Statistisk sentralbyrå
Statistics Norway

The marginal (opportunity) cost of public funds

Geir H. M. Bjertnæs

TALL SOM FORTELLER

DISCUSSION PAPERS

925

Discussion Papers No. 925, April 2020
Statistics Norway, Research Department

Geir H. M. Bjertnæs

The marginal (opportunity) cost of public funds

Abstract:
Several studies show cases where the Samuelson rule holds, or where the marginal cost of public funds (MCF) equals one within optimized tax systems. The conditions for the original Samuelson rule to hold in these studies are quite restrictive, and MCF measures employed are not consistent with MCF measures employed within real-world cost-benefit tests. The aim of the present study is to remove such restrictive conditions, and to construct a MCF measure designed for real-world cost-benefit tests. The study shows that such a MCF exceeds one within optimized tax systems. Hence, the optimal supply of public goods is below the supply obtained by the Samuelson rule. The study further shows that income taxation below optimum requires an even higher MCF to prevent that public goods provision crowd out social security transfers with a higher marginal welfare gain

Keywords: Marginal cost of public funds, The Samuelson rule, Optimal taxation, Social security transfers

JEL classification: H21; H23; H41

Acknowledgements: I am highly grateful for valuable comments by Vidar Christiansen, Andreas Peichl, Erling Holmøy, participants at the IIPF conference in 2018, and participants at the OFS Christmas seminar 2019.

Address: Geir H. M. Bjertnæs, Statistics Norway, Research Department, P.O. BOX 8131 Dep, NO-0033 Oslo, Norway, E-mail: ghb@ssb.no

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

Abstracts with downloadable Discussion Papers
in PDF are available on the Internet:
http://www.ssb.no/en/forskning/discussion-papers
http://ideas.repec.org/s/ssb/dispap.html

ISSN 1892-753X (electronic)

Sammendrag

Mange offentlige prosjekter finansieres med skatter som har negative virkninger på økonomiens effektivitet mht. ressursbruk. Kostnadene av slike negative virkninger implementeres i nytte-kostnadsanalyser ved at kostnadene multipliseres med en faktor, MCF. Norge har benyttet en MCF lik 1.2 de senere årene.

Flere nyere studier viser at tilbudet av kollektive goder bør bestemmes av nytte-kostnadsanalyser uten korreksjoner for negative virkinger av skatter, dvs. at MCF er lik 1. Definisjonen på MCF som benyttes i flere av disse studiene forutsetter imidlertid at betalingsviljen for kollektive goder i nytte-kostnadsanalyser vektes med en faktor som korrigerer for fordelingseffektene av kollektive goder. En slik vekting inngår imidlertid ikke i anvendte nytte-kostnadsanalyser. Forskningsbidraget i denne artikkelen er å beregne et MCF-mål som er tilpasset anvendte nytte-kostnadsanalyser. Utgangspunktet for beregningen er at kollektive goder er fordelingsnøytrale. Faktoren som korrigerer for fordelingseffektene av kollektive goder er derfor fjernet. Beregningen av MCF tar imidlertid hensyn til at skattene tilpasses optimalt, dvs. slik at velferden i samfunnet maksimeres. MCF beregnes også i tilfellet der skatteinntektene er underoptimale.

Studien finner at MCF bør settes slik at velferdsgevinsten av investeringer i kollektive goder på marginen er lik alternativkostnaden av penger i offentlig sektor. Denne alternativkostnaden er lik velferdsgevinsten av sosiale overføringer. Studien finner at MCF har en størrelsesorden rundt hhv. 1.04 og 1.15 i optimale løsninger for Norge med uniforme overføringer, gitt at arbeidstilbudselastisiteten er lik hhv. 0.1 og 0.2. Studien finner at MCF bør settes marginalt høyere når det antas at overføringer er forbeholdt svakerestilte grupper i samfunnet. Studien finner i tillegg at MCF bør settes enda høyere når inntektsskatten settes lavere enn den optimale løsningen. Forklaringen er at lavere skatteinntekter øker velferdsgevinsten av omfordeling. En økning i MCF forhindrer at kollektive goder fortrenger omfordeling som gir en høyere velferdsgevinst. Disse resultatene gir samlet sett støtte for en videreføring av dagens praksis med en MCF lik 1.2. Forenklede forutsetninger innebærer at de numeriske resultatene bør tolkes som illustrasjoner.

1. Introduction

Many public projects are financed with tax revenue from distorting taxes. The cost of such distortions is incorporated into cost-benefit analyses of public projects by multiplying costs with a factor labeled the Marginal Cost of Public Funds (MCF). Recent studies however argue that the MCF equals one within optimized tax systems, and that the Samuelson rule should determine the supply of public goods, see Hylland and Zeckhauser (1979), Boadway and Keen (1993), Kaplow (1996), Sandmo (1998), Christiansen (1981), Christiansen (2007), Jacobs (2018) and Stiglitz (2018). The present study challenges the arguments which lead to these results. It is argued that these results eighter rely on restrictive assumptions or a definition of MCF which is inconsistent with applied cost-benefit tests. The definitions of MCF employed require a modification of the Samuelson rule where the willingness to pay for public goods is adjusted with distributional characteristics of public goods, see Sandmo (1998) and Jacobs (2018). Such adjustments are excluded from applied cost-benefit tests, however. The contribution of the present study is to derive a measure of MCF which is relevant in such an applied cost-benefit analyses. The analytical expressions are supplemented with numerical illustrations.

The present study also challenges the view that the MCF equals one within optimized tax systems. The study shows that the MCF is determined by the opportunity cost of public funds, i.e. so that the welfare gain of spending additional public funds on public goods provision equals the welfare gain of spending additional public funds on social security transfers. The study shows that the MCF designed for applied cost-benefit studies exceeds one in the case of optimized linear income taxation, and in the case of optimized non-negative categorical social security transfers. Hence, the optimal supply of public goods will be lower than the supply implied by the Samuelson rule. Income taxation below optimum requires an even higher MCF to prevent that public goods provision crowds out social security transfers with a higher marginal welfare gain.

The studies referred to above, in which the original Samuelson rule holds, rely on quite restrictive assumptions. The pioneering contribution of Christiansen (1981) shows that the original Samuelson rule holds within optimized non-linear tax systems when individuals have identical preferences that are weakly separable in labor and the bundle of public and private goods. Boadway and Keen (1993) confirm and generalize Christiansens result. They show that the original Samuelson rule should determine the level of public goods when the marginal rate of substitution between private and public goods are equalized among high- and low productive individuals. This condition is violated in the plausible case where individuals have equal additive preferences for public goods, and where poor individuals have higher marginal utility of private consumer goods. The present study contributes by quantifying the MCF within such plausible cases.

Definitions of MCF in the literature imply a modification of the original Samuelson rule, see excellent contributions by Sandmo (1998), Jacobs (2018), and Slemrod and Yitzhaki (2001). The modification consists of incorporating distributional characteristics of public goods into the benefit-side of the rule. Sandmo (1998) shows that the most frequently used definition of MCF, i.e. the ratio between the marginal value of money in the public sector and the average marginal value of money in the private sector, is less than one when leisure is a normal good and taxation distorts the supply of labor. Uniform transfers expand the distortion in the supply of labor, and hence, drives MCF below one in this case. A MCF below one raises the optimal supply of public goods. This result is consistent with Wilson (1991), who shows that the optimal level of public goods provision with optimized linear taxation exceeds the first-best level obtained by the original Samuelson rule. This optimized level is unchanged when sub-optimal consumer tax rates are implemented.

Jacobs (2018) shows that a Diamond based definition of MCF, which equals the marginal value of public funds divided by the average of the social marginal value of private income, equals one within optimized linear and non-linear tax systems with uniform transfers. The modification of the original Samuelson rule described above is also present with this definition. The MCF equals one, he argues, because it is sub-optimal to distort public goods provision when the welfare cost of raising tax revenue equals the welfare gain of redistributing public funds. Fixed sub-optimal transfers implies that the government resorts to distortionary taxation as a strategy to finance public goods provision. Jacobs (2018) shows that the MCF is below one in this case when the income tax is below the optimal level. The present study contributes by calculating MCF designed for applied cost-benefit tests, i.e. where distributional characteristics of public goods are excluded from cost-benefit tests. Distributional characteristics of public goods is excluded by assuming equal additive preferences for public goods.

The present study explores scenarios where linear taxation is optimized. It also studies the case of sub-optimal income tax rates, since taxes are unpopular with voters and politicians. The study shows that the most frequently used definition of MCF equals one in scenarios with both optimal and sub-optimal taxation. This definition of MCF equals one because the value of money in the public sector is determined by the value of uniform social transfers. Such uniform transfers do not distort the supply of labor when quasilinear utility is assumed. Such a utility function seems to be consistent with empirical studies of labor supply, showing negligible income elasticities, see Thoresen and Vattø (2015), Dagsvik et al. (2018) and Blau and Kahn (2007). These assumptions also imply that public goods provision does not magnify labor supply distortions. The value of money in the public sector therefore equals the average marginal value of money in the private sector in scenarios with both optimal and sub-optimal income taxation. Hence, the study elaborates on the finding in Jacobs (2018) by demonstrating that the most frequently used MCF measure equals one even though the proportional income tax rate is sub-optimal. These results deviate from those in Sandmo (1998) and Wilson (1991) because transfers do not distort the supply of labor.

The MCF measure relevant for applied cost-benefit tests equals the welfare maximizing adjustment in resource costs within the original Samuelson rule. A numerical illustration shows that this MCF measure approximately equals 1.04 and 1.16-1.25 within optimized solutions with linear income taxation and uniform transfers when the uncompensated labor supply elasticity equals 0.1 and 0.2, respectively. Hence, the optimal supply of public goods is below the supply obtained by the Samuelson rule. The MCF measure is even higher when the income tax rate is fixed below optimum. The opportunity cost of public funds is determined by the value of the alternative use of public funds in this case, see Massiani and Picco (2013). The explanation is that the opportunity cost of public goods provision, i.e. the welfare gain of social security transfers, increases as the tax is reduced below optimum. The MCF is increased to prevent that public goods provision crowd out social security transfers with a higher marginal welfare gain.

Most previous studies analyze MCF within models where uniform transfers are given to all individuals. However, real world social security transfers are typically given to specific groups of poor individuals, with a marginal utility of income above average. This scenario emerges in optimized solutions when it is assumed that categorical transfers are non-negative, see Slack (2015)[1]. The present study contributes by showing that the most frequently used definition of MCF is larger than one when non-negative categorical transfers are offered to a poor group with a marginal utility of income above average. The MCF designed for applied cost-benefit tests also exceeds one in cases with non-negative categorical transfers. These results hold in scenarios with extensive margin labor choices, see Bjertnæs (2018). The MCF exceeds one because the opportunity cost of public funds, the welfare gain of categorical transfers, exceeds the average value of money in the private sector. The MCF exceeds one even though the cost of raising tax revenue equals the welfare gain of redistributing public funds. Hence, this result contradicts the explanation of why the MCF equals one in Jacobs (2018). Numerical

[1] Positive lump-sum taxes are excluded in most countries. The Thatcher-government imposed lump-sum poll taxes in 1990 in England. It created social turmoil and riots in several cities before it was abandoned later that year.

sensitivity tests show that the MCF equals 1.05-1.07 and 1.11-1.16 in scenarios with non-negative categorical transfers to specific groups when the uncompensated labor supply elasticity equals 0.1 and 0.2, respectively. These results hold when the proportional income tax is replaced with a uniform tax on consumption, and when quasilinear preferences are replaced with Stone-Gary preferences. The MCF is increased when the income tax rate is reduced below optimum within these scenarios.

Section 2 present MCF-measures. Section 3 present the model framework and analytical results. Section 4 present numerical illustrations, and section 5 concludes.

2. MCF measures employed

The most frequently used approach is to define the MCF as the marginal value of money in the public sector divided by the (average) marginal value of money in the private sector, see e.g. Atkinson and Stern (1974) and Sandmo (1998). Jacobs (2018), on the other hand, employs a Diamond-based definition of MCF. The Diamond-based definition of MCF is identical with the definition in Sandmo (1998) when the supply of labor is unaffected by transfers. The definition of MCF in Sandmo (1998) is presented in equation (1).

(1) $MCF_1 = \frac{\mu}{\bar{\lambda}}$,

The MCF_1 is defined as the shadow value of public funds, μ, divided by the average marginal utility of income, $\bar{\lambda}$. The $\bar{\lambda}$-parameter is necessary in the definition to convert the welfare effect of public funds into units of income/ consumption goods. Hence, the MCF is sometimes interpreted as the governments marginal rate of substitution between money in the public and the private sector.

Different definitions of MCF produce different estimates of MCF. The academic discussion of MCF is consequently hampered as economists disagree on which definition that should be employed. Jacobs (2018) argue that the most regularly used definition has some undesirable properties. Holtsmark (2019) however argue that the Diamond-based definition has some undesirable properties[2]. The present study develops a new approach designed to avoids these problems. The new approach consists

[2] Valseth et al. (2019) adopts a new definition where the MCF equals the welfare cost of raising tax revenue divided by the welfare cost of increasing individualized lump-sum taxes so that the loss of utility is equalized among all individuals. This definition should however not be interpreted as the governments marginal rate of substitution between money in the public and the private sector.

of calculating the welfare maximizing adjustment in resource costs within the original Samuelson rule. Hence, the welfare maximizing MCF is found by adding consumers' marginal rate of substitution between private and public goods, MRS_{zc}, and setting this equal to the marginal rate of transformation, MRT_{zc}, multiplied with the welfare maximizing cost-adjusting factor, MCF_2. This factor is given by equation (2).

$$(2)\ \sum MRS_{zc} = MCF_2 MRT_{zc}.$$

The MCF based on this modified Samuelson rule implements the welfare maximizing supply of public goods when profitable projects are effectuated. The first-best Samuelson rule holds if MCF_2 equals one. The present study calculates MCF based on these two measures within optimal and sub-optimal tax systems.

3. The model framework

The model framework is designed to calculate MCF when a welfare maximizing government allocates public funds to public goods provision and social security transfers. A linear income tax distorts the labor/ leisure choice of working individuals. Different transfer systems are implemented to investigate implications for the MCF.

3.2. The behavior of individuals

There are two types of individuals in the economy with preferences for leisure, l_i, private consumption, c_i, and consumption of public goods, z. The utility function is identical for all individuals. $n_1$individuals are working. The n_2 non-working individuals receive social transfers. Utility functions are quasilinear for consumption above a given level, $\hat{c}$. The quasilinear utility function excludes tax base effects due to income effects. The utility function of working individuals, u_1, are given by

$$(3)\ u_1 = c_1 + g(l_1) + f(z).$$

Both $f(z)$ and $g(l_1)$ are increasing and strictly concave. Consumption is given by transfers, a, and the after-tax wage income

$$(4)\ c_1 = (1-t)wh_1 + a,$$

Where w equals a fixed wage rate, h_1, equals hours of work, and t equals the tax rate. The consumption of a working individual exceeds the consumption level $\hat{c}$. The private consumer good is the numeraire with a price equal to one. The time constraint is given by

$$(5)\ h_1 = T - l_1.$$

Working individuals maximize utility, given by equation (3), conditional on their budget, equation (4), and their time constraint, equation (5). First order conditions for this optimization problem imply that

$$(6)\ \lambda = 1.$$

The marginal utility of income, λ , equals one for all levels of consumption above $\hat{c}$. First order conditions also imply that

$$(7)\ \frac{\partial g}{\partial l_1} = (1-t)w.$$

The marginal rate of substitution between leisure and private consumption equals the after-tax wage rate. The quasi linear utility function and the fixed wage rate implies that leisure is given by the tax rate, *t*.

$$(8)\ l_1 = l_1(t) \qquad \frac{\partial l_1}{\partial t} \geq 0$$

This illustrates the distortion of the labor income tax. A later section shows that the welfare cost of raising tax revenue by increasing the labor income tax exceeds the resource cost with such preferences. The indirect utility of the working individual equals

$$(9)\ v_1 = (1-t)w(T - l_1(t)) + a + g(l_1(t)) + f(z).$$

The utility of the non-working individual is given by

$$(10)\, u_2 = S(c_2) + g(l_2) + f(z), \text{ where S' > 1 and S'' < 0 when } c_2 < \hat{c}.$$

The consumption of the non-working individual equals transfers, b. These transfers can be lower than $\hat{c}$. Hence, the indirect utility of the non-working individual is given by

$$(11)\, v_2 = S(b) + g(T) + f(z).$$

The utility derived from public goods provision is equalized between individuals. This assumption is implemented to illuminate the case with public goods without a specific distributional profile. The study also assumes that productivity and tax revenue is unaffected by the provision of public goods. These assumptions are crucial for results, see e.g. Sandmo (1998) and Kaplow (1996).

3.1. The government

The government maximizes an individualistic welfare function given the budget constraint of the government. Indirect utility functions are given by equation (9) and (11). The maximization problem of the government differs between scenarios with different social transfer schemes, and between scenarios with optimal and sub-optimal income taxation. Different scenarios are presented below.

3.2.1 Uniform transfers

The case with uniform transfers is implemented by assuming $a = b$. The maximization problem with optimal income taxation is given by

$$(12)\, \underset{z,t,b}{\text{Max}}\; n_1(1-t)w(T - l_1(t)) + n_1 b + n_1 g(l_1(t)) + n_1 f(z) + n_2 S(b) + n_2 g(T) + n_2 f(z)$$

Given the budget constraint

$$(13)\, n_1 t w(T - l_1(t)) = qz + n_1 b + n_2 b.$$

The price of public goods measured in units of the consumer good is denoted q. The Lagrangian is given by

$$(14) L = n_1(1-t)w(T-l_1(t)) + n_1 b + n_1 g(l_1(t)) + n_1 f(z) + n_2 S(b) + n_2 g(T) + n_2 f(z) + \mu[n_1 t w(T-l_1(t)) - qz - n_1 b - n_2 b].$$

The shadow value of public funds is denoted μ. The restrictions on $g(l_1)$, $l_1(t)$, $f(z)$ and $S(b)$ imply that the Lagrangian is concave, see appendix A. The first order conditions and calculations of MCF are presented in appendix B. Key equations to calculate MCF is presented below.

The MCF was defined as the shadow value of public funds divided by the average marginal utility of income, see equation (1). The shadow value of public funds, μ , which is the numerator on the right-hand side of equation (1), is determined by the labor supply elasticity and the income tax rate

$$(15) \mu = \frac{1}{\left[1 - \frac{\partial h_1}{\partial w_a} \frac{w_a}{h_1} \frac{t}{(1-t)}\right]},$$

where the after-tax wage rate equals $w_a = (1-t)w$. It follows directly from the first order conditions that the shadow value of public funds, μ, equals the welfare cost of raising tax revenue. The shadow value of public funds equals 1.11 and 1.25 when the tax rate equals 0.5 and the labor supply elasticity equals 0.1 and 0.2, respectively. The average marginal utility of income, which is the denominator on the right- hand side of equation (1), is defined as

$$(16) \bar{\lambda} = \frac{n_1 + n_2 \frac{\partial S}{\partial c_2}}{n_1 + n_2}.$$

Equation (1) and (16) implies that

$$(17)\ MCF_1 = \frac{\mu}{\frac{n_1 + n_2 \frac{\partial S}{\partial c_2}}{n_1 + n_2}}$$

$\partial S / \partial c_2$ is given by the first order condition with respect to transfers, see appendix B.

$$(18)\frac{\partial S}{\partial c_2} = \frac{(n_1+n_2)\mu - n_1}{n_2}.$$

Equation (16) and (18) implies that $\bar{\lambda} = \mu$. Hence MCF_1 equals one. This confirms the result in Jacobs (2018). The opportunity cost of public goods funds is crucial for this result. The uniform-transfers-to-all assumption imply that the value of money in the public sector equals the average marginal utility of money in the private sector. Hence, MCF_1 equals one.

The first order conditions also imply that the welfare cost of raising tax revenue, μ, equals the welfare gain of spending public funds on public goods provision

$$(19)\frac{(n_1+n_2)\frac{\partial f}{\partial z}}{q} = \mu.$$

Hence, the welfare gain of public goods provision exceeds the resource cost by a factor which equals the welfare cost of raising tax revenue. The MCF_1, which is designed to implement this solution, however deviates from this factor because it is transformed into units of private consumer goods. This insight illuminates on the difference between the traditional approach in Browning (1976), and the optimal policy approach in e.g. Sandmo (1998).

3.2.2 Uniform transfers and sub-optimal income taxation

The maximization problem with sub-optimal income taxation is found by excluding the first order condition w.r.t. the income tax rate, t. Hence, equation (15) is excluded. Equation (15) is however redundant in calculations of MCF_1. Hence, MCF_1 equals one with both optimal and sub-optimal income taxation when uniform transfers are optimized. The economic intuition is that MCF_1 is determined by the opportunity cost of public funds. The uniform-transfers-to-all assumption imply that the value of money in the public sector equals the average marginal utility of money in the private sector.

The modified Samuelson rule in equation (2) becomes

$$(20) n_1\frac{\partial f}{\partial z} + n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}} = MCF_2 q.$$

Equation (20) and the first order condition w.r.t. public goods provision, z, implies that

$$(21)\, MCF_2 = \frac{\mu}{\frac{n_1 + n_2 \frac{\partial S}{\partial c_2} + \left(\frac{\partial S}{\partial c_2} - 1\right) n_1}{n_1 + n_2 + \left(\frac{\partial S}{\partial c_2} - 1\right) n_1}},$$

according to appendix C. This condition is identical in all scenarios. A comparison of equation (17) and (21) shows that the MCF_2 is larger than MCF_1 defined in equation (1) when $\partial S / \partial c_2$ is larger than one. Equation (18) and (21) implies that

$$(22)\, MCF_2 = \frac{1}{\frac{(n_1+n_2) + \frac{(\mu-1)(n_1+n_2)n_1}{n_2 \mu}}{(n_1+n_2) + \frac{(\mu-1)(n_1+n_2)n_1}{n_2}}} > 1 \text{if } \mu > 1.$$

Equation (22) shows that MCF_2 exceeds one when the shadow price of public funds, μ, exceeds one. Equation (15) implies that μ is larger than one if t is larger than zero. Finally, t is larger than zero within an optimized solution because the marginal welfare gain of public goods provision and transfers exceeds one at low levels, and because tax revenue finances such public spending according to the government budget constraint. Hence, MCF_2 exceeds one when uniform transfers are offered to all individuals. A MCF larger than one is required to implement the welfare maximizing supply of public goods. A downscaling of benefits within cost-benefit tests is required to implement the optimal supply of distribution neutral public goods if the MCF is set equal to one. An alternative interpretation is that the MCF_2 exceeds MCF_1 when non-workers are willing to sacrifice fewer private consumer goods for one public good compared to workers, i.e. MRS_{zc} for non-workers is lower than for workers. The MRS_{zc} between private and public goods is lower for non-working individuals as their marginal utility of private goods is higher. This theoretical outcome is consistent with the result in Boadway and Keen (1993).

The solution for MCF_2 within optimized income tax systems differ from the solution within sub-optimal tax systems as the shadow value of public funds, μ , differ between these solutions. A sub-optimal income tax rate generates an amount of tax revenue, which alters the shadow value of public funds, μ. The proof acknowledges that the first order conditions implies a positive connection between z and b. This positive connection and the budget constraint imply that a reduction in the tax rate entails a reduction in both transfers and public goods provision. The first order conditions imply that a reduction of transfers (or public goods provision) leads to an increase in the shadow value of public spending, μ. It follows from equation (22) that an increase in the shadow value of public funds generates an increase in MCF_2. Hence, an income tax rate below optimum requires a higher MCF_2 compared to an optimized solution. The magnitude of the change in μ, and hence in MCF_2, is determined by n_1, $n_2$and the shape of both $S(c_2)$ and $f(z)$.

3.2.3 Categorical transfers

The case with categorical transfers, i.e. group specific transfers, is analyzed by assuming that workers and non-workers belong to different observable groups. Hence, the government chooses transfers to workers, a, transfers to non-workers, b, the supply of public goods, as well as the income tax rate. Note that this scenario also represents a scenario with individualized lump-sum transfers as individuals within each group are identical. The problem is

$$(23) \max_{z,t,a,b} n_1(1-t)w(T-l_1(t)) + n_1 a + n_1 g(l_1(t)) + n_1 f(z) + n_2 S(b) + n_2 g(T) + n_2 f(z)$$

Given the budget constraint

$$(24) n_1 tw(T-l_1(t)) = qz + n_1 a + n_2 b.$$

The first order conditions imply that ${\partial S}/{\partial c_2} = \mu = 1$, see appendix D. Transfers to non-workers therefore equals or exceeds $\hat{c}$. Hence, this solution, equation (1) and equation (16) imply that MCF_1 equals one. This solution and equation (21) also imply that MCF_2 equals one. Categorical transfers are chosen to eliminate inequality in the average social marginal value of income between tagged groups. Hence, the result in Viard (2001) is confirmed. The income tax rate equals zero due to the condition in equation (15). This solution and the government budget constraint imply that $n_1 a = -qz - n_2 b$. Hence, public goods provision and transfers to non-workers are finances by positive lump-sum taxes on workers.

Positive lump-sum taxes imposed by Margaret Thatcher in 1990 lead to social turmoil and riots. One may argue that positive lump-sum taxes is excluded from tax system to prevent potential costs of such turmoil. Hence, exclusion of such lump-sum taxes might be part of an optimal tax system. Many countries have on the other hand implemented income tax systems with tax credits offered to low income earners. A reduction in such tax credits resembles a lump-sum tax on all workers if the tax credit is offered to all workers. Medium- and high-income earners are however excluded from the tax credit in countries where the tax credit is phased out as income exceeds some threshold. The earned income tax credit in the US is an example. Hence, adjustments in such tax credits do not resemble a lump-sum tax on all workers.

The solution with a positive lump-sum tax on workers also imply that the average marginal utility of private consumption is equalized among workers and non-workers. The empirical relevance of this outcome is questionable because consumption levels differ substantially between workers and non-workers in most countries. Hence, the solution with positive lump-sum taxes face substantial objections. The next section resolves these objections by considering tax systems where positive lump-sum taxes are excluded.

3.2.4 Non-negative categorical transfers

A scenario which excludes positive lump-sum taxes is analyzed by assuming zero transfers to workers. Note that this assumption excludes the Kaplow (1996) argument as the government is no longer able to impose a small lump-sum tax on all individuals. The government maximization problem is

$$(25) \underset{z,t,b}{\text{Max}}\, n_1(1-t)w(T-l_1(t)) + n_1 g(l_1(t)) + n_1 f(z) + n_2 S(b) + n_2 g(T) + n_2 f(z),$$

given the budget constraint

$$(26) n_1 t w(T - l_1(t)) = qz + n_2 b.$$

The first order condition w.r.t. transfers implies that the value of public funds equals the marginal utility of income for non-workers, ${\partial S}/{\partial c_2} = \mu$. The first order condition w.r.t. the supply of public goods and ${\partial S}/{\partial c_2} = \mu$ implies that

$$(27)\ \frac{(n_1+n_2)\frac{\partial f}{\partial z}}{q} = \frac{\partial S}{\partial c_2} = \mu.$$

Equation (27) shows that the marginal welfare cost of raising tax revenue, μ, equals the marginal welfare gain of redistributing tax revenue, ${\partial S}/{\partial c_2}$. The marginal welfare cost of raising tax revenue also equals the marginal welfare gain of investing money in public goods provision. This marginal welfare gain exceeds the cost of investing in public goods provision by a factor equal to the cost of collecting tax revenue, μ.

The marginal welfare gain of public goods provision is divided with the average marginal utility of income, $\bar{\lambda}$. Hence, equation (27) is transformed to

$$(28) \frac{(n_1+n_2)\frac{\partial f}{\partial z}}{\bar{\lambda}} = MCF_1 q.$$

The left-hand side of equation (28) is a measure of the governments' willingness to pay for public goods measured in units of the private goods. This equals the marginal rate of transformation between public and private goods, q, multiplied with MCF_1 defined as the governments' marginal rate of substitution between money in the public and the private sector. Equation (1), (16) and (27) implies that

$$(29) MCF_1 = \frac{n_1+n_2}{\frac{n_1}{\mu}+n_2} > 1 \text{if } \mu > 1.$$

The numerator in the definition of MCF_1, μ, equals the welfare gain of transferring public funds to non-working, ${\partial S}/{\partial c_2}$. This gain exceeds the average marginal utility of income. Hence, MCF_1 exceeds one as the value of public funds, μ, exceeds the average marginal utility of income, $\bar{\lambda}$. This outcome is consistent with the result in Slack (2015), which shows that the average social marginal value of income for the poor group is higher than average within solutions where positive lump-sum taxes are excluded and the size of the poor group is sufficiently large. MCF_1 exceeds one in this scenario so that public goods provision matches the welfare gain of transfers to non-workers. MCF_1 exceeds one even though the marginal welfare cost of collecting tax revenue equals the marginal welfare gain of transfers. Hence, this finding violates the claim in Jacobs (2018) that the MCF equals one because the marginal welfare cost of collecting tax revenue equals the marginal welfare gain of transfers.

Equation (21) and $\partial S/_{\partial c_2} = \mu$ implies that

$$(30)\, MCF_2 = \frac{n_1\mu + n_2}{n_1 + n_2}.$$

MCF_2 also exceeds one because μ is given by the condition in equation (15), and public spending is financed by distorting labor income taxation.

The maximization problem with sub-optimal income taxation is found by following the method described in Section 3.2.1. The solution for MCF_2 and MCF_1 within optimized income tax systems, however, differ from the solution within sub-optimal tax systems. A sub-optimal income tax rate generates a scarce amount of tax revenue which alters the shadow value of public funds, μ. The proof acknowledges that the first order conditions implies a positive connection between z and b. This positive connection and the budget constraint imply that a reduction in the tax rate leads to a reduction in both transfers and public goods provision. The first order conditions imply that a reduction of transfers, or public goods provision, leads to an increase in the shadow value of public spending, μ. It follows from equation (30) that an increase in the shadow value of public funds generates an increase in MCF_2. It follows from equation (29) that an increase in the shadow value of public funds generates an increase in MCF_1. Hence, an income tax rate below optimum requires a higher MCF_2 and MCF_1 compared to an optimized solution in this case. The magnitude of the change in μ, and hence in MCF_2 and MCF_1, is determined by n_1, $n_2$and the shape of both $S(c_2)$ and $f(z)$.

3.2.5 A tax on consumption

This section investigates how the MCF is affected when the tax on labor earnings is replaced with a proportional tax on consumption. The section calculates MCF when the tax on labor earnings is replaced with a proportional tax on consumption, t_c. The behavior of individuals described in sections 3.1 is repeated with the tax on consumption implemented into budget constraints. Indirect utility functions are obtained by following the same approach as in section 3.1. The government maximization problem is found by following the approach in section 3.2.4. The problem is

$$(31) \max_{z,t_c,b}\; n_1 \frac{w(T - l(t_c))}{(1+t_c)} + n_1 g\big(l(t_c)\big) + \; n_1 f(z) + n_2 S\left(\frac{b}{1+t_c}\right) + n_2 g(T) + n_2 f(z)$$

given the budget constraint

$$(32)\ \frac{t_c w(T-l(t_c))n_1}{(1+t_c)} + \frac{t_c b n_2}{(1+t_c)} = qz + n_2 b$$

Note that tax payments on government investments in public goods provision is paid to the government. Hence, such tax payments cancel out of the government budget constraint. First order conditions w.r.t. public goods provision and transfers are identical to corresponding conditions in the case with labor income taxation. Hence, formulas for both MCF_2 and MCF_1 are identical to formulas in section 3.2.4. The welfare maximizing solution therefore satisfies the modified Samuelson condition, equation (20), where MCF_2 is given by equation (30).

The accumulated willingness to pay for an additional public good measured in units of consumer goods is identical with the case where the tax was levied on labor income, section 3.2.4. The tax on the consumer good is however no longer normalized to zero. The tax on labor income is normalized to zero. The accumulated willingness to pay for an additional public good measured in units of (labor) income therefore equals the accumulated MRS_{zc} multiplied with the price of the consumer good, $(1 + t_c)$. The cost of an additional public good measured in units of the consumer good equals q. The cost measured in units of labor income also equals q if investments in public goods are exempt from the tax on consumption. The cost however equals $q(1 + t_c)$ if investments in public goods are not exempt from the tax on consumer goods.

An applied cost-benefit test evaluates the willingness to pay for public goods against the cost of producing public goods measured in money, which in this scenario is transformed to units of income. It is therefore crucial to separate between cases where the consumer tax is levied on investments in public goods provision, and cases where such investments are exempt from the tax. Multiplying the modified Samuelson rule in equation (20) with $(1 + t_c)$ transforms the condition so that the left-hand side equals the accumulated willingness to pay for an additional public good measured in units of income. The right-hand side equals $MCF_2(1 + t_c)q$. Hence, the right-hand side equals the cost of an additional public good multiplied with MCF_2 in scenarios where the consumer tax is also levied on public goods investments. The MCF is however scaled up with $(1 + t_c)$ in scenarios where public goods investments are exempt from the tax on the consumer good.

3.2.6 Income effects on the supply of labor

The quasilinear utility function employed in the analyzes above excludes income effects on the supply of labor. This section explores how results are affected when this utility function is replaced with a

Stone-Geary utility function, which includes income effects on the supply of labor. The utility function equals

$$(33)\, u_i = (c_i + \bar{c})^{\alpha} {l_i}^{1-\alpha} + f(z) \text{ for } i = 1{,}2.$$

$\bar{c}$ is a parameter. Other assumptions within the non-negative categorical transfer scenario are unchanged. The government maximization problem is found by following the approach in section 3.2.4. The government maximization problem is

$$(34)\, \underset{z,t,b}{\text{Max}}\, n_1 \left[\left((1-t)w(\alpha T - \tfrac{(1-\alpha)\bar{c}}{(1-t)w}) + \bar{c} \right)^{\alpha} \left((1-\alpha)T + \tfrac{(1-\alpha)\bar{c}}{(1-t)w} \right)^{1-\alpha} + f(z) \right] + n_2[(b + \bar{c})^{\alpha} T^{1-\alpha} + f(z)]$$

Given the budget constraint

$$(35)\, n_1 t w \left(\alpha T - \tfrac{(1-\alpha)\bar{c}}{(1-t)w} \right) = qz + n_2 b.$$

Appendix E shows that MCF_1 equals

$$(36)\, MCF_1 = \frac{n_1 + n_2}{n_1\left(1 - \frac{t}{1-t} El_w h_1\right) + n_2}.$$

Implementing equation (15) into equation (29) and comparing this with equation (36) shows that formulas for MCF_1 are identical in scenarios with Stone-Geary and scenarios with quasilinear preferences. Appendix E shows that MCF_2 equals

$$(37)\; MCF_2 = \frac{n_1 + n_2\left(1 - \frac{t}{1-t} El_w h_1\right)}{(n_1 + n_2)\left(1 - \frac{t}{1-t} El_w h_1\right)}.$$

Implementing equation (15) into equation (30) and comparing this with equation (37) shows that formulas for MCF_2 are identical in scenarios with Stone-Geary and scenarios with quasilinear preferences. The explanation is that social transfers are given to non-workers only within these scenarios. Such transfers do not influence the supply of labor in these scenarios. Hence, the MCF, which is determined by the welfare gain of transfers, is not altered when quasilinear preferences are replaced with Stone-Geary preferences.

4. Numerical illustrations

This section calculates MCF based on parameter values and labor force data from the US and Norway. The aim is to illuminate how MCF is affected by changes in parameters and assumptions. Parameters and functional forms are calibrated so that the model fit with aggregate labor force data and relevant labor supply responses within each scenario. This approach generates MCF estimates for each scenario that are consistent with data. The calibration method may however lead to inconsistencies between scenarios as changes in assumptions may lead to changes in optimal policy and labor force outcome which is excluded by the calibration method. Scenarios are therefore not directly comparable.

4.1 Uniform transfers

The case with uniform transfers to all do not resemble real world social transfer systems. Hence, adjustments compared to real world tax and transfer systems are required to construct this scenario. The current total US tax wedge on labor earnings includes an average tax on labor earnings of approximately 30 percent in 2018 according to OECD data, a sales tax ranging from 0 to 10 percent in different states, as well as a corporate income tax, real estate taxes, and other indirect taxes. The tax revenue required to finance uniform transfers to all individuals is however likely to exceed the current tax revenue by a substantial margin as current transfers are given to specific groups only. It is difficult to pinpoint the optimal tax rate. Piketty and Saez (2012) however find an optimal linear tax of approximately 60 percent in a scenario with uniform transfers, a utilitarian welfare function and elasticities based on empirical studies. The scenario is therefore calibrated to a 60 percent tax rate to finance uniform transfers to all.

The number of employed individuals, full time or part time, in the US in 2018 amounts to 155.6 million according to US Bureau of Labor Statistics. Some individuals also work less than a part time job. The scenario is therefore calibrated to a labor force of 156.76 million workers. The US Bureau of Labor Statistics further report that 94.1 million over the age of 16 do not participate in the labor force. A substantial share of these individuals receives some type of social security transfer as their main source of income. It is however difficult to pinpoint the exact number of individuals which receives some type of transfer from the government. The main scenario assumes that 70 million individuals receive some type of social transfer. A scenario of 35 million is also presented to uncover whether this number is crucial for calculations of MCF.

The average estimate of the Hicksian labor supply elasticity approximately equals 0.3 for males but is larger for women mainly due to entry-exit according to the survey in Keane (2011). Marshallian elasticities are more modest, and elasticities are declining over time, see Blau and Kahn (2007). Two scenarios are analyzed where the aggregate elasticity equals 0.1 and 0.2. Table 1 present data and results for the USA economy.

Table 1. Labor force data and MCF for the USA

Transfer system	n_1	n_2	$El_w h$	t	$\frac{\partial S}{\partial c_2}$	μ	MCF_1	MCF_2
Uniform transfers	156.76	70	0.1	0.6	1.57	1.176	1	1.04
Uniform transfers	156.76	70	0.2	0.6	2.388	1.428	1	1.17
Uniform transfers	156.76	35	0.2	0.6	3.348	1.428	1	1.25
Uniform transfers, sub-optimal	156.76	70	0.2	0.5	2.850	1.571	1	1.26
Non-negative categorical trans.	156.76	70	0.1	0.4	1.071	1.071	1.05	1.05
Non-negative categorical trans.	156.76	70	0.2	0.4	1.154	1.154	1.10	1.11
Non-negative trans. sub-opt	156.76	70	0.2	0.3	1.269	1.269	1.17	1.19

Uniform transfers within optimized solutions implies that the value of money in the public sector, μ, equals the average marginal utility of money in the private sector, $\bar{\lambda}$. This condition also holds when the income tax rate is sub-optimal. Hence, the MCF_1 equals one with optimal and sub-optimal income taxation.

The MCF_2 exceeds one when uniform transfers are offered to all individuals. Table 1 shows that MCF_2 in the US economy equals 1.04 and 1.17-1.25 when tax- and transfers are optimized, and the labor supply elasticity equals 0.1 and 0.2, respectively. The gain of transfers, $\bar{\lambda}$, equals the cost of raising tax revenue, μ. This cost is substantial due to the 60 percent tax rate. The gain equals the average marginal utility of income, where the stock of workers has a marginal utility of income of one, and the smaller stock of non-workers has a marginal utility of income of ${}^{\partial S}/_{\partial c_2}$. Hence, the marginal utility of income for non-workers must be substantial to satisfy the optimal tax condition, see table 1. This explains why the difference between MCF_2 and MCF_1 is substantial within the uniform transfer scenario.

A tax rate below the optimal level leads to a reduction in tax revenue which lowers both transfers and public goods provision. Reduced transfers and public goods provision increase the marginal welfare gain of both transfers and public goods provision. Hence, the shadow value of public funds increases. The magnitude of the change in μ, and hence in MCF_2, is determined by n_1, $n_2$and the shape of both $S(c_2)$ and $f(z)$. Empirical evidence on the shape of these functions is unavailable. The illustration assumes that a 10-percentage point reduction in the tax rate generates a 10 percent increase in the value of public funds. Table 1 show that MCF_2 approximately equals 1.26 when the tax rate is 10 percentage points below optimum, and the labor supply elasticity equals 0.2.

4.2 Non-negative categorical transfers

Non-negative categorical transfers are implemented by assuming zero transfers to workers and categorical transfers to non-workers. It is assumed that the total optimal tax rate on labor earnings equals 40 percent in the US. Each scenario is calibrated to current labor force data presented above. The MCF_1 exceeds one within optimized systems as the numerator in the definition of MCF_1, μ, exceeds the average marginal utility of income. Table 1 shows that MCF_1 in the US economy equals 1.05 and 1.10 when taxes- and transfers are optimized, given that labor supply elasticities equal 0.1 and 0.2, respectively. The MCF_1 is even larger when the income tax rate is below optimum. An income tax rate below optimum leads to an increase in the shadow value of public fund. This increases the numerator in MCF_1 more than the denominator. Table 1 shows that MCF_1 in the US economy equals 1.17 when the tax rate is sub-optimal, and the labor supply elasticity equals 0.2.

The MCF_2 also exceeds one within optimized systems in this case. Table 1 shows that MCF_2 in the US economy approximately equals 1.05 and 1.11 when taxes- and transfers are optimized, for labor supply elasticities equal to 0.1 and 0.2, respectively. The formula for MCF_2 in equation (21) resembles the formula for MCF_1 in equation (17). Hence MCF_2 exceeds one for the same reasons as for MCF_1. Bjertnæs (2018) shows that these results hold with extensive margin labor choices. Table 1 also shows that MCF_2 approximately equals 1.19 when the tax rate is 10 percentage points below optimum, and the labor supply elasticity equals 0.2. Replacing the income tax with a tax on consumer goods do not affect calculations of MCF when the consumer tax is also levied on public goods investments.

The case of Norway is illustrated by implementing data from 2017 and 2018. The total number of working individuals, n_1, amounts to 2.682 million. The Norwegian government reports that 687000 individuals between the age 18 to 66 received some type of social security transfer in 2017. The number of individuals receiving public pensions in 2018 was 890000 individuals. Some of these are younger than 66 years old. Hence, approximately 1.5 million individuals received social security transfers in Norway.

The current Norwegian tax wedge on labor earnings is set equal to 50 percent. This tax wedge includes an average tax on labor earnings of approximately 36 percent in 2018 according to OECD data. The VAT on most consumer goods in Norway equals 25 percent. There is also a corporate income tax, and indirect taxes. Table 2 present data and results for Norway.

Table 2. Labor force data and MCF for Norway

Transfer system	n_1	n_2	$El_w h$	t	$\frac{\partial S}{\partial c_2}$	μ	MCF_1	MCF_2
Uniform transfers	2.682	1.5	0.1	0.6	1.491	1.176	1	1.04
Uniform transfers	2.682	1.5	0.2	0.6	2.193	1.428	1	1.15
Uniform transfers, sub-optimal	2.682	1.5	0.2	0.5	2.592	1.571	1	1.22
Non-negative categorical trans.	2.682	1.5	0.1	0.5	1.111	1.111	1.07	1.07
Non-negative categorical trans.	2.682	1.5	0.2	0.5	1.25	1.25	1.15	1.16
Non-negative cat. trans, sub-opt.	2.682	1.5	0.2	0.4	1.375	1.375	1.21	1.24

Scenarios with optimized income tax rate and uniform transfers to all individuals are calibrated to a 60 percent tax rate on labor earnings. Calculations show that the MCF for the Norwegian economy are almost identical with calculations of MCF for the US economy in this case. Calculations based on scenarios with non-negative categorical transfers shows that MCF for the Norwegian economy are slightly above calculations for the US economy. The main reason is the higher optimal tax on wage earnings.

5. Conclusion

Several studies show cases where the Samuelson rule holds, or where the MCF, defined as the ratio between the marginal value of money in the public and the private sector, equals one, see e.g. Hylland and Zeckhauser (1979), Boadway and Keen (1993), Kaplow (1996), Sandmo (1998), Christiansen (1981), Christiansen (2007) and Jacobs (2018). Such definitions of MCF require a modification of the Samuelson rule, however, where the willingness to pay for public goods is adjusted with distributional characteristics of public goods, see Sandmo (1998) and Jacobs (2018). Distributional characteristics of public goods are however excluded from both applied cost-benefit tests and the original Samuelson rule. The present study contributes by calculating MCF designed for applied cost-benefit tests, i.e. where distributional characteristics of public goods are excluded from benefits within cost-benefit tests. The study shows that the MCF approximately equals 1.04 and 1.17-1.25 within optimized solutions with uniform transfers and identical additive preferences for public goods when the uncompensated labor supply elasticity equals 0.1 and 0.2, respectively. Similar results are found in cases with categorical transfers to poor groups. Hence, the optimal supply of public goods is below the supply obtained by the Samuelson rule. The MCF is designed so that the welfare gain of public goods provision equals the welfare gains of social security transfers. Hence, income taxation below optimum requires an even higher MCF as the opportunity cost of public funds is increased.

The cost of raising tax revenue by distorting taxes is incorporated into cost-benefit analyses of public projects by multiplying costs with a factor, the MCF. The present study however concludes that the MCF is determined by the opportunity cost of public funds. The opportunity cost of public funds is determined by the marginal welfare gain of social security transfers. The MCF exceeds one in several scenarios to satisfy this condition. Future research could illuminate on implications for MCF and public goods provision of more realistic social security transfer schemes. Such schemes include social security transfers combined with employment programs, social security fraud and mobility between tagged groups, see Parson (1996) and Jacquet (2014).

References

Akerlof, G. A., 1978, The economics of "tagging" as applied to the optimal income tax, welfare programs, and manpower planning, *The American Economic Review*, **68. No1**, 8-19.

Atkinson, A. B. and Stern, N. H., 1974, Pigou, taxation and public goods. *Review of Economic Studies*, **41**, 119-128.

Bjertnæs, G. H. M., 2018, Social Security Transfers and the Marginal Cost of Public Funds, *Discussion paper* no **879**. Statistics Norway.

Blau, F., Kahn, L., 2007, Changes in the labor supply behavior of married women: 1980-2000, *Journal of labor economics*, vol. **25**, 393-438.

Boadway, R. and Keen, M, 1993, Public goods, Self-Selection and Optimal Income Taxation, *International Economic Review*, Vol. **34**, No. **3**, 463-478.

Browning, E. K., 1976, The marginal cost of public funds, *Journal of Political Economy*, **84**, 283-298.

Christiansen, V., 1981, Evaluation of Public Projects under Optimal Taxation, *Review of Economic Studies*, **48** (3):447-457

Christiansen, V., 2007, Two approaches to determine public good provision under distortionary taxation, *National Tax Journal*, **60**(1), 25-43.

Dagsvik, J., Jia, Z. and Thorsen, T. O., 2018, Preferences or constraints? Understanding developments in the labor supply responsiveness over time, unpublished note.

Holtsmark, B., 2019, Is the marginal cost of public funds equal to one? *Discussion Paper*s no. **893**, Statistics Norway.

Hylland, A. and Zeckhauser, R., 1979, Distributional Objectives Should Affect Taxes but Not Program Choices or Design, *Scandinavian Journal of Economics*, **81**, No **2**. 264-84.

Jacobs, B., 2018, "Marginal Cost of Public Funds is One at the Optimal Tax System", *International Tax and Public Finance*, https://doi.org/10.1007/s10797-017-9481-0.

Jacquet, L., 2014, Tagging and redistributive taxation with imperfect disability monitoring, *Soc Choice. Welf*, **42**; 403-435.

Kaplow, L., 1996, The Optimal Supply of Public Goods and the Distortionary Cost of Taxation, *National Tax Journal*, **49**, 513.533.

Keane, M.P., 2011, Labor Supply and Taxes: A Survey, *Journal of Economic Literature*, **49**, 961–1075.

Massiani, J. and Picco, G., 2013, The opportunity Cost of Public Funds: Concepts and Issues, *Public Budgeting & Finance*, Vol. **33**, Issue 3.

Parson, D. O., 1996, Imperfect "tagging" in social insurance programs, *Journal of Public Economics*, **62**, 183-207.

Piketty, T. and Saez, E., 2012, Optimal labor income taxation, *NBER working paper series*, nr. **18521**.

Sandmo, A., 1998, Redistribution and the marginal cost of public funds, *Journal of public economics*, **70**, 365-382.

Slack, S. E., 2015, Revisiting the optimal linear income tax with categorical transfers, *Economics Letters*, **134**, 73-77.

Slemrod, J. and Yitzhaki, S., 2001, Integrating Expenditure and Tax Decisions: The Marginal Cost of Funds and the Marginal Benefit of Projects, *National Tax Journal*, Vol. **54**. No **2**.

Stiglitz, J.E. and Dasgupta, P., 1971, Differential taxation, public goods and economic efficiency, *Review of Economic Studies*, **38**, 151-174.

Stiglitz, J. E., 2018, Pareto efficient taxation and expenditures: Pre- and re-distribution, *Journal of Public Economics*, **162**, 101-119.

Thoresen T. O. and Vattø, T., 2015, Validation of the discrete choice labor supply model by methods of the new tax responsiveness literature, *Labor Economics*, **37**, 38-53.

Valseth, Å. S., Holtsmark, B. and Holtsmark, K., 2019, The cost of taxation in the presence of inequality, *Discussion Papers* no. **908**, Statistics Norway.

Viard, A. D., 2001, Optimal Categorical Transfer Payments: The Welfare Economics of Limited Lump-Sum Redistribution, *Journal of Public Economic Theory*, **3** (4), pp. 583-500.

Wilson, J. D., 1991, Optimal Public Goods Provision with Limited Lump-sum Taxation, *The American Economic Review*, Vol. **81** no. 1.

Appendix

A. The second order condition:

The second order condition is satisfied if the Lagrangian is concave. The Lagrangian consists of separable one variable functions. Hence, the condition is satisfied if the second order derivatives with respect to each variable are negative.

(A 1) $\frac{\partial^2 L}{\partial z \partial z} = (n_1 + n_2)\frac{\partial^2 f}{\partial z \partial z} < 0$

(A 2) $\frac{\partial^2 L}{\partial t \partial t} = (1-\mu)n_1 w \frac{\partial l_1}{\partial t} + (1-\mu)n_1 w \frac{\partial l_1}{\partial t} + n_1 \frac{\partial^2 g}{\partial l_1 \partial l_1}\frac{\partial l_1}{\partial t}\frac{\partial l_1}{\partial t} - \mu n_1 tw \frac{\partial^2 l_1}{\partial t \partial t} < 0$ if $\mu \geq 1$,

$t \geq 0$ and $\frac{\partial^2 l_1}{\partial t \partial t} \geq 0$.

(A 3) $\frac{\partial^2 L}{\partial b \partial b} = n_2 \frac{\partial^2 S}{\partial c_2 \partial c_2} < 0$

B. The first order conditions:

The Envelope Theorem is employed to calculate the impact of a marginal change in the tax rate.

(B 1) $\frac{\partial L}{\partial z} = (n_1 + n_2)\frac{\partial f}{\partial z} - \mu q = 0$

(B 2) $\frac{\partial L}{\partial t} = [-n_1 w(T - l_1(t))] - \left[n_1(1-t)w\frac{\partial l_1}{\partial t} - n_1 \frac{\partial g}{\partial l_1}\frac{\partial l_1}{\partial t}\right]$

$$+\mu\left[n_1 w(T - l_1(t)) - n_1 \frac{\partial l_1}{\partial t} tw\right] = 0$$

(B 3) $\frac{\partial L}{\partial b} = n_1 + n_2 \frac{\partial S}{\partial c_2} - (n_1 + n_2)\mu = 0$

The budget constraint implies that

(B 4) $n_1 tw(T - l_1(t)) = qz + n_1 b + n_2 b$

Hence, equation (B 1) gives

(B 5) $(n_1 + n_2)\frac{\partial f}{\partial z} = \mu q$

Equation (B 2) gives

(B 6) $\mu = \frac{1}{\left[1 - \frac{\frac{\partial l_1}{\partial t} tw}{w(T - l_1(t))}\right]} > 1$ if $\frac{\partial l_1}{\partial t} > 0$ and $t > 0$.

Equation (B 3) gives

(B 7) $\frac{\partial S}{\partial c_2} = \frac{(n_1+n_2)\mu - n_1}{n_2}$

The numerical illustration, however, require some additional calculations. First, the definition of leisure,

(B 8) $l_1 = T - h_1$,

imply that

(B 9) $\frac{\partial l_1}{\partial t} = -\frac{\partial h_1}{\partial t}$

Second, the definition of the after-tax wage rate,

(B 10) $w_a = (1-t)w$,

imply that

(B 11) $\frac{\partial w_a}{\partial t} = -w$

Equation (B 8)- (B 11), together with the definition

(B 12) $-\frac{\partial h_1}{\partial t} = -\frac{\partial h_1}{\partial w_a}\frac{\partial w_a}{\partial t}$

Imply that

(B 13) $\frac{\partial l_1}{\partial t} = w\frac{\partial h_1}{\partial w_a}$

Inserting (B10) and (B 13) into equation (B 6) gives

(B 14) $\mu = \frac{1}{\left[1 - \frac{\partial h_1}{\partial w_a}\frac{w_a}{h_1}\frac{t}{(1-t)}\right]}$

C. MCF based on the modified Samuelson rule

The point of departure is the first order equation (B 5)

(C 1) $n_1\frac{\partial f}{\partial z} + n_2\frac{\partial f}{\partial z} = \mu q$

Hence,

(C 2) $n_1\frac{\partial f}{\partial z} + n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}} + n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}}(\frac{\partial S}{\partial c_2} - 1) = \mu q$

Hence,

(C 3) $n_1\frac{\partial f}{\partial z} + n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}} = \mu q - n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}}(\frac{\partial S}{\partial c_2} - 1)$

Substituting $\frac{\partial f}{\partial z}$ on the right-hand side with equation (B 5) gives

(C 4) $n_1 \frac{\partial f}{\partial z} + n_2 \frac{\frac{\partial f}{\partial z}}{\frac{\partial S}{\partial c_2}} = \left[\mu - \mu \frac{\frac{n_2}{(n_1+n_2)}}{\frac{\partial S}{\partial c_2}} \left(\frac{\partial S}{\partial c_2} - 1\right)\right] q$

Hence, MCF_2 is given by the expression

(C 5) $MCF_2 = \frac{\mu}{\frac{n_1 + n_2 \frac{\partial S}{\partial c_2} + \left(\frac{\partial S}{\partial c_2} - 1\right) n_1}{n_1 + n_2 + \left(\frac{\partial S}{\partial c_2} - 1\right) n_1}}$

Inserting equation (B 7) into equation (C 5) gives

(C 6) $MCF_2 = \frac{1}{\frac{(n_1+n_2)\mu + \frac{(\mu-1)(n_1+n_2)n_1}{n_2}}{(n_1+n_2)\mu + \frac{(\mu-1)(n_1+n_2)n_1}{n_2}\mu}} > 1$ if $\mu > 1$.

D. First order conditions with categorical transfers

Equation (B 3) is replaced with the following conditions

(D 1) $\frac{\partial L}{\partial a} = n_1 - n_1 \mu = 0$

(D 2) $\frac{\partial L}{\partial b} = n_2 \frac{\partial S}{\partial c_2} - n_2 \mu = 0$

E. Stone-Geary utility

The lagrangian is

(E 1) $L = n_1 \left[\left((1-t)w(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}) + \bar{c}\right)^{\alpha} \left((1-\alpha)T + \frac{(1-\alpha)\bar{c}}{(1-t)w}\right)^{1-\alpha} + f(z)\right] +$

$n_2[(b+\bar{c})^{\alpha} T^{1-\alpha} + f(z)] + \mu(n_1 tw \left(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}\right) - qz - n_2 b)$

(E 2)) $\frac{\partial L}{\partial z} = n_1 \frac{\partial f}{\partial z} + n_2 \frac{\partial f}{\partial z} - \mu q = 0$

(E 3) $\frac{\partial L}{\partial t} = n_1 \left[\alpha \left((1-t)w(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}) + \bar{c}\right)^{\alpha-1} \left((1-\alpha)T + \frac{(1-\alpha)\bar{c}}{(1-t)w}\right)^{1-\alpha}\right] x$

$\left[-w\left(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}\right) - (1-t)w \frac{w(1-\alpha)\bar{c}}{(1-t)^2 w^2}\right]$

$+n_1 \left[(1-\alpha)\left((1-t)w(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}) + \bar{c}\right)^{\alpha} \left((1-\alpha)T + \frac{(1-\alpha)\bar{c}}{(1-t)w}\right)^{-\alpha}\right] x \left[\frac{w(1-\alpha)\bar{c}}{(1-t)^2 w^2}\right] +$

$\mu \left[n_1 w \left(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}\right) - n_1 tw \frac{w(1-\alpha)\bar{c}}{(1-t)^2 w^2}\right] = 0$

(E 4) $\frac{\partial L}{\partial b} = n_2 \alpha (b + \bar{c})^{\alpha-1} T^{1-\alpha} - n_2 \mu = 0$

The budget constraint implies that

(E 5) $n_1 tw\left(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}\right) = qz + n_2 b$

The definition of MCF_1 and $\bar{\lambda}$ implies that

(E 6) $MCF_1 = \frac{\mu}{\frac{n_1\frac{\partial u_1}{\partial c_1}+n_2\frac{\partial u_2}{\partial c_2}}{n_1+n_2}}$

Implementing the first order condition for working individuals' choices of labor supply into equation (E 3) implies that

(E 7) $\alpha\left((1-t)w\left(\alpha T - \frac{(1-\alpha)\bar{c}}{(1-t)w}\right)+\bar{c}\right)^{\alpha-1}\left((1-\alpha)T + \frac{(1-\alpha)\bar{c}}{(1-t)w}\right)^{1-\alpha}$

$= \mu\left(1 - \frac{t}{1-t} El_{w_a} h_1\right)$

Implementing (E 7) and equation (E 4) and the definition of the labor supply elasticity from appendix B into equation (E 6) gives

(E 8) $MCF_1 = \frac{n_1+n_2}{n_1\left(1-\frac{t}{1-t}El_{w_a}h_1\right)+n_2}$

Equation (2) implies that

(E 9) $n_1\frac{\frac{\partial f}{\partial z}}{\frac{\partial u_1}{\partial c_1}} + n_2\frac{\frac{\partial f}{\partial z}}{\frac{\partial u_2}{\partial c_2}} = MCF_2 q$

Implementing equation (E 2), equation (E 7), equation (E 4) and the definition of the labor supply elasticity from appendix B into equation (E 9) gives

(E 10) $MCF_2 = \frac{n_1+n_2\left(1-\frac{t}{1-t}El_{w_a}h_1\right)}{(n_1+n_2)\left(1-\frac{t}{1-t}El_{w_a}h_1\right)}$