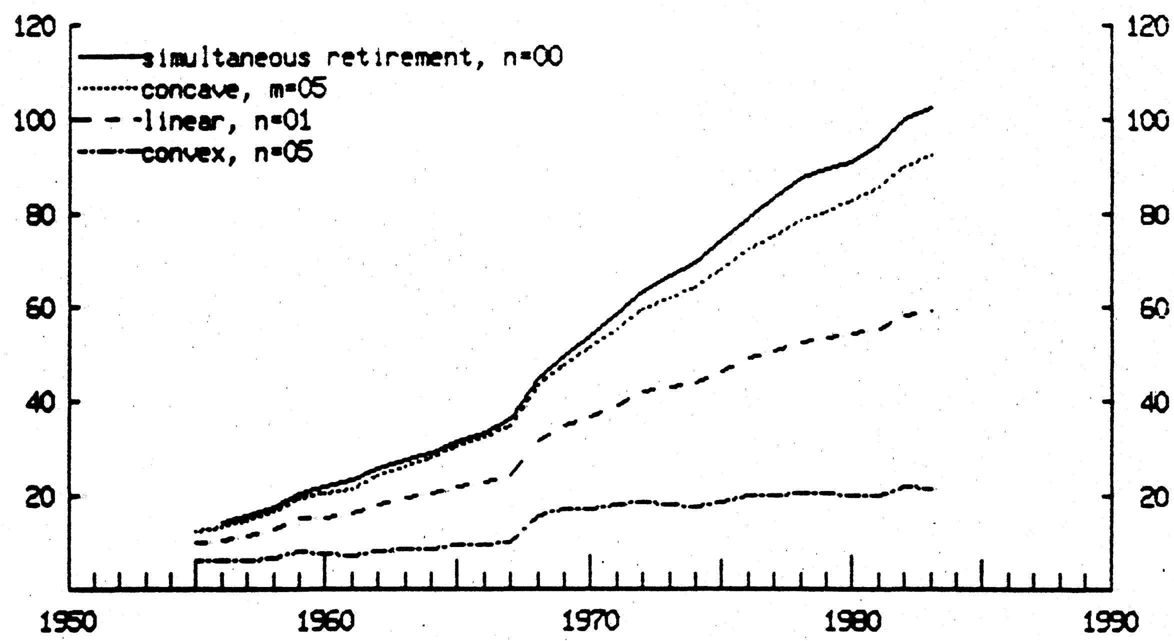



Figure 6 : Gross capital for 4 different survival functions,
in 10 million 1975 kroner.
Manufacture of textiles etc. Machinery etc.
Maximal lifetime 25 years.

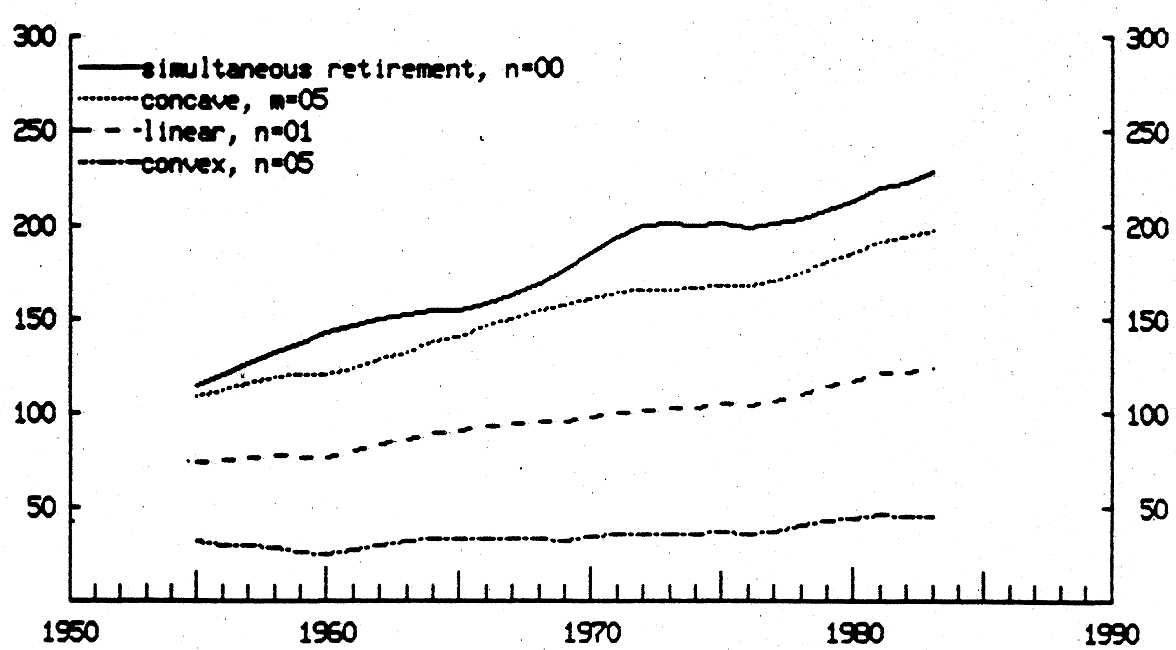
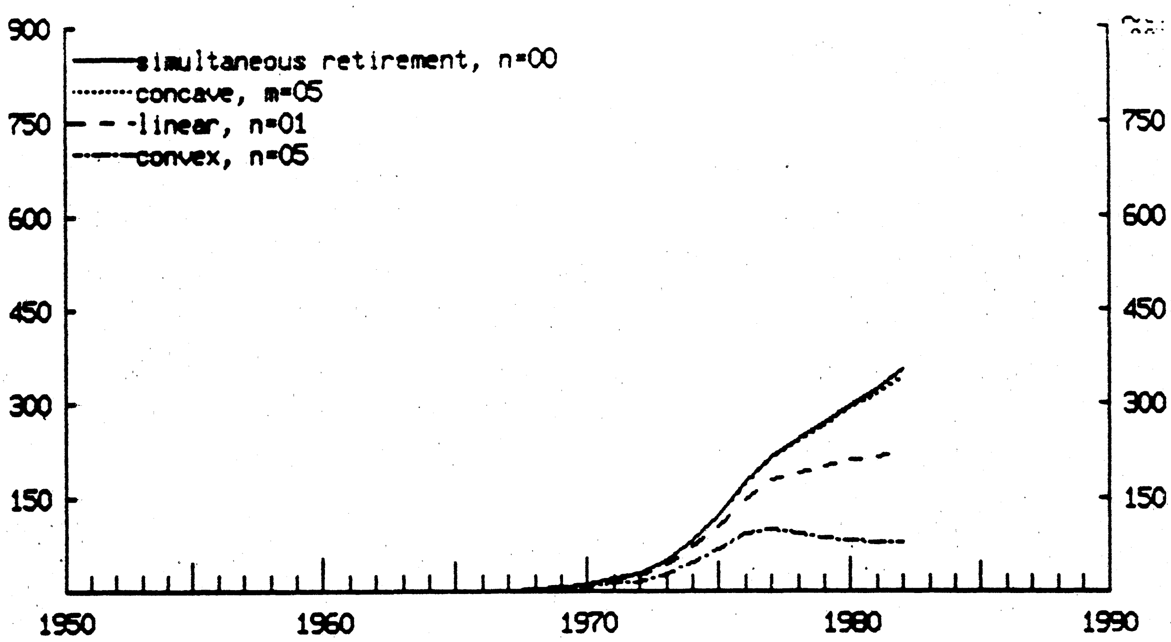


Figure 7 : Gross capital for 4 different survival functions, in 100 million 1975 kroner. Petroleum sector. Buildings and constructions. Maximal lifetime 16 years.




during the entire period in the case of a one horse shay retirement pattern, whereas capital stock attains a peak in 1977 and then decreases if a convex profile is assumed. (Figure 7.) Production of textiles etc., which exemplifies a sector with stagnation in investment activity, gives a markedly different picture; gross capital stock (machinery) is nearly constant in the convex case, moderately increasing in the linear case, and exhibiting a pattern of cyclical growth in the one horse shay case. (Figure 6.) To a large extent, these differences, of course, reflect the different age distribution of the capital stock implied by the four survival functions considered. The more convex the survival profile, the relatively larger are the weights given to investments in the current and recent vintages as compared with older vintages. Regarding the oil industry, it should be recalled that a major part of the production capacity in Norway was built up during the 1970's. A closer look at the investment data reveals that gross investment increased strongly from 1973 to 1977, but decreased thereafter until 1981, when a new peak in investment was attained. This investment path explains the development of the gross capital stock in the convex case, with an observed peak in 1977. When the survival profile is assumed to be of the simultaneous exit type, the gross capital stock increases throughout the period, because this assumption implies that no "heavy" vintages have been scrapped during the present period of observation.

Figures 8 and 9 serve to illustrate the difference between the gross and net capital concepts. Here we consider a capital category with a long service life (dwellings, N=90 years), which explains the smoothness of the growth curves in these figures. We see that net capital is close to gross capital if the retirement and decline in efficiency follows a convex pattern (B(s) convex). (Figure 8.) As noted in section 3, the weighting function for net capital, G(s), is in this case also convex. If, on the other hand, the survival function is specified to be concave, we find - at least when a zero rate of discount is applied - a substantial difference between the numerical values of the two capital measures. (Figure 9.) This reflects the basically different curvature of their weighting functions - gross capital is constructed from a concave function, net capital from a convex function (cf. figure 3) - which implies that new and old capital vintages are given widely different weights in the two capital measures in this case.

Figure 8: Gross and net capital stock in billion 1975 kroner.
Dwellings. Buildings and constructions.
Survival function: Convex, n=05. Interest rate 00 per cent.
Maximal life time 90 years.

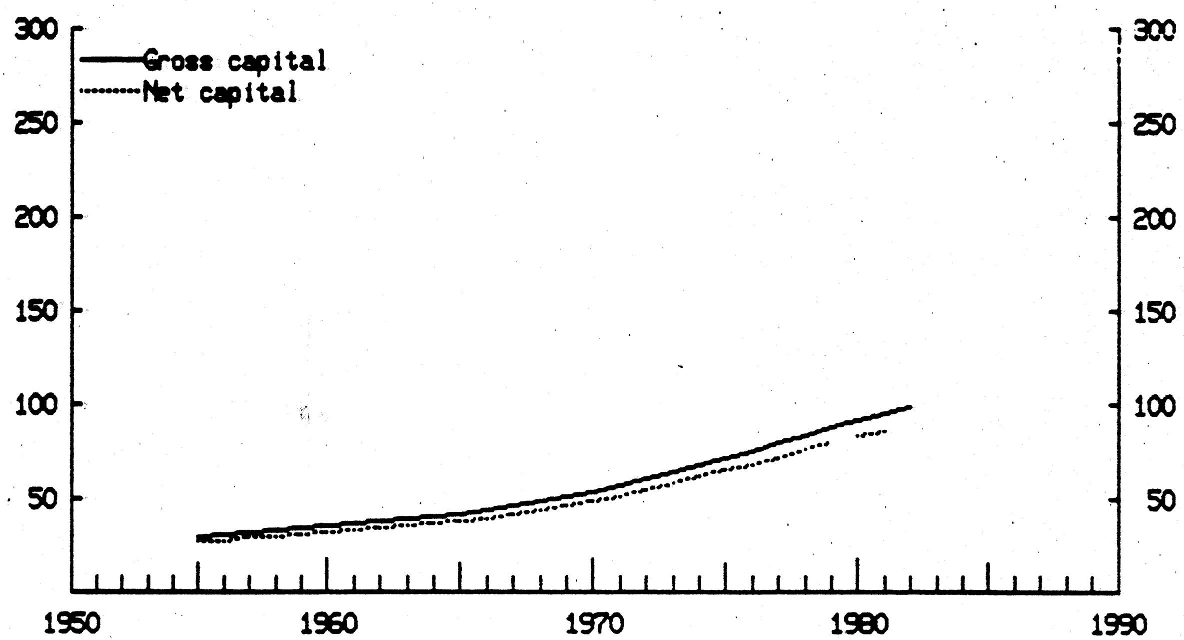


Figure 9 : Gross and net capital stock in billion 1975 kroner.
Dwellings. Buildings and constructions.
Survival function: Concave, m=05. Interest rate 00 per cent.
Maximal life time 90 years.

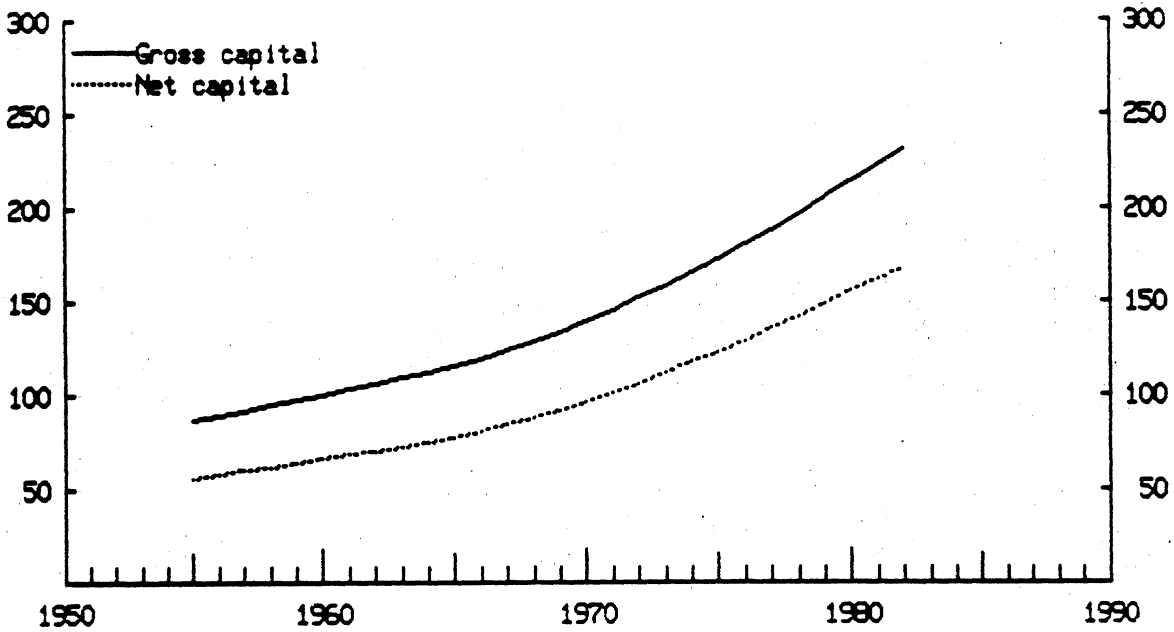
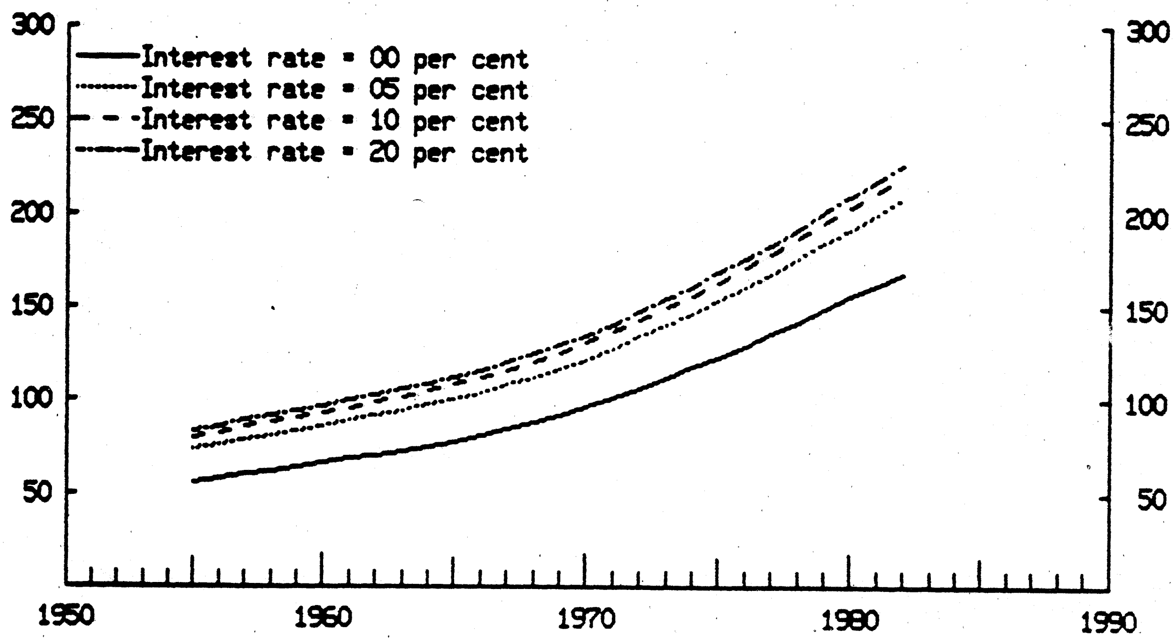



Figure 10 : Net capital stock in billion 1975 kroner.
Dwellings. Buildings and constructions.
Survival function: Concave, m=05.
Maximal lifetime 90 years.



The sensitivity of the net capital stock to variations in the rate of discount, $\rho$, is illustrated in figure 10. A change in this parameter alters the relative weights of different vintages because it affects the agents' relative evaluation of future flows of capital services under perfect market conditions. As noted in section 2, the larger the rate of discount, the closer is net capital to gross capital, and in the degenerate case when $\rho$ goes to infinity, they coincide. An increase in $\rho$ from 0 to 5 per cent leads to a substantial rise in the net capital stock; for instance, the estimates are increased by about 25 per cent for the capital item dwellings, which is illustrated in figure 10. At higher levels of the interest rate, changes in this parameter have a far less impact on the net capital stock; its values for $\rho$ = 20 per cent only slightly exceed those for $\rho$ = 10 per cent. In fact, the net capital stock for $\rho$ = 20 per cent is very close to the gross capital stock, as can be seen by comparing figures 9 and 10.

Figures 11 and 12 illustrate the effect of changing the form of the survival profile on the implied <u>retirement rate</u>, defined as the ratio between retirement and gross capital stock, i.e.,

$$(29) \qquad \delta = \frac{D}{K}.$$

We find, not surprisingly, that the level of this rate strongly depends on the curvature of the survival profile, taking its lowest average value in the one horse shay case, and its highest value in the convex case. It can be shown (cf. Biørn (1983a, section 7)) that in a situation with constant gross investment, we get the following expressions for the retirement rates (based, for simplicity, on the formulae in continuous time):

$$\text{Class I: convex:} \quad \delta = \frac{n+1}{N}.$$

$$\text{Class II: concave:} \quad \delta = \frac{m+1}{mN},$$

where n and m are the curvature parameters of the two classes of survival functions (confer section 3 above). This would imply the following retirement rates for the four cases illustrated in figures 11 and 12:

Figure 11: Retirement rate. Retirement in per cent
of gross capital stock.
Manufacture of metals. Machinery etc.
Maximal lifetime 25 years.

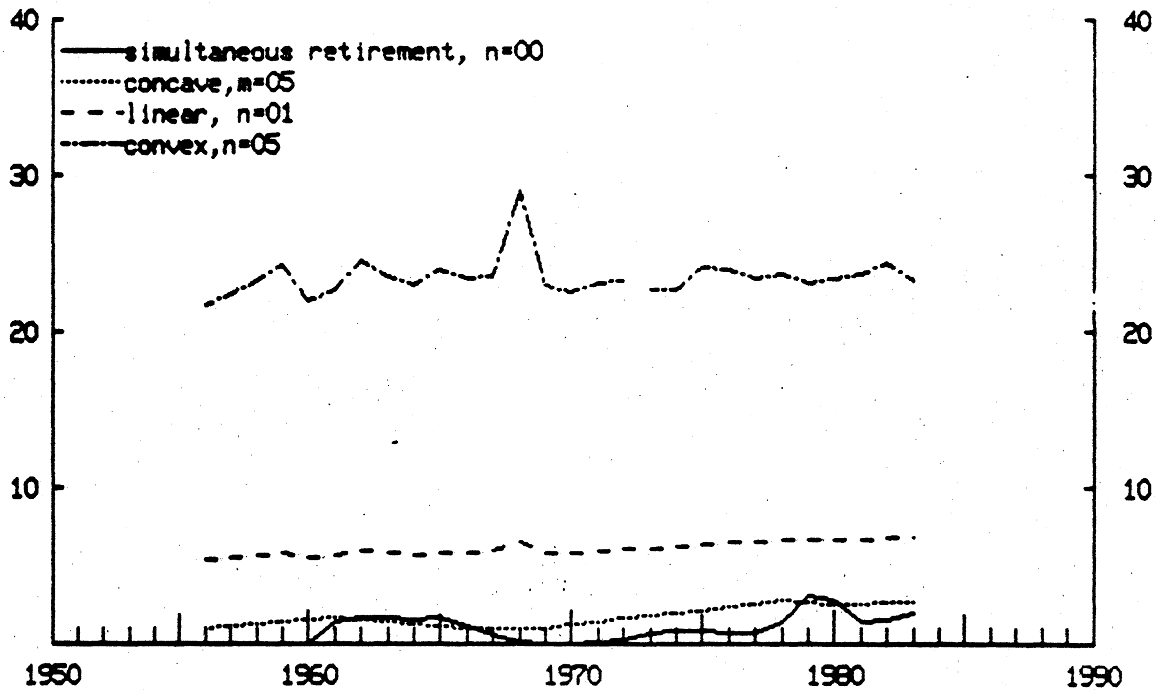
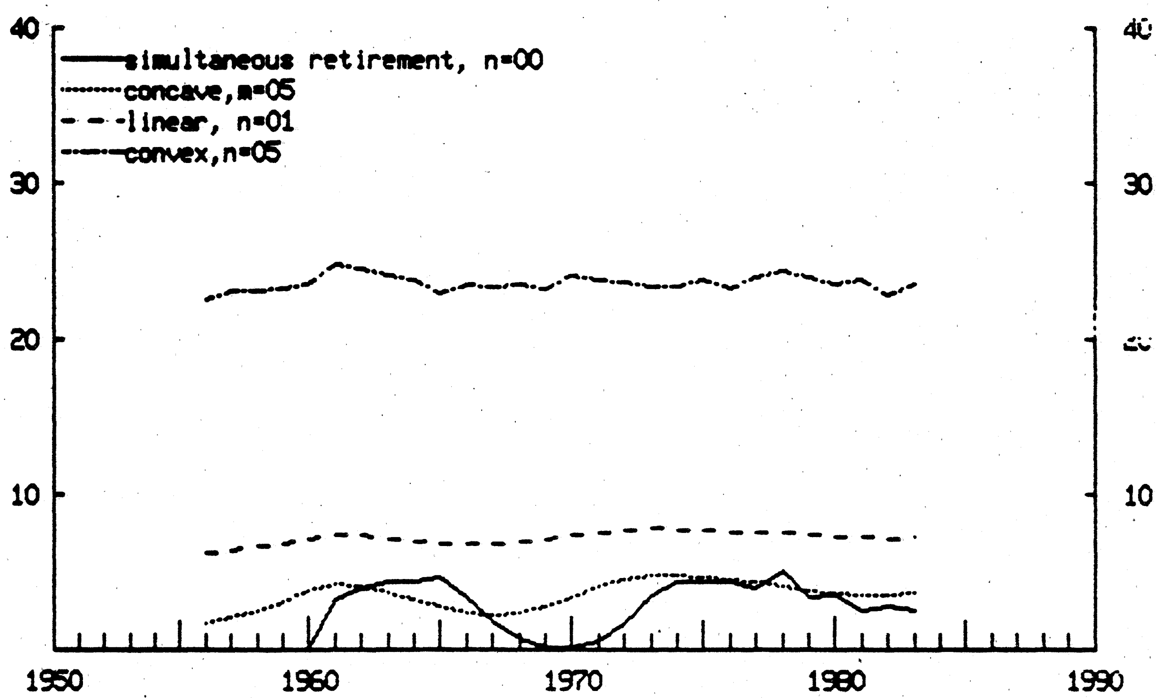



Figure 12: Retirement rate. Retirement in per cent
of gross capital stock.
Manufacture of textiles etc. Machinery etc.
Maximal lifetime 25 years.



(30)

1. simultaneous retirement: $\delta = 1/N$,

2. concave: $\delta = 1.2/N$,

3. linear: $\delta = 2/N$,

4. convex: $\delta = 6/N$.

The departure of the actual retirement rates from these "theoretical" values reflects the growth and cyclical variations in gross investment and the resulting variations in the age distribution of the capital stock over the period of observation.

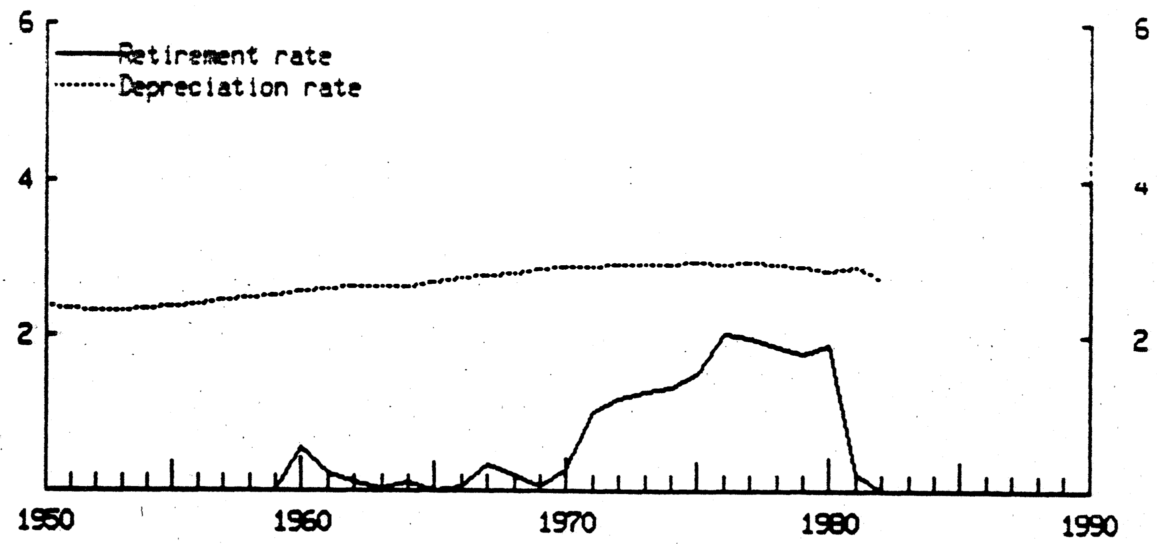
The relative fluctuations of the retirement rates about their average values are widely different in the four cases. Statistically interpreted, the retirement rates are ratios of two moving average processes in gross investment, their length and weighting system reflecting the maximal life time and the form of the survival function (cf. (3) and (4)). Both these lag distributions imply a high degree of smoothing of the investment profiles in the linear and convex case, which explains the smoothness of their retirement rates. In the simultaneous retirement case, however, retirement coincides with gross investment lagged a number of years equal to the (constant) life time N, i.e. the moving average process in the numerator of $\delta$ degenerates to a process with a constant lag. Its denominator is simply the cumulated flow of investment effectuated during the past N years. This explains the volatility of the retirement rate in this case.

Next we define the <u>depreciation rate</u> as the ratio between depreciation and net capital stock, i.e.,

$$(31) \qquad \delta_N = \frac{D_N}{K_N}.$$

An illustration of the difference between the retirement and depreciation rates for buildings and structures in Production of textiles and wearing apparels, is given in figure 13. Here a simultaneous exit survival profile is assumed, and the calculations are performed with a zero discount rate. With these assumptions it was shown in section 3 that depreciation follows a linear profile. Combining this fact with the results shown in (30), it follows that if investment were constant, the retirement rate would be half the depreciation rate. From figure 13 it is confirmed that the retirement rate is far smaller than the depreciation rate. The latter is fairly stable about 2.5-3 per cent, while the former shows considerable fluctuations, reflecting primarily the cyclical variations in past gross investment.

**Figure 13: Retirement rate and depreciation rate. Per cent. Manufacture of textiles etc. Buildings and constructions. Survival function: Sim. retirement, n=00. Rate of interest=00. Maximal lifetime 60 years.**




As mentioned in the introduction to this paper, an interesting application of the presented framework is to investigate the impact of changes in the form and curvature of the survival function on the implied measures of capital productivity, e.g. gross production at constant prices per unit of gross capital. Examples of such calculations are given in figures 14 to 16. The differences in productivity for an arbitrary sector/category of course simply reflect the differences - both with respect to level and growth profile - in the corresponding gross capital stock. Thus, productivity is highest in the convex case, in which the most rapid deterioration of the capital stock takes place. In the sector Manufacture of metals, rather strong fluctuations in productivity for machinery are observed in the convex case (figure 14). These variations reflect, to a large extent, the fact that output in this industry has varied considerably over time; the estimated capital stock has been rather stable (cf. figure 5). Calculations based on the three other survival profiles lead to higher estimates of gross capital stocks, lower average productivity and dampened relative fluctuations.

The estimated time profiles for capital productivity in the oil sector also deserve attention. Recalling the highly different, "outwear-dependent" development of the capital stock shown in figure 7, we find that the various time paths for productivity depicted in figure 15. constitutes a consistent picture: In the simultaneous exit and the concave cases, the productivity figures are relatively low and almost identical.

Figure 14 : Capital productivity. Gross production in per cent of gross capital.
Manufacture of metals. Machinery etc.
Maximal lifetime 25 years.

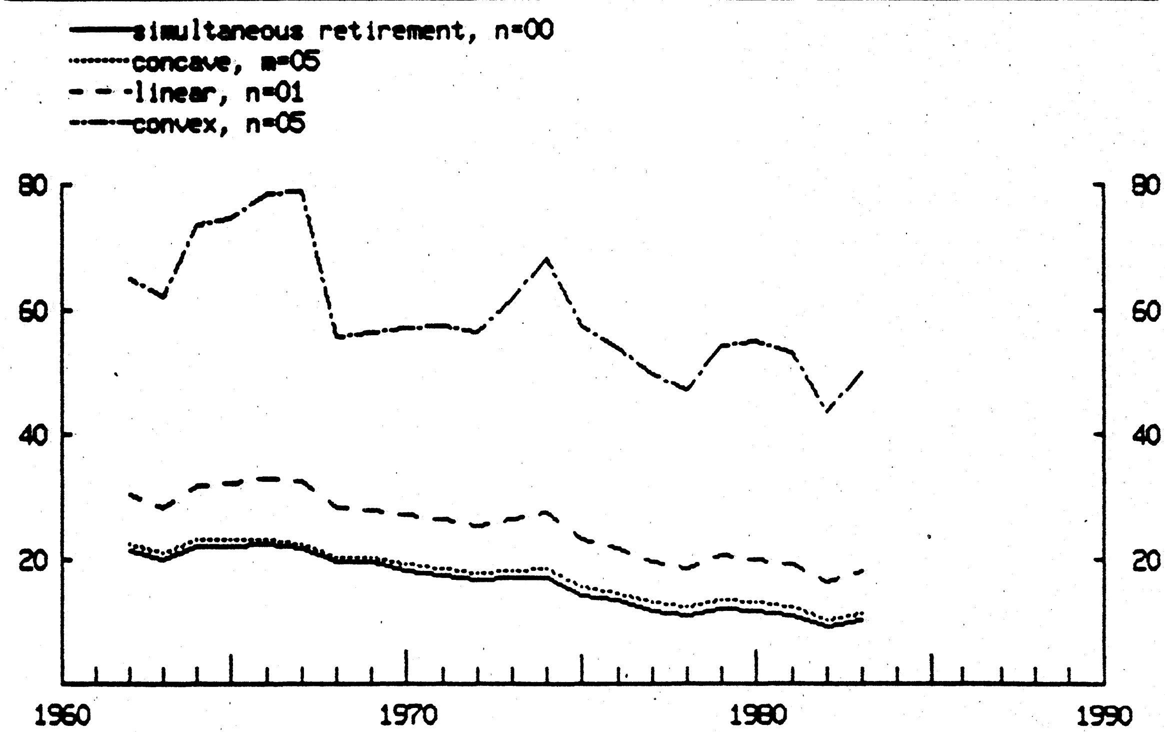


Figure 15: Capital productivity. Gross production in per cent of gross capital. Petroleum sector. Buildings and constructions. Maximal lifetime 16 years.

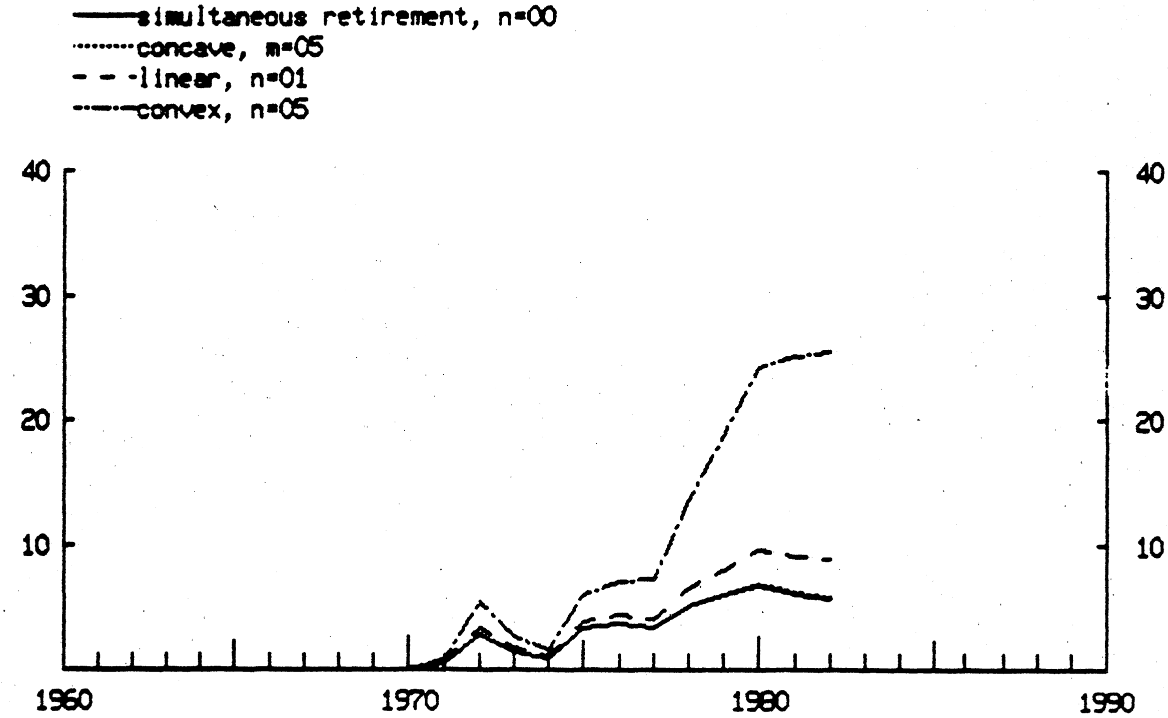
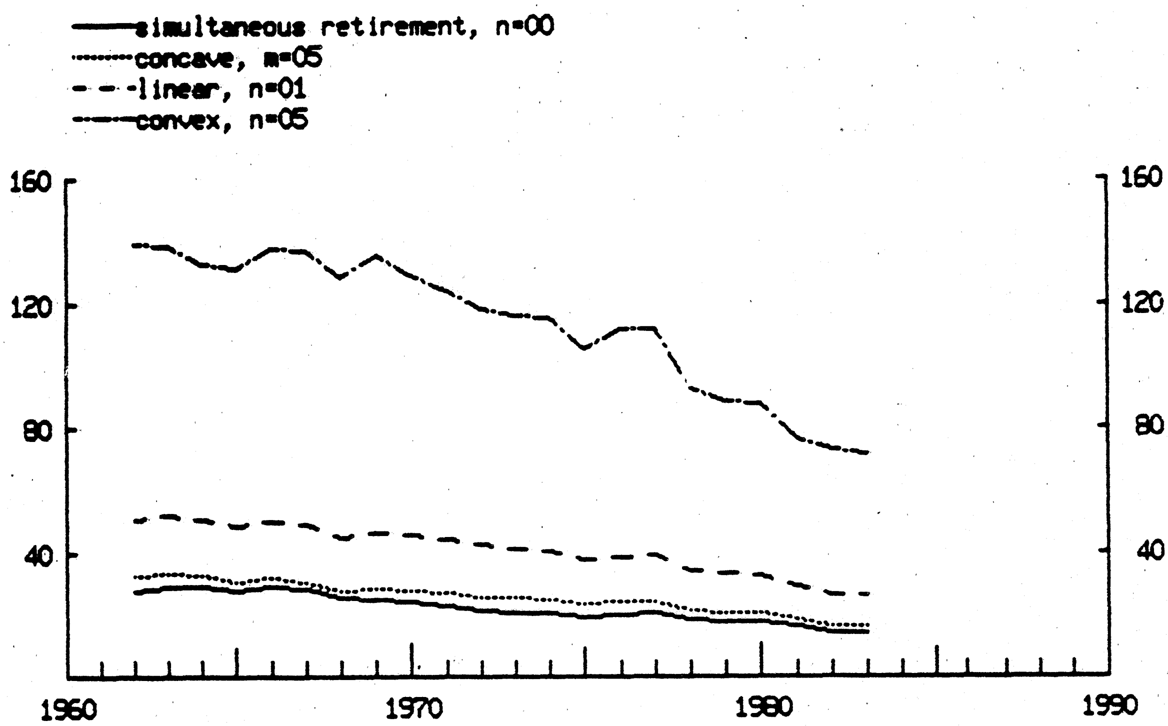



Figure 16: Capital productivity. Gross production in per cent of gross capital. Manufacture of textiles etc. Machinery etc. Maximal lifetime 25 years.



Changing to a linear structure decreases the gross capital stock and therefore increases productivity somewhat. In the convex case, productivity is still higher, and furthermore it increases strongly from 1977 which reflects the peak in the gross capital stock observed in figure 7.

Finally, let us examine how our choice of capital measure affects the implied estimates of rates of return to capital. The formula used for calculating this variable is

$$(32) \qquad r = \frac{E - qD_N}{qK_N},$$

where E is the gross capital income, i.e. the gross operating surplus (exluding remuneration to self-employed persons) as recorded in the national accounts, before deduction of the value of depreciation. The other symbols have the same meaning as before. Since the numerical values of $D_N$ and $K_N$ depend on the assumed survival function B(s) and the rate of discount $\rho$, the value of r will also depend on these parameters. We will not discuss and interpret these relationships in depth here. What we shall do is to select one single survival profile and try to shed some light on the numerical relationship between r and $\rho$ for this profile only. The survival profile selected for this investigation is profile 2, i.e. concave with m=5. Numerical results for the sectors Manufacture of metals and Manufacture of textiles etc., with all capital types taken together, are given in figures 17 and 18, respectively.

It should be admitted that the interpretation of rates of return calculated in this way is not obvious. The resulting figures may be characterized as ex post rates of return, in the sense that they are ratios between observed capital revenues and the computed market value of the capital stock. This may be a natural procedure for calculating rates of return for e.g. national accounting purposes. There is, however, a theoretical inconsistency between this approach and the traditional neo-classical theory of producers' market behaviour. This basically stems from the fact that we take $\rho$ as an exogenous and time invariant parameter and estimate sector specific rates of return as time functions conditional on this parameter value. In principle, $\rho$ and r should have been considered as simultaneously determined, as market equilibrium interest rates, within the framework of a multi-sectoral model of market behaviour.

The nature of this problem is, perhaps, best explained with reference to the following simple case: If all production functions were linearly homogeneous, if perfect market conditions with no uncertainty prevailed in all markets (including the credit market), and if all errors of measurement in our data on E and q could be disregarded, then the theory would predict equality between the observed rate of return and the interest rate applied by market agents when discounting prospective incomes and costs "in the long run", i.e. we would have $r=\varrho$. If we had chosen to stick entirely to these neo-classical assumptions, then we should have estimated net capital, depreciation, and the rate of return simultaneously by setting $r=\varrho$ in the equations from the outset for each sector under consideration. A partial <u>sector</u> <u>specific</u> "equilibrium value" for the rate of return, $\varrho^*$, could then, in principle, have been obtained from

(33) $$E = \varrho^* qK_N + qD_N,$$

with all variables interpreted as time functions, when we recall that $K_N$ and $D_N$ are functions of $\varrho^*$. From (15) and (33) it then follows that in such an equilibrium situation we would have $E = cK$, i.e. the gross capital income would be equal to the current user value of the capital stock.

Figure 17 and 18 show the variations in the rate of return in the two sectors by successively choosing 0, 5, 10, and 20 per cent as discounting rates. In all cases, the rates of return show strong fluctations over time. This is caused mainly by the variations in the observed operating surplus. In the sector "Manufacture of textiles", the rates of return decrease sharply over time, reflecting the stagnation in the activity and profitability in this industry in Norway during the last 15 years. For "Manufacture of metals" there is also a downward trend in the estimated rates of return, but this tendency is not as clear as for the "textile industry". With respect to the question of how variations in the discount rate influence the rate of return, we can make the following observations from the two figures.

- In both sectors, there seems to exist a level at which the rate of return is independent of variations in the interest rate, in the sense that all graphs intersect for this value. Furthermore, this level seems to be rather stable over time. (The "intersection levels" for the rate of return differ however between the two sectors.)

- The cyclical movements of the rate of return about this "intersection level" are dampened when the interest rate increases.

Figure 17 : Rate of return to capital. Per cent.
Manufacture of textiles etc.
Survivalfunction: Concave, m=05.

——Interest rate = 00 per cent
········Interest rate = 05 per cent
- - -Interest rate = 10 per cent
-·-·-Interest rate = 20 per cent

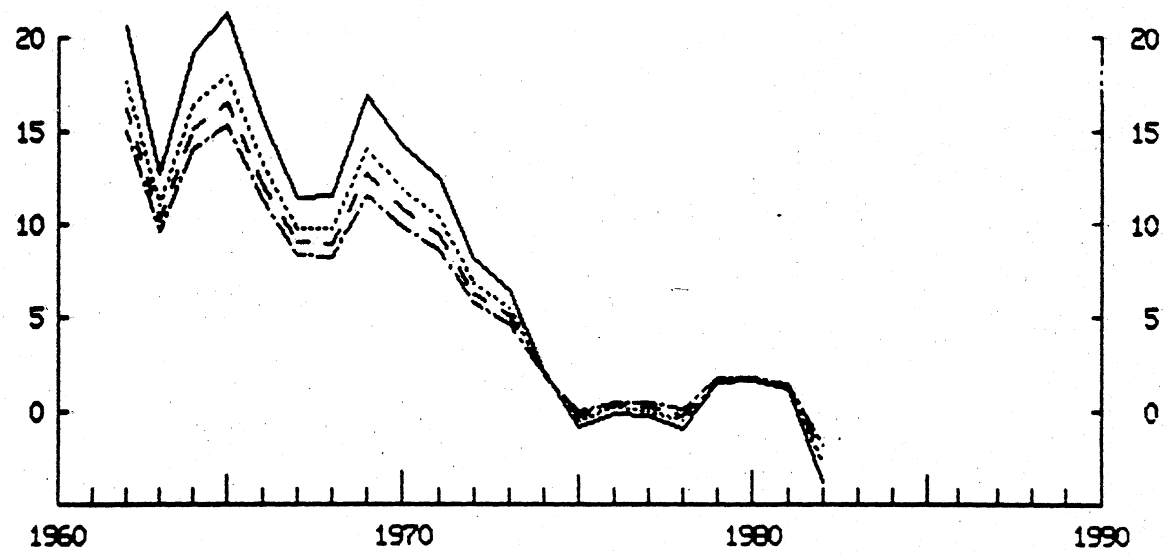
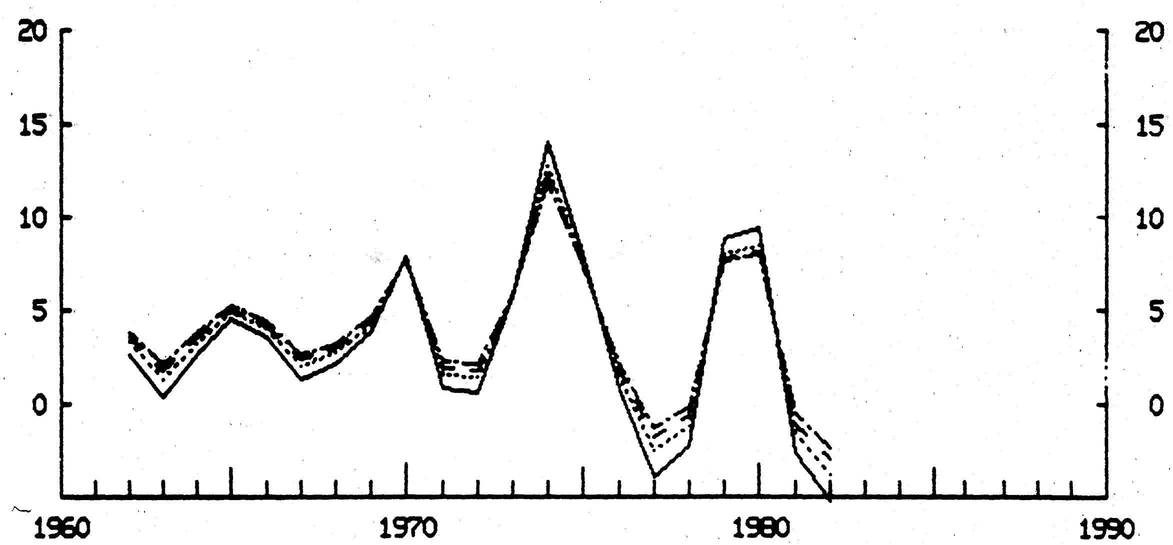



Figure 18 : Rate of return to capital. Per cent.
Manufacture of metals.
Survivalfunction: Concave, m=05.

——Interest rate = 00 per cent
········Interest rate = 05 per cent
- - -Interest rate = 10 per cent
-·-·-Interest rate = 20 per cent



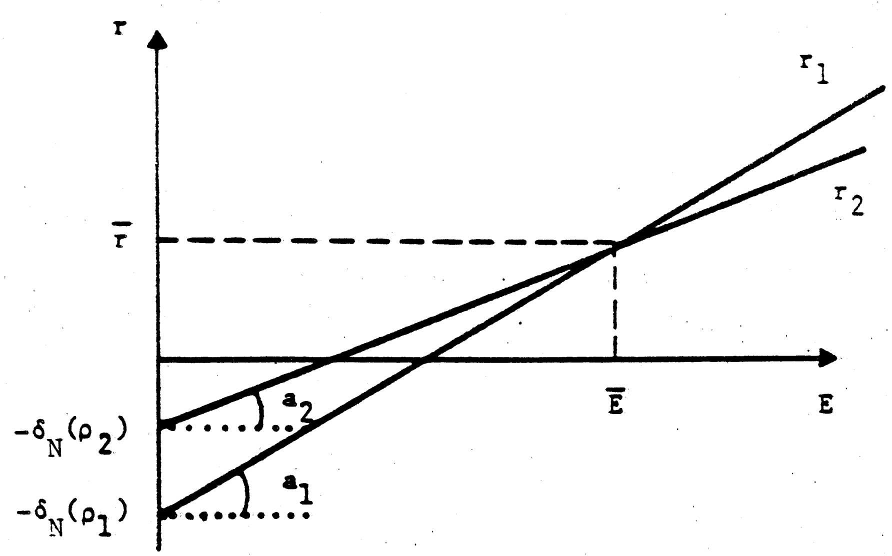
In order to get a further understanding of these findings, it is convenient, by using (31), to rewrite (32) as

$$(34) \qquad r = \frac{E}{qK_N(\varrho)} - \delta_N(\varrho),$$

where we have indicated explicitly the functional dependence of $K_N$ and $\delta_N$ on $\varrho$. Mathematically, r is a linear function of E (gross capital revenue) with slope $a = 1/qK_N(\varrho)$. For two interest rates, $\varrho_1$ and $\varrho_2$, where $\varrho_1 < \varrho_2$, we know that $a_1 = a(\varrho_1) > a_2 = a(\varrho_2)$, since an increase in the discount rate will increase the net capital stock. (Cf. section 2.) Furthermore, the depreciation rate will typically decrease with increasing interest rate, so that $\delta_N(\varrho_1) > \delta_N(\varrho_2)$.[8)]

In figure 19 we have, for illustrative purposes, depicted r as a function of E for the two arbitrarily chosen interest rates.

Figure 19: Rate of return functions indicated for two different interest rates.




The intersection point corresponds to a specific level of gross capital revenue, denoted by $\bar{E}$. When $E > \bar{E}$, then $r_1 > r_2$, while the inequality is reversed when $E < \bar{E}$.

The intersection point indicated in figure 19 is, in general, dependent on the two chosen interest rates. Still, we find that the four graphs in figure 17 and in figure 18 all intersect at about the same value of the rate of return.

Figure 19 also serves to visualize that an increase in the discount rate will dampen the fluctuations in the estimated rate of return caused by varations the gross capital revenue around the "stationary level", $\bar{E}$. As mentioned above this is "confirmed" empirically by the results in figure 17 and 18.

The intersection point in figure 19 and the corresponding values of $\bar{E}$ and $\bar{r}$ will in general depend on the time path of gross investment. From figures 17 and 18, we observe that the intersection points are rather stable over time within each of the two sectors, but there is a considerable difference between these levels, reflecting primarily the different development of past investment in the two sectors.

The relationship between the level of the intersection points and the pattern of past investments can be examined formally if we assume gross investment to grow exponentially over time at an arbitrary rate $\alpha \gtrless 0$, so that

$$(35) \qquad J(t) = J(0)e^{\alpha t} \qquad \text{for all } t.$$

Then (34) can be rewritten as[9)]

$$(36) \qquad r(t) = \lambda_G(\alpha, \varrho)\left[\frac{E(t)}{q(t)J(t)} - 1\right] + \alpha,$$

where $\lambda_G(\alpha, \varrho) = \dfrac{1}{\int_0^\infty G_\varrho(s)e^{-\alpha s}ds}$.

From (36) we see that $r(t)$ is independent of $\varrho$ if $E(t) = q(t)J(t)$, i.e. if gross operating surplus equals the current investment outlay. At this intersection point, we have $r(t) = \alpha$, i.e. the rate of return equals the rate of investment growth. Since

$$\frac{\partial \lambda_G(\alpha, \varrho)}{\partial \varrho} < 0,$$

(36) confirms that if

$$E(t) \gtreqless q(t)J(t), \text{ then } \frac{\partial r(t)}{\partial \varrho} \lesseqgtr 0.$$

It is rewarding to reexamine figures 17 and 18 with this inequality in mind.

## 6. CONCLUDING REMARKS

From the empirical results presented above, two main conclusions emerge: First, the distinction between the capacity dimension and the wealth dimension of the capital stock - i.e. between the gross and the net capital - is not only of theoretical interest; it may be empirically very important. How important it is, depends on the form of the survival profile. The difference between the two capital measures is larger for strongly concave profiles than for strongly convex ones, and is larger the smaller is the interest rate at which the future flow of capital services is discounted when constructing net capital stock. Second, the chosen form of the survival profile may have a strong influence on derived macro-economic variables like retirement rates, depreciation rates, and measures of capital productivity. This is the case not only for the level of these variables; their cyclical behaviour may also be strongly affected. These results have obvious implications for the specification of capital accumulation in macro-econometric models.

The four survival functions used as illustrations throughout this paper, represent possible ways in which we could imagine the retirement of capital units or decline in technical efficiency with age to take place. Probably, the one horse shay profile and the convex profile should be regarded as extreme cases from this point of view. Needless to say, we strongly need empirical evidence on survival profiles from which we could further constrain the class of specifications relevant to empirical work. Such information could be obtained along two directions of research: by observing the actual age distribution of the capital stock and the firms' actual scrapping behaviour, or by observing the development of vintage prices for sufficiently homogeneous capital units and exploiting the assumed law of indifference between vintages, eq. (8), which underlies the construction of the net capital stock. A closer examination of the econometric implications of these research strategies is outside the scope of the present paper.

# FOOTNOTES

1) This assumption is, of course, also related to our neoclassical model specification, in which we do not distinguish between different vintages of capital when describing the technology.

2) This is a convenient normalization for the present purpose, but this choice is, strictly speaking, arbitrary.

3) For the commonly applied exponentially declining survival function, $B(s) = e^{-\delta s}$, it can easily be shown that net capital equals gross capital for all parameter values. (Confer Biørn (1983a), section 7.)

4) However, in that case an <u>inconsistency</u> of the accounting practice in Norway is that the same (linear) survival function is used both for the estimation of depreciation and the construction of capital figures presented and used as if they were gross capital stocks.

5) In the literature this survival profile is also denoted as "one-horse-shay" or "sudden death".

6) Note that for other constellations of the parameters affecting net capital - the "curvature parameter"($m$) and the interest rate ($\varrho$) - the $G_{\varrho}(s)$ function may be concave. This function increases with both $m$ and $\varrho$. However, with an interest rate equal to zero the weighting function for the net capital is convex for all values of $m$ (degenerating to a linear function in the simultaneous exit case).

7) Solwing (33) for $\varrho^*$ is basically the same procedure for calculating rates of return to capital as the method applied in Holmøy and Olsen (1985).

8) This will certainly be the case if gross investment has been growing at a constant rate, $\alpha$, over time. Then we will have

$$J(t) = J(0)e^{\alpha t} \quad \text{for all } t,$$

and the formula for net capital becomes

$$K_N(t) = \int_0^{\infty} G_{\varrho}(s)e^{-\alpha s}J(t)ds = \frac{J(t)}{\lambda_G(\alpha,\varrho)},$$

where $$\lambda_G(\alpha,\varrho) = \frac{1}{\int_0^{\infty} G_{\varrho}(s)e^{-\alpha s}ds} > 0.$$

By using (13), the formula for the depreciation rate can be rewritten as

$$\delta_N(t) = \frac{D_N(t)}{K_N(t)} = \frac{J(t)}{K_N(t)} - \frac{\dot{K}_N(t)}{K_N(t)} = \lambda_G(\alpha,\varrho) - \alpha \quad \text{for all } t.$$

We know that $\frac{\partial G_\varrho(s)}{\partial \varrho} > 0$ for all s. This implies

$$\frac{\partial \delta_N}{\partial \varrho} = \frac{\partial \lambda_G(\alpha,\varrho)}{\partial \varrho} < 0.$$

The result holds regardless of the value of $\alpha$.

9) Confer footnote 8.

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