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Aggregation when Markets do not Clear

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Abstract:
This paper presents a method for aggregation across markets in a Non-Walrasian model, focusing mainly on labor markets. The method utilizes a probabilistic approach based on aggregating across virtual price functions instead of demand functions or budget shares as is normally done. By assuming log-linear virtual price functions and using the GEV distribution, it is possible to identify most of the micro structure of an economy in disequilibrium from observed aggregate variables. The paper discusses different possible indicators of disequilibrium in the labor market and presents some illustrative estimation results.

Keywords: Aggregation, disequilibrium, non-Walrasian models.

JEL classification: E1, C5, D5.

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1 Introduction

This paper presents an empirical method for aggregating micro markets in disequilibrium. Aggregation is done using a probabilistic (stochastic) approach based on the virtual price formulation of a non-Walrasian model presented in Andreassen (1995). This approach can be viewed both as an extension of the virtual price approach of Lee (1986) and of the smoothing by aggregation approach first suggested by Muellbauer (1978). The use of virtual prices makes it possible to take into account that some agents may be rationed in some markets or that their optimization may lead to corner solutions. Such considerations are especially important in the labor market.

The present aggregation approach is based on many restrictive assumptions, and should therefore be viewed as a tentative step towards understanding how different distributions of technology and tastes (and thereby of rationing) influence aggregate relationships. An important aspect of the paper is that it discusses the empirical interpretation of the probability of a match in a market and looks at the general restrictions inherent in such an approach. The main part of the paper gives conditions under which it is possible to estimate the structural parameters in a multi-sectoral macroeconomic model from aggregate data when there is rationing in the labor market. In addition the aggregation approach used leads to different indicators of disequilbrium. The results of the paper are illustrated through some rudimentary estimation results leading to the preliminary conclusion that the model is misspecified due to its static nature and that future research should therefore concentrate on extending the model to a dynamic setting.

There is a large literature on the estimation of econometric disequilibrium models, starting with the seminal works of Fair and Jaffee (1972) and Rosen and Quandt (1978) which examined single markets under fixed prices. Estimation methods for two- market models, such as the theoretical neo-Keynesian models first developed by Barro and Grossman (1971) and Malinvaud (1977), were developed by among others Ito (1980). Recent work within the neo-Keynesian econometric framework take as a starting point micro markets and use the so-called *smoothing by aggregation* approach. This method has been utilized in among others Burkett (1988), Lambert (1988) and Drèze and Bean (1990). Micro markets are taken to be small efficient markets where it seems reasonable to postulate that only one side of the market can be rationed at one time. This assumption is commonly referred to as the *min* condition and implies that all advantageous trades are carried out in the market. The smoothing by aggregation approach assumes that supply and demand in the micro markets can be modeled as consisting of a structural part and a stochastic residual. Assuming that the residuals are distributed in the same manner in all markets, aggregation to the macro level is achieved by integrating over micro markets. At any one time some micro markets will be in excess demand while others will be in excess supply, so that at the macro level both sides of the market may be partially rationed at the same time. In a neo-Keynesian macro model this means that there can be Keynesian unemployment at the same time as there is classical unemployment. The smoothing by aggregation approach starts with micro demands and supplies and is not based on explicit utility and profit maximization.

A survey of theoretical and econometric modeling of disequilibrium is given in Andreassen (1993). A recent review of the main advances made in the applied disequilibrium litterature is given in Laroque and Salanie (1993). They especially discuss the problems connected with aggregation and price dynamics.

Lee (1986) suggests an alternative method for estimating econometric models with many markets based on using virtual prices to describe disequilibrium. By using the notion of virtual prices he overcomes earlier difficulties in finding a computationally tractable method for estimating models with more than two markets. Lee's method is based on representative agents without explicitly considering aggregation. Andreassen (1995) extends Lee's approach by taking into account that there can be many agents, including government production, an open economy and by explicitly discussing the impact corner solutions have on estimation. Despite the use of simple specifications, it is apparent from this paper that when there are a large number of markets, the computational burden of estimation becomes heavy due to the large number of possible rationing regimes. It therefore seems necessary in multi-market econometric work to either work within a representative agent framework or, as in this paper, to develop methods for aggregating across micro markets.

The aggregation method presented in the present paper is based on specifying the distribution of virtual prices across the population. Assuming, as in Lee (1986), that they are log-linear and, as in Andreassen (1995), that they are extreme value distributed, it is possible to use aggregate time series data to estimate and analyze structural relationships in the labor market in the presence of rationing and corner solutions. The exogenous wage distribution is also assumed to be extreme value distributed. The aggregation method discussed in this paper is explicitly based on utility and profit maximization through the use of virtual prices.

The economic model on which aggregation is based is the non- Walrasian model presented in Andreassen (1995). The model is only concerned with short term equilibria and considers investment, exports, and government behavior (tax rates and the budget constraint of the government firms) as exogenous, along with prices and wages. The main reason for considering these as exogenous is analytical tractability, but it can be argued that decisions regarding these variables cover a longer time period than decisions regarding consumption and production. Imports, the trade surplus, tax revenue, the public budget deficit, and changes in the money supply are endogenous. Production and consumption inputs of a good are transacted on the same product market at the same price. There is one non-produced commodity in the model which will be referred to as money. Money enters both the utility and production functions as a means to facilitate transactions and because it is the sole means of transferring liquidity over time (there are no financial markets in the model).

The model is based on the assumptions that prices and wages do not instantaneously clear markets (though they may be flexible over time) and that the economy at any time is in a Drèze equilibrium. In a Drèze equilibrium all transactions are the result of utility and profit maximization subject to all quantity constraints that exist. In addition only one side of each market can be rationed at a given time.

An important assumption is that each combination of firm and individual is considered a separate *micro labor market*, implying that the number of micro labor markets in the model is equal to the number of consumers times the number of firms. This assumption plays a vital role in the aggregation procedure because it implies that there will be only one seller and one buyer in each micro labor market. In principle it is possible for a firm to use all types of labor and all types of commodities as inputs and it is possible for an individual to be employed in all the firms in the economy and to consume all types of commodities. Such a large and general opportunity set both for the firm and especially for the individual will naturally lead to a large number of corner solutions which it is necessary to take into consideration. The assumption that each combination of firm and worker is a separate labor market can be seen as an extreme way of modeling the heterogeneity of jobs and labor. It does not seem plausible to make a similar assumption for goods. Firms are for example rarely interested in the characteristics of their customers. This is the main reason why we in the following concentrate on aggregating across labor markets.

Government firms are included in the model to explicitly take into account that a significant share of employment in many economies takes place in the public sector. Another reason to include government production is that it constitutes a significant portion of the output in many economies and is often subject to severe rationing (for example health care). Government production is assumed to be used only by consumers, consists only of individual products (we assume there are no public goods), is not an input to other production, is not an investment good, and is never exported. The government sector may buy investment goods from private firms and from abroad. It is assumed that the government levies two types of taxes, one on labor income and one on commodities (production and consumption inputs) and hands out lump-sum subsidies. There is no tax on investment goods or on exports.

The next section discusses some general issues which arise in the aggregation approach used in this paper. For this approach to be empirically useful it is necessary to interpret empirically the probability that a certain individual works in a particular firms, the probability that the individual wishes to work in this firm, and the probability that the firm wishes to hire this individual. An important conclusion drawn from this discussion is that seemingly innocuous assumptions lead to fairly strong restrictions which apply irrespective of how the details of the model are formulated.

Section 3 discusses the virtual prices on which the aggregation method is based under assumptions that imply log-linear virtual prices. These virtual prices are those that arise in the model discussed in more detail in Andreassen (1995). The parameters in these relationships can be interpreted as structural parameters in the agents' utility and production functions.

Section 4 describes how unobserved heterogeneity is assumed to be distributed across individuals and firms. Section 5 derives the labor force probabilities implied by the model and functional forms discussed in section 3 and the distributional assumptions made in section 4. It also presents aggregation results for the goods markets and discusses estimation based on the derived aggregate relationships. Furthermore, in this section different indicators of disequilibrium in the labor market are discussed and the papers aggregation approach is compared to the smoothing by aggregation method.

Finally, in section 6 som empirical illustrations are presented for a model with one type of individual and four types of firms. The estimation is based on fairly short time series covering 16 years and some of the data are fairly poor (for example data on money demand). For this reason the empirical results presented are mainly of an illustrative nature.

2 Empirical interpretation of labor market probabilities

Before discussing the specific assumptions and specifications employed in this paper, it is important to understand some general restrictions inherent in the probabilistic approach used. Central to this approach is the probability, as seen by the econometrician, that there is a match in a micro market, $\mathrm{P}(l_{ik}^* > 0)$, where l_{ik}^* is the transacted amount of labor supplied by individual i to firm k. If there is such a match we must have that the *min* condition is satisfied,

$$l_{ik}^* > 0 \quad = \quad \min(l_{ik}^S, l_{ik}^D), \tag{2.1}$$

where l_{ik}^S and l_{ik}^D are respectively the Clower supply and the Clower demand for labor in the micro market consisting of consumer i and firm k. *Clower demands (supplies)* were first suggested in Clower (1965). They are the demand (supply) for a type of labor which arises when maximizing utility subject to the budget constraint and all quantity constraints except the quantity constraint which applies to the market in question. This in constrast to *Drèze demands (supplies)* which are the result of maximzation subject to all the constraints which apply. Since the Drèze demands take all constraints into account they will be equal to the observed transactions and will not signal any rationing. As long as we assume that the *min* condition $l_{ik}^* = \min(l_{ik}^S, l_{ik}^D)$ applies, the use of Clower demands (supplies) to signal rationing will not contradict our assumption of a Drèze equilibrium.

In the present context a match does not have any search theoretical content (though it is easy to agree that ideally it should), but only implies that $\min(l_{ik}^S, l_{ik}^D) > 0$. The probability $\mathrm{P}(l_{ik}^* > 0)$ therefore does not reflect uncertainty on the part of the agents, but only describes the econometrician's knowledge (or lack of knowledge) about the behavior at the micro level.

The probability $\mathrm{P}(l_{ik}^* > 0)$ is not readily observable, but under certain assumptions can be indirectly observed from observations of the number of persons employed, Q. One of these assumptions is that Q can be given the probabilistic interpretation

$$Q = N \cdot \mathrm{P}\left(\sum_{k=1}^{M} l_{ik}^{*} > 0\right), \tag{2.2}$$

where N is the total number of potential employed (which can for example be assumed to be all individuals between the ages of 16 and 74) and M is the total number of potential firms. Equation (2.2) says that total number of employed persons is equal to the number of potential employed times the probability that any one individual is employed. The employment rate Q/N can in the same manner be interpreted as the probability of an individual working in at least one firm,

$$\begin{aligned} \frac{Q}{N} &= \mathrm{P}(\sum_{k=1}^{M} l_{ik}^{*} > 0) \\ &= 1 - (1 - \mathrm{P}(l_{ik}^{*} > 0))^{M}, \end{aligned} \tag{2.3}$$

where $(1 - \mathrm{P}(l_{ik}^{*} > 0))^{M}$ is the probability of individual i not working in any firm. The econometrician does not observe any differences among the different persons or firms (any heterogeneity is unobserved) and therefore takes the probability $\mathrm{P}(l_{ik}^{*} > 0)$ to be the same for all i and k. The expected (average) number of jobs held by an individual given that he is employed, Λ, can be written

$$\begin{aligned} \Lambda &= \sum_{k=1}^{M} \mathrm{P}\left(l_{ik}^{*} > 0 \mid \sum_{k=1}^{M} l_{ik}^{*} > 0\right) \\ &= \frac{M \cdot \mathrm{P}(l_{ik}^{*} > 0)}{\mathrm{P}(\sum_{k=1}^{M} l_{ik}^{*} > 0)} \\ &= \frac{MN}{Q} \mathrm{P}(l_{ik}^{*} > 0) \end{aligned} \tag{2.4}$$

where the last equalities follow from equation (2.2). Combining equations (2.3) and (2.4) and rearranging leads to

$$\log(1 - \frac{Q}{N}) = \frac{\Lambda \cdot Q}{N} \cdot \frac{\log(1 - \mathrm{P}(l_{ik}^{*} > 0))}{\mathrm{P}(l_{ik}^{*} > 0)}. \tag{2.5}$$

When there are many micro labor markets we can assume that $\mathrm{P}(l^*_{ik} > 0)$ is small[1] and therefore that

$$\frac{\log(1 - \mathrm{P}(l^*_{ik} > 0))}{\mathrm{P}(l^*_{ik} > 0)} \approx -1 \tag{2.6}$$

when log denotes the natural logarithm. Setting this expression equal to -1 implies that the average number of jobs held by an employed individual will be

$$\Lambda = -\frac{N}{Q}\log(1 - \frac{Q}{N}). \tag{2.7}$$

Assuming as an example that the employment rate is $Q/N = 0.685$ (as observed in Norway in 1991), we get that the average number of jobs held by an employed individual will be $\Lambda = 1.65$. From the Norwegian Survey of Living 1991 we have that over a five year period the average number of jobs held was 1.76. Equation (2.7) implies that the employment rate determines the average number of jobs held by the individuals so that an increase in the employment ratio Q/N will increase Λ. It is important to note that these results do not depend on how we specify our model later in the paper, though the above results do depend on $\mathrm{P}(l^*_{ik} > 0)$ being small. If the approximation in equation (2.6) holds, we have that equations (2.4) and (2.7) imply that

$$M \cdot \mathrm{P}(l^*_{ik} > 0) = -\log(1 - \frac{Q}{N}). \tag{2.8}$$

From equation (2.8) it is apparant that we cannot identify M and $\mathrm{P}(l^*_{ik} > 0)$ separately. The number of potential firms M is unobservable and at the same time constitutes the set over which the probability $\mathrm{P}(l^*_{ik} > 0)$ is defined. Giving the variable M an arbitrary value can therefore be viewed as a necessary normalization of the size of the potential economy. If we exogenously determine the value of M, we then can calculate $\mathrm{P}(l^*_{ik} > 0)$ as

$$\mathrm{P}(l^*_{ik} > 0) = -\frac{1}{M}\log(1 - \frac{Q}{N}), \tag{2.9}$$

[1] If $\mathrm{P}(l^*_{ik} > 0) = 0.1$ then $\frac{\log(1-\mathrm{P}(l^*_{ik}>0))}{\mathrm{P}(l^*_{ik}>0)} = -1.05$, while if $\mathrm{P}(l^*_{ik} > 0) = 0.9$ then $\frac{\log(1-\mathrm{P}(l^*_{ik}>0))}{\mathrm{P}(l^*_{ik}>0)} = -2.56$. In section 6 we find that $\mathrm{P}(l^*_{ik})$ is of a magnitude of $10^{-}5$ to $10^{-}6$.

and the number of firms actually observed, $\hat{M}$, will be

$$\begin{aligned}\hat{M} &= M \cdot \mathrm{P}(\sum_{i=1}^{N} l_{ik}^{*} > 0) \\ &= M \cdot (1 - (1 - \mathrm{P}(l_{ik}^{*} > 0))^{N}). \end{aligned} \tag{2.10}$$

There are two other important probabilities which are central to the aggregation approach discussed in the following. One is the probability that individual i wishes to work in firm k, $\mathrm{P}(l_{ik}^{S} > 0)$, and the other is the probability that firm k wishes to hire individual i, $\mathrm{P}(l_{ik}^{D} > 0)$. Assuming that a value for M has been specified, these probabilities can be found from the number of unemployed persons U and the number of vacancies V. It seems reasonable to interpret these two observations differently. To see this, let I_{ik} be a variable that is equal to 1 if firm k wishes to employ person i (implying that $l_{ik}^{D} > 0$) and 0 otherwise. The number of persons the firms wish to employ can then be written $\sum_{i=1}^{N} \sum_{k=1}^{M} I_{ik}$. A reasonable probabilistic interpretation of V will therefore be

$$\begin{aligned} V &= NM \cdot \mathrm{P}(I_{ik} = 1) - NM \cdot \mathrm{P}(l_{ik}^{*} > 0) \\ &= M \cdot \left[N \cdot \mathrm{P}(l_{ik}^{D} > 0) - N \cdot \mathrm{P}(l_{ik}^{*} > 0)\right]. \end{aligned} \tag{2.11}$$

In equation (2.11) we have that $N \cdot \mathrm{P}(l_{ik}^{D} > 0)$ is the expected number of employees firm k wishes to employ and $N \cdot \mathrm{P}(l_{ik}^{*} > 0)$ is the expected number the firm actually employs. Taking the difference and multiplying with the total number of potential firms gives us the number of vacancies.

The number of unemployed, U, must be interpreted differently, because in most labor force surveys individuals are only asked whether they desire a job or not. Letting J_i be a variable that is equal to 1 if individual i wishes to work at least in one firm (implying that $\sum_{k=1}^{M} l_{ik}^{S} > 0$) and 0 otherwise, the number of individuals who desire a job will be $\sum_{i=1}^{N} J_i$ and the number of unemployed will be $U = \sum_{i=1}^{N} J_i - Q$. A probabilistic interpretation of U is thereby

$$U = N \cdot [\mathrm{P}(\sum_{k=1}^{M} l_{ik}^{S} > 0) - \mathrm{P}(\sum_{k=1}^{M} l_{ik}^{*} > 0)]. \tag{2.12}$$

Equation (2.12) says that the total number of unemployed is equal to the number of individuals times the probability that a random individual wishes to work without finding employment.

In the same manner as for $\mathrm{P}(\sum_{k=1}^{M} l_{ik}^{*} > 0)$, we can write $\mathrm{P}(\sum_{k=1}^{M} l_{ik}^{S} > 0)$ as

$$\mathrm{P}(\sum_{k=1}^{M} l_{ik}^{S} > 0) \quad = \quad 1 - (1 - \mathrm{P}(l_{ik}^{S} > 0))^{M}. \tag{2.13}$$

For a given M the probability $\mathrm{P}(l_{ik}^{S} > 0)$ can be calculated by substituting from (2.3) and (2.13) into (2.12) leading to

$$\mathrm{P}(l_{ik}^{S} > 0) \quad = \quad 1 - \left(1 - \frac{Q+U}{N}\right)^{\frac{1}{M}}. \tag{2.14}$$

Substituting from equation (2.9) into (2.11) gives us the probability $\mathrm{P}(l_{ik}^{D} > 0)$,

$$\mathrm{P}(l_{ik}^{D} > 0) \quad = \quad \frac{V}{MN} - \frac{1}{M}\log(1 - \frac{Q}{N}). \tag{2.15}$$

It is important to note that equations (2.14) and (2.15) are slightly different in structure, due to the different interpretations of U and V.

In the following we will present an aggregation method based on the assumption that the probabilities $P(l_{ik}^{S} > 0)$, $P(l_{ik}^{D} > 0)$, and $P(l_{ik}^{*} > 0)$ can be observed for groups of individuals and firms as discussed above. The relationships derived in this section and the inherent restrictions of our approach will therefore apply to our model irrespective of our choices of functional forms or distributional assumptions.

3 Virtual prices

Aggregation is in the following based on the model presented in Andreassen (1995), which has four types of agents: Private firms competing in world markets, private firms sheltered from international competition, government firms, and consumers/workers. Consumers maximize utility, private firms maximize profits constrained by their revenues in the previous period while the government firms maximize profits given an exogenously (politically) set budget constraint. All agents are price takers in both input and output markets.

Stochastic aggregation assumes that unobserved heterogeneity among firms and individuals can be modeled by treating the unobservables as random variables with conventional distribution functions. It can be argued that such a parsimonious modeling of unobserved heterogeneity is more realistic for parts of the labor market than for the labor market as a whole. Different segments of the labor market are also of interest in themselves. The total labor market is therefore assumed to be divided into different *aggregate submarkets* characterized by combinations of type of consumer and type of firm. All consumers who are of certain type have similar utility functions and all firms which are of a certain type have similar production functions. It is at the level of the labor submarket that the paper attempts to derive aggregate expressions. In principle there is no limit to how many such submarkets on has, but the number of micro markets in each aggregate submarket must be fairly large, since we assume that unobserved heterogeneity within each submarket can be approximated by continuous probability distributions. The submarkets must also be such that there are reasonable aggregate data available. The similarities within each type can be thought of as being the result of earlier choices regarding education in the case of the consumer and choice of production capital for firms.

In the following we utilize a different indexation from that of Andreassen (1995) to take into account labor submarkets. Types of private firms are indexed $j = 1, \cdots, J$ where the number of firms of type j is M_j, while types of consumers are indexed $h = J+1, \cdots, J+H$ where the number of consumers of type h is N_h. Firms of type j are in addition indexed by $k = 1, \ldots, M_j$ and consumers of type h by $i = 1, \ldots, N_h$. Each combination of consumer type and type of firm, (h, j), is considered an aggregate labor submarket. There are $H \cdot J$ such aggregate submarkets, each consisting of $N_h \cdot M_j$ micro markets. We assume that firm types $1, \ldots, J_p$ are private firms while firm types $J_p + 1, \ldots, J$ are government firms.

In the following we assume that consumers and firms are never rationed in their demand for private goods and that the consumers are always rationed in their demand for government goods (firms do not use government goods). This can be considered a special case of the model in Andreassen (1995), where consumers and firms could be rationed in their demand for non-traded goods and where the government could be rationed in it's supply of goods. Private firms may be rationed in their supply of goods.

All private firms are assumed to be profit maximizers and price takers in both input and output markets. Each firm uses labor and output from other firms as inputs and produces one product. The private firms do not use government goods and services as inputs. The firms' investments, inv_{jk}, and capital stock, K°_{jk} are considered exogenous. Production that goes to investment or export is considered to be governed by long term contracts (longer than the period we look at) and is therefore also considered exogenous. We let y_{jk} be firm k of type j's production for consumption and for use as inputs in other firms and let Y_{jk} be this firm's production for investment and export. The price for the first type of production, $(1 - t_2)p_{jk}$, may be different from the price of the second. The commodity tax rate t_2 is the same for all goods.

If there are imports of a good not produced nationally, we assume that there is a hypothetical firm which could have produced the good but which finds such production unprofitable. We assume a asymmetry between imports and exports. Exports are governed by longer term contracts and are exogenous, while imports are residually determined and endogenous.

We assume that the length of production is one period, implying that for all inputs that are chosen in the current period, output and the resulting revenues will first accrue in the next period. This results in the private firm being constrained by a budget constraint similar to that of the consumer. This approach is similar to that taken in for example Böhm and Lévine (1979). It is a shortcoming of the model that money is the only means of transferring purchasing power over several periods. The firms cannot, for example, borrow to finance purchases of investment goods or other inputs. It is important in future research to include a more realistic modeling of financial markets and a dynamic modeling of the firms' behavior.

Let l_{hijk} be consumer i's supply of labor to firm k, when consumer i is of type h and firm k belongs to firms of type j. Consumers have preferences both over how many hours they work and where they work. This might reflect the different working conditions in the different firms or the location of the firm in relationship to the worker. The wage, w_{hijk}, varies both over individuals and firms within the aggregate labor submarket hj. The variable t_{1h} denotes the tax on wage income for individuals of type h.

Consumer i of type h's use of good k' of type j' is denoted $x_{j'k'hi}$, while firm k of type j's use of this good is denoted $x_{j'k'jk}$. Both consumers and firms face the same price for this good, $p_{j'k'}$. We let m_{chi} denote money holdings by consumer i of type h and m_{fjk} denote money holdings by firm k of type j. C°_{hi} denotes the initial resources held by consumer i of type h. It is equal to money holdings at the end of the previous period plus consumer i's share of profits from the preceding period plus lump-sum transfers from the government. C°_{jk} denotes the initial resources held by firm k of type j. It is equal to money holdings at the end of the previous period plus revenue in the previous period minus last periods profits (which are paid out to the consumers) plus lump-sum transfers from the government. For a more detailed description of the model, but without the indexation necessitated by the introduction of labor submarkets, see Andreassen (1995).

Using the above notation we have that the consumer's budget constraint is

$$C^{\circ}_{hi} = -\sum_{j=1}^{J}\sum_{k=1}^{M_j}(1-t_{1h})w_{hijk}l_{hijk} + \sum_{j=1}^{J}\sum_{k=1}^{M_j} p_{jk}x_{jkhi} + m_{chi}, \tag{3.1}$$

while the firm's constraint is

$$C^{\circ}_{jk} - inv_{jk} = \sum_{h=J+1}^{J+H}\sum_{i=1}^{N_h} w_{hijk}l_{hijk} + \sum_{j'=1}^{J_p}\sum_{\substack{k'=1 \\ j'k'\neq jk}}^{M_j} p_{j'k'}x_{j'k'jk} + m_{fjk}. \tag{3.2}$$

For notational convenience we let $\mathbf{x}_{hi}$ be a vector of the goods and $\mathbf{l}_{hi}$ be a vector of the labor supplied by consumer i of type h, where we have

$$\mathbf{x}_{hi} = [x_{11hi}, \cdots, x_{1M_1hi}, x_{21hi}, \cdots, \cdots, x_{JM_Jhi}]$$

and

$$\mathbf{l}_{hi} = [l_{hi11}, \cdots, l_{hi1M_1}, l_{hi21}, \cdots, \cdots, l_{hiJM_J}].$$

In the same manner we introduce corresponding notation for the vectors for firm k of type j given by

$$\mathbf{x}_{jk} = \left[x_{11jk}, \cdots, x_{jk-1\,jk}, , x_{jk+1\,jk}, \cdots, x_{J_pM_{J_p}jk}\right]$$

and

$$\mathbf{l}_{jk} = \left[l_{J+1\,1jk}, \cdots, l_{J+1\,N_{J+1}jk}, l_{J+2\,1jk}, \cdots, \cdots, l_{J+H\,N_{J+H}jk}\right].$$

An important assumption in the following is that the virtual prices are log-linear. This assumption is consistent with consumer hi having the utility function

$$\begin{aligned} \mathcal{U}_{hi}(m_{chi}, \mathbf{x}_{hi}, \mathbf{l}_{hi}) &= (m_{chi})^{1-\alpha_{1h}} - \sum_{j=1}^{J}\sum_{k=1}^{M_j} a^{*}_{hijk}(l_{hijk}+1)^{1+\alpha_{2hj}} \\ &\quad + \sum_{j=1}^{J}\sum_{k=1}^{M_j} c^{*}_{jkhi}(x_{jkhi}+1)^{1-\alpha_{3jh}} \end{aligned} \tag{3.3}$$

and firm jk having the production function

$$\begin{aligned}
F_{jk}(y_{jk}, m_{fjk}, \mathrm{x}_{jk}, \mathrm{l}_{jk}, K^{\circ}_{jk}) &= y_{jk} + Y_{jk} - (m_{fjk})^{1-\beta_{1j}} \\
&\quad - \sum_{h=J+1}^{J+H} \sum_{i=1}^{N_h} b^*_{hijk}(l_{hijk}+1)^{1-\beta_{2hj}}(K^{\circ}_{jk})^{1-\beta_{3hj}} \\
&\quad - \sum_{j'=1}^{J_p} \sum_{\substack{k'=1 \\ j'k' \neq jk}}^{M_j} c^*_{j'k'jk}(x_{j'k'jk}+1)^{1-\beta_{4j'j}}(K^{\circ}_{jk})^{1-\beta_{5j'j}} \\
&= 0, \qquad (3.4)
\end{aligned}$$

where the parameters satisfy

$$\begin{array}{llll}
0 < \alpha_{1h} < 1, & a^*_{hijk} > 0, & 0 < \beta_{1j} < 1, & b^*_{hijk} > 0, \\
0 < \alpha_{2hj}, & c^*_{jkhi} > 0, & 0 < \beta_{2hj} < 1, & c^*_{j'k'jk} > 0, \\
0 < \alpha_{3jh} < 1, & & 0 < \beta_{3hj} < 1, & \\
& & 0 < \beta_{4j'j} < 1, & \\
& & 0 < \beta_{5j'j} < 1. &
\end{array}$$

Both the utility and production function are continuously differentiable, separable, and strictly concave. The assumption of separability implies that spillovers only occur through the budget constraint. The above production function implies non-increasing returns to scale in m_{fjk}, x_{jk}, and l_{jk}. If we assume that $\beta_{2ij}+\beta_{3ij} > 1$ and $\beta_{4ij} + \beta_{5ij} > 1$ there will be non- increasing returns to scale in all the variables m_{fjk}, x_{jk}, l_{jk}, and K°_{jk}. It should be noted that the concavity of the utility function implies that the consumer prefers to work in a variety of firms rather than working in only one firm.

Notice that the α and β parameters are assumed to only vary between different types of consumers and firms, while the a, b, and c parameters are assumed to vary over all individuals and firms.

The above functional forms give a structural interpretation to the log-linear virtual prices which will be used in the following. Consumers maximize utility while the firms maximize profits. As shown in Andreassen (1995) this behavior can be described through the use of virtual (shadow) prices and wages. Let ξ^l_{hijk} be the consumer hi's virtual wage for labor supplied to firm jk, ξ^x_{jkhi} the virtual price for the good supplied by firm jk, and ξ^m_{hi} the virtual price for money (which as numeraire is always equal to 1). We define similar virtual prices for the firm. Let η^l_{hijk} be firm jk's virtual wage for labor supplied to firm jk from consumer hi, $\eta^x_{j'k'jk}$ the virtual price for the good $j'k'$, η^m_{jk} the virtual price for money, and η^y_{jk} the virtual price for the good produced by firm jk.

The virtual prices give us inverse demand and supply functions which depend on observed transactions. They give the prices at which the observed transactions would have been purchased if there had been no rationing. We let the variables m^*_{chi}, m^*_{fjk}, l^*_{hijk}, x^*_{jkhi}, $x^*_{j'k'hi}$, and y^*_{jk} denote the transacted quantity of money, labor, and goods. As noted earlier, these transacted quantities will be the same as the Drèze demands[2].

From (3.3) and (3.4 it follows that

$$\begin{aligned}\log \xi^l_{hijk}(l^*_{hijk}, m^*_{chi}) &= -\frac{\partial \mathcal{U}_{hi}(m^*_{chi}, \mathbf{x}^*_{hi}, \mathbf{l}^*_{hi})/\partial l_{hijk}}{\partial \mathcal{U}_{hi}(m^*_{chi}, \mathbf{x}^*_{hi}, \mathbf{l}^*_{hi})/\partial m_{chi}} \\ &= a_{hijk} + \alpha_{1h}\log m^*_{chi} + \alpha_{2hj}\log(l^*_{hijk}+1), \end{aligned} \tag{3.5}$$

$$\begin{aligned}\log \xi^x_{jkhi}(x^*_{kjhi}, m^*_{chi}) &= \frac{\partial \mathcal{U}_{hi}(m^*_{chi}, \mathbf{x}^*_{hi}, \mathbf{l}^*_{hi})/\partial x_{jkhi}}{\partial \mathcal{U}_{hi}(m^*_{chi}, \mathbf{x}^*_{hi}, \mathbf{l}^*_{hi})/\partial m_{chi}} \\ &= c_{jkhi} + \alpha_{1h}\log m^*_{chi} - \alpha_{3jh}\log(x^*_{jkhi}+1), \end{aligned} \tag{3.6}$$

$$\xi^m_{hi} = 1, \tag{3.7}$$

$$\begin{aligned}\log \eta^l_{hijk}(l^*_{hijk}, m^*_{fjk}, K^\circ_{jk}) &= \frac{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial l_{hijk}}{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial m_{fjk}} \\ &= b_{hijk} + \beta_{1j}\log m^*_{fjk} - \beta_{2hj}\log(l^*_{hijk}+1) + \beta_{3j'j}\log K^\circ_{jk}, \end{aligned} \tag{3.8}$$

[2]This notation is slightly different from Andreassen (1995).

$$\log \eta^{x}_{j'k'jk}(x^*_{j'k'jk}, m^*_{fjk}, K^\circ_{jk}) = \frac{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial x_{j'k'jk}}{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial m_{fjk}}$$

$$= c_{j'k'jk} + \beta_{1j} \log m^*_{fjk} - \beta_{4j'j} \log (x^*_{j'k'jk} + 1) + \beta_{5jk} \log K^\circ_{jk}, \tag{3.9}$$

$$\log \eta^{y}_{jk}(m^*_{fjk}) = -\frac{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial y_{jk}}{\partial F_{jk}(y^*_{jk}, m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}, K^\circ_{jk})/\partial m_{fjk}}$$

$$= d_{jk} + \beta_{1j} \log m^*_{fjk}, \tag{3.10}$$

and

$$\eta^{m}_{jk} = 1, \tag{3.11}$$

where we have the following relationship between the parameters in the virtual price functions and those in the utility and production functions,

$$\begin{aligned}
a_{hijk} &= \log\left(a^*_{hijk} \cdot \frac{1+\alpha_{2hj}}{1-\alpha_{1h}}\right), \\
b_{hijk} &= \log\left(b^*_{hijk} \cdot \frac{1-\beta_{2hj}}{1-\beta_{1h}}\right), \\
c_{j'k'jk} &= \log\left(c^*_{j'k'jk} \cdot \frac{1-\alpha_{4j'j}}{1-\beta_{1j}}\right), \quad \text{when } j = 1, \ldots, J\ , \\
c_{jkhi} &= \log\left(c^*_{hijk} \cdot \frac{1-\alpha_{3jh}}{1-\alpha_{1h}}\right), \quad \text{when } h = J+1, \ldots, J+H\ .
\end{aligned}$$

The parameter d_{jk} in equation (3.10) is equal to one plus the Lagrange multiplier pertaining to the firm's budget constraint.

In the case of non-rationed goods the virtual prices will be equal to the observed prices. For a discussion of virtual prices see Deaton and Muellbauer (1980) pp. 109-114. A more detailed discussion of the use of virtual prices in econometric disequilibrium models can be found in Lee (1986).

The assumption that the *min* condition must apply in all micro markets leads to there being two situations which can occur in each micro labor market when $l^*_{hijk} > 0$ (no corner solution). For labor in micro market $hijk$ we have either

1. $\xi^l_{hijk} = (1 - t_{1i})w_{hijk}$ and $\eta^l_{hijk} \geq w_{ij}$: The producer is rationed;

or

2 $\xi^l_{hijk} < (1 - t_{1i})w_{hijk}$ and $\eta^l_{hijk} = w_{ij}$: The consumer is rationed.

The situation where there is equilibrium will in the following approach have a probability of measure zero and is therefore trivially included in case 1 above. A corner solution in a micro labor market will imply that either $\xi^l_{hijk} > (1 - t_{1i})w_{hijk}$ or $\eta^l_{hijk} < w_{ij}$ (or both).

In the micro markets for private goods we have assumed that the demanders are never rationed. Assuming an interior solution for the demand of all agents in the micro market for private good $j'k'$, this implies:

3 $\xi^x_{j'k'hi} = p_{j'k'}$ for all h and i, $\eta^x_{j'k'jk} = p_{j'k'}$ for all j and k, and $\exp(-d_{j'k'})\eta^y_{j'k'} \leq (1 - t_{2j'})p_{j'k'}$: The producer may be rationed.

The term $(\exp(-d_{j'k'}))$ enters the above expression because the firm is constrained by it's budget. As long as a private firm's budget constraint is binding we have that $\eta^y_{kj}(m^*_{fjk}, \mathbf{x}^*_{jk}, \mathbf{l}^*_{jk}) > (1 - t_{2j})p_{jk}$. This reflects the fact that our requirement that purchases of inputs be based on last years sales imposes an inefficiency on the firm.

In the case of government goods recall that we assume that only consumers demand these and that they are always rationed in their demand. This implies that for government good jk we always have:

4 $\xi^x_{jkhi} < p_{jk}$ for all h and i and $\exp(-d_{jk})\eta^y_{jk} = (1 - t_{2j})p_{jk}$: The producer is never rationed.

When we allow for corner solutions the above becomes more complicated, as discussed in Andreassen (1995).

4 The distribution of unobserved heterogeneity

In this section we decompose each of the parameters from the virtual price functions, the demand for money holdings, the capital stock, the prices and the wages into an aggregate structural component and a stochastic component. These distributional assumptions form the basis for the aggregation procedure discussed in the next section.

To derive aggregate expressions for the different labor submarkets, we must make assumptions about the distributions of the parameters, a_{hijk}, across individuals and, b_{hijk}, across firms. Aggregation in the goods markets is based on similar assumptions about the parameters c_{jkhi} and $c_{j'k'jk}$. We assume that these parameters can be decomposed into a structural component and a stochastic component,

$$
\begin{aligned}
a_{hijk} &= \bar{a}_{hj} + u_{2hijk}, && (4.1)\\
b_{hijk} &= \bar{b}_{hj} + v_{2hijk}, && (4.2)\\
c_{jkhi} &= \bar{c}_{jh} + u_{3jkhi}, \qquad \text{when } h = J+1, \ldots, J+H \ , && (4.3)\\
c_{j'k'jk} &= \bar{c}_{j'j} + v_{3j'k'jk}, \qquad \text{when } j = 1, \ldots, J \ , && (4.4)
\end{aligned}
$$

where the stochastic variable u_{3jkhi} varies across all individuals of type h, $v_{3j'k'jk}$ varies across all firms of type j, and u_{2hjik} and v_{2hjik} vary within the combination of consumers of type h and firms of type j. These stochastic variables are assumed to be homoscedastic and serially independent. They may however be correlated with each other. This decomposition is related to the structural parameters of the utility and production functions as follows,

$$
\begin{aligned}
a^*_{hijk} &= \frac{1-\alpha_{1h}}{1+\alpha_{2hj}} \cdot \exp(\bar{a}_{hj}) \cdot \exp(u_{2hijk}) &&= \bar{a}^*_{hj} \cdot \exp(u_{2hijk}), && (4.5)\\
b^*_{hijk} &= \frac{1-\beta_{1h}}{1-\beta_{2hj}} \cdot \exp(\bar{b}_{hj}) \cdot \exp(v_{2hijk}) &&= \bar{b}^*_{hj} \cdot \exp(v_{2hijk}), && (4.6)\\
c^*_{hijk} &= \frac{1-\alpha_{1h}}{1-\alpha_{3hj}} \cdot \exp(\bar{c}_{jh}) \cdot \exp(u_{3jkhi}) &&= \bar{c}^*_{jh} \cdot \exp(u_{3jkhi}), && (4.7)
\end{aligned}
$$

$$c^*_{j'k'jk} \;=\; \frac{1-\beta_{1j}}{1-\beta_{4j'j}} \cdot \exp(\bar{c}_{j'j}) \cdot \exp(v_{3j'k'jk}) \;=\; \bar{c}^*_{j'j} \cdot \exp(v_{3j'k'jk}), \tag{4.8}$$

where $\bar{a}^*_{hj}$, $\bar{b}^*_{hj}$, $\bar{c}^*_{jh}$, and $\bar{c}^*_{j'j}$ are the structural components of the parameters in the utility and production functions (this notation will be used later in the paper). We see that the restrictions which apply to the parameters in the utility and production functions need not apply to these derived parameters

We assume that the realized demand for money holdings at a given time also can be decomposed into a structural component and a stochastic component, where the stochastic component describes the distribution of money demand over the population of individuals and firms. From the equations for the agents budget constraints (3.1) and (3.2) we have that money demand is given by

$$m^*_{chi} \;=\; \sum_{j=1}^{J}\sum_{k=1}^{M_j}(1-t_{1h})w_{hijk}l^*_{hijk} - \sum_{j=1}^{J}\sum_{k=1}^{M_j} p_{jk}x^*_{jkhi} + C^\circ_{hi}, \tag{4.9}$$

and

$$m^*_{fjk} \;=\; -\sum_{h=J+1}^{J+H}\sum_{i=1}^{N_h} w_{hijk}l^*_{hijk} - \sum_{j'=1}^{J_p}\sum_{\substack{k'=1\\ j'k'\neq jk}}^{M_j} p_{j'k'}x^*_{j'k'jk} + C^\circ_{jk} - inv_{jk}. \tag{4.10}$$

It is assumed that the logarithm of money demand can be written as

$$\log m^*_{chi} \;=\; \bar{m}_{ch} + u_{1hi}, \tag{4.11}$$

and

$$\log m^*_{fjk} \;=\; \bar{m}_{fj} + v_{1jk}. \tag{4.12}$$

In empirical work we must be able to observe the variables $\bar{m}_{ch}$ and $\bar{m}_{fj}$. In addition we must take into account that they will be correlated with the observed aggregate supply and demand variables. We therefore assume that the variables $\bar{m}_{ch}$ and $\bar{m}_{fj}$ are determined by the reduced form functions g_{ch} and g_{fj} in the following manner:

$$\begin{aligned}\bar{m}_{ch} \;=\; & g_{ch}\{\log(\mathrm{E}_{ki}(x^*_{1khi})), \cdots, \log(\mathrm{E}_{ki}(x^*_{Jkhi})),\ \log(\mathrm{E}_{ki}(l^*_{hi1k})), \cdots, \\ & \log(\mathrm{E}_{ki}(l^*_{hiJk})), \log(\mathrm{E}_{k}(p_{1k})), \cdots, \log(\mathrm{E}_{k}(p_{Jk})), \\ & \log(\mathrm{E}_{ki}(w_{hi1k})), \cdots, \log(\mathrm{E}_{ki}(w_{hiJk})), \log(\mathrm{E}_{i}(C^\circ_{hi}))\}\end{aligned} \tag{4.13}$$

and

$$
\begin{aligned}
\bar{m}_{fj} &= g_{fj}\Big\{\log\left(\mathrm{E}_{k'k}(x^*_{1k'jk})\right), \cdots, \log\left(\mathrm{E}_{k'k}(x^*_{Jk'jk})\right),\ \log\left(\mathrm{E}_{ki}(l^*_{J+1\,ijk})\right), \cdots, \\
&\quad \log\left(\mathrm{E}_{ki}(l^*_{J+H\,ijk})\right),\ \log\left(\mathrm{E}_k(p_{1k})\right), \cdots, \log\left(\mathrm{E}_k(p_{Jk})\right),\ \log\left(\mathrm{E}_{ki}(w_{J+1\,ijk})\right), \\
&\quad \cdots, \log\left(\mathrm{E}_{ki}(w_{J+H\,ijk})\right),\ \log\left(\mathrm{E}_k\left(C^{\circ}_{jk} - inv_{jk}\right)\right)\Big\}, \qquad (4.14)
\end{aligned}
$$

where E_{ki} denotes the expectation operator over all variables bearing the subscript i and k, and u_{1hi} and v_{1jk} are random variables independently distributed of each other such that $\bar{m}_{ch} = \log\left(\mathrm{E}_i\left(m^*_{chi}\right)\right)$ and $\bar{m}_{ch} = \log\left(\mathrm{E}_i(m^*_{fjk})\right)$. It is important to note that we assume that the reduced form equations only depend on aggregate variables. When we later estimate the aggregate relationships we have derived, these variables can be viewed as being instrument variables for money demand. All the aggregate variables are assumed to have the form $\log(\mathrm{E}(x))$ instead of $\mathrm{E}(\log(x))$. This greatly simplifies the use of aggregate data because averages of variables are more readily available than averages of the logarithm of these variables[3]. Because money holdings and labor are jointly determined, u_{1hi} and v_{1jk} will not be independently distributed of the other stochastic variables. This decomposition of money demand into an aggregate and a disaggregate stochastic component has implications for the specification of spillovers in the model. Because of the assumption that the utility and production functions are separable, spillovers from one market to another occur only through the budget constraint and thereby through the money demand variable in the virtual prices.

The distribution of capital K_{jk} across firms of type j is also decomposed into a structural and a stochastic component,

$$
\log\left(K^{\circ}_{jk}\right) = \bar{K}_j + v_{4jk},
$$

where $\bar{K}_j = \log\left(\mathrm{E}_k\left(K^{\circ}_{jk}\right)\right)$ and v_{4jk} is a random variable.

[3]The expected log of a variable will not be the same as the log of the expected value of this variable. If for example the stochastic variable x is normally distributed then $\mathrm{E}(\log x) = \log(\mathrm{E}\,x) - \frac{1}{2}\mathrm{var}(\log x)$.

Using the above specifications of unobserved heterogeneity we can now write equations (3.5) to (3.11) as:

$$\begin{aligned}\log \xi^{l}_{hijk}(l^{*}_{hijk}, m^{*}_{chi}) &= \bar{a}_{hj} + \alpha_{1h}\,\bar{m}_{ch} + \alpha_{2hj}\log(l^{*}_{hijk}+1) \\ &\quad + u_{2hijk} + \alpha_{1h}\cdot u_{1hi}, \end{aligned} \tag{4.15}$$

$$\begin{aligned}\log \xi^{x}_{jkhi}(x^{*}_{kjhi}, m^{*}_{chi}) &= \bar{c}_{jh} + \alpha_{1h}\,\bar{m}_{ch} - \alpha_{3jh}\log(x^{*}_{jkhi}+1) \\ &\quad + u_{3jkhi} + \alpha_{1h}\cdot u_{1hi}, \end{aligned} \tag{4.16}$$

$$\begin{aligned}\log \eta^{l}_{hijk}(l^{*}_{hijk}, m^{*}_{fjk}, K^{\circ}_{jk}) &= \bar{b}_{hj} + \beta_{1j}\,\bar{m}_{fj} - \beta_{2hj}\log(l^{*}_{hijk}+1) + \beta_{3hj}\,\bar{K}_{j} \\ &\quad + v_{2hijk} + \beta_{1j}\cdot v_{1jk} + \beta_{3hj}\cdot v_{4jk}, \end{aligned} \tag{4.17}$$

$$\begin{aligned}\log \eta^{x}_{j'k'jk}(x^{*}_{j'k'jk}, m^{*}_{fjk}, K^{\circ}_{jk}) &= \bar{c}_{j'j} + \beta_{1j}\,\bar{m}_{fj} - \beta_{4j'j}\log(x^{*}_{j'k'jk}+1) + \beta_{5j'j}\,\bar{K}_{j} \\ &\quad + v_{3j'k'jk} + \beta_{1j}\cdot v_{1jk} + \beta_{5j'j}\cdot v_{4jk}, \end{aligned} \tag{4.18}$$

and

$$\log \eta^{y}_{jk}(m^{*}_{fjk}) = d_{jk} + \beta_{1j}\,\bar{m}_{fj} + v_{5jk} + \beta_{1j}\cdot v_{1jk}. \tag{4.19}$$

The virtual price functions given by equations (4.15) to (4.19) give a description of the behavior of the agents as functions of money demand (pseudo-demand) even under rationing or when they choose corner solutions. We now wish to use these to describe desired supply and demand in each micro market in the case of an interior solution. Aggregation will be based on the relationships derived in this manner, also when taking into account corner solutions. Recall that such supply and demand can be represented by Clower supply and demand. The variables l^{S}_{hijk} and l^{D}_{hijk} were defined earlier as the Clower supply and demand for labor. The Clower demand for goods is similarly denoted x^{D}_{jkhi} and $x^{D}_{j'k'jk}$ for individuals and firms respectively.

Since the Clower demands and supplies assume that an agent is not rationed in the market under consideration, we can set the corresponding virtual wages and prices equal to the actual wages and prices. Let the distribution of the exogenous wage in the submarket consisting of consumers of type h and firms of type j be described by

$$\log w_{hjik} \quad = \quad \log \bar{w}_{hj} + \mu_{hjik}, \tag{4.20}$$

where $\log \bar{w}_{hj} = \mathrm{E}_{ik} (\log w_{hjik})$ and μ_{hjik} is a random variable with a cumulative distribution function which will specified later. In the same manner the prices of goods from firms of type j are described by

$$\log p_{jk} \quad = \quad \log \bar{p}_j + \mu^*_{jk}, \tag{4.21}$$

where $\log \bar{p}_j = \log (\mathrm{E}_k (p_{jk}))$ and μ^*_{jk} is a random variable zero mean. Note that we choose a slightly different specification for wages than for prices. In equation (4.20) we utilize $\mathrm{E}_{ik} (\log w_{hjik})$ while in equation (4.21) we utilize $\log (\mathrm{E}_k (p_{jk}))$. These assumptions are entirely practical, reflecting our needs later.

Assuming that Clower demands and supplies apply in each market and that there are no corner solutions, we can substitute the above wage and price equations in the virtual price functions (4.15) to (4.18) to get

$$\begin{aligned}
\log (l^S_{hijk} + 1) - \kappa_{hj} \cdot \mu_{hijk} & \\
= \ & \frac{1}{\alpha_{2hj}} [\log ((1 - t_{1h}) \bar{w}_{hj}) - \bar{a}_{hj} - \alpha_{1h} \bar{m}_{ch} - u_{2hijk} \\
& \quad - \alpha_{1j} \cdot u_{1hi} + \mu_{hijk}] - \kappa_{hj} \cdot \mu_{hijk} \\
= \ & S^l_{hj} - \varepsilon_{1hijk},
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
\log (x^D_{jkhi} + 1) & = \frac{1}{\alpha_{3jh}} \left[-\log \bar{p}_j + \bar{c}_{jh} + \alpha_{1h} \bar{m}_{ch} + u_{3jkhi} + \alpha_{1h} \cdot u_{1hi} - \mu^*_{jk} \right], \\
& = D^x_{jh} - \varepsilon_{4jkhi}
\end{aligned} \tag{4.23}$$

$$
\begin{aligned}
\log(l^D_{hijk}+1) - & \kappa_{hj}\cdot\mu_{hijk} \\
&= \frac{1}{\beta_{2hj}}[-\log\bar{w}_{hj} + \bar{b}_{hj} + \beta_{1j}\,\bar{m}_{fj} + \beta_{3hj}\,\bar{K}_j + v_{2hijk} \\
&\qquad +\beta_{1j}\cdot v_{1jk} + \beta_{3hj}\cdot v_{4jk} - \mu_{hijk}] - \kappa_{hj}\cdot\mu_{hijk} \\
&= D^l_{hj} - \varepsilon_{2hijk}, \qquad (4.24)
\end{aligned}
$$

$$
\begin{aligned}
\log(x^D_{j'k'jk}+1) &= \frac{1}{\beta_{4j'j}}\left[-\log\bar{p}_j + \bar{c}_{j'j} + \beta_{1j}\,\bar{m}_{fj} + \beta_{5j'j}\,\bar{K}_j + v_{3j'k'jk}\right. \\
&\qquad \left. +\beta_{1j}\cdot v_{1jk} + \beta_{5j'j}\cdot v_{4jk} - \mu^*_{jk}\right], \\
&= D^x_{j'j} - \varepsilon_{4j'k'jk} \qquad (4.25)
\end{aligned}
$$

where

$$S^l_{hj} = \frac{1}{\alpha_{2hj}}\left[\log\left((1-t_{1h})\bar{w}_{hj}\right) - \bar{a}_{hj} - \alpha_{1h}\,\bar{m}_{ch}\right], \qquad (4.26)$$

$$D^l_{hj} = \frac{1}{\beta_{2hj}}\left[-\log\bar{w}_{hj} + \bar{b}_{hj} + \beta_{1j}\,\bar{m}_{fj} + \beta_{3hj}\,\bar{K}_j\right], \qquad (4.27)$$

$$D^x_{jh} = \frac{1}{\alpha_{3jh}}\left[-\log\bar{p}_j + \bar{c}_{jh} + \alpha_{1h}\,\bar{m}_{ch}\right], \qquad (4.28)$$

$$D^x_{j'j} = \frac{1}{\beta_{4j'j}}\left[-\log\bar{p}_j + \bar{c}_{j'j} + \beta_{1j}\,\bar{m}_{fj} + \beta_{5j'j}\,\bar{K}_j\right], \qquad (4.29)$$

$$\varepsilon_{1hijk} = (\kappa_{hj} + \frac{1}{\alpha_{2hj}})\mu_{hijk} - \frac{1}{\alpha_{2hj}}(u_{2hijk} + \alpha_{1j}u_{1hi}), \qquad (4.30)$$

$$\varepsilon_{2hijk} = (\kappa_{hj} - \frac{1}{\beta_{2hj}})\mu_{hijk} + \frac{1}{\beta_{2hj}}(v_{2hijk} + \beta_{1j}v_{1jk} + \beta_{3jk}v_{4jk}), \qquad (4.31)$$

$$\varepsilon_{4jkhi} = \frac{1}{\alpha_{3jh}}\left[u_{3jkhi} + \alpha_{1h}\cdot u_{1hi} - \mu^*_{jk}\right], \qquad (4.32)$$

$$\varepsilon_{4j'k'jk} = \frac{1}{\beta_{4j'j}}\left[v_{3j'k'jk} + \beta_{1j}\cdot v_{1jk} + \beta_{5j'j}\cdot v_{4jk} - \mu^*_{jk}\right], \qquad (4.33)$$

where κ_{hj} is a parameter. The term $\kappa_{hj}\cdot\mu_{hijk}$ has been subtracted from both sides of the labor market equations (4.22) and (4.24) for reasons which will be explained below. For notational simplicity we introduce a third variable defined by

$$\varepsilon_{3hijk} = \kappa_{hj}\mu_{hijk}. \tag{4.34}$$

One should note that there is no equation for the supply of goods. This is because this is given from the production function when all the inputs have been determined. In the case on an interior solution, we must either have that the Clower demand or Clower supply is realized (because of the *min* condition). In the aggregation method discussed in the following we will utilize this property.

In the following we will make assumptions about the joint distribution of the stochastic variables ε_{1hijk}, ε_{2hijk}, and ε_{3hijk}. The aggregation results we get on the basis of these distributional assumptions enable us to estimate the unknown parameters. However we are not able to identify the parameters in the distributions of the underlying stochastic variables, u_{2jk} and v_{2jk}. It is straight forward to derive the wage distribution given by μ_{hijk} from the distribution of ε_{3hijk} since equation (4.34) must apply. From equation (4.30) and the fact that we know the distribution of ϑ_{hijk} and μ_{hijk}, we see that in principle one can find (either analytically or through simulation) the distribution of the sum $u_{2hijk} + \alpha_{1j}u_{1hi}$ but not the individual distributions of u_{2hijk} and u_{1hi}. To do this would require additional assumptions about these distributions. The same applies to the sum $v_{2hijk} + \beta_{1j}v_{1jk} + \beta_{3jk}v_{4jk}$ in equation (4.31). This means that the micro relations are not fully recoverable from the aggregate model.

We assume that the the stochastic variables ε_{1hijk}, ε_{2hijk}, and ε_{3hijk} are jointly distributed according to a particular version of the generalized extreme value (GEV) distribution F (cf. Ben-Akiva and Lerman (1985)) given below

$$\log F = -\left(e^{-\varepsilon_{1hijk}/\theta_{hj}\tau_{hj}} + e^{-\varepsilon_{2hijk}/\theta_{hj}\tau_{hj}}\right)^{\tau_{hj}} - e^{-\varepsilon_{3hijk}/\theta_{hj}}, \tag{4.35}$$

where $\tau_{hj} > 0$ and $\theta_{hj} > 0$ are parameters. The GEV class implies $\operatorname{var}\varepsilon_{1hijk} = \operatorname{var}\varepsilon_{2hijk} = \operatorname{var}\varepsilon_{3hijk}$. The particular specification above implies that the stochastic variables ε_{1hijk} and ε_{2hijk} are independent of ε_{3hijk}. Even though these assumptions give us a fairly restrictive specification of the relationship between ε_{1hijk}, ε_{2hijk}, and ε_{3hijk}, they do not necessarily imply severe restrictions on how the u-s, v-s and μ_{hjik} are related to each other. This can be seen if we calculate $\operatorname{var}\varepsilon_{1hijk}$, $\operatorname{var}\varepsilon_{2hijk}$, and $\operatorname{var}\varepsilon_{3hijk}$. For example, we can use equation (4.30) to calculate

$$\begin{aligned}\operatorname{var}\varepsilon_{1hijk} &= (\kappa_{hj}+\frac{1}{\alpha_{2hj}})^2\operatorname{var}\mu_{hijk}) - \frac{1}{(\alpha_{2hj})^2}(\operatorname{var}u_{2hijk}+(\alpha_{1j})^2\operatorname{var}u_{1hi})\\ &\quad +2\frac{\alpha_{1j}}{(\alpha_{2hj})^2}\operatorname{cov}(u_{1hi},u_{2hijk})+2\frac{\alpha_{1j}}{\alpha_{2hj}}(\kappa_{hj}+\frac{1}{\alpha_{2hj}})\operatorname{cov}(\mu_{hijk},u_{1hi})\\ &\quad +2\frac{1}{\alpha_{2hj}}(\kappa_{hj}+\frac{1}{\alpha_{2hj}})\operatorname{cov}(\mu_{hijk},u_{2hijk}). \qquad (4.36)\end{aligned}$$

Note that the expected values of the ε-s are not necessarily equal to zero. The interpretation of τ_{hj} is as $1-\tau_{hj}^2 = \operatorname{corr}(\varepsilon_{1hijk},\varepsilon_{2hijk})$. The parameter κ_{hj} is important because it gives a more flexible relationship between the variances of the extreme value distributed random variables ε_{1hijk}, ε_{2hijk}, and ε_{3hijk} and the variance of wages w_{hijk}. Since ε_{3hijk} is extreme value distributed we have that $\operatorname{var}\varepsilon_{3hijk} = \kappa_{hj}^2\cdot\operatorname{var}(\log w_{hijk}) = (1/6)\pi^2\theta_{hj}^2$ implying that $\theta_{hj}/\kappa_{hj} = \pi^{-1}\sqrt{6\cdot\operatorname{var}(\log w_{hijk})}$. A more detailed discussion of the GEV distribution is given in appendix A.

The generalized extreme value distribution is assumed mainly for analytical convenience, but it can be motivated as being a natural way of describing the distribution of parameters and wages when they are the indirect result of earlier decisions concerning choice of education and choice of production technology. As an example consider the choice of education. Let $\varepsilon_i(a)$ be the utility of working (or a parameter determining the utility of working) given type of education a among A possible choices for consumer i. The value of $\varepsilon_i(a)$ is known to the consumer, but is unobservable to the econometrician, who views it as being stochastically distributed across the population of consumers. The resulting indirect utility of working as a function of chosen education will then be $\varepsilon_i^* = \max(\varepsilon_i(1),\cdots,\varepsilon_i(A))$. The distribution of ε_i^* will under certain regularity conditions be extreme value distributed across the population. It can be argued that this type of argument also applies to the firm's choice of capital.

5 Aggregation and estimation

Aggregation across labor submarket hj is based on the Clower demands and supplies in equations (4.22) and (4.24). These equations can be viewed as quasi-demand (supply) functions under the condition that there is no rationing and no corner solutions. They relate the endogenous demand (supply) for labor to the endogenous demand for money under such conditions. The simple structure of these equations is a consequence of our assumption of separability, implying that it is only through the money demand variables (determined by the budget constraint) that spillovers are transmitted from one market to another. The Clower demands and supplies together with the *min* condition and the distributional assumptions made earlier lead to fairly simple aggregate labor market relationships which can be estimated using ordinary least squares. In the following we also discuss similar estimation in the goods market.

5.1 Aggregate labor market relationships

We consider aggregation across the micro markets which make up the submarket consisting of consumers of type h and firms of type j. Since we concentrate on one such submarket we drop the subscripts h and j in the following. We retain the micro subscripts i denoting the individual and k denoting the firm. Since we at present are only concerned with the labor market we also let $D = D^l_{hj}$ and $S = S^l_{hj}$

If transactions are positive and $l^S_{ik} < l^D_{ik}$, then the consumer isn't rationed and we have that $\xi^l_{ik} = w_{ik}$ and $l^S_{ik} = l^*_{ik}$. If $l^S_{ik} > l^D_{ik}$ then we have that $\eta^l_{ik} = (1-t_1)w_{ik}$ and $l^D_{ik} = l^*_{ik}$. Our assumption of the economy being in a Drèze equilibrium therefore implies, as discussed in section 2, that as long as we have an interior solution, $l^*_{ik} > 0$, the following *min* condition applies

$$\log(l_{ik}^{*}+1) \;=\; \min\Big(\log(l_{ik}^{S}+1), \log(l_{ik}^{D}+1)\Big). \qquad (5.1)$$

The *min* condition links the transacted quantity to the latent supplies and demands. Together with equations (4.22) and (4.24) it describes the outcome in a situation with rationing and no corner solution. We now wish to derive the probabilities discussed in section 2 assuming that the model and functional forms discussed in section 3 and the distributional assumptions in section 4 apply. The details of how the different probabilities are derived are however deferred to appendix A. An important concept is the probability distribution of the number of hours worked by worker i in firm j, given that the worker is employed by firm k,

$$\mathrm{P}(l_{ik}^{*} > \tilde{z} \mid l_{ik}^{*} > 0) \;=\; \mathrm{P}(\log(l_{ik}^{*}+1) > \log(\tilde{z}+1) \mid \log(l_{ik}^{*}+1) > 0). \qquad (5.2)$$

For notational simplicity we denote $z = \log(\tilde{z}+1)$. Substituting from equation (5.1) for $\log(l_{ik}^{*}+1)$ in (5.2) we get

$$\begin{aligned}
&\mathrm{P}(l_{ik}^{*} > \tilde{z} \mid l_{ik}^{*} > 0) \\
&\quad = \mathrm{P}\Big(\min\Big(\log(l_{ik}^{S}+1), \log(l_{ik}^{D}+1)\Big) > z \mid \min\Big(\log(l_{ik}^{S}+1), \log(l_{ik}^{D}+1)\Big) > 0\Big) \\
&\quad = \mathrm{P}\Big(\max\Big(-\log(l_{ik}^{S}+1), -\log(l_{ik}^{D}+1)\Big) < -z \mid \max\Big(-\log(l_{ik}^{S}+1), \\
&\qquad\qquad -\log(l_{ik}^{D}+1)\Big) < 0\Big). \qquad (5.3)
\end{aligned}$$

We now introduce the parameter κ (cf. equation (4.34)) into this equation giving us

$$\mathrm{P}(l_{ik}^{*} > \tilde{z} \mid l_{ik}^{*} > 0) =$$

$$\mathrm{P}\left(\max\left(-\log\left(l_{ik}^{S}+1\right)+\kappa\mu_{ik},-\log\left(l_{ik}^{D}+1\right)+\kappa\mu_{ik}\right)<\kappa\mu_{ik}-z\right.$$
$$\left.\mid \max\left(-\log\left(l_{ik}^{S}+1\right)+\kappa\mu_{ik},-\log\left(l_{ik}^{D}+1\right)+\kappa\mu_{ik}\right)<\kappa\mu_{ik}\right), \qquad (5.4)$$

where μ_{ik} is the stochastic variable introduced earlier to describe the distribution of wages in submarket hj. As mentioned earlier, the introduction of κ gives us an extra degree of freedom, leading to a more flexible modeling of the variances in our model.

Substituting from the Clower supply and demand equations (4.22) and (4.24) into the expression for the probability distribution (5.4) using the above notation we get

$$\begin{aligned}
&\mathrm{P}(l_{ik}^{*}>\tilde{z}\mid l_{ik}^{*}>0)\\
&\quad= \mathrm{P}(\max(\varepsilon_1-S,\varepsilon_2-D)<\varepsilon_3-z\mid \max(\varepsilon_1-S,\varepsilon_2-D)<\varepsilon_3)\\
&\quad= \frac{\mathrm{P}(\varepsilon_1-S<\varepsilon_3-z,\varepsilon_2-D<\varepsilon_3-z)}{\mathrm{P}(\varepsilon_1-S<\varepsilon_3,\varepsilon_2-D<\varepsilon_3)} \qquad (5.5)
\end{aligned}$$

Under the distributional assumptions made in section 4 we find that the probability distribution in equation (5.5) becomes

$$\begin{aligned}
\mathrm{P}(l_{ik}^{*}>\tilde{z}\mid l_{ik}^{*}>0) &= \frac{1}{\mathrm{P}(l_{ik}^{*}>0)}\cdot\frac{e^{-z/\theta}}{e^{-z/\theta}+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}}\\
&= \frac{1}{\mathrm{P}(l_{ik}^{*}>0)}\cdot\frac{e^{-\log(\tilde{z}+1)/\theta}}{e^{-\log(\tilde{z}+1)/\theta}+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}} \qquad (5.6)
\end{aligned}$$

where

$$\mathrm{P}(l_{ik}^{*}>0) = \frac{1}{1+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}}. \qquad (5.7)$$

We see that the probabilities have a logistic distribution. Equation (5.7) is the probability of consumer i working in firm k. These assumption imply that the probability that the Clower supply l_{ik}^S and the Clower demand l_{ik}^D are positive will be

$$\begin{aligned} \mathrm{P}(l_{ik}^S > 0) &= \mathrm{P}(\varepsilon_1 - S < \varepsilon_3) \\ &= \frac{1}{1+e^{-S/\theta}} \end{aligned} \tag{5.8}$$

and

$$\begin{aligned} \mathrm{P}(l_{ik}^D > 0) &= \mathrm{P}(\varepsilon_2 - D < \varepsilon_3) \\ &= \frac{1}{1+e^{-D/\theta}}. \end{aligned} \tag{5.9}$$

In section 2 we found the that equations (2.9), (2.14), and (2.15) were the empirical counterparts to the theoretical equations (5.7), (5.8), and (5.9). Substituting equation (2.14) into equation (5.8) and solving for S gives us

$$\begin{aligned} S &= \theta\left\{\log\left(\mathrm{P}(l_{ik}^S > 0)\right) - \log\left(1 - \mathrm{P}(l_{ik}^S > 0)\right)\right\} \\ &= \theta\left\{\log\left(1 - \left(1 - \frac{Q+U}{N}\right)^{\frac{1}{M}}\right) - \frac{1}{M}\log\left(1 - \frac{Q+U}{N}\right)\right\} \end{aligned} \tag{5.10}$$

and substituting equation (2.15) into equation (5.9) and solving for D gives us

$$\begin{aligned} D &= \theta\left\{\log\left(\mathrm{P}(l_{ik}^D > 0)\right) - \log\left(1 - \mathrm{P}(l_{ik}^D > 0)\right)\right\} \\ &= \theta\left\{\log\left(\frac{V}{MN} - \frac{1}{M}\log\left(1 - \frac{Q}{N}\right)\right)\right. \\ &\qquad \left. -\log\left(1 - \frac{V}{MN} + \frac{1}{M}\log\left(1 - \frac{Q}{N}\right)\right)\right\} \end{aligned} \tag{5.11}$$

where M is the number of potential firms and V is the number of vacancies of type j as defined in section 2. We also defined N as the number of individuals, Q the number of employed, and U the number of unemployed among individuals of type h.

In appendix A we also derive an expression for the expected number of hours worked by a consumer at a firm given that the consumer has a job there, $\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0)$. This yields

$$\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0) \;=\; \frac{-\theta}{\mathrm{P}(l_{ij}^* > 0)} \log\left[1 - \mathrm{P}(l_{ij}^* > 0)\right]. \tag{5.12}$$

We assume that the empirical counterpart to $\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0)$ is the average number of hours worked in each filled job which we denote L. When there are many micro labor markets, $\mathrm{P}(l_{ij}^* > 0)$ will be very small and, as noted in section 2, $\mathrm{P}(l_{ij}^* > 0)/\log\left[1 - \mathrm{P}(l_{ij}^* > 0)\right]$ will be very close to one. Assuming that it is equal to one and rearranging equation (5.12) gives us

$$\theta \;=\; -\frac{L \cdot \mathrm{P}(l_{ij}^* > 0)}{\log\left[1 - \mathrm{P}(l_{ij}^* > 0)\right]} \;=\; L. \tag{5.13}$$

We thereby have that the parameter θ is simply equal to the average number of hours worked. An interesting implication of this is that the variances of the ε-s will vary over time in proportion to L^2. The mean wage across consumers and firms is found in appendix A to be

$$W \;=\; \mathrm{E}(w_{ik} \mid l_{ik}^* > 0) \;=\; \bar{w}\left(\mathrm{P}(l_{ik}^* > 0)\right)^{-\theta/\kappa} \Gamma\left(1 - \frac{\theta}{\kappa}\right), \tag{5.14}$$

where Γ denotes the gamma function[4]. The empirical counterpart to $\mathrm{E}(w_{ik} \mid l_{ik}^* > 0)$ is assumed to be the average wage in each job held by an employed person and is denoted W. It is important to note that W is different from the mean wage $\bar{w}$ covering all potential wage offers. Taking the logarithm of (5.14), substituting in for $\mathrm{P}(l_{ik}^* > 0)$ from equation (2.9), and rearranging gives us

$$\log \bar{w} \;=\; \log W + \frac{\theta}{\kappa} \log\left(-\frac{1}{M} \log\left(1 - \frac{Q}{N}\right)\right) - \log \Gamma\left(1 - \frac{\theta}{\kappa}\right). \tag{5.15}$$

[4]The gamma function is defined by the integral $\Gamma(t) = \int_0^\infty u^{t-1} e^{-u} du$ for all $t > 0$.

We see that the parameter κ is important in determining how much the expected wage over all potential jobs differs from the expected wage in those jobs that actually are filled. In the estimation procedure discussed later we will wish to calculate $\bar{w}$ directly using equation (5.15). To do this we need observations of κ from cross-sectional data. In appendix A we show that if we have observations of $\mathrm{E}\left(w_{ik}^2 \mid l_{ik}^* > 0\right)$ and $\mathrm{var}\left(w_{ik} \mid l_{ik}^* > 0\right)$, then we can get κ from the non-linear equation:

$$1 + \frac{\mathrm{E}\left(w_{ik}^2 \mid l_{ik}^* > 0\right)}{\mathrm{var}\left(w_{ik} \mid l_{ik}^* > 0\right)} = \frac{\Gamma\left(1 - 2\frac{\theta}{\kappa}\right)}{\Gamma\left(1 - \frac{\theta}{\kappa}\right)^2}. \tag{5.16}$$

The parameters θ and κ together determine the variances of the generalized extreme value distribution which we have used. The aggregation procedure presented above therefore entails the need to know the variance of wages. The simple distributional assumptions made enable us to derive both the distribution of the virtual demand wage and the virtual supply wage on the basis of this.

5.2 Identification and estimation

The parameters in the model can now be estimated using the aggregate relationships we found in the preceding section. Introducing the subscripts h, j, and t denoting consumer type, firm type and time period respectively, we have that the following set of equations allows us to identify the structural parameters concerning labor submarket hj,

$$S_{hjt}^l = \frac{1}{\alpha_{2hj}} \log\left((1 - t_{1ht})\bar{w}_{hjt}\right) - \frac{\bar{a}_{hj}}{\alpha_{2hj}} - \frac{\alpha_{1h}}{\alpha_{2hj}} \bar{m}_{cht} + \vartheta_{1hjt}, \tag{5.17}$$

$$D_{hjt}^l = -\frac{1}{\beta_{2hj}} \log \bar{w}_{hjt} + \frac{\bar{b}_{hj}}{\beta_{2hj}} + \frac{\beta_{1j}}{\beta_{2hj}} \bar{m}_{fjt} + \frac{\beta_{3hj}}{\beta_{2hj}} \bar{K}_{jt} + \vartheta_{2hjt}, \tag{5.18}$$

where

$$S^l_{hjt} = L_{hjt}\left\{\log\left(1-\left(1-\frac{Q_{hjt}+U_{hjt}}{N_{ht}}\right)^{\frac{1}{M_{jt}}}\right)\right.$$
$$\left.-\frac{1}{M_{jt}}\log\left(1-\frac{Q_{hjt}+U_{hjt}}{N_{ht}}\right)\right\}, \quad (5.19)$$

$$D^l_{hjt} = L_{hjt}\left\{\log\left(\frac{V_{hjt}}{M_{jt}N_{ht}}-\frac{1}{M_{jt}}\log\left(1-\frac{Q_{hjt}}{N_{jt}}\right)\right)\right.$$
$$\left.-\log\left(1-\frac{V_{hjt}}{M_{jt}N_{ht}}-\frac{1}{M_{jt}}\log\left(1-\frac{Q_{hjt}}{N_{ht}}\right)\right)\right\} \quad (5.20)$$

$$\log\bar{w}_{hjt} = \log W_{hjt}+\frac{L_{hjt}}{\kappa_{hjt}}\log\left(-\frac{1}{M_{jt}}\log(1-\frac{Q_{hjt}}{N_{ht}})\right)-\log\Gamma\left(1-\frac{L_{hjt}}{\kappa_{hjt}}\right) \quad (5.21)$$

and

$$\frac{\Gamma\left(1-2\frac{L_{hjt}}{\kappa_{hjt}}\right)}{\Gamma\left(1-\frac{L_{hjt}}{\kappa_{hjt}}\right)} = 1+\frac{\mathrm{E}_{ik}\left(w^2_{hijkt}\mid l^*_{hijkt}>0\right)}{\mathrm{var}_{ik}\left(w_{hijkt}\mid l^*_{hijkt}>0\right)}, \quad (5.22)$$

Variations over time in the relationships (5.17) and (5.18) are specified by adding the stochastic variables ϑ_{1hjt} and ϑ_{2hjt} which take into account variation in the relationships over time. They are assumed to have white noise properties. These equations have a linear form making them easy to estimate using OLS. From equations (5.19) to (5.22) we see that we can indirectly observe the theoretical variables S^l_{hjt}, D^l_{hjt}, and $\bar{w}_{hjt}$ in addition to variables $\bar{m}_{cht}=\log\left(\mathrm{E}\left(m_{chit}\right)\right)$, $\bar{m}_{fjt}=\log\left(\mathrm{E}\left(m_{fjkt}\right)\right)$, and $\bar{K}_{jt}=\log\left(\mathrm{E}\left(K^\circ_{jkt}\right)\right)$.

A problem with using OLS directly on equations (5.17) and (5.18) is that the money variables $\bar{m}_{cht}$ and $\bar{m}_{fjt}$ are correlated with the demand and supply variables S^l_{hj} and D^l_{hj} because money demand and labor supply or demand are assumed to be chosen simultaneously at the micro level. To get around this problem we use the money demand functions introduced earlier as instrumental variables in estimating the above equations. These demand functions can formally be approximated by

$$\bar{m}_{cht} = g^*_{ch}\left\{\log\left(\mathrm{E}_{ki}(x^*_{1khit})\right),\cdots,\log\left(\mathrm{E}_{ki}(x^*_{Jkhit})\right),\ \log\left(\mathrm{E}_{ki}(l^*_{hi1kt})\right),\cdots,\right.$$

$$\log\left(\mathrm{E}_{ki}(l^*_{hiJkt})\right), \log\left(\mathrm{E}_k(p_{1kt})\right), \cdots, \log\left(\mathrm{E}_k(p_{Jkt})\right),$$
$$\log\left(\mathrm{E}_{ki}(w_{hi1kt})\right), \cdots, \log\left(\mathrm{E}_{ki}(w_{hiJkt})\right), \log\left(\mathrm{E}_i(C^\circ_{hit})\right)\} + \vartheta_{3hjt}, \quad (5.23)$$

and

$$\begin{aligned}\bar{m}_{fjt} \;=\; & g^*_{fj}\Big\{\log\left(\mathrm{E}_{k'k}(x^*_{1k'jkt})\right), \cdots, \log\left(\mathrm{E}_{k'k}(x^*_{Jk'jkt})\right),\ \log\left(\mathrm{E}_{ki}(l^*_{J+1\,ijkt})\right), \cdots, \\ & \log\left(\mathrm{E}_{ki}(l^*_{J+H\,ijkt})\right),\ \log\left(\mathrm{E}_k(p_{1kt})\right), \cdots, \log\left(\mathrm{E}_k(p_{Jkt})\right),\ \log\left(\mathrm{E}_{ki}(w_{J+1\,ijkt})\right), \\ & \cdots, \log\left(\mathrm{E}_{ki}(w_{J+H\,ijkt})\right),\ \log\left(\mathrm{E}_k\left(C^\circ_{jkt} - inv_{jkt}\right)\right)\Big\} + \vartheta_{4hjt}, \qquad (5.24)\end{aligned}$$

where the random variables ϑ_{3hjt} and ϑ_{4hjt} are independently distributed of each other and of the stochastic variables ϑ_{1hjt}, and ϑ_{2hjt}. The reduced form functions g^*_{ch} and g^*_{fj} will in practical applications be specified as a polynomial in a subset of the exogenous variables in the model. Since the variables in these functions are only meant as instruments in estimating equations (5.17) and (5.18) misspecification of these functions need not seriously impair the estimates as long as the included variables are highly correlated with the money variables. Even in the case of misspecification, we get consistent estimates of the parameters in equation (5.17) and (5.18). One should note that the money variables $\bar{m}_{cht}$ and $\bar{m}_{fjt}$ are most likely to be correlated with the wage variable $\log \bar{w}_{hjt}$, which can give multicollinearity problems (though the dependency is probably non-linear).

It is apparent that the parameters α_{1h}, α_{2hj}, $\bar{a}_{hj}$, β_{1j}, β_{2hj}, β_{3hj}, and $\bar{b}_{hj}$ are all identifiable from OLS estimation (when the OLS-estimates are consistent) on equations (5.17) and (5.18). Using equations (5.17), (5.18), (5.23), and (5.24) to estimate the parameters in one labor submarket leads to unique estimates. If we expand the system to include two or more labor submarkets, which have either h or j in common, the estimates of the money demand parameters α_{2h} and β_{3j} will be overidentified. This is because the money demand function $\bar{m}_{cht}$ enters all labor submarket relationships which include labor of type j and $\bar{m}_{fjt}$ enters all labor submarket relationships including firms of type j. This implies that one should use simultaneous equation estimation techniques such as maximum-likelihood when estimating the relationships in more than one labor submarket. The same situation arises when estimating the labor market together with the goods market.

It is possible to calculate the parameters

$$\bar{a}^*_{hj} = \frac{1-\alpha_{1h}}{1+\alpha_{2hj}}\exp(\bar{a}_{hj}) \tag{5.25}$$

and

$$\bar{b}^*_{hj} = \frac{1-\beta_{1h}}{1-\beta_{2hj}}\exp(\bar{b}_{hj}). \tag{5.26}$$

but not the distribution of the original parameters a^*_{hijk} and b^*_{hijk}. If we normalize such that $\mathrm{E}\left(\exp(u_{2hijk})\right) = 0$ and $\mathrm{E}\left(\exp(v_{2hijk})\right) = 0$ then we have that $\mathrm{E}\left(a^*_{hijk}\right) = \bar{a}^*_{hj}$ and $\mathrm{E}\left(b^*_{hijk}\right) = \bar{b}^*_{hj}$. It should be noted that this implies that $\mathrm{E}\left(u_{2hijk}\right)$ and $\mathrm{E}\left(v_{2hijk}\right)$ will not be equal to 0.

5.3 Disequilibrium indicators

The observed variables for unemployment U and vacancies V are indicators of disequilibrium in the labor market, but do not give a complete picture of the extent of mismatch or rationing. They are exclusively concerned with work/do not work and hire/do not hire situations. Rationing leads to many other types of mismatch. The individual observes that there are other jobs open to individuals with his or her characteristics which are more desirable *given* the going wage. In the same manner the firm observes individuals which are preferable to those it hires given the going wage. Another shortcoming of the variables U and V is that they do not reflect the latent nature of many jobs. There are many potential jobs in an economy which are never done because the potential wage is too small. The aggregation approach discussed above leads to a greater number of indicators which can be used to characterize how disequilibrium changes over time in an economy. In the following we drop the subscript t denoting time period.

An indicator which is closely related to the observed variables for unemployment and vacancies, but which covers all latent micro markets, is the difference between the probability that a firm wishes to hire a person and the probability that this person wishes to work there. Weighted by the number of micro markets in submarket hj this difference can be written

$$M_j N_h \left(\mathrm{P}(l^S_{hijk} > 0) - \mathrm{P}(l^D_{hijk} > 0)\right), \tag{5.27}$$

giving the number of micro labor markets in which individuals wish to work minus the number of micro markets in which the firms wish to hire. This indicator is only concerned with whether a individual wishes to work or a firm wishes to hire and does not take into account how demand and supply interact (the correlation between the individual's desire to work at a firm and this firm's desire to hire the individual).

An alternative indicator which gives a more complete picture of the extent of rationing is a comparison of the virtual wages to the actual wages. Equation (4.15) can be rewritten to give

$$\begin{aligned}
&\log \xi^l_{hijk}(l^*_{hijk}, m^*_{chi}) - (\kappa_{hj} + \frac{1}{\alpha_{2hj}})\mu_{hijk} \cdot \alpha_{2hj} \\
&\quad = \bar{a}_{hj} + \alpha_{1h}\bar{m}_{ch} + \alpha_{2hj}\log(l^*_{hijk} + 1) + u_{2hijk} + \alpha_{1h} \cdot u_{1hi} - (\kappa_{hj} + \frac{1}{\alpha_{2hj}})\mu_{hijk} \cdot \alpha_{2hj} \\
&\quad = \bar{a}_{hj} + \alpha_{1h}\bar{m}_{ch} + \alpha_{2hj}\log(l^*_{hijk} + 1) - \alpha_{2hj} \cdot \varepsilon_{1hijk}
\end{aligned} \tag{5.28}$$

and equation (4.20) can be represented as

$$\begin{aligned}
\log w_{hjik} - \mu_{hijk} - \alpha_{2hj} \cdot \kappa_{hj} \cdot \mu_{hijk} &= \log \bar{w}_{hj} - \alpha_{2hj} \cdot \kappa_{hj} \cdot \mu_{hijk} \\
&= \log \bar{w}_{hj} - \alpha_{2hj} \cdot \varepsilon_{3hijk}.
\end{aligned} \tag{5.29}$$

Substituting equation (5.29) into (5.28) leads to

$$\begin{aligned}
&\log \xi^l_{hijk}(l^*_{hijk}, m^*_{chi}) - \log w_{hjik} \\
&\quad = \bar{a}_{hj} + \alpha_{1h}\bar{m}_{ch} + \alpha_{2hj}\log(l^*_{hijk} + 1) - \log \bar{w}_{hj} - \alpha_{2hj} \cdot (\varepsilon_{1hijk} - \varepsilon_{3hijk}).
\end{aligned} \tag{5.30}$$

Taking expectations on both sides of this equation gives us

$$\mathrm{E}\left(\log\left(\frac{\xi^l_{hijk}(l^*_{hijk}, m^*_{chi})}{w_{hjik}}\right)\right)$$
$$= \bar{a}_{hj} + \alpha_{1h}\,\bar{m}_{ch} + \alpha_{2hj}\cdot \mathrm{P}(l^*_{hijk} > 0)\cdot \mathrm{E}\,\left(\log(l^*_{hijk}+1) \mid \mathrm{P}(l^*_{hijk} > 0)\right) - \log\bar{w}_{hj}$$
$$= \bar{a}_{hj} + \alpha_{1h}\,\bar{m}_{ch} - \alpha_{2hj}\cdot\theta_{hj}\cdot\log(\mathrm{P}(l^*_{hijk} > 0)) - \log\bar{w}_{hj} \tag{5.31}$$

where we have substituted for $\mathrm{E}\,\left(\log(l^*_{hijk}+1) \mid \mathrm{P}(l^*_{hijk} > 0)\right)$ from equation (A.28) in appendix A and used the fact that the expected value of ε_{1hijk} and ε_{3hijk} are equal. In the same manner we can find from equation (4.17) that

$$\mathrm{E}\left(\log\left(\frac{\eta^l_{hijk}(l^*_{hijk}, m^*_{fjk}, K^\circ_{jk})}{w_{hjik}}\right)\right) \tag{5.32}$$
$$= \bar{b}_{hj} + \beta_{1j}\,\bar{m}_{fj} + \beta_{2hj}\cdot\theta_{hj}\cdot\log(\mathrm{P}(l^*_{hijk} > 0)) - \log\bar{w}_{hj}.$$

Using the parameter estimates and observations of θ_{hj} and $\mathrm{P}(l^*_{hijk} > 0))$ discussed above the price ratios in equations (5.31) and (5.32) can be calculated. An increase in the absolute value of these ratios will indicate increased rationing in the labor submarket under study. They capture the total effects of rationing on each side of the market, but do not consider the simultaneous distribution of demand and supply.

A third indicator is the probability that supply is larger than demand in a particular labor market when supply is greater than zero, $\mathrm{P}(l^S_{hijk} > l^D_{hijk}, l^S_{hijk} > 0)$. This indicator reflects only the number of hours which individuals wish to supply and firms wish to employ. It does not take the working/not working or the hiring/not hiring decision into account, but looks at supply and demand simultaneously in each micro labor market. In this manner it supplements the two indicators suggested above. In appendix A this probability was found to be

$$\mathrm{P}(l^S_{hijk} > l^D_{hijk}, l^S_{hijk} > 0) = \mathrm{P}(l^S_{hijk} > 0) - \left(1 - \mathrm{P}(l^S_{hijk} > l^D_{hijk})\right)\,\mathrm{P}(l^*_{hijk} > 0), \tag{5.33}$$

where

$$P(l_{ik}^S > l_{ik}^D) \;=\; \frac{1}{1 + (\exp(D_{hj} - S_{hj})^{1/(\theta_{hj}\tau_{hj})})}. \tag{5.34}$$

The rationing constraints are related to the prices and wages in the economy. We have not modeled such relationships, but the above aggregation approach does give some indication how wages affect many important variables. It can therefore be interesting in a given situation to see how the above indicators change if the wage distribution changes. This can be done by simulating these indicators' response to a small change in κ_{hj}, implying a shift in the expectation and variance of the wage distribution. Such a change will alter both the expectation and the variance of the wage distribution. It is important to note that all the relationships dicussed above are conditional on the set of observed variables and that we do not know which behavioral changes a change in a parameter will lead to. Any changes which occur in the indicators as a result of a change in κ_{hj} therefore only says whether such a change would decrease disequilibrium *given the present situation.* If this is the case we can conjecture that it will also decrease disequilibrium after behavioral changes are taken into account, since it is still possible for the agents to choose the present situation. But since the degree of disequilibrium may increase in some micro markets and decrease in others, we can not be sure of how this will influence the aggregate indicators we have suggested above.

5.4 Aggregation and estimation in the markets for goods

The aggregation method described above could also be used to aggregate the demand for goods across consumers and firms when there is rationing, but requires the assumption that each combination of buyer and seller is a separate market. This seems a more unrealistic assumption in the goods market than in the case of the labor market, because labor supply and demand are much more heterogenous than the supply and demand for goods. It is also difficult to find disequilibrium data for goods of the type we have for the labor market, where we for example have observations of unemployment and vacancies. These considerations leads us to only aggregate in the labor market under rationing and to treat the goods market in a simplified manner by assuming that demand for private goods is never rationed while demand for government goods is always rationed.

The assumption that individuals are always rationed in their demand for government goods (firms do not demand government goods) implies that it is impossible to estimate the parameters concerning such goods. Each individual's purchase of government goods will be solely determined by the rationing constraints and not by preferences. In the following we therefore only concern ourselves with the demand for private goods. We discuss only the demand by individuals in detail since the same type of relationships will characterize the demand by firms.

We assume that we can observe the average consumption of goods of type j by individuals of type h, which we denote $\bar{X}_{jh}$, and that the theoretical counterpart to this variable is the expected consumption of good jk by individual hi, $\mathrm{E}\left(x_{kjhi}\right)$. The stochastic variable ε_{4jkhi} was earlier assumed to describe the variation in the virtual price of goods of type j while μ^*_{jk} described the variation in prices. These random variables are now assumed to be distributed according to Φ,

$$\log \Phi(\varepsilon_{4jkhi}, \mu^*_{jk}) \quad = \quad -\exp(-\varepsilon_{4jkhi}/\psi_{jh}) - \exp(-\mu^*_{jk}/\psi_{jh}), \tag{5.35}$$

where $\psi_{hj} > 0$ is a parameter. This is a special case of the GEV distribution assumed for the labor market in equation (4.35), and therefore can get similar aggregation results for the goods market as for the labor market. Doing the same calculations for the goods markets as was done for the labor markets in appendix A and remembering that Clower demands equal transactions when there is no rationing and no corner solutions, we can derive the following relationships:

$$\begin{aligned} \mathrm{P}(x^*_{jkhi} > 0) &= \mathrm{P}(D^x_{jh} - \varepsilon_{4jkhi} > \mu^*_{jk}) \\ &= \frac{1}{1 + exp(-D^x_{jh}/\psi_{jh})} \end{aligned} \tag{5.36}$$

and

$$\mathrm{E}(x^*_{jkhi} \mid x^*_{jkhi} > 0) \;=\; \frac{-\psi_{hj}}{\mathrm{P}(x^*_{jkhi} > 0)} \log\left[1 - \mathrm{P}(x^*_{jkhi} > 0)\right]. \qquad (5.37)$$

These two equations are simplified versions of equations (5.7) and (5.12) which were found for the labor market. These equations taken together give us

$$\begin{aligned} \bar{X}_{jh} &= \mathrm{P}(x^*_{jkhi} > 0) \cdot \mathrm{E}(x^*_{jkhi} \mid x^*_{jkhi} > 0) \\ &= -\psi_{jh} \log\left(1 - \frac{1}{\exp(-D^x_{jh}/\psi_{jh}) + 1}\right) \end{aligned} \qquad (5.38)$$

Solving this equation for D^x_{jh} leads to

$$D^x_{jh} \;=\; -\psi_{jh} \log\left(\frac{\exp(-\bar{X}_{jh}/\psi_{jh})}{1 - \exp(-\bar{X}_{jh}/\psi_{jh})}\right). \qquad (5.39)$$

According to Johnson and Kotz (1970) p. 283, one can estimate ψ_{jh} from the following equation

$$\hat{\psi}_{jh} \;=\; \bar{X}_{jh} - \left[\sum_{ik} x_{jkhi} \exp(-x_{jkhi}/\hat{\psi}_{jh})\right] \left[\sum_{ik} \exp(-x_{jkhi}/\hat{\psi}_{jh})\right]^{-1}. \qquad (5.40)$$

It is necessary to solve this equation numerically. From equation (5.37) we see that if $\mathrm{P}(x^*_{jkhi} > 0)$ is small then $\hat{\psi}_{jh} \approx \bar{X}_{jh}/\mathrm{P}(x^*_{jkhi} > 0)$.

The above results imply that the Clower demands can be estimated in the same manner as in the labor market. Introducing the time subscript t we have that

$$D^x_{jht} \;=\; -\frac{1}{\alpha_{3jh}} \log \bar{p}_{jt} + \frac{\bar{c}_{jh}}{\alpha_{3jh}} + \frac{\alpha_{1h}}{\alpha_{3jh}} \bar{m}_{cht} + \vartheta_{5jht}, \qquad (5.41)$$

and

$$\begin{aligned} \bar{m}_{cht} \;=\; & g_{ch}\left\{\log\left(\mathrm{E}_{ki}(x^*_{1khit})\right), \cdots, \log\left(\mathrm{E}_{ki}(x^*_{Jkhit})\right),\ \log\left(\mathrm{E}_{ki}(l^*_{hi1kt})\right), \cdots,\right. \\ & \log\left(\mathrm{E}_{ki}(l^*_{hiJkt})\right), \log\left(\mathrm{E}_k(p_{1kt})\right), \cdots, \log\left(\mathrm{E}_k(p_{Jkt})\right), \\ & \left.\log\left(\mathrm{E}_{ki}(w_{hi1kt})\right), \cdots, \log\left(\mathrm{E}_{ki}(w_{hiJkt})\right), \log\left(\mathrm{E}_i(C^\circ_{hit})\right)\right\} + \vartheta_{3hjt}, \end{aligned} \qquad (5.42)$$

where equation (5.41) is equation (4.28) with the white noise random variable ϑ_{5jht} added. Estimation will now be exactly the same as in the case of the labor market. Notice that equation (5.41) has the parameter α_{1h} in common with equation (5.17) indicating, as mentioned, that the goods and labor markets ought to be estimated simultaneously.

5.5 A comparison with the smoothing by aggregation approach

The aggregation approach described above is in some ways similar to the smoothing by aggregation approach, first suggested by Muellbauer (1978). Lambert (1988) derives an aggregate relationship at the macro level which represents an explicit aggregation of micro markets. Gourieroux (1984) derives similar aggregated relationships as those in Lambert (1988) using different distributional assumptions. In the following we drop the subscripts h and j denoting types of individual and firm and the subsript t denoting time period.

Lambert (1988) starts by modeling micro markets in the same manner as we did above,

$$\log l_{ik}^S = \lambda^S + \varepsilon_1, \tag{5.43}$$

$$\log l_{ik}^D = \lambda^D + \varepsilon_2, \tag{5.44}$$

$$\log l_{ik} = \min(\log l_{ik}^S, \log l_{ik}^D), \tag{5.45}$$

where the λ-s are structural relationships (including for example prices). In contrast to our approach he does not derive the λ-s from the individual optimization of the agents and assumes that the stochastic variables ε_1 and ε_2 are bivariate normally distributed. The aggregate transacted quantity $\tilde{l}$ is given by:

$$\tilde{l} = \sum_i^N \sum_k^M \min(\log l_{ik}^S, \log l_{ik}^D) \tag{5.46}$$

where N is then number of consumers and M is the number of firms. Lambert shows that the CES function:

$$\mathrm{E}(\tilde{l}) \approx NM \cdot [\mathrm{E}(l_{ik}^S)^{-\nu} + \mathrm{E}(l_{ik}^D)^{-\nu}]^{-\frac{1}{\nu}} \tag{5.47}$$

gives a good approximation to equation (5.46) for $\nu > 0$. The parameter ν is the correlation coefficient in the bivariate normal distribution. He succeeds thereby in getting a fairly simple expression for aggregate transactions.

The main objective of the smoothing by aggregation approach is to derive a relationship which permits a gradual (smooth) increase of rationing in an aggregate market. The above expression is usually used in connection with the two market neo-Keynesian macro model, permitting some parts of the economy to experience Keynesian unemployment while other parts experience classical unemployment.

An interesting aspect of the aggregation approach of Lambert (1988) is the importance attached to the weighted probability $\mathrm{P}_w(l_{ik}^S > l_{ik}^D)$, which is the probability that supply is greater than demand when each micro market is weighted by its contribution to aggregate transactions. The same assumptions used to derive the CES function above lead to this probability being given by

$$\mathrm{P}_w(l_{ik}^S > l_{ik}^D) = \frac{1}{1 + (\mathrm{E}(l_{ik}^D)/\mathrm{E}(l_{ik}^S))^{\nu}}. \tag{5.48}$$

This probability can be compared to the probability $\mathrm{P}(l_{ik}^S > l_{ik}^D)$ which is found in appendix A as being equal to

$$\mathrm{P}(l_{ik}^S > l_{ik}^D) = \frac{1}{1 + (\exp(D_{hj} - S_{hj})^{1/(\theta_{hj}\tau_{hj})})}. \tag{5.49}$$

If we can assume that $\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) \approx D/\theta$ and $\mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0) \approx S/\theta$, then we can derive an expression for $\mathrm{P}(l_{ik}^S > l_{ik}^D)$ which is easy to compare to Lambert's expression for $\mathrm{P}_w(l_{ik}^S > l_{ik}^D)$ in equation (5.48). In appendix A we found that

$$\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) = \frac{\theta}{\mathrm{P}(l_{ik}^* > 0)} \log\left(\frac{1 + \exp(-D/\theta)}{\exp(-D/\theta)}\right) \tag{5.50}$$

and

$$\mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0) = \frac{\theta}{\mathrm{P}(l_{ik}^* > 0)} \log\left(\frac{1+\exp(-S/\theta)}{\exp(-S/\theta)}\right), \tag{5.51}$$

where D and S are defined as in equations (5.17) and (5.18). Taking the exponent of these two equations and then dividing them by each other and rearranging gives us

$$\exp(D/\theta - S/\theta) = \exp\left(\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) - \mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0)\right) \frac{1+\exp(-D/\theta)}{1+\exp(-S/\theta)}. \tag{5.52}$$

Combining this with equation (5.49) gives us

$$\mathrm{P}(l_{ik}^S > l_{ik}^D) = \frac{1}{1+\left(\frac{L^D}{L^S}\right)^{\tau}}. \tag{5.53}$$

where

$$L^D = \exp\left(\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0)\right) + \exp\left(\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) - D/\theta\right) \tag{5.54}$$

and

$$L^S = \exp\left(\mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0)\right) + \exp\left(\mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0) - S/\theta\right).$$

If one where to assume that $D/\theta \approx \mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0)$ and $S/\theta \approx \mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0)$ then equation (5.53) will become[5]

$$\begin{aligned}\mathrm{P}(l_{ik}^S > l_{ik}^D) &= \frac{1}{1+\left[\exp\left(\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) - \mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0)\right)\right]^{\tau}} \\ &= \frac{1}{1+\left[\exp\left(D/\theta - S/\theta\right)\right]^{\tau}}.\end{aligned} \tag{5.55}$$

From this we see that even though this probability has a functional form close to that proposed by Lambert (in equation (5.48)), they are different. The probability

[5] Even though this is not the case in our model, it is an assumption close to the type we make in equations (4.22) to (4.25).

derived on the basis of our model has a logistic form and the arguments are conditional on there being no corner solution ($l_{ik}^* > 0$). One should note how large a role the probabilities of there being a positive l_{ik}^S, l_{ik}^D, and l_{ik}^* play in all the aggregate variables we find. This indicates how important it is in an approach such as ours to take explicitly into account the possibility of corner solutions.

The main difference between our approach and the smoothing by aggregation approach is that we have strived for a more structural approach based on explicit utility and profit maximization enabling us to characterize disequilibrium in many submarkets, while the smoothing by aggregation approach is concerned with finding an equation to describing aggregate transactions when the demand side is rationed in some micro markets while the supply side is rationed in others. The above illustrates that it is possible in the aggregation approach discussed in this paper to find the same type of aggregate probabilities as in the smoothing by aggregation approach.

6 Data and empirical results

In the following an example is given of empirical use of the aggregation approach presented above. Estimation results are presented for the case when there is one type of individual and four types of firms and thereby four labor submarkets. Aggregation is done across all private goods so that there is only one aggregate goods market. It must be emphasized that the following results are mainly of an illustrative nature, since the model is very parsimonious and the quality of the data are fairly poor, requiring much ad hoc adjustments. Especially the money variables and the distribution of unemployment and vacancies across sectors required strong apriori assumptions. For example, the distribution of vacancies across sectors has been imputed using fixed coefficients, tending to make the time series for different sectors follow each other (which is seen in the figures we will discuss later). The following is still interesting in that it illustrates the extent to which the approach discussed earlier is relevant and where further research is most needed.

The labor market is, as mentioned, divided into four aggregate submarkets with one type of individual and the following four types of firms (sectors):

1. A public sector (central and local government);
2. A sector for traded goods except oil and gas;
3. A sector for non-traded goods (except those produced by the public sector);
4. A sector for oil and gas production.

In the following we also consider the total aggregate labor market.

Most of the data used are reported for the period 1971 to 1992, but because of some gaps in the data, the estimation results are based only on data for the period 1976 to 1991 (16 years). Data from the National Accounts are at core of the estimation though they are supplemented by other data.

Table 1: Individuals age 16-74, unemployment, vacancies, and employment. 1000

	N_t	U_t	$V_{\cdot t}$	V_{1t}	V_{2t}	V_{3t}	V_{4t}	$Q_{\cdot t}$	Q_{1t}	Q_{2t}	Q_{3t}	Q_{4t}
1972	2732	28	22.1	2.1	11.0	8.8	0.12	1673	302	339	979	53
1973	2749	26	21.3	2.4	10.1	8.7	0.11	1685	316	341	975	53
1974	2768	25	25.9	2.6	12.8	10.3	0.14	1706	324	350	983	49
1975	2809	33	15.7	1.8	7.4	6.3	0.07	1738	346	352	994	46
1976	2825	33	17.5	2.7	5.9	8.9	0.09	1796	374	350	1027	45
1977	2844	27	22.9	2.9	8.8	11.0	0.11	1843	392	349	1054	48
1978	2866	34	18.1	2.5	7.0	8.5	0.09	1877	415	342	1075	45
1979	2885	38	16.2	3.2	5.1	7.8	0.08	1905	433	338	1089	45
1980	2875	32	20.9	3.7	8.1	9.0	0.10	1947	452	338	1110	47
1981	2909	40	17.0	3.7	6.4	6.7	0.09	1967	471	335	1110	51
1982	2937	52	13.1	3.4	4.6	5.0	0.08	1969	480	324	1114	51
1983	2958	69	8.4	3.3	1.4	3.7	0.06	1963	492	308	1113	50
1984	2979	64	11.1	3.9	2.0	5.2	0.05	1976	497	307	1122	50
1985	3004	54	15.1	5.4	2.9	6.7	0.09	2028	510	309	1161	48
1986	3024	42	27.3	8.8	5.8	12.5	0.15	2089	518	312	1215	44
1987	3046	45	32.3	10.4	5.9	15.8	0.14	2133	536	311	1249	37
1988	3071	70	22.6	9.3	3.1	10.0	0.12	2118	544	297	1239	38
1989	3087	106	18.0	7.0	2.7	8.2	0.10	2069	554	278	1190	47
1990	3094	112	17.1	7.4	2.2	7.5	0.13	2051	567	271	1160	53
1991	3105	116	16.9	7.5	2.3	7.1	0.13	2034	584	261	1134	55
1992	3116	126	16.7	7.8	1.9	6.9	0.14	2029	600	260	1115	54

Table 2: Average hours worked per person and average capital per firm (in millions Nkr.).

	$L_{\cdot t}$	L_{1t}	L_{2t}	L_{3t}	L_{4t}	$\bar{K}_{\cdot t}$	$\bar{K}_{1t}$	$\bar{K}_{2t}$	$\bar{K}_{3t}$	$\bar{K}_{4t}$
1972	1707	1491	1734	1740	2167	4.9	114	3.6	3.2	22.5
1973	1694	1473	1727	1728	2164	5.1	121	3.8	3.4	24.2
1974	1672	1452	1709	1706	2192	5.3	128	4.0	3.5	25.6
1975	1653	1440	1711	1685	2143	5.6	136	4.2	3.7	28.0
1976	1599	1397	1662	1629	2102	6.0	143	4.4	3.8	32.3
1977	1564	1358	1636	1600	1928	6.4	151	4.6	4.0	35.5
1978	1525	1317	1598	1571	1802	6.6	160	4.8	4.2	35.7
1979	1501	1304	1569	1544	1818	6.8	168	4.9	4.3	35.0
1980	1499	1297	1579	1543	1847	7.0	176	5.0	4.5	33.6
1981	1489	1286	1587	1529	1845	7.3	183	5.2	4.7	35.0
1982	1477	1284	1577	1516	1820	7.4	190	5.3	4.9	34.7
1983	1472	1282	1572	1513	1779	7.7	196	5.3	5.0	35.4
1984	1464	1278	1579	1502	1770	7.9	203	5.4	5.2	36.8
1985	1458	1276	1565	1496	1770	8.1	209	5.6	5.4	33.7
1986	1453	1268	1570	1492	1730	8.4	216	5.9	5.7	34.0
1987	1424	1254	1534	1461	1682	8.7	224	6.3	5.9	34.2
1988	1428	1250	1542	1471	1685	9.0	233	6.6	6.1	35.2
1989	1422	1245	1546	1463	1738	9.3	241	6.7	6.2	37.1
1990	1414	1244	1535	1453	1767	9.4	249	6.8	6.3	35.8
1991	1407	1239	1538	1446	1771	9.5	258	6.7	6.4	35.3
1992	1414	1244	1547	1458	1775	9.6	267	6.7	6.4	35.4

The number of individuals considered to be active in the labor market, N_t, is the total population age 16-74. Unemployment among these individuals, U_t, is obtained from the Norwegian Labor Force Surveys. It is assumed that each unemployed wishes to work in all the above sectors.

Time series of vacancies, V_{jt}, have been derived in a rather ad hoc manner. They are based on the number of vacant jobs by occupation reported at the employment offices at the end of the year. We have then made ad hoc assumptions about the relationship (held constant throughout the period) between occupation and the demand of the above sectors so as to distribute vacancies across these sectors. The resulting numbers are multiplied by a factor to take into account that not all vacancies are reported to the employment offices and that a given vacancy can represent a larger number of available jobs.

The average number of man-hours worked in each sector is given by the variable Q_{jt} derived from the National Accounts. Table 1 shows the magnitudes involved for the variables discussed so far. The average number of hours worked per person, L_{jt} and the average amount of capital in each firm, $\bar{K}_{jt}$, are also obtained from the National Accounts and are illustrated in table 2.

From the Survey of Level of Living 1991 we find that an estimate of $\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0)$ is 9.5 and of $\mathrm{var}(w_{ik} \mid l_{ik}^* > 0)$ is 1.8. These figures make it possible to calculate the relationship $\theta_{jt}/\kappa_{jt} = 0.5$. Lacking data for most of the other years, it is assumed that this relationship applies to all sectors we are looking at and does not change over time.

The average nominal wage compensation per hour worked, W_{jt} is found in the National Accounts and is illustrated in figure 1. Wages generally have followed each other with the exception of the oil and gas sector. One might also note that public employees have had a greater growth in wage compensation than those in the sector producing non-traded goods. The general trend can in part be explained by reduced working hours.

Table 3: Calculated values of the parameter τ

	$\tau_{\cdot t}$	τ_{1t}	τ_{2t}	τ_{3t}	τ_{4t}
1972	0.086	0.040	0.21	0.11	0.21
1973	0.083	0.037	0.19	0.10	0.21
1974	0.088	0.035	0.20	0.11	0.23
1975	0.072	0.036	0.17	0.10	0.24
1976	0.073	0.036	0.15	0.11	0.26
1977	0.075	0.031	0.18	0.10	0.22
1978	0.071	0.032	0.18	0.10	0.26
1979	0.067	0.035	0.16	0.09	0.27
1980	0.069	0.031	0.19	0.09	0.26
1981	0.067	0.034	0.19	0.09	0.25
1982	0.067	0.040	0.19	0.08	0.27
1983	0.064	0.047	0.15	0.09	0.31
1984	0.070	0.046	0.16	0.10	0.29
1985	0.069	0.042	0.17	0.10	0.32
1986	0.084	0.041	0.20	0.11	0.35
1987	0.091	0.044	0.21	0.12	0.43
1988	0.091	0.055	0.21	0.12	0.49
1989	0.106	0.066	0.27	0.14	0.46
1990	0.108	0.068	0.27	0.14	0.44
1991	0.112	0.068	0.29	0.15	0.44
1992	0.113	0.071	0.28	0.16	0.47

We now have the observations needed to calculate the probabilities discussed in section 2 and some of the aggregate relationships discussed in section 5. The parameter τ can be calculated, determining the correlation between desired supply and demand. The results are given in table 3. We find that the correlation between supply and demand has fallen in the first part of the period we are looking at and risen in the latter part.

Before proceeding we must make a normalization assumption about the size of the potential economy as discussed in section 2. This is done by assuming that there are a total of 240000 potential firms, with 2000 large units in the public sector, 30000 firms producing traded goods, 200000 firms producing non-traded goods and 8000 firms in the oil and gas sector. These are ad hoc assumptions setting the number of firms one would have if all possible firms were to be active. It is also assumed that the number of potential firms is constant over the period we are looking at. This need not imply that the observed number of firms is constant, but in our case we have that almost all the potential firms are active and therefore that the number of firms is constant.

Figure 1. Average nominal worker compensation per hour. Nkr

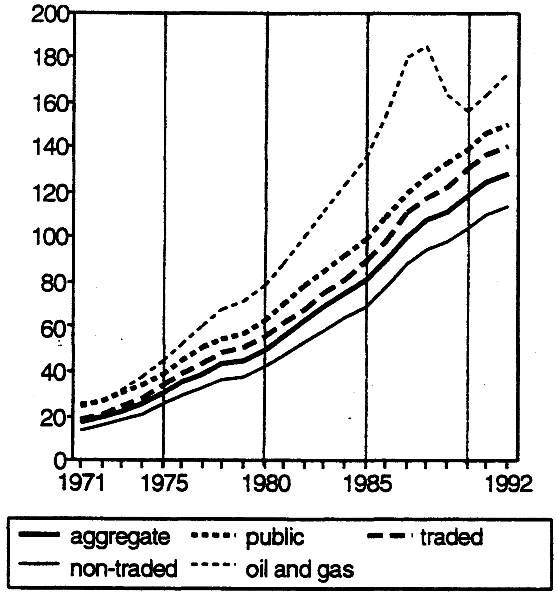


Figure 2. Probability of a match in a micro market (x 1,000,000), part 1

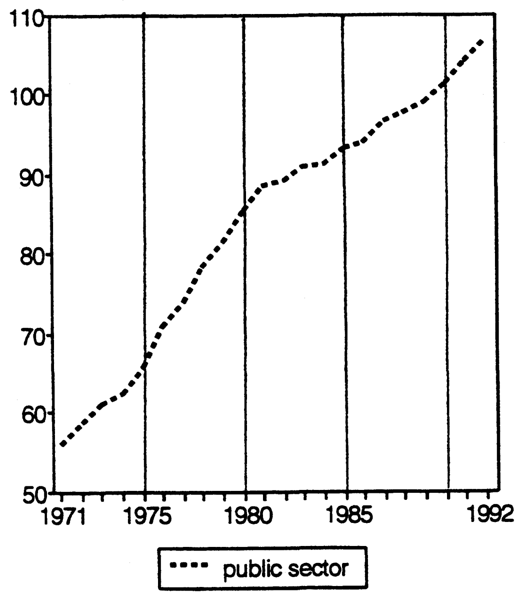
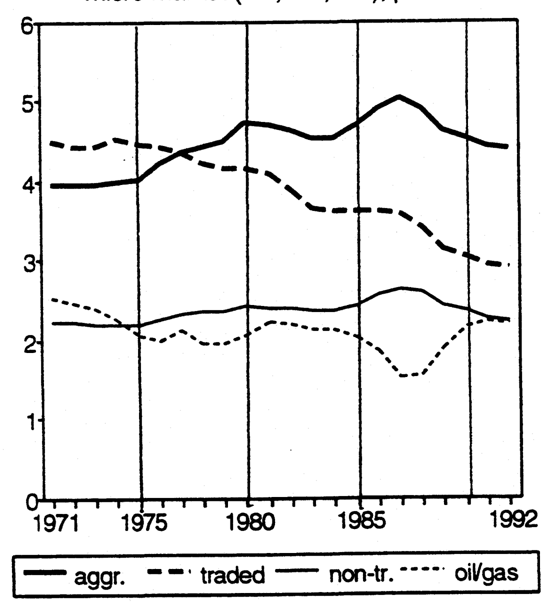


Figure 3. Probability of a match in a micro market (x 1,000,000), part 2



The development in the probability of a match in a micro market, $P(l^*_{hijk} > 0)$, is shown in figures 2 and 3 for the different sectors. This probability is by far greatest in the public sector due to the few but large units we have assumed there. It is also the only sector in which this probability is increasing, while for the non-traded sector it is declining.

Another way of viewing disequilibrium is by using the indicator given by equation (5.33) in section 5. This gives the estimated probability that supply is greater than demand in a micro market when supply is positive shown in figures 4 and 5. It takes into account both situations where there is unemployment or vacancies and disequilibrium situations where the agents are constrained in the number of hours worked. In aggregate and for the traded and non-traded goods sectors this indicator follows the business cycle showing a clear deterioration during the economic slumps of 1983/84 and during the last six years. The public sector and the oil and gas sectors do not follow the business cycle as closely. It is interesting to note that the probability of excess supply in the public sector is increasing at the same time as there is an increase in the probability of a match as seen in figure 2. This indicates that the public sector has been attracting well qualified individuals who are relatively likely to get employment there.

It is important to note that the lines in the figures denoting aggregate data are *not* an aggregation of the disaggregate lines. The aggregate line is from a model based on aggregate time series, while the disaggregate lines are from a different model based on disaggregate time series. The main difference is that in the first case the heterogeneity assumptions discussed earlier are made for the whole economy, while in the second case they are made separately for each production sector.

Figure 4. Prob. that supply is greater than demand (x1,000,000), part 1

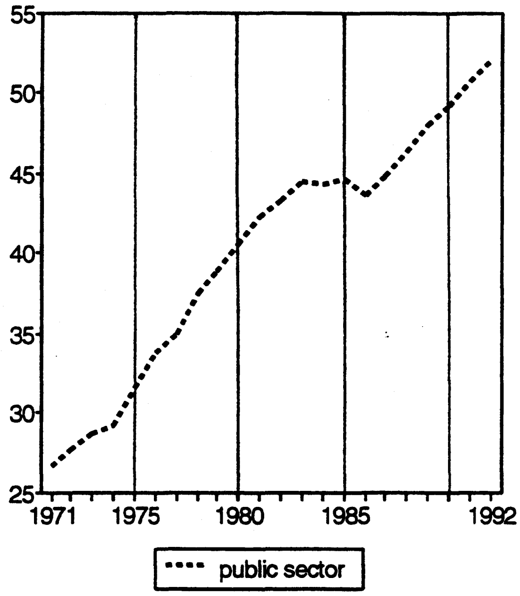


Figure 5. Prob. that supply is greater than demand (x1,000,000), part 2

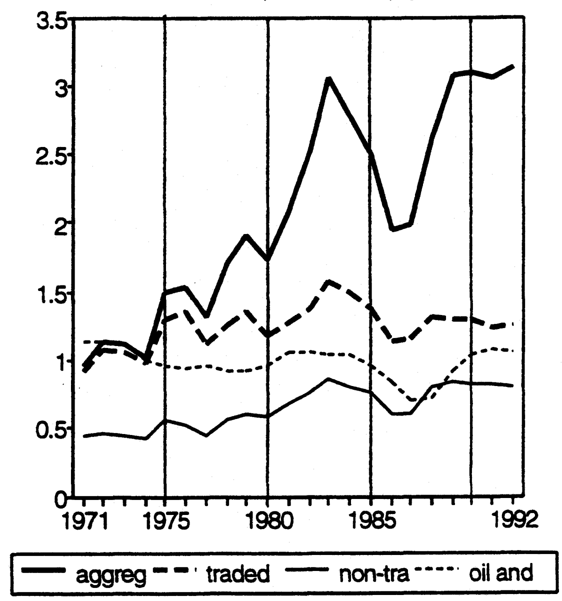


Figure 6. Nr. micro markets with pos. supply minus the nr. with pos. demand

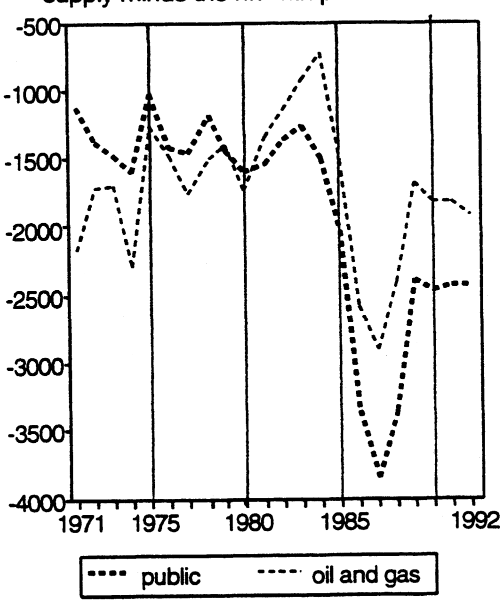
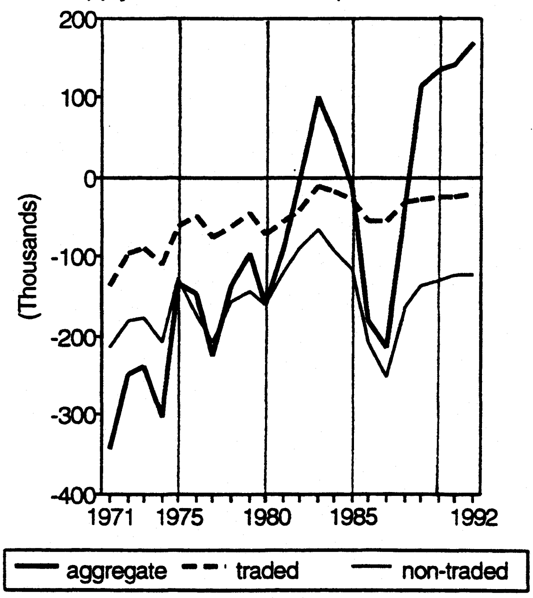


Figure 7. Nr. micro markets with pos. supply minus the nr. with pos. demand



The indicator shown in figures 4 and 5 is based on the econometric model we have discussed earlier. Another indicator, discussed in section 2, is the number of micro markets with positive supply minus the number with positive demand, $M_{jt}N_t\left[P(l^S_{hijk} > 0) - P(l^D_{hijk} > 0)\right]$, shown in figures 6 and 7. These figures basicly confirm the picture given by the indicator shown in figures 4 and 5, although it is interesting to note that the two indicators give a different picture of the development in the public sector (figures 4 and 6).

From the observed wages given in figure 1 we can calculate the theoretical wage variable $\bar{w}_{jt}$ shown in figures 8 and 9. The public sector behaves differently from the others. The theoretical wage is increasing much more rapidly there than in the other sectors. From figure 1 it is apparent that this is not just a reflection of the increase in the observed wages, but is also also explained by the econometric model we have developed. Increased relative wages are needed to draw workers to the public sector so that increased matching probabilities (figure 2), increasing employment (table 1) and increasing excess supply (figure 4) will be consistent with each other. From figure 8 it is apparent that the non-traded goods sector has grown less attractive over the period we are looking at.

Figure 8. The theoretical wage variable w, part 1

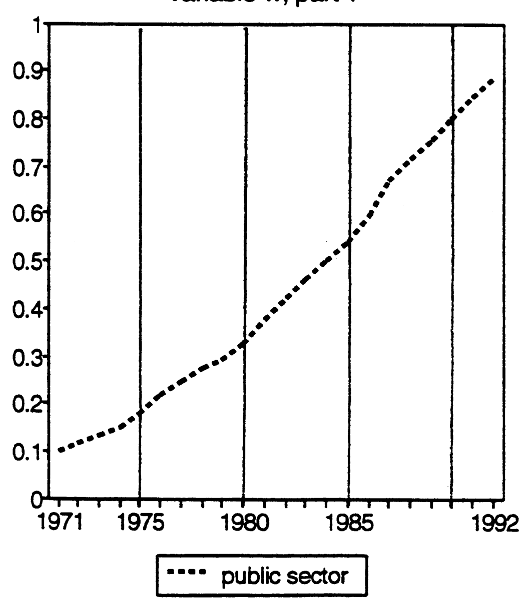
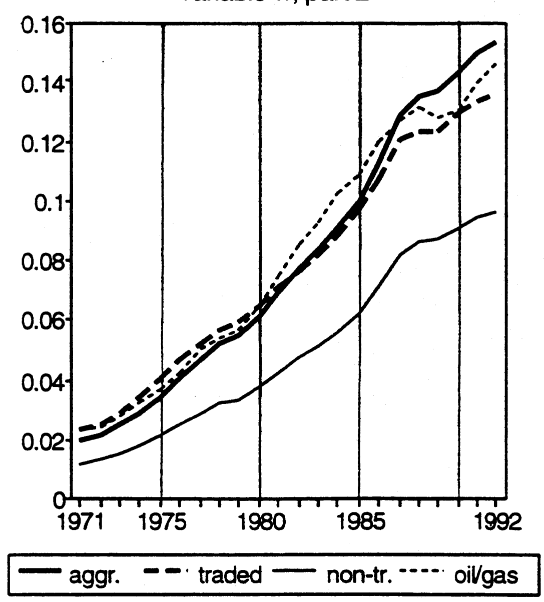


Figure 9. The theoretical wage variable w, part 2



We now consider estimation based on the model presented in the preceding parts of the paper. The equations we wish to estimate are given by

$$S^l_{jt} = \frac{1}{\alpha_{2j}} \log\left((1-t_t)\bar{w}_{jt}\right) - \frac{\bar{a}_j}{\alpha_{2j}} - \frac{\alpha_1}{\alpha_{2j}} \bar{m}_{ct} + \vartheta_{1jt}, \tag{6.1}$$

$$D^l_{jt} = -\frac{1}{\beta_{2j}} \log \bar{w}_{jt} + \frac{\bar{b}_j}{\beta_{2j}} + \frac{\beta_{1j}}{\beta_{2j}} \bar{m}_{fjt} + \frac{\beta_{3j}}{\beta_{2j}} \bar{K}_{jt} + \vartheta_{2jt}, \tag{6.2}$$

for the labor markets and

$$D^x_{ct} = -\frac{1}{\alpha_3} \log \bar{p}_t + \frac{\bar{c}_c}{\alpha_3} + \frac{\alpha_1}{\alpha_3} \bar{m}_{ct} + \vartheta_{5t}, \tag{6.3}$$

$$D^x_{fjt} = -\frac{1}{\beta_{4j}} \log \bar{p}_t + \frac{\bar{c}_{fj}}{\beta_{4j}} + \frac{\beta_1}{\beta_{4j}} \bar{m}_{fjt} + \frac{\beta_{5j}}{\beta_{4j}} \bar{K}_{jt} + \vartheta_{5jt}, \tag{6.4}$$

for the goods markets, where $\bar{p}_t$ is the consumer price index and $\bar{p}_{jt}$ is a price index for the inputs used by firms in sector j. The variables S^l_{jt}, D^l_{jt} and D^x_{ct} are calculated using equations (5.19), (5.20), and (5.39). The variable D^x_{ft} is derived in an analogous manner to D^x_{ct}. The goods market is treated as a single aggregate market. As mentioned in section 5, we also specify money demand equations as functions of variables which function as instrument variables in estimating the above equations. We choose to specify these by

$$\begin{aligned} \bar{m}_{ct} = {} & \gamma_1 + \gamma_2 \cdot \log(G_t) + \gamma_3 \cdot \log(\bar{p}_t) + \sum_j^4 \gamma_{4j} \cdot \log\left((1-t)\bar{w}_{jt}\right) \\ & + \gamma_5 \cdot \log\left(\mathrm{E}(C^\circ_{it})\right) + \vartheta_{3t}, \end{aligned} \tag{6.5}$$

for the individuals and

$$\begin{aligned} \bar{m}_{fjt} = {} & \gamma_{6j} + \gamma_{7j} \cdot \bar{K}_{jt} + \gamma_{8j} \cdot \log(\bar{p}_{jt}) + \gamma_{9j} \cdot \log(\bar{w}_{jt}) \\ & + \gamma_{10j} \cdot \log\left(\mathrm{E}(C^\circ_{jkt})\right) + \vartheta_{3jt} \end{aligned} \tag{6.6}$$

for the firms, where G_t is the average consumption of public goods. For the consumer the money variable $\bar{m}_{ct}$ mainly reflects the changes in fixed capital formation from the National Accounts. Even though this is not a normal money concept, it must be remembered that money represents the whole financial system in our model.

Table 4: OLS estimation of money demand 1976-1991 (16 observations)

Parameter	Estimated	Standard error	t-statistic
$\bar{m}_{ct} = \gamma_1 + \gamma_2 \cdot \log(G_t) + \gamma_3 \cdot \log(\bar{p}_t) + \sum_j^4 \gamma_{4j} \cdot \log((1-t)\bar{w}_{jt}) + \gamma_5 \cdot \log(\mathrm{E}(C^{\circ}_{it})) + \vartheta_{3ct}$ (R^2: 1.00 DW: 1.50)			
γ_1	-10.81	3.80	-2.84
γ_2	0.69	0.33	2.09
γ_3	-0.66	0.31	-2.09
γ_{41}	0.46	0.22	2.10
γ_{42}	-0.83	0.40	-2.07
γ_{43}	-0.09	0.36	-0.26
γ_{44}	0.27	0.92	3.00
γ_5	1.11	0.26	4.25
$\bar{m}_{f1t} = \gamma_{61} + \gamma_{71} \cdot \bar{K}_{1t} + \gamma_{81} \cdot \log(\bar{p}_{1t}) + \gamma_{91} \cdot \log(\bar{w}_{1t}) + \gamma_{101} \cdot \log(\mathrm{E}(C^{\circ}_{1kt})) + \vartheta_{31t}$ (R^2: 1.00 DW: 1.53)			
γ_{61}	8.21	5.15	1.60
γ_{71}	-0.53	0.24	-2.19
γ_{81}	-0.74	0.14	-5.37
γ_{91}	0.61	0.18	3.44
γ_{101}	1.10	0.03	32.3
$\bar{m}_{f2t} = \gamma_{62} + \gamma_{72} \cdot \bar{K}_{2t} + \gamma_{82} \cdot \log(\bar{p}_{2t}) + \gamma_{92} \cdot \log(\bar{w}_{2t}) + \gamma_{102} \cdot \log(\mathrm{E}(C^{\circ}_{2kt})) + \vartheta_{32t}$ (R^2: 0.92 DW: 1.22)			
γ_{62}	-46.17	7.13	-6.48
γ_{72}	1.35	0.58	2.32
γ_{82}	0.53	0.18	2.94
γ_{92}	-2.52	0.29	-8.69
γ_{102}	2.10	0.36	5.88
$\bar{m}_{f3t} = \gamma_{63} + \gamma_{73} \cdot \bar{K}_{3t} + \gamma_{83} \cdot \log(\bar{p}_{3t}) + \gamma_{93} \cdot \log(\bar{w}_{3t}) + \gamma_{103} \cdot \log(\mathrm{E}(C^{\circ}_{3kt})) + \vartheta_{33t}$ (R^2: 0.99 DW: 1.50)			
γ_{63}	5.31	4.00	1.33
γ_{73}	-0.38	0.24	-1.60
γ_{83}	0.06	0.07	0.82
γ_{93}	-0.006	0.09	-0.07
γ_{103}	1.03	0.08	13.1
$\bar{m}_{f4t} = \gamma_{64} + \gamma_{74} \cdot \bar{K}_{4t} + \gamma_{84} \cdot \log(\bar{p}_{4t}) + \gamma_{94} \cdot \log(\bar{w}_{4t}) + \gamma_{104} \cdot \log(\mathrm{E}(C^{\circ}_{4kt})) + \vartheta_{34t}$ (R^2: 0.99 DW: 2.18)			
γ_{64}	24.38	21.7	1.13
γ_{74}	-1.25	1.11	-1.12
γ_{84}	1.56	0.41	3.82
γ_{94}	0.42	0.81	0.52
γ_{104}	0.84	0.45	1.87

The resources available at the beginning of the period, $\mathrm{E}(C^{\circ}_{it})$, are also obtained from the National Accounts. The money variable of the firm $\bar{m}_{fjt}$ is derived in a similar manner to that of the individual and is based on fixed capital formation from the National Accounts while taking into account our assumption that the firm's revenues accrue in the period after production has taken place. To mention all the minor ad hoc assumptions made in deriving these financial variables would take too much space, but it is important to note that these variables are not very reliable. The money variables can be constructed in many different ways based on different interpretations of what they represent (to what degree they represent "pure" money and to what degree they represent a complicated financial system). The tax rate t_t is the average tax rate faced by individuals in a given year including contributions to the social security system.

The estimation results vary greatly across the different sectors. In general we get better results for the labor markets than for the goods market. As mentioned earlier, these results must just be taken as illustrations of the econometric model discussed in the paper. The model itself has too little flexibility to give good estimates in all markets and the data are in many cases poor.

Estimation of money demand was done using OLS, mainly because the short time series available required a parsimonious econometric model. The results are presented in table 4. Money demand of individuals and firms in the public and traded goods sector are estimated reasonably, though the Durbin-Watson statistic indicates the presence of positive autocorrelation. The income variable C° is found in general to be the most important determinant of money demand. Since these demand functions represent reduced form equations resulting from a complicated system, it is difficult to be sure which sign all the coefficients should have. Even so, one would expect most of the parameters to be positive.

Table 5: ML estimation of individual labor supply and demand for goods 1976-1991

Parameter	Estimated	Standard error	t-statistic
α_1	-0.56516	0.15406	-3.67
α_{21}	0.00085	0.00015	5.71
α_{22}	0.00172	0.00073	2.36
α_{23}	0.00063	0.00007	9.23
α_{24}	0.00038	0.00010	3.79
α_3	-0.00007	0.00003	-2.37
$\bar{a}_1$	15.26	2.57	5.95
$\bar{a}_2$	37.11	14.71	2.52
$\bar{a}_3$	15.21	2.19	6.95
$\bar{a}_4$	12.43	3.39	3.66
$\bar{c}_c$	- 0.43	2.74	-0.16

$$S^l_{1t} = \frac{1}{\alpha_{21}}\log\left((1-t_t)\bar{w}_{1t}\right) - \frac{\bar{a}_1}{\alpha_{21}} - \frac{\alpha_1}{\alpha_{21}}\bar{m}_{ct} + \vartheta_{11t}$$

(R^2: 0.89 DW: 0.49)

$$S^l_{2t} = \frac{1}{\alpha_{22}}\log\left((1-t_t)\bar{w}_{2t}\right) - \frac{\bar{a}_2}{\alpha_{22}} - \frac{\alpha_1}{\alpha_{22}}\bar{m}_{ct} + \vartheta_{12t}$$

(R^2: 0.44 DW: 0.89)

$$S^l_{3t} = \frac{1}{\alpha_{23}}\log\left((1-t_t)\bar{w}_{3t}\right) - \frac{\bar{a}_2}{\alpha_{23}} - \frac{\alpha_1}{\alpha_{23}}\bar{m}_{ct} + \vartheta_{13t}$$

(R^2: 0.94 DW: 0.80)

$$S^l_{4t} = \frac{1}{\alpha_{24}}\log\left((1-t_t)\bar{w}_{4t}\right) - \frac{\bar{a}_2}{\alpha_{24}} - \frac{\alpha_1}{\alpha_{24}}\bar{m}_{ct} + \vartheta_{14t}$$

(R^2: 0.70 DW: 0.89)

$$D^x_{ct} = -\frac{1}{\alpha_3}\log\bar{p}_t + \frac{\bar{c}_c}{\alpha_3} + \frac{\alpha_1}{\alpha_3}\bar{m}_{ct} + \vartheta_{5t}$$

(R^2: 0.67 DW: 0.82)

$\alpha_1 = 0$

Parameter	Estimated	Standard error	t-statistic
α_1	0		
α_{21}	0.00083	0.00016	5.13
α_{22}	0.00162	0.00074	2.20
α_{23}	0.00061	0.00007	8.48
α_{24}	0.00036	0.00099	3.64
α_3	-0.00007	0.00003	-2.47
$\bar{a}_1$	8.48	2.03	4.19
$\bar{a}_2$	28.75	14.66	1.96
$\bar{a}_3$	8.39	1.47	5.70
$\bar{a}_4$	5.44	2.50	2.18
$\bar{c}_c$	- 6.63	2.56	-2.59

An especially worrying result is that the parameter γ_{42} is significantly negative (the t-statistics should of course be used very carefully considering the many indications that our model is incorrectly specified). This implies that an increase in $\bar{w}_{3t}$ will increase the individuals' supply of labor to sectors other than sector 3. (a decrease in m_{ct} decreases the reservation wage in these sectors). The estimation of money demand in the non-traded and oil and gas sectors leads to few significant estimates. This together with the high R^2 indicates serious multicollinearity problems.

Estimation of the individuals' supply of labor to the four sectors has been done using maximum-likelihood estimation with the estimated results for money from table 3 being used to calculate instruments for the endogenous money variables. These estimation results are given in table 5. Note that the common parameter α_1 is derived simultaneously from these equations. The fit is fairly good according to the t-statistics with all the parameters except $\bar{c}_c$ being significant. If we consider the parameter restrictions which applied to the utility and production functions discussed earlier, then we see that the parameter α_1, relating the influence of money demand on labor supply, has the wrong sign. The individuals' labor has therefore also been estimated under the restriction that $\alpha_1 = 0$. Results from this estimation are given in the lower part of table 5. The R^2 and Durbin-Watson statistics are approximately the same in the two cases. Both estimations lead to estimates of the parameters α_{21} to α_{24} of the right sign and size. The Durbin-Watson statistics imply that there is positive autocorrelation in most the estimated equations. This also indicates that the variances are probably underestimated. Since the results presented here are only meant to be illustrative, we do not pursue this using more sophisticated estimation techniques. Instead we simply conclude that important shortcomings of the model are the dynamic properties and the specification and interpretation of money demand.

Table 6: ML estimation of the firms' demand for labor and goods 1976-1991

Parameter	Estimated	Standard error	t-statistic
$D^l_{1t} = -\frac{1}{\beta_{21}}\log\bar{w}_{1t} + \frac{\bar{b}_1}{\beta_{21}} + \frac{\beta_{11}}{\beta_{21}}\bar{m}_{f1t} + \frac{\beta_{31}}{\beta_{21}}\bar{K}_{1t} + \vartheta_{21t}$ (R^2: 0.95 DW: 0.91)			
$D^x_{f1t} = -\frac{1}{\beta_{41}}\log\bar{p}_t + \frac{\bar{c}_{f1}}{\beta_{41}} + \frac{\beta_{11}}{\beta_{41}}\bar{m}_{f1t} + \frac{\beta_{51}}{\beta_{41}}\bar{K}_{1t} + \vartheta_{51t}$ (R^2: 0.99 DW: 1.49)			
β_{11}	0.3618	0.1179	3.07
β_{21}	-0.0003	0.0002	-1.59
β_{31}	0.7054	0.8533	0.83
β_{41}	0.12E-06	0.3E-07	4.10
β_{51}	4.9022	0.7618	6.43
$\bar{b}_1$	-17.30	16.49	-1.05
$\bar{c}_{f1}$	-98.37	15.15	-6.49
$D^l_{2t} = -\frac{1}{\beta_{22}}\log\bar{w}_{2t} + \frac{\bar{b}_2}{\beta_{22}} + \frac{\beta_{12}}{\beta_{22}}\bar{m}_{f2t} + \frac{\beta_{32}}{\beta_{22}}\bar{K}_{2t} + \vartheta_{22t}$ (R^2: 0.60 DW: 1.14)			
β_{12}	-0.9215	0.6179	-1.49
β_{22}	0.0004	0.0004	1.04
β_{32}	3.4040	0.7304	4.66
$\bar{b}_2$	-49.44	11.67	-4.24
$D^l_{3t} = -\frac{1}{\beta_{23}}\log\bar{w}_{3t} + \frac{\bar{b}_3}{\beta_{23}} + \frac{\beta_{13}}{\beta_{23}}\bar{m}_{f3t} + \frac{\beta_{33}}{\beta_{23}}\bar{K}_{3t} + \vartheta_{23t}$ (R^2: 0.97 DW: 1.31)			
β_{13}	3.9434	5.2671	0.75
β_{23}	-0.0009	0.0012	-0.75
β_{33}	-2.0267	6.2585	-0.32
$\bar{b}_3$	-14.51	42.60	-0.34
$D^l_{4t} = -\frac{1}{\beta_{24}}\log\bar{w}_{4t} + \frac{\bar{b}_4}{\beta_{24}} + \frac{\beta_{14}}{\beta_{24}}\bar{m}_{f4t} + \frac{\beta_{34}}{\beta_{24}}\bar{K}_{4t} + \vartheta_{24t}$ (R^2: 0.66 DW: 0.80)			
$D^x_{f4t} = -\frac{1}{\beta_{44}}\log\bar{p}_t + \frac{\bar{c}_{f4}}{\beta_{44}} + \frac{\beta_{14}}{\beta_{44}}\bar{m}_{f4t} + \frac{\beta_{54}}{\beta_{44}}\bar{K}_{4t} + \vartheta_{54t}$ (R^2: 0.97 DW: 1.71)			
β_{14}	0.2596	0.0127	20.39
β_{24}	-0.0001	0.6E-04	-1.84
β_{34}	-0.9725	1.1724	-0.83
β_{44}	0.5E-07	0.1E-07	4.60
β_{54}	0.1682	0.3646	0.43
$\bar{b}_4$	13.15	21.55	0.61
$\bar{c}_{f4}$	-6.79	6.31	-1.08

If one views the inclusion of money in the utility and production functions as arising from an underlying dynamic optimization behavior, as in for example section 3.2 of Andreassen (1993), then the above two shortcomings can both be seen to reflect the same fact that the model does not adequately reflect the underlying dynamics. Money enters the utility and production functions (which more properly should be viewed as indirect functions where intertemporal considerations have been maximized out) even though money has no intrinsic value. It's importance is as a means of transferring purchasing power over time.

A tentative conclusion is therefore that future research should concentrate on extending the above framework to a dynamic setting. In general this will involve looking at the formation of expectations, price and wage determination, and at how these are reflected in an economy's financial markets.

While far from satisfactory, the estimation results for the individuals' labor supply is better than what was found for the firms' demand for labor. Table 6 gives the results from maximum-likelihood estimation for the firms' demand for labor and goods.

For the public sector and the oil and gas sector it was possible to simultaneously estimate the demand for labor and for goods. In the case of the traded and non-traded goods sectors this leads to some coefficients not being estimated due to singularity of the data. For these two sections we therefore only report single equation estimates of the labor demand equations.

The results are generally characterized by low significance levels, especially in the non-traded goods sector, and low Durbin-Watson statistics, implying positive autocorrelation. As for the individuals' supply of labor and demand for goods, the high R^2-s indicate high multicollinearity in the data. One might note that the money variable seems better behaved than in the case of the individuals and that the wage does not significantly affect labor demand.

The conclusions drawn from this are largely the same as those derived from looking at the equations characterizing the individuals' behavior. There is a need for a better understanding of the underlying dynamics. In the case of the firms this includes taking explicitly into account investment behavior, leading to the capital variable $\bar{K}$ capturing some of the firms dynamic considerations.

An attempt was made to estimate both the supply and demand for labor at the aggregate level. For the individuals' supply of labor and demand for goods the maximum-likelihood procedure did not converge, and for the firms' demand for labor and goods some coefficients were not estimated. Disaggregation lead therefore to more satisfactory results. One might therefore speculate that further disaggregation might lead to a further improvement in results. One might for example look at several types of goods, instead of aggregating all private goods together, or disaggregate individual labor supply by educational level.

The above results illustrate how the method presented can be used, but it is apparent that much further research is needed before it can become a useful econometric tool. In addition it is necessary to have better data than those used above.

7 Summary

This paper has presented an aggregation method based on a stochastic approach using virtual prices which has lead to aggregate relationships which can be used for estimation and to describe disequilibrium in the labor market. The method developed is explicitly multi-market, enabling one to analyze different submarkets simultaneously. The labor market was divided into aggregate submarkets, where we assumed that a given subset of the parameters in the utility and production functions were equal for the agents within one submarket. The remaining parameters, money holdings and the capital stock were assumed to be distributed across each submarket in such a way that the conditional demand and supply functions (conditional on a set of transactions) were extreme value distributed. These assumptions allow us to aggregate across each labor submarket, making it possible to identify the parameters in the model using mainly aggregate data. Identification was obtained using assumptions of separability and a knowledge of the reduced form structure of the demand for money.

The econometric model derived in this paper rests on a set of fairly strict assumptions. These may seem implausible, but enable us to derive fairly simple aggregate relationships based on explicit utility and profit maximization. One of the fundamental insights of past work on aggregation is that almost any macro economic structure can be generated by a "reasonable" micro economic structure. It is possible that it in the future will be deemed preferable to specify the macroeconomic structure directly instead of basing it on aggregation over micro units, but at present we know too little about what type of aggregate macroeconomic structure should be used. In an economy where prices do not clear the markets and where there is rationing, the specification and interpretation of aggregate relationships depends heavily on how rationing is distributed among individuals and firms. This paper can be viewed as a tentative step towards a better understanding of how different distributions of technology and tastes (and thereby of rationing) influence such aggregate relationships. The results obtained indicate that the main shortcoming of the present approach is the lack of dynamic specification. A more sophisticated modeling of the agents' intertemporal behavior, including the formation of expectations, price and wage formation and investment behavior, would therefore seem to be an important area for future research.

A Deriving aggregate expressions using the GEV distribution

A.1 The probabilities discussed in section 5

In the following we derive the aggregate expressions presented in section 5. In the same manner as in that section, we drop the subscripts denoting type of consumer and type of firm. In section 4 we found the following equations (equations (4.22) and (4.24):

$$\log(l_{ik}^S + 1) - \kappa \cdot \mu_{ik} = S - \varepsilon_{1ik}, \tag{A.1}$$

and

$$\log(l_{ik}^D + 1) - \kappa \cdot \mu_{ik} = D - \varepsilon_{2ik}, \tag{A.2}$$

where

$$S = \frac{1}{\alpha_2}\left[\log((1-t_1)\bar{w}) - \bar{a} - \alpha_1 \bar{m}_c\right], \tag{A.3}$$

$$D = \frac{1}{\beta_2}\left[-\log\bar{w} + \bar{b} + \beta_1 \bar{m}_f + \beta_3 \bar{K}\right], \tag{A.4}$$

$$\varepsilon_{1ik} = (\kappa + \frac{1}{\alpha_2})\mu_{ik} - \frac{1}{\alpha_2}(u_{2ik} + \alpha_1 u_{1i}), \tag{A.5}$$

$$\varepsilon_{2ik} = (\kappa - \frac{1}{\beta_2})\mu_{ik} + \frac{1}{\beta_2}(v_{2ik} + \beta_1 v_{1k} + \beta_{3k} v_{4k}). \tag{A.6}$$

We now start with the probability given in equation (5.5),

$$\begin{aligned}
&\mathrm{P}(l_{ik}^* > \tilde{z} \mid l_{ik}^* > 0) \\
&= \mathrm{P}(\max(\varepsilon_1 - S, \varepsilon_2 - D) < \varepsilon_3 - z \mid \max(\varepsilon_1 - S, \varepsilon_2 - D) < \varepsilon_3) \\
&= \frac{\mathrm{P}(\varepsilon_1 - S < \varepsilon_3 - z, \varepsilon_2 - D < \varepsilon_3 - z)}{\mathrm{P}(\varepsilon_1 - S < \varepsilon_3, \varepsilon_2 - D < \varepsilon_3)}
\end{aligned} \tag{A.7}$$

where $z = \log(\tilde{z} + 1)$ and l^*_{ik} is the transacted amount of labor. The stochastic variables ε_1, ε_2, and ε_3 are jointly distributed according to the generalized extreme value distribution expressed as

$$F(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta) = \exp(-H(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta)). \tag{A.8}$$

where $H(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta)$ is a nonnegative function which has the property $e^{-z}\, H(\gamma_1 - z, \gamma_2 - z, \gamma_3 - z) = H(\gamma_1, \gamma_2, \gamma_3)$. As any one of the arguments go towards infinity, H will do so also. The marginal distributions of the variables ε_1, ε_2, and ε_3 are extreme valued. The extreme value distribution can be obtained as the limiting distribution (as $n \to \infty$) of the largest value among n independent random variables each having the same distribution. In this sense the extreme value distribution plays the same role concerning maxima (minima) as the normal distribution plays concerning averages. For a more detailed discussion of the extreme value distribution see Galambos ((1978).

For analytical convenience we introduce the transformed variables $\delta_1 = (\varepsilon_1 - S)/\theta$, $\delta_2 = (\varepsilon_2 - D)/\theta$, and $\delta_3 = (\varepsilon_3 - z)/\theta$. The distribution function F^* for these transformed variables can be written[6]

$$\begin{aligned} &F^*(\delta_1, \delta_2, \delta_3) \\ &\quad = F(\delta_1 + S/\theta, \delta_2 + D/\theta, \delta_3 + z/\theta) \\ &\quad = \exp(-H(\delta_1 + S/\theta, \delta_2 + D/\theta, \delta_3 + z/\theta)). \end{aligned} \tag{A.9}$$

The first step is to find an expression for the numerator in equation (A.7), $\mathrm{P}(\varepsilon_1 - S < \varepsilon_3 - z, \varepsilon_2 - D < \varepsilon_3 - z) = \mathrm{P}(\delta_1 < \delta_3, \delta_2 < \delta_3)$. We have that

[6]Note that the Jacobian determinant of the transformation from the δ-s to the ε-s equals 1.

$$
\begin{aligned}
&\mathrm{P}(\delta_1 < \delta_3, \delta_2 < \delta_3) \\
&\quad = \int_{-\infty}^{\infty} \mathrm{P}(\delta_1 \le y, \delta_2 \le y, \delta_3 \in dy) \\
&\quad = \int_{-\infty}^{\infty} F_3^*(y,y,y)dy \\
&\quad = -\int_{-\infty}^{\infty} \exp(-e^{-y} H(S/\theta, D/\theta, 0)) e^{-y} dy \, H_3(S/\theta, D/\theta, 0), \\
&\quad = -\frac{H_3(S/\theta, D/\theta, 0)}{H(S/\theta, D/\theta, 0)} \Big|_{-\infty}^{\infty} \exp\left(-e^{-y} H(S/\theta, D/\theta, 0)\right) \\
&\quad = -\frac{H_3(S/\theta, S/\theta, 0)}{H(S/\theta, D/\theta, 0)}. \qquad \text{(A.10)}
\end{aligned}
$$

where we have used the property that $e^{-y} H(\gamma_1 - y, \gamma_2 - y, \gamma_3 - y) = H(\gamma_1, \gamma_2, \gamma_3)$. The functions F_3^* and H_3 are the derivatives of F^* and H with respect to the third argument.

According to section 5 the function H is specified as

$$
H(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta) = \left(e^{-\varepsilon_1/\theta\tau} + e^{-\varepsilon_2/\theta\tau}\right)^{\tau} + e^{-\varepsilon_3/\theta}, \qquad \text{(A.11)}
$$

where $\tau > 0$ is a parameter. This specification implies that the stochastic variables ε_1 and ε_2 are independent of ε_3 and that $\operatorname{var}\varepsilon_1 = \operatorname{var}\varepsilon_2 = \operatorname{var}\varepsilon_3$. The interpretation of τ is as $\tau^2 = 1 - \operatorname{corr}(\varepsilon_1, \varepsilon_2)$.

From equations (A.10) and (A.11) we obtain

$$
\mathrm{P}(\varepsilon_1 - S < \varepsilon_3 - z, \varepsilon_2 - D < \varepsilon_3 - z) = \frac{e^{-z/\theta}}{(e^{-S/\theta\tau} + e^{-D/\theta\tau})^{\tau} + e^{-z/\theta}}. \qquad \text{(A.12)}
$$

The probability of l_{ik}^* being positive is derived by setting $z = 0$ ($\tilde{z} = 0 \Rightarrow z = 0$) in the expression above,

$$
\mathrm{P}(l_{ik}^* > 0) = \mathrm{P}(\varepsilon_1 - S < \varepsilon_3, \varepsilon_2 - D < \varepsilon_3)
$$

$$= \frac{1}{1+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}}, \tag{A.13}$$

and the conditional probability $\mathrm{P}(l_{ik}^{*} > \tilde{z} \mid l_{ik}^{*} > 0)$ is thereby given by

$$\begin{aligned} \mathrm{P}(l_{ik}^{*} > \tilde{z} \mid l_{ik}^{*} > 0) &= \frac{1}{\mathrm{P}(l_{ik}^{*} > 0)} \cdot \frac{e^{-z/\theta}}{e^{-z/\theta}+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}} \\ &= \frac{1}{\mathrm{P}(l_{ik}^{*} > 0)} \cdot \frac{e^{-\log(\tilde{z}+1)/\theta}}{e^{-\log(\tilde{z}+1)/\theta}+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}} \end{aligned} \tag{A.14}$$

The probability $\mathrm{P}(\log(l_{ik}^{*}+1) > h \mid l_{ik}^{*} > 0)$ can be found by transforming the variable $\tilde{z}$ to $h = \log(\tilde{z}+1)$ to get

$$\begin{aligned} \mathrm{P}(\log(l_{ik}^{*}+1) > h \mid l_{ik}^{*} > 0) &= \frac{1}{\mathrm{P}(l_{ik}^{*} > 0)} \frac{-H_3(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\exp(h)-1}{\theta})}{H(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\exp(h)-1}{\theta})} \\ &= \frac{1}{\mathrm{P}(l_{ik}^{*} > 0)} \cdot \frac{e^{-h/\theta}}{e^{-h/\theta}+\left(e^{-S/\theta\tau}+e^{-D/\theta\tau}\right)^{\tau}}. \end{aligned} \tag{A.15}$$

Earlier in the paper we defined the Clower supply and demand variables l_{ik}^{S} and l_{ik}^{D}, which denote desired supply and demand given the realized transactions of the agents. Following the same method as above we can derive the probabilities that these conditional supplies and demands are positive as

$$\begin{aligned} \mathrm{P}(l_{ik}^{S} > 0) &= \mathrm{P}(\varepsilon_1 - S < \varepsilon_3) \\ &= \frac{-H_3(\frac{S}{\theta}, \infty, 0)}{H(\frac{S}{\theta}, \infty, 0)} \\ &= \frac{1}{1+e^{-S/\theta}} \end{aligned} \tag{A.16}$$

and

$$\begin{aligned} \mathrm{P}(l_{ik}^{D} > 0) &= \mathrm{P}(\varepsilon_2 - D < \varepsilon_3) \\ &= \frac{-H_3(\infty, \frac{D}{\theta}, 0)}{H(\infty, \frac{D}{\theta}, 0)} \end{aligned}$$

$$= \frac{1}{1+e^{-D/\theta}}. \tag{A.17}$$

The probability that supply is greater than demand in a micro market when supply is greater than zero is given by

$$\begin{aligned}
& \mathrm{P}(l_{ik}^S > l_{ik}^D, l_{ik}^S > 0) \\
&= \mathrm{P}(\varepsilon_{1ik} - S < \varepsilon_{2ik} - D, \varepsilon_{1ik} - S < \varepsilon_{3ik}) \\
&= \mathrm{P}(\varepsilon_{1ik} - S < \varepsilon_{3ik}) - \mathrm{P}(\varepsilon_{1ik} - S > \varepsilon_{2ik} - D, \varepsilon_{1ik} - S < \varepsilon_{3ik}) \\
&= \mathrm{P}(\varepsilon_{1ik} - S < \varepsilon_{3ik}) - \mathrm{P}(\varepsilon_{1ik} - S > \varepsilon_{2ik} - D) \\
&\quad + \mathrm{P}(\varepsilon_{1ik} - S > \varepsilon_{2ik} - D, \varepsilon_{1ik} - S > \varepsilon_{3ik}) \\
&= \mathrm{P}(l_{ik}^S > 0) - \left(1 - \mathrm{P}(l_{ik}^S > l_{ik}^D)\right) + \mathrm{P}(l_{ik}^S < l_{ik}^D, l_{ik}^S < 0)
\end{aligned} \tag{A.18}$$

The probability $\mathrm{P}(l_{ik}^S > 0)$ was found above and $\mathrm{P}(l_{ik}^S > l_{ik}^D)$ is given by

$$\begin{aligned}
& \mathrm{P}(l_{ik}^S > l_{ik}^D) \\
&= \mathrm{P}(\varepsilon_1 - S < \varepsilon_2 - D) \\
&= \mathrm{P}(\delta_1 < \delta_2) \\
&= \int_{-\infty}^{\infty} \mathrm{P}(\delta_1 < z, \delta_2 \in dz) \\
&= \int_{-\infty}^{\infty} F_2^*(z, z, \infty) dz \\
&= -\frac{H_2(\frac{S}{\theta}, \frac{D}{\theta}, \infty)}{H(\frac{S}{\theta}, \frac{D}{\theta}, \infty)} \\
&= \frac{\exp(-D/(\theta\,\tau))}{\exp(-S/(\theta\,\tau)) + \exp(-D/(\theta\,\tau))} \\
&= \frac{1}{1 + (\exp(D-S))^{1/(\theta\tau)}}.
\end{aligned} \tag{A.19}$$

The last probability in equation (A.18) is given by

$$\mathrm{P}(l_{ik}^S < l_{ik}^D, l_{ik}^S < 0) = \mathrm{P}(\varepsilon_{1ik} - S > \varepsilon_{2ik} - D, \varepsilon_{1ik} - S > \varepsilon_{3ik})$$

$$
\begin{aligned}
&= \mathrm{P}(\delta_2 < \delta_1, \delta_3 < \delta_1) \\
&= \frac{-H_1(S/\theta, D/\theta, 0)}{H(S/\theta, D/\theta, 0)} \\
&= \frac{(e^{-S/(\theta\tau)} + e^{-D/(\theta\tau)})^{\tau-1}\, e^{-S/(\theta\tau)}}{(e^{-S/(\theta\tau)} + e^{-D/(\theta\tau)})^{\tau} + 1} \qquad \text{(A.20)}
\end{aligned}
$$

Substituting in from equations (A.16), (A.19), and (A.20) into equation (A.18) and rearranging gives us

$$
\begin{aligned}
&\mathrm{P}(l_{ik}^S > l_{ik}^D, l_{ik}^S > 0) \\
&\quad = \mathrm{P}(l_{ik}^S > 0) - \left(1 - \mathrm{P}(l_{ik}^S > l_{ik}^D)\right) \mathrm{P}(l_{ik}^* > 0). \qquad \text{(A.21)}
\end{aligned}
$$

A.2 The expected values $\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0)$ and $\mathrm{E}(w_{ik} \mid l_{ik}^* > 0)$

We now derive the expected value of l_{ik}^* when $l_{ik}^* > 0$. Using the results above we have[7]:

$$
\begin{aligned}
\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0) &= \int_0^\infty \mathrm{P}(l_{ik}^* > \tilde{z} \mid l_{ik}^* > 0)\, d\tilde{z} \\
&= \frac{1}{\mathrm{P}(l_{ik}^* > 0)} \int_0^\infty -\frac{1}{\tilde{z}+1} \frac{H_3(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\log(\tilde{z}+1)}{\theta})}{H(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\log(\tilde{z}+1)}{\theta})}\, d\tilde{z} \\
&= \frac{\theta}{\mathrm{P}(l_{ik}^* > 0)} \left(\log H(\frac{S}{\theta}, \frac{D}{\theta}, 0) - \log H(\frac{S}{\theta}, \frac{D}{\theta}, \infty)\right). \qquad \text{(A.22)}
\end{aligned}
$$

Assuming that $H(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta)$ has the same specification as before we get

$$
\begin{aligned}
\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0) &= \frac{\theta}{\mathrm{P}(l_{ik}^* > 0)} \left(\log\left(\frac{1}{\mathrm{P}(l_{ik}^* > 0)}\right)\right. \\
&\qquad \left. - \log\left(\frac{1}{\mathrm{P}(l_{ik}^* > 0)} - 1\right)\right)
\end{aligned}
$$

[7]We use the fact that for any probability distribution f with the corresponding cumulative distribution function F we have that $\mathrm{E}\, x = \int_0^\infty x f(x)\, dx = \int_0^\infty (1 - F(x))\, dx$.

$$= \frac{-\theta}{\mathrm{P}(l_{ik}^* > 0)} \log\left(1 - \mathrm{P}(l_{ik}^* > 0)\right), \tag{A.23}$$

where $\mathrm{P}(l_{ik}^* > 0)$ is given by equation (A.13). In the same manner we can derive the expected Clower demands and supplies conditional on these demands and supplies being positive,

$$\mathrm{E}(l_{ik}^S \mid l_{ik}^S > 0) = \frac{-\theta}{\mathrm{P}(l_{ik}^S > 0)} \log\left(1 - \mathrm{P}(l_{ik}^S > 0)\right) \tag{A.24}$$

and

$$\mathrm{E}(l_{ik}^D \mid l_{ik}^D > 0) = \frac{-\theta}{\mathrm{P}(l_{ik}^D > 0)} \log\left(1 - \mathrm{P}(l_{ik}^D > 0)\right), \tag{A.25}$$

and conditional on the transacted quantity being positive,

$$\mathrm{E}(l_{ik}^S \mid l_{ik}^* > 0) = \frac{-\theta}{\mathrm{P}(l_{ik}^* > 0)} \log\left(\frac{1 + \exp(-S/\theta)}{\exp(-S/\theta)}\right) \tag{A.26}$$

and

$$\mathrm{E}(l_{ik}^D \mid l_{ik}^* > 0) = \frac{-\theta}{\mathrm{P}(l_{ik}^* > 0)} \log\left(\frac{1 + \exp(-D/\theta)}{\exp(-D/\theta)}\right). \tag{A.27}$$

From equations (A.23), (A.24), and (A.25) we see that when the probabilities $\mathrm{P}(l_{ik}^* > 0)$, $\mathrm{P}(l_{ik}^S > 0)$, and $\mathrm{P}(l_{ik}^D > 0)$ are small then $\mathrm{E}(l_{ik}^* \mid l_{ik}^* > 0) \approx \mathrm{E}(l_{ik}^S \mid l_{ik}^S > 0) \approx \mathrm{E}(l_{ik}^D \mid l_{ik}^D > 0) \approx \theta$.

The expected value of the logarithm of l_{ik}^* conditional on transactions being positive is given by

$$\begin{aligned}
&\mathrm{E}(\log(l_{ik}^* + 1) \mid l_{ik}^* > 0) \\
&\quad = \int_0^\infty \mathrm{P}(\log(l_{ik}^* + 1) > h \mid l_{ik}^* > 0)\, dh \\
&\quad = \frac{1}{\mathrm{P}(l_{ik}^* > 0)} \int_0^\infty -\frac{H_3(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\exp(h)-1}{\theta})}{H(\frac{S}{\theta}, \frac{D}{\theta}, \frac{\exp(h)-1}{\theta})}\, dh
\end{aligned}$$

$$= \frac{\theta}{\mathrm{P}(l_{ik}^* > 0)} \Big(\exp(0) \log H(\frac{S}{\theta}, \frac{D}{\theta}, 0) - \exp(-\infty) \log H(\frac{S}{\theta}, \frac{D}{\theta}, \infty) \Big) .$$

$$= \frac{-\theta}{\mathrm{P}(l_{ik}^* > 0)} \log (\mathrm{P}(l_{ik}^* > 0)) . \qquad \text{(A.28)}$$

In deriving the expected value of w_{ik} when $l_{ik}^* > 0$, we start with the decomposition $w_{ik} = \bar{w} \exp(\mu_{ik})$ in equation (4.20) and the extreme valued stochastic variable $\varepsilon_3 = \kappa \mu_{ik}$. Taken together these lead to the following expression for $\mathrm{E}(w_{ik} \mid l_{ik}^* > 0)$:

$$\begin{aligned}
\mathrm{E}(w_{ik} \mid l_{ik}^* > 0) &= \bar{w} \, \mathrm{E} \, (e^{\varepsilon_3/\kappa} \mid l_{ik}^* > 0) \\
&= \frac{\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} \int_{-\infty}^{\infty} e^{y \frac{\theta}{\kappa}} \mathrm{P}(\delta_1 < y, \delta_2 < y, \varepsilon_3/\theta \in dy) \\
&= \frac{\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} \int_{-\infty}^{\infty} e^{y \frac{\theta}{\kappa}} F_3^*(y, y, y) dy \\
&= \frac{\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} H_3(\frac{D}{\theta}, \frac{S}{\theta}, 0) \int_{-\infty}^{\infty} e^{-y(1-\frac{\theta}{\kappa})} \exp(-e^{-y+\lambda}) \, dy, \qquad \text{(A.29)}
\end{aligned}$$

where $\lambda = \log H(\frac{S}{\theta}, \frac{D}{\theta}, 0)$. Introducing the variable $\tilde{y} = \exp(-y + \lambda)$ leads to

$$\begin{aligned}
&\mathrm{E}(w_{ik} \mid l_{ik}^* > 0) \\
&= \frac{-\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} H_3(\frac{D}{\theta}, \frac{S}{\theta}, 0) \int_0^{\infty} \frac{1}{\tilde{y}} \exp((\log \tilde{y} - \lambda)(1 - \frac{\theta}{\kappa})) e^{-\tilde{y}} \, d\tilde{y} \\
&= \frac{-\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} e^{-\lambda(1-\frac{\theta}{\kappa})} H_3(\frac{D}{\theta}, \frac{S}{\theta}, 0) \int_0^{\infty} \tilde{y}^{-\frac{\theta}{\kappa}} e^{-\tilde{y}} \, d\tilde{y} \\
&= \frac{-\bar{w}}{\mathrm{P}(l_{ik}^* > 0)} H_3(\frac{D}{\theta}, \frac{S}{\theta}, 0) \left[\exp\left(- \log H(\frac{S}{\theta}, \frac{D}{\theta}, 0) \, (1 - \frac{\theta}{\kappa}) \right) \right] \Gamma \left(1 - \frac{\theta}{\kappa} \right) . \qquad \text{(A.30)}
\end{aligned}$$

From the definition of $H(\varepsilon_1/\theta, \varepsilon_2/\theta, \varepsilon_3/\theta)$ it is apparent that

$$\exp[-\log H(\frac{S}{\theta}, \frac{D}{\theta}, 0) \, (1 - \frac{\theta}{\kappa}) \;\; = \;\; (\mathrm{P}(l_{ik}^* > 0))^{1-\frac{\theta}{\kappa}} . \qquad \text{(A.31)}$$

From (A.11), (A.30), and (A.31) we thus get

$$\mathrm{E}(w \mid l_{ik}^* > 0) \quad = \quad \bar{w}\,(\mathrm{P}(l_{ik}^* > 0))^{-\theta/\kappa}\,\Gamma\left(1 - \frac{\theta}{\kappa}\right). \tag{A.32}$$

A.3 Deriving an expression for the second order moment

In section 5 of the paper it is stated that if we have knowledge of the variables $\mathrm{E}(w_{ik} \mid l_{ik}^* > 0)$, $\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0)$, and θ we can determine the parameter κ. The expression $\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0)$ can be derived in the same manner as $\mathrm{E}(w_{ik} \mid l_{ik}^* > 0)$ and it is readily seen that

$$\begin{aligned}
\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0) &= \bar{w}^2 \mathrm{E}(e^{\frac{2}{\kappa}\varepsilon_3} \mid l_{ik}^* > 0) \\
&= \frac{\bar{w}^2}{\mathrm{P}(l_{ik}^* > 0)} \int_{-\infty}^{\infty} e^{\frac{2y}{\kappa}} F_3(y, y, y)dy \\
&= \bar{w}^2\,[\mathrm{P}(l_{ik}^* > 0)]^{-\frac{2\theta}{\kappa}}\,\Gamma\left(1 - \frac{\theta}{\kappa}\right).
\end{aligned} \tag{A.33}$$

Hence equations (A.32) and (A.33) imply that

$$\frac{1}{2}\log \mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0) - \log \mathrm{E}(w_{ik} \mid l_{ik}^* > 0) \quad = \quad \frac{1}{2}\log\left(\frac{\Gamma(1 - \frac{2\theta}{\kappa})}{\Gamma(1 - \frac{\theta}{\kappa})^2}\right). \tag{A.34}$$

Since the following applies in general

$$\begin{aligned}
&\frac{1}{2}\log \mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0) - \log \mathrm{E}(w_{ik} \mid l_{ik}^* > 0) \\
&\quad = \frac{1}{2}\log\left(1 + \frac{\mathrm{var}(w_{ik} \mid l_{ik}^* > 0)}{\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0)}\right).
\end{aligned} \tag{A.35}$$

we obtain

$$1 + \frac{\mathrm{var}(w_{ik} \mid l_{ik}^* > 0)}{\mathrm{E}(w_{ik}^2 \mid l_{ik}^* > 0)} \quad = \quad \frac{\Gamma(1 - 2\frac{\theta}{\kappa})}{\Gamma(1 - \frac{\theta}{\kappa})^2}, \tag{A.36}$$

which is a non-linear equation in θ/κ. Due to the fact that the Γ function is increasing amd convex it can be demonstrated that (A.36) determines θ/κ uniquely provided $\theta/\kappa < 0.5$.

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