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Consumer Demand in a General Equilibrium Model for Environmental Analysis

Discussio

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Abstract
The system of consumer demand functions for the 22 consumption goods in the general equilibrium model MSG-EE is presented. The consumer model has the following specific features. (i) It is based on a three level utility tree. At the lowest level there is much substitution between say air transport and other forms of public transportation; at the intermediate level there is considerable substitution between public transport and private transport; and at the top level there are moderate possibilities for substitution between say transport and food. (ii) The utility tree is based on non-homothetic weak separability, taking account of the fact that the Engel elasticity of say bus transport is much less than the Engel elasticity of air transport. There is perfect aggregation over goods in the utility tree, which makes it necessary to have more than one price index for commodity groups at the intermediate and the top level. (iii) The utility functions are household specific and we use perfect aggregation across all households in Norway to derive the macro demands, in which the number of households, the number of children, and the number of adults are important variables. (iv) The model is calibrated exploiting both microeconometrics and macroeconometrics, taking both random and systematic measurement errors into account.

Keywords: non-homothetic utility trees, household size and composition, aggregation over households, microeconometrics in macromodels, demand for transport and energy

JEL classification: D12, E21, Q43

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1 Introduction

This paper presents a complete system of consumer demand funtions for the Norwegian economy. The specific features of the model are outlined in the abstract and in the conclusion of the paper. In this introduction we give some information on the project and on the organization of the paper.

The consumer demand functions are implemented in the general equilibrium model MSG-EE, which is a Multi Sectoral Growth model with emphasis on Energy and Environment, developed in the research department at Statistics Norway. The basic structure of the MSG-EE model is presented in Holmøy and Strøm (1992), and an overview and analysis of a closely related model (MSG-5) can be found in Holmøy (1992). The consumption model in MSG-EE consists of demand functions for 22 goods, which have been aggregated to demand functions for the 13 consumption goods in MSG-5, as documented in Aasness and Holtsmark (1993). The latter commodity grouping is used also in other policy simulation models developed in Statistics Norway, in particular the macroeconomic model MODAG (cf Cappelen (1992)) and the microsimulation model LOTTE-KONSUM (cf Aasness (1993)). Features of the present consumer demand functions can easily be compared, tested and/or implemented in relation to these other policy simulation models.

The theoretical starting point is the standard static consumer theory (cf section 2.1), with utility trees (cf section 2.2), and parametric forms of the direct utility function (cf section 2.3-4). We interpret the consumer to be a household, where we take into account economies of scale in household production and that children and adults have different needs. We derive the macro demand functions by perfectly aggregating the demand functions over all households in Norway (cf section 2.5). The macro demand functions depend on the price vector, the macro total expenditure, the number of households, the number of children and the number of adults in Norway.

The empirical model is presented in section 3, including all parameters and an extensive set of elasticities. The simulation model is programmed in terms of a recursive equation system which is presented in appendix A. The model is calibrated using empirical information from several types of data sources and econometric studies with emphasis on microeconometrics. The calibration of the model is presented in appendix C. The methods of this calibration procedure is developed in Aasness (1993b). The construction and application of the model can be considered as a step in a research program for developing and testing consumer demand models, as described in Aasness (1993c).

2 Theory

2.1 Standard consumer theory

Let us consider a consumer with a utility function

$$u = \upsilon(q,a) = \upsilon(.., q_j, .., a), \tag{1}$$

and a linear budget constraint

$$\sum_{j \in J} p_j q_j = y, \tag{2}$$

where q_j is the quantity and p_j the price of commodity j, q and p are the corresponding row vectors, J is the set of commodities, y is total expenditure (income for short), and a is some vector of attributes of the consumer. In our application we shall consider the consumer to be a household and $a = (a_1,a_2)$ to be the number of children and adults in the household. We assume that the quantities consumed must be non-negative, and that the vector of prices and total expenditure belongs to some subspace of the non-negative orthant of the $n + 1$ dimensional Euclidean space, called the price-income space.

The assumption that a unique solution to the problem of maximising utility subject to the budget constraint exists, gives the Marshallian demand functions

$$q_j = g_j(y,p,a), \quad j \in J. \tag{3}$$

From the assumptions above we obtain several properties of the demand functions (adding-up, homogeneity, non-negative demands), and by introducing quasi-concavity and smoothness of the utility functions we obtain further properties of the demand functions (differentiability, the Slutsky equation, symmetry and negative semidefineteness of the matrix of Slutsky derivatives), see for example Katzner (1970), Deaton and Muellbauer (1980) and Aasness (1990, Essay 2).

In order to get more substance to the consumer theory, with more testable hypotheses and less data requirements for estimation, we shall impose structure on the preferences through assumptions on separability and parametric forms. But before doing this we will partition the commodities into groups and introduce some notation.

Let us divide the set of commodities J into a set of exhaustive and mutually exclusive groups of commodities, using the notation

$$j \in J_r. \quad r \in R, \quad J = \cup_r J_r. \quad J_r \cap J_s = \phi, \quad r \neq s, \quad r,s \in R, \tag{4}$$

thus J_r means the set of commodities within group r and R is the set of groups of commodities. Define the expenditure on group r by

$$y_r = \sum_{j \in J_r} p_j q_j, \quad r \in R. \tag{5}$$

The Marshallian group expenditure functions are defined by

$$y_r = g_{yr}(y,p,a) \equiv \sum_{j \in J_r} p_j g_j(y,p,a), \quad r \in R. \tag{6}$$

These group expenditure functions are important tools when working with utility trees.

2.2 Utility trees

The assumption that the direct utility function is weakly separable in the set R of commodity groups can be symbolised by

$$u = f(..,\upsilon_r(q_r,a),..,a), \tag{7}$$

where q_r is the vector of consumed quantities of commodities in group *r*, and $\upsilon_r(.,a)$ is the subutility function for group *r*. This assumption implies that the conditional demand functions,

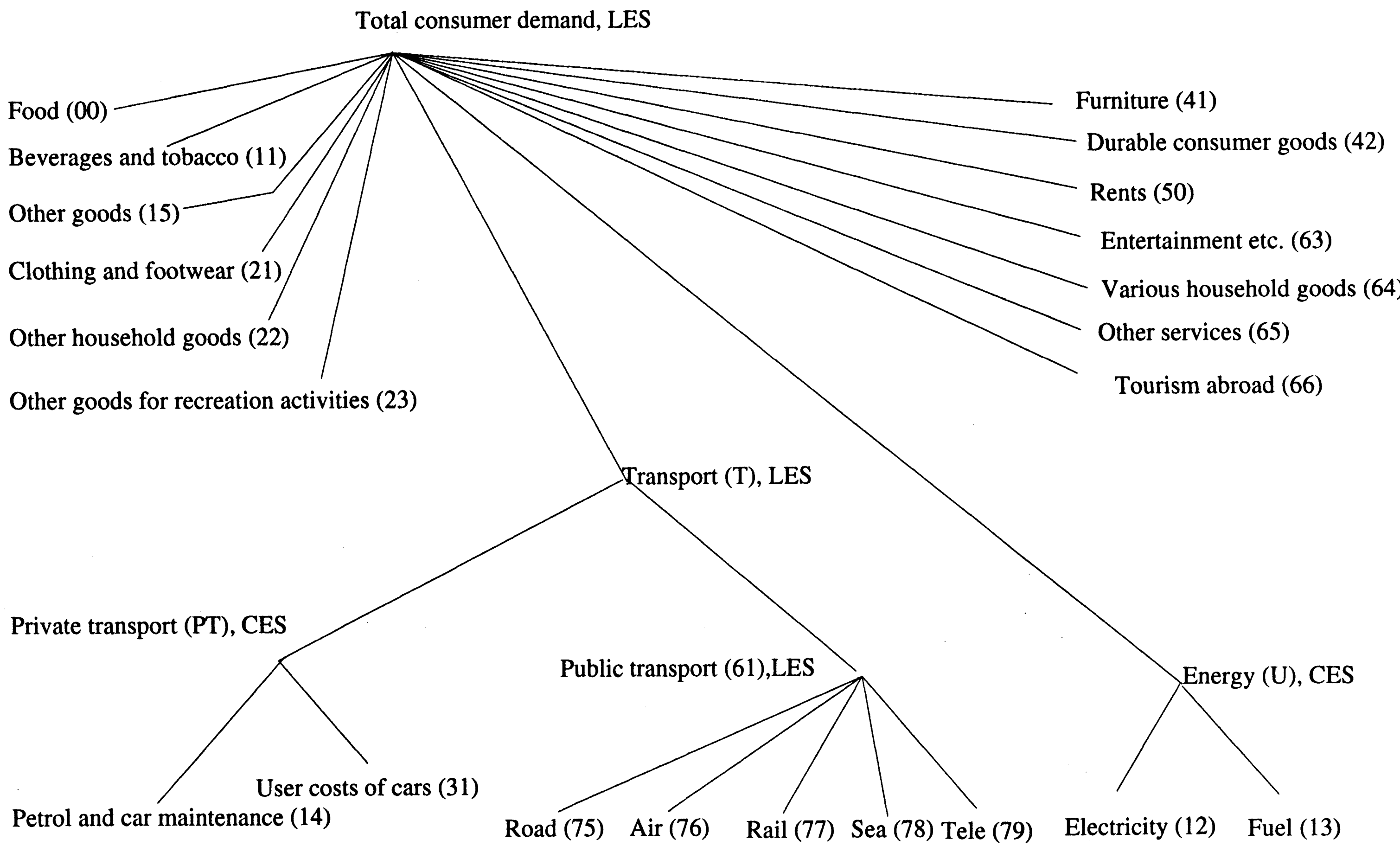
$$q_j = g_{jr}(y_r, p_r, a), \quad j \in J_r, r \in R, \tag{8}$$

i.e. the demand for commodity *j* as function of group expenditure and prices within the group, is independent of total expenditure (or utility) and prices (or possible rations) of other goods. The latter factors enters only through the group expenditure functions (6) and do not change the form of the conditional demand functions (8), cf e.g. Deaton and Muellbauer (1980). These implications of weak separability can be used to test the assumption empirically and/or introspectively. For example, it may be reasonable to assume that the conditional demand system for automobiles, gasoline and tram transportation is independent of prices (or rations) of fish and meat, but it seems unreasonable to assume that such a conditional demand system is independent of prices (or rations) of bus transportation.

The assumption of weak separability (7) can be extended by assuming that one or more of the subutility functions, $\upsilon_r(.,a)$, $r \in R$, are also weakly separable in some sub groupings, and so on in as many levels one may prefer. Such preference structures are often called utility trees, see e.g. Deaton and Muellbauer (1980). The utility tree that is the starting point for the consumer model in MSG-EE is presented in figure 1. The model is specified by specification of each subutility functions and the top level utility function *f*, including the way household size and composition affects the preferences. The full specification of the consumer model is given in section 3 and Appendix A, together with parameter values. But the main ingredients are presented below, with CES (Constant Elasticitity of Substitution) and LES (Linear expenditure System) as sub functions.

Figure 1
The utility tree in the complete demand system

Commodity codes in parantheses



2.3 Homotheticity and CES subutility functions

Let us now consider the assumption that the subutility functions in (7) are homogeneous of degree one i.e.

$$u_r = \upsilon_r(q_r, a), \quad \text{where } \upsilon_r(sq_r, a) = s\upsilon_r(q_r, a), \quad r \in R, \tag{9}$$

where s is some scalar. Note that (9) is equivalent to assuming that the subutility functions are homothetic, since we may always make monotone transformations of the subutility functions as long as we neutralise by opposite changes in the top level utility function $f(\cdot)$ in (7) such that the overall preference ordering is constant. However, it is convenient to use subutility functions which are homogenous of degree one to represent the homothetic preferences for each group of commodities. (Note that we below will present an alternative model not assuming (9), and in the empirical model we will use the homotheticity assumption only for two subgroups. But to make the exposition easier, with less symbols, we assume for a while that all subutility functions are homothetic.)

Assumption (9) implies that the subcost function of group r can be written

$$y_r = p_{ur} u_r, \quad r \in R \tag{10}$$

where

$$p_{ur} = b_r(p_r), \quad sp_{ur} = b_r(sp_r), \quad r \in R \tag{11}$$

where s is a scalar and $b_r(\cdot)$ is thus a function homogeneous of degree 1. This means that group expenditure y_r can be written as a product of the consumption aggregate u_r and the price index p_{ur} both homogenous of degree 1, given by (9) and (11) respectively. We may also say that p_{ur} is the unit cost of utility from consumption of group r. From (7) and (9) it follows that utility can be written as a function of the consumption aggregates u_r,

$$u = f(.., u_r, .., a). \tag{12}$$

From (2) and (10) we derive a linear budget for the aggregates

$$y = \sum_{r \in R} p_{ur} u_r . \tag{13}$$

By standard assumptions we may then derive Marshallian demand functions for group consumption as function of group prices, $u_r = g_{ur}(y, p_u, a)$, $r \in R$ and corresponding group expenditure functions

$$y_r = g_{yr}(y, p_u, a), \quad r \in R \tag{14}$$

where $p_u = (.., p_{ur}, ..)$ is the vector of group prices.

Note that the above results imply that we can program the demand functions (3) for single commodities by the following two stage procedure (i) calculate the group prices from (11) and the group expenditure from (14), (ii) calculate the demand for single commodities from (8). This procedure can obviously be generalised to utility trees with many levels provided that all subutility functions are homothetic, while the top level utility function $f(\cdot)$ may take any form conforming to basic requirements on utility functions.

The CES function is a popular form of a homothetic function and which we shall exploit in our empirical model. Assume that the price function (11) can be written

$$p_{ur} = \left(\sum_{j \in J_r} \omega_{jr} p_j^{1-\sigma_r} \right)^{1/(1-\sigma_r)}, \quad r \in R \tag{15a}$$

$$\sum_{j \in J_r} \omega_{jr} = 1 \quad r \in R, \tag{15b}$$

where (15b) is a convenient normalisation. Using Shephard's lemma we can from (15) easily derive the conditional demand functions

$$q_j = \omega_{jr} \left(\frac{p_{ur}}{p_j} \right)^{\sigma_r} \left(\frac{y_r}{p_{ur}} \right), \quad j \in J_r \; r \in R, \tag{16}$$

which many readers will recognise as CES demand functions. Note that σ_r is the elasticity of substitution, which according to (16) also can be interpreted as the price elasticity of demand w.r.t. the relative price p_{ur}/p_j given group utility $u_r=(y_r/p_{ur})$.

From (16) we can see that all the Engel elasticities are equal to one in the conditional demand functions, which imply that the Engel elasticities in the total demand system are equal within each subgroup. This implication of homothetic separability is contradicted by much empirical evidence, at least for many of the relevant subgroups. Thus there is a trade off between the simplicity and convenience of homothetic separability and empirical relevance. We make use of the homotheticity assumption in some part of the utility tree, but we find it unacceptable for other parts, in particular for public transport. For example, both empirical evidence and introspection tell us that the Engel elasticity for bus transportation is much less than the Engel elasticity for air transportation. Next we present a parametric model of a utility tree which can capture such empirical characteristics.

2.4 A two-level linear expenditure system with household size and composition

Let us now assume that the utility function for a household h is given by (17a) and the following specifications and normalisations in (17b-f),

$$u_h = B \prod_{r \in R} (u_{rh} - \gamma_{rh})^{\beta_r}, \quad h \in H, \tag{17a}$$

$$u_{rh} = B_r \prod_{j \in J_r} (q_{jh} - \gamma_{jh})^{\beta_j}, \quad r \in R, \qquad h \in H, \tag{17b}$$

$$\sum_{r\in R}\beta_r = 1, \quad \sum_{j\in J_r}\beta_j = 1, \quad r\in R, \tag{17c}$$

$$B = 1/\prod_{r\in R}\beta_r^{\beta_r}, \qquad B_r = 1/\prod_{j\in J_r}\beta_j^{\beta_j}, \qquad r\in R, \tag{17d}$$

$$\gamma_{jh} = \gamma_{j0} + \sum_{i\in I}\gamma_{ji}a_{ih}, \quad j\in J_r, \quad r\in R, \quad h\in H, \tag{17e}$$

$$\gamma_{rh} = \gamma_{r0} + \sum_{i\in I}\gamma_{ri}a_{ih}, \qquad r\in R, \qquad h\in H, \tag{17f}$$

where a_{ih} is the number of individuals of type i in household h, I is a set of types of individuals (in our application I consists of children and adults only), and H is the set of households in the population (Norway). Note that (17a) is a Stone-Geary utility function at the top level of the utility tree while (17b) are Stone-Geary functions at each branch of the tree at the lower level. The β-parameters are assumed to sum to one for each subutility function (17c), which makes it possible to interpret them as conditional marginal budget shares (cf the demand functions below). The normalisation (17d) turns out to be convenient w.r.t. price indexes and cost functions. Note that the marginal budget shares (β) are assumed to be the same for all households, which facilitates aggregation over households. The γ-parameters will be called minimum consumption as usual, although we do not restrict them to be positive. Smaller γ's mean more possibilities for substitution. The minimum consumption (γ) vary between households, but in a restrictive way being linear functions of the number of children and adults in the household (17e-f). This implies, among other things, convenient aggregation properties (cf theorem 2 below). The constant terms (γ_0) can capture economies of scale in household production.

Note that there are minimum quantities both at the upper level (γ_{rh}) and at the lower level (γ_{jh}). If say group r is transport and J_r={private transportation (cars and gasoline), bus and train, air transport, other transport} then we may well have that the γ_j's are all (or mostly) negative reflecting large substitution possibilities between different types of transportation. At the same time γ_r could be positive (and rather large) reflecting the necessity of some transportation for all households and rather small substitution possibilities between say food and transportation.

This model, and special versions and extensions of it, is analysed in Aasness (1993b). We shall only present results in terms of two theorems below. (This two-level linear expenditure system model (abstracting from the demographic effects) is somewhat similar to but not nested in the *S*-branch utility tree of Brown and Heien (1972). This *S*-branch utility tree has been generalised by several authors, and some may nest this type of two level linear expenditure systems. However, we have not seen any literature focusing on the chosen form of two-level linear expenditure system, and in particular not with the demographic modelling above.)

Theorem 1

Maximisation of the utility function (17) subject to the linear budget constraint (2), assuming an interior solution, implies that the Marshallian demand functions are given by the following recursive equation system (18-20).

Price indexes (of marginal utility from commodity groups):

$$p_{u_r} = \prod_{j \in J_r} p_j^{\beta_j}, \quad r \in R. \tag{18}$$

Minimum expenditures (for fixed household consumption (0) and marginal consumption for each person in different age groups (i)):

$$m_{Jr0} = \sum_{j \in J_r} p_j \gamma_{j0}, \quad m_{Jri} = \sum_{j \in J_r} p_j \gamma_{ji}, \quad r \in R, i \in I, \tag{19a}$$

$$m_{r0} = m_{J_r 0} + p_{ur}\gamma_{r0}, \quad m_{ri} = m_{Jri} + p_{ur}\gamma_{ri}, \quad r \in R, i \in I, \tag{19b}$$

$$m_0 = \sum_{r \in R} m_{r0}, \quad m_i = \sum_{r \in R} m_{ri}. \tag{19c}$$

Marshallian group expenditure functions:

$$y_{rh} = m_{r0} + \sum_{i \in I} m_{ri} a_{ih} + \beta_r \left(y_h - m_0 - \sum_{i \in I} m_i a_{ih} \right), \quad r \in R, h \in H. \tag{20a}$$

Conditional demand functions:

$$q_{jh} = \gamma_{j0} + \sum_{i \in I} \gamma_{ji} a_{ih} + \frac{\beta_j}{p_j} \left(y_{rh} - m_{Jr0} - \sum_{i \in I} m_{Jri} a_{ih} \right), \quad j \in J_r, r \in R, h \in H. \tag{20b}$$

Proof of Theorem 1

The expenditure and demand functions can be derived in several ways. One approach is to transform the consumed quantities and expenditures by subtracting the corresponding minimum consumptions and expenditures, and derive a corresponding maximization problem in the transformed variables, where we can exploit well known results for homogeneous separability and Cobb-Douglas utility. Then we can transform back to the original variables. One should start at the bottom level of the utility tree, and move upwards. See Aasness (1993b, proof of Theorem 4.8.1) for details.

Comments to Theorem 1

1. It is easy to program the demand functions on a computer by following the steps in the recursive equation system. The main steps are: (i) defining prices and minimum expenditures, starting at the bottom of the utility tree and ending at the top; (ii) computing expenditures on commodity groups starting at the top level of the utility tree, (iii) compute demand of commodities at

the bottom level of the utility tree. This procedure can be generalized to utility trees with more than two levels, and is used in our empirical model with three levels, see appendix A .

2. The conditional demand function (20b) corresponds to a Linear Expenditure System (LES) with demographic effects. The unconditional demand functions, defined by the recursive equation system, are also linear in total expenditure, but the price effects can be very different from those implied by a one level LES. Only if the minimum quantities at the top level (γ_{r0}, γ_{ri}) are set to zero, the system is reduced to a one-level LES model with demographics.

2.5 Aggregation over households

Definitions of macro variables

Let Q_j be the quantity consumed of commodity j by all households in the economy, i.e.

$$Q_j = \sum_{h\in H} q_{jh}, \quad j \in J. \tag{21a}$$

Let Y_r be the expenditure on commodity group r of all the household in the economy, i.e.

$$Y_r = \sum_{h\in H} y_{rh}, \quad r \in R. \tag{21b}$$

Let Y be the total expenditure of all the households in the economy, i.e.

$$Y = \sum_{h\in H} y_h. \tag{21c}$$

Let A_i be the total number of individuals of type i in the economy, i.e.

$$A_i = \sum_{h\in H} a_{ih} \qquad i \in I \tag{21d}$$

Theorem 2

The macro demands in a population H of households, where each household is maximising a utility function of the type (17) subject to the linear budget constraint (2), assuming interior solutions for all households, are given by the recursive equation system (18), (19), and (22), where

$$Y_r = m_{r0}N + \sum_{i\in I} m_{ri}A_i + \beta_r\left(Y - m_0 N - \sum_{i\in I} m_i A_i\right), \quad r \in R, \tag{22a}$$

$$Q_j = \gamma_{j0}N + \sum_{i\in I}\gamma_{ji}A_i + \frac{\beta_j}{p_j}\left(Y_r - m_{Jr0}N - \sum_{i\in I} m_{Jri}A_i\right), \quad j \in J, r \in R, \tag{22b}$$

N is the number of households in the population, and the other variables are defined in (21).

Proof of Theorem 2

The macro allocation functions (22a) are derived by inserting the household allocation functions (20a) in the definition of macro group expenditure (21b), and doing some simple transformations exploiting (21c) and (21d). The macro conditional demand functions (22b) are derived by inserting the household conditional demand functions (20b) in the definitions of the macro demands (21a), and doing some simple transformations exploiting (21b) and (21d).

Comments to Theorem 2

1. Note that the only type of income variable that enters the macro demand functions is total expenditure (Y). How total expenditure is distributed among different households does not affect the macro demands. This is due to our assumption of equal marginal budget shares (β_j) for all households. This is a convenient property when applied in a general equilibrium model (or other types of macro models), since we then do not need to model how the different variables affects the distribution of total expenditure across households.

2. Note further that the only demographic variables that enters the macro demand functions are the number of households (N) and the number of persons in the different age groups (A_i). How the different types of persons are distributed among different types of households do not affect the macro demands. This is due to our assumption of constant marginal minimum consumption of each type of person (17e). This is a convenient property, since good historical data and good future predictions of the number of households of different types are seldom available. It is easier to obtain data and predictions on the total number of households (N). Our model predicts the effects of this variable, which is connected to economies of scale in household production. If the constant term terms in equation (17e) are set equal to zero, with no economies of scale, the number of households disappear from the macro demands.

3. The macro demand functions (22) are analogous to the household demand functions (20), but the corresponding "preferences of the macro household" will change as the number of households and the number of persons in each age group changes.

3 Empirical model

Figure 1 describes the utility tree behind the demand model. Table 1-5 present the values of the parameters in the household utility function. Table 6-8 and figure 2-3 give different types of demand elasticities for the average household, which are equal to the macro elasticities, in the base year (1991). Table 6 also include budget shares and direct Cournot elasticities for two specific households, a "poor" couple with three children and a "rich" couple without children. Table 9 and 10 give examples of how sensitive the Cournot elasticities are to changes in some basic substitution parameters used in the calibration procedure. Appendix A gives the demand functions in terms of a recursive equation system. Appendix C presents the details of the calibration procedure. Below we give some comments that may help the reader to digest the results.

The calibration of the model is grounded on some basic principles described in Aasness (1993b) and exploits several econometric studies. These include (i) estimates of Engel functions with demographic variables, with the same approach and panel data as in Aasness, Biørn and Skjerpen (1993), but with 28 commodity groups, (ii) estimation of energy demand by Bye (1990-92), (iii) estimation of transport demand by Magnussen and Stoltenberg (1991), (iv) estimation of Engel functions for 135 different commodity groups from two different time series in Aasness and Li (1991). Given the theoretical model and the calibration principles, we have exploited the empirical evidence above according to our best judgement. Needless to say there are many uncertainties involved, and the model will be tested and improved upon within a research program in consumer econometrics, see Aasness (1993c). In this paper we focus on description and interpretation of the calibrated model, not on the uncertanties, although we include a simple sensitivity analysis at the end of this section.

Table 1 presents the parameter values for the upper level linear expenditure system. The relative large total minimum expenditures implies that low income households has small possibilities for substitution among these broad commodity groups, while the ability to substitute will be larger for richer households. The necessity expenditures are much smaller for the lower branches in the utility tree (they are partly negative in table 2-3 and implicitly zero in tables 4-5), implying larger possibilities for substitution among detailed goods. These aspects of the model are reflected in e.g. the direct Cournot elasticicites for the average, the poor and the rich household in table 6.

The fixed minimum consumption (γ_{j0}), which is independent of household size and composition, is relatively large for Energy, Rent and Various household services (including insurance on household property), reflecting economies of scale in housing and heating (cf table 1). This explains the large household elasticities for these goods in table 6 and figure 3. Thus the time trend towards smaller households in Norway, as in allmost all OECD countries, implies a tendency towards more consumption of these goods.

Table 1
Parameter values in the top level LES

Commodity group		Minimum consumption[a)] Fixed	Extra child	Extra adult	Marginal budget share
Code	Name	γ_0	γ_1	γ_2	β
00	Food	6503	8776	10026	0,062
11	Beverages and tobacco	3557	1389	1292	0,070
U	Energy b)	7058	1082	1537	0,018
T	Transport c)	-7841	2283	10613	0,168
15	Other goods	-790	1386	2149	0,035
21	Clothing and footware	-1386	2836	3926	0,063
22	Other household goods	923	585	233	0,015
23	Other goods for recreation activites	1112	956	1427	0,049
41	Furniture etc.	1484	545	582	0,059
42	Durabel consumer goods	256	391	396	0,021
50	Rents	8199	3689	-1171	0,171
63	Entertainment, education etc.	-424	399	1930	0,017
64	Various household services	1360	578	-142	0,010
65	Other services	-1830	1219	2551	0,101
66	Tourism abroad	-2143	56	1102	0,140
	Sum	16039	26170	36452	1,000

a) Measured in 1991 kroner.
b) A CES aggregate, see table 4.
c) Based on the intermediate level and bottom level LES in table 2 and 3 and the bottom level CES in table 4. Note that minimum consumption at the intermediate level comes in addition to those tabulated here, cf equation (19b).

Table 2
Parameters in the intermediate level LES for Transport

Commodity group		Minimum consumption[a)] Fixed	Extra child	Extra adult	Marg. budget share
Code	Name	γ_0	γ_1	γ_2	β
PT	Private transport	-4100	1388	349	0,7754
61	Public transport	3498	-1070	-69	0,2246
	Sum	-602	318	280	1,000

a) Measured in 1991 kroner.

The fixed minimum consumption (γ_{j0}) is negative for Private transport and of large value compared to the effects on the minimum consumption of an extra child and an extra adult (cf table 2). This implies that small households which are poor will not buy a private car, when taking proper account of corner solutions. Thus a negative γ_{j0} in this case can also reflect economies of scale, since it is not compulsory for a household to have a car, in contrast to housing and heating. (The discrete choice of having a car, with fixed costs independent of use, and aggregation across all housholds, is not modelled explicitly and properly in our demand model. The utility function is, however, fully consistent with such an approach and our demand model reflects several of these aspects.)

From table 2 we see that the relative preference for private transportation versus public transportation increases when the household gets larger, reflecting economies of scale in private transportation. This explains that the household elasticities in table 6 are positive for the different forms of public transportation and negative for petrol and cars.

Table 3
Parameters in the bottom level LES for public transport

Commodity group		Minimum consumption a) Fixed	Extra child	Extra adult	Marginal budget share
		γ_0	γ_1	γ_2	β
75	Bus transport, transport by taxi etc.	0	443	886	0,047
76	Air transport	0	-189	-378	0,245
77	Railway, tramway and subway transport	0	179	357	0,019
78	Transport by boat and ferry	0	58	116	0,052
79	Postage, telephone and telegram	0	-376	-752	0,638
	Sum	0	114	229	1,000

a) Measured in 1991 kroner.

Table 4
Parameters in the bottom level CES for Energy

Commodity group		Distribution parameter
Code	Name	ω
12	Electricity	0,865
13	Fuels	0,135
	Sum	1,000
	Elasticity of substitution	0,5

Table 5
Parameters in the bottom level CES for Private Transport

Commodity group		Distribution parameter
Code	Name	ω
14	Petrol and car maintenance	0,456
31	User cost of cars	0,544
	Sum	1,000
	Elasticity of substitution	0,1

Table 3 expresses the parameters in the bottom level linear expenditure system for public transport. In the calibration of these parameters, we assumed that the minimum consumption of each of these services is proportional to the number of children and adults in the household, thus γ_{j0} is zero by assumption (in lack of relevant microeconometric results on these commodity groups). As in the intermediate linear expenditure system, the small total minimum consumption reflects large substitution possibilities.

Table 4 and 5 list the values of the parameters in the CES-aggregate for stationary energy use in the households and the parameters in the CES-aggregate for private transport respectively.

Table 6 shows the budget shares and some important elasticities in the base year[1]. The household elasticities are commented upon above. The Engel elasticities have many properties as found in other empirical research, e.g. low Engel elasticities for Food and for Energy (stationary) and e.g. high Engel elasiticities for private transport, for Air transport, and for Tourism abroad. For luxuries, i.e. goods with Engel elasticities larger than one, the child and/or the adult elasticities are negative, and for necessities the child and/or the adult elasticities are positive. This empirical fact reflects that the households level of living decreases when the number of household members increases while total expenditure is kept constant (in accordance with the definition of person elasticities).

In table 6 we have also included the direct Cournot-elasticities and the budget shares for two specific types of households, a relative low income couple with three children and a relative high income couple without children. For Food the direct Cournot elasticitiy is rather small, in absolute terms, for the rich household, but even much smaller for the poor household. This may be interpreted as follows. The poor household has a tight food budget and cuts down almost only on non-food items when food prices increase. The rich household has enjoyed quite much luxurious food items and cuts down on some of these when food prices increase. For Air transport the direct Cournot elasticity is high, in absolute terms, for the rich houshold and even much higher for the poor household. This may be interpreted as follows. The poor household has a very small consumption of Air transport to begin with, say one ticket a year for one of the children for travelling to its grand parents, and when the air plane ticket increases they immediately consider to substitute to say railway transport. The rich

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[1] The household elasticity is defined as the elasticity of the macro consumption with respect to the number of households for a given number of children and adults and total macro expenditure on consumer goods. The definition of the child and adult elasticities with respect to the consumption of good j is (cf Aasness (1993b,sec2.4)):

$$P_{ji} = \frac{\partial q_j}{\partial a_i} \frac{a_1 + a_2}{q_j}.$$

The calibration procedure secures that the Engel elasticity and the person elasticities in a normal year are the same as used in the input in the calibration procedure, cf table C.4. There is an exception for the commodities in the CES-aggregate for private transport. Because CES is a utility function with homogeneous preferences, the functional form imposed here imply that commodity 31 User costs of cars and 14 Petrol and Car Maintenance have the same Engel- and demographic elasticities in the model.

Table 6

Elasticities in the complete demand system[a]

Commodity group		Budget share	Engel elasticity	House-hold elasticity	Child elasticity	Adult elasticity	Direct Slutsky elasticity	Direct Cournot elasticity Average	Direct Cournot elasticity Poor[c]	Direct Cournot elasticity Rich[d]	Budget share Poor[c]	Budget share Rich[d]
12	Electricity	0,054	0,279	0,558	0,122	0,177	-0,186	-0,201	-0,146	-0,380	0,054	0,034
13	Fuels	0,008	0,279	0,558	0,122	0,177	-0,451	-0,453	-0,445	-0,481	0,008	0,005
14	Petrol and Car Maintenance	0,046	1,290	-0,621	-0,037	0,463	-0,360	-0,420	-0,310	-0,475	0,049	0,054
31	User costs of cars	0,055	1,290	-0,621	-0,037	0,463	-0,410	-0,481	-0,350	-0,547	0,058	0,065
75	Bus transport, transport by taxi etc.	0,012	0,154	0,024	0,387	0,979	-0,178	-0,180	-0,088	-0,319	0,015	0,007
76	Air transport	0,007	1,352	0,212	-1,084	-0,376	-1,460	-1,469	-7,524	-1,154	0,001	0,009
77	Railway, tramway and subway transport	0,005	0,150	0,024	0,392	0,983	-0,176	-0,177	-0,085	-0,314	0,006	0,003
78	Transport by boat and ferry	0,004	0,550	0,086	-0,099	0,531	-0,637	-0,639	-0,442	-0,790	0,003	0,003
79	Postage, telephone and telegram	0,020	1,185	0,186	-0,878	-0,187	-1,091	-1,115	-2,851	-1,011	0,006	0,024
00	Food	0,185	0,336	0,155	0,478	0,520	-0,157	-0,219	-0,125	-0,457	0,247	0,115
11	Beverages and tobacco	0,070	1,007	0,183	-0,083	-0,228	-0,467	-0,537	-0,345	-0,796	0,064	0,070
15	Other goods	0,037	0,927	-0,183	0,156	0,292	-0,446	-0,481	-0,245	-0,763	0,043	0,036
21	Clothing and footware	0,069	0,905	-0,175	0,211	0,291	-0,423	-0,486	-0,255	-0,766	0,082	0,065
22	Other household goods	0,016	0,922	0,218	0,142	-0,242	-0,453	-0,468	-0,239	-0,774	0,018	0,015
23	Other goods for recreation activites	0,046	1,061	0,039	-0,093	-0,102	-0,503	-0,552	-0,342	-0,805	0,043	0,048
41	Furniture etc.	0,044	1,336	0,067	-0,286	-0,445	-0,627	-0,686	-0,491	-0,881	0,034	0,053
42	Durabel consumer goods	0,017	1,261	-0,021	-0,125	-0,281	-0,615	-0,637	-0,383	-0,865	0,015	0,019
50	Rents	0,129	1,327	0,222	-0,082	-0,717	-0,548	-0,720	-0,490	-0,918	0,120	0,148
63	Entertainment, education etc.	0,025	0,684	-0,141	-0,030	0,632	-0,335	-0,353	-0,202	-0,618	0,025	0,022
64	Various household services	0,012	0,782	0,500	0,318	-0,498	-0,386	-0,396	-0,184	-0,741	0,015	0,010
65	Other services	0,068	1,482	-0,256	-0,265	-0,211	-0,664	-0,765	-0,527	-0,915	0,057	0,087
66	Tourism abroad	0,069	2,030	-0,321	-0,656	-0,727	-0,870	-1,010	-0,978	-0,999	0,039	0,109
	Sum (weighted)[b]	1,000	1,000	0,000	0,000	0,000	-	-	-	-	1,000	1,000

a) If no other spesidication, elasticities for the average household and macro demands
b) The elasticities are weighted with the budget shares
c) Household with two adults, three children and a total expenditure of kr. 230.000.
d) Household with two adults, no children and a total expenditure of kr. 400.000.

Figure 2
Budget shares, Engel and Cournot elasticities

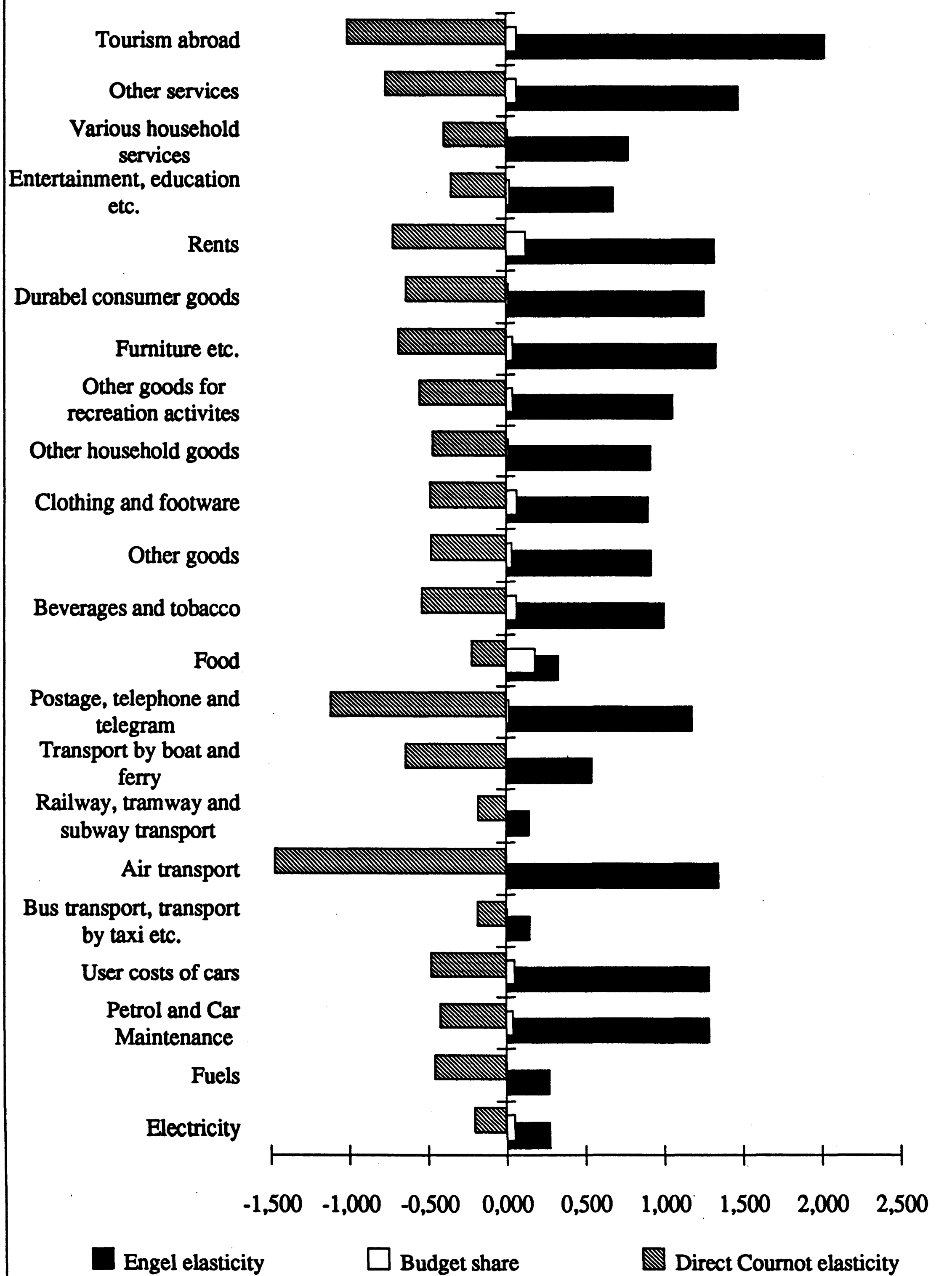
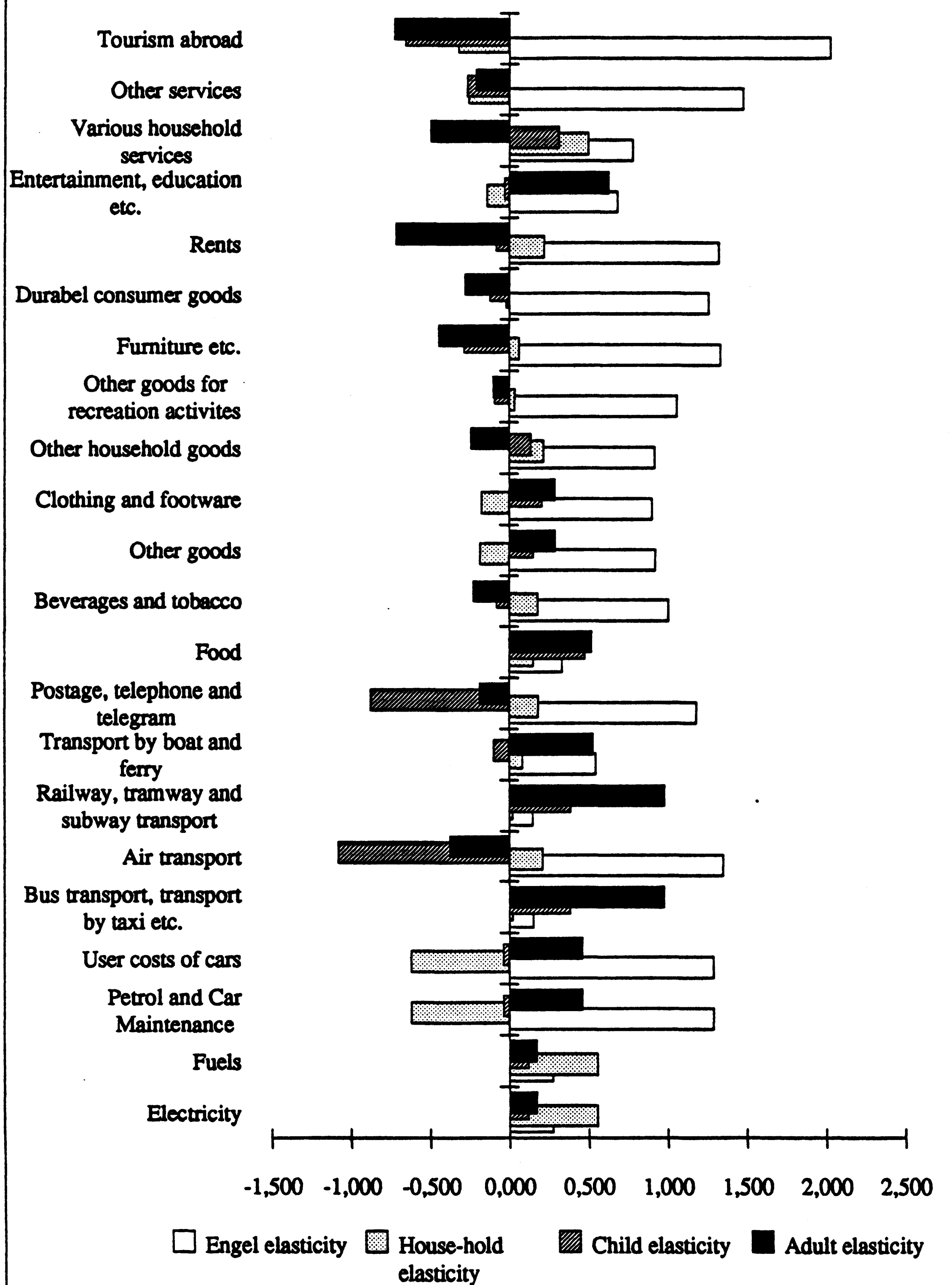


Figure 3
Engel, child, adult and household elasticities



household is also sensitive to price increases in air plane tickets, but still use Air transport in the cases when this is considerably more convenient.

In additive utility functions the direct Slutsky elasticity is approximately proportional to the Engel elasticity, cf e.g. Frisch (1959). There are also such tendencies in our utility tree where the branches consist of additive utility functions, but the rules are more complex. For example, within the branch of public transport the direct Slutsky elasticity is proportional to the Engel elasticity. But the ratio of the Slutsky and the Engel elasticitiy is much larger (in absolute value) within this group than for commodities that enter directly in the upper level of the utility tree, cf figure 1 and table 6. Note that Air transport has the highest Slutsky elasticity (in absolute value), which is due both to a high Engel elasticity and to good substitution possibilities with other types of transportation. (The direct price elasticities for petrol and cars are small compared to their Engel elasticity since they are complements, see below.)

Table 7 expresses a complete Slutsky-matrix for the demand model. In our comments we will focus on the substitution between transport services. An advantage of the model is that the functional form allows for relative little substitution between petrol and cars, while at the same time it is possible to have a lot of substitution between private transport and public transport. We see that the cross price elasticities between petrol and cars are negative, thus these goods are complements in Slutsky terms.[2]. The income effect works in the same direction as the substitution effect in this case. From table 8 we can thus observe that the cross Cournot elasticity between these goods are larger, in absolute terms, than the corresponding Slutsky elasticity. Because the cross effect is large, the direct Slutsky and Cournot elasticities within the CES-aggregate are smaller than in a corresponding linear expenditure system. If this is a good description of the reality, taxes on petrol alone are not very effective in reducing pollution from cars, but combined tax increases on both petrol and cars will be quite effective.

In our model the five types of public transportation services, including Postage, telephone and telegram, are substitutes within their own branch of the utility tree. From table 7 we see that an increase in the price of one of these goods, for a constant utility level, has little impact on the non-transportation goods, but somewhat larger impact on the other public transportation services. Furthermore, the substitution effect dominates the income effect and consequently all the cross Cournot elasticities are positive within this group. Air transport and Postage etc. are the two most income elastic goods within the group, cf table 6, and therefore price changes within this group affect these two goods the most. It is interesting that there are considerable possibilities for substitution between the environmentally clean Postage, telephone and telegram and the much polluting Air transport.

Private transportation and Public transportation are substitutes within an intermediate branch of our utility tree, and we notice that all the cross Slutsky elasticities between goods are positive with a

[2] This cross Slutsky elasticity would for example in a standard one-stage linear expenditure system be bound to be positive due to the functional form.

Table 7

Slutsky-elasticities in the complete demand system[a)]

Commodity group		sj.12	sj.13	sj.14	sj.31	sj.75	sj.76	sj.77	sj.78	sj.79	sj.00	sj.11	sj.15	sj.21	sj.22	sj.23	sj.41	sj.42	sj.50	sj.63	sj.64	sj.65	sj.66	Sum b)
12	Electricity	-0,186	0,049	0,008	0,010	0,000	0,001	0,000	0,000	0,003	0,009	0,010	0,005	0,009	0,002	0,007	0,008	0,003	0,024	0,002	0,001	0,014	0,020	0,000
13	Fuels	0,314	-0,451	0,008	0,010	0,000	0,001	0,000	0,000	0,003	0,009	0,010	0,005	0,009	0,002	0,007	0,008	0,003	0,024	0,002	0,001	0,014	0,020	0,000
14	Petrol etc.	0,010	0,002	-0,360	-0,309	0,006	0,033	0,002	0,007	0,086	0,040	0,045	0,022	0,040	0,010	0,032	0,038	0,014	0,110	0,011	0,006	0,065	0,090	0,000
31	User costs of cars	0,010	0,002	-0,260	-0,410	0,006	0,033	0,002	0,007	0,086	0,040	0,045	0,022	0,040	0,010	0,032	0,038	0,014	0,110	0,011	0,006	0,065	0,090	0,000
75	Bus transport etc.	0,001	0,000	0,025	0,030	-0,178	0,015	0,001	0,003	0,040	0,005	0,005	0,003	0,005	0,001	0,004	0,005	0,002	0,013	0,001	0,001	0,008	0,011	0,000
76	Air transport	0,010	0,002	0,221	0,264	0,026	-1,460	0,010	0,028	0,350	0,042	0,047	0,023	0,042	0,010	0,033	0,040	0,014	0,115	0,012	0,007	0,068	0,095	0,000
77	Railway, tramway etc.	0,001	0,000	0,025	0,029	0,003	0,015	-0,176	0,003	0,039	0,005	0,005	0,003	0,005	0,001	0,004	0,004	0,002	0,013	0,001	0,001	0,008	0,011	0,000
78	Transport by boat etc.	0,004	0,001	0,090	0,107	0,011	0,055	0,004	-0,637	0,142	0,017	0,019	0,009	0,017	0,004	0,014	0,016	0,006	0,047	0,005	0,003	0,028	0,038	0,000
79	Postage etc.	0,009	0,001	0,194	0,231	0,023	0,118	0,009	0,025	-1,091	0,037	0,041	0,020	0,037	0,009	0,029	0,035	0,012	0,101	0,010	0,006	0,060	0,083	0,000
00	Food	0,003	0,000	0,010	0,012	0,000	0,002	0,000	0,000	0,004	-0,157	0,012	0,006	0,010	0,003	0,008	0,010	0,004	0,029	0,003	0,002	0,017	0,024	0,000
11	Beverages and tobacco	0,008	0,001	0,030	0,036	0,001	0,005	0,000	0,001	0,012	0,031	-0,467	0,017	0,031	0,008	0,025	0,030	0,011	0,086	0,009	0,005	0,051	0,070	0,000
15	Other goods	0,007	0,001	0,028	0,033	0,001	0,004	0,000	0,001	0,011	0,029	0,032	-0,446	0,029	0,007	0,023	0,027	0,010	0,079	0,008	0,005	0,047	0,065	0,000
21	Clothing and footware	0,007	0,001	0,027	0,032	0,001	0,004	0,000	0,001	0,011	0,028	0,032	0,016	-0,423	0,007	0,022	0,027	0,010	0,077	0,008	0,004	0,046	0,063	0,000
22	Other household goods	0,007	0,001	0,027	0,033	0,001	0,004	0,000	0,001	0,011	0,029	0,032	0,016	0,029	-0,453	0,023	0,027	0,010	0,079	0,008	0,004	0,046	0,065	0,000
23	Goods for recreation etc.	0,008	0,001	0,032	0,038	0,001	0,005	0,000	0,001	0,013	0,033	0,037	0,018	0,033	0,008	-0,503	0,031	0,011	0,091	0,009	0,005	0,053	0,074	0,000
41	Furniture etc.	0,010	0,002	0,040	0,047	0,001	0,006	0,000	0,001	0,016	0,041	0,047	0,023	0,042	0,010	0,033	-0,627	0,014	0,114	0,012	0,006	0,067	0,094	0,000
42	Durable goods	0,010	0,001	0,037	0,045	0,001	0,006	0,000	0,001	0,015	0,039	0,044	0,022	0,039	0,009	0,031	0,037	-0,615	0,108	0,011	0,006	0,064	0,088	0,000
50	Rents	0,010	0,002	0,039	0,047	0,001	0,006	0,000	0,001	0,016	0,041	0,046	0,023	0,041	0,010	0,033	0,039	0,014	-0,548	0,011	0,006	0,067	0,093	0,000
63	Entertainment etc.	0,005	0,001	0,020	0,024	0,001	0,003	0,000	0,001	0,008	0,021	0,024	0,012	0,021	0,005	0,017	0,020	0,007	0,058	-0,335	0,003	0,034	0,048	0,000
64	Various h.h. services	0,006	0,001	0,023	0,028	0,001	0,004	0,000	0,001	0,009	0,024	0,027	0,013	0,024	0,006	0,019	0,023	0,008	0,067	0,007	-0,386	0,039	0,055	0,000
65	Other services	0,011	0,002	0,044	0,052	0,001	0,007	0,001	0,001	0,018	0,046	0,052	0,026	0,046	0,011	0,036	0,044	0,016	0,127	0,013	0,007	-0,664	0,104	0,000
66	Tourism abroad	0,015	0,002	0,060	0,072	0,002	0,009	0,001	0,002	0,024	0,063	0,071	0,035	0,063	0,015	0,050	0,060	0,021	0,173	0,018	0,010	0,102	-0,870	0,000
	Sum (weighted) c)	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	

a) Elasticities for the average household and macro demands in the base year (1991).

b) We apply that $\Sigma_j s_{ij}=0$, i.e. homogeniety of demands, for control.

c) We apply that $\Sigma_j w_{ij} s_{ij}=0$, i.e. an adding up contition, for control.

Table 8

Cournot-elasticities in the complete demand system

	Commodity group	ej.12	ej.13	ej.14	ej.31	ej.75	ej.76	ej.77	ej.78	ej.79	ej.00	ej.11	ej.15	ej.21	ej.22	ej.23	ej.41	ej.42	ej.50	ej.63	ej.64	ej.65	ej.66	Sum b)
12	Electricity	-0,201	0,047	-0,005	-0,005	-0,003	-0,001	-0,001	-0,001	-0,002	-0,043	-0,010	-0,006	-0,011	-0,002	-0,006	-0,004	-0,002	-0,012	-0,005	-0,002	-0,005	0,000	0,000
13	Fuels	0,299	-0,453	-0,005	-0,005	-0,003	-0,001	-0,001	-0,001	-0,002	-0,043	-0,010	-0,006	-0,011	-0,002	-0,006	-0,004	-0,002	-0,012	-0,005	-0,002	-0,005	0,000	0,000
14	Petrol etc.	-0,060	-0,009	-0,420	-0,380	-0,009	0,024	-0,004	0,002	0,059	-0,198	-0,045	-0,026	-0,049	-0,011	-0,028	-0,019	-0,008	-0,056	-0,022	-0,010	-0,023	0,001	0,000
31	User costs of cars	-0,060	-0,009	-0,319	-0,481	-0,009	0,024	-0,004	0,002	0,059	-0,198	-0,045	-0,026	-0,049	-0,011	-0,028	-0,019	-0,008	-0,056	-0,022	-0,010	-0,023	0,001	0,000
75	Bus transport etc.	-0,007	-0,001	0,018	0,022	-0,180	0,014	0,000	0,003	0,037	-0,024	-0,005	-0,003	-0,006	-0,001	-0,003	-0,002	-0,001	-0,007	-0,003	-0,001	-0,003	0,000	0,000
76	Air transport	-0,063	-0,010	0,159	0,189	0,010	-1,469	0,004	0,024	0,322	-0,208	-0,047	-0,027	-0,051	-0,012	-0,030	-0,020	-0,008	-0,059	-0,023	-0,010	-0,024	0,001	0,000
77	Railway, tramway etc.	-0,007	-0,001	0,018	0,021	0,001	0,014	-0,177	0,003	0,036	-0,023	-0,005	-0,003	-0,006	-0,001	-0,003	-0,002	-0,001	-0,007	-0,003	-0,001	-0,003	0,000	0,000
78	Transport by boat etc.	-0,026	-0,004	0,065	0,077	0,004	0,051	0,002	-0,639	0,131	-0,085	-0,019	-0,011	-0,021	-0,005	-0,012	-0,008	-0,003	-0,024	-0,009	-0,004	-0,010	0,000	0,000
79	Postage etc.	-0,055	-0,009	0,139	0,166	0,009	0,110	0,003	0,021	-1,115	-0,182	-0,041	-0,024	-0,045	-0,010	-0,026	-0,017	-0,007	-0,052	-0,020	-0,009	-0,021	0,001	0,000
00	Food	-0,016	-0,002	-0,006	-0,007	-0,004	-0,001	-0,001	-0,001	-0,003	-0,219	-0,012	-0,007	-0,013	-0,003	-0,007	-0,005	-0,002	-0,015	-0,006	-0,003	-0,006	0,000	0,000
11	Beverages and tobacco	-0,047	-0,007	-0,017	-0,020	-0,011	-0,002	-0,004	-0,003	-0,008	-0,155	-0,537	-0,020	-0,038	-0,009	-0,022	-0,015	-0,006	-0,044	-0,017	-0,008	-0,018	0,001	0,000
15	Other goods	-0,043	-0,007	-0,015	-0,018	-0,010	-0,002	-0,004	-0,002	-0,008	-0,142	-0,032	-0,481	-0,035	-0,008	-0,020	-0,014	-0,006	-0,041	-0,015	-0,007	-0,017	0,001	0,000
21	Clothing and footware	-0,042	-0,007	-0,015	-0,018	-0,010	-0,002	-0,004	-0,002	-0,008	-0,139	-0,031	-0,018	-0,486	-0,008	-0,020	-0,013	-0,006	-0,040	-0,015	-0,007	-0,016	0,001	0,000
22	Other household goods	-0,043	-0,007	-0,015	-0,018	-0,010	-0,002	-0,004	-0,002	-0,008	-0,142	-0,032	-0,019	-0,035	-0,468	-0,020	-0,014	-0,006	-0,040	-0,015	-0,007	-0,016	0,001	0,000
23	Goods for recreation etc.	-0,050	-0,008	-0,017	-0,021	-0,011	-0,002	-0,005	-0,003	-0,009	-0,163	-0,037	-0,021	-0,040	-0,009	-0,552	-0,016	-0,007	-0,046	-0,018	-0,008	-0,019	0,001	0,000
41	Furniture etc.	-0,062	-0,010	-0,022	-0,026	-0,014	-0,003	-0,006	-0,003	-0,011	-0,206	-0,046	-0,027	-0,051	-0,012	-0,029	-0,686	-0,008	-0,058	-0,022	-0,010	-0,024	0,001	0,000
42	Durable goods	-0,059	-0,009	-0,021	-0,025	-0,014	-0,003	-0,005	-0,003	-0,011	-0,194	-0,044	-0,025	-0,048	-0,011	-0,028	-0,019	-0,637	-0,055	-0,021	-0,010	-0,022	0,001	0,000
50	Rents	-0,062	-0,010	-0,022	-0,026	-0,014	-0,003	-0,006	-0,003	-0,011	-0,204	-0,046	-0,027	-0,050	-0,012	-0,029	-0,020	-0,008	-0,720	-0,022	-0,010	-0,024	0,001	0,000
63	Entertainment etc.	-0,032	-0,005	-0,011	-0,013	-0,007	-0,002	-0,003	-0,002	-0,006	-0,105	-0,024	-0,014	-0,026	-0,006	-0,015	-0,010	-0,004	-0,030	-0,353	-0,005	-0,012	0,001	0,000
64	Various h.h. services	-0,036	-0,006	-0,013	-0,015	-0,008	-0,002	-0,003	-0,002	-0,007	-0,120	-0,027	-0,016	-0,030	-0,007	-0,017	-0,012	-0,005	-0,034	-0,013	-0,396	-0,014	0,001	0,000
65	Other services	-0,069	-0,011	-0,024	-0,029	-0,016	-0,003	-0,006	-0,004	-0,012	-0,228	-0,051	-0,030	-0,056	-0,013	-0,032	-0,022	-0,009	-0,065	-0,025	-0,011	-0,765	0,001	0,000
66	Tourism abroad	-0,095	-0,015	-0,033	-0,040	-0,022	-0,005	-0,009	-0,005	-0,017	-0,312	-0,070	-0,041	-0,077	-0,018	-0,044	-0,030	-0,013	-0,089	-0,034	-0,015	-0,036	-1,010	0,000
	Sum (weighted) c)	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	

a) Elasticities for the average household and macro demands in the base year (1991).

b) We apply that $\Sigma_j e_{ij} + E_i = 0$, i.e. homegeniety of demands, for control.

c) We apply that $\Sigma_j w_j e_{ij} + E_i + w_j = 0$, i.e. an adding up condition, for control.

non-negligible magnitude, when one good is within Private transportation and the other within Public transportation. As far as an increase in the price of private transportation is concerned, the cross-Cournot-elasticities of all the public transportation services are also positive, i.e. the substitution effect dominates the income effect, cf table 8. When the prices of the different public transportation services increases, however, the total effects on private transportation are ambiguous. Due to the relatively small Slutsky elasticities, the income effect dominates when the prices of the two services 76 Bus transport etc. and 77 Railway etc., increases. On the other hand, we have positive Cournot elasticities on private transportation with respect to price increases of the three other public transportation services, because the substitution effect dominates in these cases.

Notice also that when the Food price increases, consumption of all commodities decrease due to the income effect, and that goods such as Petrol and car maintenance and Air transport decrease even more than Food itself due to their high Engel elasticities.

The calibration technique used makes it easy to perform sensitivity analyses of different types of results with respect to the values of the different input parameters in the calibration procedure. Tables 9 and 10 give examples of how sensitive the Cournot elasticities are w.r.t to the values of the substitution parameters in the three subutility functions for transport. (See appendix C for definition of these parameters and how they enter the calibration procedure.) We see that the direct Cournot elasticities outside the transport sector are almost not affected. The Cournot elasticities within the transport group are, as one would expect, more sensitive, but still quite robust. Note that the difference betweeen the Cournot elasticities for say Bus transport and Air transport, being determined by their differences in Engel elasticities, is very robust and much larger than the differences in direct Cournot elasticities due to changes in the substitution parameters.

Table 9

Sensitivity of direct Cournot elasticities w.r.t substitution parameters for transport[a]

Commodity group		s_{Tn}			σ_{PT}			s_{61n}		
		0,7	1	1,3	0,05	0,1	0,3	0,7	1	1,3
12	Electricity	-0,174	-0,174	-0,174	-0,174	-0,174	-0,174	-0,174	-0,174	-0,174
13	Fuels	-0,449	-0,449	-0,449	-0,449	-0,449	-0,449	-0,449	-0,449	-0,449
14	Petrol and Car Maintenance	-0,359	-0,404	-0,451	-0,378	-0,404	-0,511	-0,404	-0,404	-0,404
31	User costs of cars	-0,408	-0,462	-0,518	-0,439	-0,462	-0,556	-0,462	-0,462	-0,462
75	Bus transport, transport by taxi etc.	-0,159	-0,164	-0,169	-0,164	-0,164	-0,164	-0,115	-0,164	-0,214
76	Air transport	-1,519	-1,563	-1,604	-1,563	-1,563	-1,563	-1,064	-1,563	-2,171
77	Railway, tramway and subway transport	-0,157	-0,161	-0,165	-0,161	-0,161	-0,161	-0,112	-0,161	-0,211
78	Transport by boat and ferry	-0,600	-0,612	-0,624	-0,612	-0,612	-0,612	-0,416	-0,612	-0,824
79	Postage, telephone and telegram	-0,975	-1,131	-1,275	-1,131	-1,131	-1,131	-1,011	-1,131	-1,242
00	Food	-0,185	-0,185	-0,185	-0,185	-0,185	-0,185	-0,185	-0,185	-0,185
11	Beverages and tobacco	-0,468	-0,468	-0,468	-0,468	-0,468	-0,468	-0,468	-0,468	-0,468
15	Other goods	-0,413	-0,413	-0,413	-0,413	-0,413	-0,413	-0,413	-0,413	-0,413
21	Clothing and footware	-0,420	-0,420	-0,420	-0,420	-0,420	-0,420	-0,420	-0,420	-0,420
22	Other household goods	-0,396	-0,396	-0,396	-0,396	-0,396	-0,396	-0,396	-0,396	-0,396
23	Other goods for recreation activites	-0,483	-0,483	-0,483	-0,483	-0,483	-0,483	-0,483	-0,483	-0,483
41	Furniture etc.	-0,621	-0,621	-0,621	-0,621	-0,621	-0,621	-0,621	-0,621	-0,621
42	Durabel consumer goods	-0,567	-0,567	-0,567	-0,567	-0,567	-0,567	-0,567	-0,567	-0,567
50	Rents	-0,659	-0,660	-0,660	-0,660	-0,660	-0,660	-0,659	-0,660	-0,660
63	Entertainment, education etc.	-0,293	-0,293	-0,293	-0,293	-0,293	-0,293	-0,293	-0,293	-0,293
64	Various household services	-0,326	-0,326	-0,326	-0,326	-0,326	-0,326	-0,326	-0,326	-0,326
65	Other services	-0,713	-0,713	-0,713	-0,713	-0,713	-0,713	-0,713	-0,713	-0,713
66	Tourism abroad	-1,017	-1,017	-1,017	-1,017	-1,017	-1,017	-1,017	-1,017	-1,017

[a] Elasticities for the average household and macro demands in the base year (1991). When no other specification, the substitution parameters are as in the base year, i.e. s_T=1.0 (T=Transport), σ_{PT}=0.1 (PT = Private transport), s_{61}=1.0 (61=Public transport).

Table 10

Sensitivity of selected cross Cournot elasticities w.r.t. substitution parameters for transport[a]

		e_{j14}			e_{j31}			e_{j76}		
	Substitution parameter in the intermediate level LES for transport (sTn):	0,7	1,0	1,3	0,7	1,0	1,3	0,7	1,0	1,3
14	Petrol and Car Maintenance	-0,359	-0,404	-0,451	-0,308	-0,362	-0,418	0,006	0,028	0,051
31	User costs of cars	-0,258	-0,304	-0,351	-0,408	-0,462	-0,518	0,006	0,028	0,051
75	Bus transport, transport by taxi e	0,004	0,018	0,033	0,004	0,022	0,039	0,021	0,015	0,008
76	Air transport	0,038	0,189	0,326	0,046	0,225	0,388	-1,519	-1,563	-1,604
77	Railway, tramway and subway t	0,003	0,018	0,032	0,004	0,021	0,038	0,021	0,014	0,008
78	Transport by boat and ferry	0,014	0,069	0,122	0,016	0,082	0,145	0,082	0,055	0,029
79	Postage, telephone and telegram	0,033	0,162	0,280	0,039	0,192	0,334	0,198	0,130	0,067
	Substitution parameter in the bottom level LES for public transport (s61n):	0,7	1,0	1,3	0,7	1,0	1,3	0,7	1,0	1,3
14	Petrol and Car Maintenance	-0,404	-0,404	-0,404	-0,362	-0,362	-0,362	0,028	0,028	0,029
31	User costs of cars	-0,304	-0,304	-0,304	-0,462	-0,462	-0,462	0,028	0,028	0,029
75	Bus transport, transport by taxi e	0,018	0,018	0,018	0,022	0,022	0,022	0,002	0,015	0,027
76	Air transport	0,172	0,189	0,210	0,205	0,225	0,250	-1,064	-1,563	-2,171
77	Railway, tramway and subway t	0,018	0,018	0,018	0,021	0,021	0,021	0,002	0,014	0,027
78	Transport by boat and ferry	0,066	0,069	0,071	0,079	0,082	0,085	0,008	0,055	0,106
79	Postage, telephone and telegram	0,169	0,162	0,155	0,201	0,192	0,185	0,021	0,130	0,230
	Substitution elasticity in the bottom level CES for private transport (σPT):	0,05	0,1	0,3	0,05	0,1	0,3	0,05	0,1	0,3
14	Petrol and Car Maintenance	-0,378	-0,404	-0,511	-0,388	-0,362	-0,256	0,028	0,028	0,028
31	User costs of cars	-0,328	-0,304	-0,210	-0,439	-0,462	-0,556	0,028	0,028	0,028
75	Bus transport, transport by taxi e	0,018	0,018	0,018	0,022	0,022	0,022	0,015	0,015	0,015
76	Air transport	0,190	0,189	0,187	0,225	0,225	0,227	-1,563	-1,563	-1,563
77	Railway, tramway and subway t	0,018	0,018	0,018	0,021	0,021	0,021	0,014	0,014	0,014
78	Transport by boat and ferry	0,069	0,069	0,068	0,082	0,082	0,082	0,055	0,055	0,055
79	Postage, telephone and telegram	0,162	0,162	0,160	0,192	0,192	0,194	0,130	0,130	0,130

a) Elasticities for the average household and macro demands in the base year (1991). When no other specification, the substitution parameters are as in the base year, i.e. s_T=1.0 (T=Transport), σ_{PT}=0.1 (PT = Private transport), s_{61}=1.0 (61=Public transport).

4 Conclusion

The paper has presented a new system of consumer demand functions for the Norwegian economy with the following specific features. (i) It is disaggregated with 22 commodity groups, with emphasis on commodity groups particularily relevant for problems related to energy use and environmental issues. (ii) The demand system is integrated into a large scale general equilibrium model of the Norwegian economy which is used for policy analysis and long term projections, in particular designed to analyze energy and environmental issues. (iii) The model captures effects of prices, income, and demographics including the number of households, children and adults in Norway. (iv) The macroeconomic demand system is derived from a microeconomic model with utility maximizing households, with perfect aggregation across all households in Norway. This feature is of great advantage both for positive demand analysis and for normative welfare analyses. (v) The direct utility function is an example of a three level nonhomothetic utility tree, which implies strong testable restrictions on the 22x22 matrix of price elasticities. It can capture important features of households ability to substitute between specific goods. In particular, there is much substitution between different types of public transport and between private and public transport. (vi) The household utility function is given a simple and transparent parameterization by combining the well known CES and LES functional forms as subutility functions in the somewhat complex utility tree. The direct utility function, the demand functions, the indirect utility function, the cost function etc can all be given an explicit functional form and they can be programmed as a transparent recursive equation system. (vii) The model captures economies of scale in household production, making it possible to e.g. analyze the effects on consumer demand and standard of living of the tendency towards smaller households. (viii) The model is calibrated exploiting both microeconometrics and macro-econometrics, taking both random and systematic measurement errors into account. In general this approach makes it possible to exploit all available data sources for testing and estimation of parts of and/or the full model, using simple or advanced econometric techniques.

The model can be considered as a case study which represents a fruitful approach to modelling consumer demand in general and which is particularly relevant for policy analysis of energy and environmental problems. The theoretical model, the empirical work (in terms of testing, estimation, and sensitivity analysis), and the analysis and application of the empirical model can of course be extended and refined in many dimensions. Similar models could be constructed for almost any country exploiting available data on national accounts and household expenditure surveys.

Appendix A: Recursive equation system of the demand model

In this appendix we present the equations of the recursive simulation model. In parentheses we point to the corresponding equations in the theory presented in section 2. Figure 1 gives an overview of the the utility tree behind the demand model, where LES denotes a branch of utility with functional form corresponding to a Linear Expenditure System, and CES denotes a branch of utility with a Constant Elasticity of Substitution. The numerical values of the parameters are given in table 1-5. Elasticities of the complete demand system are given in table 6-8. The commodity grouping is presented in these tables and in more detail in appendix B.

Price indexes

The price index for the marginal utility of Public transport (61) in period t is given by

$$p_{61t} = \prod_{j \in J_{61}} p_{jt}^{\beta_j}, \tag{A.1}$$

where β_j is the conditional marginal budget share of commodity group j (cf equation (18)).

The price-index for the CES-aggregate for Energy (U), in time period t, aggregating prices for electricity (12) and fuels (13), is defined by

$$p_{Ut} = \left\{\omega_U p_{12t}^{(1-\sigma_U)} + (1-\omega_U) p_{13t}^{(1-\sigma_U)}\right\}^{\frac{1}{1-\sigma_U}}, \tag{A.2}$$

where σ_U is the elasticity of substitution between electricity and fuels and ω_U is a distribution parameter (cf equation (15)).

The price-index for the CES-aggregate for Private transport (PT), aggregating prices for petrol and car maintenance (14) and user cost of cars (31) , is given by

$$p_{PTt} = \left\{\omega_{PT} p_{14t}^{(1-\sigma_{PT})} + (1-\omega_{PT}) p_{31t}^{(1-\sigma_{PT})}\right\}^{\frac{1}{1-\sigma_{PT}}}, \tag{A.3}$$

where σ_{PT} is the elasticity of substitution between petrol (14) and cars (31) and ω_{PT} is a distribution parameter (cf equation (15)).

The price index for the marginal utility of Transport (T), in the intermediate LES-system for Private transport (PT) and Public transport (61), is defined by

$$p_{Tt} = \prod_{j=PT,61} p_{jt}^{\beta_j}, \tag{A.4}$$

where β_j is the conditional marginal budget share of commodity group j (cf equation (18)).

The price index for foreigners consumption in Norway is

$$p_{70t} = \sum_j a_j p_{jt}, \tag{A.5}$$

i.e. a Laspeyres price index which corresponds to a Leontief utility function with the following real consumption index $Q_{70t}=Y_{70t}/P_{70t}$, where Y_{70t} is foreigners consumption expenditure in Norway. The values of the parameters are $a_{00}=0.1$, $a_{11}=0.04$, $a_{14}=0.15$, $a_{21}=0.08$, $a_{23}=0.08$, $a_{63}=0.02$, $a_{65}=0.47$, $a_{75}=0.01$, $a_{76}=0.01$, $a_{77}=0.01$, $a_{78}=0.01$, $a_{79}=0.02$, which adds to one.

Minimum expenditures

At the bottomlevel LES for Public transport, fixed minimum expenditure for each household and marginal minimum expenditure for each person of different age groups are given by

$$m_{61it} = \sum_{j \in J_{61}} p_{jt}\gamma_{ji}, \qquad i = 0,1,2. \tag{A.6}$$

where γ_{j0} is the fixed minimum consumption of commodity j for a household, γ_{j1} is the additional minimum consumption of commodity j for each child in the household, and γ_{j2} is the additional minimum consumption of commodity for each adult in the household (cf equation (19a)).

The macro minimum expenditure of the bottom level LES for Public transport is

$$M_{61t} = m_{610t}N_t + m_{611t}A_{1t} + m_{612t}A_{2t} \tag{A.7}$$

where N_r is the number of households, A_1 the number of children and A_2 the number of adults in Norway.

At the intermediate level LES for Transport (T), fixed minimum expenditure for each household (m_{T0t}) and marginal minimum expenditure for each person of different age groups (m_{Tit}) are given by

$$m_{Tit} = \sum_{j=PT,61} p_{jt}\gamma_{ji} + m_{61it}, \qquad i = 0,1,2, \tag{A.8}$$

where γ_{j0} is the fixed minimum consumption of commodity j for a household, γ_{j1} is the additional minimum consumption of commodity j for each child in the household, and γ_{j2} is the additional minimum consumption of commodity for each adult in the household (cf equation (19b-c)). Note that for Public transport (61) there two components of the minimum expenditures, $p_{61t}\gamma_{61i}$ from the intermediate level and and m_{61it} from the bottom level.

The macro minimum expenditure of the intermediate level LES for Transport is

$$M_{Tt} = m_{T0t}N_t + m_{T1t}A_{1t} + m_{T2t}A_{2t}, \tag{A.9}$$

where N_t is the number of households, A_{1t} is the number of children, and A_{2t} is the number of adults in Norway.

At the top level LES, the fixed minimum expenditure for each household (m_{0t}) and the marginal minimum expenditure for each person of different age groups (m_{it}) are given by

$$m_{it} = \sum_{j \in R} p_{jt}\gamma_{ji} + m_{Tit}, \qquad i = 0,1,2, \tag{A.10}$$

where R is the commodity grouping at the top level, cf section 4, and the γ's are analogous to those above (cf equation (19b-c)). Note that for Transport there are two components of the minimum expenditures, $p_{Tt}\gamma_{Ti}$ from the top level and m_{Tit} from the intermediate level, and the latter includes minimum expenditures at the bottom level.

The macro minimum expenditure at the top level is

$$M_t = m_{0t}N_t + m_{1t}A_{1t} + m_{2t}A_{2t}, \tag{A.11}$$

analogous to equation (A.7) and (A.9).

The top level LES

The expenditure on Transport (cf equation (22a)):

$$Y_{Tt} = (m_{T0t} + p_{Tt}\gamma_{T0})N_t + \sum_{i=1,2}(m_{Tit} + p_{Tt}\gamma_{Ti})A_i + \beta_T(Y_t - M_t). \tag{A.12}$$

The utility aggregate of Energy consumption (cf equation (22b)):

$$Q_{Ut} = \gamma_{U0}\cdot N_t + \sum_{i=1,2}\gamma_{Ui}A_{it} + \beta_U\frac{Y_t - M_t}{p_{Ut}}. \tag{A.13}$$

Commodity demand of the other goods (cf equation (22b)):

$$Q_{rt} = \gamma_{ro}\cdot N_t + \sum_{i=1,2}\gamma_{ri}A_{it} + \beta_r\frac{Y_t - M_t}{p_{rt}} - a_rQ_{70t} + Q_{rt}^E, \qquad \forall r \in R\text{-}\{U,T\}. \tag{A.14}$$

We have included a term ($-a_rQ_{70t}$) for foreigners consumption of commodity r in Norway. The parameter a_r is the share of foreigners consumption in Norway spent on good r, cf equation (A.5). The negative sign is due to the fact that foreigners consumption in Norway (Q_{70t}) is measured as a negative number using the conventions of the national accounts. The variable Q_{70t} is exogenously given in the model. Q_{jt}^E are exogenous variables. In the base year these can be interpreted as residuals, and are calibrated to make the model fit exactly to the national account data in the base year.

The intermediate level LES for Transport

The expenditure on Public transport (cf equation (22a)):

$$Y_{61t} = (m_{610t} + p_{61t}\gamma_{610})N_t + \sum_{i=1,2}(m_{61it} + p_{61t}\gamma_{61i})A_i + \beta_{61}(Y_{Tt} - M_{Tt}). \tag{A.15}$$

The utility aggregate of Private transport (cf equation (22b)):

$$Q_{PTt} = \gamma_{PT0}\cdot N_t + \sum_{i=1,2}\gamma_{PTi}A_{it} + \beta_{PT}(Y_{Tt} - M_{Tt})/p_{PTt}, \tag{A.16}$$

The bottom level LES for Public transport

Commodity demand (cf equation (22b)):

$$Q_{jt} = \gamma_{j0}N_t + \sum_{i=1,2}\gamma_{ji}A_{it} + \beta_j(Y_{61t} - M_{61t})/p_{jt} - a_jQ_{70t} + Q_{jt}^E, \tag{A.17}$$

for $\forall j \in J_{61} = \{75,76,77,78,79\}$.

The bottom level CES for Private transport

Commodity demand (cf equation (16) and (10)):

$$Q_{14t} = Q_{PTt}\omega_{PT}\left(\frac{p_{PTt}}{p_{14t}}\right)^{\sigma_{PT}} - a_{14}Q_{70t} + Q_{14t}^{E}, \tag{A.18}$$

$$Q_{31t} = Q_{PTt}(1-\omega_{PT})\left(\frac{p_{PTt}}{p_{31t}}\right)^{\sigma_{PT}} + Q_{31t}^{E}. \tag{A.19}$$

The bottom level CES for Energy

Commodity demand (cf equation (16) and (10)):

$$Q_{12t} = Q_{Ut}\omega_{U}\left(\frac{p_{Ut}}{p_{12t}}\right)^{\sigma_{U}} + Q_{12t}^{E}, \tag{A.20}$$

$$Q_{13t} = Q_{Ut}(1-\omega_{U})\left(\frac{p_{Ut}}{p_{13t}}\right)^{\sigma_{U}} + Q_{13t}^{E}. \tag{A.21}$$

Purchase of cars

The variable Q_{31t} should be understood as a stream of services from the households stock of cars. From Q_{31t} we have to calculate the purchase of cars, Q_{30t}. This is done as follows, using standard procedures for handling purchases of durables in MSG,

$$Q_{30t} = \frac{1}{K_{31}}\left[(1+\delta)Q_{31t} - Q_{31t-1}\right], \tag{A.22}$$

where δ is the depreciation rate of the stock of cars and K_{31} is a constant explained below.

The price index for the user cost of cars is determined by:

$$P_{31t} = \frac{1}{K_{31}}(\delta + R_{Bt})\frac{P_{Kt}}{C_{30t}}\left[(C_{30t} - C_{K30t})P_{30t} + C_{K30t}P_{J40t}\right], \tag{A.23}$$

where R_{Bt} is an exogenous interest rate in the equlibrium model, P_{Kt} is an index reflecting changes in the average user cost of capital, P_{J40t} is a price index for used cars, C_{K30t} is the households purchase of used cars, and K_{31}=(δ+R_{B0})P_{K0} is a constant which normalizes the user cost of cars to 1 in the base year.

Appendix B: Commodity classifications

In this appendix, we give the detailed definitions of the commodity groups used in the calibration procedure. One starting point is the standard three-digit classification used in the Surveys of Consumer Expenditure, cf Statistics Norway (1990), and the aggregation to the 28 commodity groups used in Biørn and Jansen (1980, see table A1.1) on which the microeconometrics is based. Another starting point is the three digit classification in the National Account, cf Statistics Norway (1989), and the commodity classifications used in the macroeconomic models MSG and MODAG. Because some of the commodity-aggregates in the consumer demand system in MSG are not direct aggregates of the 28 commodity aggregates in Biørn and Jansen (1980), we split some of the groups (14, 16, 17, 23, 27 and 28) to obtain the 33 commodity groups that can be directly aggregated to the 22 commodity groups in MSG-EE, cf table B.1,

$$K = \{1_K,..,13_K,14A_K,14B_K,15,16A_K,16B_K,17A_K,17B_K,...,28B_K\}. \qquad \text{(B.1)}$$

The 22 commocity groups in MSG-EE can be aggregated further to the 13 commodity groups in MSG-5 and MODAG, cf table B.2.

In addition we have commodity aggregates corresponding to branches in the utility tree, namely U: Energy, PT:Private Transport, 61:Public transport and T:Transport. The set of commodities corresponding to these aggregates are denoted by $J_U=\{12,13\}$, $J_{PT}=\{14,31\}$, $J_{61}=\{75,76,77,78,79\}$ and $J_T=\{PT,61\}$, where the commodities within the sets are defined in table B.1 and B.2.

In the upper level LES in the empirical model we have a set R of commodities and aggregates:

$$R = \{00,11,U,T,15,21,22,23,41,50,63,64,65,66\}, \qquad \text{(B.2)}$$

cf table B.2.

Table B.1
Commodity classifications.

Connections between groups in household expenditure surveys (HES), national accounts (NA) and the macromodel MSG-EE

Micro data		Commodity codes		MSG-EE (1988)		
Codes		HES	NA	Code	Names	Aggregating equation
1	1 Flower and bread	00	00a			
2	2 Meat and eggs	01-013+035	01a-012+034			
3	3 Fish	02-024-025	02a-024- 025			
4	4 Canned meat and fish	013+024+025	012+024+025			
5	5 Dairy products	03-035	03a-034			
6	6 Butter and margarine	04	04a			
7	7 Potatoes and vegetables	05+06	05a+06a			
8	8 Other foods	07+08+09	07a+08a+09a	00	Food	C00=Cm1+..+Cm8
9	9 Beverages	11	11a			
10	10 Tobacco	12	12a	11	Beverages and tobacco	Cm9+Cm10
11	11 Clothing	21+22	21a+22a			
12	12 Footwear	23	23a	21	Clothing and footware	C21=Cm11+Cm12
13	13 Housing	31	31a	50	Rents	C50=Cm13
	14 Energy	32	32a	U	Energy (stationary)	
14	14A Electricity	321	321	12	Electricity	C12=Cm14A
15	14B Fuel	32-321	32a-321	13	Fuel	C13=Cm14B
16	15 Furniture	41+42	41a+42a			
17	16A Electric appliances	43	43a	41	Furniture etc.	C40=Cm15+Cm16A
18	16B Kitchen utensils	44	44a			
19	17A Misc. Com.	451+452+453	451+452	22	Other household goods	C22=Cm16B+Cm17 A
20	17B Misc. services	454+455+ 456+46	453+454+ 461+471	64	Various household services	C64=Cm17B
21	19 Motorcars, bicycle	61	61a	30	Purchases of cars	C30=Cm19
22	20 Running costs of vehicles	62	62a	14	Petrol and Car Maintance	C14=Cm20
23	21 Public transport[a]	63	63a			
24	22 P.T.T charges	64	64a	61	Public transport	C61=Cm21+Cm22
25	23A TV, boats etc.	710+711+ 712+714	711+712+ 713+714	42	Durable consumer goods	C42=Cm23A
26	23B Sports equipment etc.	713+715+716 +717+718+719	715+716+ 717+718			
27	24 Public enterteiment	72+74	72a+74a	63	Entertainment, education etc.	C63=Cm24
28	25 Books and newspapers	73	73a	23	Other goods for recreation activites	C23=Cm25+Cm23B
29	26 Personal care	81	81a			
30	27A Jewellery etc.	82	82a	15	Other goods	C15=Cm26+Cm27A
31	27B Other services	84+85	84a+85a			
32	28A Restaurants, hotels etc.	831+833	83a	65	Other services	C65=Cm27B+Cm28 A
33	28B Package tours etc	832	991	66	Tourism abroad	C66=Cm28B

a MSG-EE has a more detailed classification of public transport, see Table B.2.

Table B.2
Commodity classifications in MSG-EE, MSG-5, and MODAG.

MSG-EE (1988)			MSG-5 and MODAG		
Code	Names	NA-codes	Code	Names	Aggregation from MSG-EE
00	Food	0aa	00	Food	
11	Beverages and tobacco	11a+12a	11	Beverages and tobacco	
21	Clothing and footware	21a+22a+23a	21	Clothing and footware	
50	Rents	31a	50	Rents	
12	Electricity	321	12	Electricity	
13	Fuel	32a-321	13	Fuel	
41	Furniture etc.	41a+42a+43a			
			40	Furniture etc.	C40=C41+C42
22	Other household goods	44a+451+452			
			20	Other goods	C20=C22+C23+C15
64	Various household services	453+454+461+471			
			60	Other services	C60=C64+C63+C65
30	Purchases of cars	61a	30	Purchases of cars	
14	Petrol and Car Maintance	62a	14	Petrol and Car Maintance	
75	Bus transport,transport by taxi etc.	635+636+0.9*637			
76	Air transport	634			
77	Railway, tramway and subway transport	631+632			
78	Transport by boat and ferry	633+0.1*637			
79	Postage, telephone and telegram	64a			
			61	Public transport	C61=C75+C76+C77+C78+C79
42	Durable consumer goods	711+712+ 713+714			
63	Entertainment, education etc.	72a+74a			
23	Other goods for recreation activites	73a+715+716+ 717+718			
15	Other goods	81a+82a			
65	Other services	83a+84a+85a			
66	Tourism abroad	991	66	Tourism abroad	

Appendix C: Calibration procedure

C.1 Introduction

The calibration procedure is based on some basic principles which are developed in Aasness (1993b). Here we shortly describe the main idea, and how this appendix is organized.

Let θ be the vector of unknown parameters in the utility function. It can be shown that these parameters can be identified from a set of characteristics of the demand function at one point , i.e. there exist a function f,

$$\theta = f(p_n, q_n, y_n, a_n, E_n, P_{1n}, P_{2n}, S_n), \qquad (C.1)$$

where the arguments in the funcion is the set of characteristics we apply. The first set of variables (p_n, q_n, y_n, a_n), i.e. prices, quantities, total expenditure, and demographic variables in a "normal year" (n), is described in section C.2. The second set of variables (E_n, P_{1n}, P_{2n}), i.e. Engel-, child-, and adult elasticities, is described in section C3. Section C4-C7 describes the procedure for calibrating each subutility function, starting at the bottom level and moving up to the top level. The last set of variables (S_n), i.e. a set of "substitution parameters", is described in this connection.

It is easy to recalibrate the model when new empirical evidence on some of the input variables is obtained. Correspondingly, one can do inexpensive sensitivity analysis, see table 9-10 for an example.

C.2 Consumption, expenditure and prices

By assumption, a vector of normal year variables fits the demand functions exactly, i.e. all the residuals are zero. In many general equilibrium models one estimate such a vector by simply using the corresponding national account data in a selected base year. Since all experience tells us that econometric models in general do not fit exactly to data from one period, one should look for alternatives. In this paper we have chosen to estimate the normal year variables by five year averages, as defined in table C.1. Results on applying these definitons on our data is presented in table C.2 and C.3. We have adjusted the data for foreigners consumption in Norway, using the model and coefficients described in appendix A, cf (A.5).

Table C.1
Definitions of variables in the normal year

Equation	Explanation
$Q_{jn} = \sum_{t=1987}^{1991} \frac{1}{5} Q_{jt}, \quad j \in K$	consumption of commodity *i*
$Y_{jn} = \sum_{t=1987}^{1991} \frac{1}{5} Y_{jt}, \quad j \in K$	expenditure on commodity *i*
$p_{jn} = Y_{jn}/Q_{jn}, \quad j \in K$	price on commodity *j*
$Y_n = \sum_{j \in K} Y_{jn}$	total expenditure on consumer goods
$w_{jn} = Y_{jn}/Y_n, \ j \in K$	budget shares
$N_n = N_{1989}$	number of households
$A_{in} = A_{i1989}$, i=1,2	number of adults/children
$a_{in} = A_{in} / N_n, \quad i = 1, 2$	number of children and adults per household
$q_{jn} = Q_{jn}/N_n, \ j \in K$	per household consumption of commodity j
$y_{jn} = Y_{jn}/N_n, \ j \in K$	per household expenditure of commodity j
$y_n = Y_n/N_n$	per household total expenditure
$q_{rn} = \Sigma_{j \in Jr} q_{jn}, \quad r \in R$	per household group consumption
$y_{rn} = \Sigma_{j \in Jr} y_{jn}, \quad r \in R$	per household group expenditure
$w_{jrn} = y_{jn}/y_{rn}, j \in J_r, \quad r \in R$	within group budget shares

Table C.2

Consumption, prices, expenditure and budget shares in the normal year[a]

Commodities	Incl. foreigners consumption		Foreigners	Foreigners cons. excluded			
	Cons.	Exp.	shares	Prices	Cons.	Exp.	Shares
1 Flower and bread	73125	68213	0,012	0,933	71730	66937	0,021
2 Meat and eggs	152951	148793	0,029	0,974	149676	145798	0,047
3 Fish	53493	50128	0,009	0,938	52525	49242	0,016
4 Canned meat and fish	15090	14734	0,003	0,978	14772	14443	0,005
5 Dairy products	108556	94748	0,017	0,872	106609	92968	0,030
6 Butter and margarine	15295	13968	0,003	0,913	15006	13704	0,004
7 Potatoes and vegetables	96837	95782	0,016	0,991	95026	94125	0,030
8 Other foods	116447	109840	0,011	0,944	115209	108709	0,035
9 Beverages	156001	138453	0,026	0,887	153070	135773	0,044
10 Tobacco	89268	78079	0,014	0,874	87702	76647	0,025
11 Clothing	200039	190149	0,067	0,952	192516	183270	0,059
12 Footwear	39756	36879	0,013	0,928	38285	35534	0,011
13 Housing	446554	400298	0,000	0,896	446554	400298	0,128
14A Electricity	181146	167591	0,000	0,925	181146	167591	0,054
14B Fuel	30960	23844	0,000	0,770	30960	23844	0,008
15 Furniture	122308	117712	0,000	0,962	122308	117712	0,038
16A Electric appliances	22964	22323	0,000	0,972	22964	22323	0,007
16B Kitchen utensils	31071	29606	0,000	0,953	31071	29606	0,009
17A Misc. Com.	23616	21828	0,000	0,924	23616	21828	0,007
17B Misc. services	43133	37783	0,000	0,876	43133	37783	0,012
19 Motorcars, bicycle	190334	180112	0,000	0,946	190334	180112	0,058
20 Running costs of vehicles	178286	153690	0,150	0,857	161422	138271	0,044
21 Public transport	98646	88316	0,040	0,894	94149	84204	0,027
22 P.T.T charges	53809	62423	0,020	1,171	51561	60367	0,019
23A TV, boats etc.	55279	53058	0,000	0,960	55279	53058	0,017
23B Sports equipment etc.	89166	83876	0,040	0,942	84669	79764	0,026
24 Public enterteiment	87516	80990	0,020	0,926	85267	78934	0,025
25 Books and newspapers	77933	68228	0,040	0,873	73436	64116	0,021
26 Personal care	74332	69235	0,000	0,931	74332	69235	0,022
27A Jewellery etc.	50268	48365	0,000	0,962	50268	48365	0,016
27B Other services	143603	113727	0,185	0,771	122826	94729	0,030
28A Restaurants, hotels etc.	156361	142571	0,285	0,911	124298	113254	0,036
28B Package tours etc	246635	216503	0,000	0,878	246635	216503	0,069
Sum	3520779	3221844	1,000		3408355	3119046	1,000

a) Average national account data from the five year period 1987-1991, see table C.1 for definitions. Direct purchases in Norway by non resident households; consumption: -112.424 , expenditure: -102.798.

Table C.3

Demographic variables in the normal year[a]

	Total	Children	Adults
Number of persons	4180458	1128860	3051598
Number of households	1736008	-	-
Household average	2,41	0,65	1,76

a) In this case the normal year is defined as 1989. Number of persons in institutions (40222) is excluded.

C.3 Engel, child and adult elasticities

An important part of the empirical basis for our calibration is Engel functions with demographic variables for 28 commodity groups, estimated with the same panel data and the same approach as in Aasness, Biørn and Skjerpen (1993). The latter contains estimates for five broad commodity groups only, the micro econometric analysis with 28 commodity groups will be reported elsewhere. Table C.4 contains the estimates of Engel-, child-, and adult elasticities for the average household in our micro data, which we used as input in our calibration procedure. Some of the 28 commodity groups in the micro data is divided in two sub groups, assuming equal elastisticities, in order to match the commodity groups in macro data, which leaves us with 33 commodity groups (cf appendix B).

The unadjusted Engel, child, and adult elasticities (E_{jm},$P_{j1m}P_{j2m}$) satisfy the adding-up conditions using budget shares for the average household in our micro data. In order to satisfy the adding-up conditons for the macro data in the normal year we adjust the elasticities in the following way (cf Aasness (1993b, sec. 3.3))

$$E_{jn} = E_{jm}\, E_{just}, \qquad j \in K, \qquad \text{where } E_{just} = 1/\Sigma_{j\in K}E_{jm}w_{jn}, \tag{C.2}$$

$$P_{jin} = P_{jim} + P_{ijust}, \qquad j \in K, \qquad \text{where } P_{ijust} = -\,\Sigma_{j\in K}P_{jim}w_{jn},\ i = 1,2, \tag{C.3}$$

where w_{jn} is the budget share of commodity j for the macro data in the normal year. These elasticities, adjusted for the normal year, is presented in the right part of table C.4.

We also need elasticities for several aggregated commodity groups, which are computed as follows (cf Aasness (1993b, sec. 3.3))

$$E_r = \Sigma_{j\in J_r}E_{jn}w_{jrn}, \quad P_{ri} = \Sigma_{j\in J_r}P_{jin}w_{jrn}, \quad i = 1,2, \qquad \forall\, r \in R. \tag{C.4}$$

Results on aggregated elasticities can be found in table C.8-9.

Furthermore we need within group elasticities. Let E_{jr} and P_{jir} be the Engel- and person elasticities w.r.t. the within group demand functions (8). From (3), (6) and (8) it follows that (cf Aasness (1993b sec. 3.3))

$$E_{jr} = E_j/E_r, \qquad j \in J_r,\ r \in R \tag{C.5}$$

$$P_{jir} = P_{ji} - E_{jr}\,P_{ri}, \qquad j \in J_r,\ r \in R,\ i=1,2 \tag{C.6}$$

Such within group elasticities are found in tables C.7 and C.8.

Table C.4
Engel, child and adult elasticities

		Unadjusted elasticities			Adjusted elasticities		
	Commodities	Engel	Child	Adult	Engel	Child	Adult
1	Flower and bread	0,279	0,509	0,544	0,263	0,585	0,636
2	Meat and eggs	0,518	0,357	0,317	0,488	0,433	0,409
3	Fish	0,336	-0,062	0,307	0,317	0,014	0,399
4	Canned meat and fish	0,453	0,264	0,237	0,427	0,340	0,329
5	Dairy products	0,095	0,637	0,666	0,090	0,713	0,758
6	Butter and margarine	-0,008	0,565	0,727	-0,008	0,641	0,819
7	Potatoes and vegetables	0,596	0,411	0,155	0,562	0,487	0,247
8	Other foods	0,217	0,428	0,611	0,204	0,504	0,703
9	Beverages	1,376	-0,309	-0,597	1,296	-0,233	-0,505
10	Tobacco	0,590	0,080	0,130	0,556	0,156	0,222
11	Clothing	0,935	0,117	0,258	0,881	0,193	0,350
12	Footwear	1,013	0,308	-0,013	0,954	0,384	0,079
13	Housing	1,418	-0,164	-0,808	1,336	-0,088	-0,716
14B	Fuel	0,303	0,045	0,082	0,285	0,121	0,174
15	Furniture	1,462	-0,405	-0,596	1,377	-0,329	-0,504
16A	Electric appliances	1,052	-0,034	-0,130	0,991	0,042	-0,038
16B	Kitchen utensils	1,052	-0,034	-0,130	0,991	0,042	-0,038
17A	Misc. Com.	0,854	0,223	-0,595	0,805	0,299	-0,503
17B	Misc. services	0,854	0,223	-0,595	0,805	0,299	-0,503
19	Motorcars, bicycle	1,229	-0,243	0,633	1,158	-0,167	0,725
20	Running costs of vehicles	1,526	0,068	0,056	1,438	0,144	0,148
22	P.T.T charges	0,286	-0,406	0,231	0,269	-0,330	0,323
23A	TV, boats etc.	1,318	-0,183	-0,356	1,242	-0,107	-0,264
24	Public enterteiment	0,728	-0,103	0,543	0,686	-0,027	0,635
25	Books and newspapers	0,905	-0,153	-0,005	0,853	-0,077	0,087
26	Personal care	0,979	0,106	0,208	0,922	0,182	0,300
27A	Jewellery etc.	0,964	0,071	0,220	0,908	0,147	0,312
27B	Other services	0,964	0,071	0,220	0,908	0,147	0,312
28A	Restaurants, hotels etc.	2,148	-0,723	-0,824	2,024	-0,647	-0,732
28B	Package tours etc	2,148	-0,723	-0,824	2,024	-0,647	-0,732
	Adjustment factor	0,942	0,076	0,092	1,000	0,000	0,000

C.4 Calibration of the sublevel CES for Energy and Private transport

Conditional on the value of the substitution elasticity σ_r, there is only one parameter in the sublevel functions to be calibrated, the distribuition parameter ω_r, cf appendix A. This is done by the following equation (cf Aasness (1993b, sec. 4.3)):

$$\omega_{jr} = w_{jrn} p_{jn}{}^{\sigma_r - 1} / \Sigma_{Jr} w_{irn} p_{in}{}^{\sigma_r - 1}, \qquad j \in J_r,\ r = U,PT. \qquad (C.7)$$

The input and output of the calibration are presented in tables C.5 and C.6, for Energy and Private transport respectively.

The substitution elasticity between Electricity and Fuels is assumed to be 0,5, based on an evaluation of a time series study of Bye (1989,1992). (It would probably be efficient to improve the model for stationary energy, including the utility tree and functional form, before doing more estimation and testing of the values of the parameters in the subutility function.)

The substitution elasticity between Petrol etc and User cost of cars is assumed to be 0,1, according to our judgement on the most appropriate value. We have not had available an econometric study adressing this issue directly. To support our judgement we have perfomed a sensitivity analysis, cf table 9-10, and evaluated the implications of the value of this parameter. (The homotheticity assumption implied by the CES model is probably not realistic, and should be relaxed before investing too much resources on empirical econometrics on a subutility function for this commdity group.)

We can now compute utility based price indexes for these commodity groups, which we will exploit in the calibration of the subutility functions at the intermediate and upper level of the utility tree,

$$p_{rn} = \left\{ \omega_r p_{in}^{(1-\sigma_r)} + (1-\omega_r) p_{jn}^{(1-\sigma_r)} \right\}^{\frac{1}{1-\sigma_r}}, \qquad j \in J_r,\ r = U,PT. \qquad (C.8)$$

Table C.5

Calibration of the sub level CES for Energy

MSG-codes		Prices	Shares	Distribution parameter
12	Electricity	0,925	0,875	0,865
13	Fuels	0,770	0,125	0,135
U	Energy	0,903	1,0000	1,0000
	Substitution elasticity			0,5

Table C.6

Calibration of the sub level CES for Private transport

MSG-codes		Prices	Shares	Distribution parameter
14	Petrol and car maintenance	0,857	0,434	0,456
31	User costs of cars	0,946	0,566	0,544
PT	Private transport	0,905	1,0000	1,0000
	Substitution elasticity			0,1

C.5 Calibration of the the bottom level LES for Public transport

The parameters in the bottom level LES for Public transport are calibrated by the following recursive equation system. The theory is presented in Aasness (1993b, sec. 4.4-5), the data input to the calibration procedure is presented in table C.7, and the output in terms of calibrated parameters are given in table 3.

$$\beta_j = E_{j61n} w_{j61n}, \qquad j \in J_{61}, \qquad \text{(C.9)}$$

$$\gamma_{jH} = q_{jn} - \beta_j s_{61n} y_{61n} / p_{jn}, \qquad j \in J_{61}, \qquad \text{(C.10)}$$

$$\gamma_{j2} = \gamma_{jH} / (0{,}5\, a_{1n} + a_{2n}), \qquad j \in J_{61}, \qquad \text{(C.11)}$$

$$\gamma_{j1} = 0{,}5 \gamma_{j2}, \qquad j \in J_{61}. \qquad \text{(C.12)}$$

Equation (C.9) identifies the (within group) marginal budget shares from the (within group) Engel elasticities and (within group) budget shares in the normal year. The Engel elasticities in table C.7 are our assessment based on the different estimates in Magnussen and Stoltenberg (1991) and Aasness and Li (1991). Equation (C.10) identifies minimum consumption for the average household, based on inter alia the substitution parameter s_{61} (minus the inverse of the flexibility of the marginal utility of money on Public transport, cf Frisch (1959) and Bojer (1972)). The implemented value (1) of this parameter corresponds to one of the estimated models in Magnussen and Stoltenberg (1991). Equation (C.11-12) identifies the marginal minimum consumption of children and adults based on two apriori assumptions, in lack of relevant microeconometric estimates for these detailed groups. One may interpret these assumptions as follows. We have a per capita model, with no economies of scale (γ_{j0}=0), but where tickets for children cost half as much as tickets for adults.

We can now compute a price index of the marginal utility of Public transport, and minimum expenditures on Public transport, which we will exploit in the calibration at the intermediate and upper level of the utility tree,

$$p_{61n} = \prod_{j \in J_{61}} p_{jn}^{\beta_j}, \qquad \text{(C.13)}$$

$$m_{61in} = \sum_{j \in J_{61}} p_{jn} \gamma_{ji}, \qquad i=1,2,H. \qquad \text{(C.14)}$$

C.6. Calibration of the intermediate level LES for Transport

The calibration of the intermediate level LES for Transport is done by the following recursive equation system. The theory is presented in Aasness (1993, sec. 4.8), the data input to the calibration procedure is presented in table C.8, and the output in terms of calibrated parameters are given in table 2.

$$\beta_j = E_{jTn}w_{jTn}, \qquad j\in J_T, \tag{C.15}$$

$$\gamma_{jH} = (y_{jn} - m_{61Hn} - \beta_j s_{Tn} y_{Tn})/p_{jn}, \qquad j\in J_T, \tag{C.16}$$

$$\gamma_{ji} = [P_{jiTn}y_{jn}/(a_{1n}+a_{2n}) - m_{61in} + \beta_j(1-s_{Tn})y_{Tn}m_{irn}/m_{rn}]/p_{jn}, \qquad j\in J_T, \text{ i} = 1,2, \tag{C.17}$$

$$\gamma_{j0} = \gamma_{jH} - \Sigma_i\gamma_{ji}a_{in}, \qquad j\in J_T. \tag{C.18}$$

The within group Engel- and person elasticities (E_{jTn}, P_{jiTn}) are taken from the microeconometric estimates presented in section C.3. The substitution parameter s_T (minus the inverse of the flexibility of the marginal utility of money on Transport) is set equal to 1 according to our judgement, based on a sensitivity analysis on its implications on the system of demand elasticities, cf table 9-10. The last set of parameters needed to identify the subutility function is an equivalence scale on minimum expenditures. In lack of microeconometric evidence for this specific commodity group we have used the OECD scale (cf section C.7), in this first version of our simulation model.

We can now compute the price index of the marginal utility of Transport and minimum expenditures on Transport, which we will use in the calibration of top level of the utility tree,

$$p_{Tn} = p_{61n}^{\beta_{61}} p_{PTn}^{\beta_{PT}}, \tag{C.19}$$

$$m_{Tin} = p_{PTn}\gamma_{PTi} + p_{61n}\gamma_{61i} + m_{61in}, \ i=1,2,H. \tag{C.20}$$

C.7 Calibration of the upper level LES

The calibration of the top level LES is done correspondingly by the following recursive equation system. The theory is presented in Aasness (1993b, sec.4.8), the data input to the calibration procedure is presented in table C.9, and the output in terms of calibrated parameters are given in table 1.

$$\beta_j = E_{jn}w_{jn}, \qquad j\in R, \tag{C.21}$$

$$\gamma_{jH} = (y_{jn} - m_{jHn} - \beta_j s_n y_n)/p_{jn}, \qquad j\in R, \tag{C.22}$$

$$\gamma_{ji} = [P_{jin}y_{jn}/(a_{1n}+a_{2n}) - m_{jin} + \beta_j(1-s_n)y_n m_{irn}/m_{rn}]/p_j, \qquad j\in R, \text{ i} = 1,2, \tag{C.23}$$

$$\gamma_{j0} = \gamma_{jH} - \Sigma_i\gamma_{ji}a_{in}, \qquad j\in R. \tag{C.24}$$

The Engel-, child-, and adult elasticites are taken from the microeconometric estimates presented in section C.3. The substitution parameter s_n (minus the inverse of the flexibility of the marginal utility of money, cf Frisch (1959) and Bojer (1972)), is assumed to be 0,5 (in the normal year). There are a lot of empirical studies which have relevance to the size of this parameter. See for example Theil and Clements (1987) which quite strongly supports the hypothesis that s_n=0,5 is approximately correct in demand systems with broad aggregates, in aggreement with Frisch (1959) and Johansen

(1974, p.107). The last set of parameters needed to identify the top level utility function is an equivalence scale on minimum expenditures. In lack of firm microeconometric evidence we have used the OECD equivalence scale, which also can serve as a convenient point reference for future empirical work on this issue. The OECD scale have some support from the studies of Bojer (1977) and Herigstad (1979) based on Norwegian household expenditure surveys, cf the discussion in Aasness (1993a,p.88).

Table C.7

The sub level LES for Public transport. Calibration input[a]

MSG-codes	Normal year (macro) Prices	Exp.	Normal year (pr. household) Cons.	Exp.	Shares	Engel elasticity Unadj.	Adjusted
75 Bus etc.	0,905	35808	2280	2063	0,248	0,2	0,19
76 Air transport	0,893	23174	1495	1335	0,160	1,6	1,53
77 Railway etc.	0,88	14034	919	808	0,097	0,2	0,19
78 Boat and ferry	0,884	11189	729	644	0,077	0,7	0,67
79 Postage etc.	1,171	60367	2970	3477	0,418	1,6	1,53
61 Public transport	1,061	144572	8393	8328	1,000	1,05	1,00

a) The substitution parameter $s_{61n} = 1$.

Table C.8

The intermediate level LES for Transport. Calibration input[a]

MSG-codes	Normal year (macro) Prices	Exp.	Normal year (pr. household) Cons.	Exp.	Shares	Elasticities Engel	Child	Adult
PT Private transport	0,905	318382	20260	18340	0,688	1,128	0,165	0,041
61 Public transport	1,061	144572	7848	8328	0,312	0,719	-0,363	-0,091
Sum	0,938	462954	28108	26668	1,000	1,000	0,000	0,000

a) The substitution parameter $s_{Tn} = 1$. The equivalence scale: 0,3 (fixed), 0,5 (children) and 0,7 (adults).

Table C.9

The top level LES. Calibration input[a]

MSG-codes	Normal year (macro) Prices	Exp.	Normal year (pr. household) Cons.	Exp.	Shares	Elasticities Engel	Child	Adult
00 Food	0,944	585925	35746	33751	0,188	0,331	0,484	0,525
11 Beverages and tobacco	0,882	212420	13869	12236	0,068	1,029	-0,093	-0,243
U Energy	0,903	191435	12206	11027	0,061	0,285	0,121	0,174
T Transport	0,938	462954	28427	26668	0,148	1,135	-0,175	0,384
15 Other goods	0,944	117600	7177	6774	0,038	0,917	0,168	0,305
21 Clothing and footware	0,948	218804	13295	12604	0,070	0,893	0,224	0,306
22 Other household goods	0,941	51433	3150	2963	0,016	0,912	0,151	-0,236
23 Recreation activites etc.	0,910	143880	9107	8288	0,046	1,068	-0,094	-0,108
50 Rents	0,896	400298	25723	23059	0,128	1,336	-0,088	-0,716
63 Entertainment etc.	0,926	78934	4912	4547	0,025	0,686	-0,027	0,635
64 Various household services	0,876	37783	2485	2176	0,012	0,805	0,299	-0,503
65 Other services	0,842	207983	14235	11981	0,067	1,516	-0,285	-0,257
66 Tourism abroad	0,878	216503	14207	12471	0,069	2,024	-0,647	-0,732
Sum		3119046	196092	179668	1,000	1,000	0,000	0,000

a) The substitution parameter $s_n = 0,5$. The equivalence scale: 0,3 (fixed), 0,5 (children) and 0,7 (adults).

Appendix D: List of symbols

TROLL-code	Symbol in the text	Comments
	a_r	Share of foreigners consumption in Norway spent on commodity r
BE.r BE.j	β_r β_j	Marginal budget share of commodity r in the upper level LES Conditional marginal budget share of commodity group j in the intermediate level LES
CEj	Q_j	Exogenous consumption of commodity j
Cj	Q_j	Macro consumption of commodity j in fixed prices
CK40	C_{K40}	The households purchase of used cars
D.ELB	δ	Depreciation rate of the stock of cars
GA.jH0	γ_{j0}	Fixed minimum consumption of commodity j for each household
GA.jZ1	γ_{j1}	Marginal minimum consumption of commodity j for one child
GA.jZ2	γ_{j2}	Marginal minimum consumption of commodity j for one adult
	J_x	Set of commodities in group x, x= 61,PT,U,T, cf app. B
K.31	K_{31}	Constant in the equation for user cost of cars
NB0019	A_1	Number of children (age 0-19) in Norway
NB20	A_2	Number of adults (age 20 +) in Norway
NH	N	Number of households in Norway
O.PT	ω_{PT}	Distribution parameter in the demand for Private transport
O.U	ω_U	Distribution parameter in the demand for Energy
PCj	p_j	Price index for commodity j
PJ40	P_{J40}	Price index for used cars

TROLL-code	Symbol	Comments
PKJUST	P_K	Index reflecting average user costs of capital
	R	The set of commodity groups at the top level LES, cf table 1 and section 4.
RB	R_B	Exogenous interest rate
SU.PT	σ_{PT}	Elasticity of substitution between Petrol and Cars
SU.U	σ_U	Elasticity of substitution between Electricity and Fuels
VCB	-	Macro expenditure on consumption including purchase of cars Q_{30}
VCC	Y	Macro expenditure on consumption including services from stock of cars Q_{31}
VCMIN	M	Macro minimum expenditure
VCMINH0	m_0	Fixed minimum household expenditure at the top level LES (including minimum expenditure at the intermediate level LES for Transport)
VCMINTH0	m_{T0}	Fixed minimum household expenditure at the intermediate level LES for Transport
VCMINTZ1	m_{T1}	Marginal minimum expenditure for one child at the intermediate level LES for Transport
VCMINTZ2	m_{T2}	Marginal minimum expenditure for one adult at the intermediate level LES for Transport
VCMINZ1	m_1	Marginal minimum expenditure for one child at the top level LES
VCMINZ2	m_2	Marginal minimum expenditure for one adult at the top level LES
VCT	Y_T	Macro expenditure on Transport

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