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ENERGY PRICE CHANGES, AND INDUCED SCRAPPING AND REVALUATION OF CAPITAL - A PUTTY-CLAY APPROACH

BY

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ABSTRACT

The paper presents a framework for analyzing the effect of unanticipated energy price changes on the firms' plans for scrapping of capital and on the induced changes in the value of capital. A putty-clay vintage model with an ex ante CES function and an ARMA price expectation process is used. The interaction between the ex ante elasticity of substitution, the choice of technique, and the planned scrapping age is discussed. Empirical illustrations, based on Norwegian data, indicate that the price changes in the period 1970-1983 may have substantially affected the planned scrapping age and the value of the capital.

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CONTENTS

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I. INTRODUCTION

Considerable attention has been devoted in recent literature to the effect of the sharp increase in energy prices during the last decade on business investment, capital utilization, capital productivity, rate of return, and related issues. [See e.g. Berndt and Wood [1984, 1985].) In this paper, we present a framework for analyzing the effects of such price changes and illustrate its application by means of Norwegian data. The technology is represented by a vintage production model of the putty-clay type. Ex ante - i.e. before an investment in fixed capital is made - the firm faces a neo-classical technology; ex post - i.e. after the investment has got its specific physical form - the technique is characterized by fixed factor proportions. [For a general discussion of putty-clay vintage models, see Johansen [1959, 1972].)

The putty-clay framework is well suited to analysing the impact of rising energy cost on the firm's investment and scrapping decisions during the last 10-15 years - notably the effect of the OPEC induced energy price shocks in 1973-74 and 1979-80. There are two reasons for this. First, the OPEC shocks were substantial and to a large extent unanticipated by the market. The putty-clay model is well suited to analyzing unanticipated price changes since - in contrast to the neo-classical (putty-putty) one - it implies non-myopic decision rules. Second, the rising energy cost brought about substantial changes in the optimal factor proportions for new capital towards less energy intensive techniques. The restrictions on the factor adjustment are represented, in a consistent way, by the distinction between ex ante and ex post optimal factor proportions. Firms which had invested in relatively energy intensive equipment, assuming the old energy/capital price ratio - or the trend in this ratio - to prevail, would find themselves with capital vintages whose embodied energy efficiency was no longer optimal in a situation characterized not only by a new level of relative prices, but also by revised expectations about relative rates of increase from this new level. This, in turn, would affect their scrapping plans.

In this paper, we specify a simple two factor putty-clay model. The first factor, capital, is variable ex ante and fixed ex post, the second, energy, is variable ex ante as well as ex post, but with restrictions on the energy-capital ratio ex post. The firm is assumed to pursue a profit maximizing strategy. We shall be concerned with the effect of unanticipated changes in output and energy prices on its decisions with respect to output, energy use, investment, and scrapping. In particular, the treatment of the scrapping plans as decisions which are jointly endogenous with the choice of technique distinguishes our approach from that in most other studies in this area, e.g. Berndt and Wood (1984). Some of the problems concerning the relationship between price expectations, ex ante substitution and scrapping plans are discussed in some more detail and in a more general context in Biørn and Frenger (1986). In the present paper, we also consider the changes in the value of old, relatively energy inefficient, capital vintages which are induced by these price changes through their effect on the anticipated service life and the life cycle profit. Some tentative empirical illustrations of these revaluation effects, based on Norwegian data, will also be given.

II. NOTATION AND BASIC ASSUMPTIONS

Consider a firm at time t and let the ex ante technology available be represented by the neo-classical production function

$$(2.1)\quad X(t+s,s) = F[E(t+s,s),\ K(t+s,s)],$$

where s denotes the age of the equipment ($s \geq 0$), t+s is the time at which the capital reaches this age, and $X(t+s,s)$, $K(t+s,s)$, and $E(t+s,s)$ are respectively the output from, the flow of capital services from, and the energy input allocated to vintage t at time t+s (i.e., at age s). We assume that F is homothetic, i.e. that it can be written as

$$(2.2)\quad F(E,K) = f\{\Phi(E,K)\},$$

where f is monotone increasing, with $f' > 0$, $f'' < 0$, and Φ is linear homogeneous and quasi-concave, with $\Phi'_E > 0$, $\Phi'_K > 0$, $\Phi''_{EE} < 0$, $\Phi''_{KK} < 0$.

Let J(t) denote the volume of capital invested at time t. The efficiency of the capital units declines over time as described by the survival function B(s),

$$(2.3)\quad B(0) = 1,\ 0 \leq B(s) \leq 1,\ B'(s) \leq 0, \quad \text{for all } s \geq 0.$$

The potential flow of capital services from vintage t at age s is thus

$B(s)J(t)$. We allow, however, for the possibility that only a share $\xi(t+s,t)$ of this capacity is actually used,

$$(2.4) \quad 0 \leq \xi(t+s,t) \leq 1 \qquad \text{for all } t \text{ and } s \geq 0,$$

so that the actual flow of capital services from vintage t at age s can be expressed as

$$(2.5) \quad K(t+s,s) = \xi(t+s,t)B(s)J(t).$$

The <u>ex post technology</u> is characterized by the assumption that energy and capital must be used in fixed proportions. We formalize this as

$$(2.6) \quad \frac{E(t+s,s)}{K(t+s,s)} = h(s)\frac{E(t,0)}{J(t)} \qquad \text{for all } t \text{ and } s \geq 0,$$

where $h(s)$ is a positively valued function ($h(0) = 1$ by assumption) intended to represent the fact that the operation of each capital vintage, <u>for technical reasons</u>, may become more or less energy consuming as it grows older. The function $h(s)$ is thus, like $F(E,K)$ and $B(s)$, regarded as a technical datum by the firm. Energy input per capital unit is increasing, constant, or decreasing with age at age s, according as $h'(s) \gtreqless 0$. From (2.5) and (2.6) it follows that the time path of the energy input allocated to vintage t can be expressed in terms of the initial input as follows:

$$(2.7) \quad E(t+s,s) = \xi(t+s,t)B(s)h(s)E(t,0), \qquad s \geq 0.$$

We cannot, however, express the output as a similar function of the initial output unless the technology has specific properties. Inserting (2.5) and (2.7) in (2.1), while using (2.2), we obtain

$$(2.8) \quad X(t+s,s) = f[\xi(t+s,t)B(s)Z(t,s)],$$

where

$$(2.9) \quad Z(t,s) = \Phi[h(s)E(t,0),J(t)],$$

which does not, in general, imply proportionality between $X(t+s,s)$ and $X(t,0)$ for all t. Note, in particular, that the technology will <u>not</u> be characterized by constant output coefficients ex post, since

$$\frac{E(t+s,s)}{X(t+s,s)} \neq \frac{E(t,0)}{X(t,0)},$$

$$\frac{K(t+s,s)}{X(t+s,s)} \neq \frac{J(t)}{X(t,0)},$$

for all s and t unless f is linear homogeneous and $h(s) = 1$ for all s.

III. EX ANTE OPTIMIZATION

A basic implication of production models with ex post restrictions on factor substitution is that the firm must form expectations about future prices of output and variable inputs in order to make decisions about output, investment, and factor proportions today. Let $p_X^*(t+s,t)$ and $p_E^*(t+s,t)$ denote the output price and the energy input price, respectively, which at time t is expected to prevail at time t+s. Here and in the following, we use asterisks to represent expected values. These expectations are assumed to hold with certainty, but may be revised, as indicated by the two time arguments.

The ex ante quasi rent from vintage t at time t+s is defined as the difference between the output value and the cost of the variable input, energy, as expected by the firm at the time of installation t. Using (2.7) and (2.8), it can be written as

$$(3.1) \quad \Pi^*(t+s,t) = p_X^*(t+s,t)X(t+s,s) - p_E^*(t+s,t)E(t+s,s)$$

$$= p_X^*(t+s,t)f[\xi(t+s,t)B(s)Z(t+s,s)]$$

$$- p_E^*(t+s,t)\xi(t+s,t)h(s)B(s)E(t,0).$$

The firm's general ex ante optimization problem is to choose the investment $J(t)$, the initial energy input $E(t,0)$, and the (planned) time path of the scale of operation $\xi(t+s,t)$ in such a way that the present value of the expected net cash-flow from vintage t - its total profit - is maximized. Letting $q(t)$ denote the investment price at time t and $r(t)$ the (constant) future interest rate as expected at time t, this present value can be written as

$$(3.2) \quad \Pi^*(t) = \int_0^\infty e^{-r(t)s}\,\Pi^*(t+s,t)ds - q(t)J(t).$$

All prices and price expectations will be treated as exogenous variables in this optimization.

3.1. The general case

The general problem of maximizing the profit function (3.2) subject to (3.1) and (2.4) with all prices treated as exogenous variables is an optimal control problem, since inequality constraints are involved. We shall not specify our problem in this general way, but instead discuss the simpler case in which the firm plans to use the entire capacity of each vintage up to a certain age and then take it completely out of operation, i.e. $\xi(t+s,t)$ is either one or zero. We formalize this as

$$(3.3) \quad \xi(t+s,t) = \begin{cases} 1 \text{ for } 0 \leq N^*(t) \\ 0 \text{ for } s > N^*(t), \end{cases}$$

and accordingly interpret $N^*(t)$ as the planned ex ante service life of vintage t. Furthermore, we assume that the "outer" part of the production technology has the form

$$(3.4) \quad f(Z) = Z^{\varepsilon}, \qquad 0 < \varepsilon < 1,$$

which implies that the ex ante production technology F has a constant degree of homogeneity, ε, and decreasing returns to scale.

The first order conditions for ex ante profit maximization with respect to $E(t,0)$, $J(t)$, and $N^*(t)$, subject to (3.1), (3.2), and (3.4), are

$$(3.5) \quad \int_0^{N^*(t)} e^{-r(t)s} p_X^*(t+s,t)B(s)^{\varepsilon}\varepsilon Z(t,s)^{\varepsilon-1}\partial Z(t,s)/\partial E(t,0)ds$$
$$= \int_0^{N^*(t)} e^{-r(t)s} p_E^*(t+s,t)B(s)h(s)ds,$$

$$(3.6) \quad \int_0^{N^*(t)} e^{-r(t)s} p_X^*(t+s,t)B(s)^{\varepsilon}\varepsilon Z(t,s)^{\varepsilon-1}\partial Z(t,s)/\partial J(t)ds$$
$$= q(t),$$

$$(3.7)\quad p_X^*\{t+N^*(t),t\}B(N^*(t))^{\varepsilon}Z(t,s)^{\varepsilon}$$

$$= p_E^*\{t+N^*(t),t\}h(N^*(t))B(N^*(t))E(t,0),$$

where $Z(t,s)$ is given by (2.9). Eq. (3.5) states that the present value of the output flow following from an initial marginal increase of one unit in the energy input allocated to vintage t shall be equal to the present value of the total energy cost induced by this increase over the planned service life of this vintage. Correspondingly, (3.6) says that the present value of the output flow following from an initial marginal investment of one unit in vintage t shall be equal to the investment price at time t.[1] Finally, (3.7), which is equivalent to $\Pi^*\{t+N^*(t),t\} = 0$, says that the planned service life is chosen in such a way that the quasi rent equals zero at this age. We assume that a positive and finite solution value for $N^*(t)$ exists and is unique, and that the life cycle profit at this value is positive. Sufficient conditions for this to hold are

$$(3.8)\quad \Pi^*(t) > 0,$$

$$(3.9)\quad \Pi^*(t,t) > 0,$$

$$(3.10)\quad \frac{\partial \Pi^*(t+s,t)}{\partial s} < 0 \qquad \text{for all } s,$$

at an optimum. A quasi rent criterion for determination of optimal planned service life similar to (3.7) is given in Malcomson and Prior (1979), although they do not integrate the expectational aspect as explicitly into the model as is done here.

3.2. Simplifying assumptions. Cost minimization versus profit maximization

In general, (3.5)-(3.7) constitute a system of integral equations in $E(t,0)$, $J(t)$, and $N^*(t)$. The complexity of this system is due to the fact that $h(s)$ occurs as a multiplicative factor to $E(t,0)$ in $Z(t,s)$; cf. (2.9). If, however, the expressions on the left hand side of (3.5) and (3.6) can be factorized into two components - one involving $E(t,0)$ and $J(t)$, the other being an integral containing prices and technology parameters only, the solution can be simplified substantially. The following two cases have this attractive property:

(i) Constant energy input per capital unit ex post: $h(s) = 1$ for all s.

(ii) Cobb-Douglas production function ex ante: $\Phi(E,J) = AE^{\beta}J^{1-\beta}$.

In case (i) we have $Z(t,s) = \Phi[E(t,0),J(t)]$ for all s; case (ii) implies $Z(t,s) = h(s)^{\beta}\Phi[E(t,0),J(t)]$. In both cases, $\Phi(\cdot)$ is a common factor which can be set outside the integrals in (3.5) and (3.6). We shall in the following mainly be concerned with case (i), but we also make some references to case (ii).

Let

$$(3.11)\quad B_X(s) = \begin{cases} B(s)^{\varepsilon} & \text{in case (i)}, \\ B(s)^{\varepsilon}h(s)^{\beta\varepsilon} & \text{in case (ii)}, \end{cases}$$

$$(3.12)\quad B_E(s) = \begin{cases} B(s) & \text{in case (i)}, \\ B(s)h(s) & \text{in case (ii)}, \end{cases}$$

and define

$$(3.13)\quad P_X^*(t) = \int_0^{N^*(t)} e^{-r(t)s} p_X^*(t+s,t)B_X(s)ds,$$

$$(3.14)\quad P_E^*(t) = \int_0^{N^*(t)} e^{-r(t)s} p_E^*(t+s,t)B_E(s)ds.$$

which may be interpreted as the (ex ante) "life-cycle" output and energy price, respectively. They are weighted present values of the expected future prices, the weights reflecting the decline in the capital's efficiency and (in case (ii)) the change in its energy requirement with age.[2] The profit function (3.2) and the first order conditions (3.5)-(3.7) can now be written compactly as

$$(3.15)\quad \Pi^*(t) = P_X^*(t)Z(t,0)^{\varepsilon} - P_E^*(t)E(t,0) - q(t)J(t),$$

$$(3.16)\quad P_X^*(t)\varepsilon Z(t,0)^{\varepsilon-1}\frac{\partial Z(t,0)}{\partial E(t,0)} = P_E^*(t),$$

$$(3.17)\quad P_X^*(t)\varepsilon Z(t,0)^{\varepsilon-1}\frac{\partial Z(t,0)}{\partial J(t)} = q(t),$$

$$(3.18)\quad p_X^*[t+N^*(t),t]B_X[N^*(t)]Z(t,0)^{\varepsilon} = p_E^*[t+N^*(t),t]B_E[N^*(t)]E(t,0).$$

It is convenient to distinguish between three subproblems:

(a) Cost minimization with respect to energy use and investment, for given production scale and service life.

(b) Partial profit maximization with respect to energy use, investment, and service life, for given production scale.

(c) Full profit maximization, including the optimal choice of production scale.

The first order condition for subproblem (a) is simply

$$(3.19)\quad \frac{\frac{\partial Z(t,0)}{\partial E(t,0)}}{\frac{\partial Z(t,0)}{\partial J(t)}} = \frac{P_E^*(t)}{q(t)} .$$

Going from (a) to (b) implies that we replace the exogenously given value of $N^*(t)$ with the scrapping condition, (3.18). The final step from (b) to (c) implies that we suppress the binding on the production scale, $Z(t,0)$, and consider the complete system of optimizing equations, (3.16)-(3.18).

Note that the joint determination of energy use, investment, and planned service life is implied by both the full optimization problem (c) and the intermediate problem (b). Since our focus will be on the replanning of capital utilization in the context of unexpected energy price changes, the endogeneity of the service life is essential. To pose the problem as a narrow cost minimization problem of type (a), as for instance do Berndt and Wood (1984, 1985), would imply that this important effect of the energy price shocks is missed. Confer also Ando et al. (1974).

3.3. The relationship between price expectations, service life, and energy intensity. The CES case.

Let us, by using results from duality theory, characterize the solution in some more detail. For notational simplicity, we temporarily suppress the time subscripts and asterisks on the expectational variables and let $\psi = \psi(P_E,q)$ denote the unit cost function which is dual to the "inner", linear homogeneous, part of the production function, $\Phi(E,J)$. The minimal cost of producing output X, i.e. the solution to subproblem (a), can then be expressed as

$$(3.20)\quad C(X,P_E,q) = f^{-1}(X)\psi(P_E,q) = X^{1/\varepsilon}\psi(P_E,q),$$

confer Shephard (1953). Since profit maximization implies equality between the (life cycle) marginal cost and the output price,

$$(3.21)\quad C_X(X,P_E,q) = \frac{1}{\varepsilon} X^{(1-\varepsilon)/\varepsilon}\psi(P_E,q) = P_X,$$

the profit function, i.e. the maximal profit which solves problem (c) as a function of P_X, P_E, and q, becomes

$$(3.22)\quad \Pi = P_X X - C(X,P_E,q) = \Pi(P_X,P_E,q)$$

$$= \varepsilon^{\varepsilon/(1-\varepsilon)}(1-\varepsilon)P_X^{1/(1-\varepsilon)}\psi(P_E,q)^{-\varepsilon/(1-\varepsilon)}.$$

We can now state the optimal solution compactly as follows:

Production scale:

$$(3.23)\quad Z = X^{1/\varepsilon} = \left[\frac{\varepsilon P_X}{\psi}\right]^{1/(1-\varepsilon)},$$

Life cycle factor cost:

$$(3.24)\quad C = \psi Z,$$

Life cycle profit:

$$(3.25)\quad \Pi = (\frac{1}{\varepsilon} - 1)\psi Z.$$

By application of Shephard's lemma to (3.25) - while recalling that (3.23) implies $(\partial Z/\partial\psi)(\psi/Z) = -1/(1-\varepsilon)$ - we obtain:

Energy demand:

$$(3.26)\quad E = -\frac{\partial\Pi}{\partial P_E} = -(\frac{1}{\varepsilon} - 1)(Z + \psi\frac{\partial Z}{\partial\psi})\frac{\partial\psi}{\partial P_E} = \frac{\partial\psi}{\partial P_E}Z,$$

Investment demand:

$$(3.27)\quad J = -\frac{\partial\Pi}{\partial q} = -(\frac{1}{\varepsilon} - 1)(Z + \psi\frac{\partial Z}{\partial\psi})\frac{\partial\psi}{\partial q} = \frac{\partial\psi}{\partial q}Z.$$

Finally, by inserting (3.26) in (3.18), using $p_X(N)$ and $p_E(N)$ as short hand notation for $p_X^*[t+N^*(t),t]$ and $p_E^*[t+N^*(t),t]$, we find the following equation for determination of the optimal service life:

Optimal service life:

$$(3.28)\quad p_X(N)B_X(N) = p_E(N)B_E(N)\frac{\partial\psi}{\partial P_E} Z^{1-\varepsilon}.$$

In the simple cost minimization problem, (a), we consider the subsystem (3.24)-(3.27) only, and replace (3.23) and (3.28) by the exogenously given values of Z and N. In the partial profit maximization problem, (b), the solution is defined by (3.24)-(3.28), while (3.23) is replaced by the exogenously given value of Z. In that case, the optimal service life will depend on the production scale chosen except when the ex ante production function exhibits constant returns to scale ($\varepsilon=1$). Assuming full profit maximization (which, of course, presupposes decreasing returns to scale, $\varepsilon < 1$), we find from (3.23) and (3.28) the following relationship between the scrapping age, the expected output and energy prices at that age, the investment price, and the life cycle output and energy prices[3]

$$(3.29)\quad p_X(N)B_X(N)\psi(P_E,q) = \varepsilon p_E(N)B_E(N)\frac{\partial\psi(P_E,q)}{\partial P_E}P_X.$$

In the following, we let the ex ante substitution between energy and capital be represented by the CES (Constant Elasticity of Substitution) function

$$(3.30)\quad Z = \Phi(E,J) = a\{\beta(E/\beta)^{-\varrho} + (1-\beta)(J/(1-\beta))^{-\varrho}\}^{-1/\varrho},$$

where a is a positive constant, $\varrho \geq -1$, and $0 < \beta < 1$. Its dual unit cost function is

$$(3.31)\quad \psi(P_E,q) = a^{-1}\{\beta P_E^{1-\sigma} + (1-\beta)q^{1-\sigma}\}^{1/(1-\sigma)},$$

where

$$(3.32)\quad \sigma = \frac{1}{1+\varrho}$$

is the ex ante elasticity of substitution between energy and capital.[4] From (3.26), (3.27), and (3.31) it then follows that energy input per capital unit can be expressed in terms of the investment price, the life cycle energy price, the CES distribution parameter β, and the elasticity of substitution σ as

$$(3.33)\quad \frac{E}{J} = \frac{\frac{\partial\psi}{\partial P_E}}{\frac{\partial\psi}{\partial q}} = \frac{\beta}{1-\beta}\left(\frac{q}{P_E}\right)^{\sigma}.$$

The ratio between the energy cost and the value of the capital cost over the life cycle is given by

$$(3.34)\quad \frac{P_E E}{qJ} = \frac{\beta}{1-\beta}\left(\frac{q}{P_E}\right)^{\sigma-1}.$$

We see that the energy intensity, E/J, is constant and equal to $\beta/(1-\beta)$ in the case with a Leontief ex ante technology ($\sigma=0$) and is a decreasing function of the price ratio P_E/q in the case with ex ante substitution ($\sigma>0$). The energy intensity in life cycle cost terms, $(P_E E)/(qJ)$, is constant and equal to $\beta/(1-\beta)$ in the Cobb-Douglas case ($\sigma=1$) and is increasing or decreasing in P_E/q according as $\sigma<1$ or $\sigma>1$.

We now reintroduce time subscripts and assume that the output and energy prices are expected to grow at constant rates, $\pi_X(t)$ and $\pi_E(t)$, from time t, i.e.,

$$(3.35)\quad p_X^*(t+s,t) = p_X(t)e^{\pi_X(t)s},$$

$$(3.36)\quad p_E^*(t+s,t) = p_E(t)e^{\pi_E(t)s},$$

for all s, where $p_X(t)$ and $p_E(t)$ denote the current output and energy price at time t, respectively. The subscript t on these growth rates indicate that they can, in principle, be continuously revised by the firm. Let also B(s) be specified as an exponential function,

$$(3.37)\quad B(s) = e^{-\delta s}, \qquad s \geq 0,$$

where δ (≥ 0) represents the technical rate of retirement (decline in efficiency). Assume, moreover, a constant energy input per capital unit ex post, i.e. h(s)=1 for all s.

Define

$$(3.38)\quad \rho_X = \rho_X(t) = r(t) - \pi_X(t) + \delta\varepsilon,$$

(3.39) $\varrho_E = \varrho_E(t) = r(t) - \pi_E(t) + \delta,$

which can be interpreted as composite discount rates for the output and energy price, respectively,[5] since we can, by using (3.11)-(3.14), (3.35), and (3.36), express the relationship between the life-cycle output and energy prices and the corresponding current annual prices as

(3.40) $P_X = P_X(t) = p_X(t)N(t)D[\varrho_X(t)N(t)],$

(3.41) $P_E = P_E(t) = p_E(t)N(t)D[\varrho_E(t)N(t)],$

where

$$(3.42)\quad D(x) = \begin{cases} (1-e^{-x})/x & \text{for } x \neq 0, \\ 1 & \text{for } x = 0. \end{cases}$$

The equation for determination of the optimal service life, (3.28), now becomes

$$p_X e^{-\varrho_X N} = p_E e^{-\varrho_E N} \frac{\partial\psi}{\partial P_E} Z^{1-\varepsilon}.$$

Inserting from (3.26) and (3.31) and rearranging, we get

$$\begin{aligned}(3.43)\quad N &= \frac{1}{\varrho_X-\varrho_E}[\log(p_X/p_E) + \log(Z^{\varepsilon}/E)] \\ &= \frac{1}{\varrho_X-\varrho_E}[\log(p_X/p_E) - \log(\partial\psi/\partial P_E) - (1-\varepsilon)\log Z] \\ &= \frac{1}{\varrho_X-\varrho_E}[\log(p_X/p_E) + \log a - (1-\varepsilon)\log Z - \log\beta \\ &\qquad - \sigma G(q/P_E,\beta,\sigma)],\end{aligned}$$

where[6]

(3.44) $G(q/P_E,\beta,\sigma) = \frac{1}{1-\sigma}\log[\beta + (1-\beta)(q/P_E)^{1-\sigma}].$

If the initial quasi rent is positive, cf. (3.9), the numerator of (3.43) will be positive. The condition for a positive and finite solution for N to exist then is that its denominator is also positive, i.e.

(3.45) $\varrho_X > \varrho_E \iff \pi_E > \pi_X + \delta(1-\varepsilon)$.

This implies that the energy price should be expected to rise at least as fast as the output price. Note that G( ) = 0 when $q/P_E = 1$.[7]

Let (N^0, Z^0, P_E^0) be the value of (N, Z, P_E) which, for a given base value of $(p_X, p_E, \varrho_X, \varrho_E) = (p_X^0, p_E^0, \varrho_X^0, \varrho_E^0)$ and with $q = P_E^0$, solves the full profit maximization problem. Since $q = P_E^0$ implies G( ) = 0 and $\psi = P_E^0/a$ regardless of the value of the substitution elasticity σ, it follows that the base value (N^0, Z^0, P_E^0) will be independent of σ when this normalization of the relative input price is chosen (but it depends on a, β, and ε).[8] This is thus a convenient normalization of the price variables, which will be adopted in the following.

Consider an arbitrary value of $(p_X, p_E, \varrho_X, \varrho_E)$ and let, for short, $\varrho = \varrho_X - \varrho_E = \pi_E - \pi_X - \delta(1-\varepsilon)$ and $p = p_X/p_E$. From (3.43) we find

$$\varrho N - \varrho^0 N^0 = \log(p/p^0) - (1-\varepsilon)\log(Z/Z^0) - \sigma G(q/P_E,\beta,\sigma),$$

so that the difference between N and its base value can be written as

$$(3.46)\quad N - N^0 = \frac{1}{\varrho}\log(p/p^0) - \frac{N^0}{\varrho}(\varrho-\varrho^0) - \frac{(1-\varepsilon)}{\varrho}\log(Z/Z^0) - \frac{\sigma}{\varrho}G(q/P_E,\beta,\sigma).$$

This equation says that the change in the (ex ante) service life induced by changes in prices and price expectations from their base values can be decomposed into four effects:

a price level effect: $\frac{1}{\varrho}\log(p/p^0)$,

a price increase effect: $-\frac{N^0}{\varrho}(\varrho-\varrho^0)$,

a scale effect: $-\frac{(1-\varepsilon)}{\varrho}\log(Z/Z^0)$, and

a substitution effect: $-\frac{\sigma}{\varrho}G(q/P_E,\beta,\sigma)$.

The price level effect represents the impact on the planned service life of changes in the current output/energy price ratio. An increase/decrease in this ratio tends to lengthening/shortening the service life. The price increase effect represents the impact of changes in the anticipated relative rates of increase of the output and energy prices. An increase/decrease in $\pi_X-\pi_E$, i.e. a decrease/increase in $\varrho=\varrho_X-\varrho_E$ (cf. (3.38)-(3.39)), also tends to lengthening/shortening the service life. The scale effect represents the effect induced by price changes via changes in output. A one per cent increase/decrease in the production scale Z (i.e. a ε per cent increase/ decrease in output) tends to shortening/lenghtening the service life by approximately $(1-\varepsilon)/\varrho$ years. Finally, the substitution effect represents the effect on the service life of price induced changes in the energy/ capital ratio. Since from (3.44) we have

$$(3.47)\quad \frac{\partial G}{\partial \log(q/P_E)} = \alpha = \frac{(1-\beta)(q/P_E)^{1-\sigma}}{\beta+(1-\beta)(q/P_E)^{1-\sigma}},$$

a one per cent increase/decrease in P_E/q will, cet.par. - via the induced substitution away from energy consuming equipment (cf. (3.33)) - tend to increasing/decreasing the service life by approximately $\sigma\alpha/\varrho$ years. In the base situation, this effect is simply $\sigma(1-\beta)/\varrho$.

The effect on the energy intensity, i.e. energy input per capital unit, can be similarly decomposed. From (3.33) and (3.41) and the normalization assumption it follows that

$$(3.48)\quad \log(E/J) - \log(E/J)^0 = -\sigma \log(p_E/q) + \sigma \log(\varrho_E/\varrho_E^0) - \sigma \log \{(1-e^{-\varrho_E N})/(1-e^{-\varrho_E^0 N^0})\},$$

where $(E/J)^0 = \beta/(1-\beta)$ is the energy/capital ratio in the base situation. The first term represents the impact of changes in the current relative price of energy and capital. An increase/decrease in this ratio tends to decreasing/increasing the energy intensity. The second term represents the effect of changes in the anticipated rate of increase of the energy price, π_E. An increase/decrease in this rate, i.e. a decrease/increase in ϱ_E, tends to decreasing/increasing the energy intensity. The third term adjusts for the fact that changes in prices and price expectations will also affect the service life; cf. (3.45). This adjustment is negative/positive according as the service life is larger/smaller than its base value. The price level effect, the price increase effect, and the service life effect are all stronger the larger is the ex ante elasticity of substitution, cet. par. Analytical expressions for the elasticities of N and E/J with respect to the price variables are given in appendix B.

Numerical illustrations are given in tables 1-3 for the following values of the scale elasticity, the rate of retirement, and the CES distribution parameter:

$$\varepsilon = 0.8, \quad \delta = 0, \quad \beta = 0.3,$$

with prices normalized to

$$p_X(t) = p_E(t) = 1,$$

and with

$$r(t) = 0.1, \quad \pi_X(t) = 0.$$

The base situation is defined as that in which the energy price rises at the rate $\pi_E^0 = 0.05$. Adjusting the constant term in the production function so as to make the full profit maximizing output equal to $Z^0 = 1$ in this base situation, which implies a = 2.2, we get[9]

$$N^0 = 39.85, \; q^0 = 17.27.$$

Table 1 shows the sensitivity of the planned service life to changes in the expected rate of increase of the energy price, $\pi_E(t)$, for five values of the ex ante elasticity of substitution, ranging from $\sigma = 0.0$ (clay-clay technology, Leontief technology) to $\sigma = 4.0$ (large ex ante substitution). First of all, we find that the first order effect of an increase in π_E - i.e. the price increase effect - is a substantial decline in the service life. (Confer the first column of table 1.B in which both the scale and substitution effects have been eliminated.) The ex ante service life declines monotonically from 199.2 years to 22.1 years when the rate of increase of the energy price goes up from 1 per cent to 9 per cent. Second, we note that the service life is less sensitive to the expected rate of increase of the energy price the larger is the elasticity of substitution. Columns 2-5 of table 1.B shows the net result of the price increase effect and the substitution effect. For very high values of the elasticity of substitution, the latter may, in fact, dominate over the former. An example of this is the columns for $\sigma = 4.0$; the life time decreases monotonically from

103.1 to 39.8 years when π_E goes up from 1 to 5 per cent, but when it is increased further to 9 per cent the life time increases to 59.7 years. When σ is less than unity, the substitution effect is, however, moderate. Third, these conclusions are not substantially altered when the scale effect is also taken into account, as a comparison of parts A and B of table 1 makes clear. The scale effect has the same sign as the substitution effect - i.e. $Z \gtrless Z^0$ according as $N \gtrless N^0$ - and hence tends to make the life time less sensitive to changes in the rate of increase of the energy price.

In table 2, the corresponding values of the energy intensity, E(t,0)/J(t), are given. First of all, we see that the energy intensity is more sensitive to changes in the rate of increase of the energy price the larger is the elasticity of substitution. If $\pi_E < \pi_E^0$, higher substitution will motivate to choosing a less energy intensive technique. The latter effect goes through two channels: An increase in σ (i) makes E/J more responsive to changes in the investment cost/life cycle energy cost ratio q/P_E^* induced by changes in π_E with the service life kept constant (cf. tables 2.C and 3.C), and (ii) affects the service life in a negative or positive direction according as $\pi_E \lessgtr \pi_E^0$ (cf. table 1). Second, a comparison of tables 2.B and 2.C shows that the energy intensity is more sensitive to changes in π_E when the endogeneity of the life time is taken into account than when it is neglected. Third, these conclusions are not substantially altered when scale effects are also allowed for (compare tables 2.A and 3.A with 2.B and 3.B).

IV. EX POST OPTIMIZATION, UNANTICIPATED ENERGY PRICE CHANGES, AND CAPITAL REVALUATION

The firm's expectations about future energy and output prices, on which its ex ante optimization is based, will only, by coincidence, be satisfied ex post, i.e. $p_X^*(t+s,t)$ and $P_E^*(t+s,t)$ will not in general be equal to $p_X(t+s)$ and $p_E(t+s)$. As a consequence the firm may want to use its capital equipment for a longer or shorter time period (or with a higher or lower degree of utilization) than it originally anticipated. In this section, we discuss the firm's plan revision and the induced capital revaluation.

TABLE 1. Ex ante service life, N*(t), as a function of ex ante elasticity of substitution and rate of increase of energy price.

$\varepsilon = 0.8,\ \delta = 0,\ \beta = 0.3,\ a = 2.2$

$p_X(t) = p_E(t) = 1,\ q(t) = 17.2727$

$r(t) = 0.1,\ \pi_X(t) = 0$

A. Full profit maximization

$\pi_E(t)$	σ=0.0	σ=0.5	σ=1.0	σ=2.0	σ=4.0
0.01	186.05	169.18	153.25	124.52	81.05
0.02	94.35	88.29	82.46	71.54	52.44
0.03	63.97	61.49	59.01	54.04	43.50
0.04	48.87	48.04	47.15	45.21	40.14
0.05	39.85	39.85	39.85	39.85	39.85
0.06	33.83	34.28	34.82	36.27	41.97
0.07	29.52	30.22	31.10	33.76	46.31
0.08	26.28	27.10	28.21	31.96	52.84
0.09	23.74	24.63	25.89	30.69	61.93

B. Profit maximization for given output, $Z^0 = 1$

$\pi_E(t)$	σ=0.0	σ=0.5	σ=1.0	σ=2.0	σ=4.0
0.01	199.24	183.31	168.36	141.78	103.11
0.02	99.62	93.82	88.27	78.00	60.88
0.03	66.41	64.03	61.67	56.99	47.62
0.04	49.81	49.02	48.19	46.38	41.87
0.05	39.85	39.85	39.85	39.85	39.85
0.06	33.21	33.60	34.07	35.37	40.72
0.07	28.46	29.03	29.77	32.08	44.36
0.08	24.91	25.54	26.41	29.55	50.65
0.09	22.14	22.77	23.69	27.56	59.75

TABLE 2. Energy-capital ratio, $E(t,0)/J(t)$, as a function of ex ante elasticity of substitution and rate of increase of energy price.

$\varepsilon = 0.8,\ \delta = 0,\ \beta = 0.3,\ a = 2.2$

$p_X(t) = p_E(t) = 1,\ q(t) = 17.2727$

$r(t) = 0.1,\ \pi_X(t) = 0$

A. Full profit maximization

$\pi_E(t)$	$\sigma=0.0$	$\sigma=0.5$	$\sigma=1.0$	$\sigma=2.0$	$\sigma=4.0$
0.01	0.4286	0.5343	0.6662	1.0357	2.5097
0.02	0.4286	0.5040	0.5930	0.8237	1.6603
0.03	0.4286	0.4745	0.5266	0.6561	1.1132
0.04	0.4286	0.4491	0.4720	0.5281	0.7208
0.05	0.4286	0.4286	0.4286	0.4286	0.4286
0.06	0.4286	0.4124	0.3940	0.3490	0.2231
0.07	0.4286	0.3996	0.3661	0.2838	0.0973
0.08	0.4286	0.3894	0.3434	0.2293	0.0337
0.09	0.4286	0.3812	0.3246	0.1831	0.0084

B. Profit maximization for given output, $Z^0 = 1$

$\pi_E(t)$	$\sigma=0.0$	$\sigma=0.5$	$\sigma=1.0$	$\sigma=2.0$	$\sigma=4.0$
0.01	0.4286	0.5344	0.6662	1.0358	2.5039
0.02	0.4286	0.5039	0.5927	0.8215	1.6114
0.03	0.4286	0.4739	0.5252	0.6504	1.0592
0.04	0.4286	0.4483	0.4703	0.5230	0.6935
0.05	0.4286	0.4286	0.4286	0.4286	0.4286
0.06	0.4286	0.4143	0.3979	0.3570	0.2339
0.07	0.4286	0.4046	0.3760	0.3013	0.1055
0.08	0.4286	0.3983	0.3608	0.2568	0.0371
0.09	0.4286	0.3947	0.3509	0.2204	0.0093

C. Cost minimization for given output and service life, $Z^0 = 1$, $N^0 = 39.85$

$\pi_E(t)$	$\sigma=0.0$	$\sigma=0.5$	$\sigma=1.0$	$\sigma=2.0$	$\sigma=4.0$
0.01	0.4286	0.5419	0.6852	1.0956	2.8003
0.02	0.4286	0.5145	0.6177	0.8903	1.8494
0.03	0.4286	0.4864	0.5521	0.7113	1.1804
0.04	0.4286	0.4577	0.4889	0.5578	0.7259
0.05	0.4286	0.4286	0.4286	0.4286	0.4286
0.06	0.4286	0.3991	0.3716	0.3222	0.2422
0.07	0.4286	0.3694	0.3184	0.2366	0.1306
0.08	0.4286	0.3399	0.2695	0.1695	0.0670
0.09	0.4286	0.3107	0.2252	0.1184	0.0327

TABLE 3. Ex ante investment cost/ life cycle energy cost ratio as a function of ex ante elasticity of substitution and rate of increase of energy price.

$\varepsilon = 0.8$, $\delta = 0$, $\beta = 0.3$, $a = 2.2$

$p_X(t) = p_E(t) = 1$, $q(t) = 17.2727$

$r(t) = 0.1$, $\pi_X(t) = 0$

A. Full profit maximization

$\pi_E(t)$	σ=0.0	σ=0.5	σ=1.0	σ=2.0	σ=4.0
0.01	1.5545	1.5545	1.5545	1.5546	1.5556
0.02	1.3825	1.3830	1.3837	1.3863	1.4029
0.03	1.2230	1.2257	1.2288	1.2373	1.2695
0.04	1.0947	1.0979	1.1014	1.1100	1.1388
0.05	1.0000	1.0000	1.0000	1.0000	1.0000
0.06	0.9316	0.9259	0.9192	0.9024	0.8494
0.07	0.8819	0.8693	0.8542	0.8137	0.6902
0.08	0.8451	0.8256	0.8012	0.7314	0.5295
0.09	0.8175	0.7913	0.7574	0.6536	0.3741

B. Profit maximization for given output, $Z^0 = 1$

$\pi_E(t)$	σ=0.0	σ=0.5	σ=1.0	σ=2.0	σ=4.0
0.01	1.5545	1.5545	1.5545	1.5545	1.5547
0.02	1.3823	1.3826	1.3830	1.3845	1.3925
0.03	1.2208	1.2229	1.2254	1.2319	1.2538
0.04	1.0913	1.0941	1.0973	1.1047	1.1278
0.05	1.0000	1.0000	1.0000	1.0000	1.0000
0.06	0.9399	0.9347	0.9285	0.9127	0.8595
0.07	0.9024	0.8911	0.8773	0.8385	0.7043
0.08	0.8805	0.8637	0.8418	0.7741	0.5424
0.09	0.8698	0.8482	0.8188	0.7172	0.3840

C. Cost minimization for given output and service life, $Z^0 = 1$, $N^0 = 39.85$

$\pi_E(t)$	σ=0.0	σ=0.5	σ=1.0	σ=2.0	σ=4.0
0.01	1.5988	1.5988	1.5988	1.5988	1.5988
0.02	1.4413	1.4413	1.4413	1.4413	1.4413
0.03	1.2883	1.2883	1.2883	1.2883	1.2883
0.04	1.1408	1.1408	1.1408	1.1408	1.1408
0.05	1.0000	1.0000	1.0000	1.0000	1.0000
0.06	0.8670	0.8670	0.8670	0.8670	0.8670
0.07	0.7430	0.7430	0.7430	0.7430	0.7430
0.08	0.6289	0.6289	0.6289	0.6289	0.6289
0.09	0.5255	0.5255	0.5255	0.5255	0.5255

4.1. Ex post optimization

The ex post quasi rent from vintage t at time t+s is

$$(4.1)\quad \Pi(t+s,t) = p_X(t+s)X(t+s,s) - p_E(t+s)E(t+s,s)$$

$$= p_X(t+s)\xi(t+s,t)^{\varepsilon}B(s)^{\varepsilon}Z(t,0)^{\varepsilon}$$

$$- p_E(t+s)\xi(t+s,t)B(s)E(t,0),$$

where $p_X(t+s)$ and $p_E(t+s)$ denote the output and the energy price realized at time t+s, respectively, and where we have inserted for X(t+s,s) and E(t+s,s) from (2.7)-(2.9) with h(s)=1. The ex post optimzation problem is, in general terms, that of choosing the scale of operation $\xi(t+s,t)$ in such a way that $\Pi(t+s,t)$ is maximized, subject to the values of Z(t,0) and E(t,0) chosen ex ante and the constraint (2.4). The solution may be characterized by Kuhn-Tucker conditions. We shall not do this, but instead consider two simpler cases.

1. If we confine attention to the case where $\xi(t+s,t)$ is either zero or one - as we did when discussing the ex ante optimization problem, cf. (3.3) - the optimal ex post strategy would be to determine whether or not to keep vintage t in operation according to

$$(4.2)\quad \Pi(t+s,t) = p_X(t+s)B(s)^{\varepsilon}Z(t,0)^{\varepsilon} - p_E(t+s)B(s)E(t,0)$$

$$\begin{cases} > 0 \Rightarrow \text{Vintage t is in full use at time t+s.} \\ = 0 \Rightarrow \text{The firm is indifferent.} \\ < 0 \Rightarrow \text{Vintage t is not in use at time t+s.} \end{cases}$$

2. If an inner solution value for $\xi(t+s,t)$ exists, then it is determined by the marginal condition

$$\frac{\partial \Pi(t+s,t)}{\partial \xi(t+s,t)} = 0,$$

which implies

$$(4.3)\quad p_X(t+s)\varepsilon\xi(t+s,t)^{\varepsilon-1}Z(t,0)^{\varepsilon} = p_E(t+s)B(s)E(t,0)^{1-\varepsilon}.$$

This condition says that the marginal revenue from increasing the scale of operation of vintage t at time t+s by one unit shall be equal to the increase in the energy cost.

Let us consider these conditions more closely.

$\xi(t+s,t)$ restricted

From (3.35)-(3.39), (3.43), and (4.2) we obtain

$$(4.4)\quad \Pi(t+s,t) = \frac{p_X(t+s)}{p_X^*(t+s,t)} e^{(\pi_X(t)-\delta\varepsilon)s} p_X(t)X(t,0) - \frac{p_E(t+s)}{p_E^*(t+s,t)} e^{(\pi_E(t)-\delta)s} p_E(t)E(t,0)$$

$$= \left[\frac{p_X(t+s)}{p_X^*(t+s,t)} e^{\{\varrho_X(t)-\varrho_E(t)\}\{N^*(t)-s\}} - \frac{p_E(t+s)}{p_E^*(t+s,t)}\right] e^{\{\pi_E(t)-\delta\}s} p_E(t)E(t,0),$$

which implies that

$$(4.5)\quad \Pi(t+s,t) \lesseqgtr 0 \text{ according as}$$

$$s \gtreqless N^*(t) + \frac{\log\{p_X(t+s)/p_X^*(t+s,t)\}-\log\{p_E(t+s)/p_E^*(t+s,t)\}}{\varrho_X(t)-\varrho_E(t)}.$$

An unanticipated increase in the (level of the) energy price at time t+s, i.e. $p_E(t+s) > p_E^*(t+s,t)$, will have the effect of shortening the remaining service life. A one per cent unanticipated increase in the energy price at time t+s with the growth rate kept unchanged from the new level will shorten the service life by approximately $0.01/(\varrho_X(t)-\varrho_E(t))$ years. An unanticipated increase in the output price will have the opposite effect.[10]

$\xi(t+s,t)$ unrestricted, inner solution

By inserting (3.37) into (4.3), we can express the optimal scale of operation of vintage t at time t+s, provided that an inner solution for this variable exists, as

$$(4.6) \quad \xi(t+s,t) = \left[\varepsilon e^{\delta(1-\varepsilon)s} \frac{p_X(t+s)}{p_E(t+s)} \frac{X(t,0)}{E(t,0)}\right]^{1/(1-\varepsilon)} .$$

An unanticipated increase in the energy price at time t+s by one per cent will reduce the optimal ex post scale of operation by $1/(1-\varepsilon)$ per cent. An unanticipated increase in the output price will have the opposite effect.

4.2. Unanticipated energy price changes and capital revaluation

The revision of the planned service lives of capital goods which is induced by changes in the price expectations will affect the market value of used capital goods. In general, the value of a capital good is an increasing function of its remaining life time. Since we assume a vintage specific technology, the ex post value should be related in some way to the particular technique chosen for each vintage.[11] This is the effect that we define as the capital revaluation effect of the energy and output price changes.

Consider capital vintage t at time $t+\Theta$, where Θ is the age of the equipment $(\Theta \geq 0)$. The (potential) expected quasi rent at time t+s from this vintage, given that it has attained age Θ $(s \geq \Theta)$, is equal to

$$(4.7) \quad \Pi^*(t+s,t+\Theta,t) = p_X^*(t+s,t+\Theta)X(t+s,s) - p_E^*(t+s,t+\Theta)E(t+s,s)$$
$$= p_X^*(t+s,t+\Theta)B(s)^{\varepsilon}X(t,0) - p_E^*(t+s,t+\Theta)B(s)E(t,0), \quad s \geq \Theta \geq 0.$$

Since

$$p_X^*(t+\Theta,t+\Theta) = p_X(t+\Theta),$$
$$p_E^*(t+\Theta,t+\Theta) = p_E(t+\Theta),$$

price variables without asterisks denoting, as before, ex post values, we have in particular

$$(4.8) \quad \Pi^*(t+s,t,t) = \Pi^*(t+s,t)$$

and

$$(4.9) \quad \Pi^*(t+s,t+s,t) = \Pi(t+s,t);$$

cf. (3.1) and (4.1).

We assume that the value of a capital vintage ex post is equal to the present value of the total flow of quasi rents that it is expected to earn during its remaining service life.[12] The value of capital vintage t at time t+Θ can then be expressed as

$$(4.10)\quad V(t+\Theta,t) = \int_{\Theta}^{\infty} e^{-r(t+\Theta)(s-\Theta)} \max[\Pi^*(t+s,t+\Theta,t),0]\, ds,$$

where r(t+Θ) is the interest rate as expected at time t+Θ. From (3.2), (4.8), and (4.10) it follows in particular that

$$(4.11)\quad V(t,t) = q(t)J(t) + \Pi^*(t),$$

i.e., the initial capital value is equal to the investment cost plus the expected total profit over the life time at the time of instalment.

Let now $N^*(t+\Theta,t)$ denote the remaining service life of vintage t as it is expected at time t+Θ. In particular, we have $N^*(t,t) = N^*(t)$. We assume that the revised service life is determined by a quasi rent criterion similar to the ex ante criterion $\Pi^*[t+N^*(t),t]=0$, viz.

$$\Pi^*[t+\Theta+N^*(t+\Theta,t),\ t+\Theta,t] = 0.$$

This equation has only economic meaning for $N^*(t+\Theta,t) \geq 0$. If it comes out with a negative solution value, then $N^*(t+\Theta,t)$ should be set equal to zero, which has the interpretation that vintage t is taken out of operation immediately at time t+Θ. The strict definition of $N^*(t+\Theta,t)$ is thus

$$(4.12)\quad N^*(t+\Theta,t) = \max[N^{**}(t+\Theta,t),\ 0],$$

where $N^{**}(t+\Theta,t)$ is the solution to $\Pi^*[t+\Theta+N^{**}(t+\Theta,t),\ t+\Theta,t]=0$.

For Θ = N(t), N(t) denoting the actual scrapping age of vintage t, we have in particular

$$(4.13)\quad N^*[t+N(t),t] = 0$$

and

$$(4.14)\quad \Pi^*[t+N(t),t+N(t),t] = 0.$$

Let $\pi_X(t+\Theta)$ and $\pi_E(t+\Theta)$ denote the constant rates at which the output and energy prices are expected to grow from the replanning time $t+\Theta$, i.e.

$$(4.15)\quad p_X^*(t+s,t+\Theta) = p_X(t+\Theta)e^{\pi_X(t+\Theta)(s-\Theta)},$$

$$(4.16)\quad p_E^*(t+s,t+\Theta) = p_E(t+\Theta)e^{\pi_E(t+\Theta)(s-\Theta)}, \qquad s\geq\Theta\geq 0.$$

These equations generalize (3.35) and (3.36). From (4.7), (4.15), (4.16), and (3.37) we find

$$(4.17)\quad \Pi^*(t+s,t+\Theta,t) = p_X(t+\Theta)e^{\pi_X(t+\Theta)(s-\Theta)-\delta\varepsilon s}X(t,0) - p_E(t+\Theta)e^{\pi_E(t+\Theta)(s-\Theta)-\delta s}E(t,0),$$
$$\Theta \leq s \leq \Theta+N^*(t+\Theta,t).$$

Combining this with (4.12), we get[13]

$$(4.18)\quad N^*(t+\Theta,t) = \frac{\log[p_X(t+\Theta)X(t,0)/(p_E(t+\Theta)E(t,0))]+\delta(1-\varepsilon)\Theta}{\pi_E(t+\Theta)-\pi_X(t+\Theta)-\delta(1-\varepsilon)}.$$

Values of $N^*(t+\Theta,t)$ for Θ = 0, 10, 20, and 30 years and with the output fixed at $Z(t,0) = Z^0 = 1$, are given in table 4. We assume that the energy price follows its expected path from time t up to time $t+\Theta$ when an exogenous shock occurs.[14] This may come either in the form of a revision of the anticipated rate of increase of the energy price, i.e. $\pi_E(t+\Theta)\neq\pi_E(t)$, (shift parameter equal to 1) or as a discontinuous jump in the level of this price with its growth rate retained, i.e. $p_E(t+\Theta)\neq p_E^*(t+\Theta,t)$, (shift parameter equal to 2 or 3). Four values of the ex ante elasticity of substitution are considered, σ=0.0, 0.5, 1.0, and 2.0. For the sake of reference, alternatives based on perfect forsight ($\pi_E(t+\Theta) = \pi_E(t), p_E(t+\Theta) = p_E^*(t+\Theta,t)$) are also included (alternatives 1, 5, and 9). A revision of the expected rate of energy price increase from 6 per cent to 12 per cent at age Θ=10 years, for an elasticity of substitution σ=0.5, will, for instance, reduce the remaining expected service life from 23.6 to 11.8 years. If the capital is Θ=20 years of age when the shock occurs, the remaining service life will be reduced from 13.6 to 6.8 years. If a discontinuous shift in the energy price of 200 per cent takes place over night with the growth rate of 6 per cent retained from the new price level, then the remaining life time will be reduced from 23.6 to 5.3 years for a capital unit of age 10 years, whereas a unit of age 20 years will be taken out of operation, etc.

Inserting (4.17) in (4.10), we find that the ex post value of vintage t at time t+Θ can be expressed as

$$(4.19)\quad V(t+\Theta,t) = e^{-\delta\Theta}N^*(t+\Theta,t)\{p_X(t+\Theta)X(t,0)e^{\delta(1-\varepsilon)\Theta}\, D[\varrho_X(t+\Theta)N^*(t+\Theta,t)]$$

$$- p_E(t+\Theta)E(t,0)\, D[\varrho_E(t+\Theta)N^*(t+\Theta,t)]\},$$

where ϱ_X, ϱ_E and D( ) are defined as in (3.38), (3.39), and (3.42). This equation, after inserting for N*(t+Θ,t) from (4.18), shows how the value of vintage t is related to the choice of production scale and technique on the one hand, and the changes in expectations about future output and energy prices from time t to time t+Θ on the other.

We can characterize the capital revaluation in two ways, by comparing the ex post value of vintage t at time t+Θ either with its <u>original</u> ex ante value at time t or by the ex ante value of a <u>new</u> vintage installed at time t+Θ with an optimal energy/capital ratio based on the latest price information. This gives two sets of <u>revaluation coefficients</u>,

$$(4.20)\quad v(t+\Theta,t) = \frac{V(t+\Theta,t)}{V(t,t)}$$

and

$$(4.21)\quad w(t+\Theta,t) = \frac{V(t+\Theta,t)}{V(t+\Theta,t+\Theta)}\,.$$

Values of v(t+Θ,t) and w(t+Θ,t) corresponding to the values of N*(t+Θ,t) in table 4 are given in tables 5 and 6.

From table 5 we see, for instance, that an unanticipated change in the rate of increase of the energy price from 6 to 12 per cent at age Θ = 10, with σ = 0.5, will reduce the capital value from 74.4 per cent of the ex ante value to 51.4 per cent. The corresponding reduction at age Θ = 20 is from 38.9 per cent to 23.8 per cent. A 100 per cent unanticipated price increase over night will reduce the value of these vintages to 32.9 and 1.6 per cent of their ex ante value, respectively. The relative decline in the capital value depends on the ex ante choice of energy intensity, which again depends on the elasticity of substitution between energy and capital, σ (cf. table 2). If larger substitutability is accompanied by a <u>decline</u> in the energy intensity - which will be the case if $\pi_E(t) > \pi_E^0 = 0.05$ (cf. section 3.3) - then $v(t+\Theta,t)$ will be an increasing function of σ. Consider a capital vintage at age $\Theta = 10$ facing an unanticipated increase in π_E from 6 to 15 per cent. The ex post value of this vintage as compared with its ex ante value will then be 43.6 per cent if the ex ante technology is Leontief ($\sigma = 0.0$), 44.2 per cent if $\sigma = 0.5$, 45.0 per cent in the Cobb-Douglas case ($\sigma = 1.0$) and 46.9 per cent if $\sigma = 2.0$. In this sense, firms with large ex ante substitution are less vulnerable to energy price shocks than firms with small ex ante substitution. If, on the other hand, larger substitutability is accompanied by an increase in the energy intensity - which will happen if $\pi_E(t) < \pi_E^0 = 0.05$ - then $v(t+\Theta,t)$ will be a decreasing function of σ.

The conclusions about the relationship between ex ante substitution and capital revaluation change substantially when we, as do for instance Berndt and Wood (1984, p. 14), interpret revaluation in terms of capital utilizing the latest price information and optimal factor proportions at any time. From table 6 we see that $w(t+\Theta,t)$ is a decreasing function of σ if the ex post rate of increase of the energy price exceeds the ex ante one ($\pi_E(t+\Theta) > \pi_E(t)$) and is an increasing function of σ in the opposite case. Consider, for instance, a capital vintage at age $\Theta=10$ years, facing an unanticipated increase in π_E from 3 to 9 per cent. Its ex post value as compared with the ex ante value of the most efficient new vintage will be 85.8 per cent in the Leontief case ($\sigma = 0.0$), 80.1 per cent if $\sigma = 0.5$, 74.0 per cent in the Cobb-Douglas case ($\sigma = 1.0$), and 59.5 per cent if $\sigma = 2.0$. If the ex ante technology is Leontief, the value of a new capital good ($\Theta = 0$) will not be affected at all. In this sense, there will be a positive relationship between the degree of ex ante substitution and the degree of revaluation.

TABLE 4. Revised value of remaining service life, $N^*(t+\theta,t)$, after unanticipated change in energy price at age θ.

$\varepsilon = 0.8,\ \delta = 0,\ \beta = 0.3,\ a = 2.2$

$p_X(t) = p_E(t) = 1,\ q(t) = 17.2727,\ r(t) = 0.1,\ \pi_X(t) = 0,\ Z = Z^0 = 1$

			σ=0.0				σ=0.5			
shift	$\pi_E(t)$	$\pi_E(t+\theta)$	θ=0	θ=10	θ=20	θ=30	θ=0	θ=10	θ=20	θ=30
1.	0.03	0.03	66.41	56.41	46.41	36.41	64.03	54.03	44.03	34.03
1.	0.06	0.03	66.41	46.41	26.41	6.41	67.20	47.20	27.20	7.20
1.	0.09	0.03	66.41	36.41	6.41	0.00	68.31	38.31	8.31	0.00
1.	0.03	0.06	33.21	28.21	23.21	18.21	32.02	27.02	22.02	17.02
1.	0.06	0.06	33.21	23.21	13.21	3.21	33.60	23.60	13.60	3.60
1.	0.09	0.06	33.21	18.21	3.21	0.00	34.16	19.16	4.16	0.00
1.	0.03	0.09	22.14	18.80	15.47	12.14	21.34	18.01	14.68	11.34
1.	0.06	0.09	22.14	15.47	8.80	2.14	22.40	15.73	9.07	2.40
1.	0.09	0.09	22.14	12.14	2.14	0.00	22.77	12.77	2.77	0.00
1.	0.03	0.12	16.60	14.10	11.60	9.10	16.01	13.51	11.01	8.51
1.	0.06	0.12	16.60	11.60	6.60	1.60	16.80	11.80	6.80	1.80
1.	0.09	0.12	16.60	9.10	1.60	0.00	17.08	9.58	2.08	0.00
1.	0.03	0.15	13.28	11.28	9.28	7.28	12.81	10.81	8.81	6.81
1.	0.06	0.15	13.28	9.28	5.28	1.28	13.44	9.44	5.44	1.44
1.	0.09	0.15	13.28	7.28	1.28	0.00	13.66	7.66	1.66	0.00
1.	0.03	0.18	11.07	9.40	7.74	6.07	10.67	9.01	7.34	5.67
1.	0.06	0.18	11.07	7.74	4.40	1.07	11.20	7.87	4.53	1.20
1.	0.09	0.18	11.07	6.07	1.07	0.00	11.39	6.39	1.39	0.00
2.	0.03	0.03	43.31	33.31	23.31	13.31	40.93	30.93	20.93	10.93
2.	0.06	0.06	21.65	11.65	1.65	0.00	22.05	12.05	2.05	0.00
2.	0.09	0.09	14.44	4.44	0.00	0.00	15.07	5.07	0.00	0.00
3.	0.03	0.03	29.79	19.79	9.79	0.00	27.41	17.41	7.41	0.00
3.	0.06	0.06	14.90	4.90	0.00	0.00	15.29	5.29	0.00	0.00
3.	0.09	0.09	9.93	0.00	0.00	0.00	10.56	0.56	0.00	0.00

			σ=1.0				σ=2.0			
shift	$\pi_E(t)$	$\pi_E(t+\theta)$	θ=0	θ=10	θ=20	θ=30	θ=0	θ=10	θ=20	θ=30
1.	0.03	0.03	61.67	51.67	41.67	31.67	56.99	46.99	36.99	26.99
1.	0.06	0.03	68.14	48.14	28.14	8.14	70.74	50.74	30.74	10.74
1.	0.09	0.03	71.00	41.00	11.00	0.00	82.67	52.67	22.67	0.00
1.	0.03	0.06	30.84	25.84	20.84	15.84	28.50	23.50	18.50	13.50
1.	0.06	0.06	34.07	24.07	14.07	4.07	35.37	25.37	15.37	5.37
1.	0.09	0.06	35.54	20.54	5.54	0.00	41.33	26.33	11.33	0.00
1.	0.03	0.09	20.56	17.22	13.89	10.56	19.00	15.66	12.33	9.00
1.	0.06	0.09	22.71	16.05	9.38	2.71	23.58	16.91	10.25	3.58
1.	0.09	0.09	23.69	13.69	3.69	0.00	27.56	17.56	7.56	0.00
1.	0.03	0.12	15.42	12.92	10.42	7.92	14.25	11.75	9.25	6.75
1.	0.06	0.12	17.04	12.04	7.04	2.04	17.68	12.68	7.68	2.68
1.	0.09	0.12	17.77	10.27	2.77	0.00	20.67	13.17	5.67	0.00
1.	0.03	0.15	12.33	10.33	8.33	6.33	11.40	9.40	7.40	5.40
1.	0.06	0.15	13.63	9.63	5.63	1.63	14.15	10.15	6.15	2.15
1.	0.09	0.15	14.22	8.22	2.22	0.00	16.53	10.53	4.53	0.00
1.	0.03	0.18	10.28	8.61	6.95	5.28	9.50	7.83	6.17	4.50
1.	0.06	0.18	11.36	8.02	4.69	1.36	11.79	8.46	5.12	1.79
1.	0.09	0.18	11.85	6.85	1.85	0.00	13.78	8.78	3.78	0.00
2.	0.03	0.03	30.57	20.57	10.57	0.57	33.89	23.89	13.89	3.89
2.	0.06	0.06	22.52	12.52	2.52	0.00	23.82	13.82	3.82	0.00
2.	0.09	0.09	15.99	5.99	0.00	0.00	19.85	9.85	0.00	0.00
3.	0.03	0.03	25.05	15.05	5.05	0.00	20.37	10.37	0.37	0.00
3.	0.06	0.06	15.76	5.76	0.00	0.00	17.06	7.06	0.00	0.00
3.	0.09	0.09	11.49	1.49	0.00	0.00	15.35	5.35	0.00	0.00

TABLE 5. Revaluation coefficients $v(t+\theta,t)$: Ex post capital value at time $t+\theta$ as a share of ex ante value at time t.

$\varepsilon = 0.8$, $\delta = 0$, $\beta = 0.3$, $a = 2.2$

$p_X(t) = p_E(t) = 1$, $q(t) = 17.2727$, $r(t) = 0.1$, $\pi_X(t) = 0$, $Z = Z^0 = 1$

shift	$\pi_E(t)$	$\pi_E(t+\theta)$	σ=0.0 θ=0	σ=0.0 θ=10	σ=0.0 θ=20	σ=0.0 θ=30	σ=0.5 θ=0	σ=0.5 θ=10	σ=0.5 θ=20	σ=0.5 θ=30
1.	0.03	0.03	1.0000	0.9166	0.8057	0.6604	1.0000	0.9091	0.7884	0.6312
1.	0.06	0.03	1.1296	0.9101	0.5380	0.0661	1.1267	0.9138	0.5514	0.0000
1.	0.09	0.03	1.2998	0.8583	0.0761	0.0000	1.2856	0.8765	0.1155	0.0000
1.	0.03	0.06	0.8852	0.7808	0.6530	0.5018	0.8780	0.7661	0.6301	0.4712
1.	0.06	0.06	1.0000	0.7376	0.3765	0.0365	1.0000	0.7438	0.3886	0.0447
1.	0.09	0.06	1.1586	0.6523	0.0420	0.0000	1.1451	0.6751	0.0656	0.0000
1.	0.03	0.09	0.7694	0.6602	0.5349	0.3966	0.7583	0.6431	0.5120	0.3690
1.	0.06	0.09	0.8691	0.6042	0.2858	0.0252	0.8708	0.6109	0.2960	0.0310
1.	0.09	0.09	1.0000	0.5155	0.0290	0.0000	1.0000	0.5374	0.0457	0.0000
1.	0.03	0.12	0.6719	0.5660	0.4494	0.3261	0.6594	0.5488	0.4280	0.3017
1.	0.06	0.12	0.7590	0.5077	0.2295	0.0192	0.7616	0.5141	0.2382	0.0237
1.	0.09	0.12	0.8733	0.4238	0.0221	0.0000	0.8763	0.4437	0.0351	0.0000
1.	0.03	0.15	0.5932	0.4932	0.3863	0.2763	0.5804	0.4767	0.3667	0.2547
1.	0.06	0.15	0.6701	0.4364	0.1916	0.0156	0.6730	0.4424	0.1990	0.0192
1.	0.09	0.15	0.7710	0.3591	0.0179	0.0000	0.7754	0.3770	0.0284	0.0000
1.	0.03	0.18	0.5295	0.4361	0.3382	0.2394	0.5169	0.4205	0.3203	0.2202
1.	0.06	0.18	0.5981	0.3821	0.1643	0.0131	0.6012	0.3877	0.1708	0.0161
1.	0.09	0.18	0.6882	0.3112	0.0150	0.0000	0.6934	0.3273	0.0239	0.0000
2.	0.03	0.03	0.7645	0.6074	0.4117	0.1923	0.7438	0.5743	0.3668	0.1446
2.	0.06	0.06	0.6873	0.3159	0.0105	0.0002	0.6945	0.3285	0.0157	0.0002
2.	0.09	0.09	0.6409	0.1084	0.0000	0.0000	0.6582	0.1329	0.0000	0.0000
3.	0.03	0.03	0.5428	0.3355	0.1192	0.0000	0.5853	0.2878	0.0764	0.0000
3.	0.06	0.06	0.4422	0.0781	0.0000	0.0000	0.4536	0.0885	0.0000	0.0000
3.	0.09	0.09	0.3917	0.0000	0.0000	0.0000	0.4171	0.0022	0.0000	0.0000

shift	$\pi_E(t)$	$\pi_E(t+\theta)$	σ=1.0 θ=0	σ=1.0 θ=10	σ=1.0 θ=20	σ=1.0 θ=30	σ=2.0 θ=0	σ=2.0 θ-10	σ=2.0 θ=20	σ=2.0 θ=30
1.	0.03	0.03	1.0000	0.9008	0.7695	0.5996	1.0000	0.8816	0.7264	0.5292
1.	0.06	0.03	1.1233	0.9180	0.5669	0.0977	1.1143	0.9284	0.6069	0.1495
1.	0.09	0.03	1.2661	0.8996	0.1782	0.0000	1.1984	0.9636	0.4365	0.0000
1.	0.03	0.06	0.8703	0.7506	0.6062	0.4394	0.8540	0.7169	0.5544	0.3723
1.	0.06	0.06	1.0000	0.7511	0.4030	0.0553	1.0000	0.7780	0.4412	0.0878
1.	0.09	0.06	1.1374	0.7059	0.1051	0.0000	1.1088	0.8069	0.2945	0.0000
1.	0.03	0.09	0.7470	0.6255	0.4884	0.3409	0.7233	0.5884	0.4391	0.2833
1.	0.06	0.09	0.8729	0.6109	0.3082	0.0385	0.8785	0.6398	0.3410	0.0619
1.	0.09	0.09	1.0000	0.5676	0.0743	0.0000	1.0000	0.6745	0.2199	0.0000
1.	0.03	0.12	0.6468	0.5312	0.4062	0.2772	0.6208	0.4949	0.3614	0.2279
1.	0.06	0.12	0.7647	0.5218	0.2485	0.0296	0.7730	0.5423	0.2767	0.0478
1.	0.09	0.12	0.8804	0.4715	0.0574	0.0000	0.8970	0.5740	0.1750	0.0000
1.	0.03	0.15	0.5675	0.4600	0.3468	0.2332	0.5413	0.4258	0.3065	0.1903
1.	0.06	0.15	0.6765	0.4496	0.2080	0.0240	0.6861	0.4690	0.2325	0.0389
1.	0.09	0.15	0.7818	0.4022	0.0468	0.0000	0.8074	0.4976	0.1453	0.0000
1.	0.03	0.18	0.5043	0.4048	0.3022	0.2011	0.4790	0.3730	0.2658	0.1633
1.	0.06	0.18	0.6049	0.3944	0.1787	0.0201	0.6149	0.4124	0.2003	0.0328
1.	0.09	0.18	0.7009	0.3503	0.0394	0.0000	0.7314	0.4384	0.1241	0.0000
2.	0.03	0.03	0.7212	0.5388	0.3201	0.0994	0.6697	0.4598	0.2222	0.0259
2.	0.06	0.06	0.7031	0.3435	0.0230	0.0000	0.7252	0.3836	0.0480	0.0000
2.	0.09	0.09	0.6818	0.1700	0.0000	0.0000	0.7637	0.3268	0.0000	0.0000
3.	0.03	0.03	0.4653	0.2391	0.0398	0.0000	0.3777	0.1418	0.0003	0.0000
3.	0.06	0.06	0.4671	0.1016	0.0000	0.0000	0.5827	0.1393	0.0000	0.0000
3.	0.09	0.09	0.4525	0.0138	0.0000	0.0000	0.5812	0.1258	0.0000	0.0000

TABLE 6. Revaluation coefficients $w(t+\Theta,t)$: Capital value after energy price shock at age Θ as a share of value of new capital utilizing optimal technology at time t.

$\varepsilon = 0.8,\ \delta = 0,\ \beta = 0.3,\ a = 2.2,$

$p_X(t) = p_E(t) = 1,\ q(t) = 17.2727,$

$r(t) = 0.1,\ \pi_X(t) = 0;\ Z = Z^0 = 1$

Shift	$\pi_E(t)$	$\pi_E(t+\Theta)$	σ = 0.0			
			Θ=0	Θ=10	Θ=20	Θ=30
1.00	0.03	0.03	1.0000	0.9166	0.8057	0.6604
1.00	0.06	0.03	1.0000	0.8057	0.4763	0.0585
1.00	0.09	0.03	1.0000	0.6604	0.0585	0.0000
1.00	0.03	0.06	1.0000	0.8821	0.7376	0.5669
1.00	0.06	0.06	1.0000	0.7376	0.3765	0.0365
1.00	0.09	0.06	1.0000	0.5669	0.0365	0.0000
1.00	0.03	0.09	1.0000	0.8580	0.6952	0.5155
1.00	0.06	0.09	1.0000	0.6952	0.3288	0.0292
1.00	0.09	0.09	1.0000	0.5155	0.0290	0.0000

Shift	$\pi_E(t)$	$\pi_E(t+\Theta)$	σ = 0.5			
			Θ=0	Θ=10	Θ=20	Θ=30
1.00	0.03	0.03	1.0000	0.9091	0.7884	0.6312
1.00	0.06	0.03	1.0237	0.8303	0.5010	0.0727
1.00	0.09	0.03	1.0315	0.7033	0.0927	0.0000
1.00	0.03	0.06	0.9662	0.8432	0.6935	0.5186
1.00	0.06	0.06	1.0000	0.7438	0.3886	0.0447
1.00	0.09	0.06	1.0113	0.5962	0.0579	0.0000
1.00	0.03	0.09	0.9450	0.8014	0.6381	0.4598
1.00	0.06	0.09	0.9861	0.6918	0.3351	0.0351
1.00	0.09	0.09	1.0000	0.5374	0.0457	0.0000

Shift	$\pi_E(t)$	$\pi_E(t+\Theta)$	σ = 1.0			
			Θ=0	Θ=10	Θ=20	Θ=30
1.00	0.03	0.03	1.0000	0.9008	0.7695	0.5998
1.00	0.06	0.03	1.0505	0.8585	0.5302	0.0913
1.00	0.09	0.03	1.0705	0.7606	0.1506	0.0000
1.00	0.03	0.06	0.9306	0.8026	0.6481	0.4699
1.00	0.06	0.06	1.0000	0.7511	0.4030	0.0553
1.00	0.09	0.06	1.0282	0.6381	0.0950	0.0000
1.00	0.03	0.09	0.8834	0.7398	0.5777	0.4032
1.00	0.06	0.09	0.9656	0.6845	0.3409	0.0426
1.00	0.09	0.09	1.0000	0.5676	0.0743	0.0000

TABLE 6 (cont.)

Shift	$\pi_E(t)$	$\pi_E(t+\Theta)$	$\sigma = 2.0$			
			Θ=0	Θ=10	Θ=20	Θ=30
1.00	0.03	0.03	1.0000	0.8816	0.7264	0.5292
1.00	0.06	0.03	1.1161	0.9299	0.6079	0.1498
1.00	0.09	0.03	1.1850	0.9529	0.4316	0.0000
1.00	0.03	0.06	0.8527	0.7157	0.5535	0.3717
1.00	0.06	0.06	1.0000	0.7700	0.4412	0.0878
1.00	0.09	0.06	1.0947	0.7966	0.2908	0.0000
1.00	0.03	0.09	0.7314	0.5950	0.4440	0.2865
1.00	0.06	0.09	0.8892	0.6480	0.3453	0.0627
1.00	0.09	0.09	1.0000	0.6745	0.2199	0.0000

V. EMPIRICAL RESULTS

In this section, we illustrate the main results above by means of Norwegian data for total manufacturing and attempt to assess the effect of price changes on the scrapping plans and capital revaluation in the period 1967-1983. Our focus will be on the ex ante service life, the energy-capital ratio, the revised plans for the service life, and the revaluation coefficients.

5.1. Basic data and assumptions

The data are taken from the Norwegian annual national accounts and refer to an aggregate sector, total manufacturing. A brief description is given in appendix A. Since the putty-clay production model is basically microeconomic, related to the individual firm, or even to a single production plant within a firm [cf. Johansen [1972, section 2.2]], empirical applications utilizing aggregate data may not be easily interpreted. Our interpretation of the calculations to be presented below is that they illustrate, for a 'typical' Norwegian manufacturing firm [i.e. one facing 'average' price changes], the effect of changes in input and output prices on the scrapping behaviour and on the induced capital revaluation. Admittedly, the assumption of full ex post fixity of factor composition is not equally well suited to all the production activities occurring in manufacturing industries. In empirical work, the putty-clay model has been most often used for sectors consisting of fairly homogeneous production units, producing a homogeneous output by means of a limited number of identifiable inputs [see e.g. Førsund and Hjalmarsson [1983]], and such sectors are often relatively energy intensive. A priori, however, there is no reason to assume a relationship between the energy intensity and the usefulness of the "ex ante substitution/ex post fixity" assumption, although energy price shocks, of course, will have their strongest impact on the energy intensive sectors.

At this aggregate level, we formalize the producers' formation of price expectations as follows. Let p_t denote either the output price or the energy input price in year t and define

$\pi_t = p_t/p_{t-1} - 1$ = Rate of price increase observed in year t, pro anno rate.

π_t^* = Rate of price increase for the future time period as expected in year t, pro anno rate.

We assume that the producers construct their price expectations by smoothing the rates of price increase observed by means of

(5.1) $$\pi_t^* = (1-\gamma)\pi_{t-1}^* + \gamma\{\mu\pi_t + (1-\mu)\pi_{t-1}\},$$

where γ and μ are constants between zero and one. Formally, (5.1) defines π_t^* as an ARMA(1,1) process on π_t. The lower the values of γ and μ, the more sluggish is the adjustment of the expected future rate of price increase formed in year t to the rate of price increase actually observed in this year. Since the lag polynomial connecting π_t and π_t^* is given by

(5.2) $$\pi_t^* = \frac{\gamma(\mu+(1-\mu)L)}{1-(1-\gamma)L}\pi_t = B(L)\pi_t,$$

L being the lag operator, with coefficients adding to unity (i.e. B(1) = 1), the mean lag between the actual and expected rate of price increase can be expressed as (see e.g. Dhrymes (1971, p. 8))

(5.3) $$m = B'(1) = \frac{1 - \gamma\mu}{\gamma}.$$

This ARMA specification includes as special cases:

$\underline{\gamma = 0}$: Static price expectations, i.e. no revision of future rates of increase.

$\underline{\mu = 1,\ 0 < \gamma < 1}$: Adaptive expectations based on current rate of increase.

$\underline{\mu = 0,\ 0 < \gamma < 1}$: Adaptive expectations based on rate of increase in previous year.

$\underline{\gamma = 1,\ 0 < \mu < 1}$: Moving average of rates of increase in current and previous year.

$\underline{\gamma = 1,\ \mu = 0}$: Instantaneous adjustment of price expectations to rate of increase in previous year.

$\underline{\gamma = \mu = 1}$: Instantaneous adjustment of price expectations.

These annual rates are converted to rates expressed in continuous time by means of $\pi(t) = \log(1 + \pi_t^*)$, $r(t) = \log(1 + r_t)$. Note that we assume that expectations about the interest rate are adjusted instantaneously.

Since no data on price expectations are available by means of which we could estimate γ and μ econometrically, values must be assigned to these parameters a priori. The values selected as the base specification are

$$\gamma = 0.2, \ \mu = 1.0,$$

which correspond to an adaptive expectation formulation with a mean lag between the rates of inflation observed and expected of m=4 years. This concurs, to some extent, with the result of an econometric investigation based on genuine expectation data for British manufacturing industries (Pesaran (1985, table 2 A)). As alternatives we consider

$$\gamma = 0.1, \ \mu = 1.0, \text{ i.e. } m = 9,$$
$$\gamma = 0.5, \ \mu = 0.5, \text{ i.e. } m = 1.5.$$

The resulting values of π^*_{Xt} and π^*_{Et} are given in table 7. The necessary condition for the existence of a positive and finite solution value for N, $\pi_E > \pi_X$ (cf. (3.45)), is satisfied in all the years for all the three price expectation processes, except in 1965 for $\gamma = \mu = 0.5$.

The distribution parameter in the ex ante CES function (3.30) is set equal to the average energy/capital ratio (capital being defined as the aggregate of machinery and transport equipment) over the sample period, according to the national accounts. This gives

$$\hat{\beta} = 0.10122.$$

Further, we assume that the ex ante technology is linear homogeneous (ε=1) and that capital invested in 1980 (which is the base year of the national accounts data) is expected to earn a zero profit over its anticipated life cycle for an exogenously given output, which may conventionally be set to unity. This assumption, which is certainly strong, means that the life cycle output value of the "base vintage" (1980) exactly exhausts its life cycle cost. Its role is to permit us to determine the efficiency parameter of the ex ante production function, a - or equivalently, the constant term of its dual cost function, 1/a - of which no a priori information is otherwise available from data aggregated across vintages. For the other vintages, a non-zero profit (positive or negative) will, of course, normally occur. The resulting value of a is given in table 8.

TABLE 7. Rate of increase of output and energy prices for different price expectation processes. Annual rates.

Year t	γ=0.1, μ=1.0, m=9		γ=0.2, μ=1.0, m=4		γ=0.5, μ=0.5, m=1.5	
	π^*_{Xt}	π^*_{Et}	π^*_{Xt}	π^*_{Et}	π^*_{Xt}	π^*_{Et}
1964	-0.02506	0.01379	-0.02250	0.00112	-0.02122	-0.01128
1965	-0.02414	0.01145	-0.02118	-0.00104	-0.01510	-0.02196
1966	-0.02647	0.01524	-0.02643	0.00905	-0.02339	-0.00105
1967	-0.02913	0.02199	-0.03176	0.02380	-0.03682	0.03252
1968	-0.03119	0.01737	-0.03536	0.01423	-0.04411	0.03095
1969	-0.02807	0.01839	-0.02828	0.01686	-0.03449	0.01631
1970	-0.02184	0.01691	-0.01577	0.01421	-0.00867	0.01590
1971	-0.01795	0.02949	-0.00923	0.03991	0.00846	0.04453
1972	-0.01923	0.02933	-0.01352	0.03751	0.00080	0.06492
1973	-0.01648	0.03312	-0.00917	0.04345	-0.00521	0.05625
1974	-0.00141	0.07864	0.01953	0.13242	0.03303	0.16700
1975	0.00416	0.08191	0.02643	0.12820	0.06366	0.23340
1976	0.00509	0.08275	0.02387	0.12062	0.04876	0.16710
1977	0.00556	0.08941	0.02106	0.12637	0.03019	0.14347
1978	0.00937	0.08730	0.02557	0.11475	0.02845	0.12615
1979	0.01776	0.09156	0.03911	0.11780	0.04844	0.11264
1980	0.02779	0.11082	0.05490	0.15107	0.07705	0.15985
1981	0.03307	0.12024	0.06004	0.16185	0.08819	0.20221
1982	0.03565	0.11930	0.05982	0.15165	0.07898	0.18006
1983	0.03781	0.12064	0.05930	0.14786	0.06853	0.15092

An exogenous retirement of production capacity (and/or decline in efficiency with age) at a (continuous) rate of 10 per cent (δ = 0.10) is assumed over the capital's life cycle. Strictly interpreted, δ is a technical parameter in the model, but since only the sum of δ and the interest rate r occurs in the equations determining the ex ante service life and the energy/capital ratio, a non-zero value of δ may alternatively be interpreted as including a risk premium claimed by the firm for undertaking an uncertain investment project. Or more precisely, r+δ can be reinterpreted as the sum of the market interest rate (on approximate risk-free assets), the rate of retirement (decline in efficiency), and the risk premium. The value δ = 0.10 could thus, for instance, represent a retirement rate of 6 per cent and a risk premium of 4 per cent. From this it follows that the actual ex ante life cycle profit for the base vintage will be positive even if we assume the profit based on the interest rate inclusive of risk premium to be zero.

The investment price is normalized in such a way that it equals the life cycle energy price in the base year. This implies that the energy/capital ratio of the base vintage is equal to $\hat{\beta}/(1-\hat{\beta})$ = 0.1126 regardless of the value of the ex ante elasticity of substitution between the two inputs (cf. (3.33)). The implied values of the service life, N_{base}, and the normalization factor, q_{base}, are given in the last two columns of table 8.

TABLE 8. Coefficient values used in simulations

Alternative	δ	γ	μ	β	a	$N_{base}^{b)}$	$q_{base}^{c)}$
1.	0.1	0.1	1.0	0.10122	8.897	36.47	9.570
2.[a)]	0.1	0.2	1.0	0.10122	10.819	34.72	13.563
3.	0.1	0.5	0.5	0.10122	10.660	40.70	15.460

a) Base alternative.
b) Service life of base vintage (1980).
c) Normalization factor for the investment price ensuring $q/P_E = 1$ for the base vintage.

5.2. Ex ante service lives, energy-capital ratios etc.

Table 9 contains the simulated value of the ex ante service life, N*(t), for the vintages 1967-1983, using the base specification of price expectations and ex ante elasticities of substitution of 0, 1, 2, and 3. The alternative σ=0 represents a clay-clay technology, i.e. it illustrates the effect of changes in price expectations if no possibility for substituting energy with capital had existed. This is, of course, not a realistic alternative, as is indicated by the incredibly high values of the service life in the years 1967-1973, but is included for the sake of illustration only. The corresponding values of the energy/capital ratio and the life cycle price ratio are given in tables 11 and 12. These three tables clearly demonstrate the empirical relevance of treating scrapping plans for capital equipment and choice of energy intensity as joint decisions.

Of particular interest are the years 1973/1974 and 1979/80, in which the energy price shocks occurred. With an ex ante elasticity of substitution of σ=1, the model would predict a decline in the energy/capital ratio from 0.333 in 1973 to 0.130 in 1974 along with a <u>decrease</u> in the planned service life from 47.8 years to 29.4 years. The underlying life cycle price ratio q/P_E would decline from 2.95 to 1.15, i.e. by more than 60 per cent. A larger ex ante substitutability would imply a more price sensitive energy/capital ratio. The predicted decline from 1973 to 1974 for σ = 3 is as large as from 4.19 to 0.22, which is accompanied by an <u>increase</u> in the ex ante service life from 17.2 to 25.1 years. This has the interpretation that the (positive) substitution effect of the relative price change dominates over the (negative) price level and price increase effects (cf. eq. (3.46) and table 1). The 1979/1980 energy price shock had similar, but numerically smaller effects. Table 10 shows the sensitivity of the ex ante service life to the assumed price expectation process in the years before and after the two energy price shocks.

TABLE 9. Ex ante service life in years.[a)]
Price expectation process: γ=0.2, μ=1.0.

Vintage	Ex ante elasticity of substitution			
	σ=0	σ=1	σ=2	σ=3
1967	62.06	43.94	27.99	14.43
1968	69.62	47.70	29.08	15.23
1969	76.98	51.91	30.99	16.57
1970	119.47	77.38	43.83	25.65
1971	71.60	50.32	31.85	17.10
1972	69.53	48.29	29.98	15.74
1973	66.64	47.83	31.23	17.23
1974	30.58	29.35	27.69	25.10
1975	34.29	32.31	29.83	26.35
1976	35.16	31.73	27.61	21.64
1977	31.55	28.74	25.20	19.57
1978	37.01	30.91	24.16	*
1979	43.53	38.51	33.30	27.55
1980	34.72	34.72	34.72	34.72
1981	32.34	33.92	35.82	38.04
1982	35.40	35.70	36.03	36.40
1983	36.17	37.10	38.13	39.24

a) An asterisk (*) indicates that no (positive and finite) profit maximizing service life exists.

TABLE 10. Ex ante service life (integer value) for different combinations of price expectation process and ex ante elasticity of substitution.

γ	μ	σ	1973	1974	1979	1980
0.1	1.0	0.0	66	39	42	36
		1.0	51	36	39	36
		2.0	37	33	35	36
		3.0	26	29	32	36
0.2	1.0	0.0	66	30	43	34
		1.0	47	29	38	34
		2.0	31	27	33	34
		3.0	17	25	27	34
0.5	0.5	0.0	57	26	53	40
		1.0	40	26	44	40
		2.0	24	26	36	40
		3.0	*	26	27	40

*) No solution value exists.

TABLE 11. Vintage specific energy/capital ratio (E/J)[a)]
Price expectation process: γ=0.2, μ=1.0.

Vintage	Ex ante elasticity of substitution			
	σ=0	σ=1	σ=2	σ=3
1967	0.1126	0.3469	1.1175	5.3076
1968	0.1126	0.3824	1.3384	6.3413
1969	0.1126	0.3995	1.4463	6.6173
1970	0.1126	0.4590	1.8760	8.1288
1971	0.1126	0.3543	1.1524	5.0618
1972	0.1126	0.3710	1.2703	6.1216
1973	0.1126	0.3327	1.0193	4.1933
1974	0.1126	0.1300	0.1583	0.2171
1975	0.1126	0.1387	0.1811	0.2672
1976	0.1126	0.1589	0.2446	0.4797
1977	0.1126	0.1530	0.2284	0.4527
1978	0.1126	0.1982	0.3882	*
1979	0.1126	0.1693	0.2638	0.4476
1980	0.1126	0.1126	0.1126	0.1126
1981	0.1126	0.0958	0.0792	0.0636
1982	0.1126	0.1096	0.1063	0.1027
1983	0.1126	0.1036	0.0947	0.0858

a) An asterisk (*) indicates that no (positive and finite) profit maximizing service life exists.

TABLE 12. Ex ante life cycle price ratio:
Investment price/Energy life-cycle price.[a)]
Price expectation process: γ=0.2, μ=1.0.

Vintage	Ex ante elasticity of substitution			
	σ=0	σ=1	σ=2	σ=3
1967	3.071	3.080	3.150	3.612
1968	3.392	3.396	3.447	3.833
1969	3.546	3.547	3.584	3.888
1970	4.076	4.076	4.081	4.164
1971	3.141	3.146	3.199	3.555
1972	3.288	3.294	3.358	3.788
1973	2.947	2.954	3.008	3.339
1974	1.133	1.154	1.186	1.245
1975	1.207	1.232	1.268	1.334
1976	1.373	1.411	1.474	1.590
1977	1.320	1.359	1.424	1.590
1978	1.716	1.760	1.587	*
1979	1.487	1.503	1.530	1.584
1980	1.000	1.000	1.000	1.000
1981	0.862	0.851	0.839	0.827
1982	0.974	0.973	0.971	0.970
1983	0.924	0.920	0.917	0.913

a) The ratio is normalized to unity for the 1980 vintage. (Cf. table 8.) An asterisk (*) indicates that no (positive and finite) profit maximizing service life exists.

TABLE 13. Ex ante life cycle profit rate[a)b)].
Price expectation process: γ=0.2, μ=1.0.

Vintage	Ex ante elasticity of substitution			
	σ=0	σ=1	σ=2	σ=3
1967	-0.2079	-0.1732	-0.1061	0.0258
1968	-0.2213	-0.1817	-0.1012	0.0570
1969	-0.2255	-0.1836	-0.0963	0.0729
1970	-0.1917	-0.1394	-0.0210	0.2042
1971	-0.2006	-0.1643	-0.0936	0.0425
1972	-0.2067	-0.1681	-0.0908	0.0632
1973	-0.1825	-0.1489	-0.0858	0.0338
1974	-0.0926	-0.0921	-0.0913	-0.0901
1975	-0.0516	-0.0501	-0.0480	-0.0447
1976	-0.1479	-0.1443	-0.1394	-0.1316
1977	-0.2032	-0.2009	-0.1977	-0.1936
1978	-0.2661	-0.2578	-0.2457	*
1979	-0.0392	-0.0329	-0.0244	-0.0123
1980	0.0000	0.0000	0.0000	0.0000
1981	0.0510	0.0521	0.0533	0.0546
1982	0.0226	0.0226	0.0226	0.0226
1983	0.0654	0.0658	0.0660	0.0664

a) See footnote to table 9.
b) The profit rate is defined as the life cycle profit divided by the total life cycle cost (including investment cost). By construction (cf. section 5.1), it is restricted to be zero for the 1980 vintage.

The assumption of zero profit over the ex ante life cycle for the 1980 vintage is, of course, reflected in these simulation results. A positive shift in the constant term of the ex ante production function will, in general, induce a positive shift in the ex ante profitability and service life of all vintages. The profit rate, defined as the ratio between the life cycle profit and the life cycle cost, fluctuates substantially (see table 13). In the years 1967-1973 it also depends significantly on the assumed value of the elasticities of substitution.

5.3. Revised service lives and revaluation coefficients

In this section, we present empirical results for the revised service lives $N^*(t+\Theta,t)$ and the revaluation coefficients $v(t+\Theta,t)$ and $w(t+\Theta,t)$ for the vintages t = 1970,...,1983 in the years $t+\Theta$ = 1970,...,1983, assuming an ex ante elasticity of substitution between energy and capital of σ = 2. The main diagonal of table 14 contains the ex ante service life (confer table 9, column 3); the figures below the diagonal represent the revised values of the predicted remaining service life based on the most recent price information. We note a sharp drop from 1973 to 1974 and from 1979 to 1980 in the service life of vintages installed before 1973. Consider the vintages 1970-1973. From 1973 to 1974, their predicted remaining service lives dropped from 22.8, 29.5, 28.1, and 31.2 years to 9.0, 12.3, 11.6, and 13.1 years, respectively, i.e. by more than 50 per cent. From 1974 to 1978 the service life was virtually stable, and then showed a temporary increase, by 3-4 years, from 1978 to 1979 and a decrease by 4-5 years from 1979 to 1980, when the second energy price shock occurred.

The effect of the first energy price shock can also be seen by comparing the columns for vintages 1973 and 1974 in table 14. In 1977, for instance, the 1973 vintage had a predicted remaining service life of 12.9 years, which is less than half of the value for the 1974 vintage (28.5 years). This indicates that the energy price shock had been "absorbed" in the choice of energy/capital ratio for the latter, but not for the former vintage (cf. tables 11 and 12).

Table 15 shows the induced effects on the revaluation coefficients $v(t+\Theta,t)$. Consider again vintages 1970-1973. From 1973 to 1974 the value of these four vintages, still with the elasticity of substitution set to $\sigma = 2$, dropped from 70.1, 85.9, 89.6, and 100 per cent of the ex ante value to 46.6, 68.8, 69.5, and 83.0 per cent, respectively. The value of an old capital vintage may exceed its value at the time of instalment. This situation will occur if the output and energy price forecasts are more favourable ex post than ex ante and this more than compensates for the decline in efficiency and the inoptimal factor composition ex post. The 1978 vintage is a notable example, $v(t+\Theta,1978)$ exceeds unity for all the years $t+\Theta = 1979,\dots,1983$ (cf. tables 7, 9, and 11).

The assumption of a putty-clay technology is essential to the above conclusions. A <u>clay-clay</u> technology ($\sigma = 0$) would, under constant returns to scale ($\varepsilon = 1$), be characterized by equality of the predicted value of the remaining service life for <u>all</u> vintages in a given year:

$$(5.4)\quad N^*(t+\Theta,t) = N^*(t+\Theta) = \frac{\log(p_X(t+\Theta)/p_E(t+\Theta)) + \log(a/\beta)}{\pi_E(t+\Theta) - \pi_X(t+\Theta)}$$

for all t, which follows from (3.23), (3.26), (3.31), and (4.18). The implied clay-clay revaluation coefficients are given in table 16. They show less dramatic fluctuations than those in the putty-clay case when $\sigma = 2$ (see table 15). For the vintages 1970-1973 they in fact increase from 1973 to 1974. This is easy to understand since $N^*(t+\Theta,t) = N^*(t+\Theta)$ and $E(t,0)/X(t,0) = \beta/a$, with $X(t,0) = 1$ and $\varepsilon = 1$, give

$$(5.5) \quad V(t+\Theta,t) = e^{-\delta\Theta}V(t+\Theta,t+\Theta),$$

i.e.

$$(5.6) \quad v(t+\Theta,t) = e^{-\delta\Theta}\frac{V(t+\Theta,t+\Theta)}{V(t,t)}$$

(cf. (4.19) and (4.20)). Hence, $v(t+\Theta,t) > 1$ will occur when $V(t+\Theta,t+\Theta)/V(t,t) > e^{\delta\Theta}$ in the clay-clay case.

Finally, table 17 contains the corresponding putty-clay revaluation coefficients based on the alternative definition, $w(t+\Theta,t)$. Again, we note a sharp drop from 1973 to 1974 and from 1979 to 1980. From 1973 to 1974 the value of vintages 1970-1973 dropped from 58.8, 78.9, 84.4 and 100 per cent of the value of the most efficient new vintage to 24.4, 39.4, 40.8 and 51.8 per cent, respectively, i.e. the capital would depreciate by roughly 50 per cent in one year. In 1983, the 1970 vintage had an estimated value of only 6.7 per cent of the most efficient new vintage. For the 1976 vintage - which had "absorbed" the first, but not the second energy price shock - the value had declined to 43.6 per cent, and for the 1981 vintage - which had "absorbed" both price shocks - the ex post value is 83.1 per cent of the ex ante value of capital utilizing the most efficient 1983 technique. The corresponding clay-clay revaluation coefficients would not be affected by prices at all, since (4.21) and (5.5) imply

$$(5.7) \quad w(t+\Theta,t) = e^{-\delta\Theta} \qquad \text{for all } t \text{ and } \Theta,$$

i.e. the capital value would decline exponentially at the exogenously given rate of decline in efficiency, δ, regardless of the behaviour of the input and output prices.

TABLE 14. Service life ex ante (main diagonal) and revised value after unanticipated price change.
Ex ante technology: CES, σ=2.0
Price expectation process: γ=0.2, μ=1.0.

Year \ Vintage	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
1970	43.83													
1971	24.70	31.85												
1972	24.53	31.39	29.98											
1973	22.78	29.47	28.09	31.23										
1974	8.96	12.26	11.58	13.13	27.69									
1975	10.27	13.93	13.18	14.89	31.08	29.83								
1976	10.02	13.85	13.06	14.86	31.80	30.49	27.61							
1977	8.42	11.95	11.22	12.88	28.46	27.25	24.60	25.20						
1978	9.79	13.94	13.09	15.03	33.37	31.95	28.83	29.54	24.16					
1979	12.43	17.18	16.20	18.42	39.38	37.76	34.19	35.00	28.86	33.30				
1980	8.70	12.67	11.86	13.72	31.24	29.89	26.91	27.58	22.44	26.16	34.72			
1981	7.59	11.36	10.59	12.36	29.03	27.74	24.90	25.55	20.66	24.20	32.34	35.82		
1982	8.08	12.25	11.40	13.35	31.75	30.33	27.20	27.90	22.51	26.42	35.40	39.24	36.03	
1983	7.90	12.22	11.33	13.35	32.40	30.92	27.68	28.42	22.83	26.87	36.17	40.15	36.82	38.13

TABLE 15. Capital revaluation coefficients v(t+Θ,t): Ex post value as a share of ex ante value of vintage.
Ex ante technology: CES, σ=2.0
Price expectation process: γ=0.2, μ=1.0.

Year \ Vintage	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
1970	1.000													
1971	0.823	1.000												
1972	0.794	0.956	1.000											
1973	0.701	0.859	0.896	1.000										
1974	0.466	0.688	0.695	0.830	1.000									
1975	0.536	0.773	0.784	0.928	1.069	1.000								
1976	0.457	0.674	0.682	0.813	0.949	0.888	1.000							
1977	0.365	0.569	0.570	0.692	0.861	0.803	0.900	1.000						
1978	0.366	0.558	0.562	0.675	0.788	0.738	0.833	0.924	1.000					
1979	0.517	0.738	0.751	0.883	0.958	0.899	1.021	1.130	1.241	1.000				
1980	0.386	0.627	0.625	0.768	0.988	0.922	1.031	1.145	1.215	1.005	1.000			
1981	0.336	0.579	0.572	0.716	0.978	0.911	1.014	1.128	1.182	0.987	0.994	1.000		
1982	0.329	0.568	0.561	0.701	0.932	0.870	0.971	1.080	1.141	0.947	0.944	0.947	1.000	
1983	0.305	0.544	0.535	0.674	0.913	0.851	0.950	1.057	1.114	0.926	0.925	0.927	0.980	1.000

TABLE 16. Capital revaluation coefficients $v(t+\Theta,t)$: Ex post value as a share of ex ante value of vintage.
Ex ante technology: Leontief (σ=0.0)
Price expectation process: γ=0.2, μ=1.0.

Vintage / Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
1970	1.000													
1971	0.942	1.000												
1972	0.889	0.943	1.000											
1973	0.821	0.871	0.924	1.000										
1974	0.934	0.992	1.051	1.138	1.000									
1975	0.995	1.056	1.119	1.212	1.065	1.000								
1976	0.883	0.937	0.994	1.075	0.945	0.888	1.000							
1977	0.804	0.853	0.905	0.979	0.860	0.808	0.911	1.000						
1978	0.732	0.777	0.824	0.892	0.783	0.736	0.829	0.910	1.000					
1979	0.886	0.940	0.997	1.079	0.948	0.891	1.003	1.102	1.210	1.000				
1980	0.924	0.981	1.040	1.125	0.989	0.929	1.046	1.149	1.262	1.043	1.000			
1981	0.918	0.975	1.033	1.119	0.983	0.923	1.040	1.142	1.255	1.037	0.994	1.000		
1982	0.872	0.926	0.981	1.062	0.933	0.877	0.988	1.085	1.191	0.984	0.944	0.950	1.000	
1983	0.854	0.907	0.961	1.040	0.914	0.859	0.967	1.062	1.167	0.964	0.925	0.930	0.980	1.000

TABLE 17. Capital revaluation coefficients $w(t+\Theta,t)$: Ex post value as a share of ex ante value of a new vintage utilizing optimal technique.
Ex ante technology: CES, σ=2.0
Price expectation process: γ=0.2, μ=1.0.

Vintage / Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
1970	1.000													
1971	0.752	1.000												
1972	0.708	0.932	1.000											
1973	0.588	0.789	0.844	1.000										
1974	0.244	0.394	0.408	0.518	1.000									
1975	0.242	0.381	0.396	0.498	0.920	1.000								
1976	0.219	0.354	0.368	0.465	0.871	0.947	1.000							
1977	0.174	0.297	0.305	0.393	0.784	0.850	0.893	1.000						
1978	0.190	0.316	0.327	0.417	0.781	0.849	0.899	1.004	1.000					
1979	0.182	0.284	0.297	0.371	0.645	0.704	0.749	0.836	0.844	1.000				
1980	0.108	0.192	0.196	0.256	0.527	0.571	0.599	0.671	0.655	0.797	1.000			
1981	0.083	0.156	0.158	0.210	0.460	0.497	0.519	0.582	0.561	0.689	0.875	1.000		
1982	0.079	0.150	0.152	0.201	0.429	0.465	0.487	0.546	0.530	0.647	0.814	0.928	1.000	
1983	0.067	0.131	0.132	0.177	0.384	0.416	0.436	0.488	0.473	0.579	0.729	0.831	0.895	1.000

FOOTNOTES

1 Recall that (2.8), (2.9), (3.3), and (3.4) imply

$$\frac{\partial X(t+s,s)}{\partial E(t,0)} = B(s)^{\varepsilon}\varepsilon Z(t,s)^{\varepsilon-1}\frac{\partial Z(t,s)}{\partial E(t,0)},$$

$$\frac{\partial X(t+s,s)}{\partial J(t)} = B(s)^{\varepsilon}\varepsilon Z(t,s)^{\varepsilon-1}\frac{\partial Z(t,s)}{\partial J(t,0)}$$

for $0 \leq s \leq N^*(t)$.

2 This interpretation holds strictly for the energy price, cf. (3.12) and (3.14). For the output price, the weights are by exponentiation adjusted in accordance with the overall scale elasticity, ε, and the elasticity of energy, $\beta\varepsilon$, in the Cobb-Douglas production function. This adjustment "transforms" the functions B(s) and h(s) into "output equivalents".

3 Recall that P_X, P_E, and ψ also depend on N.

4 In the Cobb Douglas case, which is the limiting case of the CES model in which $\sigma \to 1$, i.e. $\rho \to 0$, (3.30) and (3.31) are replaced by

$$\Phi(E,J) = a(E/\beta)^{\beta}(J/(1-\beta))^{1-\beta},$$

$$\psi(P_E,q) = a^{-1}P_E^{\beta}q^{1-\beta}.$$

With a Leontief technology, i.e. $\sigma \to 0$, $\rho \to \infty$, they degenerate to

$$\Phi(E,J) = a\min\left[\frac{E}{\beta},\frac{J}{1-\beta}\right],$$

$$\psi(P_E,q) = a^{-1}[\beta P_E+(1-\beta)q].$$

The general case with an arbitrary ex ante technology in capital and m variable inputs is discussed in Biørn and Frenger (1986, sections 2 and 3).

5 If, more generally, h(s) is parametrized as $h(s)=e^{\gamma s}$, where $\gamma(\gtreqless 0)$ represents the rate of change of the energy requirement per capital unit with age, then (3.39) and (3.40) are replaced by, for the Cobb-Douglas case,

$$\rho_X(t) = r(t) - \pi_X(t) + (\delta + \beta\gamma)\varepsilon,$$

$$\rho_E(t) = r(t) - \pi_E(t) + \delta - \gamma.$$

6 In the Cobb-Douglas case (σ=1), we have in particular

$$G(P_E/q,\beta,1) = (1-\beta)\log(q/P_E).$$

7 The case β=1, which also implies G( ) = 0, is of no interest since the model then degenerates to a specification with energy as the only input.

8 This base value is determined by

$$N^0 = \{\log (p_X^0/p_E^0) + \log a - (1-\varepsilon)\log Z^0 - \log \beta\}/(\varrho_X^0 - \varrho_E^0),$$

$$(1-\varepsilon)\log Z^0 = \log(\varepsilon a) + \log(p_X^0/p_E^0) + \log\{D(\varrho_X^0 N^0)/D(\varrho_E^0 N^0)\},$$

$$q^0 = p^0 N_E^0 D(\varrho^0 N_E^0).$$

9 By solving the equations in footnote 8.

10 Confer also (3.46).

11 This property distinguishes the capital revaluation in a putty-clay context from that in a neo-classical model. In the latter type of model - with capital assumed to be completely malleable - the market value of a used capital good is not related in any way to the choice of technique. Its value reflects the flow of "capital services" that it can be expected to produce - without regard to the specific production process in which it is actually used - during its remaining service life. Confer the concept net capital for a neo-classical model, as discussed in e.g. Biørn (1983).

12 We assume that the market agents' price expectations coincide with those of the investing firm.

13 It is easy to check from this equation that if the price paths are correctly anticipated, i.e. if

$$\pi_X(t) = \pi_X(t+\Theta),$$

$$\pi_E(t) = \pi_E(t+\Theta),$$

$$p_X(t+\Theta) = p_X(t)e^{\pi_X(t)\Theta},$$

$$p_E(t+\Theta) = p_E(t)e^{\pi_E(t)\Theta},$$

then $N^*(t+\Theta,t) = N^*(t,t) - \Theta = N^*(t) - \Theta$, i.e. there will be no revision of the planned service life.

14 If $\Theta=0$, the shock is assumed to occur immediately after vintage t has been installed and the technique chosen.

APPENDIX A: DATA

The data for this analysis are aggregates for total manufacturing taken from the Norwegian annual national accounts. In this appendix, we give a brief description of these data.

Basic data

The basic data specify one output and five inputs, all referring to total manufacturing:

- Q : Gross production at constant (1980) prices, mill. NKr.
- M : Input of other materials, valued at constant (1980) prices, mill. NKr.
- L : Labour input, mill. hours worked.
- E_1 : Energy input, electricity, valued at constant (1980) prices, mill. N.kr.
- E_2 : Energy input, fuel etc., valued at constant (1980) prices, mill. Nkr.
- J : Gross investment in fixed capital equipment (aggregate of machinery and transport equipment), valued at constant (1980) prices, mill. Nkr.
- p_Q : Price index gross production, 1980 = 1.
- p_M : Price index, input of other materials, 1980 = 1.
- p_L : Hourly wage rate (wages paid), Nkr.
- p_{E1} : Price index, energy input, electricity, 1980 = 1.
- p_{E2} : Price index, energy input, fuel, 1980 = 1.
- p_J : Price index, gross investment, equipment, 1980 = 1.
- r : Interest rate (pro anno) on loans from commercial banks to companies.

The data on Q, M, J, p_Q, p_M, and p_J are taken from the Norwegian annual national accounts, L and p_L are taken from labour market statistics and E_1,

E_2, p_{E1}, and p_{E2} from energy statistics published by the Central Bureau of Statistics of Norway. The labour market data and energy data are integrated with the national accounts. The series for the interest rate r is calculated and published by the bank of Norway.

Energy aggregates

From the data on the two energy commodities, we have constructed

E : Aggregate energy input, value at constant (1980) prices, mill. Nkr., and

p_E : Price index, aggregate energy input, 1980 = 1.

as CES aggregates in the following way:

$$\text{(A.1)} \quad E = [\beta_1(E_1/\beta_1)^{-\eta} + \beta_2(E_2/\beta_2)^{-\eta}]^{-1/\eta},$$

$$\text{(A.2)} \quad p_E = [\beta_1 p_{E1}^{1-\lambda} + \beta_2 p_{E2}^{1-\lambda}]^{1/(1-\lambda)}, \qquad \beta_1 + \beta_2 = 1,$$

where β_1 is the electricity share and β_2 is the fuel share in total energy cost in 1980 (β_1 = 0.4906, β_2 = 0.5094), λ is the elasticity of substitution between electricity and fuel, and $\eta = (1-\lambda)/\lambda$. Eq. (A.1) may be interpreted as the "production function" for the aggregate energy input and (A.2) as its dual unit cost function. We assume that these aggregation functions have the same CES form ex ante and ex post, which has the interpretation that electricity can be substituted with fuel to the same degree after the capital has been installed and the technology chosen as it could before. The elasticity of substitution is set to λ=0.4, which concurs with the result of an investigation of the electricity-fuel substitution in Norwegian manufacturing (Bye (1984, table 3)).

Net output price

The basic data, after aggregation of the energy inputs as described above, refer to a four factor technology with materials, labour, energy, and capital as inputs. Our model specifies energy and capital as the only inputs. Hence, the effect of the material and labour cost must be eliminated from the gross output price index p_Q.

We define the net output price index as

$$\text{(A.3)} \qquad p_X = p_Q - \alpha_M p_M - \alpha_L p_L,$$

where α_M and α_L denote the input coefficients of materials and labour, respectively, in the four-factor technology. They are calculated by fitting the values of the input coefficients observed in each year, i.e. M/Q and L/Q, to logarithmic trends estimated by ordinary least squares regression. Our reason for doing this - rather than for instance setting α_M and α_L equal to the base year input coefficients - is that we want our output price indicator to capture changes in the input structure of materials and labour. The resulting output price p_X can thus be interpreted as having been adjusted for technical change relating to material and labour input requirement.

This procedure for obtaining the net output price, p_X, can, from the point of view of a four-factor technology, be interpreted as follows: Let Q(t+s,s) denote the output (gross production) from vintage t at age s and M(t+s,s) and L(t+s,s) the corresponding inputs of materials and labour. Let the ex ante technology be separable as

$$Q(t+s,s) = G\{M(t+s,s), L(t+s,s), F[E(t+s,s), K(t+s,s), t]\},$$

with G representing a Leontief technology, i.e.

$$Q(t+s,s) = \min\left[\frac{M(t+s,s)}{\alpha_M(t)}, \frac{L(t+s,s)}{\alpha_L(t)}, F[E(t+s,s),K(t+s,s)],t\right],$$

where $\alpha_M(t)$ and $\alpha_L(t)$ are (ex ante and ex post) input coefficients of materials and labour for vintage t.

The life cycle profit from vintage t,

$$\begin{aligned}\pi(t) = \int_0^N e^{-r(t)s}\big[&p_Q^*(t+s,t)Q(t+s,s) \\ &- p_M^*(t+s,t)M(t+s,s) \\ &- p_L^*(t+s,t)L(t+s,s) \\ &- p_E^*(t+s,t)E(t+s,s)\big]ds - q(t)J(t),\end{aligned}$$

the starred prices as before denoting expected values, can then be expressed as

$$\pi(t) = \int_0^N e^{-r(t)s}\left[p_X^*(t+s,t)Q(t+s,s) - p_E^*(t+s,t)E(t+s,s)\right]ds - q(t)J(t),$$

where

$$p_X^*(t+s,t) = p_Q^*(t+s,t) - \alpha_M(t)p_M^*(t+s,t) - \alpha_L(t)p_L^*(t+s,t)$$

is the net output price at time t+s as expected at time t. For s=0, we have in particular that

$$p_X(t) = p_Q(t) - \alpha_M(t)p_M(t) - \alpha_L(t)p_L(t)$$

is the current net output price at time t.

To summarize, the following set of assumptions for a four factor technology:

(i) clay-clay technology for materials, labour, and energy-capital aggregate,

(ii) exogenously given input coefficients for materials and labour at time t equal to the trend value of the average input coefficients for all vintages at time t, and

(iii) price extrapolations for net output price based on the observed value of this net price,

is consistent with the procedure chosen for calculating the net output price, p_X, in a two factor (energy, capital) putty-clay technology.

Rate of increase of net output price

By logarithmic differentiation of (A.3), treating the input coefficients α_M and α_L as fixed, we obtain

$$\text{(A.4)} \quad \pi_X = \frac{p_Q\pi_Q - \alpha_M p_M \pi_M - \alpha_L p_L \pi_L}{p_Q - \alpha_M p_M - \alpha_L p_L}$$

$$= \pi_Q + \frac{1}{p_X}\left[(\pi_Q-\pi_M)\alpha_M p_M + (\pi_Q-\pi_L)\alpha_L p_L\right],$$

where $\pi_X = \dot{p}_X/p_X$ etc. This expression shows that the rate of increase of the net output price, π_X, may be interpreted as the rate of increase of the gross output price less an adjustment for changes in relative prices between materials, labour, and gross output. Over the period considered, this adjustment is negative in most of the years, mainly owing to increase in productivity, resulting in $\pi_M < \pi_Q$ and $\pi_L < \pi_Q$, i.e. $\pi_X < \pi_Q$. The values of π_X are calculated from (A.4) with $\pi_Q-\pi_M$ set equal to the three year moving average of the relative output/material price, i.e.

$$(\pi_Q-\pi_M)_t = \left(\frac{p_{Qt+1}}{p_{Qt-2}} / \frac{p_{Mt+1}}{p_{Mt-2}}\right)^{1/3} - 1,$$

and $\pi_Q-\pi_L$ set equal to a similar three year moving average of the relative output/labour price.

The resulting series for the output and energy prices, their pro anno rates of increase and the interest rate are given in table A.1. The difference between the rates of increase of the gross and net output price is substantial in several years, and the former exceeds the latter ($\pi_Q > \pi_X$) in all years except 1980 when π_X exceeds π_Q by a small margin. The rate of increase of the energy price exceeds that of the net output price ($\pi_E > \pi_X$) in all years except 1964 and 1970.

TABLE A.1. Basic data for output price, energy price and interest rate.

Year t	p_{Xt}	p_{Et}	r_t	π_{Xt}	π_{Et}	π_{Qt}
1964	0.07655	0.21926	0.0534	-0.00206	-0.05559	0.04221
1965	0.07690	0.21714	0.0541	-0.01590	-0.00968	0.03608
1966	0.07578	0.22786	0.0555	-0.04745	0.04938	0.02005
1967	0.07361	0.24672	0.0565	-0.05305	0.08279	0.01106
1968	0.07386	0.24080	0.0577	-0.04976	-0.02402	0.00995
1969	0.07626	0.24738	0.0654	0.00004	0.02736	0.04423
1970	0.08375	0.24828	0.0667	0.03424	0.00362	0.08807
1971	0.08492	0.28370	0.0668	0.01695	0.14270	0.05806
1972	0.09098	0.29163	0.0676	-0.03068	0.02793	0.03399
1973	0.09160	0.31124	0.0690	0.00824	0.06724	0.06978
1974	0.10751	0.46321	0.0754	0.13430	0.48826	0.19884
1975	0.12297	0.51477	0.0804	0.05428	0.11133	0.11398
1976	0.12557	0.56126	0.0849	0.01343	0.09030	0.07374
1977	0.13352	0.64510	0.0894	0.00982	0.14939	0.07813
1978	0.14113	0.68914	0.1073	0.04360	0.06827	0.05041
1979	0.17478	0.77872	0.1095	0.09326	0.12998	0.09516
1980	0.19352	1.00000	0.1130	0.11805	0.28416	0.11469
1981	0.21883	1.20497	0.1215	0.08060	0.20497	0.10727
1982	0.23731	1.33854	0.1295	0.05894	0.11085	0.06954
1983	0.25895	1.51618	0.1303	0.05723	0.13271	0.05649

APPENDIX B: PRICE ELASTICITIES

In this appendix, we derive expressions for the elasticities of the service life and the energy/capital ratio with respect to the price variables in the base year of the model (1980).

Under constant returns to scale ($\varepsilon = 1$), with the capacity of the vintage, Z, exogenously given, it follows from eqs. (3.41) - (3.44) that the optimal service life is determined by

(B.1) $$(\varrho_X - \varrho_E)N = \log(p_X/p_E) + \log a - \log \beta - \sigma G,$$

where

(B.2) $$G = \frac{1}{1-\sigma} \log[\beta + (1-\beta)(q/P_E)^{1-\sigma}],$$

(B.3) $$P_E = p_E N D(\varrho_E N),$$

with

(B.4) $$D(x) = (1-e^{-x})/x \qquad (x \neq o).$$

The other symbols are defined as in the main text. These equations implicitly define N as a function of the price variables $\varrho_X = r - \pi_X + \delta$, $\varrho_E = r - \pi_E + \delta$, p_X, and p_E, and the parameters a and β. The base vintage is characterized by

$$q/P_E = 1,$$

$$G = 0,$$

$$\frac{\partial G}{\partial \log(q/P_E)} = 1 - \beta,$$

cf. (3.47).

By differentiating (B.1), while setting $dN = N \, d\log N$, we get the following relationship for the base vintage

(B.5) $$N(\varrho_X - \varrho_E) d\log N + N(d\varrho_X - d\varrho_E)$$

$$= d\log p_X - d\log p_E - \sigma(1-\beta)(d\log q - d\log P_E),$$

where dlog denotes logarithmic differential. From (B.2) and (B.4) we obtain

$$\text{(B.6)}\quad d\log P_E = d\log p_E + d\log N + D_E\left(\frac{1}{\varrho_E}\, d\varrho_E + d\log N\right),$$

where

$$\text{(B.7)}\quad D_E = \frac{d\log D(\varrho_E N)}{d\log(\varrho_E N)} = -1 + \frac{\varrho_E N}{e^{\varrho_E N}-1}.$$

Combining (B.5) and (B.6) it follows that

$$[N(\varrho_X-\varrho_E) - \sigma(1-\beta)(1+D_E)]d\log N$$

$$= d\log p_X - [1-\sigma(1-\beta)]d\log p_E - \sigma(1-\beta)d\log q$$

$$- N\, d\varrho_X + [N + \sigma(1-\beta)\frac{D_E}{\varrho_E}]d\varrho_E.$$

Substituting $d\varrho_X = dr - d\pi_X$ and $d\varrho_E = dr - d\pi_E$, and solving for dlog N, we get

$$\text{(B.8)}\quad d\log N = \lambda\{d\log p_X - [1-\sigma(1-\beta)]d\log p_E - \sigma(1-\beta)d\log q$$

$$+ N\, d\pi_X - [N + \sigma(1-\beta)\frac{D_E}{\varrho_E}]d\pi_E + \sigma(1-\beta)\frac{D_E}{\varrho_E}\, dr\},$$

where

$$\text{(B.9)}\quad \lambda = \{N(\varrho_X - \varrho_E) - \sigma(1-\beta)(1+D_E)\}^{-1}.$$

This equation defines the elasticities of the service life with respect to all the price variables in the model.

The energy/capital ratio is determined by (cf. (3.33))

$$\text{(B.10)}\quad \frac{E}{J} = \frac{\beta}{1-\beta}\left(\frac{q}{P_E}\right)^{\sigma}.$$

Differentiating, while using (B.6), we get

$$d\log(E/J) = \sigma(d\log q - d\log P_E)$$

$$= \sigma \text{ dlog } q - \sigma \text{ dlog } p_E - \sigma \frac{D_E}{\varrho_E}(dr - d\pi_E)$$

$$- \sigma(1+D_E)\text{dlog } N.$$

Substituting for dlog N from (B.8), we find

$$\text{(B.11)} \quad \text{dlog}(E/J) = - \sigma\lambda(1+D_E)\text{dlog } p_X$$

$$- \sigma\{1 - \lambda(1+D_E)[1-\sigma(1-\beta)]\}\text{dlog } p_E$$

$$+ \sigma\{1 + \lambda(1+D_E)\sigma(1-\beta)\}\text{dlog } q$$

$$- \sigma N\lambda(1+D_E)d\pi_X$$

$$+ \sigma\{\frac{D_E}{\varrho_E} + \lambda(1+D_E)[N+\sigma(1-\beta)\frac{D_E}{\varrho_E}]\}d\pi_E$$

$$- \sigma\{\frac{D_E}{\varrho_E} + \lambda(1+D_E)\sigma(1-\beta)\frac{D_E}{\varrho_E}\}dr.$$

This equation defines the elasticities of the energy/capital ratio with respect to the price variables, when the induced effect via the service life is taken into account.

In normal cases, we will have $\lambda > 0$ and $-1 < D_E < 0$. (The latter inequality follows from (B.7) since $e^{\varrho_E N} - 1 > \varrho_E N$ when $\varrho_E > 0$.) This implies that an increase in the output price and/or in its rate of increase will always lead to an increase in the service life and (when $\sigma > 0$) induce a decrease in the energy/capital ratio. The elasticities with respect to p_E, q, and π_E, however, can be of either sign. We also note that a proportional change in p_X, p_E, and q, or a parallell change in π_X, π_E, and r will always leave N and E/J unaffected. See Biørn and Frenger (1986, sections 3 and 5.1) for a more elaborate discussion of such price effects in a more general context.

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