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The Existence of Factor Substitution in the Primary Aluminium Industry
A Multivariate Error Correction Approach on Norwegian Panel Data
Abstract
This paper presents an econometric analysis of factor demands in the Norwegian primary aluminium industry using annual panel data for individual plants. Focus is on testing theoretical and technical restrictions. The translog cost function approach is applied, and a multivariate error correction model with the cost shares of labour, raw materials and electricity with individual fixed effects is estimated. Capital is assumed to be quasi-fixed. The hypothesis of fixed input coefficients in this industry is rejected, but the estimated own and cross price elasticities suggest that relative price variation has limited effect on conditional factor demands.

Keywords: Factor demand, Translog cost function, Dynamic specification, Aluminium industry

JEL classification: C33, D21, L61

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1. Introduction

This paper presents an econometric analysis of factor demands in the Norwegian primary aluminium industry. A panel with annual observations of individual plants over 1972-1990 is used. The translog cost function suggested by Christensen et al. (1971, 1973) is applied, and a multivariate error correction model with the cost shares of labour, raw materials and electricity is estimated, cf. Anderson and Blundell (1982). The multivariate error correction model allows a flexible adjustment process for each factor.

Capital is assumed to be quasi-fixed, which reduces the number of coefficients in the general dynamic model significantly and is attractable due to a relatively small sample. Net investments are assumed to depend on the access to favourable long term electricity contracts. Treating capital as a quasi-fixed factor implies, however, that the long-run concept of this analysis has a partial rather than a full equilibrium interpretation.

The paper focuses on testing theoretical and technical restrictions, but addresses also the question of dynamic specification. The neoclassical predictions of zero degree price homogeneity and symmetric cross price effects in the conditional cost share equations are not rejected, which support the chosen neoclassical approach. The estimated partial equilibrium own and cross price elasticities suggest that relative price variation has limited effect on conditional factor demands in this industry, but factor substitution is not absent. The paper rejects the hypothesis of fixed input coefficients. The paper also finds support for Hicks neutral technical progress and long-run homotheticity. In addition, the paper concludes that the input share for electricity is capacity independent, i.e. does not vary with the capital stock, while the input share for labour decreases and the input share for raw materials increases with the capital stock.

The theoretical model is presented in chapter 2, and the empirical results are given in chapter 3. The main conclusions are summarized in the final chapter.

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2 A multivariate error correction model has also been used to study factor demands by Holly and Smith (1989) and Friesen (1992). These analyses apply aggregate time series data, as is the case for most empirical research on dynamic factor demand. One recent exception is Wolfson (1993), who uses panel data for individual plants and the adjustment cost approach to introduce dynamics.

3 The aluminium industry has developed in Norway due to access to cheap hydroelectricity. Favourable long term contracts need not always be a necessity though, one of the plants covers around 20 per cent of its "full capacity" need for electricity from the spot market and short term contracts after a significant expansion in the smelting capacity in 1989/90.
2. The translog cost function

In their work on front production functions for the Norwegian primary aluminium industry, Førsund and Jansen (1983) argue that a vintage capital stock approach is most relevant. In the vintage model, the ex ante and ex post production technologies differ, due to more limited substitution possibilities ex post an investment than ex ante. In fact, Førsund and Jansen make use of the extreme putty-clay assumption, and the ex post production technology in each plant is characterized by fixed input coefficients. Raw materials are treated as shadow factors to output. Førsund and Jansen conclude that the ex ante substitution between labour and electricity is very limited, and that technical progress has been labour saving. One problem with this analysis is that changes, which are interpreted as being due to technical progress and investments in the best-practice equipment, may simply be due to ex post substitution if that is present.

We use a neoclassical rather than a putty-clay model. This is not because we assume the vintage capital stock argument to be irrelevant, but because the chosen approach is convenient for our purpose. Our aim is to test for the existence of substitution between variable inputs and to test formally whether technical progress has been biased or neutral. The translog cost function suggested by Christensen et al. (1971, 1973) does not impose à priori restrictions on the elasticities of substitution, and leaves the determination of the degree of substitutability to the data.

The static variable translog cost function with labour measured in man hours (L), raw materials (M) and electricity (E) as variable inputs, can be written as in equation (1). Capital is treated as a predetermined variable, and the three variable inputs are adjusted conditionally on the capital stock. Subscript \( f \) denotes plant \( f \). All variables except the price of raw materials vary across plants. The constant term and the coefficients linear in the input prices are assumed to be plant specific. The remaining coefficients are assumed to be identical for all plants.

\[
\ln C_f = \gamma_{0f} + \sum_i \alpha_{if} \ln Q_{if} + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln Q_{if} \ln Q_{jf} + \gamma_X \ln X_f + \frac{1}{2} \gamma_{XX} (\ln X_f)^2 \\
+ \sum_i \gamma_{Xf} \ln Q_{if} \ln X_f + \gamma_K \ln K_f + \frac{1}{2} \gamma_{KK} (\ln K_f)^2 + \sum_i \gamma_{Kf} \ln Q_{if} \ln K_f \\
+ \gamma_{Xf} \ln X_f \ln K_f + \gamma_T \ln T + \frac{1}{2} \gamma_{TT} (\ln T)^2 + \sum_i \gamma_{Tf} \ln Q_{if} \ln T + \gamma_{XT} \ln X_f \ln T \\
+ \gamma_{KT} \ln K_f \ln T \\
i,j=L, M, E, f=1,...,m
\]

(1)

\[
C_f = \sum_i Q_{if} V_{if} \\
i=L, M, E, f=1,...,m
\]

(2)

where \( C_f \) is total variable costs for plant \( f \); \( Q_{if} \) is the price of input \( i \) faced by plant \( f \);
V_{if} is the quantity of input i in plant f; X_{f} is real gross output in plant f; K_{f} is the real capital stock in plant f; T is a deterministic time trend to proxy the level of technology.

The translog cost function can be interpreted as a quadratic approximation to a general continuous twice differentiable cost function. By assuming price taking behaviour in the factor markets and using Shepard’s lemma, we find the cost share (S_{if}) equations defined in (3). These cost share equations include individual dummy variables for each plant due to the individual slope coefficients in the cost function. The dummy variables are assumed to capture permanent differences in cost shares across plants, i.e. fixed effects, that may be due to differences in the technology or efficiency. Thus, our general model incorporates differences in technology both among years and producing units. The price taking assumption is discussed further in chapter 3. Jorgenson (1986, p.1846) argue that this in general is a plausible assumption at the firm or plant level, however.

\[(3) \quad S_{if} = \frac{\partial \ln C_{f}}{\partial \ln Q_{if}} = Q_{if} \cdot V_{if} / C_{f} = \alpha_{if} + \sum_{j} \beta_{ij} \ln Q_{if} + \gamma_{ix} \ln X_{f} + \gamma_{ik} \ln K_{f} + \gamma_{it} \ln T_{i,j=L,M,E, f=1,...,m}\]

The static model presented above assumes that plants produce any output level in a cost effective manner, and that costs are minimized with respect to the input mix conditionally on factor prices, output, the capital stock and technology. But, due to adjustment costs and incomplete information, factor adjustments are not instantaneous, and economic agents will not always be on their long-run schedules. To introduce short-run disequilibrium factor adjustment, we apply the multivariate error correction model suggested by Anderson and Blundell (1982), which is flexible with respect to the dynamics or short-run elasticities.\(^4\)

The multivariate error correction representation of (3) is given in (4). For convenience, we present the model on vector form. The individual data for each variable are stacked in long vectors along the time dimension. This organization of the data, which is further described in appendix 2, enables us to estimate the simultaneous model with cross equation restrictions maintaining both the cross section and the time series dimension with standard tools. Our most general model includes all variables at t, t-1 and t-2.

\[(4) \quad \Delta S_{t} = \Delta S_{t-1} + B \Delta Z_{t}^* + C \Delta Z_{t-1}^* - D[S_{t-1} - \Pi(\theta)Z_{t-1}] + u_{t}\]

\(^4\) A common way to introduce dynamic factor demands in the literature is to include costs of adjustment from changes in quasi-fixed inputs. For a survey on this field, see Jorgenson (1986), see also Mahmud et al. (1987) and Gordon (1992).
where $\Delta$ is the first difference operator, $S$ is a vector of individual cost shares, and $Z$ is a vector of regressors and includes the logarithm of input prices, output, the capital stock, the trend variable and the dummy variables. $Z^*$ is $Z$ with all plant specific constant terms (dummy variables) excluded. $A$, $B$, $C$ and $D$ are short-run coefficient matrices of suitable dimensions, $\Pi(\theta)$ is a matrix of the long-run coefficients, i.e. the coefficients in (3), and $u_t$ is a vector of individual identically normally distributed random errors with mean zero. Because the cost shares always sum to unity, i.e. $\sum \Sigma_i S_{ift} = 1$ and hence $\sum \Delta S_{ift} = 0$, any one of the cost share equations can be expressed in terms of the other equations by using adding up restrictions. For each plant, the residuals in the three cost share equations must sum to zero each year, which implies a singular and non-diagonal error covariance matrix. Estimation may proceed, however, with the arbitrary deletion of one equation, cf. Anderson and Blundell (1982), who generalize the invariant proposition of Berndt and Savin (1975). The impact and long-run coefficients of the excluded equation can be found by using the adding up conditions in table 2.1. Let $S^n$, $u^n$ and $\Pi^n(\theta)$ denote the vectors $S$ and $u$ and the matrix $\Pi(\theta)$ with the $n$'th row deleted respectively. We assume that

$$ Eu^n_{ft}u^n_{es} = \begin{cases} \Omega_{n-1, n-1} & \text{for } t=s \text{ and } f=e, \\ 0 & \text{otherwise}. \end{cases} $$

The estimable system is given in (5), and a typical equation is given in (6), where we exclude the cost share equation for electricity.

(5) $\Delta S^n_t = A^n\Delta S^n_{t-1} + B^n\Delta Z^*_{t} + C^n\Delta Z^*_{t-1} - D^n[S^n_{t-1} - \Pi^n(\theta)Z_{t-1}] + u^n_t$

(6) $\Delta S_{ift} = a_{iL} \Delta S_{Lft-1} + a_{iM} \Delta S_{Mft-1} + b_{iL} \Delta \ln Q_{Lft} + b_{iM} \Delta \ln Q_{Mt} + b_{iE} \Delta \ln Q_{Ef} + b_{iX} \Delta \ln X_{ft} + b_{iK} \Delta \ln K_{ft} + b_{iT} \Delta \ln T_{t} + c_{iL} \Delta \ln Q_{Lft-1} + c_{iM} \Delta \ln Q_{Mt-1} + c_{iX} \Delta \ln X_{ft-1} + c_{iK} \Delta \ln K_{ft-1} + c_{iT} \Delta \ln T_{t-1} - d_{iL} (S_{Lft-1} - \alpha_{Lf}) - d_{iM} (S_{Mft-1} - \alpha_{Mf}) - d_{iL} \ln Q_{Lft-1} - d_{iM} \ln Q_{Mt-1} - d_{iE} \ln Q_{Ef-1} - d_{iX} \ln X_{ft-1} - d_{iK} \ln K_{ft-1} - d_{iT} \ln T_{t-1} - d_{iL} \ln Q_{Lft-1} - d_{iM} \ln Q_{Mt-1} - d_{iE} \ln Q_{Ef-1} - d_{iX} \ln X_{ft-1} - d_{iK} \ln K_{ft-1} - d_{iT} \ln T_{t-1} - d_{iL} \ln Q_{Lft-1} - d_{iM} \ln Q_{Mt-1} - d_{iE} \ln Q_{Ef-1} - d_{iX} \ln X_{ft-1} - d_{iK} \ln K_{ft-1} - d_{iT} \ln T_{t-1} + u_{ift}$

$i=L, M, f=1,..,m$

One attractive feature of the general error correction model in (5), is that it nests well known models from the literature such as the autoregressive error process model, the partial adjustment model and the static model. Table 2.1 gives the restrictions on (5) which correspond to these alternative dynamic models.
Table 2.1. Alternative dynamic models nested within the error correction model

| 1. $B^a = +\Pi^a^*$ ($\Pi^a$ is $\Pi^a$ excl. the constant) and $C^a = +A^a.\Pi^a^*$ | Autoregressive error process model |
| 2. $B^a = C^a = -D^a.\Pi^a^*$ | Partial adjustment model |
| 3. Autoregressive or partial adjustment and $d_{ii} = 1$, $d_{ij} = 0$ for $i \neq j$ | Static model |

The autoregressive error model defined in table 2.1 equals the static model with a first order autoregressive error process. The autoregressive coefficient is assumed equal across the cost share equations, cf. Berndt and Savin (1975);

$$u_{ift} = \rho u_{if,t-1} + v_{ift}, \quad i=L, M, E, \quad f=1,..,m.$$

The random errors, $v_{ift}$, are assumed to be identically normally distributed with mean zero. These errors are assumed independent across producing units and time, but are not independent across the cost shares for each unit for reasons explained earlier.

Table 2.2 gives the restrictions required for the cost share equations in (6) to be homogeneous of degree zero in input prices and the cross price effects to be symmetric, in addition to technical and adding up restrictions.

Table 2.2. Adding up, theoretical and technical restrictions

<table>
<thead>
<tr>
<th>Long-run</th>
<th>Short-run</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \alpha_{ix} = 1$</td>
<td>$\forall f$</td>
<td>Adding up condition</td>
<td></td>
</tr>
<tr>
<td>$\sum \beta_{i} = 0$</td>
<td>$\forall i$</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>$\sum \gamma_{ix} = 0$</td>
<td>$\forall i$</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>$\sum \gamma_{it} = 0$</td>
<td>$\forall i$</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>$\sum \beta_{ij} = 0$</td>
<td>$\forall i; i \neq j$</td>
<td>Scale invariant input share for factor i</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{ix} = 0$</td>
<td>$\forall i$</td>
<td>Homotheticity</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{ix} = 0$</td>
<td>$\forall i$</td>
<td>No price effects</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{it} = 0$</td>
<td>$\forall i$</td>
<td>Hicks neutrality</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{ik} = 0$</td>
<td>$\forall i$</td>
<td>Capacity invariant input share for factor i</td>
<td></td>
</tr>
</tbody>
</table>
Homotheticity implies scale invariant input shares, which in addition to no price effects, i.e. constant cost shares, suggest a Cobb-Douglas production technology with respect to variable inputs. We also test whether input shares are scale invariant for subsets of variable factors. If on the other hand the cost share of an input increases (decreases) with an expansion in output, i.e. $\gamma_{ix} > 0$ ($< 0$), this is defined as a positive (negative) bias of scale. Hicks neutrality is defined as technical change which leaves factor ratios and thus cost shares unchanged if factor prices are held constant. If conditional demand for factor $i$ increases (decreases) over time at constant factor prices and capacity, this is defined as factor $i$ using (saving) technical change, and corresponds to $\gamma_{it} > 0$ ($< 0$). In addition, we test for capacity invariant input shares, i.e. whether input shares remain constant when the capital stock changes. The share of input $i$ increases (decreases) with the capital stock if $\gamma_{ik} > 0$ ($< 0$). Capital stock dependent input shares may reflect that capital is not weakly separable from the variable inputs, or alternatively that capital from different vintages are not homogeneous. In addition, if there are trends in relative prices between variable inputs and substitution possibilities are more limited ex post than ex ante an investment, we may also find capital dependent input shares.

Following Berndt and Wood (1975), we define the long-run Hicks-Allen partial elasticities of substitution as

$$\sigma_{ij} = \left( \frac{B_{ij}}{S_i S_j} \right) + 1 \quad \text{for } i \neq j$$

$$\sigma_{ii} = \left[ \frac{B_{ii} + (S_j)^2 - S_i}{(S_j)^2} \right] \quad \text{for all } i,$$

where $S_i$ denotes sample means across both plants and time periods. Grant (1993) shows that the elasticities of substitution in the translog function case may be evaluated at any expansion point, also at sample means, as long as the restrictions of symmetry and homogeneity put forward in table 2.2 hold. The long-run own and cross price elasticities are

$$\varepsilon_{ij} = S_j \cdot \sigma_{ij} \quad \text{for } i \neq j$$

$$\varepsilon_{ii} = S_i \cdot \sigma_{ii} \quad \text{for all } i.$$

While the Hicks-Allen partial elasticities are symmetric, this is generally not the case with the cross price elasticities.

When estimating the translog cost function, we are not able to test explicitly the hypothesis of fixed input coefficients, i.e. the Leontief technology with constant input shares, but the degree of substitutability is revealed by the price elasticities.
3. Empirical results

We now present the results from estimating and testing the dynamic system (6) in chapter 2, where the cost share equation for electricity is excluded. The empirical variables are presented in appendix 1. Our panel includes seven plants for the period 1972-1990, and in addition one plant which was shut down in 1981. We use the full information maximum likelihood procedure (FIML) in TROLL, and the χ²-form of the likelihood ratio test is applied in a general to specific search. The small sample adjusted χ²(j)-statistic is reported, where j denotes the number of restrictions, see Mizon (1977). The significance level for rejecting the null hypothesis is reported in addition to the degrees of freedom for each regression (DF). We also report the implicit significance level for the accepted hypothesis when we treat all hypotheses symmetrically and use a one per cent probability of rejecting a true null hypothesis against the immediately proceeding hypothesis at each step. The implicit significance level of the rth hypothesis in a test sequence against the most general model then equals α=1-(1-0.01)^r.

There is no natural ordering of the hypotheses to be tested, and the ideal strategy is to look at all possible orderings. The chosen strategy does not avoid that results from sequential testing depend on the chosen ordering, but it reduces the problem that conclusions from testing theoretical and technical restrictions may depend on the dynamic specification and vice versa; First we determine the number of lags that should be included and choose between the two alternative capital measures. The maintained hypothesis from this first step is then tested with respect to dynamic specification, i.e. the restrictions put forward in table 2.1. Conditional on the preferred dynamic specification, we test the theoretical and technical restrictions in table 2.2. This is referred to as step two. We then go back and start with the model from step one again, but now continue by testing the theoretical and technical restrictions first, before testing the dynamic structure. The maintained hypothesis from this third step is then compared with that from step two.

All variables, except the cost shares, are normalized to equal one in 1972. This was necessary to obtain convergence when estimating. Table 3.1 gives the cost shares for the three variable inputs, and shows that raw materials are the major variable inputs in this industry.

\[ \chi^2(j) = -2(T-k_1-1+j/2)/T \cdot [\ln L_0 - \ln L_1], \]

where T denotes the number of observations, \( k_1 \) is the number of estimated coefficients in the general hypothesis, j is the number of restrictions, and \( \ln L_0 \) and \( \ln L_1 \) is the value of the log-likelihood function under the null and the general hypothesis respectively.
Table 3.1. Cost shares in the three variable input case, capital costs are not included

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour</td>
<td>0.236</td>
<td>0.149</td>
<td>0.402</td>
<td>0.060</td>
</tr>
<tr>
<td>Raw materials</td>
<td>0.586</td>
<td>0.424</td>
<td>0.709</td>
<td>0.061</td>
</tr>
<tr>
<td>Electricity</td>
<td>0.177</td>
<td>0.083</td>
<td>0.266</td>
<td>0.038</td>
</tr>
</tbody>
</table>

From the first step, which is discussed in detail in appendix 3, we conclude that $\Delta Z_{t-1}^*$ and $\Delta S_{t-1}^o$ can be excluded, i.e. $C^o=0$ and $A^n=0$. We also conclude that the capital measure based on melting capacities is preferred to that based on fire insurance values. The maintained hypothesis from step two is an autoregressive error process (AR) model with price homogeneity, symmetry, Hicks neutrality and scale invariant input shares between raw materials on one side and labour and electricity on the other, cf. appendix 4. Our overall conclusion is, however, that we prefer a restricted error correction (EC) model from step three, and we choose to concentrate on this model.

Table 3.2 gives the results from testing theoretical and technical restrictions on the reduced lag EC-model from step one. The acceptance of the AR-model in step two, makes it particularly relevant to test restrictions on both the short-run dynamics and the long-run relationships of the EC-model. The AR-model implies symmetry between short- and long-run effects, i.e. the existence of common factors between the cost shares and the regressors in the dynamic model, cf. Hendry and Mizon (1978). We find price homogeneity, Slutsky symmetry and Hicks neutrality in both the short- and the long-run when testing the EC-model. The data supports long-run but not short-run homotheticity, however, which contradicts the common factors hypothesis. The input shares for raw materials on one side and labour and electricity on the other are found to be scale invariant in the short-run. As output increases, there is a small tendency of decreasing input share for labour and increasing input share for electricity in the short-run. Zero short-run price effects, i.e. constant cost shares, is clearly rejected. On the other hand, with respect to labour and electricity, there are no cross price effects on the cost shares. In addition, we find support for capacity invariant input share for electricity in the long-run, decreasing input share for labour, and increasing input share for raw materials as capacity increases, as can be seen from table 3.4. The hypotheses of no individual fixed effects are clearly rejected. The accepted EC-model implies a more parsimonious (more restricted) partial equilibrium than the AR-model from step two, and the restrictions which give the AR form of this EC-model is rejected.
Table 3.2. Testing theoretical and technical restrictions on the reduced lag error correction (EC) model. Capital stock equals output capacity.

<table>
<thead>
<tr>
<th>Test Description</th>
<th>lnL</th>
<th>$\chi^2$ (df)</th>
<th>p-value</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC-model</td>
<td>692.06</td>
<td></td>
<td></td>
<td>83</td>
</tr>
<tr>
<td>LR-Homogeneity</td>
<td>690.97</td>
<td>1.43</td>
<td>0.00</td>
<td>85</td>
</tr>
<tr>
<td>LR-Symmetry</td>
<td>690.00</td>
<td>1.29</td>
<td>0.00</td>
<td>86</td>
</tr>
<tr>
<td>LR-homotheticity</td>
<td>689.41</td>
<td>0.80</td>
<td>0.00</td>
<td>88</td>
</tr>
<tr>
<td>SR-homotheticity</td>
<td>679.75</td>
<td>13.47</td>
<td>0.00</td>
<td>90</td>
</tr>
<tr>
<td>b$_{MX}$ = 0</td>
<td>689.41</td>
<td>0.00</td>
<td>0.00</td>
<td>89</td>
</tr>
<tr>
<td>No SR-price effects</td>
<td>634.60</td>
<td>78.54</td>
<td>0.00</td>
<td>95</td>
</tr>
<tr>
<td>LR-Hicks neutrality</td>
<td>688.50</td>
<td>1.28</td>
<td>0.00</td>
<td>91</td>
</tr>
<tr>
<td>SR-Hicks neutrality</td>
<td>687.63</td>
<td>1.43</td>
<td>0.00</td>
<td>93</td>
</tr>
</tbody>
</table>

lnL is the value of the log-likelihood function.

$\chi^2$(j) is the likelihood ratio test where j denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses.

DF is the degrees of freedom, LR and SR denote long-run and short-run respectively.
Table 3.2. continues

SR-Hicks Neutrality
\( \ln L = 687.63 \)
DF = 93

SR-Homogeneity
\( \ln L = 683.81 \)
\( \chi^2(2) = 5.59 \ (6\%) \)
DF = 95

SR-Symmetry
\( \ln L = 682.78 \)
\( \chi^2(1) = 1.54 \ (21\%) \)
DF = 96

\( \gamma_{MK} = -\gamma_{LK}; \gamma_{EK}=0 \)
\( \ln L = 682.76 \)
\( \chi^2(1) = 0.03 \ (87\%) \)
DF = 97

\* \( \beta_{LM} = -\beta_{LL}; b_{LE}=0 \)
\( \ln L = 682.72 \)
\( \chi^2(2) = 0.07 \ (97\%) \)
DF = 99

No LR-price effects
\( \ln L = 676.67 \)
\( \chi^2(2) = 9.43 \ (1\%) \)
DF = 101

\( \alpha_{lf} = \alpha_{L}, \ f=1,..,8 \)
\( \ln L = 670.86 \)
\( \chi^2(7) = 18.96 \ (1\%) \)
DF = 106

AR-model
\( \ln L = 673.07 \)
\( \chi^2(5) = 15.27 \ (1\%) \)
DF = 104

\( \alpha_{Mf} = \alpha_{M}, \ f=1,..,8 \)
\( \ln L = 670.30 \)
\( \chi^2(7) = 19.85 \ (1\%) \)
DF = 106

\( \ln L \) is the value of the log-likelihood function.
\( \chi^2(j) \) is the likelihood ratio test where \( j \) denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses.
DF is the degrees of freedom. LR and SR denote long-run and short-run respectively, and AR denotes autoregressive error process. \( \alpha_{if} \) is the individual constant term in the cost share equation for input \( i \).
\* The accepted combined hypothesis.

To test the AR-model from step two against the EC-model in table 3.2, we develop the
linear nesting model of which these models are reductions. The value of the log-
likelihood function of this nesting model equals 683.28. Testing the two alternative
models against their nesting model give the $\chi^2(5)=1.36$ for the AR-model and the
$\chi^2(2)=0.87$ for the EC-model. The rejection probabilities needed to reject these two
alternative reductions are 93 and 65 per cent respectively. Thus, we can not reject the
null hypothesis that each model is a valid reduction of the nesting model, and we are
not able to discriminate between these two models on the basis of this likelihood ratio
tests alone. Both Akaike’s information criterion (AIC), cf. Akaike (1974), and the
dominance ordering criterion (DC) and the likelihood dominance criterion (LDC), cf.
Pollak and Wales (1991), prefer the AR-model to the EC-model. This is due to the
very similar values of the log-likelihood function and a smaller number of estimated
coefficients in the AR-model. In addition, some of the long-run coefficients are more
precisely determined in the AR-model than in the EC-model. Despite this, we choose
the EC-model, primarily because of a more parsimonious description of the partial
equilibrium. These alternative models are very close with respect to in sample fit and
price elasticities.

The optimal sequential testing procedure does not include the test of the accepted
combined hypothesis directly against the most general model, cf. Mizon (1977). Despite
this, we test the accepted EC-model against the most general lag model with capital
equal to capacity, cf. appendix 3, because we are interested in determining the
significance level required for rejecting the combined hypothesis. We find the
$\chi^2(30)=30.63$, and we must accept a probability of rejecting a true null hypothesis of 43
per cent to reject the restricted EC-model directly against the most general model. The
implicit significance level for testing the accepted hypothesis against the most general
model is 12 per cent.

To test the assumption that the explanatory variables are weakly exogenous in the cost
share equations, we use the Hausman-Wu test procedure, cf. Godfrey (1988). From table
3.3 we conclude that weak exogeneity is not rejected for neither capital, output nor input
prices, i.e. these variables are not correlated with the residuals. We recognize that there

\[6 \text{AIC}_i=\ln L_i-k_i; \text{DC}_i=\ln L_i+c(k_{c}-k_{i})/2; i=\text{AR,EC}. \ln L_i \text{ is the value of the log-likelihood function for equation i, } k_i \text{ is the number of estimated coefficients, } k_{c} \text{ is the number of coefficients in the linear nesting model of the AR- and the EC-model, } c(v) \text{ is the critical value of the } \chi^2 \text{-distribution with } v \text{ degrees of freedom at a chosen significance level. We use the five per cent significance level. The equation with the highest AIC-value is preferred, the same is true with respect to the DC. We find AIC}_{AR}=657.40, \text{ AIC}_{EC}=654.72, \text{ DC}_{AR}=687.94, \text{ DC}_{EC}=685.71. \text{ For the LDC we define: } a=\ln L_{EC}-\ln L_{AR}=0.313, \text{ b}=\{c(k_{EC}+1)-c(k_{AR}+1)\}/2=1.836, \text{ c}=[c(k_{EC}-k_{AR}+1)-c(1)]/2=2.824. \text{ The LDC prefers the AR-model if } a<b, \text{ the EC-model if } a>b, \text{ and is indecisive if } c>a>b. \text{ All three measures prefer the AR-model to the EC-model.} \]
may be a problem with low power of these tests if the marginal models are poor, cf. Urbain (1992).

### Table 3.3. Testing the weak exogeneity assumption of the explanatory variables in the accepted error correction model

<table>
<thead>
<tr>
<th>Variable</th>
<th>The likelihood ratio test with j number of restrictions; $\chi^2(j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital (K=capacity)</td>
<td>$\chi^2(2) = 1.12$ (57%)</td>
</tr>
<tr>
<td>Output (X)</td>
<td>$\chi^2(2) = 2.18$ (34%)</td>
</tr>
<tr>
<td>Input prices (Q, i=L,M,E)</td>
<td>$\chi^2(6) = 7.28$ (30%)</td>
</tr>
</tbody>
</table>

The significance level where the null hypothesis of weak exogeneity is rejected is given in parentheses.

Figure 1-3 show actual and fitted changes in the cost shares for the variable inputs. The change in the cost share for electricity is calculated by the adding up restriction. We conclude that the accepted EC-model predicts well the in sample annual changes in the cost shares.

Figure 1. Actual and fitted changes in the cost share for labour for each plant
Figure 1. continues
Figure 2. Actual and fitted changes in the cost share for raw materials for each plant.
Figure 2. continues

Figure 3. Actual and fitted changes in the cost share for electricity for each plant
In table 3.4 we present the estimated coefficients of the preferred EC-model. The coefficients $b_{ie}$, $b_{je}$ and $\gamma_{je}$, $i=L,M,E$, $j=X,K,T$, are calculated by using the adding up conditions. The eight individual constant terms in each cost share equation are not reported. Our results imply that the treatment of raw materials as a shadow factor to production, as in Førsund and Jansen (1983), is not valid. Also, labour saving technical change, as concluded by Førsund and Jansen, is rejected by this analysis. The results suggest that the reduction in the input coefficient for labour is due to the increase in capacity and changes in relative input prices. We do have some problems with insignificant long-run coefficients, however.

The F-form of the lagrange multiplier (LM) test has been applied to test for first order autocorrelation in the residuals (Kiviet (1986)). First order autocorrelation is rejected for most plants, the only exception is in fact the cost share equation for raw materials for plant 1, cf. table 3.4.
Table 3.4. The estimated coefficients of the error correction model

<table>
<thead>
<tr>
<th>Short-run coefficients</th>
<th>Estimates</th>
<th>Long-run coefficients</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bLL 0.066 (.012)</td>
<td>βLL 0.050 (.036)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bLM -0.066 *</td>
<td>βLM -0.050 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bLE 0</td>
<td>βLE 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bMM 0.174 (.016)</td>
<td>βMM 0.140 (.039)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bME -0.105 (.009)</td>
<td>βME -0.090 (.022)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bEE 0.108 (.026)</td>
<td>βEE 0.090 (.074)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bLX -0.038 (.009)</td>
<td>γLX 0 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bMX 0 *</td>
<td>γMX 0 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bEX 0.038 *</td>
<td>γEX 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bEL -0.145 (.025)</td>
<td>γEL -0.178 (.024)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bMK 0.201 (.035)</td>
<td>γMK 0.178 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bEK -0.056 (.019)</td>
<td>γEK 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bLT 0 *</td>
<td>γLT 0 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bMT 0 *</td>
<td>γMT 0 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bET 0</td>
<td>γET 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>dLL 0.220 (.116)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>dLM -0.211 (.092)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>dMM 0.531 (.137)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>dML -0.089 (.170)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

lnL = 682.72  
DF = 99  
L: R² = 0.505  
M: R² = 0.501  
CR² = 0.400  
CR² = 0.396  
SER = 0.023  
SER = 0.034

<table>
<thead>
<tr>
<th>LM-test for AR(1)</th>
<th>Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>L</td>
<td>1.11</td>
</tr>
<tr>
<td>M</td>
<td>15.51s</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. lnL is the value of the log-likelihood function. DF is the degrees of freedom. The multiple correlation coefficient (R²), R² corrected for degrees of freedom (CR²), the equation standard error (SER) are given for the estimated cost share equations; L=labour, M=raw materials. AR(1) denotes first order autocorrelation. Not all right hand side variables are included in the auxiliary regressions for plant 8 when testing for autocorrelation, due to a small number of observations.

* Restricted a priori. All restrictions are supported by the likelihood ratio test. In addition, aij = cij = 0, i,j=L, M, E, by restriction.

s Significant at the five per cent significance level.

In table 3.5 we present the long-run Hicks-Allen partial elasticities of substitution in addition to long-run cross price and own price elasticities calculated at sample means as predicted by the EC-model. To simplify the calculations and avoid non-linearity, we
approximate standard errors of these elasticities assuming that sample means of the cost shares are constants. Ideally, we should use the estimated cost share equations and calculate standard errors at the stochastic sample means, cf. Toevs (1982). The EC-model shows a relatively quick adjustment process, but labour proves to more quasi-fixed than raw materials.

All the own price elasticities at sample means have the correct sign, and that conditional factor demands are inelastic. All inputs are substitutes, but the cross price elasticities between raw materials and electricity are close to zero, indicating that these inputs are approximately independent. The acceptance of $\beta_{LE}=0$, implies a HA-elasticity of substitution equal to unity between labour and electricity, while the cross price elasticities are determined by the cost shares. Table 3.5 shows that variation in relative input prices has limited effect on the conditional demand for labour, raw materials and electricity when capital is predetermined, supporting the assumption of relatively fixed input coefficients. We do not find support for a putty-clay (Leontief) production technology, however, factor substitution is present even in the short-run.

<table>
<thead>
<tr>
<th>HA elasticities</th>
<th>Own and cross price elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{LL}$</td>
<td>-2.34 (.647)</td>
</tr>
<tr>
<td>$\sigma_{LM}$</td>
<td>0.64 (.261)</td>
</tr>
<tr>
<td>$\sigma_{LE}$</td>
<td>1.00 *</td>
</tr>
<tr>
<td>$\sigma_{MM}$</td>
<td>-0.30 (.042)</td>
</tr>
<tr>
<td>$\sigma_{ME}$</td>
<td>0.14 (.210)</td>
</tr>
<tr>
<td>$\sigma_{EE}$</td>
<td>-1.78 (2.36)</td>
</tr>
<tr>
<td>$\epsilon_{LL}$</td>
<td>-0.55 (.153)</td>
</tr>
<tr>
<td>$\epsilon_{LM}$</td>
<td>0.37 (.153)</td>
</tr>
<tr>
<td>$\epsilon_{LE}$</td>
<td>0.18 *</td>
</tr>
<tr>
<td>$\epsilon_{MM}$</td>
<td>-0.17 (.067)</td>
</tr>
<tr>
<td>$\epsilon_{ML}$</td>
<td>0.15 (.061)</td>
</tr>
<tr>
<td>$\epsilon_{ME}$</td>
<td>0.02 (.037)</td>
</tr>
<tr>
<td>$\epsilon_{EE}$</td>
<td>-0.32 (.417)</td>
</tr>
<tr>
<td>$\epsilon_{EL}$</td>
<td>0.24 *</td>
</tr>
<tr>
<td>$\epsilon_{EM}$</td>
<td>0.08 (.123)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

To be well behaved, the cost function must be concave in prices of variable inputs\(^7\). We find local but not global concavity; the matrix of price derivatives is not negative semidefinite, cf. Jorgenson (1986, p. 1859). Most sample points support concavity, but problems arise in our case because of a very low cost share for electricity in a couple

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\(^7\) This is a necessary but not sufficient condition when we treat capital as a quasi fixed factor. To test the remaining conditions we need to estimate the cost function.
of years for two plants, where the own price elasticity for electricity is positive. One of these plants was shut down in 1981, and the second shows a significant but temporary fall in the capacity utilization around the problematic years. The mean industry own price elasticity for electricity is negative in all years though.

Table 3.6 gives the mean and variation in the own and cross price elasticities calculated at each sample point, and shows that the standard deviation is small for most price elasticities. These mean elasticities may differ somewhat from the elasticities calculated at sample means, cf. table 3.5. The conclusion that labour to some degree can be substituted with both raw materials and electricity holds at all sample points. With respect to raw materials and electricity, we find that these inputs are substitutes in most sample points. But, for the two problematic plants, we find that these inputs are compliments during the seventies and early eighties.

<table>
<thead>
<tr>
<th>Price elasticities</th>
<th>Mean</th>
<th>Min.</th>
<th>Max.</th>
<th>St.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{LL}$</td>
<td>-0.54</td>
<td>-0.55</td>
<td>-0.47</td>
<td>0.02</td>
</tr>
<tr>
<td>$\varepsilon_{LM}$</td>
<td>0.36</td>
<td>0.26</td>
<td>0.47</td>
<td>0.03</td>
</tr>
<tr>
<td>$\varepsilon_{LE}$</td>
<td>0.18</td>
<td>0.08</td>
<td>0.27</td>
<td>0.04</td>
</tr>
<tr>
<td>$\varepsilon_{MM}$</td>
<td>-0.17</td>
<td>-0.25</td>
<td>-0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>$\varepsilon_{ML}$</td>
<td>0.15</td>
<td>0.08</td>
<td>0.30</td>
<td>0.05</td>
</tr>
<tr>
<td>$\varepsilon_{ME}$</td>
<td>0.02</td>
<td>-0.09</td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td>$\varepsilon_{EE}$</td>
<td>-0.29</td>
<td>-0.40</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>$\varepsilon_{EL}$</td>
<td>0.24</td>
<td>0.15</td>
<td>0.40</td>
<td>0.06</td>
</tr>
<tr>
<td>$\varepsilon_{EM}$</td>
<td>0.05</td>
<td>-0.56</td>
<td>0.22</td>
<td>0.14</td>
</tr>
</tbody>
</table>

In table 3.7, we compare our estimated own price elasticities calculated at sample means with those reported by others. There is a problem of comparability though, partly because most empirical analyses of factor demands use annual aggregates of the manufacturing industry and do not include the 1980s in the data set. In addition, different analyses vary with respect to which factors that are modelled and the methodology applied. Our own price elasticities are closer to zero than most others reported in table 3.7, but we are surprised to find very small differences in many cases. Our à priori assumption was that the aluminium industry faces a relatively fixed input structure compared to most other manufacturing industries. In addition, the Le Chatelier principle implies that the own price elasticities of variable inputs decrease in absolute value with the number of quasi-fixed factors. Therefore, in a full equilibrium framework,
as is the case for most of the reported analyses in table 3.7, we would expect the estimated own price elasticities in the Norwegian aluminium industry to increase in absolute value, reducing the difference to other studies even more.

Table 3.7. A comparison of the estimated own price elasticities ($\epsilon_{ii}$) with other empirical analyses

<table>
<thead>
<tr>
<th>Source</th>
<th>$\epsilon_{LL}$</th>
<th>$\epsilon_{MM}$</th>
<th>$\epsilon_{EE}^1$</th>
<th>Data$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friesen (1992)</td>
<td>-0.29</td>
<td>-0.12</td>
<td>0.67</td>
<td>US Manuf., annual 1947-81,1971.</td>
</tr>
<tr>
<td>Bergstrøm et al. (1992)</td>
<td>-0.13$^3$</td>
<td>-0.18$^4$</td>
<td></td>
<td>Swedish Basic metals, annual 1963-80, 1980.</td>
</tr>
<tr>
<td>Bradley et al. (1990)</td>
<td>-0.78</td>
<td>-1.13</td>
<td>-0.60</td>
<td>Irish Traditional manuf., annual 1970-78, 1978</td>
</tr>
<tr>
<td>Morrison (1988)</td>
<td>-0.41</td>
<td>-0.66</td>
<td>-0.37</td>
<td>US Manuf., annual 1965-78.</td>
</tr>
<tr>
<td>Mahmud et al. (1987)</td>
<td>-0.58</td>
<td>-0.59</td>
<td>-0.81</td>
<td>US data, annual 1952-71.</td>
</tr>
<tr>
<td>Mohnen et al. (1986)</td>
<td>-0.85</td>
<td>-0.23</td>
<td></td>
<td>US Manuf., annual 1965-78, 1970.</td>
</tr>
<tr>
<td></td>
<td>-0.89</td>
<td>-0.49</td>
<td></td>
<td>Japanese Manuf.,</td>
</tr>
<tr>
<td></td>
<td>-0.42</td>
<td>-0.29</td>
<td></td>
<td>German Manuf.,</td>
</tr>
<tr>
<td>Pindyck &amp; R. (1983a)</td>
<td>-0.55</td>
<td>-0.18</td>
<td>-0.22</td>
<td>US Manuf., annual 1948-71.</td>
</tr>
<tr>
<td>Longva et al. (1983)$^5$</td>
<td>-0.66</td>
<td>-0.27</td>
<td>-0.90</td>
<td>Norwegian Basic metals, annual 1962-78, 1978.</td>
</tr>
<tr>
<td>This study</td>
<td>-0.55</td>
<td>-0.17</td>
<td>-0.32</td>
<td>Norwegian aluminium industry.</td>
</tr>
</tbody>
</table>

1) Most studies use total energy. We use electricity which covers most of the energy input in the Norwegian aluminium industry.
2) When time series are used, we give the base year for the calculated elasticities.
3) Blue-colour/Production workers.
4) White-colour/Non-production workers.
5) We report the partial equilibrium price elasticities when capital is a quasi-fixed factor implicitly defined by the full equilibrium model presented in Longva and Olsen, cf. Olsen (1983).

The most interesting analyses to compare with our results is perhaps Longva and Olsen (1983) and Bergstrøm et al. (1992), who estimate price elasticities for the Norwegian and Swedish basic metal industry respectively by using aggregate time series data. We should add that these analyses apply a static rather than a dynamic model, and that the Swedish metal industry is dominated by iron and steel with aluminium as only a small share. Our own price elasticities are smaller (in absolute value) than the partial
equilibrium own price elasticities for the Norwegian basic metal industry, but larger than the own price elasticities for labour in the Swedish basic metal industry.

The relatively fixed input structure implied by our results, suggests that an increase in the price of electricity faced by this industry will affect factor proportions only little. At a given output level and capacity, the aluminium plants will react by reducing their input of electricity and increasing their input of labour, while the input of raw materials will be very little affected.
5. **Concluding remarks**

Despite the limitations of this analysis due to the treatment of capital as a predetermined factor, the estimated cost share equations provide useful information on the structure of production in the Norwegian primary aluminium industry. The key findings are:

1. The conditional cost share equations are homogeneous of degree zero in input prices and the cross price effects are symmetric.

2. The hypothesis of fixed input coefficients for labour, raw materials and electricity is clearly rejected.

3. The partial equilibrium demand for variable inputs is inelastic, all own price elasticities are above minus one.

4. The mean industry elasticities show that all variable inputs are substitutes, but the partial equilibrium cross price elasticities between electricity and raw materials are close to zero.

5. Hicks neutral technical progress is accepted in both the short- and the long-run.

6. The conditional cost function suggests that the production technology is homothetic in the long- but not in the short-run. Scale invariant input coefficient for raw materials in the short-run is accepted, however.

7. The partial equilibrium input share for electricity is capacity invariant.

8. With respect to dynamic specification, we prefer the restricted error correction model to the alternative autoregressive error process model, primarily due to a more parsimonious description of the partial equilibrium. These two dynamic models are very similar with respect to statistical properties such as fit and price elasticities, both in the long- and the short-run. Akaike's information criterion and the ordering criteria suggested by Pollak and Wales prefer the autoregressive error process model, however.

The aluminium industry, which is extremely energy intensive, has been developed in Norway due to access to cheap hydroelectricity. In 1990, the industry accounted for about 15 per cent of domestic electricity consumption. Norwegian economists have for
many years argued that increased efficiency in the electricity market will give increased welfare and beneficial changes to the Norwegian economy, cf. Klette (1990). Cheap electricity to the energy intensive industries in Norway, including the aluminium industry, favours the development of these industries and has lead to an industry structure which may not be optimal. Furthermore, modern technology has made it possible to sell electric power on international markets and may thereby have increased the alternative price of electricity. The relatively fixed input structure implied by our results, suggests that an increase in the price of electricity faced by this industry will affect factor proportions only little. At a given output level and capacity, the aluminium plants will react by reducing their input of electricity and increasing their input of labour, while input of raw materials will be very little affected. The effects on output and investment decisions, which may be of major importance, are not captured by this analysis though.
APPENDIX 1.

The data

Primarily we use data from the manufacturing statistics database at Statistics Norway. The manufacturing statistics follows the Standard Industrial Classification (SIC) and gives annual data for firms at the 5-digit code. Our panel includes seven plants for the whole period 1972-1990. In addition, we include one plant which was shut down in 1981. A ninth plant operating in 1972, but shut down in 1974, is not included. With respect to the plants operating today, Hydro Aluminium A/S owns four of these and is a major share holder in a fifth, while Elkem Aluminium ANS owns the remaining two. We treat each plant as an independent unit, however, assuming that decisions of importance for conditional factor demand is taken at the plant level.

The manufacturing statistics gives output in tonnes (metric tons) of aluminium\(^8\), the number of man hours used, total labour costs, input of raw materials in Norwegian kroner (NOK) and total electricity consumption in kWh and in NOK for each plant. We use this to calculate individual labour costs per hour and electricity prices. The data for both labour and electricity measure "gross" input, and hence do not refer to the technical production process for aluminium alone. E.g., the electricity consumption includes lighting and office heating.

One may question how well the calculated electricity prices, which are annual averages, approximate the price of electricity faced by the plants at the margin. These average prices reflect favourable long term electricity contracts with national power plants, which include agreements on both quantity and price, in addition to electricity at low prices from own power plants and also some trading in the spot market. If plants use all their contracted electricity, we may face a measurement error problem. However, we expect the plants to take advantage of the seasonal price variation in the spot market and sell contracted electricity when the spot market price is high and buy when it is low. This secures a low price also at the margin. The practice by the Hydro concern since the late eighties to operate with an internal pricing system, where the price faced by the plants is connected to a market price, also reduces this measurement error problem. We assume that these average prices and the prices of interest follow a common trend and that the measurement error because of our choice of data is not systematic and has a finite

\(^8\) We include commodity 7601, 7604, 7605, 7606, 7616.1000 and 7616.9001 in our measure of production. The classification of commodities follows the Harmonized System recommended by the UN.
variance. In this case, the long-run elasticities obtained from cointegrating error correction models are consistent, cf. Engle and Granger (1987).

We may face a problem with the price taking assumption in the labour market for at least two reasons. First, the aluminium plants are large and the major employer in the area where they are situated. There may therefore be elements of monopsony in the local labour markets. And second, if there are wage negotiations at the plant level. On the other hand, the relatively centralized wage formation system in Norway and the very high degree of unionization in this industry are assumed to justify the price taking assumption. Observed differences across plants in labour costs per hour may be due to non-homogeneous labour, e.g. due to variation in qualifications and average age of the employees. A simple correlation study reveals a high correlation in labour costs per hour, the average correlation coefficient equals 0.943.

Figure A.1 and A.2 show individual labour costs per man hour and electricity prices respectively, and reveal that there are some input price variation across the eight plants included in our analysis. The trends as well as the levels are relatively similar though.

Figure A.1. Plant specific labour costs per man hour, 1972-1990. NOK/Hour
We apply two alternative empirical proxies for capital. First, we use a capital measure based on fire insurance values. These insurance values, which are measured at the beginning of each period, are assumed to reflect the replacement values of the existing capital stock. We deflate the insurance values with the industry price of investment goods reported by the Norwegian national accounts. It is an open question how well these "capital stock" data represent the maximum feasible output. Even if the insurance values correspond exactly to the capital stock values, measurement errors in the chosen price deflator will carry over to the calculated capital stock data. We find implausible outliers in the reported insurance values for some plants, and in those cases we adjust the insurance values by using the development in investments. The second set of capital measure uses smelting capacity in tonnes aluminium per year reported by the plants on request. This measure avoids the errors in the price deflator problem. We adjust these data for additional capacity due to upgrading of purchased second grade aluminium, which was important for some plants during the late eighties. These capacity data, which reflects potential output, may be interpreted as efficiency corrected capital measures, cf. Sato (1975). Changes in reported capacities may be due to (i) investments in capital with "old" technology, (ii) investments in capital with "new" technology (embodied technical change), or (iii) increased efficiency of existing capital (disembodied technical change).

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9 Capacity data over 1972-1986 is provided to us by Torstein Bye (Statistics Norway) and Finn Førsund (SNF, Oslo).

10 The implication of using capacity as a capital measure, is that the coefficient $\gamma_{KT}$ in the cost function (1), which is assumed to capture changes in the capital efficiency, should equal zero.
The assumption that capital is predetermined may be questioned when applying the second capital measure, because we use capacities reported for year $t$ as the capacities for that year. We argue, however, that the weak exogeneity assumption is plausible also with these measures, because observed capacity expansions are largely due to investment decisions and installations in previous periods. This assumption is tested.

The reported data for input of raw materials, which includes purchased second grade aluminium, is deflated by a common industry price index from the national accounts to obtain quantity measures. The major raw material is alumina, which is imported. To guarantee their access to alumina at competitive prices, Norwegian firms have long-term contracts with foreign mines and have invested in mines abroad. Because contracts and raw materials purchases are made at different points in time, and because of variation in transport costs, one may expect the plants to face different prices of raw materials, even if these are homogeneous goods. In that case, we face a problem with measurement errors in one of the explanatory variables, because we apply the same measure for this price for all plants. This problem is increased if the input structure for different raw materials varies significantly across plants. On the other hand, again according to Engle and Granger (1987), this measurement error problem is reduced if we can assume that all plants face raw material prices with a common trend and the error is integrated of order zero and has a finite variance.

Figure A.3. The price of raw materials and capital faced by the primary aluminium industry, 1989=1

Figure A.3 gives the development in the common price index of raw materials and capital faced by the aluminium industry as reported by the Norwegian national accounts, 1989=1. It was necessary to adjust the price index of capital over 1984-1986 to correct for changes in the method applied to calculate this variable.

Because of the assumption of ex post fixed input coefficients in each plant in the Førsund and Jansen analysis, it is of interest to check the stability of individual input coefficients over time. Stability in an input coefficient supports the treatment of that input as a shadow factor to output, which increases the degrees of freedom when
estimating. Simple trends or consistent shifts over time in input coefficients may reflect factor using or saving technical changes. Figure A.4-A.8 show individual input coefficients, while figure A.9 shows the average industry input coefficients. The ratios in figure A.8, which use capital equal to capacity, are capacity utilization ratios, i.e. inverse input ratios.

Figure A.4. Plant specific ratios of labour measured in man hours to output measured in tonnes

Figure A.5. Plant specific ratios of raw materials measured in constant 1989-kroner to output measured in tonnes
Figure A.6. Plant specific ratios of electricity consumption measured in MWh to production measured in tonnes

Figure A.7. Plant specific ratios of capital stock measured in constant 1989-kroner to output measured in tonnes. The capital stock data is based on fire insurance values
Our conclusion from studying figure A.4 is that the variability in labour-output ratios can not be explained by technical progress alone, because they shift both up and down over time. Labour productivity has increased significantly for most plants during the second half of the eighties though, primarily due to increased production. Figure A.5 shows significant changes in the raw materials-output ratios, and we therefore choose to treat raw materials as a variable factor in the econometric analysis rather than as a shadow factor to output. The validity of the shadow factor assumption is tested however. Figure A.6, which gives the electricity-output ratios, shows that this input coefficient has decreased over time for several plants. From figure A.7, we see that the capital-output
ratios have increased significantly during the seventies but stabilized somewhat during the eighties for most plants. Figure A.8 shows that the capacity utilization ratios have stabilized during the late eighties at a utilization ratio close to one for most plants. Figure A.4-A.8 also reveals that plant 8 has relatively large input coefficients and a low capacity utilization rate during the seventies, which may explain why this unit was shut down in 1981.

Figure A.9 shows that the average industry input coefficient is clearly decreasing during the late eighties for labour, is relatively stable over time despite annual variation for raw materials, is decreasing for electricity and is clearly increasing for capital with respect to the fire insurance measure. The capacity utilization ratio stabilizes at a high level during the late eighties after a lower capacity utilization during most of the seventies. Thus, while the fire insurance measure predicts increased capital input coefficient, the capacity measure predicts decreased capital input coefficient.
APPENDIX 2.

The organization of the data

The individual data for each variable are stacked in long vectors along the time dimension. Each long vector order the observations of a variable for each period for plant 1 first, then follows the observations for each period for plant 2 etc. $V$ denotes an arbitrary variable, and $\Delta$ is the first difference operator.

\[
\begin{array}{ccc}
\Delta V_t & \Delta V_{t-1} & V_{t-1} \\
\hline
\Delta V_{1,1974} & \Delta V_{1,1973} & V_{1,1973} \\
. & . & . \\
. & . & . \\
\Delta V_{1,1990} & \Delta V_{1,1989} & V_{1,1989} \\
\Delta V_{2,1974} & \Delta V_{2,1973} & V_{2,1973} \\
. & . & . \\
. & . & . \\
\Delta V_{2,1990} & \Delta V_{2,1989} & V_{2,1989} \\
. & . & . \\
. & . & . \\
\Delta V_{7,1974} & \Delta V_{7,1973} & V_{7,1973} \\
. & . & . \\
. & . & . \\
\Delta V_{7,1990} & \Delta V_{7,1989} & V_{7,1989} \\
\Delta V_{8,1974} & \Delta V_{8,1973} & V_{8,1973} \\
. & . & . \\
. & . & . \\
\Delta V_{8,1981} & \Delta V_{8,1980} & V_{8,1980} \\
\hline
127x1 & 127x1 & 127x1 \\
\end{array}
\]
APPENDIX 3.

Testing the lag structure and choosing capital measure

This appendix reports the results from testing the general model (6) in chapter 2 with respect to lag structure, i.e. the number of lags that should be included. The choice between the two alternative capital stock measures described in appendix 1 is also discussed.

According to table A.1, the exclusion of $\Delta Z_{t-1}^*$ is a valid reduction of the general model both when capital is based on fire insurance values (FIV) and on reported capacity (CAP) data. The exclusion of $\Delta S_{t-1}^n$ is valid according to the CAP-model, but not according to the FIV-model at the three per cent significant level.

To help us choose between the alternative capital measures, we test the alternative capital measure models with reduced lags against their linear nesting models. From table A.2 we see that neither model I or II are rejected against their linear nesting model, but we conclude that the CAP-alternative is preferred to the FIV-alternative because of a

<table>
<thead>
<tr>
<th>Table A.1. Testing the lag length, the general lag model refers to system (6) in chapter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General lag</strong></td>
</tr>
<tr>
<td>K equals FIV</td>
</tr>
<tr>
<td>$\ln L = 701.35$</td>
</tr>
<tr>
<td>DF = 69</td>
</tr>
<tr>
<td><strong>C^0 = 0</strong></td>
</tr>
<tr>
<td>$\ln L = 696.91$</td>
</tr>
<tr>
<td>$\chi^2(10) = 5.22 (88%)$</td>
</tr>
<tr>
<td>DF = 79</td>
</tr>
<tr>
<td><strong>I C^0 = 0, A^0 = 0</strong></td>
</tr>
<tr>
<td>$\ln L = 688.33$</td>
</tr>
<tr>
<td>$\chi^2(4) = 10.69 (3%)$</td>
</tr>
<tr>
<td>DF = 83</td>
</tr>
<tr>
<td><strong>General lag</strong></td>
</tr>
<tr>
<td>K equals CAP</td>
</tr>
<tr>
<td>$\ln L = 706.15$</td>
</tr>
<tr>
<td>DF = 69</td>
</tr>
<tr>
<td><strong>C^0 = 0</strong></td>
</tr>
<tr>
<td>$\ln L = 698.92$</td>
</tr>
<tr>
<td>$\chi^2(10) = 8.32 (60%)$</td>
</tr>
<tr>
<td>DF = 79</td>
</tr>
<tr>
<td><strong>II C^0 = 0, A^0 = 0</strong></td>
</tr>
<tr>
<td>$\ln L = 692.06$</td>
</tr>
<tr>
<td>$\chi^2(4) = 8.64 (7%)$</td>
</tr>
<tr>
<td>DF = 83</td>
</tr>
</tbody>
</table>

$\ln L$ is the value of the log-likelihood function.

$\chi^2(j)$ is the likelihood ratio test where $j$ denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses.

DF denotes the degrees of freedom.
higher value of the log-likelihood function. This is consistent with the dominance ordering criterion and the likelihood dominance criterion put forward by Pollak and Wales (1991), when two alternative hypotheses contain the same number of estimated coefficients. For similar reasons, the CAP-model is preferred to the FIV-model also for the models where $C^n = 0$ but $A^n \neq 0$. The maintained hypothesis from this first step is therefore model II.

Table A.2. Testing model I and II in table A.1 against their linear nesting model to discriminate between the two alternative capital measures

<table>
<thead>
<tr>
<th>The linear nesting model of I and II in table A.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^n = 0, A^n = 0$</td>
</tr>
<tr>
<td>$DF = 79$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>$C^n = 0, A^n = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$lnL = 688.33$</td>
<td></td>
</tr>
<tr>
<td>$\chi^2(4) = 6.05$ (20%)</td>
<td></td>
</tr>
<tr>
<td>$DF = 83$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II</th>
<th>$C^n = 0, A^n = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$lnL = 692.06$</td>
<td></td>
</tr>
<tr>
<td>$\chi^2(4) = 1.36$ (85%)</td>
<td></td>
</tr>
<tr>
<td>$DF = 83$</td>
<td></td>
</tr>
</tbody>
</table>

$lnL$ is the value of the log-likelihood function.
$\chi^2(j)$ is the likelihood ratio test where $j$ denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses.
$DF$ denotes the degrees of freedom.
APPENDIX 4.

Testing the dynamic specification and theoretical and technical restrictions on the autoregressive error process model

We now test the restrictions on the dynamic structure put forward in table 2.1 on the maintained hypothesis (model II) from step one, cf. appendix 3. From table A.3 we see that both the partial adjustment model and the static model are rejected, while the first order autoregressive error process (AR) model is not. The support for the AR-model suggests the existence of common factors between the cost shares and the regressors in the dynamic model, cf. Hendry and Mizon (1978). Taking into account these common factors increases the degrees of freedom by 12, and implies a gain in estimation efficiency.

Table A.3. Testing the alternative dynamic structures defined in table 2.1 on model II in appendix 3. The capital stock equals capacity (CAP)

<table>
<thead>
<tr>
<th>Model</th>
<th>Method</th>
<th>InL</th>
<th>$\chi^2(12)$</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>EC-model</td>
<td>692.06</td>
<td>2.61 (100%)</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>AR-model</td>
<td>690.18</td>
<td>52.84 (0%)</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>PA-model</td>
<td>653.93</td>
<td>118.45 (0%)</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>Static model</td>
<td>611.83</td>
<td>118.45 (0%)</td>
<td>99</td>
</tr>
</tbody>
</table>

lnL is the value of the log-likelihood function. $\chi^2(j)$ is the likelihood ratio test where j denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses. DF denotes the degrees of freedom.

EC, AR and PA denote error correction, autoregressive error process and partial adjustment respectively.

Table A.4 summarizes the results from testing the theoretical and technical restrictions put forward in table 2.2 on the AR-model III in table A.3.
Table A.4. Testing theoretical and technical restrictions on the autoregressive error process (AR) model III in table A.3. The capital stock equals capacity (CAP)

<table>
<thead>
<tr>
<th>III AR-model</th>
<th>lnL = 690.18</th>
<th>DF = 95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneity</td>
<td>lnL = 686.84</td>
<td>( \chi^2(2) = 5.00 ) (8%)</td>
</tr>
<tr>
<td>Symmetry</td>
<td>lnL = 685.51</td>
<td>( \chi^2(1) = 2.02 ) (16%)</td>
</tr>
<tr>
<td>No price effects</td>
<td>lnL = 629.85</td>
<td>( \chi^2(3) = 87.09 ) (0%)</td>
</tr>
</tbody>
</table>

\( \gamma_{MX} = 0 \)

\( \gamma_{MK} = -\gamma_{LK} \); \( \gamma_{EK} = 0 \)

\( \alpha_\ell = \alpha_\ell, \ f=1,\ldots,8 \)

\( \beta_{LM} = -\beta_{PL}; \beta_{LE}=0 \)

\( \alpha_{MF} = \alpha_M, \ f=1,\ldots,8 \)

lnL is the value of the log-likelihood function.
\( \chi^2(j) \) is the likelihood ratio test where \( j \) denotes the number of restrictions. The significance level where the null hypothesis is rejected is given in parentheses.
DF is the degrees of freedom.
\( \alpha_i \) is the individual constant term in the cost share equation for input \( i \).

Table A.4 shows that price homogeneity, symmetric cross price effects and Hicks neutrality are supported by the data, while homotheticity and no price effects, i.e. a Cobb-Douglas production technology, are clearly rejected. With respect to Hicks
neutrality, the data clearly supports the hypothesis that $\gamma_{MT}=0$, but the additional hypothesis that $\gamma_{LTE}=\gamma_{ET}=0$ is rejected at the four per cent significance level. We find support for scale invariant input shares between raw materials on one side and labour and electricity on the other. Between labour and electricity, there is a small tendency of decreasing input share for labour and increasing input share for electricity as output increases. The data does not support capacity invariant input shares, and we only report the result from testing this restriction on electricity. The results suggest that the input share for labour and electricity decreases with capacity, while the input share for raw materials increases with capacity. The null hypotheses that the constant terms in the cost share equations for labour and raw materials are identical across producing units are rejected. The implicit significance level for testing the accepted hypothesis in table A.4, i.e. model IV, against the most general model in table A.1 with capital equal to capacity, is eight cent.

The estimated coefficients are presented in table A.5. The coefficients $\beta_{iE}$, $i=L,M,E$, and $\gamma_{jE}$, $j=X,K,T$, are calculated by using the adding up conditions. The 8·3 number of individual constant terms are not reported.

| Table A.5. The estimated coefficients of the autoregressive error process model |
|---|---|
| Coefficients | Estimates | Coefficients | Estimates |
| $\beta_{LL}$ | 0.067 (.012) | $\gamma_{LX}$ | -0.042 (.010) |
| $\beta_{LM}$ | -0.067 * | $\gamma_{MX}$ | 0 * |
| $\beta_{LE}$ | 0 | $\gamma_{EX}$ | 0.042 (.010) |
| $\beta_{MM}$ | 0.175 (.016) | $\gamma_{LK}$ | -0.135 (.020) |
| $\beta_{ME}$ | -0.108 (.026) | $\gamma_{MK}$ | 0.194 (.023) |
| $\beta_{EE}$ | 0.108 (.026) | $\gamma_{EK}$ | -0.059 (.017) |
| $\gamma_{LT}$ | 0 * | $\gamma_{MT}$ | 0 * |
| $\gamma_{ET}$ | 0 * |

$\ln L = 682.40$  \quad  L: $R^2 = 0.505$  \quad  M: $R^2 = 0.498$

$DF = 102$  \quad  $CR^2 = 0.406$  \quad  $CR^2 = 0.398$

SER = 0.023  \quad  SER = 0.034

DW = 2.037  \quad  DW = 2.054

Standard errors in parentheses. $\ln L$ is the value of the log-likelihood function. DF is the degrees of freedom. The multiple correlation coefficient ($R^2$), $R^2$ corrected for degrees of freedom ($CR^2$), the equation standard error (SER) and the Durbin-Watson (DW) statistic are given for the estimated cost share equations. The DW-statistic must be interpreted with some care, because in 7 of the 127 sample points, the t-1 residual is the residual of a different firm due to our organization of the data in long vectors. This may bias the DW-statistic towards two.

* Restricted a priori. All restrictions are supported by the likelihood ratio test.
Table A.6 presents the partial equilibrium Hicks-Allen partial elasticities of substitution in addition to cross price and own price elasticities calculated at sample means. We approximate standard errors of these elasticities by assuming that sample mean cost shares are constants rather than stochastic variables. At sample means, the AR-model predicts that labour and raw materials as well as labour and electricity are substitutes, while raw materials and electricity are weak compliments. The cross price elasticities between the two latter inputs are approximately zero.

<table>
<thead>
<tr>
<th>HA elasticities</th>
<th>Own and cross price elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{LL}$</td>
<td>$\varepsilon_{LL}$</td>
</tr>
<tr>
<td>-2.03 (.219)</td>
<td>-0.48 (.052)</td>
</tr>
<tr>
<td>$\sigma_{LM}$</td>
<td>$\varepsilon_{LM}$</td>
</tr>
<tr>
<td>0.51 (.088)</td>
<td>0.30 (.052)</td>
</tr>
<tr>
<td>$\sigma_{LE}$</td>
<td>$\varepsilon_{LE}$</td>
</tr>
<tr>
<td>1.00 *</td>
<td>0.18 *</td>
</tr>
<tr>
<td>$\sigma_{MM}$</td>
<td>$\varepsilon_{MM}$</td>
</tr>
<tr>
<td>-0.20 (.045)</td>
<td>-0.11 (.027)</td>
</tr>
<tr>
<td>$\sigma_{ME}$</td>
<td>$\varepsilon_{ML}$</td>
</tr>
<tr>
<td>-0.03 (.255)</td>
<td>0.12 (.021)</td>
</tr>
<tr>
<td>$\sigma_{EE}$</td>
<td>$\varepsilon_{ME}$</td>
</tr>
<tr>
<td>-1.22 (.839)</td>
<td>-0.01 (.045)</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{EE}$</td>
</tr>
<tr>
<td></td>
<td>-0.22 (.148)</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{EL}$</td>
</tr>
<tr>
<td></td>
<td>0.24 *</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{EM}$</td>
</tr>
<tr>
<td></td>
<td>-0.02 (.149)</td>
</tr>
</tbody>
</table>
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