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Abstract:
In this paper we point out that using a two-state Markov chain to describe change in regime makes it difficult to interpret the model since there is a bias towards frequent shifts. However, by using a finite Markov chain with a transition matrix satisfying certain restrictions it is possible to circumvent the difficulty and at the same time use the established procedures for estimation and filtering. The methods are applied to a couple of time series from the Norwegian quarterly national accounts.

Keywords: Change in regime, Markov-switching models, alternating renewal processes

JEL classification: C22, E32

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1 Introduction

A common way to deal with changes in regime in models for economic time series is to let the parameters switch between several possible values according to an unobserved stochastic process. Often this is taken to be a Markov chain with finite state space. This implies that the times the process stays in a particular state will be geometrically distributed. In this paper we shall study what modifications are necessary to allow for more general distributions of the unobserved stochastic process.

To be specific we shall consider models where the observations, $Y_1, \cdots, Y_T$, are given by the following autoregression

$$(1) \qquad Y_t = \mu_{S_t} + \phi_1 Y_{t-1} + \cdots + \phi_k Y_{t-k} + \delta' D_t + \epsilon_t, t = 1, \cdots, T$$

where D_t represents a vector of fixed regressors, which in the following mainly will be seasonally centered dummies. The term μ_{S_t} can take two values $\mu_0 = \mu$ and $\mu_1 = \mu + \Delta$ according to whether a stochastic process $\{S_t\}$ is in state 0 or 1. The distribution of $\{S_t\}$ depends on the parameters p. The error terms $\epsilon_t, t = 1, \cdots, T$ are independently $N(0, \sigma^2)$ distributed, and independent of the process $\{S_t\}$. We furthermore consider the k initial observed values $y_{-k+1}, \cdots, y_0$ as given.

The interpretation of the model is that the states 0 and 1 represent different regimes governing the data generation process . It is furthermore an implication of the model that the process $\{S_t\}$ for the changes is independent of the past observations. Which regime is the prevailing one does not constitute part of the data, but is represented by the latent or unobservable variables $S_1, \cdots, S_T$.

The model where the stochastic process $\{S_t\}$ is a homogeneous finite Markov chain has in the economic context been introduced and developed by Hamilton in a series of papers (1989,1990,1993,1994a and 1996). It is however not surprising that similar ideas have been used in other fields, since models of this type are natural for sequential data with clustering and abrupt changes. Hidden Markov chain model is a suggestive denomination that is often used. In a recent monograph MacDonald and Zucchini (1996) claim that the discrete versions of the hidden Markov models are suitable as models for a wide range of discrete-valued time series. For applications to speech recognition Rabiner (1989) gives a nice survey.

Stripped of its autoregressive and regressive parts and assuming that the Markov chain is just independent switching between the two states, the model (1) is a mixture of two normal distributions. Such models have been extensively studied. Although they can often increase the goodness-of-fit substantially, the estimation is frequently complicated by the existence of several local maxima of the likelihood function. When dynamics are introduced through the Markov chain and an autoregressive structure, additional problems arise. Solving model (1) backwards one can see that stationarity of the process $\{S_t\}$ and the usual conditions on the autoregressive parameters, ensure that the model is stationary. However, under slight modifications of formulation (1), e.g. letting one of the autoregressive parameters depend on the state of $\{S_t\}$, there is no known characterization of the parameters corresponding to the stationary solutions. Some discussion and partial results on this point can be found in Holst, Lindgren, Holst and Thuvesholmen (1994). Rydén (1994) and Bickel and Ritov (1996) investigate efficient estimation in hidden Markov models with no autoregressive part.

The motivation for fitting a model of type (1) may vary. Sometimes there is no particular reason for the choice of this type of models beyond a desire to improve the fit by using a flexible class. When studying business cycles, however, one typically wants to interpret the process $\{S_t\}$ as indicating the phase of the cycle, whether there is a recession or low growth, or a recovery or high growth at time t. The conditional probability for being in state 1, say, given the observations up to time t, or given all the available observations, is a crucial element in this respect. Using a two-state homogeneous Markov chain to model the latent process may pose some problem, however. As already mentioned, an implication of the Markov hypotheses is that the distribution of the length of time the process $\{S_t\}$ spends in each state must be geometric. A consequence is that the probability that the process $\{S_t\}$ stays for short periods in each state will be larger than the probability for longer sojourns. This does not correspond to the general conception of the business cycles, according to which the lengths should be concentrated around four to five years for recessions, and due to the generally perceived asymmetry of the business cycles, somewhat longer for recoveries.

The distribution of the durations can also be expressed by the hazard rate, which is the conditional probability for a stay to end at duration w given that duration w is achieved. The geometric distribution is characterized by having constant hazard rate. An increasing hazard rate indicates positive duration dependence. The longer the process has stayed in the present state, the higher is the probability that it will switch.

The problem of possible duration dependency of business cycles has been noticed earlier, and there have been several studies of the American business cycle based on the NBER chronology. The conclusions seem to be somewhat contradictory. Diebold and Rudebusch (1990), using nonparametric techniques, did not find strong evidence for duration dependence, while Sichel (1991) concluded with positive dependence for pre-war expansions and post-war contractions. There seems now to be a general agreement on the positive duration dependence of US post-war contractions (Diebold and Rudebusch 1996).

Recently several modifications of the homogeneous two-state Markov chain as a model for the latent process have been suggested. In an interesting paper Durland and McCurdy (1994) considered a modification where the first order transition probabilities of $\{S_t\}$ were allowed to depend on how long the process has been in the current state. Filardo (1994) allowed the transition matrix to depend on economic-indicator variables. In Diebold,Lee and Weinbach (1994) a similar model is studied.

In this paper we shall consider another modification, where the process $\{S_t\}$ is a so-called alternating renewal process, see e.g. Cox and Isham (1980). This is a process $\{S_t\}$ which alternates between the two states 0 and 1. The process is therefore described by a sequence of intervals indicating the length of time the process has spent in each state. These intervals are independently distributed and sequentially drawn from distributions f_0 and f_1 on the positive integers. The two state homogeneous Markov chain arises as a special case when the distributions f_0 and f_1 are both geometric.

From a modeling point of view what need to be specified is therefore the distributions of the lengths of time the process $\{S_t\}$ spends in each state.

The technique that can be used to handle the generalization, is just to introduce a new process $\{W_t\}$, which counts how long the latent process $\{S_t\}$ has been in its present state. Then the joint process $\{S_t, W_t\}$ is a Markov chain. Once the transition matrix of this chain is taken into account, the usual estimation procedures can be employed provided the Markov chain is finite.

In the applications we will assume that f_0 and f_1 are truncated geometric and binomial densities. This entails that the indicated Markov chain is finite. The values where these are truncated are so large, however, that for suitable values of the parameters, the distributions that are used can be viewed as approximations to the geometric and the Poisson distribution. In addition one has to specify an initial distribution for the alternating renewal process. As will be explained later, specifying the sojourns to start at the first time period is an easy solution.

Apart from the intrinsic interest of the more general formulations, the models yield a framework that will make it possible to carry out some sensitivity evaluations of the unrestricted two-state Markov chain hypothesis.

Since the model contains latent variables, the EM (expectation-maximization)-algorithm can also be used for obtaining maximum likelihood estimates. One of the advantages of using this method instead of a direct maximization of the likelihood by numerical optimization techniques, is that the likelihood increases in each step. It is well known that the

EM-algorithm provides a robust, although slow, method for finding the maximum likelihood estimates. We will therefore implement a version of the EM-algorithm. A reference to the use of the EM-algorithm in this kind of time series is Hamilton (1990).

The model in (1) is not the only possible. As pointed out by Hamilton (1993), the following specification may have more plausible dynamic properties in some contexts

$$(2) \quad Y_t - \mu_{S_t} = \phi_1(Y_{t-1} - \mu_{S_{t-1}}) + \cdots + \phi_k(Y_{t-k} - \mu_{S_{t-k}}) + \delta' D_t + \epsilon_t, t = 1, \cdots, T.$$

This is a more complicated model to estimate in the framework outlined above, however. In addition to $\{(S_t, W_t)\}$ the state of $\{S_t\}$ for the k previous periods has to be taken into account. Also the log likelihood of the complete set of variables will no longer be linear in the parameters $\mu, \Delta, \phi_1, \ldots, \phi_k$, so the EM-algorithm will be more complicated.

For macroeconomic time series, which often are highly aggregated, gradual shift are frequent. Specification (1) may be more suited to capture this phenomenon than (2), so we have chosen the former. Hansen (1992,1996) contain some results comparing the two specifications for the American GNP. His analysis suggested a model of form (1), where in addition to the intercept, the second autoregressive parameter was allowed to switch between the two states.

There are several directions for extending the models presented in this paper. One of the most immediate is to augment the number of possible states occupied by the latent process $\{S_t\}$. Sichel (1994) argues that a three-phase pattern is necessary to describe US real output.

The plan for the paper is as follows. In the next section we show how the extended models are defined, and how the maximum likelihood estimates can be computed. In section three we apply the proposed models to some macroeconomic time series taken from the Norwegian quarterly national accounts.

2 Definition of the model and computation of the maximum likelihood estimators

We shall in the following estimate the parameters of the model by the method of maximum likelihood. I turns out that it is convenient to split the parameter vector $\lambda' = (\mu, \Delta, \phi_1, \cdots, \phi_k, \delta', \sigma, p')$ into two parts, $\lambda' = (\theta', p')$, where p contains the parameters of the process S_t.

If we let $\mathcal{Y}_t$ denote the observations $Y_1, \ldots, Y_t$, the conditional likelihood given the initial observations may be written by using successive conditioning

$$\prod_{t=1}^{T}\sum_{i=0}^{1}\frac{1}{\sqrt{2\pi}\sigma}exp(-\frac{1}{\sigma^2}(y_t-\mu_i-\phi_1 y_{t-1}\cdots-\phi_k y_{t-k}-\delta' D_t)^2)P(S_t=i|\mathcal{Y}_{t-1};\lambda).$$

Thus provided the conditional probabilities, $P(S_t=i|\mathcal{Y}_{t-1};\lambda), t=1,\ldots,T$, can be evaluated, maximum likelihood estimates can be found by using a suitable numerical optimization algorithm.

Remark that this result is quite general and does not depend on any properties of the distribution of the process $\{S_t\}$ beyond the independence of $\{S_t\}$ and the error terms $\{\epsilon_t\}$. For the case where $\{S_t\}$ is a renewal process, and the distributions of the time of sojourn in each state are concentrated on finite intervals, the distribution of $\{S_t\}$ may be formulated by the help of a finite Markov chain. The recursions given in Hamilton (1994b) can therefore still be used to compute $P(S_t=i|\mathcal{Y}_{t-1};\lambda), t=1,\cdots,T, i=0,1$. But some restrictions on the transition matrices have to be taken into account.

We shall explain the modifications that are necessary. Let the time of sojourn in state 0 and 1 be given by two random variables V_0 and V_1 taking values in $\{1,\ldots,K_0\}$ and $\{1,\ldots,K_1\}$. Let the densities be $f_0(.;p_0)$ and $f_1(.;p_1)$ respectively. If we let W_t be a random variable denoting the length of time the process $\{S_t\}$ has been in the present state, i.e. $W_t=sup\{k:S_{t-k}\neq S_t\}$, the process $\{(S_t,W_t)\}$ is a finite homogeneous Markov chain. If the process $\{(S_t,W_t)\}$ is in state $(0,j)$, say, either the process is in state $(0,j+1)$ at time $t+1$ or the sojourn in state 0 ended and the process is in state $(1,1)$. Therefore the rows of the transition matrix contain only two non-zero entries and the transition matrix must have the form

$$\left|\begin{array}{llllllll}
0 & 1-h_{1,0} & \cdots & 0 & h_{1,0} & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & h_{2,0} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 1-h_{K_0-1,0} & h_{K_0-1,0} & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\
h_{1,1} & 0 & \cdots & 0 & 0 & 1-h_{1,1} & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
h_{K-1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 1-h_{K-1,1} \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0
\end{array}\right|$$

where $K=K_0+K_1$ and $h_{j,i}, j=1,\ldots,K_i, i=0,1$ are the hazard rates for the densities f_0 and f_1, i.e.

$$h_{j,i}=P(V_i=j|V_i\geq j;p_i), j=1,\ldots,K_i.$$

The hazard $h_{j,i}$ is precisely the conditional probability that a sojourn in state i lasts for exactly j periods, given that the duration is j or more periods.

In addition, the initial distribution of the process $\{(S_t, W_t)\}$ is given by the distribution μ_λ. One possibility is to assume that the process $\{(S_t, W_t)\}$ starts in $(0,1)$ or $(1,1)$.

Another possibility would have been to consider the situation where the alternating renewal process is a stationary process. This corresponds to the first run being distributed according to

$$\frac{m_0}{m_0+m_1}\left(\frac{w}{m_0}f_0(w)\right)+\frac{m_1}{m_0+m_1}\left(\frac{w}{m_1}f_1(w)\right)=\frac{w(f_0+f_1)(w)}{m_0+m_1},$$

where m_1 and m_2 are the expectations in the distributions having densities $f_i, i=0,1$.

Now, to compute the value of the likelihood at specific values of the parameters, what is needed is only that the restrictions on the transition matrix just described, are taken into account. Since $\{(S_t, W_t)\}$ is a finite homogeneous Markov chain, the formulas in Hamilton (1994b), i.e. equations 22.4.5-6 can be used to compute $P(S_t = i, W_t = w|\mathcal{Y}_{t-1};\lambda), t = 1,\cdots,T, w = 1,\ldots,K_i, i = 0,1$ using the transition matrix above. The conditional probabilities $P(S_t = i|\mathcal{Y}_{t-1};\lambda), t = 1,\cdots,T, i = 0,1$ are then found by summation over w. Due to the presence of all the zero entries in the transition matrix, it is possible to simplify the formulas considerably.

We shall now describe how the EM-algorithm can be implemented for the models for change in regime we have been considering.

The likelihood of the total sets of variables, both observed and latent, conditioned on the initial observations $y_{-k+1},\cdots,y_0$, can, using that $\{S_t\}$ is independent of the shocks, be written

$$\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^T exp(-\frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t-\mu_{s_t}-\phi_1 y_{t-1}-\cdots-\phi_k y_{t-k}-\delta' D_t)^2) \tag{3}$$

on the set $\{S_1 = s_1,\ldots,S_T = s_T\}$.

Then, if the likelihood is denoted by $f(y,s;\lambda)$ the EM-algorithm consists of computing recursively

$$Q(\lambda_{n+1},\lambda_n) = \arg\max Q(\lambda,\lambda_n) \tag{4}$$

where

$$Q(\lambda,\lambda_n) = E_{\lambda_n}[\log(f(\mathcal{Y},S;\lambda))|\mathcal{Y}] \tag{5}$$

with $S = (S_1,\cdots,S_T)'$ and $\mathcal{Y} = \mathcal{Y}_T$.

Noticing that $f(y, s; \lambda) = f(y|s; \lambda)f(s; \lambda) = f(y|s; \theta)f(s; p)$ the part of Q that depends on θ can be expressed as

$$-(T/2)\log 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \mu - \phi_1 y_{t-1} - \cdots - \phi_k y_{t-k} - \delta' D_t)^2)P(S_t = 0|\mathcal{Y}; \lambda_n)$$

$$-\frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \mu - \Delta - \phi_1 y_{t-1} - \cdots - \phi_k y_{t-k} - \delta' D_t)^2)P(S_t = 1|\mathcal{Y}; \lambda_n). \tag{6}$$

Thus, when the estimation is based on the EM-algorithm, it is crucial to obtain the smoothed probabilities, $P(S_t = i|\mathcal{Y}; \lambda), t = 1, \cdots, T, i = 0, 1$. These can be obtained from formulas 22.4.14 in Hamilton (1994b) in the same way as described above for the probabilities $P(S_t = i|\mathcal{Y}_{t-1}; \lambda)$.

The updated values of the first part of λ, i.e. the parameters contained in θ, are determined directly by maximizing the expression (6) with respect to these parameters. This is straightforward, since the first order conditions found by differentiating (6) can be explicitly solved.

The parameters of the process S_t is contained in p. In the situations we shall consider $p' = (p_0, p_1)$, where p_0 and p_1 are the parameters of the densities f_0 and f_1. To find the updated values for p_0 and p_1 note that there is a correspondence between the elements of the set $\mathcal{S} = \{(s_1, \ldots, s_T) : s_i \in \{0, 1\}\}$ and the elements of the set $\mathcal{R} = \{\{r_{jwt}\} : w \in \{1, \ldots, K_j\}, j = 0, 1, t = 1, \ldots, T\}$, where r_{iwt} denotes a sojourn in state i of length w ending at time t. Thus

$$\begin{aligned} Q(\lambda, \lambda_n) &= E_{\lambda_n}[\log(f(\mathcal{Y}, S; \lambda))|\mathcal{Y}] \\ &= E_{\lambda_n}[\log(f(\mathcal{Y}, R; \lambda))|\mathcal{Y}] \\ &= E_{\lambda_n}[\log(f(\mathcal{Y}|R; \theta)) + \log(f(R; p)|\mathcal{Y}]. \end{aligned} \tag{7}$$

Only the last term depends on p_0 and p_1, so that only this term needs to be maximized to find updated versions of p_0 and p_1. The maximization can be explained as done by Hamilton (1990,p.50), as a weighting of each of the first order conditions for the logarithm of the probability of the possible events in $\mathcal{R}$ where the weights are the conditional probabilities, given the observations, for each of these events to take place.

Then $f(R; p)$ may be written on the set $\{R = r_{iwt}\}$

$$\begin{aligned} P(V_i = w; p_i) = f_i(w; p_i) &= \frac{P(V_i > 1; p_i)}{P(V_i \geq 1; p_i)} \cdots \frac{P(V_i = w; p_i)}{P(V_i \geq w; p_i)} \\ &= (1 - h_{1,i}) \cdots (1 - h_{w-1,i})h_{w,i} \\ &\quad \text{if } w < t, t = 1, \ldots, T, \end{aligned}$$

and denoting the initial distribution with $\mu_{j,i,\lambda}, j = 1, \ldots, K_i, i = 0, 1$,

$$\begin{aligned}
& \mu_{w-t+1,i,\lambda} \frac{P(V_i > w-t+1; p_i)}{P(V_i \geq w-t+1; p_i)} \cdots \frac{P(V_i = w; p_i)}{P(V_i \geq w; p_i)} \\
= & \mu_{w-t+1,i,\lambda} \frac{P(V_i = w; p_i)}{P(V_i \geq w-t+1; p_i)} \\
= & f_i(w; p_i) \mu_{w-t+1,i,\lambda} / P(V_i \geq w-t+1; p_i) \\
& \text{if } w \geq t, t = 1, \ldots, T.
\end{aligned}$$

The last term in (7) may be found by summing these terms with weights $P(R = r_{iwt}|\mathcal{Y}; \lambda_n)$.

From the last equation we see that the case where $w \geq t$ can cause some problems, since both $f_i(w; p_i)$, $\mu_{w-t+1,i,\lambda}$ and $P(V_i \geq w-t+1; p_i)$ enter. The two last terms can be complicated functions of the parameters. In the empirical applications below we therefore suppress the dependence on these two terms by assuming that $\mu_{1,i,\lambda} = 1$ for either $i = 0$ or 1. Hence $W_t \leq t$ and the two terms are independent of $p_i, i = 0, 1$. This means that the alternating renewal process always starts with a complete run.

It is also possible to introduce additional independent parameters for the quantities $\mu_{w-t,i,\lambda}/P(V_i \geq w-t+1; p_i)$ if $w - t = 1, \ldots, K_i$ along the lines suggested by Hamilton (1990). When K_0 or K_1 are large, this may be problematic, however, since a lot of new parameters will be introduced.

In either case the first order conditions for maximizing (7) with respect to p_i can now be expressed as a weighted sum of the first derivatives of $f_i(w; p_i)$.

In the updating of the parameters both the quantities $P(S_t = i|\mathcal{Y}; \lambda_n), t = 1, \ldots, T$ and $P(R_{iwt} = r_{itw}|\mathcal{Y}; \lambda_n), t = 1, \ldots, T + w, w = 1, \ldots, K_i, i = 0, 1$ enter. We have already discussed how to compute the former. To compute the latter a modification is necessary, see e.g. Storvik, Bølviken and Solberg (1994). The event that $S_t = i, W_t = w$, i.e. that the latent process has been in state i for w periods, corresponds to the union of the event that it changes to the other state next period, which is just $R = r_{itw}$ and the event that the process remains in state i at time $t + 1$, which is the event $S_{t+1} = i, W_{t+1} = w + 1$. Hence

$$\begin{aligned}
P(R = r_{itw}|\mathcal{Y}; \lambda_n) &= P(S_t = s_i, W_t = w|\mathcal{Y}; \lambda_n) \\
&- P(S_{t+1} = s_i, W_{t+1} = w + 1|\mathcal{Y}; \lambda_n), \\
& \quad t = 1, \ldots, T-1, w = 1, \ldots, K_i, i = 0, 1.
\end{aligned}$$

In case a stay starts before T and ends after we have to use

$$P(R = r_{itw}|\mathcal{Y}; \lambda_n) = P(S_T = i, W_T = w - t + T|\mathcal{Y}; \lambda_n) \times$$

$$(1-h_{w-t+T,i})\cdots(1-h_{w-1,i})h_{w,i},$$
$$t=T,\ldots,T+w-1, w=1,\ldots,K_i, i=0,1.$$

As is well-known in this kind of models there is an identification problem. For example, the models where $(S_1, W_1) = (1,1), f_0 = bin(n_0, p_0)), f_1 = bin(n_1, p_1)$ and $\Delta > 0$ are observationally indistinguishable from models where $(S_1, W_1) = (0,1), f_0 = bin(n_1, p_1), f_1 = bin(n_0, p_0)$ and $\Delta < 0$. As pointed out by Hamilton (1989) this ambiguity can be solved by associating state 1 with high growth, which means that $\Delta > 0$.

If we estimate the models without imposing the restriction $\Delta > 0$, an estimated value of Δ which is negative means that we have to reinterpret the model using a correspondence as indicated in the previous paragraph.

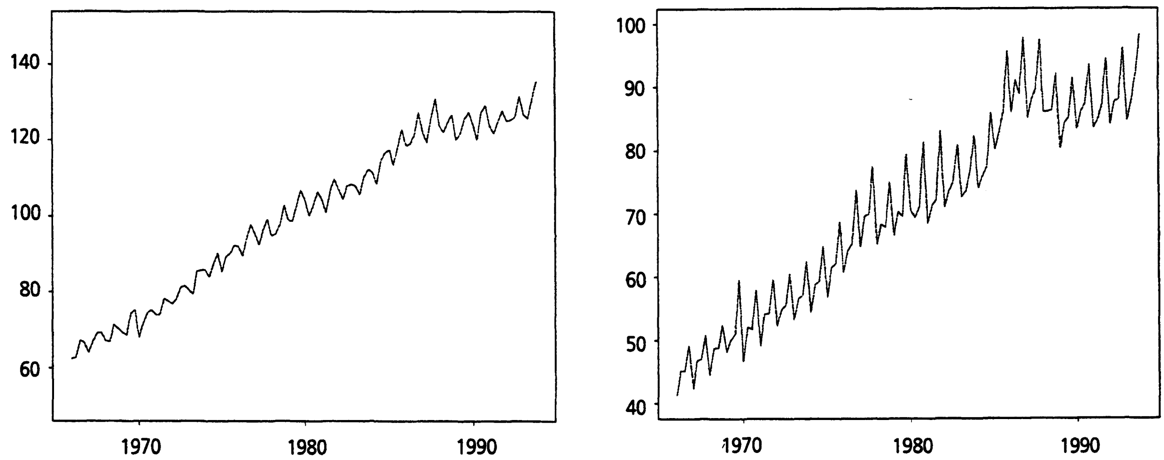
A further consequence is that if $\hat{\Delta}$ is positive, the maximal value of the likelihood will be greater or equal to what can be obtained by imposing the restriction $\Delta > 0$ and estimate the model with the alternative initial state. Hence, estimating the unrestricted model with initial value say $(0,1)$, and getting a positive $\hat{\Delta}$, yields a no smaller value of the likelihood than what can be obtained by using initial state $(1,0)$ and estimating with the constraint $\Delta > 0$ imposed. This argument will work when f_0 and f_1 belong to the same class of distributions. If they do not, both possible combinations must be estimated, and we can be certain to have obtained the largest possible likelihood in the restricted models, i.e. where $\Delta > 0$, only for the combination resulting in the largest maximum likelihood.

3 Application to two Norwegian macroeconomic time series

We shall in this section report on some applications of the methods described above and compare various specifications of the alternating renewal processes. The series used in the following estimation are both taken from the Norwegian quarterly national accounts. They are mainland GDP (i.e excluding the oil sector) (1966:1-1993:4) and total private consumption (1966:1-1993:4). The series are displayed in Figure 1.

For each series, models of the type (1) are fitted to the first difference of the logarithm of the observations. This means that the two states of the process $\{S_t\}$ will correspond to different rates of growth of the original series. In all the models centered, quarterly dummies have been used as regressors, and in addition separate sets have been used before and after 1978:1, at which point there is a break in the seasonal pattern. The truncation points for the distributions, i.e. K_0 and K_1, have been set equal to 100, which should allow for reasonable approximation to the pure geometric and the Poisson distributions for suitable values of the parameters.

Figure 1. The mainland GDP and total private consumption in billions Norwegian crowns



Several types of models have been considered where the time of sojourn is truncated geometrically and/or binomially distributed. We denote the combinations geo/geo, geo/bin etc. where the first distribution refers to state 0. In all cases reported the initial values were $(S_1, W_1) = (1, 1)$ and the estimated value of Δ positive.

The frequencies are therefore given by

$$P(V_i = w; p_i) = \begin{cases} (1-p_i)p_i^{w-1}, & w = 1, \dots, K_i - 1 \\ p_i^{K_i-1}, & w = K_i \end{cases}$$

and

$$P(V_i = w; p_i) = \begin{pmatrix} K_i - 1 \\ w - 1 \end{pmatrix} p_i^{w-1}(1-p_i)^{K_i-w}, w = 1, \dots, K_i.$$

The estimates reported below were computed by implementing the EM- algorithm of the previous section in GAUSS. We have checked the results using an unconstrained numerical optimizing procedure. As starting values we used the OLS estimates from a pure autoregressive process, and a grid of the other parameters. In some cases the global maximum corresponds to values on the boundary of the parameter set. Since these values do not make sense in a business cycle context, we excluded such cases and used the values in the interior of the parameter space corresponding to the local maximum having the largest value of the likelihood.

The estimated standard deviations are computed by the method in Hamilton (1996). We have compared the standard deviations computed with this method with the inverse elements of the Hessian from the numerical optimization of the likelihood. These values tend to be somewhat smaller, but the general impressions from the two types of estimates of the standard deviations are the same.

3.1 Mainland GDP

Since the data are quarterly, a reasonable starting point is a forth order autoregressive model with seasonal dummies. Deleting the dummies results in a decrease in twice the logarithm of the maximal value of the likelihood of 2x(289.84-268.00) = 43.68. Compared to a χ^2 distributed value with six degrees of freedom, this is clearly significant at all reasonable significance levels. This is compatible with the fact that most of the t-values for the seasonal dummies are larger than two in absolute value. Concerning the autoregressive part the coefficients before the third and fourth lag are numerically small, and also statistically insignificant judged by the t-values. Hence it is not surprising that there is almost no decrease in the likelihood, 289.91-289.84=0.07, when they are deleted. It therefore seems that an AR(2) model with seasonal dummies is appropriate in this case. Considering the other models, geo/geo etc. yields the same conclusion. The results from fitting models with two autoregressive lags and including seasonal dummies are, with the exception of the coefficients of the dummies, reported in Table 1.

The estimates of the seasonal effects and the coefficients of the autoregressive lags are practically the same for the four switching models.

To evaluate the significance of the estimates of parameters p_0,p_1 and Δ with a likelihood-ratio test is well known to be problematic in cases like this, since even if the pure autoregressive model is nested in the models for change in regime, the parameters are present only under the alternative. Hence the usual assumptions for likelihood ratio testing are invalid. A more informal judgment reveals that there is some improvement in the fit as judged by the increase in the likelihood, and the decrease in the standard deviation of the errors, σ. This is most pronounced in the bin/geo model. Comparing the t-values of the estimates, the t-values of the estimate of Δ is larger than two in all the models. For the parameters p_0 and p_1, it is only in the geo/bin and bin/bin models, that the t-values, using a 5% level, indicate that one of them belongs to the interior of the parameter set.

To compare the distributions it may be more relevant to consider the estimated means, which are 2.5 and 10.4 (quarters) in the geo/geo model, 9.3 and 24.9 in the geo/bin model, 19.4 and 55.0 in the bin/geo model and 18.6 and 82.7 in the bin/bin model.

The conditional probabilities for $S_t = 1, t = 1, \dots, T$ are presented in Figure 2. The estimates for the upswings in the beginning in the 80's and 90's for the geo/geo and

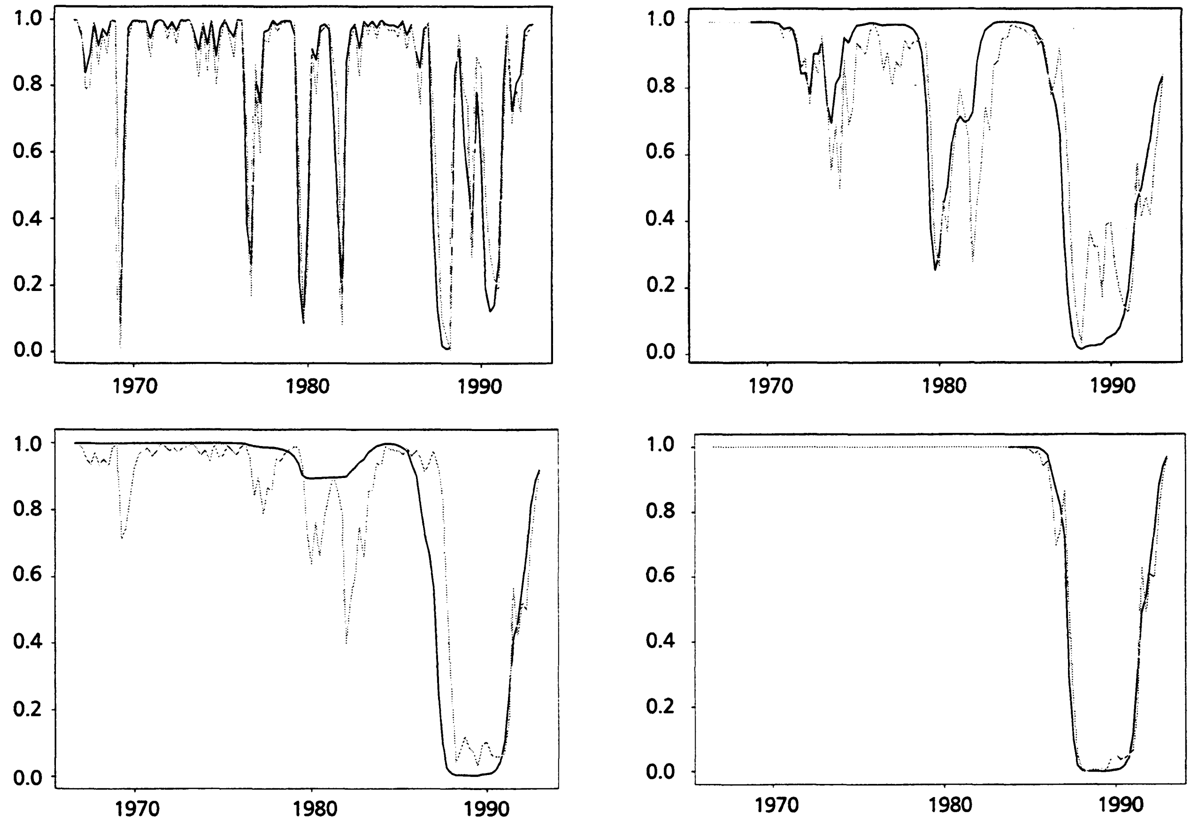
	geo/geo	geo/bin	bin/geo	bin/bin	AR(2)
log L	292.28	291.63	292.26	295.21	289.84
Δ	0.024	0.013	0.013	0.015	-
	(0.005)	(0.006)	(0.005)	(0.006)	-
μ	-0.006	0.003	0.002	0.010	0.012
	(0.005)	(0.006)	(0.005)	(0.005)	(0.002)
ϕ_1	-0.757	-0.691	-0.690	-0.694	-0.619
	(0.109)	(0.123)	(0.116)	(0.109)	(0.093)
ϕ_2	-0.338	-0.327	-0.328	-0.332	-0.259
	(0.089)	(0.104)	(0.099)	(0.099)	(0.093)
σ	0.013	0.015	0.015	0.015	0.016
	(0.002)	(0.001)	(0.001)	(0.001)	-
p_0	0.598	0.892	0.186	0.178	-
	(0.219)	(0.115)	(0.106)	(0.082)	-
p_1	0.904	0.242	0.987	0.825	-
	(0.083)	(0.067)	(0.020)	(0.073)	-

Table 1: Estimates of the parameters in models for mainland GDP. Estimated standard deviations in parentheses.

geo/bin models agree with the general conceptions of the Norwegian business cycle. The bin/geo and especially the bin/bin model are more peculiar, as is also evidenced in the estimated means reported previously.

As one can expect, using a geometric distribution instead of a binomial for the length of stay in state 0 and 1, favors short sojourns in each state. This is reflected in the smoother appearance of the conditional probabilities as functions of time when the binomial distribution is introduced.

Figure 2. The mainland GDP and the conditional probabilities for being in recovery. Solid lines indicate the conditional probabilities given all the observations, and dotted lines indicate the conditional probabilities given all the observations up to time t. The geo/geo model is in the upper left-, the geo/bin in the upper right-, the bin/geo in the lower left and the bin/bin in the lower right-hand corner



3.2 Total private consumption

In this case it is sufficient to include the first autoregressive lag. The estimates of the parameters, except the coefficients of the seasonal dummies are shown in Table 2.

The highest value of the likelihood is now for the bin/bin model. For this model judged by the significance of the t-values of the coefficients, we can conclude that Δ is different from zero at the 5% level. This is less evident in the geo/bin and bin/geo model and not at all the case in the geo/geo model. Concerning the parameters p_0 and p_1 it is only in the bin/geo and bin/bin models, that we can conclude that both belong to the interior of the parameter space by comparing the significance of the t-values at the 5% level. This is consistent with the fact that the largest increase in the log likelihood compared to the pure AR(1) model is found for this model. In this case the estimated means for the distributions f_0 and f_1 are 19.1 and 4.8 in the geo/geo-, 29.6 and 10.2 in the geo/bin-, 30.0 and 5.8 in the bin/geo- and 25.9 and 9.3 in the bin/bin model.

The conditional probabilities are displayed in Figure 3. Also in this case it appears that the conditional probabilities as functions of time, are smoother for the models using binomial distributions. The plots for the different models are more similar than for the GDP series.

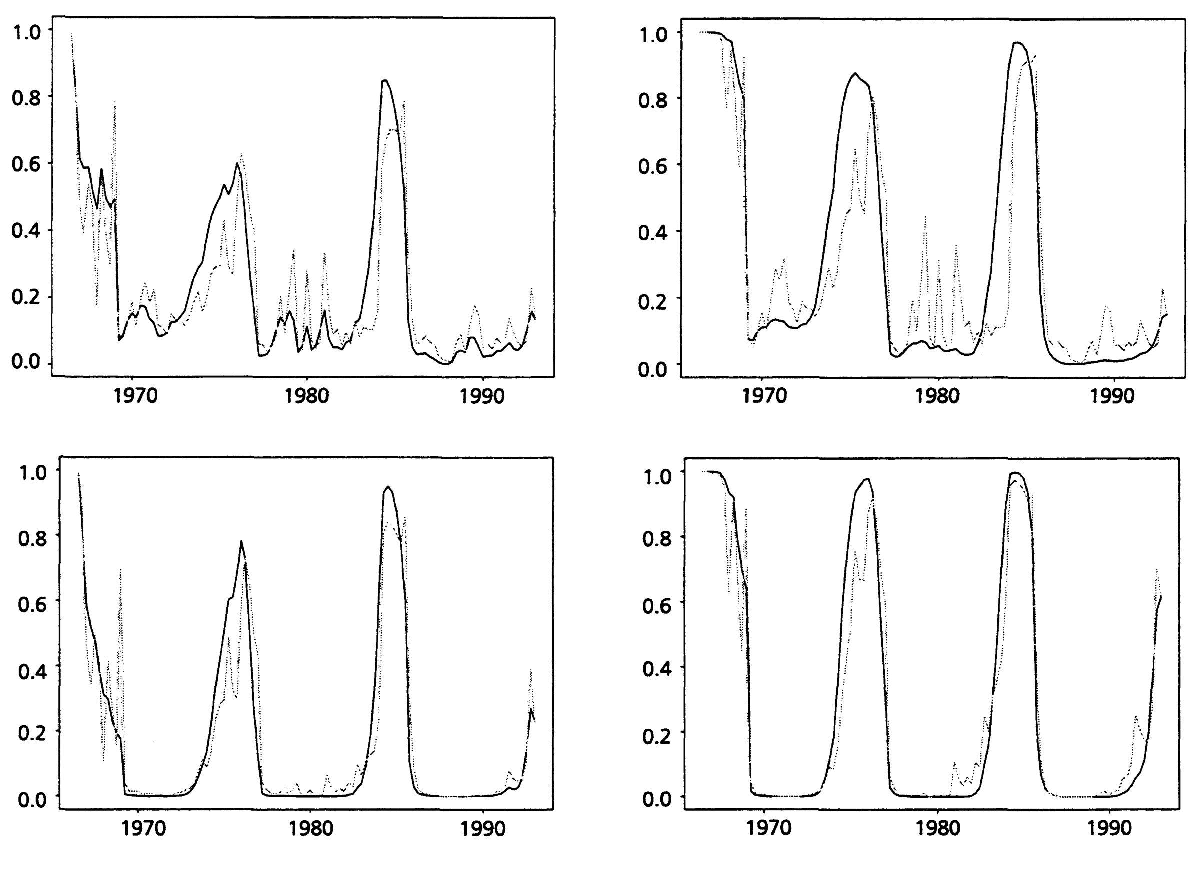
	geo/geo	geo/bin	bin/geo	bin/bin	AR(1)
log L	260.82	261.92	262.55	264.28	260.05
Δ	0.016	0.017	0.019	0.018	-
	(0.015)	(0.008)	(0.011)	(0.007)	-
μ	0.006	0.004	0.006	0.005	0.009
	(0.007)	(0.005)	(0.003)	(0.003)	(0.002)
ϕ_1	-0.461	-0.482	-0.474	-0.492	-0.419
	(0.126)	(0.101)	(0.109)	(0.095)	(0.088)
σ	0.020	0.020	0.020	0.020	0.021
	(0.002)	(0.002)	(0.002)	(0.002)	-
p_0	0.948	0.968	0.293	0.251	-
	(0.156)	0.045	(0.143)	(0.068)	-
p_1	0.793	0.093	0.827	0.084	-
	(0.317)	0.064	(0.132)	(0.039)	-

Table 2: Estimates of the parameters in models for total private consumption. Estimated standard deviations in parentheses.

4 Conclusion

In this paper we have pointed out a problem in interpreting the usual models for change in regime based on a hidden Markov model when it is applied to the analysis of business cycles. There is a bias towards short cycles. We point out how the problem can be circumvented by introducing using a finite Markov chain having transition matrices satisfying certain restrictions. Some of the proposed models are compared using time series from the Norwegian quarterly national accounts. For one of the series, the total private consumption, it seems that using one or more binomial distributions yields a better fit, and one may even question the appropriateness of a model based on a two-stage Markov chain for the process S_t indicating the prevailing regime. The conditional probabilities of being in a particular state show, as predicted, a more erratic behavior for the models based on a pure two-stage Markov chain than the models that use one or two binomial distributions to describe the lengths the latent state process is in each state. The sensitivity to the choice of latent model seems to be somewhat different. For the GDP series the results seem to be most variable, while for the the total private consumption, all four fitted models yield more similar results.

Figure 3. Total private consumption and the conditional probabilities for being in the high growth state. Solid lines indicate the conditional probabilities given all the observations, and dotted lines indicate the conditional probabilities given all the observation up to time$t$. The geo/geo model is in the upper left-, the geo/bin in the upper right-, the bin/beo in the lower left- and the bin/bin in the lower right-hand corner



References

[1] Bickel, P.J. and Ritov, Y.(1996). Inference in hidden Markov models I: Local asymptotic normality in the stationary case. *Bernoulli* **2**, 199-228.

[2] Cox, D.R. and Isham, V.(1980) *Point Processes.* London: Chapman and Hall.

[3] Diebold, F.X. and Rudebusch, G.D.(1990). A nonparametric investigation of duration dependency in the American business cycle. *Journal of Political Economy* **98**, 596-616.

[4] Diebold, F.X. and Rudebusch, G.D.(1996). Measuring business cycles: A modern perspective. *Review of Economics and Statistics* **78**, 67-77.

[5] Diebold, F.X., Lee, J.H. and Weinbach, G.C.(1994). Regime switching with time-varying transition probabilities. In *Nonstationary Time-Series Analysis and Cointegration*, ed. C. Hargreaves, Oxford, U.K.:Oxford University Press.

[6] Durland, J.M. and McCurdy, T.H.(1994). Duration-dependent transitions in a Markov Model of U.S. GNP growth. *Journal of Business & Economic Statistics* **12**, 279-288.

[7] Filardo, A.J.(1994). Business-cycle phases and their transitional dynamics. *Journal of Business & Economic Statistics* **12**, 299-308.

[8] Hamilton, J.D.(1989). A new approach to the economic analysis of non-stationary time series and the business cycle. *Econometrica* **57**, 357-384.

[9] Hamilton, J.D.(1990). Analysis of time series subject to changes in regime. *Journal of Econometrics* **45**, 39-70.

[10] Hamilton, J.D.(1993). Estimation, inference and forecasting of time series subject to change in regime. In *Handbook of Statistics, Vol 11*,eds. G.S. Maddala, C.R. Rao and H.D. Vinod, Amsterdam: Elsevier Science Publishers.

[11] Hamilton, J.D.(1994a). State-Space Models. In *Handbook of Econometrics, Vol 4*, eds. R.F. Engle and D. McFadden, Amsterdam: Elsevier Science Publishers.

[12] Hamilton, J.D.(1994b). *Time Series Analysis.* Princeton, N.J.: Princeton University Press.

[13] Hamilton, J.D.(1996). Specification testing in Markov-switching time-series models *Journal of Econometrics* **70** , 127-157.

[14] Hansen, B.E.(1992). The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP. *Journal of Applied Econometrics* **7**, S61-S82.

[15] Hansen, B.E.(1992). Erratum: The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP. *Journal of Applied Econometrics* **11**, 195-198.

[16] Holst, U., Lindgren, G., Holst, J. and Thuvesholmen, M.(1994). Recursive Estimation in switching autoregressions with a Markov regime. *Journal of Time Series Analysis* **15**, 489-506.

[17] MacDonald, I.L. and Zucchini, W.(1997). *Hidden Markov and Other Models for Discrete-valued Time Series.* London: Chapman and Hall.

[18] Rabiner, L.E.(1989). A tutorial on hidden Markov Models and selected applications in speech recognition. *Proc. IEEE* **337**, 407-428.

[19] Rydén, T.(1994). Consistent and asymptotically normal parameter estimates for hidden Markov models. *Annals of Statistics.* **22**, 1884-1895.

[20] Sichel, D.E.(1991). Business cycle duration dependence: A parametric approach. *Review of Economics and Statistics* **73**, 254-260.

[21] Sichel, D.E.(1994). Inventories and the three phases of the business cycle. *Journal of Business & Economic Statistics* **12** , 269-277.

[22] Storvik,G., Bølviken, E. and Solberg, A.S.(1994). Hidden Markov chains for recognition of structures in time series. Report no. 880, Norwegian Computing Center.

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