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## A random scale framework

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**Abstract:**
Amartya Sen has developed the so-called capability approach to meet the criticism that income alone may be insufficient as a measure of economic inequality. This is because knowledge about people's income does not tell us what they are able to acquire with that income. For example, people with the same income may not have the same access to health and transportation services, schools and opportunities in the labor market. Recently, there has been growing interest in empirical studies based on the capability approach. Most of these, however, are only loosely related to quantitative behavioral theory, at least in a concrete and empirically operational way. The purpose of this paper is to demonstrate that the theory of random scale (utility) models (RSM) offers a powerful theoretical and empirical framework for representing and accounting for key aspects of Sen's theory.

**Keywords:** Capability approach, Random scale, Discrete choice, Welfare function

**JEL classification:** C25, C35, D31, D63

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## Sammendrag

Amartya Sen har utviklet den såkalte capability-tilnærmingen for å imøtekomme kritikken at inntekt alene kan være et utilstrekkelig mål på økonomisk ulikhet. Grunnen til dette er at kjenskap til individers inntekt ikke sier noe om hva individene er i stand til å oppnå med sin inntekt. Personer med samme inntekt vil for eksempel ikke alltid ha samme tilgang til helsetjenester, transport, skoler og jobbmuligheter. I de senere årende har det vært en voksende interesse i å benytte capability tilnærmingen i empiriske analyser av ulikhet og velferd. Imidlertid er de fleste av disse studiene kun løselig knyttet til kvantitativ atferdsteori, i det minste på en konkret og empirisk operasjonell måte. Formålet med denne artikkelen er å vise at teorien for modeller basert på stokastisk nytte, eller stokastisk skala, representerer et effektivt teoretisk og empirisk rammeverk som gjør det mulig å ta hensyn til vesenlige aspekter ved Sens teori, og som det er svært vanskelig å ta hensyn til i tradisjonell økonomisk teori.

# 1. Introduction

The capability approach was initially conceived in the 1980s as a new way of looking at welfare economics. In it, Amartya Sen brought together a range of ideas that seemed to be neglected in traditional welfare economics. In traditional "welfarism", the distribution of income has been the central focus. Sen (1997) argues, however, that economic inequality is not necessarily the same as income inequality. The reason for this is that knowledge about people's income in itself does not tell us about other things that matter for their welfare. People may be restricted in their choices as a result of discrimination, customs, moral codes, political regime, climate, infrastructure, transport, organization of health care, etc. For example, in many cities the risk of becoming a victim of violence restricts sports and social activities for women.

The capability theory proposed by Sen has been discussed in several papers and books: see Sen (1979, 1980, 1982, 1984, 1985a, 1985b, 1987, 1992, 1993, 1997, 1979, 1998) and Drèze and Sen (2002); see also Robeyns (2003) and Robeyns and Kuklys (2005). Central to it is the notion of "functions" and "capabilities" (or capability sets). Functions are "beings and doings": that is, various states of human beings and activities that a person can undertake. The capability set represents a person's opportunities to achieve functionings. Thus, while "travelling" is a functioning, the opportunity to travel is an element of the person's capability set. Although Sen is not entirely clear in some of his writings, a reasonable interpretation might be that a normative evaluation of well-being should depend upon both the individual's achieved functionings and his or her capability sets, where the capability set represents the extent of freedom, whereas the achieved functionings measure aspects of welfare other than freedom (Sugden 1993, p. 1952).

Recently, there has been a growing interest in empirical studies based on Sen's capability approach: see, for example, Anand, Hunter and Smith (2005), Anand, Santos and Smith (2008), and Kuklys (2005), together with the references given there. These studies address central issues of the capability approach, such as the role of socio-economic and environmental factors, the ability of individuals to take advantage of opportunities that are objectively available to them, etc. However, these studies are typically reduced-form analyses of achieved functionings and do not address the challenge of establishing welfare measures, with explicit representation of the distribution of household preferences and capability sets, at least not in a concrete and empirically operational way. One obvious reason for this state of affairs is that Sen himself is rather vague on matters regarding structural empirical strategies, and it is far from evident how such strategies should be devised. Specifically, if one goes along with the view that a welfare function should depend on both achieved functionings and the capability sets, how should such a measure be formulated and justified on theoretical grounds? Since Sen does not address empirical methodology, researchers who attempt to apply the capability approach in an empirical context are left pretty much without guidance. This void easily leads to ad hoc procedures.

Sen seems to reject conventional economic choice theory as a useful methodological framework for generating welfare measures based on his capability approach: see, for example, Sen (1997). Conventional textbook theory typically deals with unconstrained choice (subject to the budget constraint), where preferences are deterministic and imply perfectly transitive choice behavior. Evidently, conventional unconstrained choice theory is hardly relevant for the purposes of the capability theory. In principle, constrained choice theory could offer an alternative methodology, because it would allow the researcher to account for the restrictions represented by the capability set. Constrained (deterministic) choice theory is, however, extremely complicated and impractical, apart from in very simple cases. As will be discussed further in Section 4.1, a second drawback with conventional constrained choice theory is, according to Sen, that the value of the capability set, and consequently the notion of "freedom", are, in Sen's interpretation, viewed as inadequately represented.

The purpose of this paper is to discuss the application of a theoretical and empirical framework, denoted a random scale model (RSM), as a strategy for constructing welfare measures based on the capability approach. A RSM is based on probabilistic choice theory of how representations of rank orderings of alternatives within a finite set should be formulated and what corresponding choice models should look like.[1] Here the role of randomness is to allow for uncertainty, noise or unobservables in the representation of the attractiveness of the alternatives. A great advantage with RSM is that it is practical and yet allows for quite flexible representations of preferences and choice constraints. This type of probabilistic choice model was in large part developed by psychologists, going back at least to Thurstone (1927). Since the pioneering work of Luce (1959a) and McFadden (1973), these models have become indispensable in the theory and applications of choice behavior in situations where the alternatives are qualitative, or discrete.

In several papers Sen seems to go a long way toward endorsing an "objective" approach to valuing functionings. But he does not say who should undertake such evaluations. The methodological issues discussed in this paper, however, do not depend on whether the approach is "subjective" (based on a sample of individuals) or "objective" (based, for example, on a sample of selected representative experts). In this paper we discuss how to apply information from rank orderings of functionings to establish a RSM, either from (i) choice data from a sample of agents, where by agent we mean an individual or a selected expert, or from (ii) data on agents' evaluations of utility levels that correspond

[1] The reason we deviate from using the notion of random utility is to avoid the mistaken idea that we are following a traditional utilitarian approach.

to stated income levels, or data on agents' reporting utility levels that correspond to stated alternatives. Furthermore, we discuss how welfare measures based on RSM can be constructed.

The paper is organized as follows. In Section 2 we discuss key features of the capability approach. Section 3 reviews basic aspects of the RSM framework. Specifically, we focus on the well-known result that choice—or ranking probabilities for choosing—or rank ordering alternatives depend on the choice set of available alternatives and their attributes, through suitable scale representations of alternatives. In Section 4 we show how the RSM framework can be used to derive different types of welfare measures as functions of the choice set, scale parameters that characterize preferences and depend on income, prices and other attributes that characterize the alternatives. Section 5 discusses an alternative RSM methodology, originally proposed by van Praag (1968), which is known as the Leyden School approach. Van Praag's approach is based on the dual preference representation, which is similar to a particular expenditure function representation. Section 6 contains a brief description of an approach that is similar to the one proposed by the Leyden School.

## 2. A discussion of key features of the capability approach

The capability approach is a theoretical framework that puts emphasis on the importance of freedom to achieve well-being, where freedom to achieve well-being is understood in terms of people's opportunities to choose and how to be. Amartya Sen pioneered the approach and several others have significantly developed it. For an introduction to the capability approach and how the approach has been applied in practice, see Robeyns (2003, 2006).

Central to the capability approach is the concept of *functioning*. Functionings are defined as "beings and doings": that is, various states of human beings and activities that a person can undertake (Sen 1979). Examples are "being well nourished", "being housed in a particular kind of house or apartment", "having obtained a specific education", "being literate", "consuming a specific amount goods", "choosing particular leisure activities", "consumption of energy for heating", and "participating in the labor market".

A second key concept is termed *capability*. Capabilities (or the capability set) represent a person's opportunities to achieve functionings. Thus, while "travelling" is a functioning, the opportunity to travel is an element of the person's capability set. The capability approach views functionings and capabilities as the best metric for most kinds of interpersonal evaluations. This is so because the capability approach aims to take into account individual heterogeneity in choice opportunities in addition to income. In Sen's original terminology, the notion of capability (or capability set) is equivalent to the set of functionings available to the individual (choice set).[2] However, as Robeyns (2003) and Robeyns and Kuklys (2005) have pointed out, the notion of capability has been used in several different ways in the theoretical literature.[3] Robeyns (2003) also notes that Sen in his later writings has used different definitions of capability.[4]

A third concept used in the capability approach is the notion of *conversion factors*. The relation between a good and the achievement of certain beings and doings is represented by a "conversion factor", which is defined as the degree to which a person can transform a resource into a functioning. For example, a person who is handicapped or someone who was never taught to ride a bike has a very low conversion factor as regards riding a bike.

The notions of capability set and conversion factor thus, in principle, allow non-preference information that is excluded by traditional utilitarianism to be accounted for. This includes, for example, a person's additional physical needs due to being physically disabled.

For the sake of illustrating these concepts, it may be instructive to consider an example. Assume that the most important factors determining the well-being of a household are the opportunities available in housing, schooling, and the labor market. Thus the relevant functionings in this context are housing alternatives (apartments, studios, and houses), types of school, and jobs. Assume that the schools, housing alternatives, and jobs are each classified into types that are observable to the social planner. This classification may also include location alternatives. The space of functionings therefore consists of the housing, schooling and job alternatives in the respective markets, and possibly location opportunities. However, the capability set of a particular household or person will be a subset of this space: in other words, the set of combinations of housing, schooling, and job alternatives available to the household. We shall return to this example in more detail below.

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[2] As Sen (1998, p. 15) writes: "A person's capability is, then, given by the set of alternative functioning vectors, from which the person can choose any one vector."

[3] For example, Nussbaum (1999) has labeled potential functionings "capabilities". Nussbaum also distinguishes between basic capabilities, internal capabilities, and combined capabilities. A basic capability set consists of the basic capabilities of human beings, such as the ability to reason, to imagine, to eat and speak, etc. An internal capability set represents restrictions on the set of capabilities that come from internal physical and psychological factors due to handicaps, deformed preferences, and fears, such as the inability to break out of a violent relationship. A combined capability set represents restrictions due to both internal psychological factors and restrictions imposed by the family, social and religious conventions, and the legal system.

[4] In his formalism, Sen (1985a, pp. 11–15) distinguishes between the commodity vector, the characteristics of the commodities, and the utilization function (which is the different patterns of use of the characteristics associated with the commodities). This may be fine in a theoretical discussion for the sake of clarifying concepts, but it is questionable whether it is very helpful in the context of making the theory operational in practice.

A frequent misunderstanding in the secondary literature concerns the use of the term freedom. Especially in his more recent work, Sen often equates capabilities with freedoms, without always specifying in more detail what kind of freedoms he is referring to. A careful reading of Sen's work clarifies that capabilities are freedoms conceived as real opportunities (Sen 1985a). For Sen, capabilities as freedoms refer to the *presence* of valuable options or alternatives in the sense of opportunities that exist not only formally or legally, but also effectively because they are available to the individual.[5]

As mentioned above, the motivation for Sen's capability approach stems from some problematic aspects of standard welfare theory. Traditional welfare analysis focuses on resources such as income and neglects important aspects related to "qualitative" alternatives and restrictions on the set of available alternatives that matter for well-being. The choice sets of such alternatives may vary substantially across individuals due to qualifications, social networks, discrimination, environmental factors, geographical location, etc. These aspects, among others, provide the motivation for why Sen introduces such concepts as functionings and capabilities in his theory of welfare.

Sen does not seem to regard the methodological apparatus of utility theory as a particularly useful framework for undertaking welfare analysis. His main objection seems to be that utility theory is not a satisfactory approach in the context of welfare analysis for a variety of reasons. First, he does not believe that the standard assumptions of utility theory, such as completeness and transitivity, hold. Second, he seems to be skeptical about the conventional subjective position of welfare assessment. Sen (1985a, pp. 33–35) declares his belief that a purely subjective view of well-being is "ultimately rejectable" and that "the limits of objectivity extend well into the assessment of well-being". Third, and as mentioned in the introduction, important variables that matter for well-being, such as choice constraints, are not explicitly addressed. Fourth, knowledge of choice behavior does not, without further assumptions, identify utility levels. As regards utility, and choice theory, Sen's position is somewhat surprising, since welfare analysis *also* involves statements about rank ordering of functionings, where rankings are undertaken by either individuals or "objective" experts representing the social planner. Thus one cannot avoid dealing with *some* notion of preferences and choice when making welfare judgments.[6] Furthermore, violations of transitivity may also be a problem within Sen's

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[5] See Sen (2002) for a general discussion on aspects of freedom.

[6] In his earlier writings, Sen seems to view preferences as related only to mental states, to be interpreted in terms of either pleasure or desire. He also argues that choice alternatives cannot always be rank-ordered in a manner perfectly consistent with that assumed by traditional utility theory. However, as Sugden (1993) points out, a more general interpretation is possible in which preferences are viewed as the mental states that precede choice. Thus to prefer alternative *b* to *c*, say, is to be in a state of mind in which one is disposed to choose *b* rather than *c*. This interpretation is clearly closer to common usage in contemporaneous economic theory. In later writings, however, such as Sen (1997), he acknowledges that modern usage of utility theory can be interpreted more broadly.

objective approach. This is because there is no guarantee that different experts will share the same opinion about rank orderings of functionings. In fact, it seems likely that different members of a representative expert group will rank functionings differently.

In this paper we define choice alternatives as vectors of functionings and the capability set (sometimes called the choice set) as the set of available choice alternatives, possibly individual-specific. Evidently, there is no essential loss of generality in defining the choice alternatives in such a way that they become mutually exclusive because this is solely a matter of suitable breakdown of the space of functionings.

Thus a person’s choice set is supposed to account for the individual’s ability, capability or will to take advantage of the opportunities offered to her or him. The functionings in the capability set may be characterized by a vector of characteristics (attributes), see Gorman (1956) and Lancaster (1966). The agent (individual, or selected expert) is viewed as having preferences over the universal set of alternatives. As a result, the agent’s preferences can be represented by a scale function (utility function), as we shall elaborate in detail in the next section. The value of the chosen alternative (vector of functionings) will consequently be the maximum of the scale function, subject to the choice set. The capability set may be interpreted either as the set of alternatives that are objectively available to the individual or as the set (subjective) of opportunities that are effectively available to the individual in the sense that she or he is able to make full use of all the options in the choice set, consistent with the notion of conversion mentioned above. In concrete empirical applications the extent to which the researcher is able to accommodate conversion aspects may, however, depend on how much detailed information is available about individual abilities and health status, as well as about the choice environment.

# 3. RSM and probabilistic choice models

The Random Scale Model is based on a theory of stochastic scale representations of an individual’s preference rank orderings of alternatives, and how the probability of the most preferred alternative can be characterized, as a function of the scale and the choice set. The RSM is analogous to random utility models (RUM), which were originally developed by psychologists, pioneered by Thurstone (1927). Whereas random utility models were developed to analyze individual choice behavior, an RSM may have a wider interpretation and allow applications that are not necessarily explicitly related to choice behavior in the traditional sense. The fact that the scales involved are random is of particular relevance because it allows the researcher to accommodate aspects, or variables, that are unobservable to him or her.[7]

To fix ideas, we shall now recapitulate some key concepts and developments within probabilistic choice theory which are directly useful for the RSM we have in mind. Consider a countable set, $S$, of "universal" choice objects. By this we mean that $S$ is the absolute maximal set of alternatives that are relevant, regardless of whether or not they are available to every agent in the population. Consider a particular agent (for simplicity we drop the indexation of the agent for now) with choice set $C$, possibly agent-specific. For some agents $C$ may be equal to $S$, but often the choice set will be a proper subset of $S$. The agent is assumed to have preferences over the alternatives in $S$. A central part of the theory is how choices from one choice set are related to choices from another choice set, given that the representation of preferences in some sense remains fixed. In particular, Luce (1959a) and subsequently others have proposed an axiomatic approach to obtain theoretically justified structures on the choice probabilities.

## 3.1. Probabilistic rationality and the Luce approach

Luce (1959a) proposed a fundamental axiom that is known as "Independence from Irrelevant Alternatives" (IIA). One way of presenting the axiom goes as follows: Let $J(C)$ denote the choice function—that is, the index of the most preferred alternative in $C$. As indicated above, the choice function $J(C)$ is perceived as stochastic due to taste variables that are unobserved to the researcher but known to the agent, and variables that are unclear to the researcher as well as to the agent in the sense that tastes may fluctuate in a way that is not foreseeable by the agent. Then, for choice alternative $j$ and choice sets $A$ and $C$ such that $j \in A \subset C \subseteq S$, IIA asserts that

(1) $$P(J(C) = j \mid J(C) \in A) = P(J(A) = j).$$

The relation in (1) states that the probability of choosing $j$ from $C$, given that the most preferred alternative belongs to $A$, is equal to the probability of choosing $j$ when $A$ is the choice set. Note that it is implicit in the notation that the probability on the right-hand side of (1), $P(J(A) = j)$, does *not* depend on alternatives in $C \setminus A$; they are irrelevant. The statement in (1) is clearly a rationality assertion. Note that since it is a probabilistic statement it does not mean that IIA needs to hold in every single experiment. Instead it is meant to hold on average, when the choice experiment is replicated a large number of times, or alternatively it should hold on average in a large sample of "identical"

[7] Note that an empirical model is also needed in the case of Sen's objective approach, because a sound methodology is necessary to accommodate the fact that different members of the selected expert group may rank functionings differently.

agents: that is, agents with identically distributed tastes. We may therefore think of IIA as an assumption of "probabilistic rationality". Another way of expressing IIA is that the rank ordering within any subset of the choice set is, on average, independent of alternatives outside the subset.

Luce (1959a) has demonstrated that IIA holds if and only if there exists a positive scale $\{v_j\}$, such that

$$P(J(C)=j)=\frac{v_j}{\sum_{r\in C} v_r}, \tag{2}$$

for $j\in C.$ Equation (2) expresses how the probability of choosing alternative *j* from *C* depends on the scale and the choice set *C*. The empirical counterpart is the fraction of observably identical agents that have chosen alternative *j* from *C*. Furthermore, the scale $\{v_j\}$ is unique up to multiplication by an arbitrary positive constant. This latter property is easily verified since $v_j/v_1 = P(J(S)=j)/P(J(S)=1),$ which implies that only the ratio $v_j/v_1$ is identified.

Subsequent authors have investigated the issue of a random scale or utility representation of IIA: see McFadden (1984). A random utility representation that rationalizes IIA is given as follows: Let $U_j = v_j\varepsilon_j,\ j\in S,$ where $\varepsilon_j,\ j=1,2,..,$ are i.i.d. positive random terms with c.d.f., $\exp(-1/x)$ (standardized type I extreme value distribution), for positive *x*.[8] This then means that the choice probability that follows from utility maximization—namely, $P(U_j = \max_{r\in C} U_r)$ – is equal to the expression on the right-hand side of (2). In other words, this random utility representation is consistent with the choice probabilities that follow from IIA.[9] As with the discussion above, the error terms $\varepsilon_j,\ j\in S,$ represent possible unpredictable variations in the agent's tastes across replications of identical choice experiments, whereas the corresponding scale values $v_j$, *j* = 1, 2, …, represent the mean or representative values of the alternatives.[10] Alternatively, the error terms may be viewed as representing unobserved variations in tastes across agents as well as agent-specific temporal uncertainty in tastes.

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[8] The classification of extreme value distributions used here follows Resnick (1987). The distributional assumption assumed here is equivalent to the assumption that $\log\varepsilon_j$ has c.d.f. $\exp(-\exp(-x)).$ The latter is the relevant one in a corresponding additive formulation of the utility function.

[9] The approach taken here is somewhat analogous to Sen (1991), where he proposes an axiom system to characterize preferences over choice sets. However, whereas IIA yields an explicit functional form characterization of the choice model in terms of preference terms and choice set, Sen does not demonstrate how his axioms can lead to operational results of relevance for empirical relations.

[10] Here, "mean" is understood as the representative or systematic term and not the expected value as defined in probability theory. Mathematically, the mean $EU_j$ is infinite when the c.d.f. of $U_j$ is standardized type I extreme value.

The model in (2) is the well-known Luce model (see Luce 1959a) that McFadden (1973) developed into a practical econometric framework known as the Multinomial Logit model: see, for example, McFadden (1973, 1984, 2001). McFadden and others have subsequently extended the model to accommodate situations where IIA is too restrictive. Bear in mind that both the choice set $C$ and the systematic terms $\{v_j\}$, representing the mean preferences, may be individual-specific. In empirical analysis, the term $v_j$ will often be specified as a function of observed characteristics of the agent and observed attributes of alternative $j$, such as alternative-specific prices and possibly other attributes characterizing the alternatives.

Once the scale $\{v_j\}$ has been given an empirical specification one can estimate the unknown parameters in the specification using well-known methods, such as the maximum likelihood procedure. The estimated model, represented by choice probabilities as in (2), can be applied for conducting policy simulation experiments to assess the effect on choice behavior by changing the choice/capability set $C$, or by changing some or all of the attributes characterizing the alternatives and entering the model through the systematic parts, $\{v_j\}$, of the random scale function.

The stochastic choice model reviewed above can also be used to analyze data where one has information about rank orderings of alternatives. Recently, so-called Stated Preference (SP) surveys have become increasingly popular: see, for example, Louviere, Hensher and Swait (2000) for a discussion of this methodology. In SP surveys individuals are asked to make choices, or alternatively to state the rank ordering of hypothetical alternatives presented. The advantage of the SP method is that it enables the analyst to obtain information about agents' rank orderings among all alternatives in a stated choice set and not just the most preferred one.[11]

Suppose for simplicity that the choice set consists of four alternatives: that is, $C = \{1, 2, 3, 4\}$. Let $Q_C(h, j, k)$ be the probability that alternative $h$ is the most preferred, alternative $j$ is the second most preferred, and alternative $k$ is the third most preferred. Then it can be shown that under the same distributional assumptions as above one obtains (Beggs, Cardell and Hausman 1981)

(3) $$Q_C(h, j, k) = P\{U_h > U_j > U_k\} = \frac{v_h}{\sum_{r \in C} v_r} \cdot \frac{v_j}{\sum_{r \in C \setminus \{h\}} v_r} \cdot \frac{v_k}{\sum_{r \in C \setminus \{h, j\}} v_r}.$$

[11] Some researchers express scepticism toward SP data because such data are not viewed as reliable as revealed choice data. It is also relevant that preferences may be experience-dependent, implying that decision-makers may find it hard to evaluate alternatives of which they have no experience.

We notice that (3) can be expressed as

(4) $$Q_C(h, j, k) = P(J(C) = h)P(J(C \setminus \{h\}) = j)P(J(C \setminus \{h, j\}) = k).$$

The interpretation of the right-hand side of (4) is quite intuitive. It conveys that the ranking of the alternatives in $C$ can be interpreted as if the agent first selects alternative $h$ as the most preferred one from $C$, second selects alternative $j$ as the most preferred one from the alternatives in $C\setminus\{h\}$, and finally selects $k$ as the most preferred one from the alternatives in $C\setminus\{h, j\}$.

We have seen above that probabilistic choice theory offers new opportunities to relax the rather strong consistency assumptions so central to conventional deterministic utility theory. As is well known, the motivation of psychologists such as Thurstone (1927) for proposing a random utility framework was to deal with the observational fact that individuals often violate transitivity when faced with replications of (seemingly) identical choice experiments (Thurstone 1927, Tversky,1969). His explanation was that decision-makers may be ambiguous about the precise value of the respective alternatives, in the sense that if the same choice setting is repeated they may choose a different alternative. This unpredictable temporal variation in tastes is represented by the stochastic error terms in the utility representation. However, at each moment in time (each choice setting presented) the agent (individual or member of the expert group) will of course choose the alternative that maximizes *momentary* utility. Thus the so-called random utility—or discrete choice—theory initiated by Thurstone (1927), and further developed by Luce (1959a), McFadden (1984), and others, is particularly designed to allow for this type of seemingly bounded rational behavior. Moreover, part of this theory has been developed with particular reference to practical methods for carrying out empirical analyses in cases where the choice set is discrete: see McFadden (2001).

Due to the random terms in the utility function, we realize that this type of stochastic choice models imply patterns of intransitivity. In particular, the Luce model presented above is consistent with the following type of cyclical behaviour, see Luce and Suppes (1965, p. 350): Let $j$, $k$ and $r$ be any three alternatives in $S$. For short, let $P(j,k) = P(J(\{j,k\}) = j),$ that is, $P(j,k)$ is the probability of preferring $j$ over $k$. The Luce model then implies the so-called *product rule*, namely,

(5) $$P(j,k)P(k,r)P(r, j) = P(j,r)P(r,k)P(k, j).$$

The intuition behind the product rule is as follows: Suppose an agent is making binary choices from the set $\{j,k,r\}.$ If we let $\succ$ denote "preferred to", the left-hand side of (5) is the probability of the intransitive chain $j \succ k \succ r \succ j,$ and the right-hand side of (5) is the probability of the intransitive chain $j \succ r \succ k \succ j.$ Thus the product rule can be interpreted as a particular assertion about cyclical behaviour: namely, that intransitive chains in different "directions" are equally probable.

## 3.2. Latent choice sets

The framework outlined above can readily be extended to settings with latent choice sets, as developed by Ben-Akiva and Watanatada (1981), Dagsvik (1994), and Dagsvik and Strøm (2006). A good example of this type of setting is as follows: Consider an agent facing the choice of which region to settle in. Let $S$ be the total set of regions. Assume that not all the regions are available to the agent. This will, for example, be relevant in the context of international migrations, where people from developing countries have choice opportunities as regards migration that differ from those of people from developed countries. It is also relevant for migration within some countries, where there are restrictions on long-term residence in major cities. Let the regions be indexed by $j \in C.$ Given the choice of region, the agent must make other decisions related to job opportunities, schooling, recreational and child-care facilities, etc. Assume that these sub-alternatives within regions are not observable to the researcher and denote them by elemental alternatives (Ben-Akiva and Watanatada, 1981). Let $k$ = 1, 2, …, be an enumeration of all combinations of elemental alternatives, $K_j$ the corresponding choice set of latent elemental alternatives within region $j$, and $n_j$ the number of elemental alternatives in $K_j$. Let $U_{jk} = v_j \varepsilon_{jk},$ be the utility of elemental alternative $k$ in region $j$, where $v_j$ is the representative utility of region $j$. Only the random error terms $\{\varepsilon_{jk}\}$ depend on the respective elemental alternatives, since they represent unobservable attributes of the elemental alternatives (unobservable to the researcher). Under the same assumptions about the error terms as above, it follows that the probability that the agent will choose elemental alternative $k$ in region $j$ is equal to

(6) $$P(U_{jk} = \max_{r \in C, q \in K_r} \max U_{rq}) = \frac{v_j}{\sum_{r \in C} \sum_{q \in K_r} v_r} = \frac{v_j}{\sum_{r \in C} n_r v_r}$$

for $k \in K_j.$ Note that the expressions on the right-hand side of (6) do not depend on $k$ because the observing researcher has no information about the elemental alternatives, and they therefore have equal probability of being selected, as perceived by the researcher. From this it follows that the probability of choosing to live in region $j$ is given by

(7) $$P(J(C) = j) = \sum_{r \in K_j} \frac{v_j}{\sum_{r \in C} n_r v_r} = \frac{n_j v_j}{\sum_{r \in C} n_r v_r},$$

where $J(C)$ denotes the choice of region from $C$. We note that the choice probability in (7) (of the observable choice) has a form where the representative scale terms are weighted by the size of the respective choice set of latent elemental alternatives. Remember also that the scale values $\{n_j v_j\}$ are aggregate ones, representing the mean value of region $j$, in contrast to the respective scale values $\{v_j\}$ of the elemental alternatives. As noted in Section 3.1 both the elemental as well as the aggregate scales are unique up to multiplication of a constant. Although the terms $\{n_j\}$ are unobservable, they can be represented by variables characterizing the size of the choice sets of elemental alternatives. How depends on the specific topic being studied. Another possibility is to conduct SP surveys so as to obtain data on the distribution of pure preferences. Such data will enable the researcher to estimate the parameters of the deterministic terms of the scale (utility) function.

## 3.3. Discrete/Continuous choice

In many choice situations a discrete choice is made jointly with an associate continuous choice. For example, a worker may face the problem of which job to choose and how many hours to work (conditional on the choice of job). Another example is a consumer who considers purchasing electric versus gas appliances, as well as how much electricity or gas to consume. A third example is a household that chooses which type of car to own and the intensity of car use. Such choice situations are called discrete/continuous, reflecting the fact that the choice set along one dimension is discrete while it is continuous along another. See McFadden (1984) and Hanemann (1984) for further examples and analyses. Without going into details, the discrete/continuous choice setting can in many cases be formally viewed as a two-stage choice where the agent chooses the discrete alternative in the first stage and the continuous one in the second. Here it is implicit that in the first stage the agent takes into account the fact that the subsequent second-stage choice is made optimally. Thus in the discrete/continuous choice settings the utility $U_j$ may be interpreted as a conditional indirect utility given the discrete alternative $j$. That is, $U_j$ is the highest utility the agent can attain when maximizing utility over the continuous alternatives, subject to the budget constraint and given alternative $j$. The demand function that corresponds to the continuous choice, given discrete alternative $j$, can be obtained by applying Roy's identity on $U_j$.[12]

## 3.4. Relaxation of IIA

It is well known that IIA may be too restrictive in some choice settings. It is in most cases an empirical matter whether IIA is viewed as reasonable or not. See McFadden (1978, 1981, 1984) for a discussion of random utility models that do not depend on the IIA property. It

[12] Recall that Roy's identify yields the demand of a (continuous) good as the negative partial derivative of the indirect utility with respect to the good's price, divided by the partial derivative of the indirect utility with respect to income.

turns out that the IIA property depends crucially on the utilities being independent and identically distributed across alternatives. To allow for more general behavioral patterns, McFadden (1978) proposed what he called the Generalized Extreme Value model (GEV). The GEV class is very general and contains the Luce model as a special case. Specifically, Dagsvik (1994, 1995) has demonstrated that the class of RSM can be approximated arbitrarily closely by GEV models. The GEV class is specified as follows: Let

$$G(x_1, x_2, .., x_M) = -\log F(1/x_1, 1/x_2, .., 1/x_M),$$

where $x_j > 0$ for all *j*, and where *F* is the joint c.d.f. of the random error terms $(\varepsilon_1, \varepsilon_2, ..., \varepsilon_M)$, and *M* is the size of *S*. The mapping *G*, from $(0,\infty]^M$ to $[0,\infty],$ is assumed to be linear homogeneous and in addition to satisfy suitable regularity conditions to ensure that *F* is a continuously differentiable c.d.f., see McFadden (1978). It follows that the one-dimensional marginal distributions are equal to $\exp(-1/x)$, $x > 0$. Let *C* be a subset of *S* and $G^C = -\log F^C$, where $F^C$ is the joint c.d.f. of the random error terms associated with alternatives in *C*. Then it follows that the corresponding choice probability for selecting alternative j from *C* is given by

(8) $$P(J(C) = j) = \frac{v_j G_j^C(v_1, v_2, ..)}{G^C(v_1, v_2, ..)},$$

where $G_j^C$ denotes the partial derivative of $G^C$ with respect to the j-th argument. For example, if $G(x_1, x_2, .., x_M) = x_1 + x_2 + .. + x_M$, the Luce model follows. Another example that leads to a nested multinomial logit model follows from the special case with $M = 3$, and $G(x_1, x_2, x_3) = x_1 + (x_1^{1/\rho} + x_2^{1/\rho})^\rho$, where $0 < \rho \le 1$. The parameter $\rho$ can be interpreted as $\rho^2 = 1 - corr(\log\varepsilon_2, \log\varepsilon_3)$. Thus this specification allows the random error terms associated with alternatives 2 and 3 to be correlated, whereas $\varepsilon_1$ is independent of $(\varepsilon_2, \varepsilon_3)$.

# 4. Welfare measurement

## 4.1. The constrained indirect random scale

Recall that in the capability approach the evaluation of well-being is a function of both the chosen functioning and the capability set. Sen even argues that in some instances capabilities may be more relevant than functionings (1992, p. 41). The challenge is thus how an index of well-being can be constructed to accommodate these requirements. This issue will now be addressed.

Define the constrained indirect random scale as the scale of the achieved alternative in $C$, which is the maximum of the scale function taken over the alternatives in the choice set $C$. As reviewed in Section 2, we adopt Thurstone's setting by interpreting the random terms of the scale function as representing unobserved heterogeneity in preferences as well as being random to the agent him- or herself, in the sense that under repetition of seemingly identical choice experiments the agent may choose different alternatives on each occasion due to his or her difficulties in evaluating the precise value of the alternatives once and for all. Bear in mind, therefore, that although the agent maximizes the momentary scale function, the error terms may change over repeated choice settings. Now it follows from the distributional assumptions above for the GEV case (see Section 3.4) that

$$P\left(\max_{k\in C}(v_k\varepsilon_k)\le u\right)=P\left(\bigcap_{k\in C}(\varepsilon_k\le u/v_k)\right)=F^C(u/v_1,u/v_2,...)$$
$$=\exp(-G^C(v_1/u,v_2/u,...))=\exp(-u^{-1}G^C(v_1,v_2,...)).$$

The last equality above follows from the linear homogeneity property of $G^C$. Thus the result above implies that

(9) $$\max_{k\in C}(v_k\varepsilon_k)\overset{d}{=}\eta G^C(v_1,v_2,...),$$

where $\overset{d}{=}$ means equality in distribution and $\eta$ is a random term that has c.d.f. equal to $\exp(-1/x)$, $x > 0$, and is independent of $\{v_k\}$.

In many empirical settings $v_j$ can be expressed in the form

(10) $$v_j=g_j(y,w_j,z_j,X),$$

where $y$ is income, $w_j$ is the cost associated with alternative $j$, $z_j$ is a vector of alternative $j$-specific attributes other than cost, $X$ is a vector of individual characteristics, and $g$ is a suitable parametric function. In the case of discrete/continuous choice settings, the function $g_j(y,w_j,z_j,X)$ may also depend on a vector of prices, $p$, say, of (implicit) continuous goods, but for notational simplicity we shall suppress such prices in the notation.

Here we have abstracted from concrete practical aspects on how functionings, capability sets, costs, non-pecuniary attributes, and characteristics should be defined and how the corresponding data should be collected. These issues are by no means simple in the context of welfare analysis. In this paper, we shall simply assume that these problems have been resolved and that one has obtained data on agents’ choices (or rank orderings), where the sample consists of either individuals or a group of representative experts. In this situation there exists a well-established statistical methodology for estimation and testing of the concrete parametric specification of the function $g$ appearing in (9). Let $w=(w_1,w_2,...,w_m)$ and $z=(z_1,z_2,...,z_m)$. By applying the result in (9) it follows that the constrained indirect random scale function, say $V_C(y,w,z,X,\tilde{\varepsilon})$, can be written as

(11) $$V_C(y,w,z,X,\tilde{\varepsilon})=\max_{k\in C}U_k=\bar{V}_C(y,w,z,X)\eta,$$

where $\tilde{\varepsilon}=(\varepsilon_1,\varepsilon_2,...,)$ and

(12) $$\bar{V}_C(y,w,z,X)=G^C(g_1(y,w_1,z_1,X),g_2(y,w_2,z_2,X),...).$$

In the special case where IIA holds, (12) reduces to

(13) $$\bar{V}_C(y,w,z,X)=\sum_{r\in C}v_r=\sum_{r\in C}g_r(y,w_r,z_r,X).$$

In the more general case with latent alternatives, the deterministic part of the welfare function in (12) takes the form

(14) $$\bar{V}_C(y,w,z,X)=\sum_{r\in C}n_r g_r(y,w_r,z_r,X).$$

The term $\bar{V}_C(y,w,z,X)$ is the deterministic part (representative part) of the constrained indirect scale, conditional on individuals with characteristics $X$, and, as mentioned above, the random term $\eta$ is independent of $\bar{V}_C(y,w,z,X)$ and has c.d.f. equal to $\exp(-1/x)$, $x>0$. Note, furthermore, that the representative constrained indirect scale functions in (12), or (13) or (14), depend on the entire choice set, in contrast to the traditional theory. Recall that in conventional deterministic utility theory the indirect utility will depend on the choice set only in cases where the highest-ranked alternative in $S$ is not contained in the choice set $C$. Otherwise it will be independent of the choice set. Consequently, in the conventional theory the indirect utility does not depend on the choice set apart from in special cases. Sen argues that even when alternatives that are ranked lower than the currently chosen one are removed from the choice set, the agent may still experience a psychological loss of “freedom”. This is equivalent to the property that the welfare function should increase when the choice set increases. In other words, Sen argues that an increasing capability set of a person is associated with increasing freedom for the person. Indeed, the representative constrained indirect scale function has this property. To realize this, let $C$ and $D$ be two capability sets such that $C\subset D$. Let $A=D\setminus C$. Then we have

$$V_D(y,w,z,X,\tilde{\varepsilon})=\max_{k\in D}U_k=\max(\max_{k\in C}U_k,\max_{k\in A}U_k)$$

$$=\max(V_D(y,w,z,X,\tilde{\varepsilon}),\max_{k\in A}U_k)\geq V_C(y,w,z,X,\tilde{\varepsilon}).$$

Hence, since by (9) and (11) the constrained indirect scale has type I extreme value c.d.f. $\exp(-1/u)$, the inequality above yields, for any positive $u$, that

$$\exp(-\bar{V}_D(y,w,z,X)/u) = P(V_D(y,w,z,X,\tilde{\varepsilon}) \le u)$$
$$\le P(V_C(y,w,z,X) \le u) = \exp(-\bar{V}_C(y,w,z,X)/u),$$

which is equivalent to $\bar{V}_D(y,w,z,X) \ge \bar{V}_C(y,w,z,X)$.[13] Thus the RSM approach proposed in this paper has the advantage of offering a possible interpretation of how and why welfare evaluation in this case will depend on the entire capability set and what a corresponding representation of the indirect scale function looks like. Furthermore, we have shown that the welfare function is non-decreasing as a function of the choice set. When IIA holds we realize immediately from (13) that when $D \setminus C$ is non-empty, then $\bar{V}_D(y,w,z,X) > \bar{V}_C(y,w,z,X)$. In general, the inequality need not be strict even if $D \setminus C$ is non-empty, because the alternatives in $D \setminus C$ may have the exact same value as alternatives in $C$. Specifically, the reason why the representative constrained indirect scale function depends on the capability set in our approach stems from the property that the random scale function depends on stochastic terms. Intuition suggests that since the error terms may fluctuate randomly across replications of choice experiments, then in some instances agents may not be constrained in their choice, whereas in other instances they may be constrained. As a result, the representative constrained indirect scale function will depend on the choice set because this part represents "average" behavior. In contrast, if perfect transitivity were to hold, then the welfare function would not depend on $C$ when the highest-ranked alternative in $S$ belongs to $C$.

One possible explanation for the seemingly irrational perceptions of experiencing a loss of freedom when alternatives other than the most preferred one are removed from the choice set is the following: Since the agents' preferences are uncertain in the sense used by Thurstone, they know that there is a chance that they may revise the evaluations of the alternatives several times in the future. Accordingly, the alternative that is chosen currently may not be the most preferred one at all future points of time, due to the influence of whims in perceptions and problems with assessing the precise value of the alternatives once and for all. Even in seemingly identical choice settings, an alternative other than the currently chosen one may thus be the most preferred at some future point in time. Thus, if an alternative other than the currently chosen one is removed, the agent may experience a loss of freedom because the alternative being removed might very well turn out to be the most preferred one at some time in the future. The above argument is applicable in the general case in which the error

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[13] Since the expectation of a random variable with c.d.f. $\exp(-1/x), x > 0$, is infinite one cannot apply the familiar result that for random variable $X$ and $Y$ if $X > Y$ then $EX > EY$.

terms are interpreted as representing intra- as well as inter-agent randomness (due to unobservable variables that affect tastes).

## 4.2. Toward a general characterization of the welfare function

Whereas the indirect scale function discussed above can be used to represent rank orderingsof alternatives, it cannot without further assumptions be taken as a representation of welfare that accommodates interpersonal comparisons in a rigorous sense. We shall now apply principles from the theory of meaningfulness, as developed by Falmagne and Narens (1983), Roberts and Rosenbaum (1986), and Aczél, Roberts and Rosenbaum (1986) to propose a characterization of the welfare function. Luce (1959b) observed that the general form of a functional relationship between variables is greatly restricted if we know the scale type of the dependent and independent variables, where the scale type is defined by a class of admissible transformations. The most common scale types are the ratio scales, interval scales, and the ordinal scales. For ratio scales, for example, the class of admissible transformations consists of the transformations $x \rightarrow \gamma x,$ where $\gamma$ is a positive constant. Briefly, a statement involving scales of measurement is called *meaningful* if its truth or falsity is unchanged whenever all scales used in the statement are transformed by admissible transformations. In practice, however, the class of admissible transformations may be unknown. In particular, it is not clear on which scale the welfare function should be measured and thus which is the corresponding class of admissible transformations. Note, furthermore, that the constrained indirect utility is an ordinal concept, so in general the individual welfare function of individual *i* has the form $\lambda_i(V_{C_i}(y_i, w, z, X_i, \tilde{\varepsilon}_i)),$ where $\lambda_i$ is a strictly increasing mapping, possibly individual-specific. For convenience, let *DC* and *Dy* denote the distribution of the choice sets and incomes across the population respectively, and let $W(w, z, DC, Dy)$ denote the corresponding aggregate welfare function. We shall now provide a characterization of *W*. The function *W* depends on $(w, z, DC, Dy)$ through the respective individual constrained indirect scales: that is,

$$W(w, z, DC, Dy) = \tilde{\xi}(\lambda_1(V_{C_1}(y_1, w, z, X_1, \tilde{\varepsilon}_1)), \lambda_2(V_{C_2}(y_2, w, z, X_2, \tilde{\varepsilon}_2)), \ldots),$$

for some suitable mapping $\tilde{\xi}$ that remains to be determined. Due to (11), we can express the welfare function above as

$$W(w, z, DC, Dy) = \tilde{\xi}(\lambda_1(\bar{V}_{C_1}(y_1, w, z, X_1)\eta_1), \lambda_2(\bar{V}_{C_2}(y_2, w, z, X_2)\eta_2), \ldots),$$

But since the mapping $\tilde{\tilde{\xi}}$ so far is completely general, the equation above is equivalent to

$$W(w,z,DC,Dy) = \xi(\bar{V}_{C_1}(y_1,w,z,X_1),\bar{V}_{C_2}(y_2,w,z,X_2),...),$$

where $\xi(x_1,x_2,...) = \tilde{\tilde{\xi}}(\lambda_1(x_1\eta_1),\lambda_2(x_2\eta_2),...).$ Recall next that the scale $\{v_j\}$ is unique only up to an arbitrary positive multiplicative constant. This means that $\{v_j\}$ is measured on a ratio scale. It follows that the corresponding representative constrained indirect scale $\bar{V}_C(y,w,z,X)$ given in (12), (13) or (14) is therefore also measured on a ratio scale. Let $(C_i, y_i, w, z, X_i)$ and $(\tilde{C}_i, \tilde{y}_i, \tilde{w}, \tilde{z}, X_i)$ represent two different capability sets, incomes, prices, and attributes for household $i$. A fundamental assumption we shall now postulate is that it is *meaningful* to assert, for any positive vectors $\mu = (\mu_1, \mu_2,...,)$ and $\theta = (\theta_1,\theta_2,...,),$ then

$$\xi_\mu(\mu_1\bar{V}_{C_1}(y_1,w,z,X_1),\mu_2\bar{V}_{C_2}(y_2,w,z,X_2),...) > \xi_\mu(\mu_1\bar{V}_{\tilde{C}_1}(\tilde{y}_1,\tilde{w},\tilde{z},X_1),\mu_2\bar{V}_{\tilde{C}_2}(\tilde{y}_2,\tilde{w},\tilde{z},X_2),...)$$

holds if and only if

$$\xi_\theta(\theta_1\bar{V}_{C_1}(y_1,w,z,X_1),\theta_2\bar{V}_{C_2}(y_2,w,z,X_2),...) > \xi_\theta(\theta_1\bar{V}_{\tilde{C}_1}(\tilde{y}_1,\tilde{w},\tilde{z},X_1),\theta_2\bar{V}_{\tilde{C}_2}(\tilde{y}_2,\tilde{w},\tilde{z},X_2),...)$$

holds. The interpretation of this statement is that a change in the input scale (corresponding to a change in the unit of measurement) will not change the initial inequality. Note that here the mapping $\xi_\mu$ is allowed to depend on the unit of measurement. This definition of meaningfulness was proposed by Falmagne and Narens (1983) and seems to be a minimal, yet essential, requirement that scientific laws should satisfy. A stronger statement is obtained when this mapping is independent of the input scale. Following Falmagne and Narens (1983) and Falmagne (1985), the welfare function is said to be *dimensional-invariant* if

$$\xi(\mu_1\bar{V}_{C_1}(y_1,w,z,X_1),\mu_2\bar{V}_{C_2}(y_2,w,z,X_2),...) > \xi(\mu_1\bar{V}_{\tilde{C}_1}(\tilde{y}_1,\tilde{w},\tilde{z},X_1),\mu_2\bar{V}_{\tilde{C}_2}(\tilde{y}_2,\tilde{w},\tilde{z},X_2),...)$$

holds if and only if

$$\xi(\theta_1\bar{V}_{C_1}(y_1,w,z,X_1),\theta_2\bar{V}_{C_2}(y_2,w,z,X_2),...) > \xi(\theta_1\bar{V}_{\tilde{C}_1}(\tilde{y}_1,\tilde{w},\tilde{z},X_1),\theta_2\bar{V}_{\tilde{C}_2}(\tilde{y}_2,\tilde{w},\tilde{z},X_2),...)$$

holds. What distinguishes dimensional invariance from meaningfulness is that in the latter case the mapping $\xi$ does not depend on the respective scales.

The interpretation of dimensional invariance in our context is that different ratio scale representations of the individual representative constrained indirect scales are viewed as equivalent. The motivation is twofold. First, the social planner has information only about the (estimated) representative constrained indirect scale values and thus has no basis for assigning particular individual utility levels. Second, it is consistent with a social planner who asserts that differences in individual utility levels should not matter for comparing welfare policies as long as the structure of the representative constrained indirect scale is given, as revealed by data on preference orderings. In other words, individual utility levels, as subjectively perceived by the individuals, should not matter for comparisons of welfare policies. The latter assertion appears consistent with Sen's objective position.

Falmagne and Narens (1983) have proved that dimensional invariance implies that

(15) $$W(w,z,DC,Dy)=T\left(\prod_{i=1}^{N}\bar{V}_{C_i}(y_i,w,z,X_i)^{\beta_i}\right),$$

where $T$ is a strictly increasing continuous function, $N$ is the size of the population, and $\{\beta_i\}$ are positive weights: see, for example, Theorem 14.17 in Falmagne (1985).

The welfare function representation given in (15) is sufficient for ordinal comparisons of the welfare of different regimes, and it does not require knowledge of $T$. However, the social planner may also have the additional ambition of comparing changes in welfare. In this case the welfare function in (15) is not adequate. To this end we now postulate that for any $c$ > 0, $(w,z,DC,Dy)$ and $(w',z',D'C,D'y),$ it is meaningful to assert that

(16) $$W(w',z',D'C,D'y)=cW(w,z,DC,Dy).$$

The assertion in (16) means that one believes it is meaningful to measure relative changes: that is, to assert that a change of input variables from $(w,z,DC,Dy)$ to $(w',z',D'C,D'y)$ implies some relative change of welfare, in this case equal to $c-1.$ For example, if $c$ = 1.1, (16) asserts that it is meaningful to say that a change of input from $(w,z,DC,Dy)$ to $(w',z',D'C,D'y)$ yields a 10 percent increase in welfare. Roberts and Rosenbaum (1986) demonstrate that the assumption of meaningfulness in (16) implies that the welfare function is measured on a ratio scale. It now follows from (11) and the theorem proved in Aczél, Roberts and Rosenbaum (1986) that the welfare function is equivalent to a weighted geometric mean: that is, (15) holds with $T(x) = Ax$, where $A$ is an arbitrary positive constant.

Although the individual-specific weights $\{\beta_i\}$ are independent of the individual constrained indirect utilities, they can be specified (by the social planner) as function of individual characteristics such as income.

We shall next provide an interpretation and justification for the special case where the weights do not depend on income and where the weights are equal: that is, $\beta_i=\beta$ for all $i$. We can of course, with no loss of generality, normalize such that $\beta=1$ in this case. Assume first that the weights are independent of income. Consider two individuals, indexed by $i$ and $h$. Then from (10) we get that the respective values of functioning $j$ for these individuals are given by

$$g_j(y_i, w_j, z_j, X_i)^{\beta_i} \quad \text{and} \quad g_j(y_h, w_j, z_j, X_h)^{\beta_h},$$

where we recall that the parameters of the functions $\{g_j\}$ are identified from choice behavior. Suppose now that there is a "basic" functioning, say functioning 1, such that it makes sense to assert that for all $i$ and $h$,

(17) $$g_1(y_i, w_1, z_1, X_i)^{\beta_i} = g_1(y_h, w_1, z_1, X_h)^{\beta_h},$$

in the special case where $y_i = y_h$.[14] Evidently, since (17) is supposed to hold for any $i$ and $h$ it implies that $\beta_i = \beta$. The assertion in (17) means that, from the point of view of the social planner, the basic functioning is viewed as equally valuable to each member of society, provided that everyone has the same income. In other words, the above assertion reflects an aspect of the social planner's views on fairness and justice. Note that this assertion does not entail any judgment about the subjective individual values, as perceived by the agents.

## 4.3. The capability-adjusted income distribution

In many instances it is of interest to apply distributional measures such as the income distribution or the Gini coefficient. This is because a single aggregate welfare function of the type discussed above does not discriminate between cases with various degrees of inequality as long as the (weighted) sum of the constrained indirect scales are the same. In this section we shall introduce a new welfare measure called the *capability-adjusted income distribution*. The capability-adjusted income distribution is a money metric measure that is supposed to account for inequality due to *both* the distribution of income and capability sets. Consider a social planner who faces the problem of rank-ordering two combinations of capability sets and incomes, namely $(C, y)$ and $(C', y')$, given prices, non-pecuniary attributes, and individual characteristics. In other words, the social planner must decide whether or not he or she views the combination $(C, y)$ to be better for the individual than $(C', y')$. To the social planner, the vector of random variables $\tilde{\varepsilon}_i$ is uncertain. Assume, furthermore, that the axioms of the expected utility theory hold. In this case the value of $(C, y)$, conditional on $(w, z, X_i)$, can be represented by $E\psi(V_{C_i}(y_i, w, z, X_i, \tilde{\varepsilon}_i))$, where $\psi$ is a strictly increasing (unknown) function and the expectation is taken with respect to $\tilde{\varepsilon}_i$, given the information of the social planner. As is well known, the function $\psi$ is unique up to an affine transformation. Here it is assumed that the social

[14] Examples of basic functioning may be "clean air" and "clean water".

planner knows the distribution of $\tilde{\varepsilon}_i$.[15] Let $C_0$ be a "reference" capability set, consisting of functionings viewed by the social planner as necessary for maintaining a reasonable level of well-being, and let $(w^0, z^0)$ be the corresponding prices and non-pecuniary attributes associated with the alternatives in $C_0$. We define the *capability-adjusted income*, $\tilde{Y}_i$, of individual *i* by

(18) $$E\psi(V_{C_i}(y_i, w, z, X_i, \tilde{\varepsilon}_i)) = E\psi(V_{C_0}(\tilde{Y}_i, w^0, z^0, X_i, \tilde{\varepsilon}_i)).$$

If the representative constrained indirect scale is strictly increasing in income, the capability-adjusted income will be uniquely determined by (18). Thus $\tilde{Y}_i$ is the income, determined by the social planner, to be necessary and sufficient for achieving the expected welfare when the capability set is replaced by the reference set $C_0$, the reference price, and attributes, $(w^0, z^0)$. The interpretation of (18) is that the social planner determines the capability-adjusted income such that the expected value, given the capability set, current income, prices, and non-pecuniary attributes equals the expected value, given the reference capability set, adjusted income, price, and non-pecuniary attributes. The definition in (18) implies the reasonable property that $\tilde{Y}_i$ increases as the actual capability set $C_i$ increases. Recall that from (11) we have that $V_{C_i}(y_i, w, z, X_i, \tilde{\varepsilon}) = \bar{V}_{C_i}(y_i, w, z, X_i)\eta_i$, where the random variable $\eta_i$ is positive and has c.d.f. that is independent of $(C_i, y_i, w, z, X_i)$. Hence, (18) is equivalent to

(19) $$E\psi(\bar{V}_{C_i}(y_i, w, z, X_i)\eta_i) = E\psi(\bar{V}_{C_0}(\tilde{Y}_i, w^0, z^0, X_i)\eta_i).$$

From the Lemma in Appendix A it follows that (19) is equivalent to

(20) $$\bar{V}_{C_i}(y_i, w, z, X_i) = \bar{V}_{C_0}(\tilde{Y}_i, w^0, z^0, X_i).$$

The result in (20) is very important because it demonstrates that one can compute the adjusted incomes without knowing the transformation $\psi$. For the sake of interpretation we realize that (20) implies that $\tilde{Y}_i = y_i - CV_i$, where $CV_i$ is the Compensating Variation that corresponds to a "reform" that replaces the choice set $C_i$ by the reference set $C_0$, reference price, and attributes $(w^0, z^0)$.[16] On the basis of the capability-adjusted income distribution one can compute corresponding capability-adjusted inequality measures such as the capability-adjusted Gini coefficient.

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[15] A rationale for assuming that the distribution of $\tilde{\varepsilon}_i$ is known to the social planner is that he or she has been informed by the researcher that this follows from IIA.

[16] Provided the social planner knows that (11) holds, then the result in (20) does not depend on the c.d.f. of $\eta$. In other words, (20) holds even if the social planner uses any subjective c.d.f. when calculating the expectation.

The capability-adjusted income distribution allows the researcher to assess the welfare effect of heterogeneity in capability sets in a practical way. We shall illustrate how this can be done in the examples below.[17]

## 4.4. Examples

*Example 1*

Consider a woman who faces the choice of participating in a combination of available social, cultural, and leisure activities. Let *S* be the index set of this list of combinations of activities (functionings). Specifically, woman *i* faces a choice set, $C_i$, of feasible combinations of activities that may be less than *S* because she may, for example, be reluctant to go out after dark. That is, the alternatives in $S \setminus C_i$ are viewed as not available by the woman. Assume that

(21) $$g_j(y_i, w_j, z_j, X_i) = \exp((y_i - w_j)\theta + X_i\delta_j),$$

where $\theta$ and $\delta_j$ are unknown parameters.[18] Then, provided that IIA holds, the probability that the woman will choose activity *j* from $C_i$ is given by

(22) $$P(J_i(C_i) = j) = P(U_{ij} = \max_{k \in C_i} U_{ik}) = \frac{\exp(-w_j\theta + X_i\delta_j)}{\sum_{k \in C_i} \exp(-w_k\theta + X_i\delta_k)}.$$

Provided that we have a sample of individual observations on women's choices, we can estimate the unknown parameters $\theta$ and $\{\delta_k\}$. From the results in Section 4.2 it follows that

(23) $$\bar{V}_{C_i}(y_i, w, z, X_i) = \left( \sum_{k \in C_i} \exp(X_i\delta_k - \theta w_k) \right) \exp(\theta y_i).$$

As regards the corresponding adjusted income distribution, it follows from (20) and (23) that

(24) $$\tilde{Y}_i = Y_i + \frac{1}{\theta} \log \left( \frac{\sum_{j \in C_i} \exp(X\delta_j - \theta w_j)}{\sum_{j \in C_0} \exp(X\delta_j - \theta w_j^0)} \right).$$

[17] An alternative approach to the capability-adjusted income distribution can be based on the methodology of Dagsvik and Karlström (2005).

[18] For the sake of identification we need to normalize such that $\delta_1$, say, is equal to zero.

*Example 2*

In this example we consider the capability-adjusted income that follows from heterogeneity in urban transportation capabilities. Assume that there are four relevant transportation modes for going to work: namely, "walking" (1), "bicycle" (2), "own car" (3), and "public transport" (4). Public transport consists of tram, subway, bus or train, or combinations. Thus $S = \{1, 2, 3, 4\}$. We assume that alternatives 1 and 2 are always available (we disregard that for some people the distance may be too large for walking or bicycling to work). However, for some people alternatives 3 or 4 are not available. Assume that

$$g_j(y_i, w_j, z_j, X_i) = \exp\left(a_j - b_1 z_{1ij} - b_2 z_{2ij} - \frac{b_3 w_j}{y_i}\right), \tag{25}$$

where $w_j$ is the cost, $z_{1ij}$ is the "in-vehicle time", and $z_{2ij}$ is the "out-of-vehicle" time of mode $j$ for individual $i$. The parameter $a_j$ represents the mean value of "comfort" of mode $j$ and the parameters $b_j$, $j$ = 1, 2, 3, are all positive. If data on individual choices, capability sets, costs, income, in-vehicle, and out-of-vehicle time use are available one can estimate the unknown parameters $a_j$ and $b_j$ for all $j$ (with $a_1$ normalized to zero) by well-known methods. From (13) and (25) it follows, provided that IIA holds, that

$$\bar{V}_{C_i}(y_i, w, z, X_i) = \sum_{j\in C_i} \exp\left(a_j - b_1 z_{1ij} - b_2 z_{2ij} - b_3 w_j / y_i\right). \tag{26}$$

Let $C_0 = S,$ and suppose that the social planner is able to assign suitable individual values of in-vehicle and out-of-vehicle time for the people who do not have alternative 3 or 4 in their choice sets. Then from (26) it follows that the corresponding capability-adjusted income according to the first approach is determined by

$$\sum_{j\in C_i} \exp\left(a_j - b_1 z_{1ij} - b_2 z_{2ij} - b_3 w_j / y_i\right) = \sum_{j\in S} \exp\left(a_j - b_1 z_{1ij}^0 - b_2 z_{2ij}^0 - b_3 w_j^0 / \tilde{Y}_i\right). \tag{27}$$

Since the right-hand side of (27) is strictly increasing in $\tilde{Y}_i$, $\tilde{Y}_i$ is uniquely determined.

*Example 3*

This example is an elaboration of the example mentioned briefly in Section 2. Recall that we consider a context where well-being is assumed to be determined by the household's opportunities in housing, schooling and the labor market. The housing, school and job alternatives are each classified into types that are observable to the social planner. The space of functionings thus consists of the housing, schooling and jobs alternatives in their respective markets. However, the capability set of a particular household, or person, will be a subset of this space: namely, the set of combinations of housing, schooling and job alternatives that are available to the household. Assume, furthermore, that the opportunities in the housing market are "priced out", whereas this is not necessarily the case in schooling and the labor market. Assume that the preferences satisfy IIA. Let $n_{jk}$ be the (unobserved) number of jobs of type $j$ and schools of type $k$ available to the agent. Let $(w_{1j}, z_{1j})$ be the wage and non-pecuniary attributes characterizing jobs of type $j$, $(w_{2k}, z_{2k})$ the pecuniary and non-pecuniary attributes characterizing schools of type $k$, and $(w_{2r}, z_{2r})$ the price and non-pecuniary attributes characterizing houses of type $r$. Now, in a similar way to (14), it follows that

(28) $$\bar{V}_C(y, w, z, X) = \sum_{(j,k,r)\in C} n_{jk} g_{jkr}(y, w_{1j}, w_{2k}, w_{3r}, z_{1j}, z_{2k}, z_{3r}, X).$$

By means of (28) one can now compute the capability-adjusted income distribution as in the examples above. In empirical analyses it is a problem that the terms $\{n_{jk}\}$ are unobserved. However, in some cases it might be possible to represent them as functions of relevant observable indicators.

# 5. Stated Preference data and the Leyden School approach

The methodological approach we have discussed above assumes that data on agents' choices among feasible alternatives are available. This is, however, not always the case. In this section we shall discuss a method based on SP data that was originally proposed by van Praag and associates (Leyden School approach) and has recently been extended by Dagsvik, Jia and Strøm (2006). This method does not require data on ranking of alternatives. The original approach attributed to the Leyden School was developed by van Praag (1968, 1971, 1991, 1993). Other contributions include van Herwaarden, Kapteyn and van Praag (1977), van Herwaarden and Kapteyn (1981), and Kapteyn and Wansbeck (1985). We shall now give a brief summary of the approach of Dagsvik, Jia and Strøm (2006).

The theoretical approach of the Leyden School is to estimate welfare measures based on a representation of the expenditure function. Let $Y_C(u)$ denote the expenditure function at utility level $u$. Let us first review how the Leyden School propose to collect SP data: Each agent (individual or selected expert) is asked to indicate (under his or her current conditions) what income level is needed for maintaining a specific utility level. The Leyden School typically uses six welfare levels, corresponding to "very bad", "bad", "insufficient", "more than sufficient", "good", and "excellent". Thus $Y_C(u_j)$ is the expenditure necessary for achieving welfare level $u_j$, $j$ = 1, 2, …, 6. This means that one can interpret the income levels the respondents report in the survey as realizations of the random variables $Y_C(u_j)$, $j$ = 1, 2, …, 6, for each individual respondent. The welfare levels the respondents associate with the questions in the survey are to be interpreted as the respective (current) mean utility levels in the population. Thus, unobserved heterogeneity in agents' perceived utility levels, is here represented by the random component of the expenditure function.

Let us now see how the notion of expenditure function fits into the RSM framework discussed above. Define the expenditure function $Y_C(u)$ by $V_C(Y_C(u), w, z, X, \tilde{\varepsilon}) = u.$ Note that since the scale function is random, so is the expenditure function. Evidently, since $V_C(Y_C(u), w, z, X, \tilde{\varepsilon}) = u,$ it follows from (11), due to the assumption of extreme value distributed random error terms, that the c.d.f. of $Y_C(u)$ is given by

(29) $$P(Y_C(u) \le y) = P(V_C(Y_C(u), w, z, X, \tilde{\varepsilon}) < V_C(y, w, z, X, \tilde{\varepsilon}))$$
$$= P(u < V_C(y, w, z, X, \tilde{\varepsilon})) = (u < \bar{V}_C(y, w, z, X)\eta)) = 1 - \exp(-\bar{V}_C(y, w, z, X)/u),$$

for $u$ > 0, with corresponding probability density function $f(y \mid u, w, z)$ given by

(30) $$f(y \mid u, w, z, X) = \frac{\partial \bar{V}_C(y, w, z, X)}{u \partial y} \exp(-\bar{V}_C(y, w, z, X)/u),$$

conditional on the utility level, prices, and other attributes. The above formal definition of the expenditure function is, however, incomplete, since it does not correspond to observations of income at several welfare levels for each person. Thus the theoretical framework with only one utility level is a special case in the Leyden School approach. In the general case Dagsvik, Jia and Strøm (2006) have developed an appropriate extension, similar to the Leyden School approach, where the scale is viewed as a stochastic process of income: that is, a stochastic process with income as parameter. The corresponding dual representation is the expenditure function as a stochastic process with scale level as parameter. In this paper, however, we shall not discuss further details of the approach taken by Dagsvik, Jia and Strøm (2006). Thus, for expository simplicity, we consider only the special case where we have answers that correspond only to one utility level. With independent observations on the expenditure function, it is possible to use the method of maximum likelihood based on the probability density in (30) to estimate the mean welfare function, $\bar{V}_C(y, w, z, X),$ provided that a suitable empirical specification of the function $g(\cdot)$ has been chosen. However, due to the fact that the Leyden School approach does not involve direct observations on the most preferred alternatives, identification of the welfare function may be more delicate and be sensitive to functional form assumptions.

We shall next consider the case where the systematic term of the random scale function is given by (21). An alternative type of specification is utilized by Dagsvik, Jia and Strøm (2006), see

Appendix B for details on this case. Then, in the case of latent choice sets, it follows that the constrained indirect random scale is given by

(31) $$\bar{V}_C(y, w, z, X) = e^{\theta y} H_C$$

where

$$H_C = \sum_{j \in C} n_j e^{-\theta w_j} h(z_j, X).$$

Consequently, (30) implies that

(32) $$P(Y_C(u) \le y) = 1 - \exp(-u^{-1} H_C e^{\theta y})).$$

The distribution function given in (32) is a Weibull-type distribution function. The structure of (29), (31), and (32) implies that we can express the expenditure function as

(33) $$Y_C(u) = \log u - \log H_C - \theta\kappa,$$

where $\kappa$ is a random variable that is independent of $u$ and $H_C$, and has extreme value c.d.f. equal to $\exp(-\exp(-x))$ for real *x*. This follows immediately by using (33) to calculate the c.d.f. of the expenditure function. Recall that $H_C$ is agent-specific. Evidently, the specification in (33) can also be estimated by OLS.[19] With data for several utility levels (as in the Leyden School approach), one obtains estimates $\log H_C - \log u_j$, corresponding to welfare level $u_j$. Thus up to a suitable normalization one can identify and estimate $u_j$ for each *j* and $H_C$ for each individual without further assumptions about $H_C$. In a second stage, the individual estimates of $\log H_C$ can be applied as dependent variables in a regression relation, with independent variables characterizing the individual's environment, including the individual's actual income (or the individual's income in the previous period). Specifically, a key point of the Leyden School approach is that it allows the researcher to assess the extent of how perceived well-being depends on current and past achieved income levels. This is obtained by introducing current income as an independent variable in the regression relation mentioned above in a second stage.

So far we have assumed implicitly that the capability set *C* is fixed in the SP survey. But this is not a necessary requirement. In fact, by making repeated SP experiments for each agent for every possible subset of the universal set *S*, one can in principle determine $H_C$, as a function of *C*, for *any* $C \in S$. If the capability set contains many alternatives, however, this does not seem practical. It may also be questionable to what extent the agents are able to assess the expenditures that correspond to the

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[19] The random term $\kappa$ has mean equal to Euler's constant, 0.5772.., and variance equal to $\pi^2/6$.

respective capability sets. But suppose now that the researcher has only a few selected policy simulation experiments (scenarios) in mind regarding the welfare effect of changing capabilities. Suppose, moreover, that these counterfactual capabilities consist of a modification of the original ones (which may be unobserved by the researcher), in that selected alternatives are added to or removed from the capability set, corresponding to the respective scenarios of interest. One could then carry out separate SP surveys corresponding to each specific scenario and subsequently estimate the scenario-specific values of $H_C$.

Provided that the specification in (32) has been estimated, one can, as in Example 1, compute the capability-adjusted income. It is given by

(34)
$$\tilde{Y}_i = Y_i^0 + \frac{1}{\theta}\log\left(\frac{H_{C_i}(\tilde{X}_i)}{H_{C_0}(\tilde{X}_0)}\right),$$

where $\tilde{X}_i$ denotes the vector of variables that affects $H_C$ and $\tilde{X}_0$ denotes the corresponding vector of reference variables.

It is clear that the Leyden School approach discussed above applies equally well to a setting where the sample of individuals is replaced by members of an expert evaluation panel and where each member of the panel is exposed to the SP questionnaire, as explained above.

# 6. Welfare measurement based on life-satisfaction data

Several researchers have conducted surveys in which individuals are interviewed about their overall life satisfaction on, for example, a six-point scale: see Anand, Hunter and Smith (2005) and Anand, Santos and Smith (2008). Subsequently, data obtained in this way have been used to analyze the relationship between life satisfaction and selected covariates supposed to be relevant to life satisfaction. One version of this approach is, from a methodological point of view, similar to the Leyden School method. Let $u_j,\ j=1,2,...,6,$ denote six welfare levels. Individuals in the survey are asked to report their satisfaction regarding welfare levels from 1 to 6. As above, let $V_C(y,w,z,X,\tilde{\varepsilon})$ denote the scale of an individual and let welfare level 1 correspond to $V_C(y,w,z,X,\tilde{\varepsilon}) \le u_1$, welfare level $j$ correspond to $V_C(y,w,z,X,\tilde{\varepsilon}) \in (u_j, u_{j+1}],$ for $j$ = 1, 2, …, 5, and welfare level 6 correspond to $V_C(y,w,z,X,\tilde{\varepsilon}) > u_6$. Whereas the scale levels $u_j,\ j=1,2,...,6,$ are to be interpreted as mean population welfare levels, the index $V_C(y,w,z,X,\tilde{\varepsilon})$ is an agent-specific random scale. From (11) it follows that

(35) $$p_j = P(u_{j-1} < V_C(y, w, z, X, \tilde{\varepsilon}) \le u_j) = F(u_j / \bar{V}_C(y, w, z, X)) - F(u_{j-1} / \bar{V}_C(y, w, z, X)).$$

Under the assumptions made in the previous section it follows from (34) that

(36) $$p_j = \exp(-u_j^{-1} H_C e^{\theta y}) - \exp(-u_{j-1}^{-1} H_C e^{\theta y}).$$

It can easily be verified, subject to some normalization (for example, with $u_1 = 1$), that one can identify $u_j$, $j = 2, 3, ..., 6,$ and $H_C$. The expressions in (35) or in (36) can be applied to estimate $\{u_j\}$, and $H_C$, given a specification of $H_C$, provided that one has data on lifetime satisfaction and covariates that are suitable for characterizing $H_C$. But as with the Leyden School approach, one cannot identify the structure of $H_C$ as a function of the choice set, prices, attributes, and person characteristics without further assumptions and further data.

As with the approaches discussed above, the methodology outlined in this section can obviously also be used where data are obtained from expert panel evaluations.

# 7. Concluding comments

A fundamental motivation for Sen's capability approach is the view that economic inequality is not necessarily the same as income inequality, because income may not always be informative about what people get from their income. In this paper we have addressed the challenge of formulating a structural decision-theoretic framework that is suitable for analyzing and assessing economic inequality based on Sen's capability theory. The point of departure is stochastic choice theory, in which the representation of choice sets and preferences in terms of random scale functions is crucial. A major advantage with this theory is that it avoids the strict assumption of transitive preferences and leads to a reasonable representation of the value of both the chosen functionings and the capability set.

A particularly interesting result obtained in this paper is the money metric welfare measure we have called the capability-adjusted income distribution. This measure accommodates the effect on the income distribution caused by variation of the capability sets across the population.

We have also discussed how a dual approach, based on the notion of random expenditure function and similar to the Leyden School approach, can be applied to obtain welfare measures. Although this dual approach is simpler to apply than the one based on data on choice of functionings, it is not particularly well suited for identification of key structural parameters. Finally, we have discussed how an RSM can be applied to estimate welfare representations in the case where one has data on life satisfaction. Also in the latter approach it seems hard to identify key structural parameters.

# References


Aczél, J., R. S. Roberts and Z. Rosenbaum (1986): On Scientific Laws without Dimensional Constants. *Journal of Mathematical Analysis and Applications*, **119**, 389–416.

Anand, P., G. Hunter and R. Smith (2005): Capabilities and Wellbeing: Evidence Based on the Sen–Nussbaum Approach to Welfare. *Social Indicators Research*, **79**, 9–55.

Anand, P., C. Santos and R. Smith (2008): The Measurement of Capabilities. In K. Basu and R. Kanbur (eds), *Arguments for a Better World: Essays in Honour of Amartya Sen.* Oxford University Press, Oxford.

Beggs, S., S. Cardell and J. Hausman (1981): Assessing the Potential Demand for Electric Cars. *Journal of Econometrics*, **16**, 1–19.

Ben-Akiva, M., and T. Watanatada (1981): Application of a Continuous Spatial Choice Logit Model. In D. McFadden and C. Manski (eds), *Structural Analysis of Discrete Data with Econometric Applications*. MIT Press, Cambridge, MA.

Dagsvik, J. K. (1994): Discrete and Continuous Choice, Max-stable Processes and Independence from Irrelevant Attributes. *Econometrica*, **62**, 1179–1205.

Dagsvik, J. K. (1995): How Large is the Class of Generalized Extreme Value Random Utility Models? *Journal of Mathematical Psychology*, **39**, 90-98.

Dagsvik, J. K., and A. Karlström (2005): Compensating Variation and Hicksian Choice Probabilities in Random Utility Models that are Nonlinear in Income. *Review of Economic Studies*, **72**, 57–76.

Dagsvik, J. K., and S. Strøm (2006): Sectoral Labor Supply, Choice Restrictions and Functional Form. *Journal of Applied Econometrics*, **21**, 803–826.

Dagsvik, J. K., Z. Jia and S. Strøm (2006): Utility of Income as a Random Function: Behavioral Characterization and Empirical Evidence. *Mathematical Social Sciences*, **51**, 23–57.

Drèze, J., and A. Sen (2002): *India: Development and Participation*. Oxford University Press, Oxford.

Falmagne, J.-C. (1985): *Elements of Psychophysical Theory*. Oxford University Press, New York.

Falmagne, J.-C., and L. Narens (1983): Scales and Meaningfulness of Quantitative Laws. *Synthese*, **55**, 287–325.

Flinn, C., and J. J. Heckman (1982): New Methods for Analyzing Structural Models of Labor Force Dynamics. *Journal of Econometrics*, **18**, 115–168.

Gorman, W. M. (1956): The Demand for Related Goods. Journal paper J3129 (Iowa Experimental Station, Ames, IA).

Hanemann, W. M. (1984): Discrete/Continuous Choice of Consumer Demand. *Econometrica*, **52**, 541–561.

Kapteyn, A., and T. Wansbeck (1985): The Individual Welfare Function: A Review. *Journal of Economic Psychology,* **6**, 333–363.

Kuklys, W. (2005): *Amartya Sen's Capability Approach. Theoretical Insights and Empirical Applications.* Springer-Verlag, Berlin.

Lancaster, K. J. (1966): A New Approach to Consumer Demand. *Journal of Political Economy,* **74**, 132–157.

Louviere, J. J., D. A. Hensher and J. D. Swait (2000): *Stated Choice Methods: Analysis and Applications.* Cambridge University Press, Cambridge UK.

Luce, R. D. (1959a): *Individual Choice Behavior.* Wiley, New York.

Luce, R. D. (1959b): On the Possible Psychophysical Laws. *Psychological Review*, **66**, 81–95.

Luce, R. D., and P. Suppes (1965): Preference, Utility and Subjective Probability. In R. D. Luce, R. R. Bush and E. Galanter (eds), *Handbook of Mathematical Psychology,* Vol III. Wiley, New York.

McFadden, D. (1973): Conditional Logit Analysis of Qualitative Choice Behavior. In P. Zarembka (ed.), *Frontiers in Econometrics*. Academic Press, New York.

McFadden, D. (1978): Modelling the Choice of Residential Location. In A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull (eds), *Spatial Interaction Theory and Residential Location* (chapter 3). North-Holland, Amsterdam.

McFadden, D. (1981): Econometric Models of Probabilistic Choice. In C. F. Manski and D. McFadden (eds), *Structural Analysis of Discrete Data with Econometric Applications*. MIT Press, Cambridge, MA.

McFadden, D. (1984): Econometric Analysis of Qualitative Response Models. In Z. Griliches and M. D. Intriligator (eds), *Handbook of Econometrics,* Vol. III, Elsevier Science Publishers BV, New York.

McFadden, D. (2001): Economic Choices. *American Economic Review,* **91**, 351–378.

Nussbaum, M. C. (1999): *Sex and Social Justice*. Oxford University Press, Oxford.

Resnick, S. I. (1987): *Extreme Values, Regular Variation and Point Processes*. Springer-Verlag, New York.

Roberts, F. S., and Z. Rosenbaum (1986): Scale Type, Meaningfulness, and the Possible Psychophysical Laws. *Mathematical Social Sciences,* **12**, 77–95.

Robeyns, I. (2003): The Capability Approach: An Interdisciplinary Introduction. Mimeo, Department of Political Science and Amsterdam School of Social Science Research, University of Amsterdam.

Robeyns, I. (2006): The Capability Approach in Practice. *Journal of Political Philosophy*, **14**, 351–376.

Robeyns, I., and W. Kuklys (2005): Sen's Capability Approach to Welfare Economics. In W. Kuklys (ed.), *Amartya Sen's Capability Approach: Theoretical Insight and Empirical Applications.* Springer-Verlag, Berlin.

Sen, A. K. (1979): Personal Utilities and Public Judgments: or What's Wrong with Welfare Economics. *Economic Journal*, **89,** 537–558.

Sen, A. K. (1980): Equality of What? In S. McMurrin (ed.), *The Tanner Lectures on Human Values.* University of Utah Press, Salt Lake City.

Sen, A. K. (1982): *Choice, Welfare and Measurement.* MIT Press, Cambridge, MA.

Sen, A. K. (1984): Rights and Capabilities. In A. K. Sen, *Resources, Values and Development*. Harvard University Press, Cambridge, MA.

Sen, A. K. (1985a): *Commodities and Capabilities*. North-Holland, Amsterdam.

Sen, A. K. (1985b): Social Choice Theory. In A. Arrow and M. D. Intriligator (eds), *Handbook of Mathematical Economics*, Vol. III, North-Holland, Amsterdam.

Sen, A. K. (1987): The Standard of Living. In G. Hawthorn (ed.), *The Standard of Living*, Cambridge University Press, Cambridge, UK.

Sen, A. K. (1991): Welfare, Preference, and Freedom. *Journal of Econometrics*, **50**, 15–29.

Sen, A. K. (1992): *Inequality Re-examined*. Clarendon Press, Oxford.

Sen, A. K. (1993): Capability and Well-being. In M. Nussbaum and A. K. Sen (eds), *The Quality of Life*, Clarendon Press, Oxford.

Sen, A. K. (1997): From Income Inequality of Economic Inequality. *Southern Economic Journal*, **64**, 384–396.

Sen, A. K. (1998): *Welfare Economics and the Quality of Life.* Chung-Hua series of lectures by invited eminent economists, no 24. Institute of Economics, Academia Sinica. Nankan, Taipei, Taiwan.

Sen, A. K. (2002): *Rationality and Freedom*. Harvard University Press, Cambridge, MA.

Sugden, R. (1993): Welfare, Resources, and Capabilities: A Review of Inequality Reexamined, by Amartya Sen. *Journal of Economic Literature*, **31**, 1947–1962.

Thurstone, L. L. (1927): A Law of Comparative Judgment. *Psychological Review*, **79**, 281–299.

Tversky, A. (1969): Intransitivity of Preferences. *Psychological Review*, **76**, 31–48.

Van Herwaarden, F. G., and A. Kapteyn (1981): Empirical Comparisons of the Shape of Welfare Functions. *European Economic Review*, **15**, 261–286.

Van Herwaarden, F. G., A. Kapteyn and B. M. S. van Praag (1977): Twelve Thousand Individual Welfare Functions. *European Economic Review,* **9**, 283–300.

Van Praag, B. M. S. (1968): *Individual Welfare Functions and Consumer Behavior: A Theory of Rational Irrationality.* North-Holland, Amsterdam.

Van Praag, B. M. S. (1971): The Welfare Function of Income in Belgium: An Empirical Investigation. *European Economic Review*, **2**, 337–369.

Van Praag, B. M. S. (1991): Ordinal and Cardinal Utility: An Integration of the Two Dimensions of the Welfare Concept. *Journal of Econometrics,* **50**, 60–89.

Van Praag, B. M. S. (1993): The Relativity of the Welfare Concept. In M. Nussbaum and A. Sen (eds), *The Quality of Life*. Clarendon Press, Oxford.

# Appendix A

In this appendix we prove the following result:

**Lemma**

*Assume that* $\psi$ *is a strictly increasing mapping from* $(0,\infty)$ *to* $(0,\infty)$ *and let* $\bar{V}(y)$ *be a positive increasing function of y and* $\bar{V}_0$ *a given positive number. Moreover, let* $\eta$ *be a positive random variable with strictly increasing c.d.f. that is independent of* $\psi$, $\bar{V}_0$ *and* $\bar{V}(y)$. *Then y determined by the equation*

*(A.1)* $$E\psi(\bar{V}(y)\eta) = E\psi(\bar{V}_0\eta)$$

*is equal to y determined by*

$$\bar{V}(y) = \bar{V}_0.$$

**Proof:**

Recall that if $X$ is a positive random variable then one has that

$$EX = \int P(X > x)dx.$$

Let $F$ be the c.d.f. of $\eta$. It follows that

$$E\psi(\bar{V}(y)\eta) = \int P(\psi(\bar{V}(y)\eta) > x)dx = \int P(\eta > h(x)/\bar{V}(y))dx = \int (1 - F(h(x)/\bar{V}(y))dx,$$

where $h$ is the inverse function of $\psi$. For positive $z$, let

$$H(z) = \int (1 - F(h(x)/z)dx.$$

Since $(1 - F(h(x)/z)$ is strictly increasing in $z$ for each fixed $x$, it implies that $H(z)$ is strictly increasing. Furthermore, $H$ is independent of $h$ and $\bar{V}(y)$. Consequently, eq. (A.1) is equivalent to $H(\bar{V}(y)) = H(\bar{V}_0)$. Since $H$ is strictly increasing this is equivalent to $\bar{V}(y) = \bar{V}_0$.

Q.E.D.

# Appendix B

In Dagsvik, Jia and Strøm (2006) it is argued on the basis of behavioral axioms and some empirical evidence that the functional form of $g$ in some instances should be a power function, that is

(B.1) $$g_j(y, w_j, z_j) = g(y, w_j, z_j) = (y - w_j)^{\alpha} h(z_j),$$

where $\alpha$ is a positive constant and $h(z_j)$ is a suitable function of the non-pecuniary attributes of alternative $j$. Assume now that the set of alternatives is very large, so that we can approximate the choice set with a suitable continuum. Assume furthermore that the costs belong to the set $[\mu_C, \infty]$ and that the non-pecuniary attributes belong to the set $K_C$. Assume there is a one-to-one mapping between the indexation of the alternatives and the values of the vector of non-pecuniary attributes. Let $n(z)$ be the counterpart of $n_r$ in the continuous case. Hence we obtain from (B.1) that

(B.2) $$\sum_{r \in C} n_r g(y, w_r, z_r) \cong \int_{w \in [\mu_C, y]} \int_{z \in K_C} n(z) h(z) (y - w)^{\alpha} dz dw = \left( y - \mu_C \right)^{\alpha+1} H_C^{\alpha+1},$$

where

$$H_C = \left[ \frac{1}{\alpha+1} \int_{z \in K_C} n(z) h(z) dz \right]^{1/\alpha+1}.$$

From (B.2) it now follows that we can express the deterministic term of the constrained indirect scale as

(B.3) $$\bar{V}_C(y) = (y - \mu_C)^{\alpha+1} H_C^{\alpha+1}$$

for $y > \mu_C$.[20] Clearly, the term $\mu_C$ can be interpreted as a subsistence level. Thus we have demonstrated that our welfare measure can be expressed as income minus subsistence level $\mu_C$, weighted with the function $H_C$ which represents the value of non-pecuniary attributes. Both the subsistence level $\mu_C$ and the function $H_C$ may depend on the choice set $C$. Consequently, it follows from (B.3) and (32) that

(B.4) $$P(Y_C(u) \le y) = 1 - \exp(-(y - \mu_C)^{\alpha+1} H_C^{\alpha+1} / u).$$

The distribution function given in (B.4) is in fact a Weibull distribution, with support above the threshold $\mu_C$. The structure of the distribution in (B.4) implies that we can express the expenditure function as

[20] In the present context we have for convenience suppressed prices, attributes and individual characteristics in the notation.

(B.5) $$Y_C(u) = \mu_C + \frac{\eta^{1/(\alpha+1)} u^{1/(\alpha+1)}}{H_C},$$

where $\eta$ is an exponentially distributed random variable with parameter equal to 1. This follows immediately by using (B.4) to calculate the c.d.f. of the expenditure function. By differentiating (B.4) with respect to $y$ one obtains the corresponding probability density for the expenditure function, and this density can be applied in a maximum likelihood estimation procedure to estimate the parameters $\alpha, \mu_C$ and $H_C$. Note, however, that unless $\mu_C$ is determined a priori, this is a non-standard inference problem, because $\mu_C$ defines the boundary of the support. Now, suppose for a moment that we have data for different population groups and that within each group the level of $\mu_C$ is the same. As Flinn and Heckman (1982) have demonstrated, the maximum likelihood estimate of $\mu_C$ for a given group is the minimum of the observations within the given group. Once $\mu_C$ has been estimated it can be inserted into the likelihood function and one can estimate $\alpha$ and $H_C$ in a second stage, given a specification of $H_C$.

If one is interested only in the aggregate welfare measure there is no need to estimate the parameter $\alpha$. In fact, a simple estimation method is to apply OLS to estimate $H_C$, with $\log(Y(u) - \mu_C)$ as dependent variable, because (B.5) implies that

$$E \log(Y(u) - \mu_C) = u/(\alpha+1) - \log H_C + 0.5772/(\alpha+1),$$

where 0.5772 is Euler's constant.

**B**

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