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THE CONTINUOUS GENERALIZED EXTREME VALUE MODEL WITH SPECIAL REFERENCE TO STATIC MODELS OF LABOR SUPPLY

BY

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ABSTRACT

The paper extends the generalized extreme value random utility model (McFadden, 1981) to the case where the choice set is continuous. One particular area of application is treated, namely the static models of labor supply. The continuous Luce model as well as several standard models that appear in the consumer demand literature emerge as special cases. The paper also provides theoretical assumptions that justify the stochastic properties of the model.

1. Introduction

This paper presents a particular econometric approach for analyzing individual choice behavior under perfect certainty, with general budget constraints and with discrete and continuous alternatives. We focus on a particular area of application, namely the econometrics for labor supply but it is clear that the field of applications is much wider. The framework developed is related to the literature on discrete/continuous choice, see e.g., McFadden (1981), Hanemann (1984) and Hausman (1985) but it departs somewhat from the models that have appeared in the literature in that we insist on a more careful treatment of the unobservables. Specifically, our approach represents an extension of the generalized extreme value random utility model to cover the case with continuous alternatives. In contrast to the traditional approach where the unobservables usually enter as "tasteshifters", we argue that these unobservables are related to latent opportunity constraints and matching conditions. Specifically, we define a space of unobservable alternatives called matches. Each match is related to particular abilities offered by the individual, skills demanded to perform specific tasks that correspond to certain market or non-market alternatives and attributes associated with these activities. An optimizing individual will try to obtain the "best" match, i.e., he will engage in an activity that yields the best position relative to his abilities and tastes. This starting point provides a fruitful step for developing a random utility model where the stochastic properties of the model can be justified on theoretical grounds and where the budget constraints can be quite general.

The paper is organized as follows: In section 2 we discuss some of the weak parts of the static labor supply models analyzed in the literature and we postulate a random utility structure that is convenient for accommodating different types of heterogeneity. We also derive some testable properties of this distribution.

In Section 3 the joint distribution of the supply function and the wage rate is derived under general restrictions on the set of available hours of work, given the match. We also derive some testable properties of this distribution.

Section 4 discusses the particular case with no constraints on hours of work within matches and Section 5 considers the other extreme when hours of work for each match is given.

The final section is devoted to a theoretical justification of the stochastic structure of the utility function. Specifically, we provide assumptions that imply the type of utility function postulated in Section 2 and hence the derived supply distribution of Section 3.

2. The utility structure

The textbook model for labor supply starts with a specification of a direct or indirect utility function in leisure and consumption. Utility is quasi-concave and nondecreasing, and the budget constraint is linear or concave. By standard marginal calculus the supply function is obtained in this case. Recently Hausman (1985) has modified this approach so as to take into account tax systems that imply a piecewise linear non-concave budget curve.

The textbook model is subject to criticism for some obvious reasons of which we mention a few:

(i) The model assumes that each individual faces only one offered wage rate and thus it neglects that, at least some individuals have the choice of several jobs with different wages.

(ii) The fact that non-pecuniary job characteristics matter when labor market decisions are made is neglected. Only the wage rate is assumed to matter.

(iii) The decision-makers are assumed to be able to adjust hours of work freely. Yet we know that for most jobs hours of work are more or less determined by the firms or by regulations set by the central authorities.

(iv) With a few exceptions (Heckman and MaCurdy,1981 , Hausman, 1980, 1979, Burtless and Hausman, 1979) the tax structure is neglected or approximated by a smooth convex function.

(v) The unobservables are treated superficially. Distributional assumptions are typically made ad hoc.

Naturally, these are quite a few of the objections that are raised against the traditional approach and they are mentioned here since we intend to develop a framework that is convenient for dealing with just these important shortcomings.

The model presented in this chapter is, however, traditional in that the individual is assumed to operate in an environment of complete certainty with a one-period budget constraint, i.e., no borrowing or saving is allowed. However, from the observing econometrician's point of view individual behavior is perceived as random because many important variables that affect the decisions usually are unobservable. Important unobservables that characterize the environment are type of "job" and types of non-market activities. We may think of a type of job as the attributes and the nature of the specific tasks performed as well as the qualifications demanded to perform the tasks. Similarly non-market alternatives may be identified by the type of tasks and skills needed to perform them. Examples of non-market alternatives are related to schooling, sports, household production and childcare.

The individual's choice set, i.e., the set of available opportunities depends on personal abilities and degree of qualifications. Also his qualifications may affect the preferences since he would, to some extend, be likely to consider opportunities that correspond to his abilities. However, the decision process is only partly determined on the basis of matching considerations because the individual usually have skills suited to perform a variety of tasks but still prefer particular tasks to others. Moreover, the most preferred tasks are not necessarily the ones for which he is best qualified.

Next we shall introduce the concept match. We define a space of abstract latent alternatives denoted matches. Each match identifies specific abilities offered and skills demanded to perform a particular combination of tasks related to certain market and non-market activities. It also identifies the nature of the activities. Thus a particular match is defined by,

(i) tasks related to a combination of market and non-market activities,
(ii) abilities offered to perform these tasks,
(iii) skills required to perform the tasks,
(iv) attributes associated with the specific combination of activities.
Such attributes are related to for example working environment, location of the jobs, commuting time, residental location, quality and prestige of an educational alternative, etc.

Without loss of generality we may identify each match by a discrete number z, i.e., z=1,2,..., is an enumeration of the space S of all the matches. This enumeration as well as the definition of the matches is "objective" in that it is not dependent of the individual. We assume that the individual has preferences over these matches and that he is perfectly certain about the outcome of a matching process.

Let U(h,C,z) be the utility of hours of work, h, consumption, C, and match, z, for a randomly selected individual.

The budget constraints are given by

$$C = f(hW(z) + I \qquad (2.1)$$

$$h \in D(z), \; z \in B \; . \qquad (2.2)$$

The first equation specifies the "economic" budget where W(z) is the wage rate that is allowed to be match-specific. I is non-labor income and f(•) is the function that transforms gross income to income after tax. This function may be non-differentiable and non-concave which corresponds to the current tax system in many countries. We define z as a pure non-market match if W(z)=0.

The second equation defines the match-specific constraints on hours of work. These constraints stem from regulations on working hours specific to the tasks performed as well as from the fact that certain activities take a minimum or a maximum of time to perform. This set also accounts for rationing in the labor market. For instance D(z)=∅ (the emty set) for some z which means that certain matches offer no possibility for work either because non-market tasks take all of the available time or simply because there are no jobs that correspond to these matches.

The third constraint defines the set of matches that are feasible to the individual. The set B account for the fact that for a particular individual the abilities offered to perform the respective tasks are given. In the short run, which is our concern here, the personal qualifications cannot be changed. Thus B is the subset of matches for which the qualifications offered coincide with those of the individual. The set B also accounts for the fact that many matches simply do not exist in the market. Specifically, the individual's choice of matches is made indirectly in that he chooses from a set of jobs (or leisure activities) that imply a particular combination of tasks and attributes. We have chosen not to let B account for rationing in the labor market. Such restrictions are, as discussed above, identified by D(z). This is of course to some extent a matter of convention. Note that the convention adopted here implies that D(z) does not depend on the individual. For a given z, D(z) is the same for everyone. The heterogeneity across individuals is accounted for by B.

The framework introduced above is sufficiently general to accommodate to the fact that there may be a trade-off between wages and non-pecuniary attributes. For instance, some jobs may be well paid but offer unattractive working conditions or tedious work (relative to the individual).

The econometric challenge is, of course, whether or not we are able to represent these structural features of the market in a tractable empirical model. We shall see below that this is possible to some extent.

Before we continue to develop the formal model we would like to emphasize that although our approach may differ from most previous work in this area there are nevertheless previous developments along these lines. One notable contribution dates back to Tinbergen (1956) in which he attempts to give the frequency distribution of incomes an economic theoretic foundation. Tinbergen introduces attributes of the jobs and abilities associated with the workers and he let the utility function depend on the tension between the skills required at the job and the abilities offered by the worker. Thus his optimizing regime implies that the worker acts so as to minimize the tension between the skills required and the abilities offered. The regime we have presented above extends Tinbergen's model in that the utility function may also depend on attributes associated with different activities. In Tinbergen's world a person's abilities coincide with his tastes while here tastes and abilities are allowed to be in conflict with each other.

The decision process may be decomposed as follows: First, the individual evaluates the highest utility of each match i.e., he evaluates the conditional indirect utility function

$$\hat{V}(z) = \max_{h \in D(z)} U(h, f(hW(z) + I), z) \tag{2.3}$$

for each z. Second he finds the match, $\hat{z}$, for which

$$\hat{V}(\hat{z}) = \max_{z \in B} \hat{V}(z) .$$

Now optimal hours of work, $\tilde{h}$, is determined by

$$U(\tilde{h}, f(\tilde{h}W(\hat{z}) + I), = \hat{V}(\hat{z}) .$$

Alternatively, the decision process may be decomposed in the following way: The budget constraint (2.2) is equivalent to

$$z \in B(h), \quad h \in \cup_z D(z) \tag{2.4}$$

where

$$B(h) = \{z | h \in D(z), z \in B\}$$

i.e., B(h) is the set of matches for which h hours are available. First the individual finds the optimal match for each h i.e., he evaluates $U^*(h)$ defined by

$$U^*(h) = \max_{z \in B(h)} U(h, f(hW(z) + I), z) \tag{2.5}$$

which is the highest utility the individual can attain at h hours. Second, he finds the optimal hours, $\tilde{h}$, by maximizing U^*.

To the observing econometrician U(h,C,z) is, as mentioned above, perceived as random due to unobserved variables that influence the preferences. In addition the set B is perceived as random because of unobserved heterogeneity with respect to skills. Let

$$U(h,C) = \max_{z \in B(h)} U(h,C,z) .$$

This is the utility function for the observable choice variable (h,C). The utility function U(h,C) is thus random due to unobserved heterogeneity with respect to skills and tastes.

The stochastic properties of U(h,C) can be justified by a set of plausible behavioral assumptions which are discussed in Section 6. These assumptions may be categorized in two equivalent sets. The first set of assumptions is, in part, of the revealed preference type. The other set is related to the distribution of the utilities. The two sets are equivalent in that they imply the same stochastic structure of the utility function. Specifically, they imply that

$$U(h,C,z) = v(h,C,T(z)) + \frac{\varepsilon(z)}{a}, \qquad a > 0 \qquad (2.6)$$

where $\{T(z),\varepsilon(z)\}$ is an enumeration of the points of the Poisson process on $R_+ \times R$ with intensity measure

$$\lambda(dt) \cdot e^{-\varepsilon} d\varepsilon$$

where

$$c \equiv \int_0^\infty \lambda(dt) < \infty$$

and $v(\cdot,\cdot,\cdot)$ is a suitable deterministic function. Recall that this means that the expected number of matches for which $(T(z),\varepsilon(z)) \in (a,b) \times (x,y)$ is given by

$$\Lambda(a,b,x,y) \equiv (e^{-x} - e^{-y}) \int_a^b \lambda(dt) \; .$$

Moreover, the probability that exactly n matches satisfy $(T(z),\varepsilon(z)) \in (a,b) \times (x,y)$ is

$$\frac{\Lambda(a,b,x,y)^n}{n!} \exp(-\Lambda(a,b,x,y)) \; .$$

In particular, the probability that there is no match for which $\varepsilon(z) > x$ is

$$P(\max_z \varepsilon(z) < x) = \exp(-\Lambda(0,\infty,x,\infty)) = \exp(-ce^{-x}) \; .$$

The variable T(z) captures the effect of the non-pecuniary aspects related to match z. Thus the choice environment is characterized by a distribution of wages, W(z), match-specific choice sets, D(z), and non-pecuniary aspects summarized in T(z). This distribution is the same for every individual and it is, as mentioned above, assumed to be known to the decision-makers.

The variables (W(z),D(z),T(z)) appear here as random variables only in the sense that the enumeration of matches is arbitrary. For a given enumeration these variables are non-stochastic. For the sake of interpretation and to link the above setup to standard models of discrete choice consider the corresponding case where the universe of matches is finite with size n. Let $\eta(z), z=1,2,\ldots, n$, be independent draws from the extreme value distribution, $\exp(-e^{-x})$, and let Y(z), $z=1,2,\ldots,n$, be a non-stochastic variable. Furthermore let the draws $\{\eta(z)\}$ be independent of $\{Y(z)\}$. Let $\hat{G}_n(y)$ be the fraction of matches for which $Y(z)<y$, i.e., $\hat{G}_n(y)$ is an "empirical" distribution function. Let

$$U_n(h,C) = \max_{z\leq n} \left(v(h,C,Y(z)) + \frac{\eta(z)}{a}\right) .$$

The variable Y(z) may be percieved as stochastic only in the sense that the probability that a randomly selected match from the set of n matches satisfies $Y(z)<y$ is $\hat{G}_n(y)$. The variable $\eta(z)$ accounts for the individual variation in tastes for a given match z. We may therefore interpret v(h,C,Y(z)) as the mean utility accross individuals given (h,C,z). It follows by standard arguments that $\{U_n(h,C)\}$ converges weakly towards $\{U(h,C)\}$ as $n \to \infty$ where

$$U(h,C) \equiv \max_{z} \left(v(h,C,T(z)) + \frac{\varepsilon(z)}{a}\right) .$$

Also $\hat{G}_n(y) \to G(y)$ in the continuity points of G where $G(y) = \frac{\lambda(y)}{c}$.

The theoretical justification of (2.6) is deferred to Section 6. As demonstrated by de Haan (1984) the decomposition (2.6) is equivalent to (under certain regularity conditions) the property that

$$(U(h_1,C_1),U(h_2,C_2),\ldots,U(h_m,C_m))$$

for arbitrary $(h_1,C_1),(h_2,C_2),\ldots,(h_m,C_m)$ and arbitrary integer m is distributed according to the multivariate extreme value distribution, type III, (see Galambos, 1978). This distribution, which is also denoted the generalized extreme value distribution (see McFadden, 1981) is presented in Section 6. In the terminology of de Haan $\{U(h,C)\}$ belongs to the class of max-stable processes.

The decomposition result (2.6) is very important because it enables us together with assumptions about W(z) and D(z) to derive a convenient expression for the realized hours of work and wage distribution. Before we consider this distribution let us look at the interpretation of (2.6). First observe that if U(h,C,z) is assumed to be quasiconcave in (h,C), non-decreasing in C and non-increasing in h for given z the same properties hold true for v(h,C,T(z)) since ε(z) does not depend on (h,C). Let

$$\hat{v}(w,t,A) \equiv \max_{h\in A} v(h,f(hw + I),t) \quad . \qquad (2.7)$$

Then

$$\hat{V}(z) = \hat{v}(W(z),T(z),D(z)) + \frac{\varepsilon(z)}{a}$$

is the match-specific indirect utility function since we just noticed that ε(z) is constant when z is kept fixed. In the case where f(•) is concave and A is the set of feasible hours per year, $\hat{v}$ would possess the standard indirect utility properties, i.e., be quasiconvex in w and nondecreasing in w and I.

Note that ε(z) only affects the level of the utility while T(z) affects both the level and the marginal utilities $\partial U/\partial h$ and $\partial U/\partial C$. In the special case where T(z) is degenerate then all matches have the same marginal utilities.

It is interesting to note that the standard textbook model is obtained as a special case of our model. As discussed above the textbook model is silent about unobservable choice opportunities. All the unobservables regardless of their origins are usually represented by a random variable that enters the utility function (direct or indirect utility) in such a way so as to preserve quasiconcavity and monotonicity in (h,C). Now let us see how the textbook model emerges. From (2.6) we see that when a is small then the effect of the "systematic" part v(h,C,T(z)) will be small. Thus for a value of a near zero the optimal match would be determined by the sequence $\{\varepsilon(z)\}$ which is purely random. Given the optimal match, hours of work is determined by maximizing $v(h,C,T(z))$ with respect to h. But this is a standard textbook utility maximization problem when $T(\hat{z})$ is given. Since the determination of $\hat{z}$ is independent of the rest of the variables that enter the utility function it follows that $(W(\hat{z}),T(\hat{z}))$ can be viewed as a draw from the same distribution as the one that generated the sequence $(W(z),T(z))$. Therefore our model is indeed not a restriction of the traditional model that appears in the econometric literature of labor supply. Instead it offers an extension which is capable of explaining a wider variety of unobserved heterogeneity than in the traditional case at the same time as it's structure is partly justified from theoretical arguments.

It is also an extension of the generalized extreme value models to the continuous case (cf. McFadden, 1981). For example, the probability that $U(h_1, C_1) > U(h_2,C_2)$ has the form of a generalized extreme value model and this example is a special case within the general framework presented here.

In the next section we shall present the distribution function for the realized wage and hours of work.

3. The distribution of realized wage and hours of work

The optimal wage, $\tilde{W}$, and hours of work, $\tilde{h}$, can formally be expressed as

$$\tilde{W} = W(\hat{z})$$

and

$$\tilde{h} = \hat{h}(W(\hat{z}),D(\hat{z}))$$

where we recall that $\hat{z}$ denotes the optimal match, i.e., the match that maximizes $\hat{V}(z)$ and $\hat{h}(w,t,A)$ is the value that maximizes $v(h,f(hw+I),t)$, $h\in A$. For simplicity assume that $D(z)$ is an interval $D(z) = [\underline{D}(z),\bar{D}(z)]$ and let $G(w,t,\underline{h},\bar{h})$ be the probability that a randomly drawn match $z\in B$ satisfies

$$(W(z)<w,T(z)<t, \underline{D}(z)<\underline{h}, \bar{D}(z)<\bar{h}|z\in B) .$$

Let $g_o = G(\infty,\infty,0,0)$, $g_1 = 1-g_o$ and let $g_2(w,t,\underline{h},\bar{h})$ be the conditional density of $G(w,t,\underline{h},\bar{h})$ given that $\bar{D}(z) > 0$. Evidently, g_o represents the (expected) fraction of pure non-market matches in B while g_1 is the (expected) fraction of feasible market matches in B. $g_2(w,t,\underline{h},\bar{h})dwdtd\underline{h}d\bar{h}$ represents the (expected) fraction of feasible market matches that have characteristics $(w,t,\underline{h},\bar{h})$. Let K denote the set of feasible hours of work (observable). For example K may be total amount of hours per year. Define the corresponding cumulative distribution function by

$$\Phi(h,w,K) = P(0<\tilde{h} < h,\tilde{W} < w), \quad h>0, \quad h\in K$$

and

$$\Phi(0,K) = P(\tilde{h} = 0) \ .$$

Also define

$$\Omega_1(h,w) = \{(y,t,\underline{h},\bar{h})|0 < \hat{h}(y,t,\underline{h},\bar{h}) < h,y < w,[\underline{h},\bar{h}]\subset K\} \qquad (3.1)$$

and

$$\Omega_o = \{(y,t,\underline{h},\bar{h})|\hat{h}(y,t,\underline{h},\bar{h}) = 0,[0,\bar{h}]\subset K\} \ . \qquad (3.2)$$

The set Ω_1 is equivalent to the set of suitable matches for which supply of hours and wages are less than or equal to h and w, respectively. The set Ω_o is equivalent to the set of matches that are unsuitable for work.

The following theorem gives the distribution of the realized wage and hours of work:

<u>Theorem 1</u>. Assume that

$\{W(z),T(z),\underline{D}(z),\bar{D}(z),\varepsilon(z)\}$, $z=1,2,\ldots,z\in B$, is an enumeration of the points in the Poisson process on $R_+^2 \times[0,M]\times[0,M]\times R$ with intensity measure

$$\lambda(dw,dt,d\underline{h},d\bar{h})\times e^{-\varepsilon}d\varepsilon$$

where

$$c \equiv \int \lambda(dw,dt,d\underline{h},d\bar{h}) < \infty .$$

and M is the upper bound on hours of work. Moreover, assume that

$$U(h,C,z) = v(h,C,T(z)) + \frac{\varepsilon(z)}{a} .$$

Then

$$\Phi(h,w,K) = \frac{N_1(h,w)}{N_o+N_1(K,\infty)} , \quad h>0, h\in K \tag{3.3}$$

and

$$\Phi(0,K) = \frac{N_o}{N_o+ N_1(K,\infty)} \tag{3.4}$$

where

$$N_1(h,w) = \int_{\Omega_1(h,w)} \exp(a\hat{v}(y,t,\underline{h},\bar{h}))G(dy,dt,d\underline{h},d\bar{h}) ,$$

$$N_o = \int_{\Omega_o} \exp(av(0,f(I),t))G(dy,dt,d\underline{h},d\bar{h})$$

and

$$G(w,t,\underline{h},\bar{h}) = \frac{\lambda(w,t,\underline{h},\bar{h})}{c} .$$

The idea behind the abstract formalism of the theorem is in fact quite simple. For the sake of expository simplicity suppose that the number of matches is finite and let n_j be the number of matches with the same values of (W(z),T(z),D(z)), j=1,2,... . The utility has the form (say)

$$u_j(z) = a_j + \varepsilon_j(z) \tag{3.5}$$

where

$$a_j = \hat{v}(W_j,T_j,D_j)$$

and $\varepsilon_j(z)$ are i.i. extreme value distributed. But this represents a standard description of a discrete choice problem. Consequently, the choice probabilities are given by the Luce model

$$p_j(z) = P\{u_j(z) = \max_{i,k} u_k(i)\} = \frac{e^{a_j}}{\Sigma_i n_i e^{a_i}}.$$

The probability of choosing a match in category j is therefore

$$p_j = \sum_z p_j(z) = \frac{n_j e^{a_j}}{\Sigma_i n_i e^{a_i}} = \frac{g_j e^{a_j}}{\Sigma_i g_i e^{a_i}} \tag{3.6}$$

where g_j is the frequency of matches in category j. If $\Omega(h)$ is the set of matches for which optimal hours are equal or less than h then the distribution of hours, p(h), is given by

$$p(h) = \sum_{j\in\Omega(h)} p_j = \frac{\Sigma_{j\in\Omega(h)} g_j e^{a_j}}{\Sigma_i g_i e^{a_i}} \tag{3.7}$$

which is precisely a simplified discrete version of Φ in Theorem 1. When the number of matches increases then (3.7) tends towards a continuous distribution that corresponds to the theorem. A more formal proof is given in Dagsvik (1987). Q.E.D.

Next we shall rewrite the result of Theorem 1 because it may illuminate the interpretation. Note that

$$N_1(h+dh,w+dw) - N_1(h,w)$$

$$= E\{\exp[av(h,f(hw+I),T(z))]|\hat{h}(w,T(z)) = h,W(z) = w,D(z)\subset K\}$$

$$\cdot P(\hat{h}(w,t,\underline{h},\bar{h})\in(h,h+dh),W(z)\in(w,w+dw))+o(dh\cdot dw).$$

Accordingly, we may express the probability density ϕ of Φ as

$$\phi(h,w,K) = \frac{\exp[\tilde{v}(h,w)]\tilde{g}(h,w)}{\int_{\substack{x>0\\x\in K}} \exp[\tilde{v}(x,y)]\tilde{g}(x,y)dxdy + \int \exp[\tilde{v}(0,y)]\tilde{G}(0,dy)} \tag{3.8}$$

where

$$\exp[\tilde{v}(h,w)] = E\{\exp[av(h,f(hw+I),T(z))] \\ |\hat{h}(w,T(z),D(z)) = h,W(z) = w\} \tag{3.9}$$

and $\tilde{g}$ is the density of $\tilde{G}$, where $\tilde{G}$ is defined by

$$\tilde{G}(x,y) = P\{\hat{h}(W(z),T(z),D(z)) \leq x,W(z) \leq y|D(z)\subset K,z\in B\} . \tag{3.10}$$

The function $\exp(\tilde{v}(h,w))$ is interpreted as the mean utility across all the matches for which (w,h) is the optimal choice. The distribution $\tilde{G}(x,y)$ is the expected fraction of feasible matches for which optimal hours and wages are less than or equal to x and y, respectively.

The probability of not working is given by

$$\phi(0,K) = \frac{\int \exp[\tilde{v}(0,y)]\tilde{G}(0,dy)}{\int_{\substack{x>0\\x\in K}} \exp[\tilde{v}(x,y)]\tilde{g}(x,y)dxdy + \int \exp[\tilde{v}(0,y)]\tilde{G}(0,dy)} . \tag{3.11}$$

From (3.8) to (3.11) it easily verified that we obtain the textbook model by letting $a \to 0$. From (3.9) we get

$$\lim_{a \to 0} \tilde{v}(h,w) = 0.$$

Hence (3.9) reduces to

$$\phi(h,w,K) = \frac{\tilde{g}(h,w)}{\int_{\substack{x \in K \\ x>0}} \tilde{g}(x,y)dxdy + \int \tilde{G}(0,dy)} = \tilde{g}(h,w)$$

which by definition is the distribution of $(\hat{h}(W(z),T(z),D(z)),W(z))$.

As already suggested above $\tilde{g}(h,w)$ is the joint density of an ordinary constrained Marshallian "demand" function, $\tilde{h}$, and the "price", $W(z)$. Thus we have expressed the density ϕ in terms of the "ordinary" supply density $\tilde{g}$ and the mean value of the latent choice variables conditional on optimal hours and wages.

Note that by (3.8) $\tilde{G}(x,y)$ depends on K but this is suppressed above for notational convenience. Below we need, however, to incorporate K into the notation.

Let $\phi_1(h,w,K)$ and $\tilde{g}(h,w,K)$ denote the densities $\phi(h,w)$ and $\tilde{g}(h,w)$ in the particular case where $D(z)=K$ and let $\exp(\tilde{v}(h,w,K))$ be the corresponding mean utility function. Then (3.9) implies that

$$\frac{\phi_1(h_1,w_1,K)}{\phi_1(h_2,w_2,K)} \Big/ \frac{\phi_1(h_1,w_1,K^*)}{\phi_1(h_2,w_2,K^*)}$$

$$= \frac{\exp(\tilde{v}(h_1,w_1,K))\tilde{g}(h_1,w_1,K)}{\exp(\tilde{v}(h_1,w_1,K^*))\tilde{g}(h_1,w_1,K^*)} \Big/ \frac{\exp(\tilde{v}(h_2,w_2,K))\tilde{g}(h_2,w_2,K)}{\exp(\tilde{v}(h_2,w_2,K^*))\tilde{g}(h_2,w_2,K^*)} \tag{3.12}$$

for $h_1>0$, $h_2>0$, $h_1,h_2 \in K \cap K^*$. From (3.12) we are able to derive testable properties which is stated in the next two theorems.

Theorem 2. Assume that the match-specific choice set $(D(z))$ is constant across matches and equal to the observed feasible set for hours (K). Moreover, assume that the budget set is convex ($f(\cdot)$ is concave) and that $v(\cdot,\cdot,t)$ is quasiconcave for given t. Then

$$\frac{\phi_1(h_1,w_1,K)}{\phi_1(h_2,w_2,K)} = \frac{\phi_1(h_1,w_1,K^*)}{\phi_1(h_2,w_2,K^*)}$$

when h_1 and h_2 are positive and belong to the interior of $K \cap K^*$.

Note that the statement of the theorem is equivalent to Luce choice axiom IIA when the choice alternatives belong to the interior of the choice set.

Proof: When f(x) is concave then the function $\hat{h}(w,t,K)$ is determined by local criteria and it is therefore independent of K provided it takes values in the interior of K. As a consequence, both $\tilde{v}(h,w,K)$ and $\tilde{g}(h,w,K)$ are independent of K when h lies in the interior of K. The theorem now follows from (3.12). Q.E.D.

In general the marginal distribution of realized hours has positive mass when h belongs to the boundary of the choice set K. In section 5 we shall consider a particular case with no jump in the marginal distribution of realized hours at the boundary of the choice set. In this case IIA holds for any choice alternative.

Consider next the case we have denoted the "textbook model" above. On page 15 we demonstrated that when $a \to 0$ then $\phi_1(h,w,K)$ reduces to $\tilde{g}(h,w,K)$. The next result provides a test for the hypothesis $a=0$.

Theorem 3. Suppose the assumptions of Theorem 2 hold. If $\phi_1(h,w,K)$ is independent of K when h is positive and lies in the interior of K then $\phi_1(h,w,K)$ reduces to $\tilde{g}(h,w,K)$.

Proof: As noted in the proof of Theorem 2 the assumptions imply that $\tilde{v}(h,w,K)$ and $\tilde{g}(h,w,K)$ are independent of K when $h>0$ and lies in the interior of K. Thus (3.8) implies that we can express $\phi_1(h,w,K)$ on the form

$$\phi_1(h,w,K) = \frac{\exp(\tilde{v}(h,w))\tilde{g}(h,w)}{Q(K)} \qquad (3.13)$$

where Q(K) is the denominator of (3.8). But since $\phi_1(h,w,K)$ is independent of K then Q(K) must be independent of K. From Theorem 1 it follows that

$$Q(K) = E \exp(a\hat{v}(W(z),T(z),K)) .$$

Now let K^* be a choice set, $K^* \subset K$. Then $\hat{v}(w,t,K) \geq \hat{v}(w,t,K^*)$. Moreover it is possible to choose K^* and K so that $\hat{v}(w,t,K) > \hat{v}(w,t,K^*)$ for (w,t) on some set with positive probability mass because otherwise v(h,f(hw+I),t) would be independent of h almost everywhere. Accordingly, if a>0

$$Q(K)=Q = E \exp(a\hat{v}(W(z),T(z),K)) > E \exp(a\hat{v}(W(z),T(z),K^*)) = Q$$

which is a contradiction. Therefore a=0 and Q=1. By (3.9) $\tilde{v}(h,w) = 0$ and the theorem now follows from (3.13). Q.E.D.

4. Unconstrained choice of hours within matches

This section is devoted to the special case where D(z) = K = [0,M] for the market matches which means that there are no match-specific restrictions on hours provided the upper bound M is not biting. This case is of particular interest because it illustrates the power of the framework. For notational simplicity let

$$\hat{v}(w,t) = \hat{v}(w,t,0,M) ,$$

$$G_2(w,t) = G_2(w,t,0,M) ,$$

$$\hat{h}(w,t) = \hat{h}(w,t,0,M) ,$$

$$W^*(t) = W^*(t,0,M)$$

and let $W^*(t)$ be the match-specific reservation wage determined by

$$\hat{v}(W^*(t),t) = v(0,f(I),t) \ .$$

Let

$$A(h,w) = \{t \mid \hat{h}(w,t) = h\}$$

i.e., A(h,w) is equivalent to the set of matches for which (h,w) is optimal. It now follows that

$$\phi(h,w) = g_1 \int_{A(h,w)} \exp(a\hat{v}(w,t))g_2(w,t)dt\Big[\int \exp(av(0,f(I),t))g^*(t)dt + g_1 \int_{y>W^*(t)} \exp(a\hat{v}(y,t))g_2(y,t)dydt\Big]^{-1} \tag{4.1}$$

for h>0 and

$$\Phi(0) = 1 - \int_{x>0} \phi(x,y)dxdy \tag{4.2}$$

where

$$g^*(t) = \partial G(\infty,t,0,0)/\partial t + g_1 \int_0^{W^*(t)} g_2(y,t)dy \ . \tag{4.3}$$

In the particular case where $\hat{h}(w,t)$ is increasing and differentiable in t for fixed w we get

$$\int_{A(h,w)} \exp(a\hat{v}(w,t))g_2(w,t)dt$$

$$= \exp[a\hat{v}(w,\hat{t}(h,w))]g_2(w,\hat{t}(h,w))\partial\hat{t}(h,w)/\partial h \tag{4.4}$$

$$= \exp[av(h,f(hw+I),\hat{t}(h,w))]g_2(w,\hat{t}(h,w))\partial\hat{t}(h,w)/\partial h$$

where $\hat{t}$ is defined by

$$\hat{h}(w,\hat{t}(h,w)) = h \ .$$

The corresponding density can thus be expressed as

$$\phi(h,w) = \frac{\exp[av(h,f(hw+I),\hat{t}(h,w))]g_2(w,\hat{t}(h,w))\partial\hat{t}(h,w)/\partial h}{g_1\int \exp(a\hat{v}(y,t))g_2(y,t)dydt + \int \exp(av(0,f(I),t))g^*(t)dt} \tag{4.5}$$

when $h>0$.

Before we consider some examples let us compare this model with the traditional one. Let $f(x)=x$. Then the match-specific hours of work function is determined by Roy's identity, i.e.

$$\hat{h}(w,t) = \frac{\partial\hat{v}(w,t)/\partial w}{\partial\hat{v}(w,t)/\partial I} \tag{4.6}$$

given that $T(z)=t$.

Let $\gamma(t|w)$ be the probability density of $T(\hat{z})$ given that $W(\hat{z})=w > W^*(t)$ where as above, $\hat{z}$ denotes the optimal match. We get directly from (4.1) that

$$\gamma(t|w) = \frac{\exp(a\hat{v}(w,t))g_2(w,t)}{\int_{w>W^*(t)} \exp(a\hat{v}(w,t))g_2(w,t)dt} \ . \tag{4.7}$$

Accordingly, we realize that the model considered in the present section differs from the standard econometric specification in that the distribution of the "unobservables", given here by γ, depends on the structural parameters of the model through the match-specific indirect utility function.

This observation suggests a two stage estimation procedure. In stage one estimate the model by approximating $\gamma(t|w)$ by

$$g_2(t|w) = \frac{g_2(w,t)}{\int_{w>W^*(t)} g_2(w,t)dt} \ .$$

This stage is therefore equivalent to the traditional econometric approach for estimating labor supply equations (cf. Heckman and MaCurdy, 1981). In the second stage the match-specific indirect utilities can be computed by using the estimates obtained in stage one. Next a and the parameters of $g_2(w,t)$ can be estimated on the basis of (4.7). If a is estimated to be small this indicates that the heterogeneity with respect to non-pecuniary attributes is not important.

Example 1. Let

$$v(h,C,t) = \alpha\log(d-h)+\beta\log(C+t)$$

where α,β and d are parameters and assume that $f(\cdot)$ is concave and twice differentiable. The first order conditions give

$$\frac{\alpha}{d-\hat{h}} = \frac{\beta m(\hat{h},w)}{C+t}$$

where

$$m(h,w) = wf'(wh+I) \ .$$

Hence

$$\hat{t}(h,w) = \frac{\beta}{\alpha} m(h,w)(d-h) - f(hw+I),$$

$$W^*(t) = \frac{\alpha(t+f(I))}{\beta f'(I)} \ ,$$

$$\frac{\partial\hat{t}(h,w)}{\partial h} = \frac{\beta}{\alpha} w^2 f'(hw+I)(d-h) - (\frac{\beta}{\alpha} + 1)m(h,w)$$

and

$$\hat{v}(w,\hat{t}(h,w)) = (\alpha+\beta)\log(d-h)+\beta \log m(h,w)+\beta \log(\frac{\beta}{\alpha}).$$

Example 2. Assume that $f(x)=x$ and

$$\hat{v}(w,t) = c(w)t+b(I)$$

where c(w) does not depend on I and b(I) does not depend on w. By Roy's identity

$$\hat{h}(w,t) = \frac{c'(w)}{b'(I)}\ t = \frac{tc(w)}{R}$$

where

$$R = \frac{c(w)b'(I)}{c'(w)} ,$$

$$\hat{t}(h,w) = \frac{hR}{c(w)} ,$$

$$\frac{\partial\hat{t}(h,w)}{\partial h} = \frac{R}{c(w)} ,$$

$$\hat{v}(w,\hat{t}(h,w)) = Rh+b$$

and W^* is determined by $c'(W^*)=0$, which means that W^* does not depend on t. In this case the density takes the form

$$\phi(h,w) = \frac{g_1 \exp(aR+ab)g_2(w, \frac{hR}{c(w)}) \frac{R}{c(w)}}{\int \exp(ab)g^*(t)dt+g_1 \int_{y>W^*} \exp(ac(y)t+ab)g_2(y,t)dydt}$$

$$= \frac{g_1 \exp(aR(w)h)g_2(w, \frac{hR(w)}{c(w)}) \frac{R(w)}{c(w)}}{\int g^*(t)dt+g_1 \int_{y>W^*} \exp(ac(y)tg_2(y,t)dydt} .$$

5. Match-specific hours of work

In this section we shall assume that once the match is given, hours of work associated with that match is completely determined. This assumption means that a certain combination of tasks take a fixed time to perform or that the firm or other institutions determine the hours of work. An extended version of this model has been applied to analyse the labor supply for married couples in Norway, see Dagsvik and Strøm (1988).

Let $H(z)$ denote hours of work specific to the match. When $H(z)=0$ then the match is either a pure non-market match or it is not available to the individual. Let

$$G_2(w,t,h) = P(W(z) < w, T(z) < h \mid H(z)>0, z\in B)$$

and assume that the density, g_2, of G_2 exists. With the notation of section 3 we have

$$g_o = G(\infty,\infty,0,0) \quad \text{and} \quad g_1 = 1 - g_o \, .$$

Also define

$$g_4(w,h) = \int g_2(w,t,h)dt$$

which is the (expected) fraction of feasible market matches with wages and hours (w,h).

In the terminology of Theorem 1 it is now clear that

$$N_1(M,\infty) = g_1 \int_{\substack{x\in D\\ x>0}} \exp(av(x,f(xy+I),t))g_2(y,t,x)dxdtdy$$

and

$$N_o = \int \exp(av(0,f(I),t))G(\infty,dt,0,0) \, .$$

Now let

$$\exp(\phi(h,C,w))$$

$$= E\{\exp(av(h,C,T(z))) \mid z\in B, H(z) = h, W(z) = w\}$$

$$= \int \exp(av(h,C,t)) \frac{g_2(w,t,h)}{g_4(w,h)} dt$$

for $h>0, h\in K$ and

$$\exp(\phi(0,C)) = E\{\exp(av(0,C,T(z)))|H(z) = 0, z \in B\}$$

$$= \int \exp(av(0,C,t)) \frac{G(\infty,dt,0,0)}{g_o} .$$

The interpretation of $\exp(\phi(h,C,w))$ is as the average utility of all feasible market matches with hours h and wage w. Observe that in general $\phi(h,C,w)$ depends on w because T(z) and W(z) may be interdependent. For instance some jobs have high wages so as to compensate for unattractive non-pecuniary attributes. One may also argue that attractive matches are correlated with high wages because persons with certain qualifications that matches certain positions are difficult to get and they therefore must be offered high wages. In the special case where T(z) and W(z) are independent then $\phi(h,C,w)$ reduces to $\phi(h,C)$, i.e., the wage only affects the average utility through consumption.

From Theorem 1 we get

$$\phi(h,w,K) = \frac{\exp(\phi(h,f(hw+I),w))g_4(w,h)g_1}{g_1 \int_{\substack{x>0 \\ x\in K}} \exp(\phi(x,f(xy+I),y))g_4(y,x)dxdy + g_o \exp(\phi(0,f(I)))} \qquad (5.1)$$

and

$$\phi(0,K) = \frac{g_o \exp(\phi(0,f(I)))}{g_1 \int_{\substack{x>0 \\ x\in K}} \exp(\phi(x,f(xy+I),y))g_4(y,x)dxdy + g_o \exp(\phi(0,f(I)))} . \qquad (5.2)$$

From (5.1) we can immediately derive certain important conditional distributions. Let $\phi_2(h,K|w)$ be the conditional distribution of hours given the wage. We get

$$\phi_2(h,K|w) = \frac{\phi(h,w,K)}{\int_{\substack{x>0 \\ x\in K}} \phi(x,w,K)dx}$$

$$= \frac{\exp(\phi(h,f(hw+I),w))g_4(w,h)}{\int_{\substack{x>0 \\ x\in K}} \exp(\phi(x,f(xw+I),w))g_4(y,x)dx} . \qquad (5.3)$$

From (5.3) we realize that when H(z) and W(z) are stochastically independent so that

$$g_4(w,h) = g_5(w)g_6(h)$$

(say), then the marginal density $g_5(w)$ cancels in (5.3). This means that in this case it is possible to estimate the parameters of ϕ and g_6 without specifying the marginal wage density g_5.

Provided the distribution G(w,t,h) is independent of K it is easy to verify that (IIA) holds. We state this result below.

Theorem 4.

When D(z) is a singleton, the IIA property holds.

Proof: From (5.1) we get

$$\frac{\phi(h_1,w_1,K\ )}{\phi(h_2,w_2,K\ )} = \frac{\phi(h_1,w_1,K^*)}{\phi(h_2,w_2,K^*)}$$

which shows that the odds ratio is independent of the choice set. But this property is equivalent to the IIA assumption. Q.E.D.

6. Justification of the stochastic structure of the utility function from theoretical assumptions

In this section we present two alternative sets of assumptions that imply the structure (2.6). The first set is partly of the revealed preference type and it is based on Luce choice axiom "independence from irrelevant alternatives".

Let S be the space of matches which is assumed to be countable. Assume that there exists a mapping T(•) from S to R_+ (say). The interpretation is that T(z) is an index that summarizes the "objective" qualitative characteristics of match z. By objective we understand that it is the same relative to every individual. T(z) is perceived as random here because the enumeration of the matches is arbitrary. Let $(R, \mathcal{B}, P_B(\cdot|h,C))$ be a probability space where $\mathcal{B}$ is the Borel field and

$$P_B(A|h,C) = P\{\max_{T(z)\in A} U(h,C,z) = \max_{T(z)\in B} U(h,C,z)\}$$

for $A, B\in\mathcal{B}$ and $A\subset B$. The interpretation of this measure is as the probability of selecting a match for which $z\in T^{-1}(A)$ when the choice set (for z) is $T^{-1}(B)$ and given (h,C).

<u>Assumption 1</u>. If $A_1, A_2, B\in\mathcal{B}$ and $A_1\subset A_2\subset B$,

$$P_B(A_1|h,C) = P_B(A_2|h,C)P_{A_2}(A_1|h,C)$$

Moreover, $P_{R_+}(\cdot|h,C)$ is absolutely continuous with respect to the probability measure of T.

We recognize this assumption as a version of IIA. In other words $P_B(A|h,C)$ is assumed to be a conditional probability measure.

<u>Assumption 2</u>. For each (h,C) the utilities, U(h,C,z), z=1,2,..., are independent draws from the same distribution. Furthermore U(h',C',z') and U(h,C,z) are independent when $z\neq z'$.

Assumption 2 states that to the observer all matches are "orthonormal" and "look the same" apart from purely random disturbances. That is, there are no hierarchical differences between the matches.

Let us now consider the implications of Assumption 1 and 2 for the structure of the utility function.

Let $\{A_j\}$ be a finite partition of a choice set $B\in\mathcal{B}$. From Assumption 1 it follows that there exists a measure, $\alpha(\cdot,h,C)$, that is proportional to

$P_S(\cdot|h,C)$ such that

$$P_B(A_j|h,C) = \frac{\alpha(A_j,h,C)}{\sum_k \alpha(A_k,h,C)} .$$

This is a version of the famous Luce model.

By Assumption 2 and from Yellott (1977) it follows that

$$\max_{T(z)\in A_j} U(h,C,z) \overset{D}{=} \log \alpha(A_j,h,C) + \eta_j(h,C) \tag{6.1}$$

where $\overset{D}{=}$ means equality in distribution and η_j, $j=1,2,\ldots$, are independent draws from $\exp(-e^{-xa})$. Let λ denote the distribution of T. Since $\alpha(\cdot,h,C)$ is absolutely continuous with respect to λ we have

$$\alpha(A_j,h,C) = \int_{A_j} \beta(t,h,C)\lambda(dt)$$

for some function $\beta(\cdot,h,C)$. Thus (6.1) implies that

$$P\{\max_{T(z)\in A} U(h,C,z) < u\} = \exp(-e^{-au} \int_A \beta(t,h,C)\lambda(dt)) . \tag{6.2}$$

But this means that we have

$$\max_{T(z)\in A} U(h,C,z) \overset{D}{=} \max_{T(z)\in A} \{v(h,C,T(z))+\varepsilon(z,h,C)\} \tag{6.3}$$

where $v(h,C,t) = \log \beta(t,h,C)$ and $\{T(z), \varepsilon(z)\}$, $z=1,2,\ldots$, is an enumeration of the points in this Poisson process on $R_+ \times R$ with intensity measure $\lambda(dt)\cdot e^{-a\varepsilon} a d\varepsilon$.

Note that for a given match the individual's preferences with respect to (h,C) are not random which means that $\varepsilon(z,h,C)$ is independent of (h,C).

To see that (6.3) holds let for notational convenience

$$m(t) = \begin{cases} u - v(h,C,t) & \text{for } t\in A \\ -\infty & \text{otherwise} \end{cases} .$$

Then

$$P\{\max_{T(z)\in A} (v(h,C,T(z)) + \varepsilon(z)) < u\}$$

$$= P\{\varepsilon(z) < u - v(h,C,T(z)),\ z,\ T(z)\in A\}$$

$$= P\{\text{There are no points of the Poisson process above the graph of } m(\cdot)\}$$

$$= \exp(-\int_{m(t)<\varepsilon} ae^{-a\varepsilon} d\varepsilon \cdot \lambda(dt)) = \exp(-e^{-u} \int_A \exp(av(h,C,t))\lambda(dt)).$$

Thus, (6.3) implies that

$$U(h,C,z) \overset{D}{=} v(h,C,T(z)) + \varepsilon(z) .$$

We have thus provided a set of assumptions that are consistent with (2.6). We state this below.

<u>Theorem 5</u>. If Assumption 1 and 2 hold then the utility function is distributed as (2.6).

Next we present an assumption that is an alternative to Assumption 1. First we need some additional definitions. Let B(h) be a finite set of matches given (h,C). Let n(h) denote the number of matches in B(h). The set B(h) may also depend on C but for notational simplicity we suppress that here. Let $B_k(h)$ denote the set of matches which is obtained by adding (k-1)n(h) matches to the original set where k is an arbitrary integer. Let

$$U^k(h,C) = \max_{z\in B_k(h)} U(h,C,z) .$$

Thus $U^1(h,C) = U(h,C)$.

<u>Assumption 3</u>. Let m be an arbitrary integer and let

$((h_1,C_1),(h_2,C_2),\ldots,(h_m,C_m))$ be arbitrary points. The joint distribution of

$$(U^k(h_1,C_1),U^k(h_2,C_2),\ldots,U^k(h_m,C_m))$$

is the same as the joint distribution of

$$(U(h_1,C_1) + d_k,U(h_2,C_2) + d_k,\ldots,U(h_m,C_m) + d_k)$$

where d_k is a location parameter that may depend on $((h_1,C_1),(h_2,C_2),\ldots,$ $(h_m,C_m))$.

The basic idea of Assumption 3 is due to Yellott (1977) who called it "invariance under uniform expansions of the choice set". The version stated here is a slightly different version from Yellott's one but the essential idea is the same. The assumption states that "in average" the number of persons that has a particular ranking of hours of work and consumption alternatives remains unchanged under expansions of the sets of matches.

Note that the assumption does not mean that the choice sets are perfectly replicated. There are random errors which imply that each duplicate is "slightly" different from the original opportunity. Our assumption is an assumption about aggregate behavior in a "large" population: Since the utility for each level of (h,C) is purely random the individual variations in the opportunity sets "cancel" and the average rankings remain unaltered.

The Assumptions 2 and 3 imply the following result which proof is given in Dagsvik (1987).

Theorem 6. If Assumption 2 and 3 hold and if $\{U(h,C)\}$ is continuous in probability then the distribution of

$$(U(h_1,C_1),U(h_2,C_2),\ldots,U(h_m,C_m))$$

is multivariate extreme value distributed (type III).

The multivariate extreme value distribution is also known as the generalized extreme value distribution and it has been applied within the field of qualitative choice models, see for instance McFadden (1981).

The essential property of this distribution, F, (say) is that

$$\log F(x_1,x_2,\ldots,x_m) = e^{-ay} \log F(x_1-y,x_2-y,\ldots,x_m-y)$$

where $a>0$ is a constant and y is arbitrary. (See Johnson and Kotz, 1972, or Galambos, 1978).

We noted in Section 2 that $\{U(h,C)\}$ is a stochastic process in the "time" parameter (h,C). A stochastic process with finite dimensional marginal distributions of the multivariate extreme value type is called a max-stable process (cf. de Haan, 1984).

In order to obtain a tractable expression for the distribution of optimal hours of work and wage it is necessary to make the next assumption.

Assumption 4. Under random sampling of matches (W(z),T(z),D(z)), z=1,2,..., are independent draws from the same distribution. Also, (W(z), T(z),D(z)) and $\varepsilon(z)$ are independent.

Observe that Assumption 4 does not rule out the possibility of correlation between the preference and W(z) since utility and wage may be interdependent through the matching variable T(z). Now Assumptions 1,2 and 4, are sufficient to give Theorem 1.

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