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Abstract:

While little attention has been paid to the role of profitability in the empirical literature on firm exit, we employ a detailed recently established database of Norwegian manufacturing firms to identify the extent to which profitability explains a firm's exit behavior. Some key characteristics of the data are: i) 25 percent of firms that exited experienced positive profits every year before exit, ii) there is no negative profitability shock immediately prior to exit, and iii) firms may continue production, even though they frequently experience negative profits. We use these data to estimate a theory-founded econometric model of exit, where the exit and investment decisions of firms are formulated as the solution to a discrete-continuous dynamic programming problem. In particular, the probability of exit depends on profitability, which is not directly observable to the econometrician. We estimate this model for six manufacturing industries and find that increased profitability lowers the probability of exit and that this effect is statistically significant in all industries. We show that the difference in annual exit probability between firms that exited during the observation period (1994–2009) and firms that did not exit is highly persistent over time, and there is no tendency for a sharp increase in the estimated exit probability just prior to exit. Hence, it is the cumulative effect of the higher risk of exit over several years, compared with the average firm, that causes firms to exit.

Keywords: Exit, investments, entrepreneurship, continuous-discrete choice, monopolistic competition, manufacturing firms, policy simulation, ownership structure, wage raise.

JEL classification: C33, C51, C61, C72, D21

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Sammendrag

Liten vekt er hittil blitt lagt på betydningen av lønnsomhet i den empiriske litteraturen om bedriftsnedleggelsen. Vi anvender en foretaksdatabase med mikrodata for å identifisere i hvilken grad lønnsomhet kan forklare nedleggelse. Vi bruker disse data for å estimere en teoribasert økonometrisk modell, der nedleggelses- og investeringsbeslutningen er formulert som løsningen på et diskret-kontinuerlig dynamisk programmeringsproblem. Vi estimerer modellen på seks industrinæringer og finner at økt lønnsomhet reduserer sannsynligheten for nedleggelse og at effekten er statistisk signifikant i alle næringer. Vi finner også at sannsynligheten for nedleggelse er persistent over tid, slik at det er den kumulative effekten av moderat høyere sannsynlighet over flere år, sammenlignet med gjennomsnittsbedriften, som forårsaker at bedriften legges ned.

1 Introduction

According to standard economic theory, low (negative) profitability is the key reason for firms to exit. Until now, however, profit has not been a key variable in empirical research on firm exit. For instance, Olley and Pakes (1996) estimate probit models of firm exit using productivity, age and capital (not profit) as covariates, while Boeri and Bellmann (1995) seek to explain firm exit also without using profit information. For the most part, this observation reflects the generally limited access to detailed information at the firm level.

In the present paper, we use a recently established database of Norwegian manufacturing firms that provides detailed information on revenues and costs during the period 1993–2009. Some key characteristics of the data are i) 25 percent of firms that exited experienced positive profits *every year* before exit, ii) there is no negative profitability shock immediately prior to exit (in fact, about 65 percent of the firms that exited had positive profits in the year prior to exit), and iii) firms may continue production even though they repeatedly experience negative profit – 30 percent of the firm-year observations (one observation for each firm in each year) for non-exiting firms have negative profit. These observations raise a number of key questions. First, what causes firms to exit? Second, what characteristics distinguish firms that exit from those that continue production? Third, is profit of key importance in explaining firm exit? The purpose of this analysis is to employ a structural microeconometric model to provide answers to these important questions.

We define exit as the state in which production at a site has come to a permanent stop. Note that a firm acquired by another firm is then *not* defined as having exited. Below we use the terms exit and closedown interchangeably. If profitability data are not available for exit studies, there is a likelihood that the importance of some covariates may be overrated or even false. Alternatively, the estimated relationships between the probability of exit and some of the covariates may be spurious because the partial effect of profitability is not controlled for. The Boeri–Bellmann study mentioned earlier, for example, specifies different indicators for the business cycle, e.g., aggregate unemployment, and rather surprisingly concludes that cyclical factors do not affect exit. In contrast, Salvanes and Tveterås (2004) using Norwegian manufacturing data, conclude that exit rates increase in severe downturns. In the latter study, profitability is a covariate that significantly lowers the exit probability of firms. Although the difference in business cycle effects between the Boeri–Bellmann and Salvanes–Tveterås studies may reflect idiosyncrasies in their data, the Boeri–Bellmann study may have drawn false conclusions largely because profitability was not included in the set of covariates.

In the present study, we build on economic theory to derive a theory-consistent econometric model of firm exit. Needless to say, our choice of econometric model should reflect the key characteristics of the data. Therefore, in Section 2, we identify stylized facts about the firms in our data set. These are firms in six export-oriented manufacturing industries: wood products, rubber and plastic products, metal products, machinery, electronic equipment, and transport equipment. We employ this detailed Norwegian micro data set of manufacturing firms to estimate their exit probability. Because the exit probability of incumbent firms may differ systematically from that of new firms because of self-selection, we restrict our attention to start-up firms during the data period employed.

We demonstrate in Section 2 that the adjustment of labor and materials from one year to the next exhibits a different pattern than that for capital adjustment. This justifies modeling labor and materials differently from capital. We also show that there is a high degree of heterogeneity between firms in the same industry with respect to profitability. In particular, in all of the industries examined, there are firms with negative profitability and firms with positive profitability in the same year. In the econometric model, we account for this heterogeneity using firm-specific productivity terms.

In all industries we observe huge aggregated profit over time. This suggests the existence of market power, and we therefore assume imperfect competition (here modeled as monopolistic competition) instead of competitive markets. We also observe that around one-third of the firms that exited during our observation period *always* had positive profit. Moreover, about three-fourths of the firms that exited in the observation period had a positive operating surplus (revenue less variable costs) in the last year before they exited. These two facts may indicate a weak relationship, if any, between profitability and exit.

In Section 3, we introduce a model of production. In this model, each firm produces a variety of a differentiated good under monopolistic price competition. Production requires the input of labor, materials, and capital. While materials and labor are assumed to be fully flexible production factors, capital is quasi-fixed, reflecting the observed patterns found in Section 2. The production function incorporates both neutral (Hicksian) and nonneutral technological progress, and we assume these productivity terms are both firm specific and time specific in order to take account of firm heterogeneity. Productivity is not observable to the econometrician, and is decomposed into a firm-specific permanent effect, reflecting productivity at the time the firm was established, and cumulative innovations following the establishment of the firm.

In Section 4, we explain how we can use stochastic dynamic programming to derive how much the firm will invest in each period of time. We extend the work of Rust (1994) by allowing for a discrete decision variable (in our model, whether to exit), and a continuous decision variable (in our model, investment). We also allow for both positive and negative investment; if the firm sells its entire stock of capital we define it as an exit. Under the standard assumption that the state vector is Markovian, we derive the exit probability function of the firm. This is a function of its scrap value (obtained if the firm exits) and the net present value of the firm if it continues production for at least one more year and makes optimal decisions now and in the future.

We present the empirical model that encompasses our behavioral model in Section 5. There we discuss the stochastic specification of the model, explain how the net present value of the firm can be approximated by a measure of profitability and the stock of capital, and provide guidelines for estimation. The main crux in estimating the model is the nonobservability of the explanatory variable profitability; this variable depends on the current productivity of firms, which is represented by latent variables. Because both the price and the investment decision depend on the productivity of the firm, there is simultaneity between these decisions. However, the handling of latent variables makes the empirical model too complicated to be estimated jointly by maximum likelihood, so we propose a two-step procedure where in the first step some of the parameters are estimated by a simplified method.

We estimate the model in Section 6. We find that for a given level of capital, improved profitability reduces the probability of exit, and that this effect is statistically significant in all industries. Moreover, ceteris paribus, a high level of capital increases the probability of exit. These findings imply that in all industries, firms that exited during the observation period have a substantially higher estimated exit probability than firms that did not. The difference between the estimated annual exit probabilities is also highly persistent over time and is not limited to the year immediately prior to exit. In fact, the exit probabilities do not increase sharply prior to exit, which reflects that there are no (negative) profitability shocks in the last few years prior to exit. Therefore, it is the *cumulative effect of the higher risk of exit* over several years, compared with the average firm, that causes exit.

In Section 6, we also run policy simulations by examining the effects of a 10 percent permanent increase in the real wage. We find that for most of our industries, the survival probabilities decrease by roughly three percentage points after 10 years. We also test whether ownership concentration and the gender composition of firm owners affect investment and exit. We find i) a weak tendency that the exit probability of firms with a high concentration of (individual) owners responds more strongly to changes in profit than do firms that are more widely held, and ii) that there is no significant effect of gender composition on the probability of exit. Finally, Section 7 concludes.

As discussed, profit has not been a key variable in empirical research on firm exit. Part of this body of research has focused on descriptive statistics; see, e.g., Dunne et al. (1988) for a study of exit rates, market shares and firm age for US manufacturing industries, and Disney et al. (2003) for a study of exit rates in UK manufacturing. In addition, there is an extensive literature on the application of reduced-form logit/probit models and Cox proportional hazard models. For example, both Doms et al. (1995) and Mata et al. (1995) find that plant size is an important determinant of firm exit. In addition, Pérez et al. (2004) conclude that the probability of exit is highest for small firms and that export-intensive firms and R&D-intensive firms have a lower exit probability. In other work, Agarwal and Audretsch (2001) and Klepper (2002) focus on the importance of age and/or size, not profit, to explain firm exit.

Our study differs from these along at least two dimensions. First, we employ data on firm profitability. Second, in contrast to the referred papers we derive a theory-consistent model, which is the starting point for our structural econometric model. One advantage of structural models is that their coefficients have a clear economic interpretation, and thus policy simulations can be performed. In the present paper, we use our estimated structural model to examine (in Section 6) how firms adjust to cost shocks, and how this response has an impact on the exit probability. Such an exercise is hardly feasible when using reduced-form models.

2 Data

Our main data source is a database from Statistics Norway based on register data, the Capital Database, which covers the entire population of Norwegian limited liability companies involved in manufacturing. The main statistical unit in this database is the firm, where a firm is defined as "the smallest legal unit comprising all economic activities engaged in by one and the same owner". We analyze the survival and dynamics of new firms as opposed to incumbent firms. We define a firm as having exited in year t if it is recorded in the Capital Database in year $t-1$, but not in year t, *and* is registered as either bankrupt or having closed down for an unspecified reason after $t-1$ according to the Central Register of Establishments and Enterprises (REE).[1] A firm may drop out of the Capital Database without having exited. This may be the case if another form acquires the firm or if it is reclassified to belong to another sector (i.e., outside manufacturing). To avoid problems with the analysis of multiplant firms (which may close down only some of their plants, see the discussion in Section 7), only single-plant firms (in the year of start-up) are included in the analysis.

The Capital Database contains annual observations on revenue, wage costs, intermediate expenses (including energy), fixed capital (tangible fixed assets) and many other variables for all Norwegian limited liability manufacturing firms during the period 1993–2009.[2] The database combines information from two sources: (i) account statistics for all Norwegian limited liability companies, and (ii) structural statistics for the manufacturing sector. In general, all costs and revenues are in nominal prices, and incorporate taxes and subsidies, excluding value-added tax (VAT). Labor costs include salaries and wages in cash and kind, social security, and other costs incurred by the employer.

[1] There may be a delay in the registration of closedowns in the REE – typically 1 or 2 years after the firm drops out from the Capital Database.

[2] See Raknerud et al. (2004).

A unique feature of the database is that it contains net capital stock in both current and fixed prices at the firm level. The data set also distinguishes between two types of capital goods: (i) buildings and land, and (ii) other tangible fixed assets. The latter consists of machinery, equipment, vehicles, movables, furniture, tools, etc., and is therefore quite heterogeneous. The method for calculating capital stock in current prices is based on combining gross investment data and the book values of the two categories of fixed tangible assets from the balance sheet; see Raknerud et al. (2007) for details.

Our econometric model contains only a single aggregate capital variable, constructed using a Törnqvist volume index, where each type of capital is proportional to the sum of: (i) the user cost of capital owned by the firm, and (ii) total leasing costs. This aggregation corresponds to a constant-returns-to-scale Cobb–Douglas aggregation function for different types of capital (see OECD, 2001).[3]

Table 1 presents summary statistics for the six industries examined in our study. These are wood products (NACE 20), rubber and plastic products (NACE 25), metal products (NACE 28), machinery (NACE 29), electronic equipment (NACE 30–33) and transport equipment (NACE 34–35). As mentioned, we only consider firms that were established during the observation period: a firm is defined to have entered the market in year $t-1$ if it was first registered in the Capital Database in $t-1$ and was also recorded in year t. We limit attention to new firms that operated for at least 2 years. In addition, we use the first observation year solely to obtain information about the initial capital stock of firms (at the end of that year). This effectively means that we only examine firms established *after* 1993.

The first and second columns in Table 1 detail the number of firms and the number of firm exits by industry (for the entire data period 1994–2009), respectively. The third column details the annual exit frequency, which is typically 7–8 percent. During the entire observation period, about 25 percent of firms exited. The fourth column in Table 1 shows the average number of man-years in the firm entry year. The value is typically around

[3]Formally, the aggregate capital stock is calculated using the Törnqvist volume index $K_{it} = (K_{it}^b)^{\nu}(K_{it}^o)^{(1-\nu)}$, where K_{it}^b and K_{it}^o are the stocks of buildings and land (b) and other tangible fixed assets (o), respectively. Further, $v = \sum_{it} R_{it}^b / \sum_{it}(R_{it}^b + R_{it}^o)$, where $R_{it}^k = (r+\delta_k)K_{it}^k$, $k = b, o$ is the annualized (user) cost of capital (including leased capital). In the latter expression, r is the real rate of return, which we calculate from the average real return on 10-year government bonds over the period 1994–2009 (4 percent), and δ_k is the median depreciation rate obtained from accounts statistics; see Raknerud et al. (2007) for details.

15, except for transport equipment where the mean is 43. The corresponding median values are, however, much lower; between 3 and 6. Thus, most firms are small – this is a feature typical of Norwegian manufacturing. They are also usually characterized by having a high ownership concentration – an issue we return to in Section 6. Firms in these six industries compete in international markets. We therefore follow the standard assumption in the international trade literature of imperfect competition, here specified as monopolistic competition. The basic premise is that firms have some degree of market power, yet there are so many firms in the industry that it is reasonable to assume that each firm neglects that its choice of price has an impact on the demand curve of its competitors.

Standard economic theory suggests that profit is (much) larger under imperfect competition (price exceeds marginal cost) than under perfect competition (price equal to marginal cost). As an informal test of our market structure assumption (monopolistic competition), we calculated wage costs, capital costs and profit aggregated over all firms in all periods (for each industry), and divided each by aggregate value added. Table 1 provides the corresponding shares.[4] We find that profits make up between 10 and 18 percent of value added in the six industries.[5] Because perfect competition can be seen as a special case of the monopolistic competition model (infinitely large demand elasticity and a homogeneous good), in Section 6.1 we use our estimates to provide more evidence that perfect competition is not an adequate description of the market structure.

[4] We calculate capital costs using the standard user-cost formula with an interest rate equal to the average yield on 10-year government bonds (see also footnote 3).

[5] According to the seminal study by Mehra and Prescott (1985), risk aversion explains at most 1 percentage point of the US equity premium, that is, the difference between the return on equities and risk-free bonds. This suggests that correcting for risk aversion will not alter the general picture suggested in Table 2.

Table 1: **Descriptive statistics for 1994–2009**

Industry (NACE)	No. of firms	No. of exits	Average exit frequency*	Mean/median man-years**	Share of value added by: labor	capital	profit***
Wood products (20)	656	220	.069	11/3	.71	.19	.10
Plastic products (25)	242	68	.051	15/5	.68	.19	.13
Metal products (28)	1094	224	.041	11/5	.72	.18	.10
Machinery (29)	961	242	.051	17/4	.74	.14	.12
Electrical equipment (30–33)	761	161	.048	19/3	.73	.12	.15
Transport equipment (34–35)	685	166	.050	43/6	.70	.12	.18
Pooled data	4399	1049	.047	19/4	.74	.14	.12

*Number of exits divided by number of firm-years

**Number of man-years in year of entry

***Labor costs, (annualized) capital costs and profit as a share of value added

In Figure 1, we examine how the use of the three production factors, labor (measured by man-hours), materials (intermediate inputs, including energy) and capital change over time. For each factor of production and each firm in each year, we first calculate the use of a factor in year t ($t = 1995, ..., 2009$) relative to the use of this factor in year $t-1$. In Figure 1, the horizontal axis measures the log of this ratio, that is, the relative change in the use of inputs, while the vertical axis measures frequency. As shown, the graphs for man-hours and materials are almost identical and resemble the normal distribution. At first glance, the graphs may give the impression that changes in man-hours and materials follow each other almost perfectly. There is, however, substitution possibilities between these two inputs: when comparing (for each industry), the within-firm variation in the (log of the) materials–labor ratio to the within-firm variation in the (log of) man-hours, we find that this ratio is around 50 percent. If materials and labor were used in a fixed ratio specific to each firm, this ratio should have been zero. (This would also hold if the firm-specific ratios change proportionally over time for all firms.) In Section 3, we therefore assume substitution possibilities between labor and materials.

Figure 1 also depicts the log of changes in the stock of capital. This graph has somewhat thicker tails than those for man-hours and materials. The thicker tails mean that observations with large (negative or positive) changes are more frequent. Moreover, the thicker right tail – the distribution is skewed to the right – reflects the intermittent and lumpy nature of investment in Norwegian manufacturing; see Nilsen and Schiantarelli (2003).

We see that net investment takes negative values for roughly 50 percent of the observations. A firm with negative net investment has a lower acquisition of capital than depreciation. In particular, net investment may be strongly negative because of the sale of capital; in our data, the value of annual sales of capital amounts to about 10 percent of gross (annual) investment, which is substantial relative to aggregate depreciation. As discussed in Section 3, this distinctive pattern of investment calls for capital to be modeled differently than labor and materials.

In our data set, a substantial share of the observations has negative profitability. This is the case both for i) firms that did not exit during the observation period ("nonexiting firms"), and ii) firms that did exit during the observation period ("exiting firms"). In

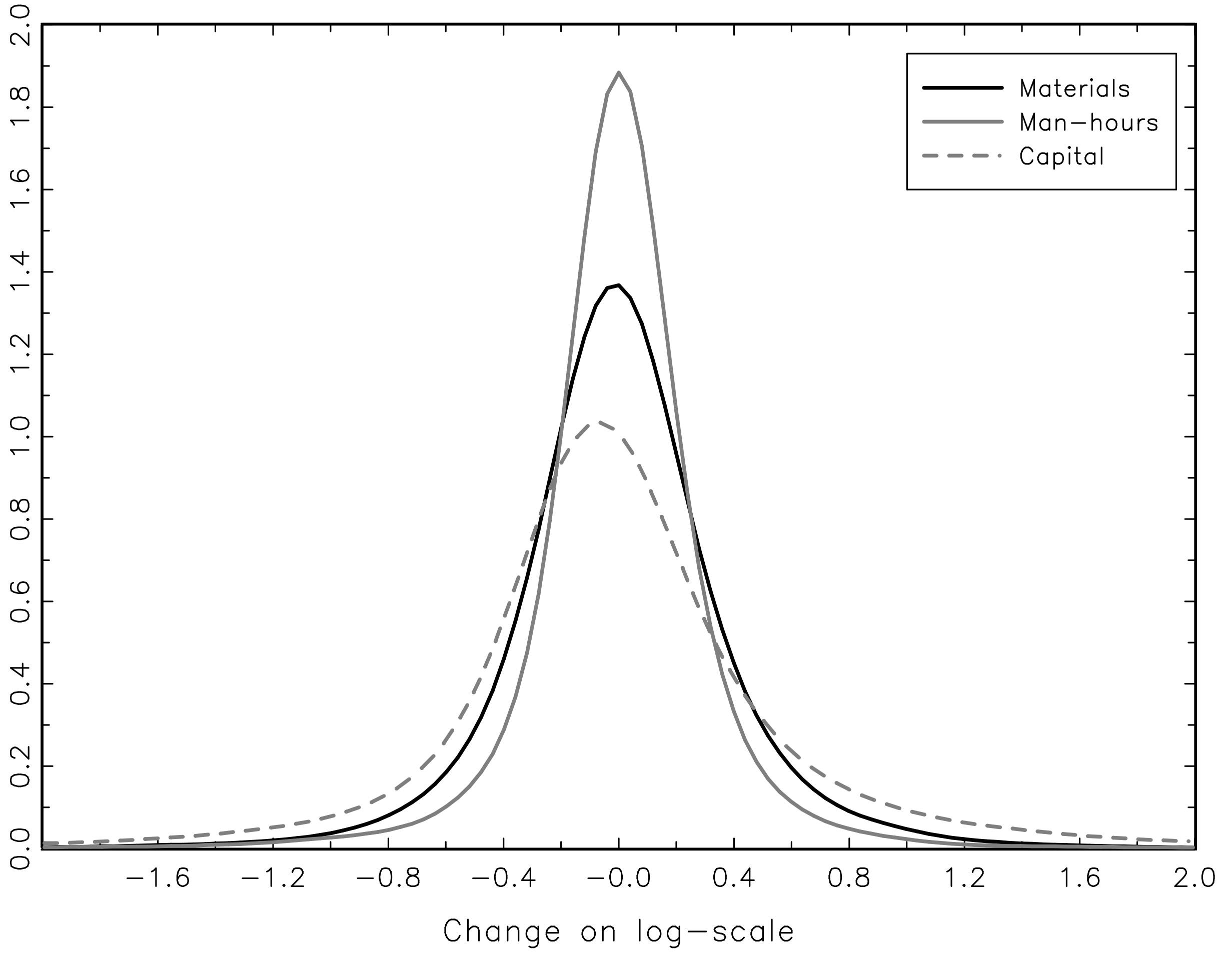

Figure 1: Distribution of log of annual changes in capital, man-hours and materials. Kernel density estimates

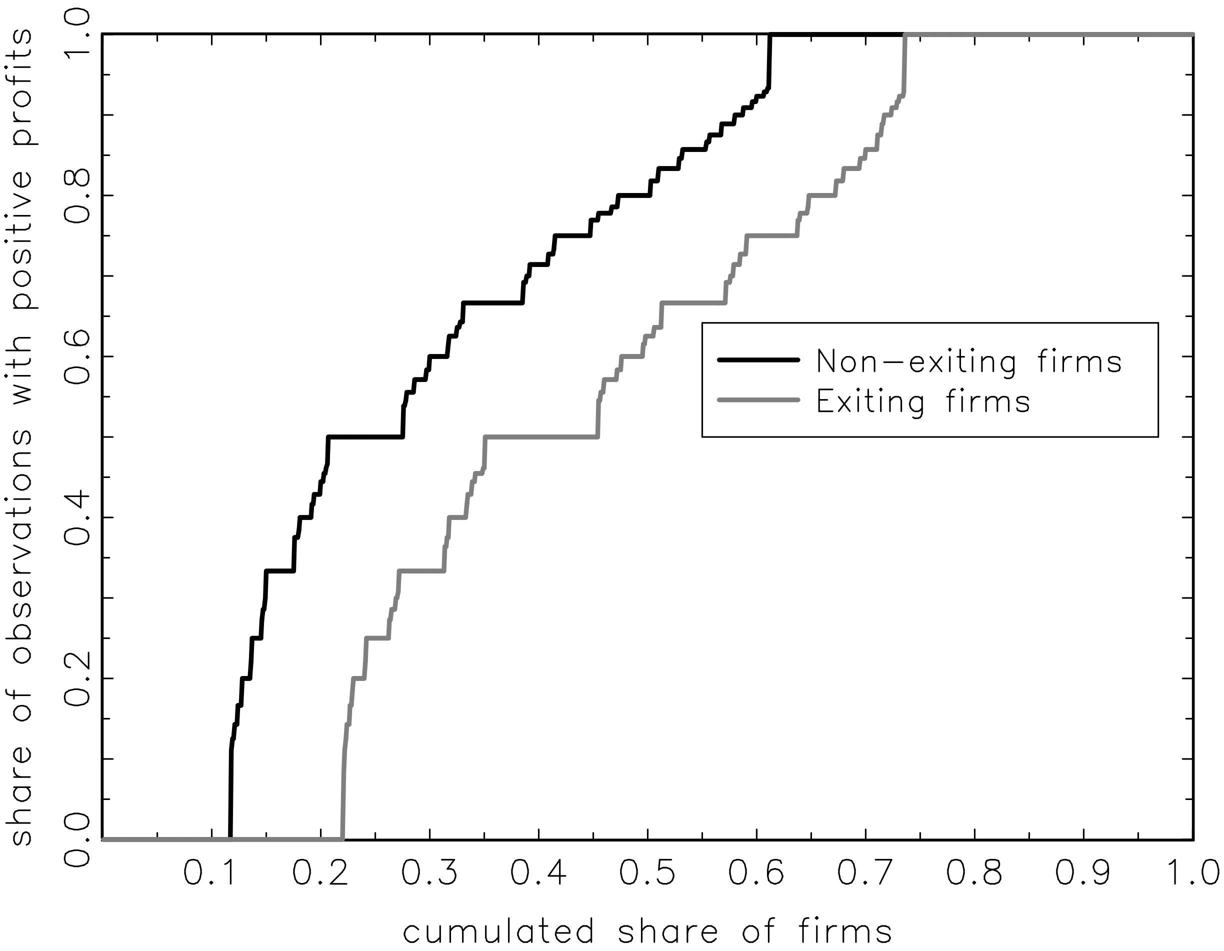

Figure 2: Distribution of share of observations (for each firm) with positive profits

fact, almost 20 percent of the firm-year observations for nonexiting firms (one observation for each operating firm in each year), and more than 25 percent of the observations of the exiting firms, have negative operating surplus. The corresponding values for profit, that is, operating surplus less capital costs, are 30 percent for nonexiting firms and 35 percent for exiting firms. Our model should therefore allow for negative profitability, in particular, negative operating surplus.

The share of observations with negative profitability may be unevenly distributed over firms. For example, some firms may have no, or just a few, observations with negative profitability, whereas others may have several observations with negative profitability. Figure 2 shows, for nonexiting and exiting firms, how the observations with *positive* profitability are distributed over firms. We construct each curve as follows. For each firm, we find its share of observations with positive profitability, henceforth termed the positive profitability share. We then sort firms by their positive profitability share (from 0 to 1), and group firms with the same positive profitability share together. In Figure 2, the horizontal axis measures the cumulative share of firms while the vertical axis measures the positive profitability share. Each curve consists of a number of steps. The length of each step indicates the share of firms with the same positive profitability share, and the height of the step depicts the positive profitability share.

Figure 2 shows that when measuring profitability by profit, about 22 percent of the exiting firms have a positive profitability share of zero, that is, all their observations have negative profit. The corresponding figure for nonexiting firms is 12 percent. Moreover, almost 45 percent of the exiting firms have a positive profitability share that is 0.5 or lower, i.e., at least half of their observations have negative profit. We also see that about 25 (45) percent of the exiting (nonexiting) firms have a positive profitability share of 1, that is, they have positive profit in every year.

Figure 2 gives a mixed picture of the importance of profitability relative to exit. On the one hand, a substantial share of the exiting firms (25 percent) always have positive profit. Moreover, most exiting firms are profitable in the last few years before the exit.[6] This may indicate that the relationship between profitability and exit is weak. On the

[6] The share of exiting firms with positive operating surplus 3 years, 2 years and 1 year prior to exit is 86, 82 and 75 percent, respectively. The corresponding shares with positive profits are about 10 percentage points lower.

other hand, the graph of nonexiting firms lies above that of the exiting firms, reflecting that the former on average have higher profitability. The area between the two graphs is considerable, suggesting that there is a negative relationship between profitability and exit. We return to the question of whether there is a significant relationship between profitability and exit in Section 6.

3 Short-run factor demand

We consider an industry with monopolistic competition. Each producer faces a demand function of the following form:

$$Q_{it} = \Phi_t P_{it}^{-e} \tag{1}$$

where Q_{it} is the output of firm i at time t, P_{it} is the output price and Φ_t is an exogenous demand shift parameter characterizing the size of the market. Furthermore, $e > 1$ is the absolute value of the direct price elasticity. The price elasticity is common to all firms and constant over time.

Let M_{it} denote materials input, L_{it} labor input, and K_{it} capital. We assume that the use of materials and labor are determined at the beginning of each time period (variable inputs), whereas capital services in year t are determined by the capital stock at the end of $t-1$, $K_{i,t-1}$. However, through investment in period t, the capital stock at the end of period t increases (capital is quasi-fixed – see discussion below). The production function of producer i is assumed to be:

$$Q_{it} = A_{it} K_{i,t-1}^{\gamma} \left[M_{it}^{\rho} + (w_t L_{it})^{\rho} \right]^{\frac{\varepsilon}{\rho}}, \ \rho < 1 \tag{2}$$

where the elasticity of scale is equal to $\varepsilon + \gamma$, the elasticity of substitution between materials and labor is $1/(1-\rho)$, and w_t is a time-varying distribution parameter. Our production function can be seen as a nested Cobb–Douglas function defined over capital and a constant elasticity of substitution (CES) aggregate over labor and materials. The specification (2) allows for heterogeneity in productivity across firms: Hicks-neutral changes in efficiency are picked up by A_{it}, which may shift over time and vary across firms, whereas a positive change in w_t can be interpreted as a labor-augmenting innovation. Thus, w_t captures the feature that the skill composition of labor typically changes over time. While

L_{it} is the use of labor as measured in man-hours, $w_t L_{it}$ should be interpreted as the use of labor measured in efficiency units.

Let $\mathbf{q}_{it} = (q_{Mt}, q_{Lit})$ be a vector of the unit price of materials and labor, respectively. The unit price of labor is firm specific, which may reflect that the composition of the different types of labor varies across firms. Producers are assumed to be price takers in all factor markets. Using Shephard's lemma, the short-run cost function is:

$$C(\mathbf{q}_{it}, K_{i,t-1}, Q_{it}) = c_{it} \left(\frac{Q_{it}}{A_{it} K_{i,t-1}^{\gamma}} \right)^{\frac{1}{\varepsilon}} \tag{3}$$

where

$$c_{it} = [(q_{Lit}/w_t)^{\varrho} + q_{Mt}^{\varrho}]^{\frac{1}{\varrho}}, \ \varrho = \frac{\rho}{\rho - 1}. \tag{4}$$

Here, c_{it} is a firm-specific price index of variable inputs, i.e., derived from the CES aggregate of labor and materials. Note that c_{it} depends on the distribution parameter w_t, where q_{Lit}/w_t is the efficiency-corrected price of labor.

The short-run optimization problem of firm i in the beginning of period t, when the producer knows $\mathbf{q}_{it}$, Φ_t, A_{it} and w_t (and also e, γ, ρ and ε), is to choose – for a given stock of capital – the price that maximizes operating surplus:

$$\Pi_{it} = \max_{P_{it}} \left\{ \Phi_t P_{it}^{1-e} - c_{it} \left(\frac{\Phi_t P_{it}^{-e}}{A_{it} K_{i,t-1}^{\gamma}} \right)^{\frac{1}{\varepsilon}} \right\} \tag{5}$$

where $\Phi_t P_{it}^{1-e} = P_{it} Q_{it}$ (from (1)) is the revenue of the firm. Solving the resulting first-order condition gives the following equations for revenue $R_{it} = P_{it} Q_{it}$ and short-run factor costs $q_{Mt} M_{it}$ and $q_{Lit} L_{it}$:

$$\begin{bmatrix} \ln R_{it} \\ \ln(q_{Mt} M_{it}) \\ \ln(q_{Lit} L_{it}) \end{bmatrix} = \mathbf{1}\theta_1 \ln A_{it} + \begin{bmatrix} \theta_2 & 0 \\ \theta_2 - \varrho & 0 \\ \theta_2 - \varrho & \varrho \end{bmatrix} \boldsymbol{\alpha}_{it} + \mathbf{1}\gamma\theta_1 \ln K_{i,t-1} + \varrho \begin{bmatrix} 0 \\ \ln q_{Mt} \\ 0 \end{bmatrix} + \mathbf{1}\frac{\theta_1}{e-1} \ln \Phi_t \tag{6}$$

where $\mathbf{1}$ is a vector of ones,

$$\boldsymbol{\alpha}_{it} = \left[\ \ln c_{it}, \ \ \ln(q_{Lit}/w_t) \ \right]' \tag{7}$$

and

$$\theta_1 = \frac{(e-1)}{(\varepsilon + e - e\varepsilon)} > 0, \ \theta_2 = \frac{-\varepsilon(e-1)}{(\varepsilon + e - e\varepsilon)} < 0. \tag{8}$$

We can see that θ_1 is a common coefficient of the Hicks-neutral efficiency term $\ln A_{it}$ in all three equations in (6). On the other hand, a change in the firm-specific price index of variable inputs, c_{it}, will have a different impact on revenues (θ_2) than on factor costs ($\theta_2 - \varrho$). Note that an increase in w_t (for given q_{Lit}) increases revenue R_{it} because $\theta_2 < 0$; see (6) and (8). An increase in w_t has no direct impact on material costs, see (6), but will through a drop in the firm-specific price index c_{it} (see (4)), increase material costs if $\theta_2 < \varrho$, see (6). An increase in w_t has an identical indirect effect, through c_{it}, on labor costs as on material costs ($\theta_2 - \varrho$), but has in addition a direct impact on labor costs (ϱ). If $\varrho > 0$, an increase in w_t will therefore lower the short-run cost share of labor, i.e., the innovation is labor saving.

If the demand parameter Φ_t is allowed to be firm–time specific, denoted Φ_{it}, the system (6) is unaltered except that A_{it} is replaced by $A^*_{it} = \Phi_{it}^{1/(e-1)} A_{it}$. Thus, neutral efficiency shocks (A_{it}) and (idiosyncratic) demand shocks (Φ_{it}) enter the two alternative systems in a completely symmetric way, and we would not be able to distinguish between them in the empirical analysis. Therefore, A_{it} may capture both technology shocks and demand shocks, but we will still refer to A_{it} as "efficiency". This should be kept in mind when interpreting the results reported in Section 6.

The operating surplus Π_{it} defined in (5) has the closed form:

$$\begin{aligned} \Pi_{it} &= \big[\exp\left(\left[\begin{array}{cc} \theta_2, & 0 \end{array}\right] \boldsymbol{\alpha}_{it}\right) \\ &\quad - \exp\left(\left[\begin{array}{cc} \theta_2 - \varrho, & 0 \end{array}\right] \boldsymbol{\alpha}_{it} + \varrho \ln q_{Mt}\right) \\ &\quad - \exp\left(\left[\begin{array}{cc} \theta_2 - \varrho, & \varrho \end{array}\right] \boldsymbol{\alpha}_{it}\right)\big] A_{it}^{\theta_1} \Phi_t^{\frac{\theta_1}{e-1}} K_{i,t-1}^{\gamma\theta_1} \\ &\equiv \pi_{it} K_{i,t-1}^{\gamma\theta_1} \end{aligned} \tag{9}$$

where π_{it} is defined by the identity in (9), that is, $\pi_{it} = \Pi_{it}/K_{i,t-1}^{\gamma\theta_1}$. To ensure that optimization with respect to capital is well defined, we need to have $\gamma\theta_1 < 1$. (Our model meets this requirement; see below.)

4 Exit and investment dynamics

The producer invests in capital during year t and the purchase price of a unit of capital is q_{Kt}. We deflate all prices by the same index, so that in any time period one dollar of any cost component has the same value as one dollar of a revenue component. (If profit components are deflated by different indexes, nominal profit and deflated profit may have different signs.) We use the price index of capital as the deflator to reflect the opportunity cost of investment. Therefore, without loss of generality, we can normalize the real price of capital by setting $q_{Kt} \equiv 1$ in all time periods.

We follow the standard assumption that it takes one period until the stock of capital adjusts. If there were no costs of adjusting capital, then the stock of capital would be found from maximizing:

$$\Pi_{it} - (r + \delta)K_{i,t-1} \tag{10}$$

with respect to $K_{i,t-1}$, where Π_{it} is a function of $K_{i,t-1}$ given by (9), and $(r + \delta)K_{i,t-1}$ is the (neoclassical) user cost of capital (r is the real interest rate and δ the depreciation rate). Below, we refer to the solution of this problem as the stock of frictionless capital, $K^*_{i,t-1}$.

We now address the more realistic case where there are costs of capital adjustment. In each period, the firm makes an investment decision. Investments can be positive or negative. In particular, if the firm decides to exit during year t, it will sell its remaining stock of capital at the end of year t; $I_t = -(1 - \delta)K_{t-1}$.

Let the choice variables be (I_t, z_t), where I_t is investment and z_t is a dummy variable that takes a value of one if the firm continues to operate throughout year t, and zero if the firm exits during year t. We take the Markovian discrete choice model of Rust (1994) as the starting point and assume that the period t utility from the choice (I_t, z_t), given the state vector $S_t = (\pi_t, K_{t-1})$, can be written as:

$$u(S_t, I_t, z_t) + \varepsilon(z_t) \tag{11}$$

where $u(S_t, I_t, z_t)$ is operating surplus minus capital expenditures and $\varepsilon(z_t)$ is a random component associated with the discrete choice z_t. By definition we have:

$$u(S_t, I_t, z_t) = \begin{cases} \Pi_t - c(I_t) & z_t = 1 \\ \Pi_t - c(-(1 - \delta)K_{t-1}) & z_t = 0 \end{cases} \tag{12}$$

where $c(I_t)$ is the total cost of capital. Below, we assume that there is only one type of capital adjustment cost, namely, that the resale price of capital is lower than the purchase price of capital, i.e., costly reversibility (see Abel and Eberly, 1996). Then, $c(I_t)$ is weakly convex with a kink at zero.[7] Operating surplus Π_t follows from S_t and is therefore not affected by z_t and I_t. If $z_t = 0$, t is the terminal period and the firm sells its remaining capital stock, $I_t = -(1-\delta)K_{t-1}$, and obtains a scrap value, $-c(-(1-\delta)K_{t-1})$, at the end of the year.

Following Rust (1994), we assume that the state vector S_t is Markovian with transition probability $g(dS_{t+1}|S_t, I_t)$ and that $\varepsilon(z) = (\varepsilon(0), \varepsilon(1))$ has a bivariate extreme value distribution with scale parameter τ and location parameters $\gamma_z = (\gamma_0, \gamma_1)$:[8]

$$h(\varepsilon) = \prod_{z\in\{0,1\}} \tau \exp\{-\tau\varepsilon(z) + \gamma_z)\} \exp\{-\exp\{-\tau\varepsilon(z) + \gamma_z\}\}. \tag{13}$$

Further, the firm's choice of whether to continue production, and if so, how much to invest, follow from the solution of the Bellman equation:

$$V(S_t, \varepsilon_t) = \max_{z_t,\, I_t} \left\{ u(S_t, I_t, z_t) + \varepsilon(z_t) + \frac{1}{1+r} E_t\left[V(S_{t+1}, \varepsilon_{t+1})\right] \right\}. \tag{14}$$

The value function $V(S_t, \varepsilon_t)$ is characterized in Proposition 1, which is an extension of the discrete choice model in Rust (1994), that is, we allow for a discrete *and* a continuous decision variable.

Proposition 1 *Assume (11)-(13) and that S_t is Markovian with transition probability $g(dS_{t+1}|S_t, I_t)$. Then the expected net present value of the firm is:*

$$V(S_t, \varepsilon_t) = \max_{z_t\in\{0,1\}} \left[\Pi_t + v(S_t, z_t) + \varepsilon(z_t)\right] \tag{15}$$

where

$$v(S_t, 0) = -c(-(1-\delta)K_{t-1}) \tag{16}$$

and

$$\begin{aligned} v(S_t, 1) &= \max_{I_t} \left\{ -c(I_t) + \frac{1}{1+r} \times \right. \\ & \left. \int \left[\Pi_{t+1} + \frac{1}{\tau} \ln\left[\exp(-\tau c(-(1-\delta)K_t) + \gamma_0) + \exp(\tau v(S_{t+1}, 1) + \gamma_1)\right]\right] g(dS_{t+1}|S_t, I_t) \right\}. \end{aligned} \tag{17}$$

Finally, the exit probability is given by:

$$P(z_t = 0|S_t) = \frac{1}{1 + \exp\{-\left[-\tau c(-(1-\delta)K_{t-1}) - \tau v(S_t, 1) + \gamma_0 - \gamma_1\right]\}}. \tag{18}$$

[7] An alternative assumption is that the total cost of capital also includes resources to adjust to a higher stock of capital. Under the standard assumption that this type of cost of adjustment is decreasing in the initial stock of capital (for a given level of investment), see Abel and Eberly (1994), all our results apply.

[8] Because $E(\tau\varepsilon(z) - \gamma_z) = \gamma$ where γ is Euler's constant, we have $E(\varepsilon(z)) = (\gamma + \gamma_z)/\tau$.

The proof is in Appendix A.

$v(S_t, 1)$ can be interpreted as an expression for the net present value of the firm if it does not exit in the current period ($z_t = 1$) and makes optimal investment decisions now (I_t) and in the future:

$$v(S_t, 1) = \max_{I_t} \left\{ -c(I_t) + \frac{1}{1+r} E_t \left[V(S_{t+1}, \varepsilon_{t+1}) \right] \right\}.$$

We show in Appendix A that $\partial v(S_t, 1)/\partial K_{t-1} \geq 0$, that is, the value function is increasing in the current stock of capital. Using a similar proof, we can also show that if $g(dS_{t+1}|S_t', I_t)$ stochastically dominates $g(dS_{t+1}|S_t, I_t)$ for all $S_t = (\pi_t, K_{t-1})$ and $S_t' = (\pi_t', K_{t-1})$ with $\pi_t' > \pi_t$,[9] then $\partial v(S_t, 1)/\partial \pi_t \geq 0$. We will use these two properties of $v(S_t, 1)$ when we discuss the sign of parameters in the econometric exit model; see Section 6.2.

5 Stochastic specification and identification

The production model We now formulate an empirical model that encompasses our structural model. We first consider the empirical implications of (17) with regard to the investment dynamics. For an operative firm, I_{it} will be the maximizer of (17). According to Bloom et al. (2007), if i) firms maximize the expected net present value of profit, ii) adjustment costs are weakly convex, and iii) π_{it} $(= \Pi_{it}/K_{i,t-1}^{\gamma\theta_1})$ is a Markovian stochastic process, then (conditional on survival) the actual capital stock $K_{i,t-1}$ and the frictionless capital stock $K_{i,t-1}^*$ (see above) have the same long-run growth rate:

$$\ln K_{i,t-1} = \ln K_{i,t-1}^* + \text{error}$$

where the error term is stationary. Note that all three assumptions are in accordance with our structural model. It can be shown that:

$$\ln K_{i,t-1}^* = \kappa_a \ln A_{it} + \left[\begin{array}{cc} \kappa_c, & 0 \end{array} \right] \boldsymbol{\alpha}_{it} + \kappa_t \tag{19}$$

where κ_t is a time-varying intercept and:

$$\begin{aligned} \kappa_a &= \frac{e-1}{\gamma - \gamma e + \varepsilon + e - \varepsilon e} \\ \kappa_c &= -\varepsilon \kappa_a. \end{aligned} \tag{20}$$

[9] That is, $G(S_{t+1}|S_t') \leq G(S_{t+1}|S_t)$ for any S_{t+1}, where $G(S_{t+1}|S_t)$ is the cumulative density function (c.d.f.) corresponding to the probability density function (p.d.f.) $g(S_{t+1}|S_t)$. In our model, this means that a higher current profit π_t uniformly shifts the c.d.f. of the next year's profit, π_{t+1}, rightwards.

As a first-order approximation of the capital formation process of a firm, we estimate a linear equilibrium correction model with (19) as the equilibrium level, conditional on $z_{it} = 1$. That is:

$$\Delta \ln K_{it} = (\phi - 1)\left(\ln K_{i,t-1} - \ln K^*_{i,t-1}\right) + e_{Kit} \quad \text{if } z_{it} = 1 \tag{21}$$

where ϕ is an unknown coefficient and e_{Kit} is a white noise error term. Note that z_{it} is an endogenous variable that is simultaneously determined with $K^*_{i,t-1}$. Hence, we cannot estimate (21) separately from (18).

Next, let:

$$\mathbf{y}_{it} = (\text{log–revenue, log–material costs, log–labor costs, log–capital})'$$

be the vector of *observed* variables corresponding to the vector of theoretical variables $(\ln R_{it}, \ln(q_{Mt}M_{it}), \ln(q_{Lit}L_{it}), \ln K_{it})'$. We assume that the observed variables are identical to the corresponding variables in the system of structural equations (6), except for the additive white noise error terms, $\mathbf{e}_{it} = \begin{bmatrix} : & e_{Rit}, & e_{Mit}, & e_{Lit}, & e_{Kit} \end{bmatrix}'$, which are assumed to be normally distributed with mean zero and unrestricted covariance matrix $\mathbf{\Sigma}_e$. Combining (6), (19) and (21), and assuming that the firm enters the sample at $t = 1$ and τ_i is the last year firm i is observed (hence, $z_{it} = 1$ for $t = 1, ..., \tau_i$), we obtain:

$$\mathbf{y}_{it} = \begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_1 \\ \widetilde{\theta}_1 \\ (1-\phi)\widetilde{\kappa}_a \end{bmatrix} a_{it} + \begin{bmatrix} \theta_2 & 0 \\ \theta_2 - \varrho & 0 \\ \theta_2 - \varrho & \varrho \\ (1-\phi)\kappa_c & 0 \end{bmatrix} \boldsymbol{\alpha}_{it} + \begin{bmatrix} \gamma\theta_1 \\ \gamma\theta_1 \\ \gamma\theta_1 \\ \phi \end{bmatrix} \ln K_{i,t-1} + \mathbf{d}_t + \mathbf{e}_{it} \text{ for } t = 1, ..., \tau_i \tag{22}$$

where $(\widetilde{\theta}_1, \widetilde{\kappa}_a) = \widetilde{k}(\theta_1, \kappa_a)$ and $a_{it} = \ln A_{it}/\widetilde{k}$ for an arbitrary proportionality factor $\widetilde{k}$, and $\mathbf{d}_t$ is a vector of variables and parameters that only depends on t (not i).

We should note that the first three equations in (22) are identical to the equations in (6) but are augmented with noise terms. Thus, whereas the solution to (6) corresponds to an ex ante production plan that is based on the information available to the firm at the beginning of t, the ex post realizations, i.e., the data, are also determined by other (unpredictable) factors, for example, measurement errors, new information obtained during the year, and optimization errors.

We cannot identify θ_1 and κ_a because $\ln A_{it}$ is unobserved: (22) is observationally equivalent for any two values of $\widetilde{k}$. The parameters $(\widetilde{\theta}_1, \widetilde{\kappa}_a)$ are identified from the stochastic assumptions we make about a_{it}. We assume that:

$$\begin{aligned} a_{it} &= a_{i,t-1} + \eta_{it},\ t = 2, ..., \tau_i \\ a_{i1} &\sim \mathcal{IN}(0, \sigma_a),\ \eta_{it} \sim \mathcal{IN}(0,1). \end{aligned} \tag{23}$$

The variable a_{i1} represents the productivity of firm i in its start-up year relative to the average productivity of all new firms in that year, and the variance σ_a of a_{i1} characterizes the cross-sectional heterogeneity across firms in their first observation year. *Observed* productivity differences between operative firms in any subsequent year is the result of initial heterogeneity, a_i, *cumulated innovations*, $\sum_{t=2} \eta_{it}$, and self-selection (the most productive firms survive). To obtain identification, both the initial value of a_{i1} and the subsequent innovations η_{it} must have zero mean since any nonzero mean will be indistinguishable from the industry-wide intercept $\mathbf{d}_t$ in (22). Moreover, the variance of the innovation η_{it} is set to one to obtain identification of $(\widetilde{\theta}_1, \widetilde{\kappa}_a)$.

In (23) we have assumed that a_{it} is a random walk. We test the random walk assumption by allowing an AR(1) structure in (23): $a_{it} = \varphi a_{i,t-1} + \eta_{it}$. Using a likelihood ratio test, we cannot reject at the 5 percent level of significance that φ is one. The assumption of a random walk, which is consistent with Gibrat's law that firm growth rates are independent of firm size,[10] is therefore retained throughout this paper.

The exit decision Assume that:

$$-c(-(1-\delta)K_{i,t-1}) = s(1-\delta)K_{i,t-1}, \qquad s \leq 1. \tag{24}$$

The rationale behind (24) is that upon selling capital, the firm may not obtain the purchase price of capital (which in the present analysis equals one by normalization). Put differently, markets for old capital may be imperfect, or there may be large transaction costs, that is, $s < 1$. For parts of the capital stock there may even be no market (i.e., zero price) because of, for example, asymmetric information. In that case, the firm will face cleanup costs when the old capital is removed from the production site. The special case $s = 1$ corresponds to the neoclassical theory of investment. We now discuss how to find the function $v(S_t, 1)$ in (17). One possibility is to specify the parametric forms of $c(I_t)$

[10] The empirical literature suggests that Gibrat's law is only valid for large- and medium-sized firms. The validity of Gibrat's law for smaller firms appears to depend on whether the analysis is restricted to surviving firms; see Sutton (1997) and Caves (1998) for a discussion.

and $g(dS_{t+1}|S_t, I_t)$ in (17) and then solve for $v(S_t, 1)$. In general, we do not know these functions. In fact, one must choose very specific functions to be able to solve (17) and the chosen parametric forms may be bad approximations of the true forms. An alternative approach is to approximate $v(S_t, 1)$ directly. We chose the latter approach because it provides greater flexibility in fitting the data. We approximate $v(S_t, 1)$ by means of a sum of power functions:

$$v(S_t, 1) \simeq \beta_0^* + \beta_{1,\pi}^* \pi_{it} + \beta_{1,k}^* K_{i,t-1} + \beta_{2,\pi}^* \pi_{it}^{\gamma_\pi} + \beta_{2,k}^* K_{i,t-1}^{\gamma_k} + \beta_{\pi,k}^* \pi_{it}^{\lambda_\pi} K_{i,t-1}^{\lambda_k}. \tag{25}$$

When $\gamma_\pi = \gamma_k = 2$ and $\lambda_\pi = \lambda_k = 1$, (25) is a second-order Taylor expansion. However, much more flexibility is obtained by letting these coefficients be free parameters. In practice, it is not possible to accurately estimate all the coefficients in (25), and we will therefore choose which terms to include based on the Akaike information criterion (AIC).

An important feature of our approach is that the state variable π_{it} is derived from our *theoretical* model: $\pi_{it} = \Pi_{it}/K_{i,t-1}^{\gamma\theta_1}$, where Π_{it} defined in (9) is the firm's operating surplus under the assumption of no uncertainty and no decision errors in the short-run optimization. Because Π_{it} is *not* observable to the econometrician, neither is π_{it}. This represents the main challenge for estimating the model: π_{it} is a latent state variable.

Π_{it} can be estimated from the first three elements of $\mathbf{y}_{it}$, that is, observed revenue less the two observed cost components of operating surplus. However, this observational counterpart of Π_{it} is contaminated by white noise error terms, e_{jit} ($j = R, L, M$). In particular, the observed operating surplus may be negative, which is in contrast to Π_{it}.[11] In addition, there is a selection problem because $\mathbf{y}_{it}$ is observed conditional on $z_{it} = 1$. Our estimation method will take both these concerns into account.

Above, we implicitly assumed that $\varepsilon(z)$ is drawn independently across firms. More flexibility is allowed by letting γ_z in (18) be random coefficients that are common across firms, but that vary randomly from year to year; γ_{zt}. Formally, γ_{zt} is included in the state vector, that is, $S_{it} = (\pi_{it}, K_{i,t-1}, \gamma_{0t}, \gamma_{1t})$. In our empirical model, $(\gamma_{0t}, \gamma_{1t})$ are treated as fixed parameters to be estimated.

Estimation We first consider the estimation of ϱ and w_t. From (6) we have:

$$\ln\left(\frac{q_{Lit} L_{it}}{q_{Mt} M_{it}}\right) = -\varrho \ln w_t + \varrho \ln\left(\frac{q_{Lit}}{q_{Mt}}\right) + e_{Lit} - e_{Mit}. \tag{26}$$

[11] This follows from a well-known property of the (nested) Cobb–Douglas production function.

We can utilize (26) to obtain simple regression estimates of ϱ and w_t. Next, we introduce $\widehat{\boldsymbol{\alpha}}_{it}$ by replacing ϱ and w_t in $\boldsymbol{\alpha}_{it}$; see (7), with their estimated values obtained from the regression, $(\widehat{\varrho},\, \widehat{w}_t)$. Hence, $\widehat{\boldsymbol{\alpha}}_{it}$ is defined as:

$$\widehat{\boldsymbol{\alpha}}_{it} = \begin{bmatrix} \ln\left[(q_{Lit}/\widehat{w}_t)^{\widehat{\varrho}} + q_{Mt}^{\widehat{\varrho}}\right]^{\frac{1}{\widehat{\varrho}}} \\ \ln(q_{Lit}/\widehat{w}_t) \end{bmatrix}.$$

It is now clear that we can identify θ_2, ϱ and $\gamma\theta_1$ (cf. (22)) because the components of $\widehat{\boldsymbol{\alpha}}_{it}$, as well as $\ln K_{it}$, are observed regressors. However, we cannot identify θ_1 or κ_a, but only $(\widetilde{\theta}_1, \widetilde{\kappa}_a) = \widetilde{k}(\theta_1, \kappa_a)$. Moreover, from the expression $\theta_2 = -\varepsilon(e-1)/(\varepsilon + e - e\varepsilon)$, see (8), we see that we cannot identify both ε and e. To obtain identification of ε and e, we need to impose an additional condition. For example, if markets are assumed competitive, that is, $e \to \infty$, then $\theta_2 = -\varepsilon/(1-\varepsilon)$ and $\gamma\theta_1 = \gamma/(1-\varepsilon)$, so both ε and γ are identified. Alternatively, we can assume that the elasticity of scale is $\varepsilon + \gamma = 1$. Then, $\theta_2/\gamma\theta_1 = -\varepsilon/(1-\varepsilon)$, so ε is identified and then e follows from θ_2.

Given the estimates $\widehat{\varrho}$ and $\widehat{\boldsymbol{\alpha}}_{it}$ obtained in the first step of the estimation, our data on firm i can be seen as the realization of a stochastic process $(\mathbf{y}_{i1}, ..., \mathbf{y}_{i\tau_i})$, where $\tau_i \leq \overline{T}$ is the stopping time and $\overline{T}$ is the last observation year, i.e., 2009, and we have assumed for simplicity of notation that the firm enters at $t = 1$. The reason for stopping is either censoring or exit: in the latter case, $z_{i,\tau_i+1} = 0$. Note that $z_{it} = 1$ for $t \leq \tau_i$, while $z_{i,\tau_i+1} = 0$ (firm has exited) or $z_{i,\tau_i+1} = 1$ (firm is censored). By a standard factorization (see Billingsley, 1986) the log p.d.f. of $(\mathbf{y}_{i1}, ..., \mathbf{y}_{i\tau_i}, \tau_i = k, z_{i,\tau_i+1} = j)$ can be written as:

$$\ln P(z_{i2} = 1, ..., z_{ik} = 1, z_{i,k+1} = j | \mathbf{y}_{i1}, ..., \mathbf{y}_{ik}) + \ln f(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik}) \tag{27}$$

where $f(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik})$ is the density of $(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik})$ when k is fixed, i.e., *not* a stopping time.

To calculate $\ln f(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik})$, our model can be cast in a state space form with $\mathbf{y}_{it}$ as the observation vector and a_{it} as the only latent variable. Then, $\ln f(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik})$ can be calculated by standard methods from the one-step- ahead predictions and prediction variances of the state vector (see Shumway and Stoffer, 2000). To obtain analytical derivatives, we utilize a decomposition of $\ln f(\mathbf{y}_{i1}, ..., \mathbf{y}_{ik})$, which is well known from the EM-algorithm; see Koopman and Shephard (1992).

As discussed above, the explanatory variable π_{it} in the exit model is not observable (to the econometrician) but depends on the latent variable a_{it}, as seen from (9). Hence, it

Table 2: **Estimates of loading coefficients.** The standard errors (in parentheses) are obtained from the inverse Hessian of the log-likelihood function

Industry	Directly identified coefficients				Derived estimates assuming:						
					$e=\infty$			$\varepsilon+\gamma=1$			
	$\gamma\theta_1$	θ_2	ϕ	ϱ	ε	γ	κ_a	ε	γ	e	κ_a
Wood products	.12 (.01)	−.56 (.12)	.73 (.13)	.26	.36	.10	.73	.79	.21	1.83	.25
Plastic products	.18 (.01)	−.85 (.12)	.77 (.13)	.30	.46	.16	.68	.73	.27	2.67	.35
Metal products	.18 (.01)	−.39 (.11)	.72 (.11)	.35	.28	.11	.77	.72	.28	1.67	.22
Machinery	.13 (.01)	−.30 (.11)	.72 (.11)	.22	.23	.04	.80	.84	.16	1.32	.15
Electrical eq	.14 (.01)	−.77 (.21)	.70 (.11)	.28	.43	.08	.69	.83	.17	2.08	.28
Transport eq.	.24 (.01)	−1.15 (.32)	.77 (.11)	.37	.53	.10	.65	.82	.18	2.74	.34

is necessary to integrate – by Monte Carlo methods – over a_{it} *given* $(\mathbf{y}_{i1},...,\mathbf{y}_{ik})$, which is normally distributed, to obtain the firm-year- specific exit probability $P(z_{i2}=1,...,z_{ik}=1,z_{i,k+1}=j|\mathbf{y}_{i1},...,\mathbf{y}_{ik})$. More details about the estimation are in Appendix B, where we outline the maximum likelihood algorithm that we have implemented in GAUSS.

6 Results

6.1 Estimates of structural coefficients

In the empirical model, $\gamma\theta_1$ is the coefficient of lagged capital, $\ln K_{i,t-1}$, in the equations for revenues, material costs and labor costs; see (22). We can identify this (composed) coefficient, which, because of the log-linear form of our model, is the elasticity of an operating surplus factor (revenue, material costs or labor costs) with respect to the capital stock. The estimates of $\gamma\theta_1$ are depicted in the first column of Table 2, and they vary between 0.12 and 0.24. The relatively low values imply considerable curvature in the profit function. In contrast, Cooper and Haltiwanger (2006) find an elasticity of profit with respect to the capital stock of 0.59 for US manufacturing firms. The difference may reflect that Cooper and Haltiwanger (2006) assume price-taking behavior, whereas we allow firms to face downward-sloping demand curves (monopolistic competition). The speed of adjustment of the log of capital toward the equilibrium level $\ln K_{it}^*$, that is, $(1-\phi)$, is moderate. The estimates of ϕ vary from 0.70 to 0.77, which indicates a slow adjustment toward the frictionless capital stock K_{it}^* and may also reflect lumpy investment; see the discussion in Section 2.

As mentioned in Section 2, perfect competition is a special case in our model. We obtain perfect competition by letting the demand elasticity e in (1) approach infinity. For this limiting case, we have $\theta_2 = -\varepsilon/(1-\varepsilon)$ and $\gamma\theta_1 = \gamma/(1-\varepsilon)$; see the discussion after (26). Hence, we now obtain an estimate of ε, and this estimate varies between 0.23 and 0.53; see Table 2. We also obtain an estimate of γ, which varies from 0.4 to 0.16. Hence, the estimate of the long-run scale elasticity $\varepsilon + \gamma$ is in the range of 0.3 to 0.6, which is much lower than most estimates of the scale elasticity as they are typically around one. We believe our low estimate reflects that the imposed assumption of a competitive market is not valid; see the discussion in Section 2.

Another special case is obtained by imposing a long-run scale elasticity of one. Then the estimate of ε is roughly around 0.75 (see Table 2), which is close to the ratio between labor costs and value added in our data set; see Table 1. In this special case, we also obtain an estimate of e, which varies from 1.3 to 2.7 $-$; this is consistent with a high degree of market power and the large profit shares reported in Table 1. We also obtain an estimate of κ_a. This estimate, which is an elasticity, is low (0.2 to 0.3), implying a weak link between technological improvement and investment.

The estimates of ϱ in Table 2 lie between 0.2 and 0.4. Note that $1-\varrho$ is the elasticity of substitution between labor and materials, which is estimated to be small in our data. These estimates may be plausible as they are roughly in line with the corresponding parameters in the large-scale computable general equilibrium model of the Norwegian economy MSG.[12]

All the coefficients in Table 2 are highly significant. Our model is parsimoniously parameterized relative to the amount of data, and we obtain a high goodness of fit as measured by (pseudo) R^2, which varies between 90 and 92 percent depending on the industry.[13]

[12] See Bye et al. (2006).

[13] The pseudo R^2 is defined as:

$$R^2 = 1 - \frac{\text{tr } \widehat{Var}(\mathbf{e}_{it})}{\text{tr } \widehat{Var}(\widehat{\mathbf{y}}_{it} - \widehat{\mathbf{d}}_t)}$$

where tr denotes the trace, that is, the sum of the diagonal elements.

6.2 Exit probabilities

The most general approximation of $v(S_t, 1)$, see (25), leads to weakly identified and generally insignificant parameter estimates. It is therefore necessary to obtain a more parsimonious parameterization. Based on the AIC model selection value, we dropped some of the terms in (25). Table 3 provides the AIC value for the different submodels. The specification with no interaction term ($\beta^*_{\pi,k} = 0$), no nonlinear term in K_{it} ($\beta^*_{2k} = 0$) and no linear term in π_{it} ($\beta^*_{1\pi} = 0$) obtained for the different industries was either the lowest AIC value or a value very close to the minimum. This is also the best model for the pooled data, i.e., when all the industries are grouped together, whereas the most general specification, as well as the specification with no interaction term, did not converge with pooled data. Therefore, our specification satisfies $\beta^*_{\pi,k} = \beta^*_{2k} = \beta^*_{1\pi} = 0$. Then, combining (24) and (25) we can rewrite (18) as:

$$\Pr(z_{it} = 0|\ S_{it}) = \frac{1}{1 + \exp\left\{-\left[\beta_{0t} + \beta_{1k}K_{i,t-1} + \beta_{2\pi}\pi_{it}^{\gamma_\pi}\right]\right\}} \tag{28}$$

where $\beta_{0t} = -\tau\beta^*_0 + \gamma_{0t} - \gamma_{1t}$, $\beta_{1k} = \tau(s(1-\delta) - \beta^*_{1k})$ and $\beta_{2\pi} = -\tau\beta^*_{2\pi}$. The parameter β_{1k} of $K_{i,t-1}$ consists of two terms. The first term is related to the scrap value of capital, $s(1-\delta)$, where s is the price of old capital relative to the purchase price of new capital. Hence, if the scrap value of the firm increases such that the firm obtains more money if it exits, then, *cet. par.*, the probability to exit increases. The second term reflects how the optimal value of the firm, $v(S_t, 1)$, depends on K_{t-1}. In Section 4, we derived that $\partial v(S_t, 1)/\partial K_{t-1} \geq 0$. That is, if the capital stock increases it becomes, *cet. par.*, more valuable to continue production, and the probability of exit decreases. Hence $\beta^*_{1k} > 0$. The parameter of β_{1k} of $K_{i,t-1}$ is positive if $s(1-\delta) > \beta^*_{1k}$. In Section 4 we also derived that $\partial v(S_t, 1)/\partial \pi_t \geq 0$. This suggests that $\beta^*_{2\pi} > 0$, and hence $\beta_{2\pi} < 0$; that is to say, improved profitability lowers the probability of exit.

The second column in Table 4 shows that π_{it} has a significant negative impact on the probability of exit. That is, the estimated value of $\beta_{2\pi}$ is negative in all industries, and varies moderately across industries, ranging from -0.68 in transport equipment to -1.32 in metal products. The exponent of the corresponding power function, γ_π, is estimated to be around 0.4 in most industries. The lowest estimate is found in plastic products (0.17) and the highest in wood products (0.54). If we pool all six industries, the estimates of $\beta_{2\pi}$

Table 3: **AIC for different model specifications**

Industry	Unrestricted model	$\beta^*_{\pi,k}=0$	$\beta^*_{\pi,k}=\beta^*_{2k}$ $=\beta^*_{1\pi}=0$	$\beta^*_{\pi,k}=\beta^*_{2k}$ $=\beta^*_{2\pi}=0$
Wood products (20)	602.3	602.5	600.4	606.4
Plastic products (25)	NA^*	219.1	216.3	220.0
Metal products (28)	667.4	667.4	667.5	682.1
Machinery (29)	788.2	813.0	785.5	809.0
Electrical eq. (30–33)	534.2	530.5	532.8	547.5
Transport eq. (34–35)	570.2	570.4	568.7	573.7
Pooled data	NA^*	NA^*	3706	3774
# parameters:	25	22	19	18

*Not converging

Table 4: **Exit probability estimates** Standard errors of estimation in parentheses

Industry	β_{1k} (coeff. of $K_{i,t-1}$)	$\beta_{2\pi}$ (coeff. of $\pi_{it}^{\gamma_\pi}$)	γ_π
Wood products	.50 (.19)	-1.11 (.15)	.54 (.10)
Plastic products	.17 (.18)	-.78 (.24)	.17 (.17)
Metal products	.28 (.12)	-1.32 (.15)	.45 (.08)
Machinery	.20 (.17)	-1.12 (.11)	.35 (.07)
Electrical eq.	.30 (.17)	-.97 (.12)	.32 (.10)
Transport eq.	.18 (.06)	-.68 (.10)	.49 (.11)
Pooled data	.11 (.02)	-1.05 (.05)	.45 (.04)

and γ_π are -1.05 and 0.45, respectively. Both estimates are significantly different from zero in that the t-value of the estimate of $\beta_{2\pi}$ is 20 whereas the t-value of the estimate of γ_π is 10.[14]

The column second to last in Table 5 shows the elasticity of the exit probability with respect to operating surplus, Π_{it} (for a given capital stock, $K_{i,t-1}$), that is, by how many percent the exit probability changes – for a firm with mean values of the explanatory variables – when the operating surplus of this firm increases by one percent. The table shows that this elasticity varies across sectors from -0.12 in plastic products to -0.69 in wood products.

Table 4 shows that the estimate of the capital coefficient, β_{1k}, which consists of two counteracting effects (see the above discussion), is positive in all industries. It varies from

[14] For some firms exit may occur for other reasons than weak profitability. For example, the owner of a firm retires and decides to close down the firm because none of his family members are ready to continue the business. In our data, we have additional information on exit in that each exit is categorized as either bankruptcy or "steered closedown". If we restrict our attention to bankruptcy, which is clearly related to weak profitability, the estimate of $\beta_{2\pi}$ is -0.91 when the six industries are pooled, that is, close to the estimate when all firms (in the six industries) are included.

Table 5: **Elasticity of exit probabilities w.r.t. operating surplus and the stock of capital**

Industry	Mean of* Π_{it}	Mean of* $(r+\delta)K_{it}$	Mean of Pr(exit)	Elasticity of Pr(exit) with respect to:** Π_{it}	$K_{i,t-1}$
Wood products	.08	.35	.057	-.69	.17
Plastic products	.12	.61	.046	-.12	.10
Metal products	.10	.40	.029	-.62	.11
Machinery	.09	.35	.051	-.43	.06
Electrical eq.	.12	.37	.056	-.29	.10
Transport eq.	.13	.56	.065	-.30	.10
Pooled data	.10	.44	.043	-.43	.05

*In millions EUR

** Calculated for a representative firm, i.e., with mean values of Π_{it} and K_{it}

0.17 in plastic products to 0.50 in wood products, but is significantly different from zero at the 5 percent level in only some industries. However, the pooled estimate of 0.11 is highly significant, with a standard error of only 0.02. Thus, the net effect of additional capital seems to be a higher exit probability. This is as expected in that our estimates of $\gamma\theta_1$ (below 0.5) suggest that the return per Euro invested in capital, $\Pi_{it}/K_{i,t-1} = \pi_{it}K_{i,t-1}^{\gamma\theta_1-1}$, is strongly decreasing in $K_{i,t-1}$.

From the last column in Table 5, we can see that the elasticity of the probability to exit with respect to $K_{i,t-1}$, that is, the impact on the exit probability of a higher stock of capital, varies between 0.10 and 0.17 percent. The estimated elasticity from the pooled data is only 0.05. This indicates that the impact of the scrap value of capital on exit is only slightly stronger than the impact of improved profitability (because of additional capital) on exit, that is, $s(1-\delta)$ is only slightly greater than β^*_{1k}.

To evaluate the aggregate performance of our model, in each year we divide firms into two groups: *closing-down firms* in year t are those that exited during $t+1$, and *nonexiting* firms are those that did not exit during the entire observation period. Our two definitions imply that firms exiting in $t+1+p$ $(p \neq 0)$ are not included in any of the two groups in year t.[15] For each firm we are able to estimate – for each (relevant) year – the exit probability over the next year.

[15] As mentioned in Section 2, we study new firms that operated in at least 2 years, and we use the first observation year solely to obtain information about the initial capital stock of firms. This implies that the first possible year of exit is 1996 (i.e., firms deciding to close down at the end of 1995). Because of lags in the registration of closedowns, we cannot identify all firms that exited during our last year of observation, 2009; some are correctly categorized as exited, others are censored.

Figure 3 plots annual averages of the estimated exit probability of the two groups of firms. Our model discriminates between the two categories of firms: the exit probabilities of the closing-down firms are generally higher than those of the nonexiting firms. However, the differences between the two groups vary a lot over time (for a given industry), with large differences in some years and quite small ones in others.

Because our estimator maximizes the probability of observing the actual data, the result that closing-down firms have a higher estimated exit probability than nonexiting firms may seem uninteresting. To understand our result better, consider the hypothetical case in which all the realized variables of the firms, including exit, are assigned in a purely random way. Then, by assumption, there is no relation between exit, profitability and the size of the capital stock (our covariates). Hence, in this constructed data set, profitability and the stock of capital will have no impact on the estimated probability of exit, and the estimated coefficients will be (approximately) zero. We find that both profitability and the capital stock have a *significant* impact on the estimated exit probability. These statistically significant relations generate a substantial difference between the exit probability of closing-down and nonexiting firms.

6.3 Survival functions

Figure 3 illustrates the difficulty of predicting the *exit time* in that the estimated exit probabilities of the closing-down firms are erratic and vary a lot over time. The interpretation of Figure 3 is, however, not straightforward as the graphs incorporate different effects. First, they reflect temporal variations in both firm-specific conditions (e.g., technological changes) and in industry-specific conditions (changes in factor prices and demand). Second, the graphs of the closing-down firms reflect a composition effect as different firms are operating in different years. That is, if entrants to the industry (on average) have a higher exit probability than the incumbents, then (cet. par.) the average exit probability tends to decrease over time as the share of incumbent firms will increase compared to 1995 when all firms in our sample were start-up firms.

To control for calendar time and self-selection effects, we use our estimated model to simulate survival functions. These show the probability that a firm has survived until the end of year t as a function of time after entry and initial conditions. We construct the survival functions as follows. The year of entry of all the firms is referred to as year 0 (the entry year). For each firm, we use the estimated logit model, and the values of the observed variables, $\mathbf{y}_{it}$, in the first year the firm is operating to calculate the exit probability, that is, the probability of exit during the next year. We then remove a proportion of the firms using the following procedure. For each firm, we draw a random number from the uniform distribution $[0,1]$. Firms with a number lower than its estimated exit probability are removed. For each of the "surviving" firms, a new exit probability is estimated using the estimated logit model and the values of $\mathbf{y}_{it}$ in the second year the firm is contained in the data set. Then a proportion of the firms is removed, and so on. If a firm "survives" longer than in the actual data set, we use the econometric model to simulate the values of $\mathbf{y}_{it}$.

We repeated this experiment 100 times. In general, a firm will experience many different exit years. We use the frequency of exit years to construct the survival function of a firm as follows. Let Z_{it}^s be an indicator function which is one if – in the s'th simulation – a firm i has not exited by year t. Note that $Z_{it}^s = 1$ is conditional on $Z_{i0}^s = 1$ since all firms are operative at the end of year 0. By repeated simulations, $s = 1, ..., 100$, a firm-specific *conditional* survival function, $S_i(t) = P(Z_{it} = 1 | Z_{i0} = 1, \mathbf{y}_{i0})$ was estimated as:

$$\widehat{S_i(t)} = \frac{1}{100} \sum_{s=1}^{100} Z_{it}^s.$$

We then grouped each firm according to whether it was an exiting firm or a nonexiting firm (see the definitions above), and constructed survival functions for each group by averaging the survival functions over all firms in each group; see Figure 4. By comparing the survival functions for exiting and nonexiting firms, we can evaluate to what extent our model is *ex ante* able to "pick" the firms that actually exited during the observation period. Overall, we find that our model clearly discriminates between the two groups. For example, for wood products, we find that the ex ante survival probability of exiting firms is about 35 percent after 10 years, compared with 60 percent for nonexiting firms. We identify similar differences in the other industries. These results suggest that the main distinguishing characteristic of an exiting firm is not that its annual exit probability is very much higher than that of a surviving firm, rather that the difference in annual exit probabilities is highly persistent. Hence, it is the cumulative effect of higher annual exit probabilities over many years – compared with the average firm – that causes a firm to exit.

6.4 Does ownership structure matter?

In line with traditional neoclassical theory, the characteristics of the owners of a firm play no role in the above model. This may be justified in the case of listed corporations, at least as a theoretical simplification. However, in most countries, including Norway, closely held firms constitute by far the majority of companies. Typically, such firms – as opposed to widely held firms – face financial constraints, and must therefore rely on cash credits and their own working capital. This may have implications for investment and exit decisions.[16] Further, owners may differ along a number of dimensions, for example, gender, which may also have importance for the decisions made by the firm. The purpose of this subsection is to apply our econometric model to test the importance of ownership structure on firm behavior where we focus on the importance of financial constraints and gender differences among owners.

It is widely believed that difficulties in raising funds limit the opportunities of individuals to set up their own business. According to Parker (2004), most start-up firms with only private owners tend to obtain funding from the entrepreneurs themselves or their families. According to Carrol (2001), a liquidity-constrained owner–manager may "borrow from himself or herself" by postponing dividend payments and will face a higher discount rate the more current consumption is forsaken to undertake investment projects. Also, in Korinek and Stiglitz (2009), investor's subjective discount rate exceeds the risk-adjusted discount rate, reflecting agency problems between owners and managers of firms. Such agency problems arise because of asymmetric information between insiders and external investors; see Myers and Majluf (1984). In general, a high discount factor tends to decrease the stock of capital, that is, $K^*_{i,t-1}$ in our model.

There is also a widespread view that women are more risk averse than men and that they differ in their emotional reaction to uncertain situations. This may be because of biological differences; see Byrnes et al. (1999) for details. Therefore, there will be gender differences in the attitudes toward risk taking; see Croson and Gneezy (2009). According

[16] According to Caggese (2007), financial constraints are mostly relevant for small and privately owned firms.

to this literature, men also tend to be more overconfident than women, and, as a result, may have a different (more optimistic) perception of the probability distribution of future events. Finally, men tend to view risky situations as challenges, as opposed to women who are more likely to interpret risky situations as threats that encourage avoidance. This literature suggests that women tend to choose fewer (uncertain) investment projects. In our econometric model, $K^{*}_{i,t-1}$ may therefore be lower the higher the ownership share of women. We now proceed to test within our empirical model whether ownership concentration or the gender composition of owners affects investment and exit. It is expedient to construct three indices that reflect ownership concentration and gender composition in the start-up year of the firm. Because we will examine each index sequentially, all indices are termed o_i.

First, we use a Herfindahl index of owner concentration, and define i) $o_i \equiv \sum_{i=1}^{N} s_i^2$, where s_i is the ownership share of *individual* i and N is the number of individual (personal) owners of the firm. We identify both direct and indirect owners, thereby including relatively complex ownership structures such as pyramids and ownership chains up to three levels. We accomplish this by matching our firm data with the Shareholder Register, which contains information about owners (both individuals and firms) and their shareholdings. The ownership data cover the period 2001–09, and hence this analysis applies only to a subsample of firms.[17] Table 6 presents information on ownership concentration in the start-up year. The Herfindahl index has a mean/median of 0.5. This corresponds to two owners, each with a 50 percent ownership share. Hence, firms in our sample tend to have a very high degree of owner concentration at start-up.

An alternative measure of ownership structure is ii) $o_i \equiv$ "Share of personal owners", that is, number of shares held by individuals relative to all shares. As shown in Table 6, this group on average owns 89 percent of the shares (the median is 100 percent). Finally, to examine the importance of gender differences, we construct the variable iii) $o_i \equiv$ "Share of female owners $> 1/3$". This is a dummy variable that takes a value of one if the number of shares held by women relative to all shares is greater than one-third (the threshold for a blocking minority). In our data, this condition is met by 9 percent of the firms. As

[17] Using these registers, we can identify about 85 percent of the ownership of unlisted Norwegian limited companies. The remaining 15 percent are unidentified owners: institutions (public enterprises, ASA firms or public funds), foreigners (these cannot be identified through a Norwegian personal number), and listed firms. Thus, the sum of personal ownership shares in a sector may be less than one.

shown in Table 6, the average share of female ownership is only 8 percent, whereas the median is zero. Hence, the ownership share of female owners in these firms is generally small.

More formally, we incorporate the effects of ownership structure on investment by adding a term, $\kappa_o o_i$, to the frictionless capital, $\ln K^*_{i,t-1}$, defined in (19), where κ_o is an unknown parameter to be estimated:

$$\ln K^*_{i,t-1} = \kappa_o o_i + \kappa_a \ln A_{it} + \left[\begin{array}{cc} \kappa_c, & 0 \end{array} \right] \boldsymbol{\alpha}_{it} + \kappa_t. \tag{29}$$

Three separate analyses are carried out, one for each operationalization of o_i. If $\kappa_o = 0$, then there is no effect of ownership or gender on investment behavior. In contrast, the discussion above suggests that $\kappa_o < 0$, that is, $K^*_{i,t-1}$ is lower, the higher is o_i. The variable o_i may also affect the exit probability directly. We modify the exit probability equation as follows:

$$\begin{aligned} \Pr(z_{it} &= 0|\, S_{it}) = \\ & \frac{1}{1 + \exp\left\{ - \left[\beta_{0t} + \beta_o o_i + \beta_{1k} K_{i,t-1} + \beta_{2\pi} \pi_{it}^{\gamma_\pi} + \beta_{2\pi o} o_i \times \pi_{it}^{\gamma_\pi} \right] \right\}}. \end{aligned} \tag{30}$$

Here we allow o_i to affect the exit probability both as a first-order effect, through the term $\beta_o o_i$, and as an interaction effect with profitability, through the term $\beta_{2\pi o} o_i \times \pi_{it}^{\gamma_\pi}$.

The null hypothesis to be tested is that there is no effect of ownership or gender: $\beta_o = \beta_{2\pi s} = 0$. The term $\beta_o o_i$ may, however, pick up nonpecuniary returns to entrepreneurship (e.g., the utility of being "your own manager"), which may be important to personal owners of closely held firms – as these are often employed in the firm. This line of reasoning suggests that $\beta_o < 0$ in cases i) and ii). Further, we expect that firms with a high share of institutional ownership have easier access to equity through capital markets than do firms held mainly by personal owners. Thus, if a firm with only a few private owners loses its equity, it may be forced to close down because the owners may not be able to raise new equity. Hence, we expect $\beta_{2\pi o}$ also to be negative in cases i) and ii). Finally, the discussion above suggests that women are more risk averse than men and may therefore place more emphasis on negative profit shocks than men. Thus, the higher the share of female owners, the more sensitive the exit probability to profit shocks, that is, $\beta_{2\pi o}$ may be negative also in iii).

Table 7 details the estimates for the pooled data of firms established in 2001 or later. Because of the small number of firms (1,963) and exits (140), we do not present results at the industry level. The first row of results in Table 7 ($o_i \equiv 0$) is comparable to the results for pooled data reported in Table 4 (4,399 firms and 1,049 exits). All estimates of the β-coefficients are close to the corresponding estimates in Table 4. Table 7 shows that the estimated first-order effects (κ_o and β_s) are clearly insignificant for all three alternative operationalizations of o_i. There is, however, a weak tendency that firms with a high ownership concentration (the case of $o_i \equiv$"Herfindahl index") respond more strongly to changes in profit than do firms that are widely held, and also that firms with a high share of personal owners (the case of $o_i \equiv$ "Share of personal owners") respond more strongly to changes in profit than do firms that mainly have institutional owners. We can see this from the negative sign of the estimates of $\beta_{2\pi s}$ in these two cases (significant at the 90 percent level when $o_i \equiv$ "Share of personal owners"). Finally, the share of female owners has no effect on firm behavior.[18]

[18] This result is not changed if we alter the threshold in the definition of o_i to 0.5, or define o_i simply as the share of female owners.

Table 6: **Number of start-up firms, exits and measures of ownership structure, 2001–09**

Industry	No. of firms	No. of exits	Mean/median Herfindahl index*	Mean/median share of personal owners**	Mean/median share of female owners***
Wood products	270	28	.54/.50	.90/1	.08/0
Plastic products	94	5	.48/.47	.89/1	.14/0
Metal products	560	36	.55/.50	.92/1	.06/0
Machinery	399	27	.50/.49	.88/1	.06/0
Electrical equipment	375	26	.47/.41	.85/1	.16/0
Transport equipment	393	21	.47/.37	.88/1	.07/0
Pooled data	1963	140	.51/.50	.89/1	.08/0

*Herfindahl index=$\sum_{i=1}^{N} s_i^2$, where s_i is the ownership share of the ultimate *individual* owner i

**Number of shares held by individuals relative to all shares

****Number of shares held by women relative to all shares

Table 7: **The impact of ownership structure on capital growth and exit probability estimates** Standard errors of estimators in parentheses

	Coeff. of Eq. (29)	Coeff. of Eq. (30)				
	κ_o	β_o	β_{1k}	$\beta_{2\pi}$	$\beta_{2\pi o}$	γ_π
Basic model ($o_i \equiv 0$)	–	–	.09(.05)	−1.15(.12)	–	.57(.10)
$o_i \equiv$ “Herfindahl index”	−1.7 (3.33)	.06 (.31)	.07 (.04)	−.86 (.26)	−.48 (.39)	.57 (.09)
$o_i \equiv$ “Share of personal owners”	−.20 (.70)	.08 (.34)	.05 (.05)	−.26 (.46)	−.88 (.11)	.57 (.09)
$o_i \equiv$ “Share of female owners > 1/3”	−.40 (.30)	−.38 (.56)	.08 (.05)	−.89 (.13)	.56 (.29)	.58 (.09)

6.5 Policy simulations

It is important for policy makers as well as unions, management and owners of firms to know to what extent a higher wage increases the exit probability of firms. We can use the estimated model to shed light on this issue. Assume that there is a *permanent* increase in the price of labor of $\Delta \ln q_L$, that is, in each year the log wage is $\Delta \ln q_L$ higher than in the base case (q_{Lit}). How would this, *ceteris paribus*, affect the exit probabilities?

As a starting point, we fix the stock of capital. Then we see from (6) that the effect measured as an elasticity of a wage increase of $\Delta \ln q_L$ on revenue and factor costs is:

$$\begin{aligned} \mathrm{El}_{q_L} R_{it} &= \theta_2 \mathrm{El}_{q_L} c_{it} \\ \mathrm{El}_{q_L} q_{Mt} M_{it} &= (\theta_2 - \varrho) \mathrm{El}_{q_L} c_{it} \\ \mathrm{El}_{q_L} q_{Lit} L_{it} &= (\theta_2 - \varrho) \mathrm{El}_{q_L} c_{it} + \varrho, \end{aligned} \tag{31}$$

where, from (4):

$$\mathrm{El}_{q_L} c_{it} = \frac{(q_{Lit}/w_t)^{\varrho}}{(q_{Lit}/w_t)^{\varrho} + (q_{Mt})^{\varrho}} \ . \tag{32}$$

We find that the elasticity of the factor price index c_{it} with respect to the wage is on average (across firms and calendar years) in the range 0.2 to 0.4 across the different industries.

It is also of interest to simulate the long-run effects of a wage increase, which requires that the effect on capital accumulation is taken into account. From the capital equation (21) we obtain the following expression:

$$\mathrm{El}_{q_L} K_{it} = \phi \mathrm{El}_{q_L} K_{i,t-1} + (1 - \phi) \kappa_c \mathrm{El}_{q_L} c_{it}. \tag{33}$$

If the wage increase starts at the beginning of year $t = 1$, then $\mathrm{El}_{q_L} K_{i0}^{K} = 0$ (as the capital stock is predetermined at the beginning of the year), and (33) can be used recursively to calculate the effect on capital in a subsequent year of the permanent wage increase.

We now simulate the effect of a 10 percent permanent wage increase ($\Delta \ln q_L = 0.1$) that takes place at the beginning of $t = 1$. The base case is the actual realization of wages in the 10-year period 2000–09, while in the simulations, wages are 10 percent higher each year. The results from the simulations are summarized in Table 8. After one year, the survival probability decreases by one percentage point in all industries, and after 10 years the survival probability is reduced by only 2–3 percentage points in most industries. The exception is machinery where the survival probability is reduced by 5 percentage points after 10 years. Nevertheless, the overall picture is that the magnitude of the effects is small for all industries.

Table 8: **Survival probabilities in percent at the end of the year.** Base case (no wage increase) and with a permanent 10 percent wage increase at the beginning of the base year

Industry	After 1 year		After 5 years		After 10 years	
	Base case	With increase	Base case	With increase	Base case	With increase
Wood products	93	92	68	66	44	41
Plastic products	94	93	74	72	61	59
Metal products	96	95	79	77	62	59
Machinery	94	93	72	70	54	49
Electrical equipment	94	93	75	74	58	57
Transport equipment	94	93	73	71	53	51

7 Conclusion

In this paper, we raised two questions: what causes firms to exit, and what are the characteristics that distinguish exiting firms from nonexiting firms? Using a structural econometric model, we derived explanatory variables from economic theory and estimated models for six Norwegian export-oriented manufacturing industries. The results show that when exit is defined as a state in which production at the site has come to a permanent stop, increased profitability significantly lowers the exit probability, or put differently, low profitability causes firms to exit. We have also found a clear difference in the estimated exit probabilities between firms that exited during the sample period (1994–2009) and firms that did not exit. According to our study, exiting firms differ from nonexiting firms in that their annual exit probabilities are *persistently* higher. Conversely, exiting firms are not characterized by having a very high exit probability immediately prior to exit, which reflects the fact that there were no (negative) profitability shocks in the last few years prior to exit.

According to our results, an increase in the size of the capital stock increases the probability of exit, and this effect is statistically significant in most industries. Because the stock of capital can be a proxy for firm size, this result appears to be in conflict with the existing literature, which concludes that firm size lowers the exit probability; see the discussion in Section 1 for details. However, our study shows the partial effect of firm size (capital) when profitability is controlled for, whereas other studies typically control for variables other than profitability, for example, firm age. This reinforces our point in Section 1 that empirical exit studies may draw false conclusions when profitability is not included in the set of covariates. In Section 2, we argue that our data suggest a weak relationship between profitability and exit. Yet, our estimation results clearly indicate such a relationship. We believe this shows the power of econometric modeling and methods. For example, in our model cost of adjustment, reflecting that the resale price of capital is lower than the purchasing price of new capital, is potentially a key factor in determining exit. However, this type of cost is not included in our capital data. By deriving an expression for the adjustment cost of capital we have been able to incorporate this factor into the expression for the probability of firm exit.

We close by outlining some topics for future research. To start with, in the introduction

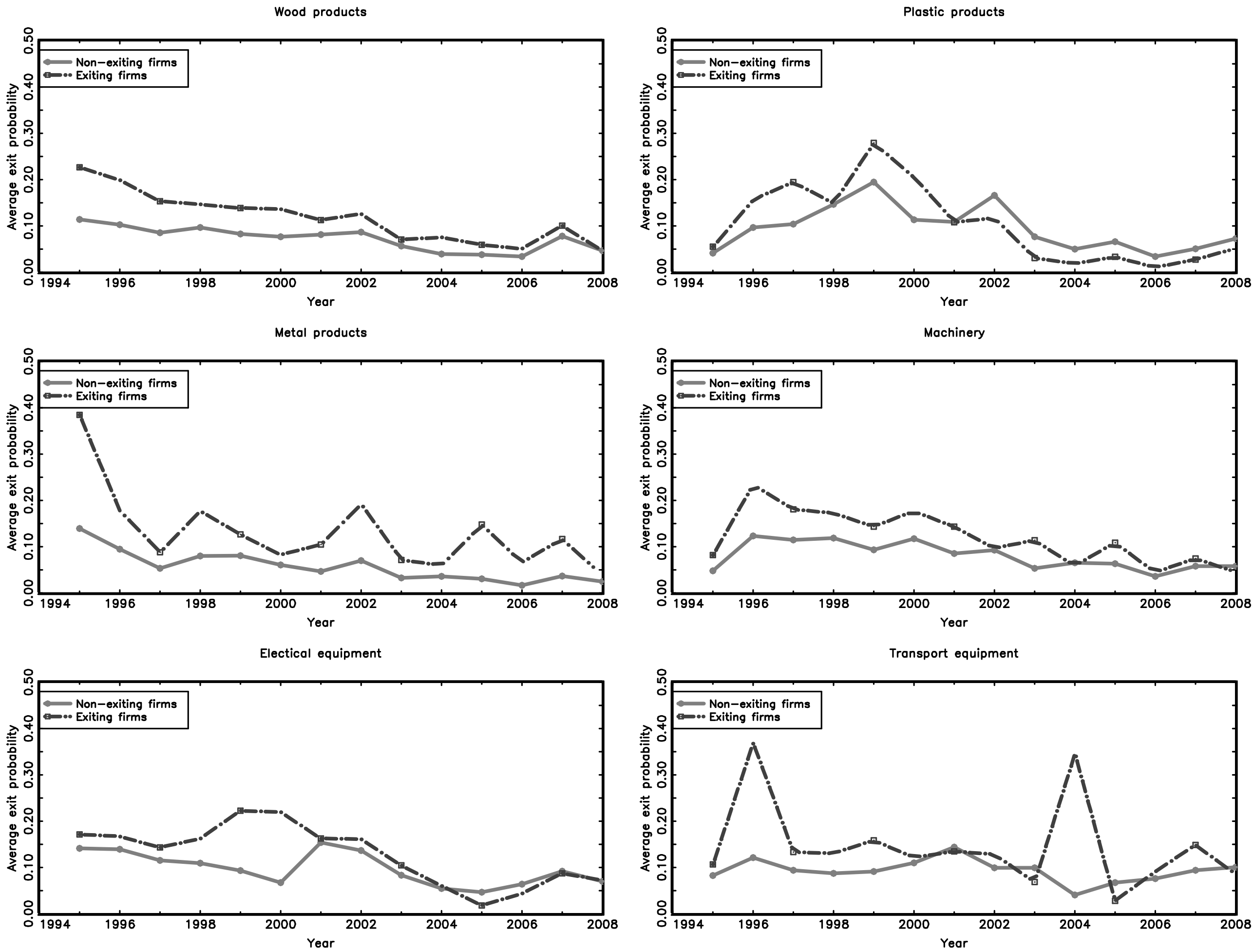

Figure 3: **Estimated aggregate exit probabilities for nonexiting and closing-down firms in** $t = 1995, ..., 2008$

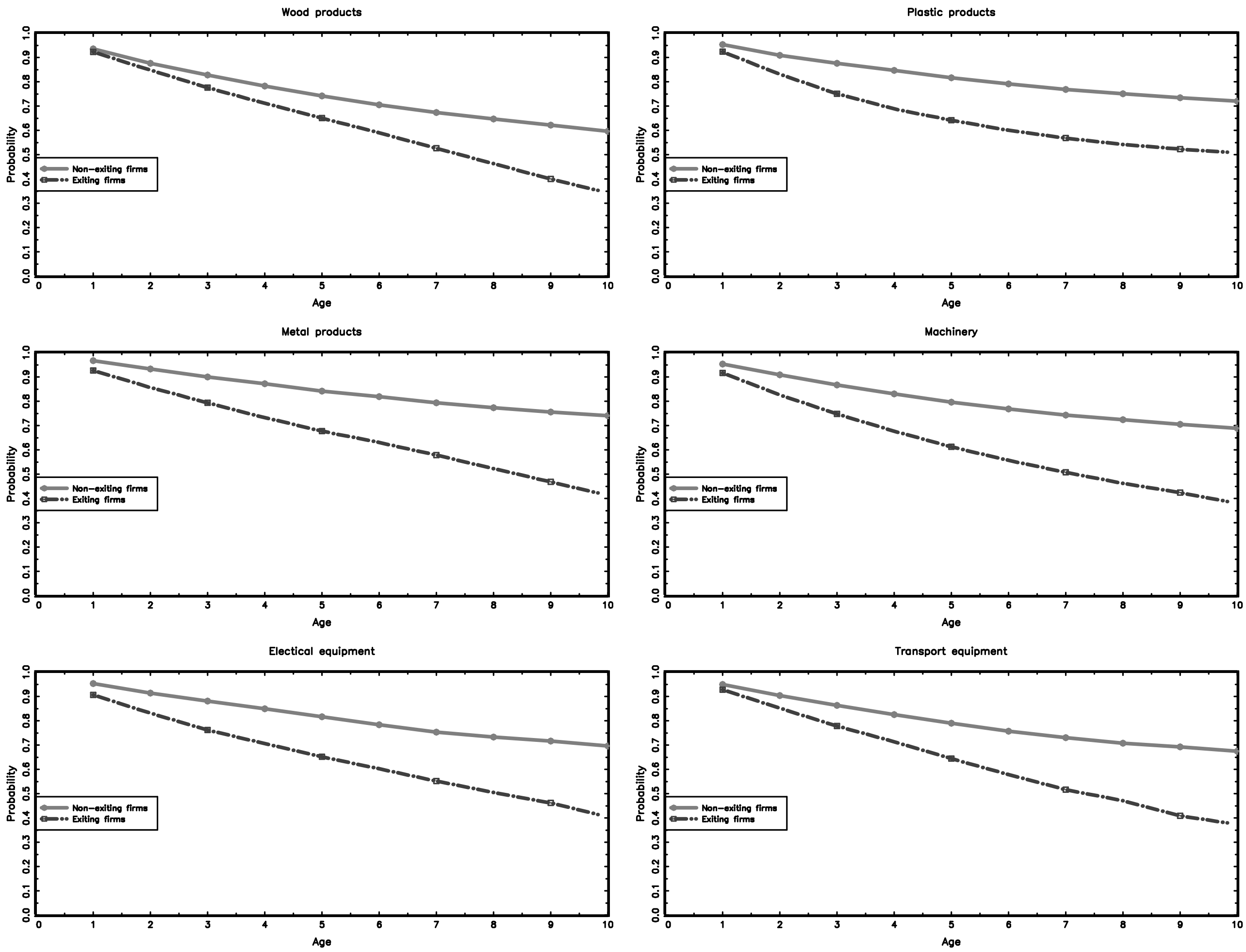

Figure 4: Estimated survival probabilities as a function of firm age for nonexiting and exiting firms

to this paper we referred to a number of articles that have empirically documented that the age of a firm is related to its probability of exit. However, in our derived model, age is not specified as an explanatory variable, simply because it plays no role in the theoretical foundation of the econometric model. Age may, however, play a role through learning processes; see Jovanovic (1982) and Pakes and Ericson (1998). Hence, one extension of our model would be to incorporate learning effects.

A second issue that remains unsolved is the treatment of multiplant firms. These may potentially exhibit a different relationship between profitability and exit than the one we have found. This is because a multiplant firm may take into consideration that increased output from one plant will lower the output price, and, hence, adversely affect the profitability of its other plants. For example, some years ago the largest Norwegian pulp and paper firm (Norske Skog), which owns several plants in Norway and abroad, announced plans to close one domestic unit. According to management, although production at this unit was profitable, continued production would lower profitability for the entire multiplant firm. Management, later supported by the majority of owners, was not even willing to sell the unit as they feared continued production under new owners would adversely affect the firm.

This example indicates that there may not be a simple relationship between plant profitability and exit for multiplant firms. For example, shocks may induce multiplant firms to reorganize production by closing (or opening) plants; see Reynolds (1988) and Whinston (1988) for analyses of exit behavior of firms operating several plants. Likewise, Bernard and Bradford Jensen (2007) employ a probit model to study exits in single and multiplant firms in the US manufacturing industry. They find that plants belonging to multiplant firms are less likely to exit, but after controlling for plant and industry attributes, these same plants are more likely to exit than are single plant firms. Similarly, using a logit analysis, Lieberman (1990) finds that after controlling for plant size, large multiplant firms are more likely to close plants. Hence, a topic for future research is to examine, using a structural microeconometric framework, the closedown of plants in multiplant firms.

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8 Appendix A: Proofs

Proof of Proposition 1 Let $v(S_t, z_t)$ denote net present value given S_t and z_t:

$$v(S_t, z_t) = \max_{I_t} \left\{ u(S_t, I_t, z_t) - \Pi_t + \frac{1}{1+r} E_t \left[V(S_{t+1}, \varepsilon_{t+1}) \right] \right\}. \tag{34}$$

Then (15) follows by definition. If $z_t = 0$, t is the terminal period and $v(S_t, 0) = u(S_t, -(1-\delta)K_{t-1}, 1) - \Pi_t = -c(-(1-\delta)K_{t-1})$, which proves (16).

To prove (17), assume a finite horizon problem and let $v_T(S_t, 1)$ be defined in the same way as $v(S_t, 1)$ in (34), except that ∞ is replaced by T in the summation limit. That is, $v_T(S_t, z_t) + \Pi_t + \varepsilon(z_t)$ is the net present value in period t of choosing z_t and then make optimal decisions with regard to I_t and $(I_{t+k},\ z_{t+k})$ for $k = 1, ..., T$, where $t + T$ is the terminal period. Thus, we consider a T-period-ahead problem. For example, $v_0(S_t, 1)$ is the solution of the static problem ($T = 0$), $v_1(S_t, 1)$ is the one-period-ahead problem ($T = 1$), etc. Let $V_T(S_t, \varepsilon_t)$ denote the value function (15) corresponding to the T-periods-ahead problem: $V_T(S_t, \varepsilon_t) = \max_{z_t}[\Pi_t + v_T(S_t, z_t) + \varepsilon(z_t)]$. Obviously, $v_0(S_t, 1) = \max_{I_t}[-c(I_t)]$. Furthermore:

$$\begin{aligned}
v_1(S_t, 1) &= \max_{I_t} \left\{ -c(I_t) + \frac{1}{1+r} \int V_0(S_{t+1}, \varepsilon) h(\varepsilon) d\varepsilon \, g(dS_{t+1}|S_t, I_t) \right\} \\
&= \max_{I_t} \left\{ -c(I_t) + \frac{1}{1+r} \int \max \left\{ \Pi_{t+1} + v_0(S_{t+1}, 0) + \varepsilon(0), \Pi_{t+1} + v_0(S_{t+1}, 1) + \varepsilon(1) \right\} h(\varepsilon) d\varepsilon \times \right. \\
&\quad \left. g(dS_{t+1}|S_t, I_t) \right\} \\
&= \max_{I_t} \left\{ -c(I_t) + \frac{1}{1+r} \int \left[\Pi_{t+1} + \frac{1}{\tau} \ln \left[\exp(-\tau c(-(1-\delta)K_t) + \gamma_0) + \exp(\tau v_0(S_{t+1}, 1) + \gamma_1) \right. \right. \right. \\
&\quad \left. g(dS_{t+1}|S_t, I_t) \right\},
\end{aligned}$$

where the integrand after the last equality is the so-called "social surplus" function. The last equality follows from (12) and a well-known property of the extreme value distribution (see Rust, 1994). By backward recursion we obtain:

$$\begin{aligned}
v_T(S_t, 1) &= \max_{I_t} \left\{ -c(I_t) + \frac{1}{(1+r)} \times \right. \\
&\quad \int \left[\Pi_{t+1} + \frac{1}{\tau} \ln \left[\exp(-\tau c(-(1-\delta)K_t) + \gamma_0) + \exp(\tau v_{T-1}(S_{t+1}, 1) + \gamma_1) \right] \right] \times \\
&\quad \left. g(dS_{t+1}|S_t, I_t) \right\}. \qquad (35)
\end{aligned}$$

Under the regularity conditions of Rust (1994), equation (35) defines contraction mapping so that:

$$\sup_S |v_T(S, 1) - v_{T-1}(S, 1)| \to 0 \text{ as } T \to \infty.$$

Then there exists a limiting function $v(S, 1)$ that satisfies (17). Finally, from (15):

$$\begin{aligned}
P(z_t &= 0|S_t) = P\left(v(S_t, 0) + \varepsilon(0) > v(S_t, 1) + \varepsilon(1)|S_t\right) \\
&= P\left(\tau v(S_t, 0) + \gamma_0 + (\tau\varepsilon(0) - \gamma_0) > \tau v(S_t, 1) + \gamma_1 + (\tau\varepsilon(1) - \gamma_1)|S_t\right) \\
&= \frac{1}{1 + \exp\{-[\tau v(S_t, 0) - \tau v(S_t, 1) + \gamma_0 - \gamma_1]\}},
\end{aligned}$$

where in the last equation $\tau\varepsilon(z) - \gamma_z$ has a standard extreme value distribution and is independent for $z = 0, 1$. Hence, (18) follows from (16).

Q.E.D.

Proof of $\partial \mathbf{v}(\mathbf{S}_t, \mathbf{1})/\partial \mathbf{K}_{t-1} \geq \mathbf{0}$. We have:

$$\begin{aligned}\frac{\partial v_1(S_t,1)}{\partial K_{t-1}} &= \left\{\frac{1}{1+r}\int\left[\pi_{t+1}\gamma\theta_1 K_t^{\gamma\theta_1-1}(1-\delta)+P(z_{t+1}=0|S_{t+1})(1-\delta)s+\right.\right.\\ &\left.\left. P(z_{t+1}=1|S_{t+1})\frac{\partial v_0(S_{t+1},1)}{\partial K_t}\right]g(dS_{t+1}|S_t,I_t)\right\}\\ &\geq 0\end{aligned}$$

where we have used the envelope theorem and that $\frac{\partial K_t}{\partial K_{t-1}} = 1-\delta$. By recursion, $\frac{\partial v_T(S_t,1)}{\partial K_{t-1}} \geq 0$, and hence in the limit, $\frac{\partial v(S_t,1)}{\partial K_{t-1}} \geq 0$.

Q.E.D.

Appendix B: The likelihood function and the ML estimator

We now outline the procedure for estimation of the parameters. We partition the parameters as follows: $\beta = (\beta_0, \beta_1, \beta_2)$ and θ contains all the remaining parameters to be estimated. For notational simplicity, assume that all firms enter the sample at $t = 1$ (the general case is a straightforward extension). All probability statements will henceforth be conditional on the initial capital stock, K_{i0}, although for simplicity this conditioning is suppressed in the notation.

The observed data on firm i consist of $\{\mathbf{y}_{it}\}; t = 1, ..., \tau_i$ and $\{z_{it}\}; t = 2, ..., \min(\tau_i + 1, \overline{T})$. We will now establish the likelihood as a function of (β, θ).

Let $a^i = \{a_{i1}, ..., a_{i,\tau_i+1}\}$. Further, let $f(a^i|Y^i;\theta)$ be the density of a^i conditional on $Y^i \equiv (\mathbf{y}_{i1}, .., \mathbf{y}_{i\tau_i})$, and let $f(Y^i;\theta)$ be the marginal density of Y^i. The position after the semicolon is used to indicate the unknown parameters of the density. The joint log-likelihood function $l(\beta,\theta)$ becomes:

$$l(\beta,\theta) = \sum_{i=1}^{N} l^i(\beta,\theta) \tag{36}$$

where N is the number of firms, and, reformulating (27):

$$\begin{aligned} l^i(\beta,\theta) = \ln \int \prod_{t=2}^{\min(\tau_i+1,\overline{T})} & P(z_{it}|\pi_{it},\ K_{it};(\beta,\theta)) \quad f(a^i|Y^i;\theta)da^i \\ & + \ln f(Y^i;\theta). \end{aligned} \tag{37}$$

A natural estimation strategy would be to maximize the joint log-likelihood with respect to the unknown parameters. We can then utilize that (22)–(23) are formulated in state space form, with $\mathbf{y}_{it}$ as the observation vector and a^i as the state vector, to obtain $f(a^i|Y^i;\theta)$ by means of the Kalman smoother. The multiple integral in (37) can be evaluated by Monte Carlo simulations using the algorithm of Durbin and Koopman (2002). However, the estimation problem remains complex because $\mathrm{P}(z_{it}|\ \pi_{it},\ K_{it};(\beta,\theta))$ depends in a complex way on the parameters θ and on the latent vector a^i through π_{it}.

To estimate β and θ, we first obtain simple preliminary estimates $(\overline{\beta},\overline{\theta})$ as follows:

$$
\begin{aligned}
\overline{\theta} &= \arg\max_{\theta} \sum_{i=1}^{N} \ln f(Y^i;\theta) \\
\overline{\beta} &= \arg\max_{\beta} \sum_{i=1}^{N} \sum_{t=2}^{\min(\tau_i+1,\overline{T})} \ln \mathrm{P}(z_{it}|\ \widehat{\pi}_{it},\ K_{it};(\beta,\overline{\theta}))
\end{aligned}
$$

where $\widehat{\pi}_{it}$ is the predicted value of π_{it} obtained by replacing a_{it} by $E(a_{it}|Y^i;\overline{\theta})$ – this expectation is obtained from the Kalman smoother. These are then used as starting values when maximizing (37) jointly with respect to (β,θ).

We find that the final estimates, $(\widehat{\beta},\widehat{\theta})$, are close to the initial estimates, $(\overline{\beta},\overline{\theta})$, and that the process converges quite quickly. Visual inspection of the log-likelihood in orthogonal directions (corresponding to the eigenvectors of the estimated covariance matrix) confirms we have found a global maximum.

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