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A GENERALIZED SINGLE EQUATION ERROR CORRECTION MODEL AND ITS APPLICATION TO QUARTERLY DATA

BY

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ABSTRACT

In this paper, we specify a class of single equation 'error correction' models on the basis of a general autoregressive-distributed lag regression equation with one regressor and a white noise disturbance. This relationship is interpreted in terms of long run trends in the regressor and regressand and short run deviations from these trends. A parametrization which is useful for quarterly seasonally unadjusted data is proposed. The model is estimated by means of a non-linear least squares algorithm. Empirical results based on Norwegian quarterly national accounts data - illustrating the relationship between (i) household consumption and income, (ii) production and demand in manufacturing, and (iii) capital accumulation and production in manufacturing - are presented. Some experiences from forecasting exercises are also reported.

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1. INTRODUCTION

With the increasing complexity of macro econometric models in the last decade - and the growing ambitions of the users of such models - problems concerning the dynamic specification of econometric equations have come strongly into focus. For recent surveys of problems and methods in this field, see Hendry and Richard (1983) and Hendry, Pagan, and Sargan (1984). A wide class of dynamic formulations of linear regression equations is defined by the unrestricted autoregressive-distributed lag model. The 'error correction' formulation, which basically relies on the idea of a 'long run' relationship between the variables and a mechanism correcting for 'short run' departures from this long run relationship, is an interesting and intuitively appealing way of parametrizing a general dynamic equation. There are at least two reasons for this. First, the model is parsimonious in terms of the number of free parameters, and these parameters can usually be given a direct economic interpretation. Second, long run relationships derived from economic theory can be far more easily implemented in this framework than in a general autoregressive-distributed lag formulation.

This paper is organised as follows: First, we specify a class of single equation error correction models, taking a general autoregressive-distributed lag regression equation with one regressor and a white noise disturbance as our point of departure. This dynamic relationship is contrasted with other dynamic specifications and interpreted in terms of long run trends in the regressor and regressand and short run deviations from these trends. Proportionality between the variables is assumed in the long run (section 2). A parametrization which is useful for handling quarterly seasonally unadjusted data is then proposed and discussed (section 3) and an estimation algorithm based on non-linear least squares is presented (section 4). Next, we report empirical results - including tests of model specification - based on Norwegian quarterly national accounts data, for the relationship between (i) household consumption and income, (ii) production and demand in manufacturing, and (iii) capital accumulation and production in manufacturing (section 5). Finally, we present some experiences from forecasting exercises, both simulations for the sample period and genuine post sample forecasts (section 6).

2. THE SINGLE EQUATION ERROR CORRECTION MODEL: GENERAL BACKGROUND

2.1. A general autoregressive-distributed lag (AD) equation

Consider the following dynamic relationship

$$(2.1) \qquad Y_t = b(L)Y_t + a_0X_t + a(L)X_t + \varepsilon_t,$$

where X_t and Y_t are the values of the exogenous (input) and endogenous (output) variables at time t, respectively, and a(L) and b(L) are lag polynomials of degree P and Q (without constant terms), i.e.

$$a(L) = \sum_{i=1}^{P} a_i L^i ,$$

$$b(L) = \sum_{i=1}^{Q} b_i L^i ,$$

L being the lag operator. The disturbance ε_t is assumed to be white noise. Eq. (2.1) is thus, in the terminology of Hendry, Pagan, and Sargan (1984, p. 1040), an autoregressive-distributed lag equation of order Q and P, or <u>AD(Q,P)</u>, for short. No restrictions are imposed on the lag polynomials so far, except that $b(1) = \Sigma b_i$ lies between zero and one.

Let $\varrho(L)$ and $\{\lambda_0, \lambda(L)\}$ be the normalized lag polynomials of Y and X, defined as

$$(2.2) \qquad \varrho(L) = \frac{b(L)}{b(1)} ,$$

$$(2.3) \qquad \lambda_0 = \frac{a_0}{a_0+a(1)} , \qquad \lambda(L) = \frac{a(L)}{a_0+a(1)} ,$$

which ensure that $\varrho(1) = \sum_{i=1}^{Q} \varrho_i = 1$ and $\lambda_0 + \lambda(1) = \sum_{i=0}^{P} \lambda_i = 1.$ Furthermore, let

$$(2.4) \qquad \gamma = 1 - b(1) ,$$

$$(2.5) \qquad \beta = \frac{a_0+a(1)}{1-b(1)} .$$

Since $0 \leq b(1) < 1$, we have that $0 < \gamma \leq 1$, and that $\beta \gtreqless 0$ according as $a_0+a(1) \gtreqless 0$. With this reparametrization, (2.1) takes the form

$$(2.6) \quad Y_t = (1-\gamma)\varrho(L)Y_t + \gamma\beta\{\lambda_0+\lambda(L)\}X_t + \varepsilon_t .$$

We see that β can be interpreted as the <u>long-run effect</u> on Y of changes in X, or the total multiplier, since $X_t = X_{t-1} = \ldots\ldots = X$ implies $Y = \beta X$ (when disregarding the disturbance term). The model thus implies <u>proportionality</u> between Y and X in the long run. The <u>short-run effect</u>, or the impact multiplier, is given by

$$(2.7) \quad \alpha = \gamma\beta\lambda_0 .$$

The number of free parameters in this unrestricted dynamic model is P+Q+1 (of which P λ's, Q-1 ϱ's, γ, and β). Let us consider two ways of restricting the lag structure in order to reduce the size of the parameter vector. The first is a common factor formulation, the second is a generalized error correction model.

<u>2.2. A model with common factors in the lag polynomials</u>

Eq. (2.6) can be rewritten as

$$(2.6a) \quad [1-(1-\gamma)\varrho(L)]Y_t = \gamma\beta[\lambda_0+\lambda(L)]X_t + \varepsilon_t .$$

Assume now that the lag polynomials of Y_t and X_t have a common factor $\phi(L)$ of degree R ($R \leq P$, $R \leq Q$), i.e.

$$(2.8) \quad \begin{aligned} \lambda_0 + \lambda(L) &= \frac{a_0+a(L)}{\gamma\beta} = \phi(L)\mu(L) , \\ 1 - (1-\gamma)\varrho(L) &= 1-b(L) = \gamma\phi(L)\eta(L) , \end{aligned}$$

where $\mu(L)$ and $\eta(L)$ are lag polynomials of degrees P-R and Q-R, respectively, satisfying $\mu(1) = \eta(1) = \phi(1) = 1$.[1] Combining (2.6a) and (2.8), we find - provided that all roots of $\phi(z)=0$ lie outside the unit circle so that $\phi(L)$ is invertible - that the common factor specification is equivalent to

$$(2.9) \quad \eta(L)Y_t = \beta\mu(L)X_t + v_t ,$$

where v_t is an autoregressive (AR) process of order R, or AR(R) for short, given by

$$\phi(L)v_t \quad = \quad \varepsilon_t \; .$$

The parameter restrictions implied by the common factor specification (2.8) thus transform the original AD(Q,P) model (2.6) with a white noise disturbance term into an AD(Q-R,P-R) model with an AR(R) disturbance term. If, in particular, the coefficients of the two lag polynomials are proportional, i.e. $\eta(L)=1$, which implies $\phi(L) = \{1-(1-\gamma)\,\varrho(L)\}/\gamma$ and R=Q, the transformed equation becomes a simple distributed lag model of order P-Q , i.e. AD(0,P-Q), with an AR(Q) disturbance process. In the latter case, the parameter γ and the polynomial $\varrho(L)$ are 'transferred' from the 'structural part' of the AD equation to its disturbance process.

2.3. A generalized error correction model

Error correction models (ECM) can be motivated as a way of formalizing economic agents' optimizing behaviour in the presence of costs of adjustment or incomplete information; see e.g. Granger and Weiss (1983) and Nickell (1985). In this paper, we take a slightly different approach, by focusing on the formal restrictions implied by this parametrization of the adjustment process in relation to a general AD(Q,P) model. This will facilitate comparisons with the common factor specification.

In the common factor specification, we restrict the lag polynomials of Y and X - inclusive of their constant terms, (i.e. inclusive of the terms representing current values) - to have a common factor, $\phi(L)$ (cf. (2.8)). In the case where the lag polynomial of Y is a factor in the polynomial of X, i.e. $\eta(L) = 1$, the model is equivalent to a simple distributed lag model with an AR disturbance term and if the coefficients of the two polynomials are proportional, i.e. $\mu(L)=\eta(L)=1$ (cf. 2.9)), the model is equivalent to a static model with an AR disturbance term.

Assume now that proportionality between the two lag polynomials holds exclusive of their constant terms, i.e. proportionality between a(L) and b(L). The resulting model can be interpreted as representing an error correction mechanism.

To show this, we write (2.6) as follows:

$$\text{(2.6b)} \quad \{1-\varrho(L)\}Y_t \quad = \quad \gamma[\beta\{\lambda_0+\lambda(L)\}X_t - \varrho(L)Y_t] + \varepsilon_t \; .$$

Assume that <u>P=Q</u> and that the coefficients of $\lambda(L)$ are <u>proportional</u> to those of $\varrho(L)$, i.e. $\lambda(L) = k\varrho(L)$, where k is a positive constant. This is equivalent to $b(L) = k(1-\gamma)/(\beta\gamma)a(L)$. Since, by construction, $\lambda(1) = 1-\lambda_0$ and $\varrho(1) = 1$, it follows that the factor of proportionality must be $k = 1-\lambda_0$, so that the restriction becomes

$$(2.10) \quad \lambda(L) \quad = \quad (1-\lambda_0)\varrho(L) \ .$$

Inserting (2.10) into (2.6b), while using (2.7), we find

$$(2.11) \quad \{1-\varrho(L)\}Y_t \quad = \quad \alpha\{1-\varrho(L)\}X_t + \gamma\varrho(L)(\beta X_t - Y_t) + \varepsilon_t \ .$$

A generalization of (2.10) would be to assume that $\lambda(L)$ and $\varrho(L)$ (or equivalently, $a(L)$ and $b(L)$) have a common factor of lower order than either of these polynomials. The resulting model would be a combined error correction-common factor specification. This generalization will not, however, be discussed further here.

Since the coefficients of $\varrho(L)$ add to unity, i.e. $\varrho(1)=1$, we can interpret

$$(2.12) \quad Y_t^* \quad = \quad \varrho(L)Y_t$$

and

$$(2.13) \quad X_t^* \quad = \quad \varrho(L)X_t$$

as filtered values, or <u>trend values</u>, of Y_t and X_t, and

$$(2.14) \quad \Delta^* Y_t \quad = \quad Y_t - Y_t^* \ ,$$

$$(2.15) \quad \Delta^* X_t \quad = \quad X_t - X_t^* \ ,$$

as the departures of the current Y and X values from their trends. Eq. (2.11) can then be written simply as

$$(2.16) \quad \Delta^* Y_t \quad = \quad \alpha \, \Delta^* X_t + \gamma(\beta X_t^* - Y_t^*) + \varepsilon_t \ ,$$

which can be given the following interpretation: In the short run, i.e. for given trend values, a departure of X from its trend by one unit affects Y's departure from its trend by α units. However, in the long run, the model imposes a restriction on the ratio between the two trend values, reflected by the fact that it has $Y = \beta X$ as its long-run solution. If the trend value of Y is "too high", i.e. if $Y_t^* - \beta X_t^* > 0$, the difference exerts a negative influence on $\Delta^* Y_t$ (since $\gamma > 0$), and if the trend value of Y is "too low", the adjustment is positive. The adjustment coefficient is γ; the larger its value, the faster is Y adjusted towards its new equilibrium. The trend departure of Y, $\Delta^* Y_t$, is thus the net result of two effects, the <u>trend departure</u> of X, $\alpha\Delta^* X_t$, and the <u>trend error correction</u>, $\gamma(\beta X_t^* - Y_t^*)$.

By making use of the moment generating function of the lag distribution connecting X and Y, which is

$$(2.17)\quad B(z) \;=\; \frac{\alpha/\beta-(\alpha/\beta-\gamma)\varrho(z)}{1-(1-\gamma)\varrho(z)} \;,$$

it is easy to show that the <u>mean lag</u> is equal to[2)]

$$(2.18)\quad m \;=\; B'(1) = \frac{\gamma(1-\alpha/\beta)\varrho'(1)}{\{1-(1-\gamma)\varrho(1)\}^2} = \frac{\beta-\alpha}{\beta\gamma}\varrho'(1) = \frac{1-\gamma\lambda_0}{\gamma}\varrho'(1) \;.$$

This shows that the response of Y to changes in X is slower (i) the smaller is the short-run coefficient in relation to the long-run coefficient, α/β, i.e. the smaller is λ_0, (ii) the smaller is the adjustment coefficient γ, and (iii) the larger is $\varrho'(1)$.

Eq. (2.11) is a generalization of the error correction model discussed in e.g. Sargan (1964), Hendry and Richard (1983, pp. 130-131), and Hendry, Pagan, and Sargan (1984, pp. 1048-1049), which can be obtained by setting $\varrho(L) = L$ in (2.11)-(2.16)[3)], giving

$$(2.19)\quad \Delta Y_t \;=\; \alpha\Delta X_t + \gamma(\beta X_{t-1}-Y_{t-1}) + \varepsilon_t \;,$$

where $\Delta Y_t = Y_t - Y_{t-1}$ etc. In the following we shall refer to the latter model as the <u>simple error correction model</u> (SEC) and let (2.11) be denoted as the <u>generalized error correction model</u> (GEC). Note, however, that SEC is only a simple reparametrization of an AD(1,1) model since (2.19) is equivalent to

$$Y_t = (1-\gamma)Y_{t-1} + \alpha X_t + (\gamma\beta-\alpha)X_{t-1} + \varepsilon_t$$

$$= (1-\gamma)Y_{t-1} + \gamma\beta\lambda_0 X_t + \gamma\beta(1-\lambda_0)X_{t-1} + \varepsilon_t \, .$$

This equation is equivalent to (2.6) for $P=Q=1$ when we take account of the normalization constraint $\lambda_0+\lambda_1 = 1$, which means that SEC does not effectively restrict the parameter vector as compared with a general AD(1,1) model. Only if $P = Q > 1$ will GEC restrict the dynamic specification as compared with an AD(Q,Q) model, the number of free parameters being Q+2 for the former and 2Q+1 for the latter. For this reason, we find GEC models more interesting than SEC models for econometric purposes.

3. ERROR CORRECTION MODELS FOR QUARTERLY DATA

3.1. A parametrization of $\varrho(L)$ which allows for seasonal effects

If seasonality is present in X_t and Y_t, it seems sensible to take this fact into account when specifying the parametric form of $\varrho(L)$. Above, we interpreted this lag polynomial as the filter by means of which the trend values of X and Y are constructed in the error correction process. With this in mind, since our data are seasonally unadjusted quarterly data, we parametrize $\varrho(L)$ as

$$(3.1) \qquad \varrho(L) = \varrho_1 L + \varrho_4 L^4 - \varrho_1\varrho_4 L^5 \, ,$$

so that the 'detrending operator' becomes

$$1-\varrho(L) = (1-\varrho_1 L)(1-\varrho_4 L^4) \, .$$

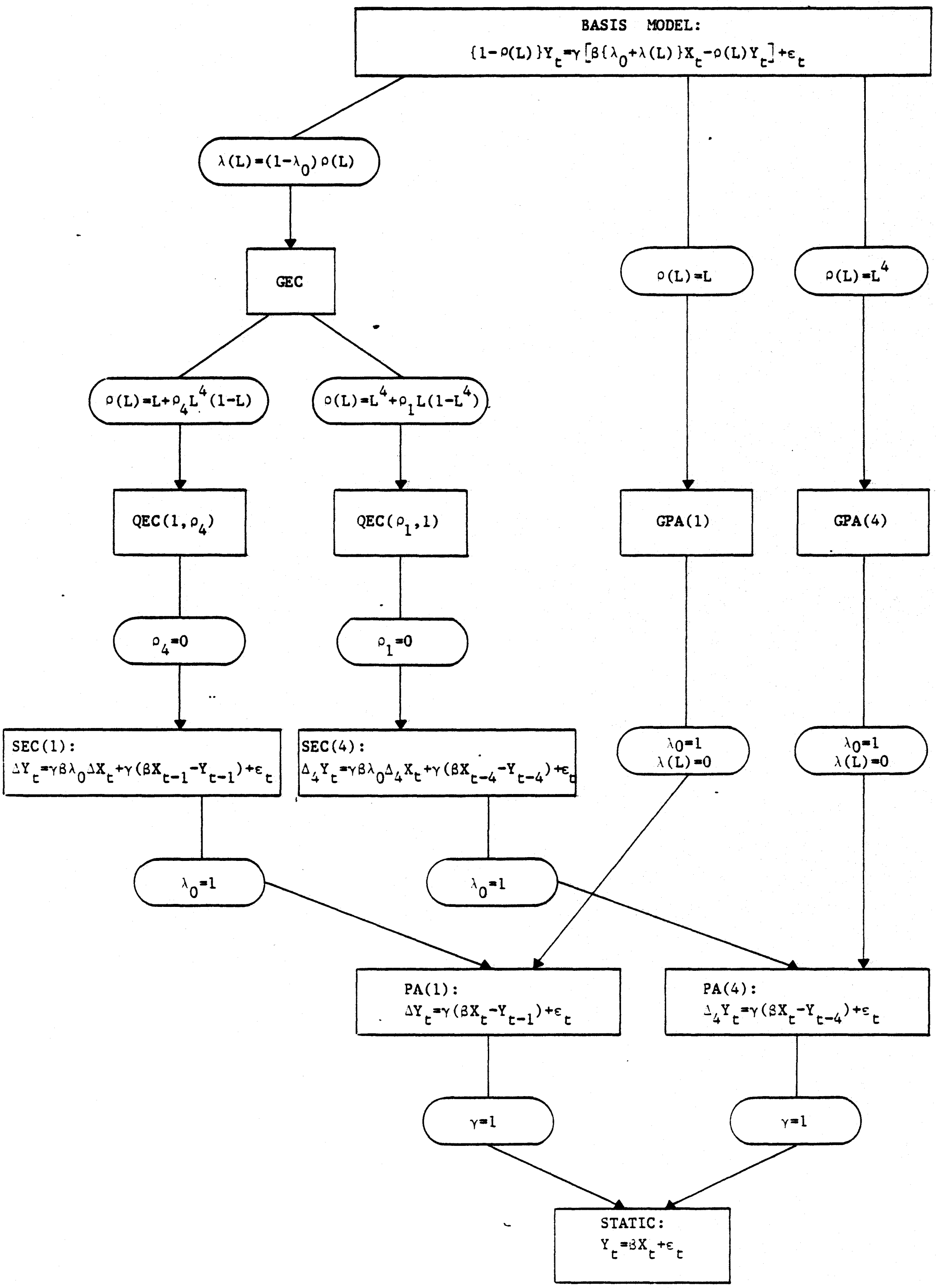
It contains both the one quarter difference operator, $\Delta = 1-L$, the four quarter difference operator, $\Delta_4 = 1-L^4$, and the combined one and four quarter difference operator, $\Delta\Delta_4$, as special cases.

In order to satisfy the normalization constraint $\varrho(1) = 1$, either ϱ_1 or ϱ_4 must be set to unity, since $\varrho(1)=1$ implies $(1-\varrho_1)(1-\varrho_4) = 0$. The interpretation of this is that the 'detrending' operation must imply a full differencing either across one or across four quarters. This gives two alternative models, which we shall denote as

$\mathrm{QEC}(1,\varrho_4)$: $\varrho_1 = 1$, ϱ_4 free,

$\mathrm{QEC}(\varrho_1,1)$: ϱ_1 free, $\varrho_4 = 1$, using QEC as an abbreviation of '<u>quarterly error correction</u>'. The explicit dynamic adjustment equation and the associated mean lag between X and Y for

FIGURE 1. TYPOLOGY OF MODELS



these two models are, respectively[4)]

QEC(1,ϱ_4)

(3.2) $$(1-L)(1-\varrho_4 L^4)Y_t = \alpha(1-L)(1-\varrho_4 L^4)X_t + \gamma\{L+\varrho_4 L^4(1-L)\}(\beta X_t - Y_t) + \varepsilon_t,$$

(3.3) $$m = \frac{\beta-\alpha}{\beta\gamma}(1-\varrho_4),$$

QEC(ϱ_1,1)

(3.4) $$(1-\varrho_1 L)(1-L^4)Y_t = \alpha(1-\varrho_1 L)(1-L^4)X_t + \gamma\{L^4+\varrho_1 L(1-L^4)\}(\beta X_t - Y_t) + \varepsilon_t,$$

(3.5) $$m = -4\,\frac{\beta-\alpha}{\beta\gamma}(1-\varrho_1).$$

An overview of the error correction models discussed above and their relation to other dynamic adjustment models, is given in Figure 1. At the top, we find the general autoregressive-distributed lag model (AD), written as in (2.6b); at the bottom is given the corresponding static equation $Y_t = \beta X_t + \varepsilon_t$. We see that the simple error correction models of order 1 and 4, denoted by SEC(1) and SEC(4), respectively, and the corresponding simple partial adjustment models, PA(1) and PA(4), are special cases of the QEC models. The figure also includes the generalized partial adjustment models of order 1 and 4:

$$\text{GPA(1)}: \quad \Delta Y_t = \gamma[\beta\{\lambda_0+\lambda(L)\}X_t - Y_{t-1}],$$

$$\text{GPA(4)}: \quad \Delta_4 Y_t = \gamma[\beta\{\lambda_0+\lambda(L)\}X_t - Y_{t-4}].$$

These do not belong to the GEC class, but are generalizations of PA(1) and PA(4) in which the target for Y is constructed on the basis of a general moving average of X, $\{\lambda_0+\lambda(L)\}X_t$.

3.2. Deterministic seasonals

In addition to the quarterly seasonal effects captured by (3.1) we also include an additive deterministic seasonal component in the econometric specification of the quarterly error correction mechanism. We have two reasons for doing this. First, our parametrization of $\varrho(L)$ does not necessarily remove all seasonals from $X_t^* = \varrho(L)X_t$ and $Y_t^* = \varrho(L)Y_t$ and hence the "trend error" $\beta X_t^* - Y_t^*$ may contain a systematic seasonal component. X_t and Y_t may have their "high" and "low" seasons in different quarters and hence $\Delta^* X_t$ and $\Delta^* Y_t$ in (2.16) may not be seasonally synchronized if the seasonals in X_t^* and Y_t^* had been completely removed.

We now augment αX_t and $\beta X_t - Y_t$ in (2.11) by seasonal dummy terms, i.e. replace them by

$$\alpha X_t + \sum_{i=1}^{4} \delta_{1i} z_{it},$$

$$\beta X_t - Y_t + \sum_{i=1}^{4} \delta_{2i} z_{it},$$

where z_{it} is equal to one if the t'th observation is from the i'th quarter and zero otherwise (i=1,2,3,4), and δ_{1i} and δ_{2i} are (so far unrestricted) constants. Eq. (2.11) then changes into

$$(3.6) \quad \{1-\varrho(L)\}Y_t = \alpha\{1-\varrho(L)\}X_t + \gamma\varrho(L)\{\beta X_t - Y_t\} + \sum_{i=1}^{4} \delta_i(L) z_{it} + \varepsilon_t,$$

where

$$(3.7) \quad \delta_i(L) = \delta_{1i} + (\gamma\delta_{2i} - \delta_{1i})\varrho(L) = \gamma\delta_{2i} + (\delta_{1i} - \gamma\delta_{2i})\{1-\varrho(L)\}.$$

The seasonal term of (3.6) can alternatively be written as $\sum_{i=1}^{4} \delta_i' z_{it}$, where $(\delta_1', \delta_2', \delta_3', \delta_4')$ is a suitable transformation of $(\delta_1, \delta_2, \delta_3, \delta_4)$ since $L^{4s} z_{it} = z_{it}$ for all (integer) values of i, s, and t, i.e. (3.6) is equivalent to a GEC model with additive quarterly seasonal dummies.

Which restrictions, if any, will this parametrization impose on the deterministic seasonal term? From (3.7) it follows that

$$(3.8) \quad \sum_{i=1}^{4} \delta_i(L) = \{1-\varrho(L)\} \sum_{i=1}^{4} \delta_{1i} + \gamma\varrho(L) \sum_{i=1}^{4} \delta_{2i},$$

and, since $\varrho(1) = 1$, that

$$(3.9) \quad \delta_i(1) = \gamma\delta_{2i} \qquad (i=1,2,3,4).$$

This shows that (i) the coefficients of the seasonal dummies in (3.6) will be unrestricted if no restrictions are imposed on δ_{1i} and δ_{2i}, (ii) the sum of the coefficients across quarters for a fixed lag L will depend on both δ_{1i} and δ_{2i}, and (iii) the sum of the coefficients across lags for a given quarter i depends on δ_{2i} only. A necessary and sufficient condition for the sum across quarters, (3.8), to be zero for any given lag is that both sets of dummy coefficients add to zero, i.e. $\Sigma_i\delta_{1i} = \Sigma_i\delta_{2i} = 0$. A necessary and sufficient condition for the sum across lags, (3.9), to be zero in any quarter is that $\delta_{2i} = 0$ $(i=1,..,4)$.

Combining (3.1) and (3.7), while recalling that $L^4z_{it} = z_{it}$, we find that the seasonal term of (3.6) for a QEC model can be written out as

$$(3.10) \quad \sum_{i=1}^{4} \delta_i(L)z_{it} = \gamma\sum_{i=1}^{4} \delta_{2i}z_{it} + (1-\varrho_4)\sum_{i=1}^{4} (\delta_{1i}-\gamma\delta_{2i})(1-\varrho_1L)z_{it}.$$

For <u>QEC(1,ϱ_4)</u> we get in particular

$$(3.10a) \quad \Sigma_i\delta_i(L)z_{it} = \gamma\Sigma_i\delta_{2i}z_{it} + (1-\varrho_4)\Sigma_i(\delta_{1i}-\gamma\delta_{2i})\Delta z_{it},$$

where $\Delta z_{it} = z_{it} - z_{it-1}$, while <u>QEC($\varrho_1$,1)</u> is characterized by

$$(3.10b) \quad \Sigma_i\delta_i(L)z_{it} = \gamma\Sigma_i\delta_{2i}z_{it}.$$

We see that the seasonal coefficients of the latter model are uniquely defined by the seasonal coefficients in its error correction term, δ_{2i}.

As noted in section 2.3, the GEC model imposes long run proportionality between Y and X, i.e. $Y=\beta X$. Since we do not want the introduction of seasonal terms to change this property, it seems sensible to impose the restriction

$$(3.11) \quad \sum_{i=1}^{4} \delta_{2i} = 0$$

and, when needed, also

$$(3.12) \quad \sum_{i=1}^{4} \delta_{1i} = 0.$$

This implies

$$(3.13) \qquad \sum_{i=1}^{4} \delta_i(L) = 0 \quad \text{for all values of } L,$$

so that (3.10) can be replaced by

$$(3.14) \qquad \sum_{i=1}^{4} \delta_i(L)z_{it} = \sum_{i=1}^{3} \delta_i(L)(z_{it}-z_{4t})$$

$$= \gamma \sum_{i=1}^{3} \delta_{2i}(z_{it}-z_{4t}) + (1-\varrho_4) \sum_{i=1}^{3} (\delta_{1i}-\gamma\delta_{2i})(1-\varrho_1 L)(z_{it}-z_{4t})$$

$$= \sum_{i=1}^{3} \bar{\delta}_i(z_{it}-z_{4t}),$$

where $\bar{\delta}_i$ is a suitable linear transformation of the δ_{1i}'s and δ_{2i}'s.[5)]

4. ESTIMATION PROCEDURE

Eq.(3.6) - with (3.1) inserted for $\varrho(L)$ and (3.14) inserted for the seasonal term - is linear in (the current and lagged values of) Y_t and X_t and non-linear in the eight parameters α, β, γ, ϱ_1, ϱ_4, $\bar{\delta}_1$, $\bar{\delta}_2$, and $\bar{\delta}_3$. For given values of γ, ϱ_1, and ϱ_4 it is, however, linear (with no constant term) in the other five parameters. The equation is estimated by the non-linear least squares routine NLS of the TROLL system (see TROLL (1981)), by means of which we can perform an unconstrained minimization of the sum of squares of residuals

$$(4.1) \qquad \sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} \{(1-\varrho_1 L)(1-\varrho_4 L^4)(Y_t-\alpha X_t)$$

$$+ \gamma(\varrho_1 L+\varrho_4 L^4-\varrho_1\varrho_4 L^5)(Y_t-\beta X_t) - \sum_{i=1}^{3} \bar{\delta}_i(z_{it}-z_{4t})\}^2$$

$$= Q(\alpha, \beta, \gamma, \varrho_1, \varrho_4, \bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3),$$

where T is the number of observations, the observations on Y_t and X_t starting at t=-4. If the disturbances are normally distributed, the estimates will be maximum likelihood estimates.

NLS solves this minimization problem iteratively by means of the NL2SOL algorithm, which is a quasi-Newton algorithm. In contrast to the Gauss-Newton method, in which the second order term in the Taylor expansion of the equation is ignored, NL2SOL approximates this second order term by an update method which ensures that the estimate satisfies the quasi-Newton equation. There is no guarantee that this iterative process will converge to the global minimum of Q. In some cases, this was checked by repeating the computation, starting from a different set of initial guesses of the coefficients. There was no indication that these guesses influenced the final result.

We did not, however, by means of this algorithm manage to perform a simultaneous minimization of Q with respect to all the eight coefficients in the model. A combined grid-search-NLS procedure was therefore used. We set $\rho_4=1$ a priori, i.e. consider the $QEC(\rho_1,1)$ specification of $\rho(L)$, and because we have some information about the a priori feasible interval for ρ_1, and γ, we specify a two-dimensional grid for these coefficients and estimate the remaining five coefficients conditionally by unconstrained NLS. The final solution is the coefficient set which minimizes Q. For ρ_1, the search is done over the interval [0.0, 1.0], with a step length of 0.1. For γ, we face the problem that the long-run coefficient β cannot be identified when $\gamma=0$, i.e. when no error correction occurs (and the mean lag m is undefined; cf. (2.18)). We therefore did the search for this parameter over the interval [0.1, 1.0], with a step length of 0.1, only. If the sum of squared residuals attains its minimum for $\gamma=0.1$, i.e. a boundary solution, there are thus indication of absence of an error correction effect and lack of identification of β.

The coefficients of the general AD(5,5) model, which we use as a standard of comparison for the QEC model (cf. section 2.3), are estimated by OLS.

Some of the variants of the QEC model are estimated with two alternative specifications of the disturbance process. At the initial stage, the estimation is performed with ε_t specified as a white noise process. If the combined grid-search-NLS procedure described above converges to the global minimum of Q, and if the disturbances are independently and normally distributed, the resulting estimates will be maximum likelihood (ML) estimates[6] for the $QEC(\rho_1,1)$ model. In some cases, however, the Durbin-Watson and Box-Ljung statistics for residual serial correlation indicated first and/or higher order autocorrelation. (Confer section 5.2.) We then reestimated the coefficients, assuming ε_t to follow the fourth order AR process

$$(4.2) \quad (1 - \phi_1 L - \phi_4 L^4)\varepsilon_t = u_t,$$

where ϕ_1 and ϕ_4 are constants and u_t is white noise. We chose this particular process in order to be able to detect potentially misspecified trends and/or seasonal effects with a minimal number of additional parameters.

The joint estimation of α, β, γ, ϱ_1, ϱ_4, $\bar{\delta}_1$, $\bar{\delta}_2$, $\bar{\delta}_3$, ϕ_1, and ϕ_4 can be carried out by minimizing the sum of squares of transformed residuals

$$
\begin{aligned}
(4.3) \qquad \sum_{t=5}^{T} u_t^2 &= \sum_{t=5}^{T} \{(1-\phi_1 L-\phi_4 L^4)\varepsilon_t\}^2 \\
&= Q^*(\alpha,\ \beta,\ \gamma,\ \varrho_1,\ \varrho_4,\ \bar{\delta}_1,\ \bar{\delta}_2,\ \bar{\delta}_3,\ \phi_1,\ \phi_4),
\end{aligned}
$$

where ε_t is defined as in (4.1). Again, this minimization will produce approximate ML estimates if the disturbances are normally distributed. If we regard Y_{-4}, $Y_{-3}, \ldots, Y_4$ as fixed, it gives exact ML estimates. (Confer Harvey (1981, pp. 121-122).) For the numerical minimization we use a stepwise algorithm similar to the Cochrane-Orcutt algorithm for a linear regression model with AR(1) disturbances. First, the coefficients are estimated, assuming white noise disturbance terms, by the combined grid-search-NLS procedure described above. In the second step, the autoregressive parameter ϕ_1 and ϕ_4, are estimated from the residuals calculated in the first step by regressing $\hat{\varepsilon}_t$ on $\hat{\varepsilon}_{t-1}$ and $\hat{\varepsilon}_{t-4}$. Thirdly, the coefficients are reestimated, by minimizing (4.3) conditionally on the estimates of the autoregressive parameters. This process is repeated until convergence.

5. EMPIRICAL APPLICATIONS

5.1. Problems and data

The quarterly error correction model QEC(ϱ_1,1) and the corresponding AD(5,5) model are estimated with three different data sets:

i) household consumption at constant prices (Y) and real disposable household income (X),

ii) gross production at constant prices in wood and printing industries (Y) and demand (final and intermediate, net of imports) at constant prices of wood and printing commodities (X), and

iii) capital stock in machinery in mining and raw-material industries (Y) and production in this sector at constant prices (X), i.e. we estimate a consumption function implying proportionality between consumption and income in the long run, an output adjustment function implying proportionality between output and demand in the long run, and an investment function implying proportionality between capital stock and output in the long run.

The data are taken from the Norwegian quarterly national accounts from the period 1967.1-1983.4, which are reconciled with the corresponding annual accounts.[7)] The quantity series in the Norwegian national accounts change base year regularly, and the quarterly data used in the present investigation are all rebased to 1984 prices at a fairly disaggregate level of sector and commodity classification. The aggregate series needed, like total household consumption, are obtained by summing the rebased series at the disaggregate level. The quarterly national accounts data are seasonally unadjusted and they are used here in this form.[8)]

A number of model formulations, belonging to the general scheme in Figure 1, are estimated. Results for the error correction models are presented in Tables 4-8 and for the general AD lag models in Tables 1-3.

The most general specification, denoted as Aa in the tables, is the autoregressive distributed lag model AD(5,5), in which no parameter restrictions are imposed, neither on the lag distribution nor on the seasonal coefficients. This is our base model. In Ab, lags of the second and third order are excluded from both Y and X, Ac and Ad are, respectively, AD(5,0) and AD(0,5) equations with second and third order terms omitted, and Ae is a static equation. Four additive seasonal dummies are included in all the equations.

The error correction models form a separate branch in our model scheme starting from the AD(5,5) specification; cf. Figure 1. Using the terminology introduced in section 3.1, the most general of these models, indicated by Ba in the tables, is a QEC(ϱ_1,1), with ϱ_1 unrestricted. In Bb, we impose $\alpha=\gamma\beta$ as an additional restriction, in Bc we impose $\varrho_1=0$, which gives a SEC(4) model, and in Bd we set γ to zero, in which case β is unidentifiable and Y and X have the same lag polynomial, $1-\varrho(L)=(1-\varrho_1 L)(1-L^4)$. This may be interpreted as a model with no error correction, since both Y and X are represented by their four quarter differences only, which makes β unidentifiable. Finally, in Be, ϱ_1 is set to unity. We see that Bb, Bc, Bd, and Be are all special cases of Ba, but none of them are nested.

5.2. Estimates of the OEC and AD models

Household consumption and income

The estimation results for the general lag equations are presented in Table 1 and the results for the error correction models in Table 4. The fit, evaluated by standard errors of regression and Durbin-Watson statistics, is not substantially different for the two models. The estimates are, on the whole, reasonable, with a long run propensity to consume of about 0.95, a short run propensity in the range 0.5-0.7, and a mean lag of 1-2 quarters. There are, however, differences that may be of importance when the model is used for simulation and forecasting - for instance the presence of negative lag coefficients in the AD specification. Thus it may be necessary to restrict the coefficients in the AD(5,5) model in some way, by restricting either the form of the lag polynomials or the lag length. It is just this sort of restrictions that the error correction formulation imposes. Furthermore, the coefficients of the latter model, β, α, and γ, have direct economic interpretations, as long run and short run propensities to consume, and speed of error correction, respectively.

In the general lag model without any restrictions on the additive seasonal terms, all the seasonal coefficients have positive values, indicating that there is a positive trend in the specified consumption function. This explains why the long run propensity to consume - interpreted as β in the error-correction-formulations and as the sum of lag-coefficients in the general lag-formulations - comes out with a larger estimate in the former formulation, which excludes a constant term in the long run, than in the latter.

The specification Bd, in which the error correction term is eliminated, is also interesting. The Durbin-Watson statistic is satisfactory, whereas the Box-Ljung statistic indicates significant fourth order autocorrelation in the disturbance process.[9)] The seasonal effects are probably inadequately captured by this specification, or the dynamics may have been misspecified in other ways.

Production and demand

The results for the general AD specification are presented in Table 2 and those for the error correction models in Table 5. Neither in this example there are large discrepancies between the two formulations in terms of goodness of fit. The long run production-demand ratio in the error correction models, β, slightly exceeds, but is not significantly different from unity. The corresponding long run effect in the general AD equation has an estimated value of about 0.9, and the short run coefficients have values in the range 0.2-0.3. The estimated mean lag between production and demand is 2-4 quarters, but the presence of negative lag-coefficients is still a problem. This again makes the error correction formulation attractive. It is, however, worth noting that in the formulation Bd - where γ is restricted to zero - the α coefficient of 0.29 is considerably lower than in the error correction formulations where γ is allowed to be positive. In Bd, on the other hand, we find indication of a positive trend, incorporated in its constant term. The Durbin-Watson statistic is acceptable, but the Box-Ljung statistics indicate fourth order autocorrelation. It thus seems that some systematic seasonal effects remain when γ is restricted to zero.

Results from estimating the coefficients in the error correction model by the modified Cochrane-Orcutt iterative method allowing for residual serial correlation are presented in Table 7. The autoregressive coefficients of the first as well as of the fourth order are significant (when judged by ordinary t statistics), with values of about -0.4 (alternatives Ba, Bb, and Be). This is an indication that the trend and seasonal effects are inadequately represented by the original formulation. The estimate of the short run coefficient α is reduced by about 50 per cent while the value of the long run coefficient β is virtually unchanged. Furthermore, the estimate of γ is decreased, which indicates a slower degree of error correction, and in alternatives Ba and Bb the estimate of ϱ_1 is considerably increased - from 0 to 0.8. We see from (3.5) that the decrease in α and γ contributes to a longer adjustment lag, whereas the increase in ϱ_1 leads to a shorter lag. The total effect of these changes in the parameter estimates is a reduction in the mean lag.[10)]

Capital in machinery and production

The estimation results for the general AD specification are presented in Table 3 and those from the error correction models in Table 6. In this case, there are clear differences between the two models. On the whole, the estimates are rather unsatisfactory even if some of the coefficient estimates are reasonable. Among the specifications with general lag formulations, alternatives Aa, Ab, and Ac are statistically acceptable, when judged by their Durbin-Watson and Box-Ljung statistics, although many coefficients are negative and the coefficient of the endogenous variable, capital, lagged one quarter is considerably larger than one. However, this formulation does not give reasonable estimates of the short run coefficients, and the estimate of the long run coefficient is also unreasonably low.

In the error-correction formulations we have - strictly speaking - not detected any error-correction effects since, as remarked in section 4, a γ estimate of 0.1 obtained by grid search probably indicates absence of such effects. On the other hand, a long run capital-production ratio of 2.1 and a marginal capital coefficient in the range 0.1-0.2 does not seem too bad. The very low values of the Durbin-Watson statistic indicates, however, that there is a trend that is not captured by the specified models. This conclusion is strengthened by the fact that in alternative Bd, in which all linear trends and seasonal effects have been eliminated, the Durbin-Watson statistic has a larger value, although the coefficient estimates are quite unreasonable.

The change in the specification of the disturbance process and the estimation by means of the modified Cochrane-Orcutt-method, improve the results considerably (Table 8); the value of the Durbin-Watson statistic is increased and the values of the Box-Ljung statistics are considerably reduced. The estimates of α, β, and γ, however, are not much changed. The value of ρ_1 larger than 1 in alternative Bb may, however, cause problems in simulation experiments. Neither does an estimated mean lag of about 25 quarters, which corresponds to this specification, inspire much confidence.

Table 1. Household consumption and disposable income. General lag distributions. Million 1984 kroner. Ordinary Least Squares estimates. LHS-mean:43517.

Aa: AD(5,5), unrestricted
Ab: AD(5,5), with zero restrictions on second and third order coefficients
Ac: AD(5,0), with zero restrictions on second and third order coefficients
Ad: AD(0,5), with zero restrictions on second and third order coefficients
Ae: Static

	Aa	Ab	Ac	Ad	Ae
A0	0.5113	0.4694	0.5017	0.4566	0.8913
	0.1890	0.1736	0.0981	0.1737	0.0208
A1	-0.0163	-0.1032	*	-0.0267	*
	0.2233	0.2095	*	0.1954	*
A2	-0.1232	*	*	*	*
	0.2301	*	*	*	*
A3	0.0297	*	*	*	*
	0.2316	*	*	*	*
A4	0.0805	0.0864	*	0.2334	*
	0.2223	0.2024	*	0.1832	*
A5	0.0905	0.1962	*	0.2146	*
	0.2200	0.1952	*	0.1603	*
B1	0.1092	0.1708	0.1804	*	*
	0.1474	0.1405	0.1268	*	*
B2	0.0203	*	*	*	*
	0.1450	*	*	*	*
B3	0.1551	*	*	*	*
	0.1390	*	*	*	*
B4	0.1520	0.1896	0.2607	*	*
	0.1444	0.1351	0.1161	*	*
B5	-0.0836	-0.1011	-0.0123	*	*
	0.1397	0.1320	0.1096	*	*
D1	1030.3	1176.7	150.5	1977.2	1003.2
	1639.2	1471.3	1242.5	954.7	957.5
D2	2758.1	3483.2	2809.3	4386.1	3272.5
	1547.9	1025.6	887.9	919.3	951.7
D3	357.6	1859.8	1108.9	2513.9	624.4
	1651.0	1434.1	932.8	1280.4	1022.8
D4	5824.2	6092.6	5220.1	7535.2	6425.8
	1506.6	1293.7	1057.5	988.3	1008.2
SER	901.4	918.6	912.1	929.4	1026.9
RSQ	0.99968	0.99964	0.99963	0.99961	0.99950
DW	1.8923	1.9368	1.8931	1.5011	1.2519
CHISQ(1)	0.0609	0.0109	0.0400	3.4643	8.0857
CHISQ(4)	0.1937	1.5402	1.7920	8.0921	21.5144
CHISQ(8)	6.1659	5.2863	7.5165	11.4977	25.8852

Table 2. Production and demand in wood and printing industries. General lag distributions. Million 1984 kroner. Ordinary Least Squares estimates. LHS-mean: 11633.

Aa: AD(5,5), unrestricted
Ab: AD(5,5), with zero restrictions on second and third order coefficients
Ac: AD(5,0), with zero restrictions on second and third order coefficients
Ad: AD(0,5), with zero restrictions on second and third order coefficients
Ae: Static

	Aa	Ab	Ac	Ad	Ae
A0	0.2650	0.2341	0.2199	0.3126	0.8552
	0.1280	0.1288	0.1145	0.1242	0.0438
A1	-0.0144	0.0779	*	0.2143	*
	0.1331	0.1372	*	0.1239	*
A2	-0.0908	*	*	*	*
	0.1264	*	*	*	*
A3	0.2728	*	*	*	*
	0.1242	*	*	*	*
A4	-0.0354	-0.0342	*	0.1688	*
	0.1388	0.1469	*	0.1188	*
A5	-0.0242	0.0448	*	0.1514	*
	0.1266	0.1250	*	0.1182	*
B1	0.1032	0.1675	0.2124	*	*
	0.1529	0.1600	0.1430	*	*
B2	0.2294	*	*	*	*
	0.1553	*	*	*	*
B3	0.1774	*	*	*	*
	0.1570	*	*	*	*
B4	-0.1095	0.1081	0.1298	*	*
	0.1586	0.1471	0.1326	*	*
B5	0.1315	0.2850	0.3232	*	*
	0.1600	0.1585	0.1201	*	*
D1	2109.5	1673.9	1710.7	2342.8	2612.4
	564.2	572.5	472.6	467.2	500.5
D2	1443.7	1550.7	1437.6	2407.0	2134.0
	627.7	606.1	470.4	445.1	509.0
D3	-462.4	605.9	555.8	1044.8	774.2
	627.5	540.2	488.5	453.3	520.6
D4	2013.3	2457.7	2484.5	2220.5	1644.4
	583.1	533.2	516.5	491.9	567.3
SER	399.3	428.2	420.0	437.6	531.2
RSQ	0.99914	0.99892	0.99890	0.99880	0.99813
DW	2.0093	2.1114	2.2430	1.8443	2.1343
CHISQ(1)	0.0072	0.2936	1.0730	0.1689	0.4655
CHISQ(4)	0.2612	1.4490	2.5639	0.6336	5.9027
CHISQ(8)	1.8256	2.8769	4.0493	2.2097	7.9927

Table 3. Capital in machinery and production in mining and raw-material industries. General lag distributions. Million 1984 kroner. Ordinary Least Squares estimates. LHS mean:23037.8.

Aa: AD(5,5), unrestricted
Ab: AD(5,5), with zero restrictions on second and third order coefficients
Ac: AD(5,0), with zero restrictions on second and third order coefficients
Ad: AD(0,5), with zero restrictions on second and third order coefficients
Ae: Static

	Aa	Ab	Ac	Ad	Ae
A0	-0.0133	-0.0188	0.0469	1.5865	2.4183
	0.0401	0.0401	0.0274	0.8698	0.4430
A1	0.0237	0.0876	*	-0.2312	*
	0.0521	0.0401	*	0.9625	*
A2	0.1020	*	*	*	*
	0.0526	*	*	*	*
A3	-0.0780	*	*	*	*
	0.0583	*	*	*	*
A4	-0.0126	-0.0191	*	-0.2745	*
	0.0608	0.0438	*	1.0126	*
A5	0.0335	0.0192	*	2.0431	*
	0.0413	0.0400	*	0.9338	*
B1	1.5346	1.3668	1.3924	*	*
	0.1471	0.0495	0.0425	*	*
B2	-0.0953	*	*	*	*
	0.2768	*	*	*	*
B3	-0.3905	*	*	*	*
	0.2658	*	*	*	*
B4	-0.1573	-0.5320	-0.6409	*	*
	0.2535	0.1801	0.1593	*	*
B5	0.0927	0.1458	0.2328	*	*
	0.1466	0.1467	0.1337	*	*
D1	-311.6	-473.2	-336.1	-11504.0	-4235.3
	245.3	244.9	227.5	4912.2	5014.2
D2	-331.6	-331.1	-139.1	-11315.0	-3449.7
	249.0	247.5	223.4	4878.1	4904.2
D3	-289.0	-399.8	-167.5	-9394.8	-1153.6
	243.4	227.6	199.1	4707.9	4512.8
D4	-13.3	-28.2	27.5	-9168.8	-3685.2
	257.1	253.4	231.9	4951.0	5019.2
SER	110.9	114.9	117.9	2824.7	3152.1
RSQ	0.99998	0.99998	0.99998	0.98731	0.98328
DW	1.9448	1.7798	1.7541	0.1446	0.1393
CHISQ(1)	0.0267	0.7430	0.9120	52.8647	53.2491
CHISQ(4)	0.2828	1.8485	2.3405	173.8880	157.3310
CHISQ(8)	6.8994	6.3992	7.4364	235.2710	207.3010

Table 4. Household consumption and disposable income. Quarterly Error Correction model. Million 1984 kroner. Nonlinear Least Squares/Grid Search estimates. LHS mean:43517.

Ba: QEC(ϱ_1,1)
Bb: PA
Bc: SEC(4)
Bd: No error correction
Be: QEC(1,1)

	Ba	Bb	Bc	Bd	Be
RHO1	0.4	0.4	0.0	0.2	1.0
	*	*	*	*	*
RHO4	1.0	1.0	1.0	1.0	1.0
	*	*	*	*	*
GAMMA	0.6	0.6	0.6	0.0	0.6
	*	*	*	*	*
ALPHA	0.6185	0.5776	0.6385	0.6201	0.7107
	0.1315	*	0.1102	0.1477	0.1282
BETA	0.9614	0.9626	0.9669	*	0.9519
	0.0059	0.0045	0.0070	*	0.0047
CONST	*	*	*	324.7	*
	*	*	*	211.7	*
D1	-1740.7	-1743.1	-1702.0	*	-1786.8
	359.0	356.0	365.8	*	377.3
D2	469.1	469.1	485.5	*	446.6
	358.9	356.0	365.8	*	377.1
D3	-2300.8	-2304.6	-2345.6	*	-2227.1
	359.2	356.1	366.1	*	377.1
SER	962.9	955.1	981.3	1091.5	1011.5
RSQ	0.99956	0.99956	0.99954	0.97245	0.99951
DW	1.7654	1.7715	1.4069	2.1885	2.2211
CHISQ(1)	0.4178	0.3868	4.2323	0.8280	1.2106
CHISQ(4)	5.8715	6.0222	9.8441	12.9313	6.8834
CHISQ(8)	7.6717	7.8788	13.5767	15.9206	11.0386

Table 5. Production and demand in wood and printing industries. Quarterly Error Correction model. Million 1984 kroner. Nonlinear Least Squares/Grid Search estimates. LHS mean: 11633.

Ba: QEC(ϱ_1,1)
Bb: PA
Bc: SEC(4)
Bd: No error correction
Be: QEC(1,1)

	Ba	Bb	Bc	Bd	Be
RHO1	0.0	0.1	0.0	0.0	1.0
	*	*	*	*	*
RHO4	1.0	1.0	1.0	1.0	1.0
	*	*	*	*	*
GAMMA	0.5	0.5	0.5	0.0	0.8
	*	*	*	*	*
ALPHA	0.5511	0.5147	0.5511	0.2937	0.6479
	0.1040	*	0.1040	0.1145	0.1182
BETA	1.0303	1.0295	1.0303	*	1.0073
	0.0123	0.0112	0.0123	*	0.0075
CONST	*	*	*	225.7	*
	*	*	*	77.7	*
D1	929.5	933.0	929.5	*	909.9
	228.3	225.7	228.3	*	151.6
D2	412.0	414.2	412.0	*	397.6
	228.0	225.6	228.0	*	151.5
D3	-1019.7	-1029.1	-1019.7	*	-1042.2
	229.0	225.6	229.0	*	153.1
SER	508.9	504.4	508.9	546.1	542.0
RSQ	0.99829	0.99829	0.99829	0.90070	0.99806
DW	1.9714	2.0740	1.9714	2.0912	2.3794
CHISQ(1)	0.0045	0.1135	0.0045	0.1310	2.4897
CHISQ(4)	3.6679	4.2743	3.6679	11.6314	4.8570
CHISQ(8)	8.1568	9.2352	8.1568	18.6322	5.1782

Table 6. Capital in machinery and production in mining and raw-material industries. Quarterly Error Correction model. Million 1984 kroner. Nonlinear Least Squares/Grid Search estimates. LHS mean: 23037.8.

Ba: QEC(ϱ_1,1)
Bb: PA
Bc: SEC(4)
Bd: No error correction
Be: QEC(1,1)

	Ba	Bb	Bc	Bd	Be
RHO1	1.0	1.0	0.0	1.0	1.0
	*	*	*	*	*
RHO4	1.0	1.0	1.0	1.0	1.0
	*	*	*	*	*
GAMMA	0.1	0.1	0.1	0.0	0.1
	*	*	*	*	*
ALPHA	0.1165	0.2107	-0.2448	-0.1360	0.1165
	0.0726	*	0.1024	0.0469	0.0726
BETA	2.1089	2.1066	2.7947	*	2.1089
	0.0372	0.0374	0.0830	*	0.0372
CONST	*	*	*	-27.9	*
	*	*	*	26.2	*
D1	-1104.1	-1119.1	-996.8	*	-1104.1
	698.6	702.8	1503.2	*	698.6
D2	-268.1	-315.5	-54.8	*	-268.1
	699.3	702.6	1503.1	*	699.3
D3	1698.0	1778.6	1995.8	*	1698.0
	701.7	703.3	1503.9	*	701.7
SER	312.3	314.2	672.0	202.7	312.3
RSQ	0.99984	0.99983	0.99924	0.99735	0.99984
DW	0.4092	0.4502	0.1508	0.8850	0.4092
CHISQ(1)	35.6055	33.6958	52.6195	16.3148	35.6055
CHISQ(4)	105.5720	94.5712	138.3210	34.7499	105.5720
CHISQ(8)	149.7960	133.0190	156.9690	46.8576	149.7960

Table 7. Production and demand in wood and printing industries. Quarterly Error Correction model with autoregressive disturbance process of fourth order. Nonlinear Least Squares/Grid Search estimates. Million 1984 kroner. LHS mean:11633.

Ba: QEC(ϱ_1,1)
Bb: PA
Bc: SEC(4)
Bd: No error correction
Be: QEC(1,1)

	Ba	Bb	Bc	Bd	Be
RHO1	0.8	0.8	0.0	0.5	1.0
	*	*	*	*	*
RHO4	1.0	1.0	1.0	1.0	1.0
	*	*	*	*	*
GAMMA	0.3	0.3	0.5	0.0	0.4
	*	*	*	*	*
ALPHA	0.3473	0.3054	0.5826	0.2865	0.3820
	0.1164	*	0.1036	0.1108	0.1131
BETA	1.0176	1.0181	1.0288	*	1.0067
	0.0097	0.0095	0.0117	*	0.0073
PHI1	-0.4200	-0.4187	0.0185	-0.3115	-0.4372
	0.1045	0.1041	0.1323	0.1107	0.1048
PHI4	-0.4011	-0.4121	-0.0877	-0.3831	-0.3910
	0.1052	0.1048	0.1323	0.1116	0.1058
CONST	*	*	*	112.5	*
	*	*	*	39.9	*
D1	959.5	964.6	933.8	*	934.9
	284.4	279.2	207.9	*	220.3
D2	349.8	351.0	408.2	*	364.5
	284.2	279.2	207.6	*	220.3
D3	-1044.0	-1063.6	-1020.7	*	-1056.3
	289.3	279.2	208.6	*	224.1
SER	469.9	466.3	506.7	477.8	477.8
RSQ	0.99854	0.99853	0.99830	0.92120	0.99849
DW	2.0866	2.0812	1.9884	2.2513	2.1175
CHISQ(1)	0.1555	0.1400	0.0000	0.9893	0.2964
CHISQ(4)	3.5830	3.9004	3.3354	8.0887	2.4240
CHISQ(8)	10.9405	11.2530	7.2824	14.7011	7.6452

Table 8. Capital in machinery and production in mining and raw-material industries. Quarterly Error Correction model with autoregressive disturbance process of fourth order. Nonlinear Least Squares/Grid Search estimates. Million 1984 kroner. LHS mean: 23037.8.

Ba: QEC(ρ_1,1)
Bb: PA
Bc: SEC(4)
Bd: No error correction
Be: QEC(1,1)

	Ba	Bb	Bc	Bd	Be
RHO1	1.0	0.3	0.0	0.9	1.0
	*	*	*	*	*
RHO4	1.0	1.0	1.0	1.0	1.0
	*	*	*	*	*
GAMMA	0.1	0.1	0.1	0.0	0.1
	*	*	*	*	*
ALPHA	0.1011	0.2111	0.0186	-0.0690	0.1011
	0.0413	*	0.0470	0.0381	0.0413
BETA	2.0875	2.1110	2.6155	*	2.0875
	0.0928	0.1983	0.1513	*	0.0928
PHI1	0.7416	1.0936	1.0556	0.6535	0.7416
	0.1093	0.0579	0.0463	0.1051	0.1093
PHI4	0.0219	-0.2062	-0.2001	-0.1443	0.0219
	0.1070	0.0564	0.0463	0.1073	0.1070
CONST	*	*	*	46.3	*
	*	*	*	41.0	*
D1	-988.0	-983.7	-1161.8	*	-988.0
	317.4	267.1	244.7	*	317.4
D2	-237.9	-288.2	-214.3	*	-237.9
	315.7	254.1	237.7	*	315.7
D3	1629.4	1717.7	2039.7	*	1629.4
	325.6	301.7	265.6	*	325.6
SER	188.9	201.8	184.8	155.7	188.9
RSQ	0.99994	0.99993	0.99994	0.99837	0.99994
DW	2.3849	1.8163	1.5792	2.2504	2.3849
CHISQ(1)	3.2387	0.0146	1.9638	1.6111	3.2387
CHISQ(4)	10.6234	1.3546	5.6021	12.0020	10.6234
CHISQ(8)	17.4603	3.0496	9.4736	27.0179	17.4603

5.3. Tests of model specification

We have performed some statistical tests in order to clarify whether or not the restrictions on the parameter vector imposed by the error correction formulation are "effective", in the sense that they can be rejected in favour of the more general autoregressive-distributed lag formulation. For simplicity, and in order to keep the power of the test at an acceptable level, we have refrained from testing all the relevant specifications, concentrating on those in which a white noise disturbance is assumed.

Our testing scheme, shown in Figure 2, specifies a total of 11 hypotheses, including the most general lag distribution and the most restrictive static equation without a constant term and seasonal dummies. The test scheme has two main branches, of which the right one (i.e. hypotheses H6-H9) relates to the general AD models and the left one (i.e. hypotheses H1-H5) to the error correction models. The remaining hypotheses, at the bottom of the testing scheme, i.e. H10 (static model with seasonal terms adding to zero) and H11 (static model without seasonal terms) are special cases of both models. Each of the 14 subtests specified - i.e. H2 against H1, H6 against H2, etc. - is performed by means of the Likelihood Ratio Test. (Confer Table 9.) The strategy consists in testing from the basic hypothesis H1 to the gradually more specialized hypotheses until a hypothesis is rejected at a chosen significance level α. With 14 subtests, and each of which tested at the level α, the "overall significance level" is at most 14α, in the sense that 14α is the maximal probability of making at least one erronoûs rejection. The values of the test statistics - which under the null hypothesis are approximately χ^2 distributed with a number of degrees of freedom equal to the number of parameter restrictions - are presented in Table 9.

Consider first the AD branch of the testing scheme. In neither of the cases can H2 be rejected against the basic specification, H1, at the level α=0.001, i.e. $a_2=a_3=b_2=b_3=0$ cannot be rejected, but it is rejected in the production-demand example for α=0.01. On the other hand, neither H4 nor H3 can be rejected against H2 in the household consumption and production example, but the static specification H5 is rejected against H4 as well as against H3 in both cases. In the capital accumulation example, H3 is definitely rejected, i.e. we can reject the hypothesis that no lagged endogenous variables occur. At the level α=0.01, H5 is rejected even in this case.

The most general error correction specification H6 is a special case of H2. For the household consumption example, the restrictions imposed cannot be rejected at $\alpha=0.01$, i.e. the restrictions imposed by the error correction specification (cf. Table 9) are not "effective" in a statistical sense. Neither can the more restricted error correction hypotheses H7, H8, and H9 be rejected against H6. On the other hand, H10 can be rejected against H7, but not against H8 and H9 at $\alpha=0.01$. From this we can conclude that the PA specification H7 seems to be the "optimal" parametrization of the consumption-income relationship, since it is significantly better than H10 in terms of goodness of fit and cannot be rejected against the more general specifications of the adjustment mechanism.

Turning next to the production-demand example, we find from Table 9 that H6 is rejected at the level of significance $\alpha=0.001$. From this one might perhaps conclude that the parametrization implied by the error correction model is too restrictive. However, the effects of adjusting for the presence of autocorrelation (confer Table 7), should be recalled. In all cases, the sum of squares of residuals is considerably reduced as compared with the 'white noise formulation' shown in Table 5. This specification cannot, however, be tested against H2, by means of classical test procedures since a QEC(ϱ_1,1) model with autocorrelated disturbances and the general AD(5,5) model with white noise disturbances are non-nested hypotheses. We can, however, easily test the 'white noise formulation' $\phi_1=\phi_4=0$ against more general formulations with autocorrelated disturbances. The Likelihood Ratio Test is performed and the value of the χ^2 statistic is 9.57. Thus, using $\alpha=0.01$, H6, $\phi_1=\phi_4=0$, is rejected in favour of a formulation correcting for first and fourth order autocorrelation. Consequently, it is still possible that an error correction formulation with a simple autoregressive disturbance process should be preferred to a general lag-distribution with white noise disturbances.

It is difficult to get firm conclusions from the test procedure for the capital-production example. From Table 9, it seems that the error correction formulation should be definitely rejected. However, in this example we have found strong residual autocorrelation (cf. Table 7). As in the previous example, a test of the hypothesis H6 with $\phi_1=\phi_4=0$ against a more general specification allowing for autocorrelated disturbances, leads to rejection. The value of the test statistic is 60.32, implying that H6 is rejected in favour of a formulation correcting for first and fourth order residual autocorrelation. Thus, also in this case, the question of whether or not a general lag distribution is preferable to an error correction formulation is still open.

FIGURE 2. TESTING SCHEME

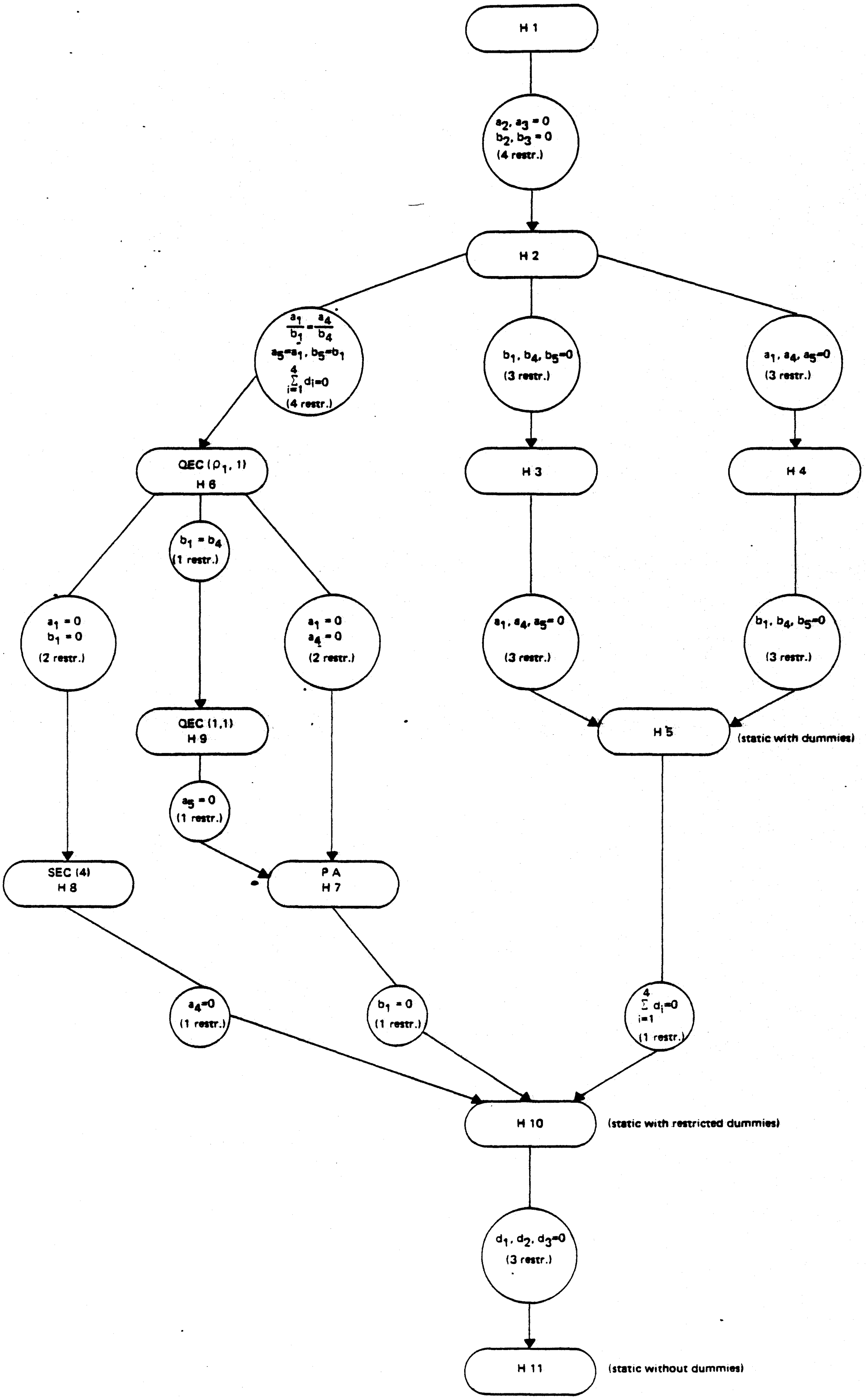


Table 9: Test statistics for Likelihood Ratio testing of the hypotheses H1,...,H11. Based on models with no correction for autocorrelated disturbances.

Likelihood Ratio test statistic = $-2 \log \lambda$.[a)]

The critical values are:

$\alpha = 0.01$: $\chi^2(1)=6.63$, $\chi^2(2)=9.21$, $\chi^2(3)=11.34$, $\chi^2(4)=13.28$

$\alpha = 0.001$: $\chi^2(1)=10.83$, $\chi^2(2)=13.81$, $\chi^2(3)=16.27$, $\chi^2(4)=18.47$

Testing hypotheses[b)]	No. of restrictions	Household consumption: Test statistic	Production: Test statistic	Capital accumulation: Test statistic
H2 vs. H1	4	7.38	13.50	9.42
H3 vs. H2	3	4.96	6.18	387.81
H4 vs. H2	3	2.71	1.26	6.68
H5 vs. H3	3	15.34	26.62	16.53
H5 vs. H4	3	17.59	31.54	397.66
H10 vs. H5	1	0.25	12.11	0.47
H6 vs. H2	4	12.58	27.66	126.93
H8 vs. H6	2	2.27	-	91.94
H9 vs. H6	1	5.91	7.55	-
H7 vs. H6	2	0.11	0.02	1.81
H7 vs. H9[c)]	1	1.28	1.92	1.81
H10 vs H7	1	7.86	17.23	276.06
H10 vs. H8	1	5.69	17.25	185.94
H11 vs. H10	3	111.57	58.28	6.71

a) $\lambda = (SSR_1/SSR_0)^{n/2}$, where SSR_0 and SSR_1 are the sum of squared residuals in the null hypothesis and the alternative, respectively, and n=60 is the number of observations. Thus, $-2 \log \lambda = n(\log SSR_0 - \log SSR_1)$.

b) See Figure 2.

c) This test can be performed only if $\varrho_1=1$, which is not necessarily the best partial adjustment model. In the previous test, the partial adjustment model giving the lowest SER is used, and thus it is not always the same model which is tested as H7 in these two cases.

6. SIMULATION RESULTS

In this final section, we briefly report the results of simulation experiments conducted on some of the estimated equations. The equations selected are respectively the general autoregressive-distributed lag and error correction models which gave the best overall performance, according to the estimation and test results in section 5. Ex post simulations of the endogenous variables are carried out for the estimation period 1969.1-1983.4 with the exogenous variables set equal to the observed values and the simulated values substituted for the lagged endogenous variables. In the starting period, the observed values of the lagged variables are used. The choice of starting period may thus affect the forecasting performance because the effect of the initial disturbance may accumulate during the simulation period.

The main results are reported in Table 10 in the form of the Relative Root Mean Square Error - RRMSE - defined as

$$\text{RRMSE} = \frac{\text{RMSE}}{\bar{Y}} = \frac{\{\frac{1}{T}\sum_{t=1}^{T}(Y_t-\hat{Y}_t)^2\}^{1/2}}{\frac{1}{T}\sum_{t=1}^{T}Y_t} \cdot 100,$$

where Y_t is observed and $\hat{Y}_t$ is simulated value in period t and T is the number of simulation periods. The results in the first three columns refer to the models in which white noise disturbances are assumed and, accordingly, with the disturbance process ignored in the simulation experiments. In the fourth and fifth columns, we report results for the error correction model with autoregressive disturbances, in which this particular structure of the disturbance process is taken into account.

The within sample period simulations seem, on the whole, to support the test results in section 5. The RRMSEs of the AD lag models is somewhat lower than those of the QEC models. The differences are, however, not very large, and this statistic ignores the problems in interpreting some of these lag models. For the consumption-income relationship, the RRMSE is about 2 per cent, the AD lag specification giving a somewhat better fit than the QEC model. The tracking performance of the latter model is illustrated in figures 4 and 6. (The corresponding input and output data are exhibited in figures 3 and 5.) The RRMSE of the production-demand relationships is 3.1 and 4.7 per cent, respectively. The RRMSE of the error correction model adjusted for residual autocorrelation is 4.2 per cent, a slight reduction compared with the 'white noise' formulation of the model. This indicates that our 'predictions' of the disturbance process serve to improve the overall tracking performance of the model. The simulation result for the capital-production relationship confirm the conclusion from section 5 that this is a far less satisfactory relationship. There are, however, no substantial difference between the various models in this case.

Table 10. Dynamic simulations of AD and QEC models. Relative Root mean Square Error (RMSE), per cent.

	Period of simulation	Model AD(5,5)	QEC(p_1,1)	PA	QEC(p_1,1), AR disturbances	PA with AR disturbances
Consumption-Income	1969.1-1983.4	1.86	2.10	-	-	-
	1984.1-1985.4	3.38	2.21	-	-	-
Production-Demand	1969.1-1983.4	3.11	4.73	4.71	4.24	4.35
	1984.1-1985.4	5.56	5.13	5.63	5.94	8.11
Capital-Production	1969.1-1983.4	10.77	14.72	14.55	14.14	15.17
	1984.1-1985.4	2.28	0.60	1.18	0.60	2.04

Additionally, we have performed post sample period simulations over eight quarters starting in 1984.1, still with the exogenous variables set equal to their observed values. In this case, the results are less conclusive than the within sample simulations.

According to the consumption-income relationship, the error-correction model now has a lower RRMSE than the AD lag model, 2.2 and 3.4 per cent, respectively. In the production-demand example, the RRMSE of the corresponding AD lag model are 5.1 and 5.6 per cent. The results for the capital-production relationship are remarkable; the RRMSE in the error correction formulation is almost 75 per cent lower than in the AD lag formulations.

Altogether, there are thus indications that in post sample simulations - which are, of course, the most relevant for judging a model's usefulness as a tool for actual forecasting - the error correction models are superior to general, unrestricted AD lag models. And, as pointed out earlier, it is also superior with respect to economic interpretability. On the other hand, we cannot conclude from these experiments that the error correction models adjusted for first and fourth order disturbance autocorrelation perform significantly better than the corresponding 'white noise' formulations. The reasons may be that in models adjusted for first and fourth order autocorrelation, there are much more influence from lagged endogenous variables because of increased lag length and thus errors in the endogenous variables may accumulate more rapidly than in the 'white noise' formulation of the model. The choice of starting period is probably also a more critical issue.

Figure 3. Household consumption and disposable income.
Million 1984 kroner.

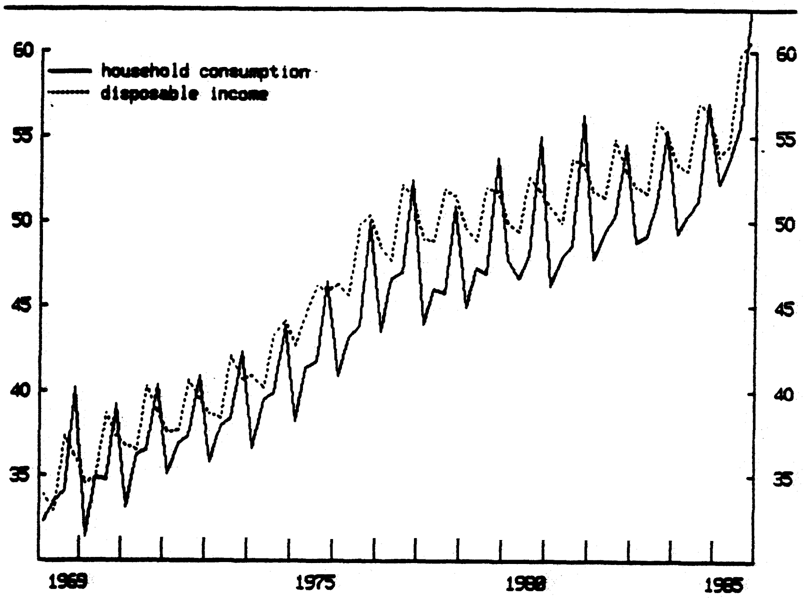


Figure 4. Household consumption. Simulated and observed. QEC(RHO1,1). Million 1984 kroner.

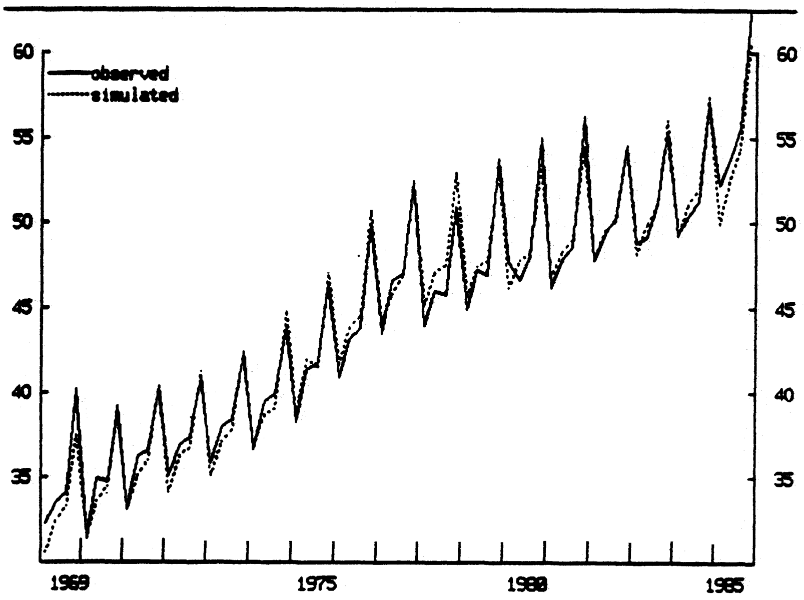


Figure 5. Production and demand in wood and printing industries. Million 1984 kroner.

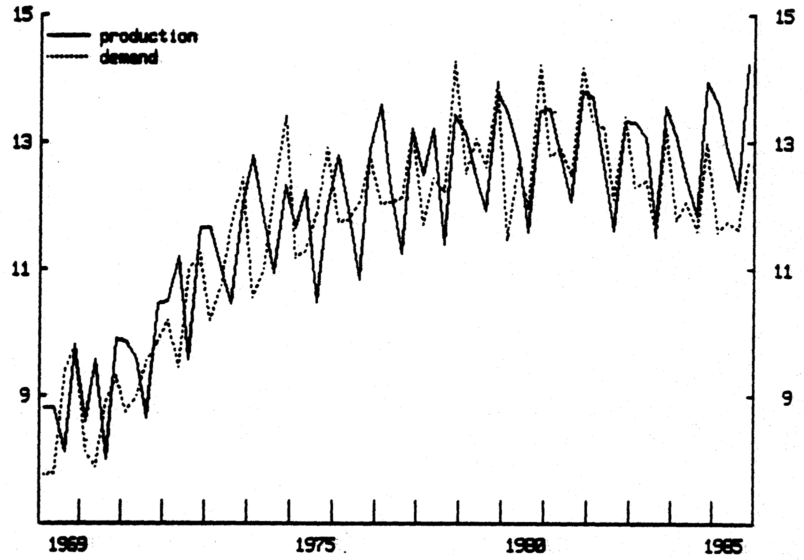
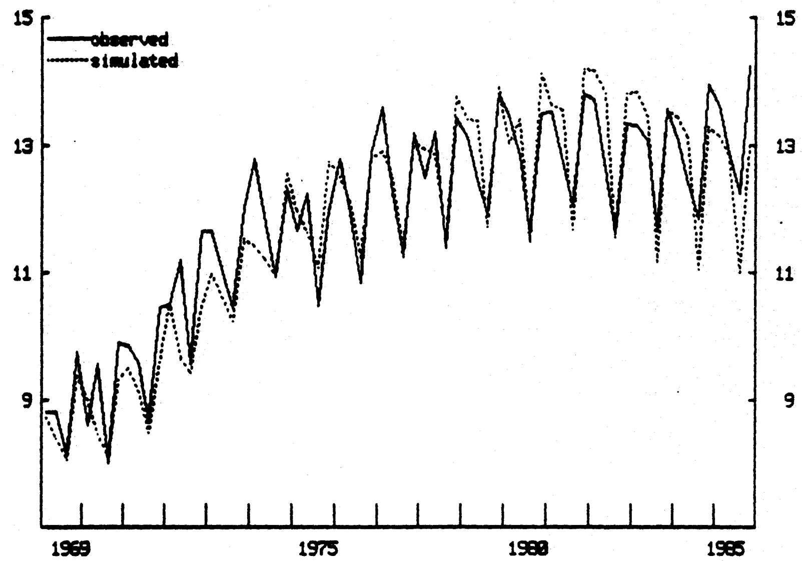


Figure 6. Production in wood and printing industries. Simulated and observed. QEC(RHO1,1). Million 1984 kroner.



FOOTNOTES

1) For a discussion of models with common factors in the lag polynomials, see Hendry and Mizon (1978) and Sargan (1980).

2) See Dhrymes (1971, p. 8). The mean lag between ε_t and Y_t, similarly defined, is $(1-\gamma)\varrho'(1)/\gamma$. Note that (2.18) defines the mean lag only if (2.11) is invertible, which implies that all roots of the polynomial equation $1-(1-\gamma)\varrho(z) = 0$ should lie outside the unit circle.

3) The alternative $\varrho(L) = L^4$ is also proposed, for models based on seasonally unadjusted data, by Davidson et al. (1978), Hendry and von Ungern-Sternberg (1981), and Hendry and Richard (1983, pp. 131-132). As noted by Hendry and Richard (1983, p. 131), the parametrization (2.19) "provides a convenient means of implementing long-run economic theories in dynamic models based on servomechanistic control principles".

4) Recall that (3.3) and (3.5) define the mean lag only if (3.2) and (3.4) are invertible, which implies that all roots of $1-(1-\gamma)(\varrho_1 z+\varrho_4 z^4-\varrho_1\varrho_4 z^5) = 0$ should lie outside the unit circle. This condition is satisfied for QEC(1,0) and QEC(0,1) (whenever $\gamma > 0$), but not for QEC(1,1). Confer footnote 2.

5) For QEC $(\varrho_1,1)$ we have, in particular, $\bar{\delta}_i = \gamma\delta_{2i}$.

6) This holds strictly if $Y_{-4}, Y_{-3},\ldots,Y_0$ are regarded as fixed (or conditionally on the value of these variables); otherwise this iterative procedure will give approximate ML estimates only.

7) A documentation of this data source is given in Olsen, Reymert, and Ulla (1985).

8) The definitions, sector classifications, and data sources are essentially the same as those used in the quarterly model KVARTS, a survey of which is given in Biørn, Jensen, and Reymert (1985). The regular quarterly national accounts contain no income account. Disposable household income is obtained by deducting net household taxes from the gross disposable income of the household sector. The latter is calculated from the data on wages, operating surplus, and depreciation in the national accounts and data on other income components collected from different sources (or estimated if no information is available). Household taxes are disaggregated to a quarterly periodicity from corre-

sponding annual data by means of an econometrically estimated tax equation.

9) The Box-Ljung statistic for testing autocorrelation in time series models is discussed in Ljung and Box (1978) as an extension of the Box-Pierce test statistic (Pierce (1971, 1972)).

10) Note, however, that the autoregressive disturbance process also affects the mean lag between X and Y, since the associated lag polynomials may be transformed into the lag polynomials of X and Y in the structural part of the equation. Eq. (3.5) does not take this contribution to the mean lag into account.

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