Brasch, T. (2017). The import price index with trade barriers: theory and evidence. *Empirical Economics*, 52(1), 123-141.
https://doi.org/10.1007/s00181-015-1064-2

# The cost-of-living index with trade barriers: Theory and evidence

**Abstract**

The standard cost-of-living index hinges on the assumption of free trade. Applying it to situations with barriers to trade yields biased results compared to a true cost-of-living index. To circumvent this problem it is common in the literature to use average prices as an aggregator function. However, average prices do not measure cost-of-living effects from trade liberalisation. In this article, I generalise the cost-of-living index to allow for barriers to trade in the form of quantity constraints and I develop an upper bound index to the true cost-of-living index. To illustrate the theoretical framework, I use the case of clothing imports to Norway and show that the Laspeyres index overestimates the true cost-of-living annual inflation rate by 1.5 percentage points between 1988 and 2005. I also show that a unit value index, which is believed to be appropriate for the aggregation of homogenous items, overestimate the inflation rate by 0.5 percentage points when goods are perfect substitutes.

*Keywords:* Cost-of-living, Index numbers, Price level, Trade barriers.

*JEL classification*: C43, E31, F14.

## 1 Introduction

The cost-of-living index is based on economic theory and the point of departure is a consumer minimising the expenditure necessary to reach a particular level of utility for a given set of prices. Given this minimum expenditure level, the cost-of-living index is defined as the ratio of the expenditures required to attain a particular indifference curve under two price regimes. Within this framework, it is assumed that the consumer is free to choose between all goods - there are no barriers to trade. If one applies the standard cost-of-living framework to situations where there are barriers to trade, it will not represent the true cost-of-living index. To illustrate this, consider the following paradox. A country imports shirts of identical quality from country $L$ and $H$. Let $p_{Lt}$ and $p_{Ht}$ denote the price level in country $L$ and $H$, respectively. It is assumed that country $L$ is a low cost country while country $H$ is a high cost country ($p_{Lt} < p_{Ht}$). Moreover, inflation in country $L$ is assumed somewhat higher than inflation in country $H$, i.e., measured in the logarithmic difference $\Delta \ln p_{Lt} > \Delta \ln p_{Ht}$. Due to trade barriers such as quantity constraints, consumers cannot import as many shirts from country $L$ as preferable. Gradually, trade barriers are reduced, and more low cost shirts are imported from country $L$. This new availability of low cost shirts reduces the average price consumers have to pay for shirts and increases their utility. The cost-of-living has been reduced. But the cost-of-living index would increase. To see this, consider the aggregate inflation rate from a Törnqvist price index. This index approximates the cost-of-living index with second order accuracy (Theil 1967, Diewert 1976). The aggregate inflation rate ($\Delta \ln p_t$) is given as a weighted average of the inflation rates in country $L$ and $H$:

$$\Delta \ln p_t = \overline{s_{Lt}}\, \Delta \ln p_{Lt} + (1 - \overline{s_{Lt}})\, \Delta \ln p_{Ht}, \qquad (1)$$

where the overscore above a variable represents the moving average operator between two time periods, $\overline{s_{Lt}} = 1/2(s_{Lt} + s_{L,t-1})$, and where $s_{Lt}$ is the value share of imports from the low cost country. The increased imports of shirts from country $L$, due to reduced trade barriers, increases the weight of the inflation rate in country $L$, and reduces the weight of the inflation rate in country $H$. Since inflation was assumed somewhat higher in country $L$ than $H$, the overall inflation rate increases. That the cost-of-living index can increase, when the true cost-of-living has decreased, is a paradox. The paradox is caused by the fact that the standard cost-of-living framework implicitly assumes free trade.

The literature analysing how a gradual lowering of trade barriers and an increased integration of low cost countries into the world economy have put downward pressure on inflation rates try to circumvent this problem by looking at a weighted sum of price levels. The geometric average price level is defined by:

$$\ln p_t = s_{Lt} \ln p_{Lt} + (1 - s_{Lt}) \ln p_{Ht}. \qquad (2)$$

Pain et al. (2006) apply this framework to identify the impact of imports from emerging markets on inflation in OECD economies; Nickell (2005) and Coille (2008) use the framework to analyse the evolution of inflation in the United Kingdom; and Benedictow and Boug (2013) use empirically a similar framework to calculate foreign price impulses to Norwegian import prices of clothing. Using an arithmetic average instead of the geometric average, Kamin et al. (2006) study the impact of Chinese exports on import prices in 26 OECD countries. Røstøen (2004) applies the arithmetic average price framework to identify external price impulses to imported consumer goods in Norway. Moreover, bureaus of statistics such as Statistics Norway use an arithmetic average price framework, with quantity shares as weights (unit values), as sub-indices for homogenous product groups to calculate import price indices, see the Export and Import Price Index Manual Manual (IMF et al. 2009, Chapter 2). The use of average prices when there is price variation for the same quality of good or service is also recommended in the SNA 2008 (European Commision et al. 2009, Paragraph 15.68). To see how the average price framework can be used to identify the impact from a gradual lowering of trade barriers on inflation, apply the quadratic approximation lemma (Diewert 1976, p. 118) to the geometric average price level (2) to get the inflation rate:

$$\Delta \ln p_t = \overline{s_{Lt}}\, \Delta \ln p_{Lt} + (1 - \overline{s_{Lt}})\, \Delta \ln p_{Ht} + \Delta s_{Lt} \left( \overline{\ln p_{Lt}} - \overline{\ln p_{Ht}} \right). \tag{3}$$

The difference between the inflation rate from the Törnqvist index (1), and the inflation rate from the geometric average price level (2), i.e., the term $\Delta s_{Lt} \left( \overline{\ln p_{Lt}} - \overline{\ln p_{Ht}} \right)$, is interpreted as the bias from applying the cost-of-living index to situations where trade barriers are present. If the value share of imports from the low cost country increases due to lowering of trade barriers, the bias is negative and the increased integration of low cost countries into the world economy is interpreted to have put downward pressure on inflation.

The main problem with this approach is that average prices, geometric average prices or unit values are not measures of cost-of-living when trade barriers are present. The inflation rate (3) is consistent with a cost-of-living index from a time varying Cobb Douglas utility function $u_t = x_{Lt}^{\alpha_t} x_{Ht}^{1-\alpha_t}$, where $x_{Lt}$ and $x_{Ht}$ are the goods from the low cost and high cost country, respectively, and $\alpha_t$ is a time varying preference parameter equal to $s_{Lt}$ in equilibrium. However, within this model, an increase in the import share is not caused by lowering of trade barriers. Rather it is caused by a preference change towards the low cost country. This use of geometric average prices is therefore not suitable if the purpose is to analyse how a gradual lowering of trade barriers has affected inflation. Unit values can only be meaningfully linked to economic theory when all commodities are homogenous (perfect substitutes) and when there is no price dispersion, i.e., $p_{Lt} = p_{Ht}$, see e.g., Balk (1998) and Bradley (2005). In contrast, cost-of-living effects from a gradual lowering of trade barriers will only exist if there is price dispersion. It is therefore difficult to interpret results from studies using average prices. A different approach is needed to identify a bias that can be interpreted as showing the cost-of-living effects from trade liberalisation.

The literature on index numbers has rarely focused on the cost-of-living bias arising from trade barriers in the form of quantity constraints. For example, The Boskin Commission highlighted four sources of bias in the Consumer Price Index: the new good bias, the outlet bias, the quality bias and the substitution bias (Boskin et al. 1996). Several articles have reviewed the results of the Boskin Commission, and others have provided ways to deal with these biases, see e.g., Diewert (1998) and Hausman (2003). Many studies have also linked a bias in the import price index to these biases. For example, Feenstra (1994) accounts for the new good bias in a constant-elasticity of substitution aggregate of import prices, and Feenstra and Shiells (1996) provide an international analogue to the outlet bias in the import price index, where foreign suppliers take the role of low cost outlets. The difference between these studies and the paradox mentioned above is that they assume tangency between the indifference curve and the budget line at some point. In the example above, however, the indifference curve and the budget line are not tangent since trade barriers hinder consumers from importing the number of shirts they would like to. Taking account of cost-of-living effects from situations where the standard first-order conditions do not hold requires a modification of the original cost-of-living framework.

The purpose of this paper is to generalise the original cost-of-living framework to allow for barriers to trade. In Section 2, I introduce the cost-of-living framework, and then generalise it to allow for barriers to trade by building upon the theory of rationed households, see e.g., Rothbarth (1941), Tobin (1952) and Howard (1977). In Section 3, I construct an index that serves as an upper bound to the true index with CES preferences. This index is based on goods being perfect substitutes, but it excludes the cases when perfect substitute preferences will no longer serve as an upper bound to CES preferences in general. The upper bound index has an intuitive interpretation and it is easy to calculate. In Section 4, I apply the upper bound index in an empirical example using data on imports of clothing to Norway between 1988 and 2012. During most of this time period, the price of clothing from China was between 40% - 80% lower than the price of clothing from other countries. Due to a gradual removal of trade barriers, the expenditure share of clothing from China increased from about 3% to about 50% over the sample period. Using a Laspeyres or a Paasche index yields a price level in 2005 that is about the same as the price level in 1988. In contrast, the upper-bound to the cost-of-living index shows that the price of clothing was at least 30% lower than the level indicated by the Laspeyres-Paasche band. This corresponds to an average annual bias between of 1.5 percentage points. The empirical example illustrates that even a conservative estimate of the trade barrier bias yields results of first order importance. Moreover, the upper bound index is compared with the geometric average price index (2), the corresponding average price index and the unit value index. In contrast to a valid measure of cost-of-living, these average price indices yield a higher inflation rate than the inflation rate from the upper bound of the cost-of-living index. Since the true cost-of-living index is below the upper bound index, the yearly underestimation of how trade liberalisation has impacted inflation from using e.g., unit values is at least 0.5 percentage points. Section 5 concludes.

## 2 The cost-of-living index

First, I outline the standard cost-of-living index based on a utility maximising consumer and then generalise the framework to allow for changes in the index due to lowering of trade barriers. Note that this framework can also be applied to describe e.g., the economic import price index for an establishment, see the Export and Import Price Index Manual Manual (IMF et al. 2009, Section 18.F1).

### 2.1 The standard index

Consider a utility maximising consumer. Let $x_t' = (x_{1t}, x_{2t}, \ldots, x_{nt})$ denote a vector of quantities at time $t$ and let $p_t' = (p_{1t}, p_{2t}, \ldots, p_{nt})$ be the corresponding price vector where $'$ indicates the transpose operator. Further, let $u_t = f(x_t)$ denote the consumer's utility function as a function of quantities and let $c(p_t, u_t)$ be the expenditure function. The expenditure function $c(p_t, u_t)$ represents the minimal amount of expenditure necessary to achieve the utility level $u_t$ at prices $p_t$:

$$c(p_t, u_t) \equiv \min_{x_t} \{p_t' x_t : u_t = f(x_t)\} . \tag{4}$$

A cost-of-living index is the ratio of the expenditures required to attain a particular level of utility under two price regimes. In particular, the standard Konüs (1939) cost-of-living index ($I_t^K$) is defined as

$$I_t^K \equiv c(p_t, u_{t-1})/c(p_{t-1}, u_{t-1}). \tag{5}$$

Together, (4) and (5) constitute the cost-of-living framework. The index shows the change in the minimal cost necessary to sustain a given level of utility when prices change between period $t-1$ and $t$. From this definition, it is obvious that if prices remain unchanged between the two time periods, the cost-of-living index is unity. If the consumer behaved optimally in period $t-1$, and if prices remain constant, the consumer will not change behaviour between time periods for a given utility level. The numerator and denominator are equal. Any change in the cost-of-living index is therefore caused by a change in prices.

The cost-of-living framework hinges on the assumption that the consumer is free to choose between all bundles of goods. There are no restrictions on the availability of goods in the definition of the expenditure function (4). As a consequence, the index (5) yields a biased estimate of cost-of-living when there are barriers to trade.

### 2.2 The index with trade barriers

In the previous section, it was shown that any change in the cost-of-living index must be caused by a change in prices. However, increased imports from low cost countries is not a phenomenon caused by changing relative prices or changing income. It is caused by increased availability of low cost goods and services. This increase in availability allows consumers to enjoy a plethora of new products which increase their utility even when income and prices remain unchanged. A cost-of-living index that takes the effects of trade liberalisation into account should therefore decrease when the amount of available goods increases.

To be more precise, a cost-of-living index should show the ratio of the expenditures required to attain a particular indifference curve under two price regimes and between two different time periods:[1]

$$I_t \equiv c_t(p_t, u_{t-1})/c_{t-1}(p_{t-1}, u_{t-1}). \tag{6}$$

Observe that the difference between this definition and the cost-of-living index (5) is the time subscript on the expenditure functions. Even when prices are unchanged, and utility is kept constant, this index can change due to exogenous factors such as lowering of trade barriers. Moreover, note that allowing the cost-of-living index to change, when prices are unchanged, violates one of the axiomatic requirements for price indices: the identity property explicitly states that if prices are constant over the two periods being compared, then the price index should equal one (Balk 2012, p. 58). The purpose of this article is to identify the welfare gains from trade liberalisation; gains that occur irrespective of price changes. To allow for such welfare gains in a cost-of-living index, the identity property must be violated.

I proceed by defining an economy with restrictions on trade. Let the index $j \in \mathcal{J}$ run across goods where such restrictions apply. The consumption of any good $j$ cannot exceed a predefined level $\bar{x}_{jt}$: $x_{jt} \leq \bar{x}_{jt}$. The nature of the process $\bar{x}_{jt}$ is exogenous. It represents the restriction that hinders the consumer from choosing freely between goods. Such restrictions can be due to direct quota restrictions or it can be due to the sluggish response of supply from the gradual removal of trade barriers. Incorporating these trade barrier restrictions yields a new definition of the expenditure function:

$$c_t(p_t, u_t) \equiv \min_{x_t} \left\{ p_t' x_t : u_t = f(x_t),\ x_{jt} \leq \bar{x}_{jt},\ j \in \mathcal{J} \right\}. \tag{7}$$

It shows the minimal expenditure necessary to reach a particular level of utility, given prices, a utility function and possible restrictions on availability. Together, (6) and (7) constitute the generalisation of the cost-of-living index framework (4) and (5). If there are no trade barriers, i.e., $\bar{x}_{jt} = \infty,\ j \in \mathcal{J}$, this expenditure function is equivalent to the expenditure function in the previous section. With respect to the topic of this paper, the cost-of-living framework (4) and (5) can thus be interpreted as a situation of free trade between countries.

[1]This definition is similar to the one adopted by Feenstra (1994). It is also equivalent to equation (4) in Balk (1989), who studied time-varying preferences. If preferences are time-varying, (6) implies a cardinal interpretation of utility. In the context of this paper, however, the utility function is assumed constant across time periods, and thus represents an ordinal entity.

The new good bias is also encompassed by this framework. Assume that a new good, say $x_{jt}$, is introduced in period $t = s$. Before period $s$, there cannot be consumption of good $j$, i.e., $\bar{x}_{jt} = 0$ when $t < s$. When the new good is introduced to the market, there is no restriction on the consumption of the good, i.e., $\bar{x}_{jt} = \infty$ when $t \geq s$. This highlights the difference between the bias resulting from the introduction of new goods and the bias resulting from gradual removal of trade barriers. The former is a one time change in availability, while the latter is a gradual change in availability.

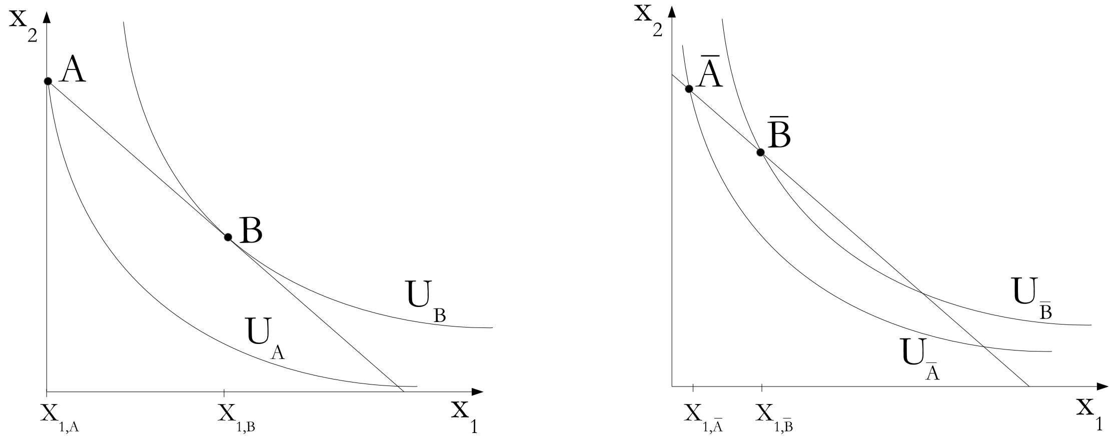
This difference between the new good bias and the trade barrier bias is crucial in terms of identification. Consider for example how Hausman (1999) identified the new goods bias of cellular telephones. After these telephones were introduced to the market, Hausman (1999) estimated the demand curve and then solved for the expenditure function using Roy's identity. He identified the bias of cellular phones by solving for the price which causes the demand for cellular phones to be zero. Note that only if consumers are free to choose between all products will their pattern of consumption reveal their underlying preferences. This approach therefore depends on consumers being free to choose between the new good and other goods, i.e., $\bar{x}_{jt} = \infty$, after the new good has been introduced, $t \geq s$. The impact from trade barriers is different in this respect since the state of free trade ($\bar{x}_{jt} = \infty,\ j \in \mathcal{J}$) has not yet been reached. We have moved gradually from a state with trade restrictions to a state with less trade restrictions. In terms of identification this is a problem. If observed consumption patterns are the result of increased availability, and not the result of income changes and relative price changes, consumption patterns will not reveal the form of the utility function.

This is illustrated graphically in Figures 1a and 1b. Point $A$ in Figure 1a shows the situation before the new good ($x_1$) has been introduced. This is tantamount to a situation of autarky in the international trade literature. If $x_1$ cannot be imported, it is only $x_2$ that is consumed. The indifference curve labeled $U_A$ corresponds to the level of utility reached at point $A$. When the economy opens up to trade, and there are no restrictions on the imports of $x_1$, the optimal consumption level will be at point $B$. Opening up for trade increases the utility of the consumer, as shown by the outward movement of the indifference curve to $U_B$. Feenstra (1994) shows how to incorporate this movement from autarky to free trade into a cost-of-living index when using a CES utility function. Feenstra and Weinstein (2010) show how to do it from a Translog expenditure function. The new good bias thus represents two extremes: the time before the new good is introduced can be viewed as a situation of infinitely high trade barriers, and the time after the good is introduced can be viewed as there being no trade barriers. The main concern of this paper is the situation between these two extremes, i.e., the case when there are some trade barriers that are gradually removed, see Figure 1b. Point $\bar{A}$ shows the consumption level when some trade restrictions are present. Point $\bar{B}$ shows the consumption level when fewer trade restrictions are present. The movement from $\bar{A}$ to $\bar{B}$ increases the utility of the consumer, i.e., the indifference curve moves outwards from $U_{\bar{A}}$ to $U_{\bar{B}}$. However, the indifference curves are not in any of the states tangent to the budget line. Since the trade barrier restriction holds, the standard means of identifying compensating variation based on observed prices and quantities cannot be applied.

Figure 1 also illustrates another important difference between the new good bias and the trade barrier bias. The new goods bias refers to the welfare increase when the new product is included in the cost-of-living index: In period $A$, the good $x_1$ is not included in the index and in period $B$, it is included in the index. In other words, the introduction of a new good into the index signalises a potential bias. In contrast, there is no such signal of a bias arising from a gradual removal of trade barriers. The good $x_1$ is included in the index in both time periods. It is only outside knowledge about the existence of trade barriers that can signal a potential bias. For example, it is a historical fact that the Multi-Fibre Arrangement imposed quota restrictions on imports of textiles from China. This fact is utilised in the empirical analysis in Section 4 to evaluate the size of the bias in the case of textile imports to Norway.

Calculating the welfare effects of barriers to trade in the form of quantity constraints dates at least back to the literature on rationed households which began during the Second World War when the essentials of life were rationed in many countries (Rothbarth 1941, Tobin 1952, Howard 1977). Ahlheim (1998) provides an excellent textbook introduction. As with the identification of the new good bias, welfare measurement with quantity constraints in general, based on either compensating variation or a distance function, hinges on the availability of data from a period without quantity constraints. For example, the method developed by Breslaw and Smith (1995) requires knowledge of the unconstrained Marshallian demand function. Only if data from a period without quantity constraints are available can the unconstrained Marshallian demand function be estimated and the welfare effects of quantity constraints be identified. In many cases, the state of no quantity constraints has not been reached, and the standard means of identifying welfare effects can not be applied. This paper takes a different route to identity the cost-of-living effects from trade barriers. The purpose is not to provide an unbiased estimate of the true cost-of-living index, but it is to

**Figure 1** – Cost-of-living effects from increased trade




**(a)** From autarky to free trade

**(b)** Gradual lowering of trade barriers

provide an upper bound to the true cost-of-living index. It is to this I now turn.

## 3 An upper bound to the true cost-of-living index

The purpose of this section is to develop an index that will serve as an upper bound to the true reductions in cost-of-living from a gradual lowering of trade restrictions. To this end, I consider a constant elasticity of substitution (CES) utility function over $n$ goods. The index $j \in \mathcal{J} = \{1, 2, \dots, n-1\}$ runs across the $n-1$ goods with trade restrictions. Since CES utility is weakly separable, I let the $n$th good, $x_n$, represent an aggregate good of all the goods that are traded freely. The expenditure function (7) in the CES economy with barriers to trade can then be written:

$$
\begin{aligned}
c_t(p_t, u_t) &= \min_{x_t} \left\{ p_t' x_t : u_t = (\sum_i^n \delta_i x_{it}^\rho)^{1/\rho},\ x_{jt} = \bar{x}_{jt} < x_{jt}^* \ \text{ for } j \in \mathcal{J} \right\} \\
&= \sum_{j \in \mathcal{J}} p_{jt} x_{jt} + \frac{p_{nt}}{\delta_n} \left( u_t^\rho - \sum_{j \in \mathcal{J}} \delta_j x_{jt}^\rho \right)^{1/\rho} . \qquad (8)
\end{aligned}
$$

The parameter $\delta_i$ in the CES utility function can be thought of as a quality parameter for good $i$ and the mapping between the parameter $\rho$ and the elasticity of substitution $\sigma$ is given by $\sigma = 1/(1-\rho)$. $x_{jt}^*$ denotes the optimal consumption of good $j$ when there are no barriers to trade, i.e., the cost minimising consumption of good $j$ in (4). Since the first $n-1$ goods are characterised by binding trade restrictions, the second equality follows from substituting the utility function for the $n$th good in the budget constraint. Utilising that $y_{t-1} = c_{t-1}(p_{t-1}, u_{t-1})$, we can write the cost-of-living index (6) in the CES economy as a function of observed prices, quantities and income:

$$I_t^{CES} = \left( \sum_{j\in\mathcal{J}} p_{jt}x_{jt} + p_{nt} \left( x_{n,t-1}^{\rho} - \sum_{j\in\mathcal{J}} (\delta_j/\delta_n)(x_{jt}^{\rho} - x_{j,t-1}^{\rho}) \right)^{1/\rho} \right) /y_{t-1}, \tag{9}$$

where the numerator is equation (8), evaluated at $u_{t-1}$.[2] To clarify concepts further, I follow how Diewert (1998, p. 51) defined the outlet substitution bias and define the cost-of-living bias due to trade barriers ($B_t^{CES}$) as the difference between the true index $I_t^{CES}$ and the Laspeyres index $I_t^L$:[3]

$$B_t^{CES} \equiv I_t^{CES} - I_t^L. \tag{10}$$

The case of perfect substitutes ($\rho = 1$) is of particular interest, for three reasons: the bias has an intuitive interpretation, it is easy to calculate, and the case of perfect substitutes will normally represent an upper bound to the true index ($I_t^{CES}$). It follows from (9) that the index when goods are perfect substitutes ($I_t^{PS}$) is given by:[4]

$$I_t^{PS} = I_t^L + \sum_{j\in\mathcal{J}} B_{jt}^{PS}, \tag{11}$$

where the good specific bias ($B_{jt}^{PS}$) is given by[5]

$$B_{jt}^{PS} = \frac{(p_{jt} - (\delta_j/\delta_n)p_{nt})}{y_{t-1}} \Delta x_{jt}. \tag{12}$$

The numerator represents the quality adjusted price difference between the low cost and the high cost good. The whole fraction represents the quality adjusted amount saved per unit of the low cost good with respect to the expenditure level in the previous period. In total, the bias when goods are perfect substitutes is defined as the (quality adjusted) amount saved from the new availability of low

[2] That is, $u_{t-1}^{\rho} = \sum_i^n \delta_i x_{i,t-1}^{\rho}$ is inserted for $u_t^{\rho}$ in (8). It is assumed that changes in $\bar{x}_{jt}$ are such that $x_{n,t-1}^{\rho} - \sum_{j\in\mathcal{J}}(\delta_j/\delta_n)(x_{jt}^{\rho} - x_{j,t-1}^{\rho}) > 0$.

[3] The Laspeyres index is defined as $I_t^L \equiv (\sum_i p_{it}x_{i,t-1})/(\sum_i p_{i,t-1}x_{i,t-1}) = \sum_i s_{i,t-1}(p_{it}/p_{i,t-1})$ for $i \in \mathcal{J} \cup \{n\}$.

[4] See Section 6.1 in the Appendix.

[5] The formula (12) cannot be used directly to calculate the bias when comparing aggregates and not price levels of specific goods. For a given spatial index, $(p_{jt}/p_{nt})$, and temporal indices, $(p_{jt}/p_{j,t-1})$ and $(x_{jt}/x_{j,t-1})$, the bias can be written in a more usable form:

$$B_{jt}^{PS} = ((p_{jt}/p_{j,t-1}) - (\delta_j/\delta_n)(p_{jt}/p_{nt})(p_{jt}/p_{j,t-1}))\,(x_{jt}/x_{j,t-1} - 1)\,s_{j,t-1},$$

where $s_{j,t-1}$ is the cost share of good $j$ in period $t-1$.

cost goods relative to the previous periods expenditure level. For example, consider the case when $n = 2$ and assume that prices do not change between two consecutive time periods. The Laspeyres index is then unity. If the total budget is $y_{t-1} = 500$, the quality adjusted price difference between the goods is $(p_{1t} - (\delta_1/\delta_2)p_{2t}) = -10$, and five more low cost goods are purchased ($\Delta x_{1t} = 5$), the bias is $B_{1t}^{PS} = -0.1$. The true index is in this case $I_t^{PS} = 0.9$.

The purpose of the following is to establish conditions when the index $I_t^{PS}$ and the bias $\sum_{j \in \mathcal{J}} B_{jt}^{PS}$ represents a conservative approach to identifying the true index $I_t^{CES}$ and the bias $B_t^{CES}$. The index $I_t^{PS}$ is said to represent an *upper bound* to the true index $I_t^{CES}$ if $I_t^{PS} - I_t^{CES} > 0$. To provide some intuition, it is appropriate to first consider the case when $n = 2$:

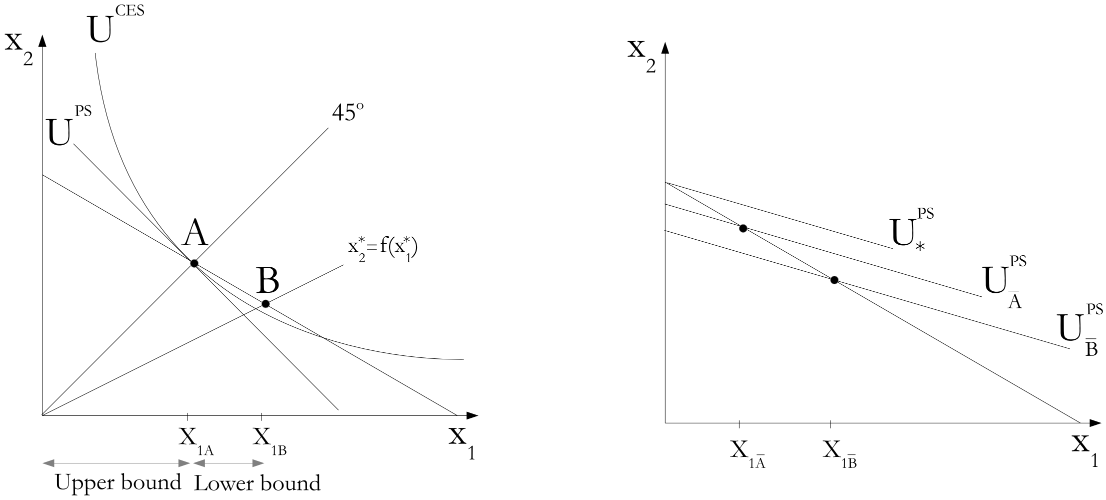
**Proposition 1** *(Upper bound, $n = 2$) Consider the cost-of-living index (9) when $n = 2$. Let the lowering of trade barriers be small, i.e., $x_{1,t} = \epsilon_1 x_{1,t-1}$ where $\epsilon_1$ is greater than, but close to unity, and let $MRS_{1t}^{CES}$ denote the marginal rate of substitution of the CES utility function: $MRS_{1t}^{CES} \equiv \frac{\delta_1}{\delta_2} \frac{x_{1t}^{\rho-1}}{x_{2t}^{\rho-1}}$. The index $I_t^{PS}$ represents an upper bound to the true index $I_t^{CES}$ if, and only if*

$$MRS_{1,t-1}^{CES} > MRS_{1,t-1}^{PS}. \tag{13}$$

*Proof: See the appendix, Section 6.2.*

It follows from Proposition 1 that the index $I_t^{PS}$ represents an upper bound to the true index $I_t^{CES}$ only if $x_{1t-1} < x_{2t-1}$. The intuition underlying this result is illustrated in Figure 2a. In this static presentation, it is assumed that prices and income are unchanged between the two periods. $U^{CES}$ shows the indifference curve for a CES utility function and the line $U^{PS}$ represents the indifference curve when goods are perfect substitutes. Both indifference curves intersect the budget line at point $A$. When availability is restricted beyond this point, i.e., $x_1 < x_{1A}$, the marginal rate of substitution for $U^{CES}$ is higher than for $U^{PS}$: $MRS^{CES} > MRS^{PS}$. A consumer with preferences $U^{CES}$ is willing to give up more units of $x_2$ in exchange for a unit of $x_1$, compared with a consumer with preferences $U^{PS}$. As a result, a lowering of trade barriers leads to a larger increase in utility, and a lower cost-of-living, when preferences are $U^{CES}$, compared with that of perfect substitutes $U^{PS}$. The index $I^{PS}$, which is based on $U^{PS}$, will thus represent an upper bound to the true index $I^{CES}$ based on preferences $U^{CES}$. When the available amount of $x_1$ exceeds $x_{1A}$,

**Figure 2** – Trade barrier bias - perfect substitutes as an upper bound: $I_t^{PS} - I_t^{CES} > 0$.




**(a)** $p_1/p_2 < \delta_1/\delta_2$

**(b)** $p_1/p_2 > \delta_1/\delta_2$

the situation changes. The marginal rate of substitution for $U^{CES}$ is then lower than the marginal rate of substitution for $U^{PS}$. In this case the index $I^{PS}$ represents a lower bound to the true index $I^{CES}$. The line going from the origin through point $B$ is the expansion path connecting the optimal consumption bundles as the budget increases. For the CES utility function, the expansion path is given by: $x_2^* = f(x_1^*) = \left(\frac{\delta_2 p_1}{\delta_1 p_2}\right)^\sigma x_1^*$. It will be to the right of the 45° degree line if $p_1/p_2 < \delta_1/\delta_2$.

Figure 2b illustrates the case when the expansion path is to the left of the 45° degree line, i.e., when $x_1$ is the high priced good, taking quality into account: $p_1/p_2 > \delta_1/\delta_2$. The indifference curve $U_*^{PS}$ shows that it is optimal to only consume $x_2$. However, if the lowering of trade barriers leads to a movement from $x_{1\bar{A}}$ to $x_{1\bar{B}}$ for the true underlying preference function, this will be interpreted as a decrease in the level of utility and an increase in cost-of-living: the trade barrier bias $B_1^{PS}$ (12) is positive. The index $I^{PS}$ when goods are perfect substitutes is still an upper bound, but for the wrong reasons. Creating an index that serves as an upper bound to the true index when trade barriers are reduced should exclude the case illustrated in Figure 2b.

Some adjustments must be made to the index (11) to make it an upper bound to the true index in the $n$ good case. As illustrated in Figure 2a, the index $I^{PS}$ represents a lower bound if $x_1 > x_{1A}$. To exclude this case, an intuitive approach is to set the good specific bias $B_j^{PS}$ to zero for all goods that lie between $x_{1A}$ and $x_{1B}$:

**Proposition 2** *(Upper bound, n) Consider the cost-of-living index (9) when* $p_{j,t-1}/p_{n,t-1} < \delta_j/\delta_n$

*for $j \in \mathcal{J}$. Let the lowering of trade barriers be small, i.e., $x_{j,t} = \epsilon_j \, x_{j,t-1}$ where $\epsilon_j$ is greater than, but close to unity. Further, separate the $n-1$ goods that are characterised by trade barriers by dividing the set $\mathcal{J} = \{1, 2, \ldots, n-1\}$ into two complement sets $\mathcal{A}_t = \{j \in \mathcal{J} : 0 \leq x_{j,t-1} \leq x_{n,t-1}\}$ and $\mathcal{A}_t^c = \left\{j \in \mathcal{J} : x_{n,t-1} < x_{j,t-1} < x^*_{j,t-1}\right\}$. The cost-of-living index*

$$I_t^L + \sum_{j \in \mathcal{A}_t} B_{jt}^{PS} \tag{14}$$

*is an upper bound to the price index $I_t^{CES}$ if $MRS_{j,t-1}^{CES} > p_{jt}/p_{nt}$ for $j \in \mathcal{A}_t^c$, where $MRS_{jt}^{CES}$ denote the marginal rate of substitution of the CES utility function: $MRS_{jt}^{CES} \equiv \frac{\delta_j}{\delta_n} \frac{x_{j,t}^{\rho-1}}{x_{n,t}^{\rho-1}}$. Proof: See the appendix, Section 6.3.*

The condition in Proposition 2 is not restrictive and it is far from necessary. Since $x_{j,t-1} < x^*_{j,t-1}$, the marginal rate of substitution is greater than or equal to the price ratio at time $t-1$: $MRS_{j,t-1}^{CES} > p_{j,t-1}/p_{n,t-1}$. The condition will thus hold if $MRS_{j,t-1}^{CES} > MRS_{jt}^{CES}$, which is equivalent to an increase in the relative consumption of the restricted good: $\Delta \left(x_{jt}/x_{nt}\right) > 0$. Alternatively, it will hold if the relative price decreases or remains unchanged between the two time periods: $\Delta \left(p_{jt}/p_{nt}\right) \leq 0$.[6]

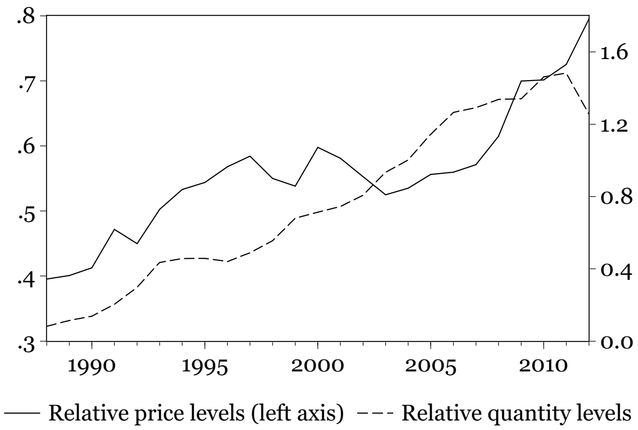
## 4 Empirical application

The purpose of this section is to illustrate the importance of the trade barrier bias when calculating price indices. To this end I use the case of clothing imports from China to Norway. The data used in this analysis are based on the two digit SITC from the external trade statistics published by Statistics Norway.[7] Let $x_{1t}$ represent the amount of imported clothing from China (measured in tonnes), and $x_{2t}$ represent the amount of imported clothing from all other countries and let $p_{1t}$ and $p_{2t}$ be the corresponding unit values. Because these measures of quantity are not adjusted for differences in quality or other characteristics, unit values are considered less reliable than price surveys, see e.g., Silver (2010). For example, the unit values of audiovisual equipment would typically be unreliable since it has decreased in weight at the same time as technological advances has been considerable. For clothing however, where technological advance has been less pronounced, it is assumed that

[6] $\Delta \left(p_{jt}/p_{nt}\right) \leq 0$ imply that $p_{j,t-1}/p_{j,t-1} \geq p_{jt}/p_{nt}$.

[7] Data are taken from the external trade statistics, Table 08809, see https://www.ssb.no/en/utenriksokonomi. Only countries with a positive level of imports across the sample are included. In the end, the data set holds 51 countries. On average, these countries account for 96% of the value of clothing imports to Norway.

**Figure 3** – Imports of clothing: $(p_{1t}/p_{2t})$ and $(x_{1t}/x_{2t})$

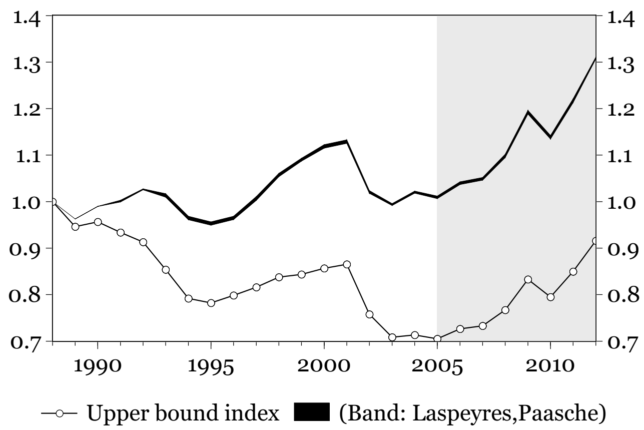



unit values are indicative of movement in trade prices. Figure 3 shows how the relative price level $(p_{1t}/p_{2t})$ and quantity level $(x_{1t}/x_{2t})$ have developed between 1988 and 2012.[8] In 1988, the price level on imported clothing from China was about 40% compared with the price of clothing from other countries. Over the time period, the relative price level has about doubled to 80% in 2012. The relative level of imported goods from China has also increased during this time period, from a level of about 8% in 1988 to 120% in 2012. This massive increase in imports from China, together with the price surge, begs the question: why has imports from China risen so much when imports from China have become so much more expensive?

Within the standard cost-of-living index framework, Figure 3 is consistent with clothing produced in China being a Giffen good, i.e., a good that people paradoxically consume more of as the price rices. Another, and more plausible explanation, is that this surge in imports is due to a gradual removal of trade restrictions. After six years of bilateral trade negotiations, Norway rejoined the Multi-Fibre Arrangement (MFA) in 1984. The MFA governed world trade in textiles and garments from 1974 through 2004 by imposing quotas on the amount developing countries could export to developed countries. These quota restrictions came in addition to already high tariff rates, ranging from 17% to 25%. Both quota restrictions and tariffs were gradually reduced during the 1990s and the quota arrangement on clothing expired in 1998 (Wilhelmsen and Høegh-Omdal 2002). This historical fact tells us that in the first ten years of the sample period, there were indeed restrictions

[8] The spatial index is calculated as $p_{1t}/p_{2t} = \sum_{i \in \mathcal{C}} w_{it} \left(p_{1t}/p_{it}\right)$, where $i$ run across all other countries than China, $\mathcal{C}$, and the weights are the import shares: $w_{it} = p_{it}x_{it}/(\sum_{i \in \mathcal{C}} p_{it}x_{it})$. The index $(x_{1t}/x_{2t})$ is calculated residually, from the product rule: $(y_{1t}/y_{2t}) = (p_{1t}/p_{2t})(x_{1t}/x_{2t})$, where $y_{1t} = p_{1t}x_{1t}$ and $y_{2t} = p_{2t}x_{2t}$.

**Figure 4** – Cost-of-living and the trade barrier bias




The upper bound index follows from Proposition 2 and equals $I_t^L + B_{1t}^{PS}$ before 2005 and since $x_{1t-1} > x_{2t-1}$ for $t \geq 2005$, it equals $I_t^L$ after 2005, where the good specific bias is defined by $B_{1t}^{PS} = \frac{(p_{1t}-(\delta_1/\delta_2)p_{2t})}{y_{t-1}}\Delta x_{1t}$ and $\delta_1/\delta_2 = 1$ is set to unity.

on availability.

Further, and maybe more importantly, the general lowering of trade barriers has led to an increase in supply of clothing from China. At the 8th round of multilateral trade negotiations, known as the Uruguay round, the Agreement on Textiles and Clothing (ATC) ended the MFA and began the process of integrating textile and clothing products into GATT/WTO rules. China entered the WTO on December 11 2001 and on January 1, 2005, the ATC, and all restrictions thereunder, were terminated. This led to a surge in Chinese exports and lower prices of textile and clothing. Harrigan and Barrows (2009) show that the prices of quota constrained categories in the U.S. fell by 38 % in 2005. Moreover, as shown by Brambilla et al. (2010), China's share of U.S. imports jumped threefold, from 10 to 33 %, between the time it joined the WTO and the end of the ATC regime. Consistent with a terms-of-trade effect, most of this growth was in existing varieties (the intensive margin). In line with these findings, Figure 3 shows that the relative demand for Chinese clothing continued to increase also in Norway after China joined the WTO.

These historical restrictions on trade in the textile industry, together with the massive increase in imports from China, support the hypothesis that consumption of clothing from China has been less than under a free trade regime.

Figure 4 shows the development in the cost-of-living index with trade barriers compared with the Laspeyres-Paasche band. It is well known that the true cost-of-living index lies within the Laspeyres-

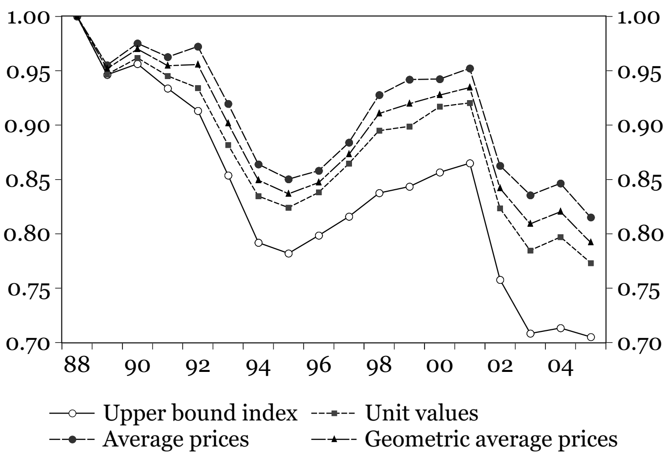
Paasche band when preferences are homothetic and when there are no barriers to trade, see e.g., the Export and Import Price Index Manual (IMF et al. 2009, p. 421). The difference between the true cost-of-living index and either the Laspeyres or Paasche index represents a substitution bias, i.e., the bias from not taking account of how consumers switch away from goods that have become relatively more expensive and toward goods that have become relatively less expensive. By comparing the trade barrier bias with the Laspeyres-Paasche band yields a visual picture of its importance with respect to the substitution bias.

If the increase in imports from China is caused by substitution and income effects, and there have been no restrictions on availability during this time period, the standard cost-of-living framework (4)-(5) is valid, and the true index lies somewhere within the Laspeyres-Paasche band. The Laspeyres-Paasche band shows that the standard cost-of-living index was about at the same level in 2005 as in 1988, and it was about 30% higher in 2012 than in 1988.

On the other hand, if the increase in imports is a result of increased availability due to lowering of trade barriers and an increase in supply, the cost-of-living index with trade barriers (Upper bound index) in Proposition 2 should be applied. From the conditions of Proposition 2, the goods specific bias ($B_{jt}^{PS}$) is only subtracted from the Laspeyres index if $x_{1t-1} < x_{2t-1}$. This does not mean that the trade barrier bias is not present when $x_{1t-1} > x_{2t-1}$. From Figure 2a and Proposition 2 it follows that the trade barrier bias is also present when $x_{1A} < x_1 < x_{1B}$, but the index $I_t^{PS}$ no longer constitutes an upper bound. In Figure 3 it can be seen that the case of perfect substitutes is an upper bound until $t = 2005$. The shaded area in Figure 4 marks the part of the sample when this condition do not hold. From 2005 to 2012 the upper bound index is the Laspeyres index. It is therefore in the period prior to 2005 that the discrepancy between the Laspeyres-Paasche band and the band of the upper bound index occurs. In 2005, the upper bound index (14) is 70% of the Laspeyres index. This amount to a mean annual inflation rate bias between of 1.5 percentage points between 1988 and 2005.

In Figure 5 the upper bound index (14) is compared with average prices. The average price with quantity shares as weights, commonly referred to as unit values, is used by many statistical bureaus to compare homogenous commodities across different countries of origin in the creation of import price indices, see Chapter 2 in the Export and Import Price Index Manual Manual (IMF et al. 2009). The rationale is that unit values are thought to be appropriate when goods are homogenous: "unit values indices are suitable - indeed they are ideal - for the aggregation of price changes of

**Figure 5** – The upper bound index vs. average prices




The upper bound index is equation (14) given in Proposition 2, using $\delta_1/\delta_2 = 1$. Unit values, average prices and geometric average prices are chained from $p_t/p_{t-1}$, where the price levels are defined by $p_t = \left(\frac{x_{1t}}{x_{1t}+x_{2t}}\right) p_{1t} + \left(\frac{x_{2t}}{x_{1t}+x_{2t}}\right) p_{2t}$, $p_t = s_{1t}p_{1t} + (1 - s_{1t})p_{2t}$ and $\ln p_t = s_{1t} \ln p_{1t} + (1 - s_{1t}) \ln p_{2t}$, c.f. equation (2), respectively.

homogenous items" (IMF et al. 2009, Section B1, 1.10). The use of unit values when there is price variation for the same quality of good or service is also recommended in the SNA 2008 (European Commision et al. 2009, Paragraph 15.68). Average prices and geometric average prices, both using value shares as weights, have been used in the literature to analyse the impact on inflation from a gradual lowering of trade barriers, see e.g., Nickell (2005), Kamin et al. (2006), Pain et al. (2006), Benedictow and Boug (2013). What is striking about this comparison is that the average prices all lie above the upper bound index. Since the true cost-of-living index, for any value of the elasticity of substitution $\sigma$, lies below the upper bound index, an alternative measure of the impact of trade liberalisation on cost-of-living should, at a minimum, also lie below the upper bound index. That the average prices lie above the upper bound index illustrates how average prices is not a measure of cost-of-living effects from trade liberalisation. The mean inflation rate between 1988 and 2005 of the average price index was -1.1%, the mean inflation rate of the geometric average price was -1.3% and the mean inflation rate of the unit value index was -1.4%. In contrast, the mean inflation rate of the upper bound index was -1.9%. In other words, the annual underestimation of how trade liberalisation has affected inflation from using average prices, geometric average prices and unit values was at least 0.8, 0.6 and 0.5 percentage points respectively. When trade barriers are present, the use of unit values to aggregate homogenous items can thus yield biased results.

## 5 Conclusions

In this article I have studied how to measure cost-of-living effects from a gradual change in buying patterns from high to low cost countries due to trade liberalisation. Applying a standard cost-of-living index to situations with trade barriers yields biased results since the standard index implicitly assumes free trade. In practice, import price indices can be particularly vulnerable to this bias since many of the goods included in these indices are characterised by either explicit or implicit trade barriers.

The literature analysing how a gradual lowering of trade barriers and an increased integration of low cost countries into the world economy have put downward pressure on inflation rates try to circumvent this problem by looking at average prices. Moreover, many bureaus of statistics use average prices with quantity shares as weights (unit values) at a low level of aggregation in the construction of the aggregate index. As is shown in this article, average prices are not measures of cost-of-living when trade barriers are present. It is therefore difficult to interpret results from the average price framework.

The main contribution of this article is the construction of a cost-of-living index that can be applied also when there are barriers to trade. In particular, the index constructed represents an upper bound to the true cost-of-living index when preferences are of CES form. The framework used is general, and it encompasses the new good bias, i.e., the welfare gain that consumers experience when a new product appears. To illustrate the theoretical framework, I used the case of clothing imports to Norway and showed that the Laspeyres index overestimates the true cost-of-living annual inflation rate by 1.5 percentage points between 1988 and 2005. I also showed that average prices, in the form of a unit value index, overestimate the inflation rate by 0.5 percentage points. This is particularly interesting since unit values is thought to be appropriate for the aggregation of price changes of homogenous items. But as this article has shown, when trade barriers are present, the use of unit values to aggregate homogenous items can yield biased results.

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## 6 Appendix

### 6.1 The cost-of-living bias due to trade barriers when goods are perfect substitutes

When goods are perfect substitutes, ($\rho = 1$), the index (9) can be written:

$$\begin{aligned}
I_t^{PS} &= \left( \sum_{j=1}^{n-1} p_{jt} x_{jt} + p_{nt} x_{n,t-1} - p_{nt} \sum_{j=1}^{n-1} (\delta_j/\delta_n)(x_{jt} - x_{j,t-1}) \right) / y_{t-1} \\
&= \left( \sum_{j=1}^{n-1} p_{jt} x_{jt} + p_{nt} x_{n,t-1} - p_{nt} \sum_{j=1}^{n-1} (\delta_j/\delta_n) \Delta x_{jt} + \sum_{j=1}^{n-1} p_{jt} x_{j,t-1} - \sum_{j=1}^{n-1} p_{jt} x_{j,t-1} \right) / y_{t-1} \\
&= \left( \sum_{j=1}^{n-1} p_{jt} \Delta x_{jt} - p_{nt} \sum_{j=1}^{n-1} (\delta_j/\delta_n) \Delta x_{jt} + \sum_{j=1}^{n} p_{jt} x_{j,t-1} \right) / y_{t-1} \\
&= \left( \sum_{j=1}^{n} p_{jt} x_{j,t-1} \right) / y_{t-1} + \left( \sum_{j=1}^{n-1} (p_{jt} - (\delta_j/\delta_n) p_{nt}) \Delta x_{jt} \right) / y_{t-1} \\
&= I_t^L + \sum_{j=1}^{n-1} B_{jt}^{PS}.
\end{aligned}$$

### 6.2 Proof: Proposition 1

The cost-of-living index when goods are perfect substitutes, defined in (11), represents an upper bound to the true index ($I_t^{CES}$) if $I_t^{PS} - I_t^{CES} > 0$. It follows from (9) that

$$I_t^{PS} - I_t^{CES} = \frac{p_{2t}}{y_{t-1}} \left[ (\delta_1/\delta_2) x_{1,t-1} + x_{2,t-1} - (\delta_1/\delta_2) x_{1t} - \left( (\delta_1/\delta_2) x_{1,t-1}^{\rho} + x_{2,t-1}^{\rho} - (\delta_1/\delta_2) x_{1t}^{\rho} \right)^{1/\rho} \right].$$

This expression is positive only if

$$(\delta_1/\delta_2) x_{1,t-1} + x_{2,t-1} - (\delta_1/\delta_2) x_{1t} > \left( (\delta_1/\delta_2) x_{1,t-1}^{\rho} + x_{2,t-1}^{\rho} - (\delta_1/\delta_2) x_{1t}^{\rho} \right)^{1/\rho}. \qquad (15)$$

Without loss of generality, I define the following relationships: $c_1 \equiv x_{1,t-1}/x_{2,t-1}$ and $d_1 \equiv x_{1,t}/x_{2,t-1}$. Inserting these relationships into (15), and taking the natural logarithm, yields

$$\ln [1 + (\delta_1/\delta_2)(c_1 - d_1)] > (1/\rho) \ln [1 + (\delta_1/\delta_2)(c_1^{\rho} - d_1^{\rho})].$$

When the increase in availability of $x_{1t}$ is small, i.e., $\epsilon_1 = d_1/c_1$ is close to unity, it follows, from a first-order Taylor approximation, that[9]

$$(c_1 - d_1) > (1/\rho)(c_1^\rho - d_1^\rho). \tag{16}$$

Inserting $c_1 = x_{1,t-1}/x_{2,t-1}$ and $d_1 = x_{1,t}/x_{2,t-1}$ yields

$$\left(\frac{x_{1,t-1}}{x_{2,t-1}} - \frac{x_{1t}}{x_{2,t-1}}\right) > (1/\rho)\left(\frac{x_{1,t-1}^\rho}{x_{2,t-1}^\rho} - \frac{x_{1t}^\rho}{x_{2,t-1}^\rho}\right).$$

Inserting $x_{1t} = \epsilon_1 x_{1,t-1}$, and rearranging, yields

$$1 < \frac{x_{1,t-1}^{\rho-1}}{x_{2,t-1}^{\rho-1}} \frac{1-\epsilon_1^\rho}{(1-\epsilon_1)}(1/\rho),$$

where $\frac{x_{1,t-1}^{\rho-1}}{x_{2,t-1}^{\rho-1}}$ is the relative marginal rates of substitution: $MRS_{1,t-1}^{CES}/MRS_{1,t-1}^{PS}$ and $\frac{1-\epsilon_1^\rho}{(1-\epsilon_1)}(1/\rho)$ goes towards unity when $\epsilon_1$ goes towards unity by L'Hôpital's rule. The opposite relationship, $MRS_{1,t-1}^{CES} < MRS_{1,t-1}^{PS}$, implies that $I_t^{PS} - I_t^{CES} < 0$, by the same arguments.

### 6.3 Proof: Proposition 2

Define the auxiliary variables $\epsilon_i \equiv x_{it}/x_{i,t-1}$, $c_i \equiv x_{i,t-1}/x_{n,t-1}$ and $d_i \equiv x_{it}/x_{n,t-1}$. The sets $\mathcal{J}$, $\mathcal{A}$ and $\mathcal{A}^c$ are given by $\mathcal{J} = \{1, 2, \ldots, n-1\}$ , $\mathcal{A}_t = \{j \in \mathcal{J} : 0 \leq x_{j,t-1} \leq x_{n,t-1}\}$ and $\mathcal{A}_t^c = \left\{j \in \mathcal{J} : x_{n,t-1} < x_{j,t-1} < x_{j,t-1}^*\right\}$. The bias (10) can then be written

$$B_t^{CES} = \frac{x_{n,t-1}}{y_{t-1}}\left(\sum_{i\in\mathcal{J}} p_{it}(d_i - c_i) - p_{nt} + p_{nt}\left(1 - \sum_{i\in\mathcal{J}}(\delta_i/\delta_n)(d_i^\rho - c_i^\rho)\right)^{1/\rho}\right)$$

The sum of individual biases when goods are perfect substitutes, for goods $i \in \mathcal{A}$, is given by

$$\sum_{i\in\mathcal{A}_t} B_{it}^{PS} = \frac{x_{n,t-1}}{y_{t-1}}\left(\sum_{i\in\mathcal{A}_t}(d_i - c_i)\left(p_{it} - (\delta_i/\delta_n)p_{nt}\right)\right)$$

[9] $\ln(1+z) \approx z$ around $z = 0$.

The index $I_t^{PS}$ is said to represent an *upper bound* to the true index $I_t^{CES}$ if $I_t^{PS} - I_t^{CES} > 0$. Since

$$I_t^{PS} - I_t^{CES} = \sum_{i \in \mathcal{A}} B_{it}^{PS} - B_t^{CES}$$
$$= \frac{x_{n,t-1}}{y_{t-1}} \left( - \sum_{i \in \mathcal{A}_t} (d_i - c_i)(\delta_i/\delta_n) p_{nt} - \sum_{i \in \mathcal{A}_t^c} (d_i - c_i) p_{it} + p_{nt} - p_{nt} \left( 1 - \sum_{i \in \mathcal{J}} (\delta_i/\delta_n)(d_i^\rho - c_i^\rho) \right)^{1/\rho} \right),$$

the index $I_t^{PS}$ is an upper bound if:

$$1 - \sum_{i \in \mathcal{A}_t} (d_i - c_i)(\delta_i/\delta_n) - \sum_{i \in \mathcal{A}_t^c} (d_i - c_i) \frac{p_{it}}{p_{nt}} > \left( 1 - \sum_{i \in \mathcal{J}} (\delta_i/\delta_n)(d_i^\rho - c_i^\rho) \right)^{1/\rho}.$$

When the changes in trade barriers are small, i.e., $d_i/c_i$ is close to unity for all $i$, it follows, from a first-order Taylor approximation, that[10]

$$\sum_{i \in \mathcal{A}_t} (c_i - d_i)(\delta_i/\delta_n) + \sum_{i \in \mathcal{A}_t^c} (c_i - d_i) \frac{p_{it}}{p_{nt}} > (1/\rho) \sum_{i \in \mathcal{J}} (\delta_i/\delta_n)(c_i^\rho - d_i^\rho).$$

This is positive if the following conditions both hold

$$\begin{aligned} &i) \quad (c_i - d_i) > (1/\rho)(c_i^\rho - d_i^\rho) \text{ for } i \in \mathcal{A}_t, \\ &ii) \quad (c_i - d_i) \frac{p_{it}}{p_{nt}} > (1/\rho)(c_i^\rho - d_i^\rho) \text{ for } i \in \mathcal{A}_t^c. \end{aligned}$$

*i)* follows from Proposition 1 and equation (16). Since $c_i^\rho - d_i^\rho = \frac{x_{i,t-1}^\rho}{x_{n,t-1}^\rho}(1 - \epsilon_i^\rho)$, it follows that *ii)* can be written as

$$MRS_{i,t-1}^{CES} > p_{it}/p_{nt} \text{ for } i \in \mathcal{A}_t^c,$$

since $\frac{1-\epsilon_i^\rho}{(1-\epsilon_i)}(1/\rho)$ goes towards unity when $\epsilon_i$ goes towards unity.

[10] $\ln(1 + z) \approx z$ around $z = 0$.